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Alexis Bergès

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ÉCOLE DES PONTS ET CHAUSSÉES

**SUPPLY FUNCTION EQUILIBRIA ON THE ELECTRICITY
MARKET**

THÈSE

pour l'obtention du titre de Docteur en Sciences Économiques

par

Alexis BERGÈS

19 juin 2018

Jury

Philippe CHONÉ, ENSAE Paris Tech

Président

Laurent LAMY, ENPC Paris Tech

Suffragant

David MARTIMORT, PSE et EHESS

Directeur de thèse

Jean-Christophe PODOU, Université de Montpellier

Rapporteur

Jérôme POUYET, Université de Cergy-Pontoise et ESSEC

Rapporteur

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Summary

Chapter 1:

The first chapter focuses on what the introduction of ramping costs in a theoretical framework brings to the table. Ramping costs represent the fact that electricity suppliers incur costs when their production varies over time. Our main contribution is to build and justify how these ramping costs can be tackled theoretically. First, we note that going to a continuous time description of the problem allows us to bring to the literature about supply function equilibria powerful mathematical tools mostly used in option pricing, that is stochastic dynamics: we want to model ramping costs, i.e. costs associated to the variation in production, while retaining the key ingredient brought by [Klemperer and Meyer, 1989], i.e. the uncertainty, through the use of brownians, and more precisely, Itô processes. In so doing we face the issue that one cannot derive a brownian, and bring our second contribution, a physical argument about how power plants function that effectively operates as a low pass filter on our stochastic processes, and allow us to continue to build a tractable model of ramping costs under uncertainty. Third, we find in the literature a specification of Itô processes that allows the model to remain tractable.

From these technical contributions we obtain our economic contributions in having a rich tractable model that yields results that contrast strongly with past results from the literature. First, in the specific case of linear demand and linear costs we obtain a unique Nash equilibria, which contrasts with the usual continuum of Nash equilibria

in the supply function equilibria literature. Second, our solutions are not ex-post optimal, meaning that gathering information about the expected future evolution of demand yields different optimal strategies for suppliers, which in turn means that producers in our framework have a motive for submitting different supply functions from one time step to the next. Third, we have closed form solutions which yield specific predictions about the evolution of bids under uncertainty, namely that when uncertainty increase, suppliers submit steeper supply schedules in order to transmit more of these shocks to changes in price and not quantities, which are costly due to the existence of ramping costs. Finally, and less importantly, our framework justifies the existence of negative prices ¹ by producers being willing to pay consumers to consume more in order to avoid facing large variations in production, in contrast to everywhere positive schedules in the case of the supply function equilibria literature. These results open the door to models being able to differentiate between day-ahead and intraday markets and therefore to offer a framework in which their interactions might be possible.

Chapter 2:

In the second chapter our main focus is on analyzing our data, on building a way to describe it, and on building proxies for the uncertainty that producers face about the residual demand they have to anticipate when bidding on the day-ahead market.

First, we note that aggregate supply functions on the day ahead market cannot be well captured by parametric functions. Therefore, we devise a way to describe them non-parametrically: we note that although they cannot be captured parametrically, they still have a rough S shape, and therefore four main parts, two extremal sections, and two interior ones separated by the inflection point of the curve in its middle section. We define the transition points between these sections as the points of maximal absolute

¹Note that such negative prices happen, a few hours a year for example in France or Germany, for example in 2017 there were 146 such hours, spread over 24 days in Germany [EPEX, 2018a]

value for the derivative and second derivative of the supply schedules. This definition relies on kernel density estimates and is therefore non-parametric. We observe that by using 5 such points, we are able to capture about 98% of the intrinsic variability of the supply schedules, and stop there although our method can be used to define more non-parametric points. This method allows us to define points that we consider comparable across auctions, that allow use to perform cross-sectional analysis of our data in the third chapter.

Second, we build proxies for the amount of weather uncertainty that producers face and variables that capture information that suppliers have before bidding and should therefore be controlled for. For the information available to suppliers, we note that the effect of weather on the demand, and more importantly temperature, is well understood and that we need to control for it. To do so we build an effective temperature for France, as an average of the localised temperature weighted by the population of the spatial region considered, in order to capture the overall effect temperature has on heating.² The rest of our focus is on building a proxy for the uncertainty concerning renewable production. To do so we analyze spatialized wind and sunlight data, and study it's spatial structure. We argue that spatial autocorrelation is a proxy for the uncertainty associated with weather forecasts, noting that if this data displays more spatial gradients, it is likely to be of a lesser quality due to the numerical nature of the weather simulations used to predict the weather, and therefore more uncertain.

Our contribution in the second chapter is to provide a non parametric way to define comparable points across auctions, and a measure of the uncertainty associated with weather forecasts.

²France has a high level of electric heating overall, which means that demand for electricity is quite sensitive to temperature.

Chapter 3:

In this empirical chapter, we study the impact that uncertainty about the demand plays on the shape of the aggregate supply functions bidden by suppliers on the French electricity market. We segment our analysis to different parts of the supply functions in order to show how the overall shape changes with respect to our explanatory variables. We test some of the predictions from our first chapter, mainly that the supply function should see its slope increase when uncertainty increases.

We note that the main uncertainty is about the shape of the demand schedules itself. Therefore, we consider data available to the producers and regress the demand schedules on these variables. Next, we study the residuals of these regressions, and more specifically note that they are heteroskedastic. We leverage this, regressing the square of these residuals on our variables, in order to predict the expected amplitude of the residuals, that is the amplitude of the uncertainty of the demand schedule regression.

We then study the effect of our different proxies for uncertainty on the slope of the supply schedules, and note that if our proxies about the weather uncertainty (through the channel of renewable production) have the expected effect, the results are less clear cut for our residuals on the demand schedules. As we are working with full-blown schedules in the quantity-price plane, we perform our residual analysis both on the prices and the quantities. We therefore obtain estimates for the uncertainty pertaining to the position of a given point of our demand schedule either in price or in quantity. In our theoretical framework, we make the strong assumptions that demand schedules are linear, and that demand shocks are additive, i.e. they do not impact the slope of the demand schedules. These assumptions yield that we cannot differentiate between shocks in price or quantity, and that they should have effects in the same direction: more uncertainty implying steeper supply curves to reduce the amount of fluctuation in production. However, we observe that the effects of price and quantity uncertainty as estimated by our residuals' method yield opposite effects. Both of these assumptions, although required to obtain

closed form results, are clearly not satisfied by our data, and we think that this is a clear path for improvement of the model.

The contribution of the third chapter is to provide a way to estimate the uncertainty about the demand schedules faced by suppliers, and to estimate how this uncertainty affects the shape of the supply schedules at different points along its overall length, i.e. we provide a framework to describe how the functional form of schedules is affected by estimates of the uncertainty faced by suppliers.

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General Introduction

General Introduction

The electricity markets and their modelization over time

Public utility pricing

The interest for modelling the electricity markets can be traced back to the reference work by Marcel Boiteux, vice president in charge of economic studies at Electricité de France, at the outset of the second world war. The question at the time was mainly that of public utility pricing: in the context of a public monopoly, which price should the consumers face in order to allow the producers to recover their costs.

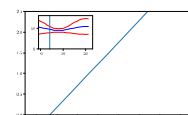
There are two main concerns that electricity producers have to face: the uncertainty of demand and the cyclicity of demand, for a commodity that essentially cannot be stored.³

The first question is addressed in [Boiteux, 1951]. In this paper, Boiteux considers a constant expected demand with fluctuations. The goal is to find the correct marginal pricing so that consumers internalize the additional cost that an uncertain demand entails for the producer. With a certain probability that demand is above its expected value by a given amount, how much more reserve capacity has to be kept in order to insure an accepted failure probability.⁴

The second question is addressed in [Boiteux, 1960]. Contrary to the previous situation, demand is now considered to change over time in a deterministic and cyclical fashion. The question is to price electricity in order for consumers to be sensitive to the additional investment cost implied by higher demand peaks.

³Electricity can be stored in hydroelectric dams, but the total energy stored is not enough to stabilize completely the demand faced by the other generation units, and only a fraction of the hydroelectric storage capacities can be actively replenished: the pumped storage facilities, which have two lakes and can therefore pump from the lower lake to the upper one on demand to store more electricity than that naturally stored in a lake that would be naturally replenished by a river.

⁴In the context of electricity, as production has to match demand at every point in time, every national grid is built with the notion of an acceptable probability of mismatch which translates in curtailments



These contributions have sparked a larger literature on the question of the pricing of economically non-storable commodities whose demand varies periodically, first in [Brown and Johnson, 1969] which studies the impact of stochastic demand on expected welfare. We refer the interested reader to the following review [Crew et al., 1995].

This literature has been mainly interested in questions of optimal pricing when the agent choosing the pricing tries to maximize the consumer's welfare, that is in the case of public monopolies.

Regulatory evolution

The previous literature took as an assumption the fact that these commodities were produced by public monopolies. Network utilities, such as gas, telecoms and electricity were thought to require to be organised as vertically integrated monopolies.

This view started to change in the 80s, with pressure to create competition. In 1984, access to gas pipelines was opened to competition in the USA and in 1990 Britain privatised electricity, separating generation and transmission. It was indeed thought that the natural monopoly emerged from the network, and that by separating generation from the network, generation could be opened to competition.

The overall argument for liberalization is that private competition is considered a safer road towards efficiency than regulation of a monopoly. In a situation of perfect competition, actors would be strongly incentivized for efficiency gains, and these gains would be transferred to consumers [Schmidt, 1996]. As perfect competition is a very rare situation, a new branch of the literature started to coalesce around the questions of modeling competition in the case of electricity markets [Newbery, 1997].

Although this liberalization movement is empirically considered to bring at least mod-

est medium-term efficiency [Fabrizio et al., 2007], it has been somewhat slowed down after the California crisis in the early 2000s [Jamash and Pollitt, 2005], which mainly concentrated on wholesale electricity markets. Because of very little price responsiveness of demand as well as interactions with forward contracts, there was very high fluctuations in price as well as shortages [Borenstein, 2002]. In Europe, the European Commission has pushed with success for the continuation of the program of liberalization and integration, and wholesale markets for electricity are now ubiquitous, without further instances of failure as in California.

The markets for electricity

The way the markets for electricity are organised stem from two main characteristics:

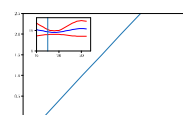
- The market has to reflect the changing demand for electricity.
- The form of the bids has to allow them to cope with the uncertain nature of demand at the time of bidding.

These ingredients have pushed for the creation of hourly or half-hourly markets, where suppliers are asked to submit supply schedules for a set number of bids (generally every 24 hours, that is 24 or 48 supply schedules once a day depending on whether the bids are hourly or half-hourly). These supply schedules take the form of a set of monotonous price quantity pairs, that can be considered as forming step functions⁵ or linear functions by parts.⁶

In the 1980s, a theoretical push was made to model competition in supply functions. The first occurrences of this approach can be found in [Grossman, 1981] and [Hart, 1982]. They consider situations where producers compete in supply curves when facing a given demand curve. The main result is that such problems can be solved and one can obtain specifications for optimal strategies in supply functions, but that there exists a very large

⁵as in the case of the England and Wales pool in the 1990s

⁶Where price-quantity pairs are considered to be joined by lines instead of steps, which is the case for the French electricity day-ahead market, as well as the UK day-ahead market (half-hourly). Both of these markets are exchanged through EPEX Spot as of 2017.



multiplicity of equilibria in this setting.

Around the same time, [Klemperer and Meyer, 1986] introduce a setting in which firms choose endogenously to compete either in quantity or prices. This too yields a large multiplicity of outcomes, but the key insight comes from the fact that this multiplicity is drastically reduced when uncertainty is introduced.

This insight brings along the seminal paper [Klemperer and Meyer, 1989] which studies supply function competition under uncertainty. In this paper, it is shown that although there is still a continuum of equilibria, this continuum has a structure that can be studied when suppliers face an uncertain demand. In the rest of this thesis, we denote supply function equilibria as SFE.

The setting introduced by Klemperer and Meyer is then rapidly put to use in the context of electricity markets, where [Green and Newbery, 1992] studies the competition in the British spot market through the SFE framework.

This use of SFE sparks some debate as to whether a smooth function approximation can or not capture the correct effects in markets which are largely at the time asking bidders to submit step functions: [von der Fehr and Harbord, 1993] argue that step functions of finite length are different to continuous functions.⁷ In addition, there is empirical evidence that strategies predicted by SFE and actual observed strategies are significantly different, see [Willems et al., 2009] and [Willems et al., 2009]. These results question whether the SFE is the correct approach that only needs to be perfected, for example by using functions that are affine by parts and not only affine [Baldick et al., 2004], or a framework that is not adapted to describing these markets.

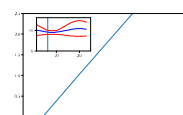
However, this approach is still considered relevant by a number of authors, although

⁷This debate is largely obsolete now that most of the market rules imply bids that are linear by parts and not step functions anymore.

the multiplicity of equilibria makes it difficult to obtain clear results. In addition, the solutions are not exactly easily usable, these functions being defined as solutions to a differential equation, therefore without an analytical formula. To overcome this issue, a number of authors either consider competition in simpler settings, for example Cournot competition settings applied to the electricity market in the case of [Borenstein and Bushnell, 1999], or choose to restrict themselves to one special solution out of the continuum of possible solutions that come out of the SFE framework: the supply function that is the unique linear solution out of this continuum. In so doing these authors pick arbitrarily one solution with a functional form and then use it to further analyze some economic questions. For example, [Green, 1996] focuses on the linear supply solution out of the SFE multiple equilibria in order to have analytical tractable forms and study the effect of three different policies on competition, where [Hobbs et al., 2000] is able to model transmission constraints with an affine supply function.

We also want to note that day-ahead markets do not exist in a vacuum, and in fact electricity can be traded through forward contracts, on the day-ahead market, as well as on the intraday markets. Capacity markets on which guaranteed online capacity is traded for also exist. All these markets interact with one another, and part of the literature focuses on modelling these interactions. Generally the SFE considered are simplified to be able to perform such analysis, for example to linear functions [Green, 1999] to study the interaction with forward markets, or to linear asymmetric function [Anderson and Hu, 2012] for the same purpose. Generally speaking, these papers focus on the interaction between day-ahead markets and forward contracts because the SFE framework does not allow differentiating between day-ahead and intraday markets.

Overall, we refer the interested reader to the review by [Ventosa et al., 2005] for a more detailed overview. In this thesis we rely heavily on the work by Klemperer and Meyer, and comment and contrast their results to ours. In order to make this easier to follow, we summarize in the following section the results of their paper that will be used



in this thesis.

Klemperer and Meyer 1989

Consider a setting in which firms bid supply functions while facing an uncertain demand.

Let $D(p, \theta)$ be the demand function as a function of demand shock θ . Consider that for all (p, θ) , $-\infty < D_p < 0$, $D_{pp} \leq 0$ and $D_\theta > 0$.

All firms are considered to be facing the same cost function $C(\cdot)$, with $C'(q) > 0$ and $0 < C''(q) < \infty$ for all $q > 0$.

The timing is such that suppliers have to bid simultaneously a supply function prior to the realization of demand shock θ being known. Consider for now two firms i and j with $S^k(p)$ the supply function of supplier k and that these supply functions are twice differentiable. After this shock is known, every firm produces quantity $S^k(p^*(\theta))$ at price $p^*(\theta)$, such that $D(p^*(\theta)) = S^i(p^*(\theta)) + S^j(p^*(\theta))$.

Firm i 's residual demand is given by the total demand from which the supply of firm j is subtracted, $D(p, \theta) - S^j(p)$. As θ is considered a scalar, the set of profit-maximizing points for every possible shock θ define a curve. If there is a unique intersection between i 's supply curve and every possible demand curve, then such a supply curve is ex-post optimal, meaning that it is pointwise optimal for every realization of the shock θ .

Given the assumption that supply curves indeed behave in this way, then maximizing the expected profit for the distribution of shocks can be abstracted away from the distribution of shocks, and i 's optimal supply curve solves for every shock θ the following program:

$$\max_p (D(p, \theta) - S^j(p)) - C(D(p, \theta) - S^j(p)) \quad (1)$$

which F.O.C writes:

$$D(p, \theta) - S^j(p) + (p - C'(D(p, \theta) - S^j(p))) (D_p(p, \theta) - S^{j'}(p)) = 0 \quad (2)$$

with eq 1 being strictly concave in p (we refer the reader to the original paper for more justifications), then eq 2 defines the unique profit maximizing $p^*(\theta)$ for every θ , which parametrizes the optimal supply function.

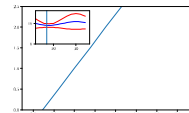
Consider that $D_{\theta p} = 0$, that is that θ is an additive shock, and that we focus on symmetric equilibria, which allows us to drop the firms' superscripts. In addition, consider the fact that eq 2 has to hold for every shock, it can therefore be rewritten as:

$$S'(p) = \frac{S(p)}{p - C'(S(p))} + D_p(p) = f(p, S) \quad (3)$$

This differential equation defines the supply function equilibria, the role of uncertainty being to ensure that this equation has to hold for every shock, therefore for every possible price. However, we can see that this differential equation is not accompanied by an initial condition. Therefore, there exists many admissible solutions to this equation.

Supply functions are therefore bounded by possible values of their slope, namely that the functions have slopes bounded between 0 and $+\infty$. By solving the differential equation, one can define the locus of points for which the solutions have slopes equal to these bounds and thus obtain a region of admissible solutions, in the context of our problem:

Therefore, the admissible set of solutions is defined by the upper bound of the demand shocks $\bar{\theta}$, in that if the solutions cross the slope boundaries before reaching the maximal shock, they cannot be accepted as solutions to the problem which constrains the solutions more strictly than the differential equation alone. In figure 1 the demand associated with the upper bound of the shocks $D(p, \bar{\theta})$ is represented in orange, and solutions A and B to the differential equation are not solutions to the problem as they reach the boundaries



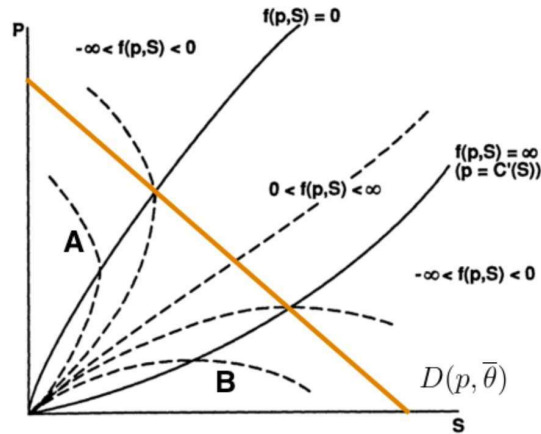


Figure 1: This graph is adapted from the original paper by Klemperer and Meyer and illustrates the admissible region of solutions to the differential equation so as to verify the constraints on the slope of the supply function.

for smaller values of shocks.

The last result we will review here, is that in the case of an unbounded support of shocks, the set of equilibria is at its smallest, as it means that solutions have to have a positive finite slope for every value of the shocks, and not only for a segment of the real line.

In some cases, for example for linear demand schedules, this set can collapse to a unique solution.

The case for ramping costs

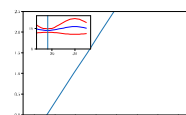
This framework models the costs as depending only on the quantity produced. In the context of electricity generation, an important type of costs that cannot be captured in such a specification of the cost function is that of ramping costs. These ramping costs refer to the fact that making production change over time induces specific costs.

To explain how such costs can arise, consider a thermal power plant (fossil fuels, nu-

clear, etc.), and more precisely its core. Physically, to produce a given level of electricity, one has to maintain the core at a given temperature. To increase production, the temperature of the core has to increase. This means that when production is increased some fuel has to be lost to simply increase the temperature, this energy expenditure is not attached to any additional production.

This issue of ramping costs is at the heart of the choice of "quick" gas power plants to match sudden peaks in demand, where nuclear plants are more generally used for low frequency adjustments. Therefore these ramping costs are important technically on the electricity market. They are important enough for the project of European Power Exchanges named "Price Coupling of Regions" (PCR), which aims to develop a single price coupling solution to be used to calculate electricity prices across Europe, to consider the possibility to use load gradient orders, that is orders that condition their availability on the change in production from one hour to the next. However, at the moment of writing, PCR is still very much a work in progress [EPEX, 2018c].

Some papers have tried to estimate their values empirically, [Wolak, 2007] and more recently [Reguant, 2011]. There is also a strand of literature concerned with ramping costs, looking at the optimal price that allows to maximize the overall social welfare [Tanaka, 2006], that is, which price schedule allows to maximize the consumer welfare from which the production costs are subtracted. This literature does not use game-theoretical frameworks, but concerns itself with the best price signal to use in order to limit the ramping costs incurred due to varying demand, while still considering that the trajectory of demand is known. To our knowledge, there is no game-theoretical framework that has been brought to take ramping costs into account, and describe their effects on optimal strategies for the agents bidding on the market.



Contribution

This thesis focuses on the question of these ramping costs. In the first chapter, I tackle this question in a theoretical framework which yields predictions on the change of shape in supply functions over time as a function of the underlying uncertainty about demand shocks. The second chapter then introduces methods to study the shape of supply functions as observed on the French electricity market for data from 2011 to 2013, as well as methods to estimate the uncertainty contributed by the weather. The third chapter applies these methods to test these theoretical predictions on actual market data. The second and third chapters have been co-written with Henri de Belsunce, who finished his PhD in 2015 at the Munich-based Max Planck Institute for Innovation and Competition, under the supervision of Prof. Dr. Klaus M. Schmidt [Belsunce, 2015].

The first chapter focuses on what the introduction of ramping costs in a theoretical framework brings to the table. Our main contribution is to build and justify how these ramping costs can be tackled theoretically. First, we note that going to a continuous time description of the problem allows us to bring to the literature about supply function equilibria powerful mathematical tools mostly used in option pricing, that is stochastic dynamics: we want to model ramping costs, i.e. costs associated to the variation in production, while retaining the key ingredient brought by [Klemperer and Meyer, 1989], i.e. the uncertainty, through the use of brownians, and more precisely, Itô processes. In so doing we face the issue that one cannot derive a brownian, and bring our second contribution, a physical argument about how power plants function that effectively operates as a low pass filter on our stochastic processes, and allow us to continue to build a tractable model of ramping costs under uncertainty. Third, we find in the literature a specification of Itô processes that allows the model to remain tractable.

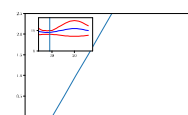
From these technical contributions we obtain our economic contributions in having a rich tractable model that yields results that contrast strongly with past results from the literature. First, our solutions are unique, which contrasts with the usual continuum

of Nash equilibria in the supply function equilibria literature. Second, our solutions are not ex-post optimal, meaning that gathering information about the expected future evolution of demand yields different optimal strategies for suppliers, which in turn means that producers in our framework have a motive for submitting different supply functions from one time step to the next. Third, we have closed form solutions which yield specific predictions about the evolution of bids under uncertainty, namely that when uncertainty increase, suppliers submit steeper supply schedules in order to transmit more of these shocks to changes in price and not quantities, which are costly due to the existence of ramping costs. Finally, and less importantly, our framework justifies the existence of negative prices⁸ by producers being willing to pay consumers to consume more in order to avoid facing large variations in production, in contrast to everywhere positive schedules in the case of the supply function equilibria literature. These results open the door to models being able to differentiate between day-ahead and intraday markets and therefore to offer a framework in which their interactions might be possible.

At the bottom of all right pages, there is a small graph showcasing the optimal strategy for a given set of parameters of the model, in the plane quantity-price, with an insert illustrating the evolution of the support of demand shocks over time. The vertical line in the insert represents the point in time for which the strategy is presented in the actual graph. This forms a flipbook, which allows the reader to get a feeling for the evolution of strategies with the evolution of the support of shocks by flipping rapidly the pages and observing the graph becomes animated, just like a cartoon. This serves only an illustrative purpose.

In the rest of the thesis, the goal is to test our predictions on data from the French day-ahead market. In so doing, as our theoretical predictions are mainly about the effect that the amount of uncertainty has on the slope of the optimal supply schedule, we separate the issue of building proxies for this uncertainty in our second chapter and the

⁸Note that such negative prices happen, a few hours a year for example in France or Germany, for example in 2017 there were 146 such hours, spread over 24 days in Germany [EPEX, 2018a]



actual analysis of the evolution of bids on these proxies in the third chapter.

In the second chapter our main focus is on analyzing our data, on building a way to describe it, and on building proxies for the uncertainty that producers face about the residual demand they have to anticipate when bidding on the day-ahead market.

First, we note that aggregate supply functions on the day ahead market cannot be well captured by parametric functions. Therefore, we devise a way to describe them non-parametrically: we note that although they cannot be captured parametrically, they still have a rough S shape, and therefore four main parts, two extremal sections, and two interior ones separated by the inflection point of the curve in its middle section. We define the transition points between these sections as the points of maximal absolute value for the derivative and second derivative of the supply schedules. This definition relies on kernel density estimates and is therefore non-parametric. We observe that by using 5 such points, we are able to capture about 98% of the intrinsic variability of the supply schedules, and stop there although our method can be used to define more points. This method allows us to define points that we consider comparable across auctions, that allow use to perform cross-sectional analysis of our data in the third chapter.

Second, we build proxies for the amount of weather uncertainty that producers face and variables that capture information that suppliers have before bidding and should therefore be controlled for. For the information available to suppliers, we note that the effect of weather on the demand, and more importantly temperature, is well understood and that we need to control for it. To do so we build an effective temperature for France, as an average of the localised temperature weighted by the population of the spatial region considered, in order to capture the overall effect temperature has on heating.⁹ The rest of our focus is on building a proxy for the uncertainty concerning renewable production. To do so we analyze spatialized wind and sunlight data, and study it's spatial structure. We

⁹France has a high level of electric heating overall, which means that demand for electricity is quite sensitive to temperature.

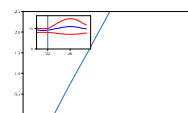
argue that spatial autocorrelation is a proxy for the uncertainty associated with weather forecasts, noting that if this data displays more spatial gradients, it is likely to be of a lesser quality due to the numerical nature of the weather simulations used to predict the weather, and therefore more uncertain.

Our contribution in the second chapter is to provide a non-parametric way to define comparable points across auctions, and a measure of the uncertainty associated with weather forecasts.

In the third chapter, we focus on building the main proxy for the uncertainty faced by producers, and then on analyzing how the bids evolve relative to these proxies.

We note that the main uncertainty is about the shape of the demand schedules itself. Therefore, we consider data available to the producers and regress the demand schedules on these variables. Next, we study the residuals of these regressions, and more specifically note that they are heteroskedastic. We leverage this, regressing the square of these residuals on our variables, in order to predict the expected amplitude of the residuals, that is the amplitude of the uncertainty of the demand schedule regression.

We then study the effect of our different proxies for uncertainty on the slope of the supply schedules, and note that if our proxies about the weather uncertainty (through the channel of renewable production) have the expected effect, the results are less clear cut for our residuals on the demand schedules. As we are working with full-blown schedules in the quantity-price plane, we perform our residual analysis both on the prices and the quantities. We therefore obtain estimates for the uncertainty pertaining to the position of a given point of our demand schedule either in price or in quantity. In our theoretical framework, we make the strong assumptions that demand schedules are linear, and that demand shocks are additive, i.e. they do not impact the slope of the demand schedules. These assumptions yield that we cannot differentiate between shocks in price or quan-



tity, and that they should have effects in the same direction: more uncertainty implying steeper supply curves to reduce the amount of fluctuation in production. However, we observe that the effects of price and quantity uncertainty as estimated by our residuals' method yield opposite effects. Both of these assumptions, although required to obtain closed form results, are clearly not satisfied by our data, and we think that this is a clear path for improvement of the model.

The contribution of the third chapter is to provide a way to estimate the uncertainty about the demand schedules faced by suppliers, and to estimate how this uncertainty affects the shape of the supply schedules at different points along its overall length, i.e. we provide a framework to describe how the functional form of schedules is affected by estimates of the uncertainty faced by suppliers.

Introduction Générale

Évolution de la modélisation des marchés de l'électricité

Tarification des services publics

L'intérêt pour la modélisation des marchés de l'électricité remonte aux travaux de référence de Marcel Boiteux, vice-président en charge des études économiques d'EDF au sortir de la Seconde Guerre Mondiale. La question principale à l'époque est celle de la tarification d'un service public : dans le contexte d'un monopole d'état, à quel prix les consommateurs devraient-ils faire face afin que les producteurs recouvrent leurs coûts.

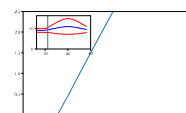
Les producteurs d'électricité font face à deux contraintes particulières : le caractère incertain de la demande ainsi que sa périodicité, le tout pour un bien qui ne peut essentiellement pas être stocké.¹⁰

La première contrainte est traitée dans [Boiteux, 1951]. Dans cet article, Boiteux considère une demande constante en moyenne, mais sujette à des fluctuations. L'objectif est de trouver la tarification marginale permettant au consommateur d'internaliser le coût supplémentaire qu'une demande incertaine fait peser sur le producteur. Étant donné une certaine probabilité que la demande soit au-dessus de sa valeur espérée d'une certaine quantité, il s'agit de trouver quelle capacité de réserve doit être maintenue en ligne pour garantir une probabilité cible de défaillance.¹¹

La seconde contrainte est traitée dans [Boiteux, 1960]. Contrairement à la situation précédente, la demande est ici considérée comme périodique et déterministe. L'objectif

¹⁰Il est possible de stocker de l'électricité grâce à des barrages, mais l'énergie totale ainsi stockable n'est pas suffisante pour complètement stabiliser la demande à laquelle les moyens de production font face. Par ailleurs, seule une fraction de l'énergie ainsi stockée est renouvelable volontairement : les stations de transfert d'énergie par pompage disposent de deux lacs ce qui permet de pomper de l'eau d'un lac situé en aval vers un lac en amont et ainsi de reconstruire les réserves plus rapidement qu'en attendant que les affluents naturels du lac amont ne le remplissent.

¹¹Dans le contexte de l'électricité, comme la production doit être égale à la demande à chaque instant, chaque réseau national est dimensionné avec un niveau de probabilité de défaillance acceptable.



est de trouver la tarification permettant de transmettre aux consommateurs le coût d'investissement supplémentaire associé à une demande présentant des pics plus élevés.

Ces contributions nourrissent une large littérature sur la question de la tarification de biens non-stockables faisant face à une demande cyclique, tout d'abord par [Brown and Johnson, 1969] qui étudie l'impact d'une demande stochastique sur le bien-être social espéré. Nous renvoyons la lectrice intéressée à la revue de littérature suivante : [Crew et al., 1995].

Cette branche de la littérature se concentre principalement sur la question de la tarification optimale lorsque l'agent fixant le prix a pour objectif de maximiser le bien-être des consommateurs, dans le cas de monopoles d'état.

Évolutions de la régulation

Cette branche de la littérature prend pour hypothèse que ces biens sont produits par des monopoles d'état. Il est alors admis que les services de réseau, comme le gaz, l'électricité ou les télécoms doivent être organisés sous la forme de monopoles verticalement intégrés.

Cette position évolue dans les années quatre-vingt, avec l'ouverture à la compétition de ces monopoles d'état. En 1984, l'accès aux gazoducs est ouvert à la compétition aux États-Unis et en 1990, la Grande-Bretagne privatise la fourniture d'électricité, en séparant production et transmission. Il est alors considéré que la condition de monopole naturel est liée au réseau, et qu'en séparant production et transmission, la production peut s'ouvrir à la compétition.

L'argument général en faveur de la libéralisation est que la compétition privée est considérée comme une voie plus sûre vers l'efficacité économique que la régulation d'un monopole. Dans une situation de compétition parfaite, les agents seraient ainsi fortement

incités à rechercher des gains d'efficacité, et ces gains seraient transmis aux consommateurs [Schmidt, 1996]. La compétition parfaite étant une situation rare, une nouvelle branche de la littérature émerge autour de la question de la modélisation de la compétition dans le marché de l'électricité [Newbery, 1997].

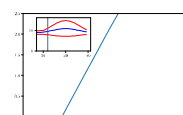
Bien que ce mouvement de libéralisation soit considéré comme permettant à minima des gains d'efficacité modestes à moyen terme [Fabrizio et al., 2007], il ralentit après la crise californienne du début des années 2000 [Jamasb and Pollitt, 2005], qui se concentre principalement sur les marchés de gros de l'électricité. De très grandes fluctuations de prix ainsi que des pénuries ont lieu, principalement induites par une très faible élasticité-prix de la demande, ainsi que l'interaction entre le marché de gros et les contrats à terme [Borenstein, 2002]. En Europe, la Commission européenne pousse avec succès pour la poursuite du programme de libéralisation et d'intégration européenne, et les marchés de gros de l'électricité sont maintenant répandus sur le continent, sans que ne se produise de défaillances semblables à celles observées en Californie.

Les marchés de l'électricité

L'organisation des marchés de l'électricité est très fortement induite par deux caractéristiques importantes :

- Le marché se doit de refléter les changements rapides de demande.
- La forme des enchères doit leur permettre de se satisfaire de la nature incertaine de la demande au moment de l'enchère.

Ces ingrédients sont à la source de la construction de marchés horaires voire même demi-horaires, au sein desquels les producteurs doivent soumettre des courbes d'offre pour un nombre déterminé d'enchères (en général une fois par jour, soit 24 ou 48 courbes d'offre à la fois selon que l'enchère est horaire ou demi-horaire). Ces courbes d'offres prennent la forme d'un ensemble de paires de prix et de quantités monotone, qui peuvent



définir des fonctions constantes par morceaux¹² ou des fonctions linéaires par morceaux.

13

Dans les années quatre-vingt, une impulsion théorique cherche à modéliser ces marchés sous forme d'équilibres en courbes d'offres. Cette approche est introduite en premier lieu par [Grossman, 1981] and [Hart, 1982]. Ils considèrent une situation où des producteurs rivalisent via des courbes d'offres en faisant face à une courbe de demande connue et donnée. Le principal résultat de cette approche est que ces modèles peuvent être résolus, que les stratégies optimales en courbe d'offre peuvent être spécifiées, mais qu'il existe une forte multiplicité d'équilibre dans ce contexte.

Peu de temps après, [Klemperer and Meyer, 1986] introduisent un modèle dans lequel les producteurs choisissent d'entrer en concurrence en prix ou en quantité de façon endogène. Cette approche donne également lieu à une grande multiplicité d'équilibre, mais le résultat clef est que cette multiplicité est drastiquement réduite lorsque de l'incertitude est introduite dans le modèle.

Ce résultat inspire le papier fondateur [Klemperer and Meyer, 1989] qui étudie une situation de compétition en courbes d'offres face à une demande incertaine. Dans cet article, bien qu'il y ait toujours un continuum d'équilibres, ce continuum possède une structure qui peut être étudiée lorsque la demande est incertaine. Dans le reste de cette thèse, nous appelons les équilibres en courbes d'offre des SFE (supply function equilibria).

Le modèle général proposé par Klemperer et Meyer est rapidement mis à profit pour décrire les marchés de l'électricité, la compétition sur le marché spot anglais est ainsi étudiée avec des SFE par [Green and Newbery, 1992].

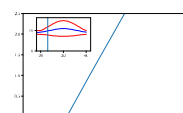
¹²Comme dans le cas du marché de gros d'Angleterre et du Pays de Galles dans les années quatre-vingt-dix

¹³Les paires prix quantité sont considérées comme étant reliées par des linéaires, comme dans le cas du marché day-ahead français, mais aussi le marché anglais actuel (demi-horaire). Ces deux marchés spot sont gérés par la bourse EPEX Spot.

Cet usage des SFE déclenche des débats sur la validité qu'il y a à décrire le marché de l'électricité, à l'époque encore principalement caractérisé par des enchères constantes par morceaux, avec des fonctions continues et dérivables : [von der Fehr and Harbord, 1993] présente un argument montrant qu'une compétition via des constantes par morceaux de tailles finies exhibe des comportements fondamentalement différents du cas de fonctions continues.¹⁴ Des résultats empiriques montrent également que les stratégies prédites par cette classe de modèles diffèrent de façon significative des stratégies effectivement observées sur les marchés, voir [Willems et al., 2009] et [Willems et al., 2009]. Ces résultats interrogent sur la validité de l'approche SFE pour décrire les marchés de l'électricité, plus précisément la question est de savoir si les modèles de SFE sont valides, mais attendent d'être perfectionnés, par exemple en les modélisant explicitement sous la forme de fonctions affines par morceaux [Baldick et al., 2004], ou si cette approche est fondamentalement incapable de capturer les stratégies observées.

Cette approche est toutefois encore considérée comme pertinente par nombre d'auteurs, bien que la multiplicité des équilibres complique l'analyse des résultats théoriques. Par ailleurs, les solutions ne sont pas directement exploitables, étant définies implicitement comme solution d'équations différentielles, et donc sans forme analytique dans la majorité des cas. Pour dépasser ces limites, la littérature cherche à décrire la compétition dans des cadres plus simples, par exemple dans des modèles de compétition de Cournot appliquée au cas des marchés de l'électricité dans le cas de [Borenstein and Bushnell, 1999], ou choisissent de se restreindre à une solution particulière parmi le continuum de solutions obtenu dans un contexte de SFE: l'unique solution linéaire du lot. Ces auteurs choisissent donc arbitrairement une solution tractable et s'en servent pour pousser le raisonnement économique plus loin qu'habituellement possible en conservant le continuum de solutions. À titre d'exemple, [Green, 1996] se focalise sur la courbe d'offre linéaire parmi le continuum obtenu dans le cadre des SFE et se sert de son expression analytique pour étudier

¹⁴Ce débat est principalement obsolète maintenant que la plupart des marchés sont passés à des linéaires par morceaux et plus des constantes par morceaux.



les effets de trois politiques d'encadrement de la compétition, quand [Hobbs et al., 2000] fait de même pour étudier les contraintes de transmission.

Par ailleurs, il est important de noter que les marchés day-ahead n'existent pas hors-sol, et que l'électricité peut être échangée sur ces marchés, mais aussi par des contrats à terme ou encore sur les marchés intraday. Il existe aussi des marchés de capacité ou une garantie de capacité disponible à une certaine date s'échange. Tous ces marchés interagissent les uns avec les autres, et une partie de la littérature s'attache à décrire ces interactions. Les SFE utilisés à cette fin sont généralement simplifiés, par exemple en ne considérant que les équilibres linéaires pour étudier l'interaction entre marché day-ahead et contrats à terme [Green, 1999], ou encore en considérant des équilibres asymétriques linéaires dans le même but [Anderson and Hu, 2012]. Ces articles se concentrent généralement sur les interactions entre contrats à terme et marché day-ahead, car les SFE ne sont pas en mesure de distinguer le marché day-ahead du marché intraday.

Nous renvoyons la lectrice intéressée vers la revue de littérature [Ventosa et al., 2005] pour une vue d'ensemble plus détaillée. Dans cette thèse, nous nous appuyons fortement sur le travail de Klemperer et Meyer, et contrastons souvent nos résultats aux leurs. Pour faciliter la lecture au lecteur qui ne serait pas familier avec leurs travaux, nous résumons ci-après leurs résultats sur lesquels nous nous appuyons.

Klemperer et Meyer 1989

Soit un marché sur lequel des producteurs enchérissent des courbes d'offre tout en faisant face à une demande incertaine.

Soit $D(p, \theta)$ la courbe de demande comme fonction du prix p et du choc θ . Considérons que pour tout (p, θ) , $-\infty < D_p < 0$, $D_{pp} \leq 0$ et $D_\theta > 0$.

Les producteurs font face à la même fonction de coût $C(\cdot)$, avec $C'(q) > 0$ et

$0 < C'''(q) < \infty$ pour tout $q > 0$.

Les producteurs soumettent leurs offres en même temps avant la réalisation du choc de demande θ . Considérons pour l'instant deux producteurs i et j avec $S^k(p)$ la courbe d'offre du producteur k différentiable deux fois. Une fois le choc connu, chaque producteur produit la quantité $S^k(p^*(\theta))$ au prix $p^*(\theta)$, tel que $D(p^*(\theta)) = S^i(p^*(\theta)) + S^j(p^*(\theta))$.

La demande résiduelle du producteur i est donnée par la demande totale à laquelle la production du producteur j est soustraite, $D(p, \theta) - S^j(p)$. Comme θ est un scalaire, l'ensemble de points maximisant le profit pour chaque choc θ possible définit une courbe. Si il existe une intersection unique entre la courbe d'offre de i et toute courbe de demande possible, alors cette courbe d'offre est ex-post optimale, c'est à dire qu'elle est optimale point à point pour chaque réalisation possible de θ .

Sous l'hypothèse que les courbes d'offres se comportent effectivement de cette manière, la maximisation du profit espéré devient indépendante de la distribution des chocs de demande, et la courbe d'offre optimale de i résout pour chaque choc θ le programme de maximisation suivant :

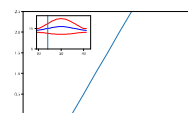
$$\max_p p (D(p, \theta) - S^j(p)) - C (D(p, \theta) - S^j(p)) \quad (4)$$

dont la condition du premier ordre s'écrit :

$$D(p, \theta) - S^j(p) + (p - C' (D(p, \theta) - S^j(p))) (D_p(p, \theta) - S^j'(p)) = 0 \quad (5)$$

Comme l'équation 4 est strictement concave en p (nous renvoyons le lecteur vers le papier original pour plus de détails), l'équation 5 définit un unique prix $p^*(\theta)$ maximisant le profit pour chaque choc θ , qui paramétrise la courbe d'offre optimale.

Considérons que $D_{\theta p} = 0$, c'est à dire que θ est un choc additif, et concentrons nous



sur les équilibres symétriques, ce qui nous permet de ne plus faire attention aux exposants i et j caractérisant la productrice. Sachant que l'éq. 5 doit être vérifiée pour tout choc, elle peut se réécrire :

$$S'(p) = \frac{S(p)}{p - C'(S(p))} + D_p(p) = f(p, S) \quad (6)$$

Cette équation différentielle définit l'équilibre en courbes d'offres, l'incertitude ayant pour conséquence que cette équation doit être vérifiée pour tout choc, et donc pour tout prix possible. Cette équation ne s'accompagne toutefois pas d'une condition initiale. Il existe donc une multiplicité de solutions admissibles.

Les courbes d'offres sont bornées par les possibles valeurs de leur pente, qui doit être comprise entre 0 et $+\infty$. Il est possible de définir le lieu des points pour lesquels les solutions de l'équation différentielle ont pour pente ces valeurs extrémales, qui définit donc la région des solutions admissibles dans notre contexte :

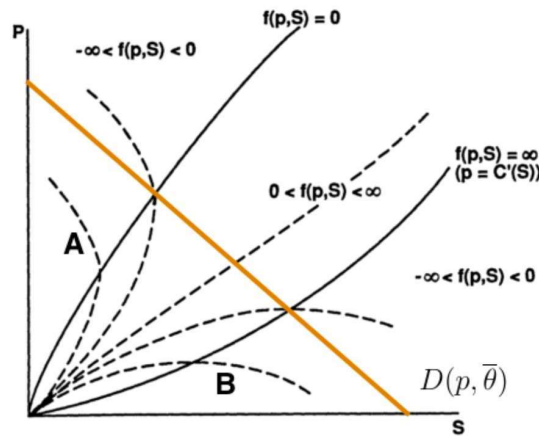


Figure 2: Ce graphique est adapté du papier original de Klemperer et Meyer et illustre la région de solutions de l'équation différentielle admissibles dans le cadre de notre problème, c'est-à-dire vérifiant les contraintes sur leur pente.

L'ensemble des solutions admissibles est donc défini par la borne supérieure de nos chocs de demande $\bar{\theta}$, en cela que si une solution de l'équation différentielle devait traverser les frontières définies ci-dessus pour un choc inférieur au choc maximal, elle ne serait pas pour autant solution de notre problème qui contraint plus fortement les solutions que

l'équation différentielle seule. Dans la figure 2, la demande associée avec le choc maximal $D(p, \bar{\theta})$ est représenté en orange et les solutions A et B de l'équation différentielle ne sont pas solutions de notre problème, car elles franchissent nos frontières pour des valeurs de chocs inférieures à ce maximum.

Le dernier résultat que nous évoquerons ici est que dans le cas d'un support de choc infini, l'ensemble d'équilibres symétriques est le plus petit possible, car il faut dans ce cas que toute solution ait une pente positive et finie pour tout choc positif et plus seulement pour un segment de la droite des réels.

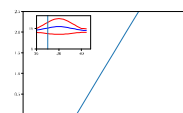
Dans certains cas, par exemple pour des courbes de demande linéaires, cet ensemble peut converger vers une solution unique.

Les coûts de variation

Ce modèle choisit de considérer des coûts ne dépendant que de la quantité produite. Dans le contexte de la production d'électricité, il existe un type de coûts qui ne peut pas se modéliser ainsi, les coûts de variation. Ces coûts sont induits lorsque la production varie dans le temps.

Afin d'expliquer ces coûts, considérons une centrale thermique (fossile, nucléaire, etc.) et plus précisément son réacteur. Physiquement, pour produire une certaine puissance, il faut maintenir le réacteur à une température donnée. Afin d'accroître la production, la température du réacteur doit augmenter. Cela implique que lorsque la production augmente, du combustible doit être perdu afin de simplement réchauffer le réacteur, cette dépense énergétique n'étant pas associée à une production d'énergie.

L'existence de ces coûts est au cœur du choix d'utiliser des centrales gaz "rapides" afin de suivre une hausse soudaine de la demande, là où les centrales nucléaires sont plus



généralement utilisées afin de suivre les changements basse fréquence de la demande. Ces coûts sont suffisamment importants pour que le projet des bourses européennes d'électricité EPEX intitulé "Price Coupling of Regions" (PCR), qui cherche à développer un unique mécanisme de couplage des prix européens de l'électricité, considère la possibilité de soumettre des ordres conditionnés sur les gradients de charge, c'est-à-dire des ordres dont la disponibilité serait conditionnée sur le respect de valeurs maximales de variation de la production d'une enchère à l'autre. Le projet PCR n'est toutefois qu'à l'état d'ébauche au moment de l'écriture de ces lignes [EPEX, 2018c].

Des articles se sont attachés à estimer la valeur de ces coûts de variation empiriquement, [Wolak, 2007] et plus récemment [Reguant, 2011]. Il existe également une branche de la littérature s'intéressant à ces coûts à travers le prisme de leur impact sur le prix optimal maximisant le bien-être social [Tanaka, 2006], c'est-à-dire s'intéressant à la chronique temporelle de prix permettant de maximiser l'utilité des consommateurs à laquelle sont soustraits les coûts de variation. Ces travaux ne s'inscrivent pas dans une approche de théorie des jeux, mais cherchent à trouver le signal prix permettant de limiter les coûts de variation induits par une demande variant de façon déterministe. À notre connaissance il n'existe pas de modèle de ces coûts dans un contexte de théorie des jeux permettant de décrire leur impact sur les stratégies optimales des producteurs jouant sur le marché de l'électricité.

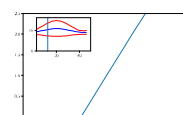
Contribution

Cette thèse se concentre sur la question des coûts de variation. Dans le premier chapitre, cette question est abordée dans un modèle théorique produisant des prédictions sur l'évolution de la forme des courbes d'offres optimales dans le temps en fonction de la dynamique sous-jacente des chocs de demande. Le deuxième chapitre introduit ensuite des techniques permettant d'étudier empiriquement la forme des courbes d'offre observées sur le marché de l'électricité français entre 2011 et 2013, ainsi que des méthodes permettant d'estimer l'incertitude sur la demande associée à la météo. Le troisième chapitre

applique ces méthodes afin de tester empiriquement les prédictions théoriques du premier chapitre. Ces deux derniers chapitres sont issus d'une collaboration avec Henri de Belsunce, qui a soutenu son doctorat en 2015 au Max Planck Institute for Innovation and Competition de Munich, sous la supervision de Prof. Klaus M. Schmidt.

Le premier chapitre se concentre sur ce que la prise en compte des coûts de variation apporte dans un contexte théorique. La contribution principale est de construire et de justifier comment tenir compte théoriquement de ces coûts. En premier lieu, nous passons à une description du problème en temps continu afin d'apporter à la littérature sur les SFE des outils mathématiques puissants surtout utilisés en finance, à savoir la dynamique stochastique : nous cherchons en effet à modéliser les coûts de variation, en conservant l'ingrédient clef introduit par [Klemperer and Meyer, 1989], i.e. l'incertitude, grâce à l'utilisation de browniens, plus précisément des processus d'Itô. Ce faisant, nous faisons face au caractère non-dérivable des processus stochastiques, et apportons notre deuxième contribution, un argument physique concernant la mode de fonctionnement des centrales de production qui opèrent effectivement comme des filtres passe-bas sur nos processus stochastiques et qui nous permet de continuer à construire un modèle tractable de coûts de variation avec incertitude. Troisièmement, nous trouvons une spécification d'un processus d'Itô nous permettant de conserver une forme analytique.

De ces contributions techniques, nous obtenons nos contributions économiques grâce à un modèle riche et tractable qui propose des résultats contrastant fortement avec les résultats passés de la littérature. Tout d'abord, nos solutions sont uniques, contrairement aux continuums d'équilibres de Nash habituels dans la littérature des SFE. Deuxièmement, nos solutions ne sont pas ex-post optimales, c'est-à-dire qu'acquérir de l'information sur l'évolution attendue de la demande induit des stratégies optimales différentes pour les producteurs, ce qui a pour conséquence que les producteurs dans notre modèle ont une justification pour soumettre des enchères variant dans le temps. Troisièmement, nous obtenons des formes analytiques faisant des prédictions précises sur



l'évolution des stratégies avec la dynamique des chocs de demande, à savoir que lorsque l'incertitude sur la demande augmente, les producteurs soumettent des courbes d'offre de plus en plus pentues afin de transmettre une plus grande part des chocs de demande aux prix plutôt qu'aux quantités dont les variations sont coûteuses. Enfin, notre modèle justifie l'existence de prix négatifs¹⁵ avec des producteurs étant prêts à subventionner la demande pour ne pas avoir à faire face de forts coûts de variation, ce qui contraste avec les courbes d'offre partout positives dans les modèles de SFE. Ces résultats ouvrent la porte à des modèles capables de différencier entre marchés day-ahead et intraday et donc potentiellement de modéliser leurs interactions.

Au bas des pages de droite de la thèse, il y a de petits graphiques présentant la stratégie optimale pour un certain jeu de paramètres de notre modèle, dans le plan quantité-prix, avec un insert illustrant l'évolution dans le temps du support des chocs de demande, dans le plan temps-choc. La ligne verticale dans l'insert représente le point dans le temps pour lequel la solution optimale est représentée dans le reste du graph. Ceci forme un folioscope, ce qui permet au lecteur de se rendre compte de la dynamique de l'évolution des stratégies optimales en fonction de l'évolution du support des chocs en feuilletant rapidement la thèse et en animant ainsi ce graphique à la manière d'un dessin animé. Ceci ne remplit qu'un rôle d'illustration.

Dans le reste de la thèse, l'objectif est de tester ces prédictions sur des données issues du marché day-ahead français. Comme ces prédictions portent avant tout sur l'impact de l'incertitude sur la forme des courbes d'offre, nous séparons en deux chapitres la construction de proxies pour cette incertitude dans le deuxième chapitre et l'analyse de l'évolution des enchères en fonction de cette incertitude dans le troisième chapitre.

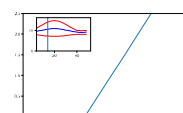
Dans le second chapitre nous nous concentrons sur l'analyse de nos données, sur le

¹⁵De tels prix négatifs s'observent sur les marchés de l'électricité quelques heures par an en moyenne pour l'Allemagne et la France, par exemple en 2017 il y a eu 146 heures en Allemagne pour lesquelles les prix étaient négatifs, répartis sur 24 jours [EPEX, 2018a]

développement d'une méthodologie permettant d'utiliser des données dont chaque observation est une fonction, et sur la construction de proxys pour l'incertitude à laquelle les producteurs font face vis-à-vis de la demande résiduelle qu'ils se doivent d'anticiper lorsqu'ils jouent leurs enchères sur le marché day-ahead.

Nous notons d'abord que les courbes d'offre agrégées sur le marché day-ahead ne sont pas bien capturées par des fonctions paramétriques. Nous construisons une approche non-paramétrique pour les décrire : nous notons que malgré leur mauvaise approximation par des fonctions paramétriques, elles sont néanmoins grossièrement en forme de S, et donc constituées de quatre grandes parties, les deux sections extrémales, et deux sections intérieures séparées par le point d'inflexion de la fonction. Nous définissons les points de transition entre ces sections comme étant les points pour lesquels les valeurs absolues des premières et secondes dérivées de nos courbes d'offres sont maximales. Cette définition repose sur des estimations de densité de kernels, et est donc bien non paramétrique. Nous remarquons qu'en utilisant 5 de ces points, nous sommes en mesure de capturer environ 98% de la variabilité intrinsèque des courbes d'offre, et nous contentons donc de ces 5 points bien que notre méthode peut servir à définir plus de points. Cette méthode nous permet de caractériser des points que nous considérons comparables d'une enchère à l'autre et nous permet de réaliser l'analyse de nos données dans le chapitre 3.

Nous construisons ensuite des proxys pour le degré d'incertitude associée à la météo à laquelle les producteurs doivent faire face ainsi que des variables capturant l'information disponible aux producteurs avant les enchères et que nous devons contrôler. La météo étant connue au moment d'enchérir et ayant une influence sur la demande résiduelle, il nous faut en tenir compte. Néanmoins la météo influence la demande par des effets locaux là où le marché a lieu au niveau national, il nous faut donc construire des proxys nationaux des variables météo localisées sur le territoire français dont nous disposons. Nous construisons notamment un proxy national de température basé sur une moyenne de la météo localisée pondérée par la population afin de capturer l'effet de la température



sur la demande à travers le chauffage notamment.¹⁶ Le reste de notre travail consiste à construire des proxies pour l'incertitude concernant la production renouvelable qui vient réduire la demande résiduelle. Pour ce faire, nous analysons la structure spatiale de nos données de vent et de rayonnement. Nous défendons le fait que l'autocorrélation spatiale de ces variables est un proxy de l'incertitude, en notant que plus les gradients spatiaux sont élevés, moins la prédiction météo sur laquelle se basent les producteurs au moment de jouer sur le marché sera de qualité.

Dans ce second chapitre nous développons une méthode non-paramétrique permettant de comparer les observations fonctionnelles dont nous disposons les unes aux autres, ainsi que des proxies pour l'incertitude associée à la météo.

Dans le troisième chapitre, nous nous concentrons sur la construction du proxy principal pour l'incertitude à laquelle les producteurs font face, l'incertitude sur la courbe de demande agrégée, et nous attachons ensuite à décrire comment les enchères évoluent avec nos différentes sources d'incertitude.

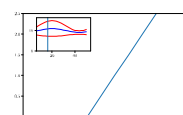
Nous considérons les informations accessibles aux producteurs au moment d'enchérir et régressons les courbes de demandes sur ces variables. Nous étudions ensuite les résidus de cette régression et notons qu'ils sont hétéroskédastiques. Nous nous servons de cette propriété en régressant le carré de ces résidus sur nos variables explicatives, afin de prédire l'amplitude attendue de ces résidus, c'est-à-dire l'amplitude de l'incertitude sur la courbe de demande au moment d'enchérir.

Nous étudions ensuite l'effet de nos différentes sources d'incertitude sur la pente de nos courbes d'offre, et notons que nos proxies pour l'incertitude météo se comportent comme attendus selon nos résultats théoriques obtenus dans le premier chapitre, alors que les résultats sur l'incertitude associée aux courbes de demande sont moins tranchés.

¹⁶Une large proportion du chauffage en France est électrique, ce qui induit que la demande électrique en France est assez météo-sensible.

Comme nos courbes de demande sont des fonctions complètes dans le plan prix-quantité, nous réalisons l'analyse de nos résidus tant en prix qu'en quantité. Nous obtenons donc des estimations de l'incertitude associée à la position d'un point de nos courbes de demande en prix et en quantité. Dans notre modèle théorique, nous faisons des hypothèses fortes sur la forme de la demande, que nous supposons linéaire, et nous supposons que les chocs de demande sont additifs, i.e., ils ne changent pas la pente de la demande. Ces hypothèses ont pour conséquence qu'il est impossible de différencier l'impact d'une incertitude en prix ou en quantité, et que ces deux incertitudes devraient avoir des effets de même signe : plus d'incertitude induit une pente plus grande de la courbe d'offre pour réduire la variation de la production. Nous observons néanmoins que les effets estimés de l'incertitude sur la quantité et le prix sont de signe opposés. Ces deux hypothèses, bien que nécessaires pour obtenir des solutions analytiques, sont évidemment violées par nos données, et nous pensons qu'il s'agit d'une piste importante d'amélioration du modèle.

La contribution du dernier chapitre est de proposer une méthodologie estimant l'incertitude directe sur la courbe de demande et d'estimer comment cette incertitude impacte la forme des courbes d'offre en différents points de son graph, i.e. nous proposons une approche permettant d'estimer comment les courbes d'offre se déforment sous l'effet de l'incertitude.



Chapter 1

Dynamics of the Electricity

Day-Ahead Market : Supply

Function Equilibria and Ramping

Costs ¹

¹JEL Classification Numbers: C62, C72, D21, D43, D84, L94, Q41
Keywords: Supply function, Equilibria, Unicity, Oligopoly, Electricity, Auctions, Day-ahead, Uncertainty, Dynamics

1.1 Introduction

In this chapter, we introduce ramping costs to the theoretical framework of supply function equilibria.

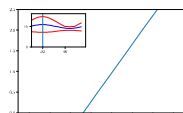
Supply function equilibria are used as a theoretical approach to describe the electricity market on which suppliers bid actual functions contrary to most markets, where one's assumptions about demand and supply curves never translate to agents actually bidding on these objects. The most striking results of this literature are that there exists many (in fact a continuum) of Nash equilibria, that all of those equilibria are ex-post optimal, and that they exhibit always positive prices.

The ex-post optimality implies that once an equilibrium is reached, this equilibrium shouldn't change from auction to auction, given that the cost structure remains constant, even when new information is gathered. However, the observation of the hourly bids on the day-ahead electricity market shows that bids indeed do change from hour to hour. There are many reasons for which the bids might change from one another.

The first is that power plants are brought online or offline to face varying levels of demand for electricity. In so doing, the cost structure of the suppliers changes, which can justify changes in bids. The second is that changes in production are costly in and of themselves, that is, there exists ramping costs associated to the production of electricity. This effect, although technically well supported, also sees support from the existence of negative prices from time to time on the electricity market, on days of high production and low demand. These cases show that subsidizing consumption is less costly than not producing for suppliers.

In this chapter, we propose a theoretical framework to account for these ramping costs.

We choose to model the discrete time bidding as a continuous time process. This



allows us to bring to the literature about supply function equilibria powerful mathematical tools mostly used in option pricing, that is stochastic dynamics: we want to model ramping costs, i.e. costs associated to the variation in production, while retaining the key ingredient brought by [Klemperer and Meyer, 1989], i.e. the uncertainty, through the use of brownians, and more precisely, Itô processes. These tools are the same introduced in the recent literature on dynamic games, see for example [Sannikov et al., 2016]. However, where the focus of this literature is to revisit classical results of repeated games in the context of a time-continuous framework as well as to describe real-world cases more appropriately captured by continuous time models (for example trading), our focus is to be able to capture the effects of ramping costs on the electricity day-ahead market, a market which is discrete in nature.

We obtain a rich and tractable model that yields results that contrast strongly with past results from the literature. First, in the specific case of linear demand and linear costs we obtain a unique Nash equilibria, which contrasts with the usual continuum of Nash equilibria in the supply function equilibria literature. Second, our solutions are not ex-post optimal, meaning that gathering information about the expected future evolution of demand yields different optimal strategies for suppliers, which in turn means that producers in our framework have a motive for submitting different supply functions from one time step to the next. Third, we have closed form solutions which yield specific predictions about the evolution of bids under uncertainty, namely that when uncertainty increase, suppliers submit steeper supply schedules in order to transmit more of these shocks to changes in price and not quantities, which are costly due to the existence of ramping costs. Finally, and less importantly, our framework justifies the existence of negative prices² by producers being willing to pay consumers to consume more in order to avoid facing large variations in production, in contrast to everywhere positive schedules in the case of the supply function equilibria literature.

²Note that such negative prices happen, a few hours a year for example in France or Germany, for example in 2017 there were 146 such hours, spread over 24 days in Germany [EPEX, 2018a]

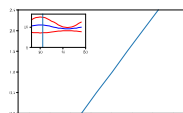
At the bottom of all right pages, there is a small graph showcasing the optimal strategy for a given set of parameters of the model, in the plane quantity-price, with an insert illustrating the evolution of the support of demand shocks over time. The vertical line in the insert represents the point in time for which the strategy is presented in the actual graph. This forms a flipbook, which allows the reader to get a feeling for the evolution of strategies with the evolution of the support of shocks by flipping rapidly the pages and observing the graph becomes animated, just like a cartoon. This serves only an illustrative purpose.

1.1.1 Litterature review

The electricity markets flourished in Europe during the 1990s during the wave of privatization. The argument for their creation was one of competition, that was supposed to bring lower prices to the end consumer of electricity.

An important specificity to the economics of electricity is that electricity cannot be stored in large amounts, which in turn implies that at every moment production and consumption have to match. This means that in order to have a working electric grid, that is one that can produce electricity at higher levels during the winter and lower levels in summer, one has to have production units ready to be turned on if the demand is high enough, but turned off otherwise. This, in turn, means that although their existence is required, it is difficult to see how marginal cost pricing can cover their investment costs, which has been a long running argument in the litterature [Boiteux, 1960]. For this reason, from the very beginning, the issue of the market design was deemed to be crucial to insure that the wished for outcome of the privatization wave came to fruition [Green, 1991].

Most countries having opened the production of electricity to competition have implemented day-ahead markets. As said above, the production and the consumption have



to match constantly. The very short term matching is done by automating tiny adjustments around what a producer is already producing in order to match the fluctuating consumption. To plan which plant should be online at which hour of the day however, the day-ahead markets come in. The idea is that producers and big consumers of electricity (either for themselves, or as aggregators of the individual consumptions) are asked to bid demand or supply functions respectively. The market operator then aggregates the demand and supply curves, which yields an equilibrium giving the price and quantities to be produced for each producer.

There has been an active literature trying to model and measure the market power of oligopolists on these newly created markets [Green and Newbery, 1992, Newbery, 1998, Green, 1999]. The models have mainly been based on Klemperer and Meyer 1989's Econometrica founding paper about supply function equilibria [Klemperer and Meyer, 1989] (henceforth known as KM).

This paper builds upon previous results about competition in supply schedules without uncertainty [Grossman, 1981], which yielded a very high multiplicity of equilibria. KM add a key ingredient: uncertainty about the demand schedule facing the suppliers. This addition greatly reduces the multiplicity, and adds more structure to it, although in this framework there is still a continuum of Nash equilibria, which are always pinned between Cournot and Bertrand outcomes.

Groundbreaking and fertile, the original model by KM studied how demand uncertainty collapses dramatically the set of available supply function equilibria to a well-defined continuum when contrasted to the case of competition in supply schedules without uncertainty [Grossman, 1981]. These equilibria are always pinned between Cournot and Bertrand outcomes. This continuum collapses further to a single Nash equilibria by considering an infinite support of demand shocks, that is an unbounded distribution of shocks. All of these equilibria are ex-post optimal, meaning that changes in anticipated

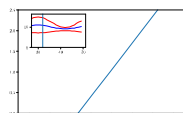
demand shocks do not impact the actual solutions, but only the parts of the solutions that are actually explored as shocks realize, a very strong result.

The electricity markets literature has embraced this framework because it is considered to capture some of the structure at play in the electricity markets: the producers do not know what demand they are going to face when they choose their supply schedule, the demand side is considered much less sophisticated than the supply side, and their demand schedules can therefore be considered to some extent as being exogenous. Some have argued that the schedules submitted in the real markets are discrete and that this discrete nature makes their modelling as continuously differentiable schedules is both incorrect and yields different results from discrete ones [von der Fehr and Harbord, 1993]. However, recent results suggest that with a sufficient number of steps both approaches converge [Holmberg et al., 2008], and indeed we see that recent implementations of the market rules increase the number of steps allowed for a single bid, and consider that these points are linearly joined instead of stepwise.

One of the most striking aspects of the supply function equilibria approaches is, as was alluded to above, the multiplicity of Nash equilibria. This result has been generally viewed as the source of the danger of tacit collusion in electricity markets: if there is a continuum of nash equilibria, repeated interactions are feared to be conducive to a convergence of bidding strategies towards the most profitable equilibria [Bolle, 1992].

Furthermore, these models abstract away some of the details of the actual markets, reason for which authors which try and evaluate the market power of producers on the electricity markets view their endeavour as painting the situation with an optimistic brush [Green and Newbery, 1992].

Here we will tackle the points raised in the last two paragraphs to some extent. We propose to consider a technical reality of the operating of power plants: their cost struc-



ture is history-dependant, more precisely, producing a quantity q_1 does not entail the same cost if the previous quantity produced was already q_1 or if the previous quantity was different from it. Raising or decreasing production in and of itself imply costs. By introducing these costs we aim to produce a model capable of capturing more precisely the competition that arises in the electricity markets, and in so doing we will show that the continuum of equilibria characteristic of supply function equilibria under uncertainty collapses to unique equilibria, which in turn allows us to comment on the question of tacit collusion.

1.1.2 The day-ahead markets

On the electricity day-ahead markets, producers are generally required to submit supply schedules once a day for all the auctions taking place during the next day. The APX (England) and the EPEX (Austria, France, Germany and Switzerland) markets allow hourly auctions [APX, 2017, EPEX, 2015], and EPEX allows for bids comprising up to 256 price quantity combinations, effectively approximating smooth supply functions. Producers can submit different supply schedules for each individual auction, but every bid must be placed at the same time one day in advance for each block of 24 hours. Customers go through the same process and submit their demand schedules, then the market operator matches supply and demand for each auction. Producers thus have to submit schedules facing uncertain demand, which is the reason for the popularity of supply function equilibria approaches to the electricity market.

However, on this market, bids change from auction to auction. From the point of view of KM's model, this should happen only through a coordination of agents agreeing to collectively swap from one Nash equilibrium to another in the available continuum. Describing these dynamics, however, is increasingly important as the energetic mix is bound to include an increasing fraction of renewables. Power production can be separated in two classes: dispatchable and non-dispatchable technologies. Nuclear, coal,

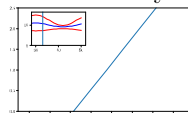
land-fill gas or hydroelectric power generation are mainly dispatchable as one can actually choose their level of production whereas the two rising stars of renewable energy, namely wind and solar, are non-dispatchable: they react to weather conditions. Having these technologies in the mix introduces uncertainty on the production side, which comes down to dispatchable units facing a more uncertain residual demand [Boyle, 2007]. In this paper we want to explore how to model these dynamics.

Electricity production faces very specific technological constraints. These constraints, generally labelled as ramping costs, vary across production technologies and have yet to be captured in a model. We propose to do so by introducing a multivariate cost function, depending as always on the quantity produced, but also on the rate at which production varies: $C(S, \frac{dS}{dt})$. We call this class of cost functions dynamic cost functions. We detail which form these costs take, and its shape in section 1.3.2.

All power plants face maintenance costs. However, part of these maintenance costs is induced by the dynamics of production, and can be seen as ramping costs. More precisely, whatever the production technology, fluctuations in production are costly. Indeed, they imply fluctuations in the temperature of the core of the power plant, thus dilation and contraction cycles of the different parts, which cause wear and tear. The industry is aware of these effects [GE, 2015], as well as the literature, although it focuses mainly on other types of effects, such as the impact on shutdowns due to maintenance [Rothwell et al., 1995], some B2B companies even specialize in minimizing the related long term costs. For example, Wartsila Power Plants, a supplier of power plants and tools to forecast long term costs, explains in a white paper [Arima, 2012]:

Increased variability in net load demand means that dispatchable generating units have to ramp considerably more steeply and deeper than traditionally, thus increasing wear and tear to components.

We are going to model these ramping costs through a dynamic cost function, increasing in the absolute value of its second argument: any change in production is costly. This



paper will focus on the implications of considering this type of ramping costs. Other types of ramping costs exist, for example startup costs, but they will not be studied in this paper.

These effects cannot be captured by traditional cost functions depending on the level of production alone. One needs to take into account the actual path leading to a given quantity produced, and more precisely in the instantaneous variation, not the whole history. This implies that we need to impose structure on the dynamics of the system while retaining uncertainty, the key ingredient of KM's paper. To do so, we use stochastic dynamics.

This seemingly small addition to KM's framework has a lot of implications on the results obtained. The solutions are not ex-post optimal anymore, allowing to account more satisfactorily for the dynamics of optimal supply schedules, and our solutions are unique, even for bounded demand shocks. We also define a novel selection rule to choose from KM's continuum of equilibria. Finally these results open the possibility to distinguish intraday and day-ahead markets.

In section 1.2 we will present a heuristic approach to get the intuition of the model. Then, in section 1.3 we will introduce the mathematical tools needed to use stochastic dynamics in this context, in section 1.4 and section 1.5 we will solve the monopoly and the symmetric oligopoly cases while considering that producers have information about the overall distribution of shocks during the day, but do not have information about differences in the shocks at different dates. Finally in section 1.6 we will discuss the dynamic variation of the optimal bids, while sections 1.7 and 1.8 will respectively cover some limits of this approach and conclude the paper.

1.2 Heuristic Description of the Model

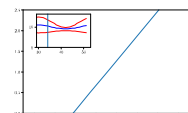
In this section the essence of the model is presented before introducing the proper mathematical tools needed to treat this problem rigorously in the next section. It is thought of as an overview of the mathematical methods that are going to be used, as a way to give a sense of the intent of the modelling choices.

As in KM's setup, the aim is to model an oligopoly facing uncertain demand, taken as exogenous. Before the demand shocks are realized, each firm needs to commit on a strategy. Firms also incur costs that not only depend on the level of production but also on the evolution of the production given its anterior level produced.

More formally, the producer, as in KM, faces uncertain demand, $D(\theta, p)$, with θ a stochastic shock to the demand and p the price. We add to that both ramping costs and uncertain dynamics of demand. As we want to keep the key ingredient of KM, the introduction of uncertainty, but take into account the dynamics of this uncertainty, of these demand shocks, we need to add more structure.

Consider the following notation, where $\theta(t)$ denotes the value of the stochastic shock at time t , whereas Θ denotes the family of all available time trajectories of our demand shocks.

In the real market, bidders submit a finite number of bids once a day, and face the ramping costs inter-period, that is, when production has to be adjusted to reach the subsequent market outcome. The first bit of structure we introduce is that we are going to assume that time is continuous. The second is that ramping costs are incurred continuously and can be thought of as costs depending on the variation of production over time. Finally, we consider that bidders are allowed to submit a different supply schedule for every point in time between 0 and T . This amounts to being asked to submit a surface of strategy in the price-quantity-time space for the next day.



The producer maximises her expected profits, and we consider the simplest case here in which the distribution of shocks is static, that is that the distribution of probability of shocks does not depend on time, and the producer is asked by the market operator to submit the same supply schedule for every point in time a day in advance. In an oligopoly, the program maximised by producer i is, therefore:

$$\max_{S_i(p)} \mathbb{E}_\Theta \left[\int_0^T \left(p(\theta(t)) S_i(p(\theta(t))) - C \left(S_i(p(\theta(t))), \frac{dS_i(p(\theta(t)))}{dt} \right) \right) dt \right] \quad (1.1)$$

with $p(\theta(t))$ the price given the demand shock $\theta(t)$ at date t , $S_i(\cdot)$ the supply schedule of producer i and $C(\cdot, \cdot)$ the dynamic cost function. Note that the price depends on t only through $\theta(t)$, i.e. a given level of demand shock implies a given price.

The goal of this section is to provide a first run through of the model, therefore we will not describe here the conditions that must be verified by the different terms of the model. We will simply assume that the dynamic cost function is additively separable between a static and a ramping term, $C(S_i, \frac{dS_i}{dt}) = C_s(S_i) + C_r(\frac{dS_i}{dt})$, and that the demand shocks θ are bounded in $[\underline{\theta}, \bar{\theta}]$. Lastly we require the ramping term $C_r(\cdot) = \frac{\gamma}{2}(\cdot)^k$ for clarity, and $k \geq 2$ an integer. We distribute the expectation operator and write that $\frac{dS_i}{dt} = \frac{dS_i}{dp} \frac{dp}{d\theta} \frac{d\theta}{dt} = S'_i \cdot \dot{p} \cdot \frac{d\theta}{dt}$, with X' the derivative of univariate function X with respect to its argument, $\dot{X} = \frac{dX}{d\theta}$.

With this setup, by distributing the expectation operator over all possible trajectories of shocks, we are able to rewrite the problem without having time t appear explicitly. This point is crucial, as it is what will let us use mathematical tools that will yield our unicity results. The maximization program can indeed be written as follows:

$$\max_{S_i(p)} T \int_{\underline{\theta}}^{\bar{\theta}} f(\theta) \left(p(\theta) S_i(p(\theta)) - C_s(S_i(p(\theta))) - \frac{\gamma}{2} (S'_i \cdot \dot{p})^k \mathbb{E}_\Theta \left[\left(\frac{d\theta}{dt} \right)^k \middle| \theta \right] \right) d\theta \quad (1.2)$$

with $f(\theta)$ the distribution of shocks, and γ the ramping cost parameter capturing the magnitude of the ramping costs. The expected value on the trajectory of shocks of any of the terms above that only depends on $\theta(t)$, that is the value of the shock at a point in time, can be rewritten simply as an integral over the possible values of the shock.

We are left with $\mathbb{E}_{\Theta} \left[\left(\frac{d\theta}{dt} \right)^k \middle| \theta \right]$ as the only term that depends on the trajectory of shocks. Take for granted that this term can only depend on θ for now, this result will be justified properly in the next section.

Note now that producer i faces a residual demand so that $S_i(p(\theta(t))) = D(\theta, p(\theta(t))) - S_{-i}(p(\theta(t)))$ which depends only on θ and p , t does not intervene directly, with S_{-i} the aggregate supply schedule of all the other producers, taken as given by producer i . This implies that the integrand in eq. 1.2 depends only on three variables: θ , p and \dot{p} . The maximization program is therefore equivalent to an Euler-Lagrange problem, a very well described mathematical object: $\max_p \int \mathcal{L}(\theta, p, \dot{p}) d\theta$.³ The information obtained from taking the first-order condition of an Euler-Lagrange problem yields a second order dif-

³The Euler-Lagrange equation is an equation satisfied by a function p of a real argument θ , which is a stationary point of the functional:

$$S(q) = \int_a^b \mathcal{L}(\theta, p(\theta), \dot{p}(\theta)) dt$$

where:

1. p is the function to be found:

$$\begin{aligned} p: [a, b] \subset \mathbb{R} &\rightarrow X \\ \theta &\mapsto x = p(\theta) \end{aligned}$$

s.t. p is differentiable

2. \dot{p} is the derivative of p w.r.t. θ :

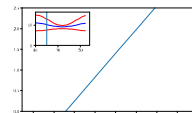
$$\begin{aligned} \dot{p}: [a, b] &\rightarrow T_{p(\theta)}X \\ \theta &\mapsto v = \dot{p}(\theta) \end{aligned}$$

$T_{p(\theta)}X$ denotes the tangent space to X at the point $p(\theta)$.

3. \mathcal{L} is a real-valued function with continuous first derivatives:

$$\begin{aligned} \mathcal{L}: [a, b] \times TX &\rightarrow \mathbb{R} \\ (\theta, x, v) &\mapsto \mathcal{L}(\theta, x, v) \end{aligned}$$

$T_{p(\theta)}X$ denotes the tangent space to X at the point $p(\theta)$.



ferential equation as well as two boundary conditions: $\frac{\partial \mathcal{L}}{\partial p} = \frac{d}{d\theta} \frac{\partial \mathcal{L}}{\partial p}$ and $\frac{\partial \mathcal{L}}{\partial p} \Big|_{\theta} = \frac{\partial \mathcal{L}}{\partial p} \Big|_{\bar{\theta}} = 0$. This is why we obtain unique solutions: if the boundary conditions are not verified, there exists profitable deviations.

In less mathematical term, taking ramping costs into account as specified above means that for a given level of shock, the producer not only cares about the optimal level of production for this shock, but also about the optimal slope of the supply schedule at this level of production. Effectively, this means that optimal levels of production cannot be chosen independently for different level of shocks as is the case in KM, thus shrinking the continuum of equilibria. The boundary conditions' argument explains why the continuum not only shrinks, but collapses to a unique equilibrium.

Note that if the ramping cost parameter γ is taken equal to 0 we are back to KM's model: one doesn't care about the slope of the supply schedule anymore, and the problem comes down to a pointwise maximization which therefore yields ex-post optimal equilibria. We want to stress that this means that it is not sufficient to specify the dynamics of the shocks to obtain a supply function model that would react to these dynamics, one needs to take into account ramping costs.

The maximization program 1.2 is a heuristic description of the situation. We want to model the stochastic nature of demand and of its dynamics. We do this by using Itô processes, a class of stochastic processes built through brownians, to describe the stochastic trajectory of the demand shocks with respect to time. The difficulty is that brownians are everywhere continuous but nowhere differentiable, therefore the way program 1.2 is written, with a term in $\frac{d\theta}{dt}$, is a shortcut.

In the next section we introduce the stochastic dynamics properly without using the concept of a derivative.

1.3 Stochastic Dynamics

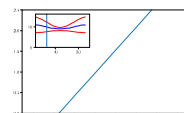
As described in the previous section, we consider that bidders submit surfaces, that is supply schedules for every point in time. The reason to describe a discrete dynamic market as a continuous one is that although discrete time is conceptually more easily understood, continuous time allows to use much more powerful mathematical tools and to obtain closed form solutions, which we think are crucial in gaining intuitive insights about these dynamics. Therefore, we consider that demand fluctuates continuously and that ramping costs are incurred instantaneously. This approximation would need to be tested, although it should be noted that day ahead markets operate with hourly or half-hourly periods and producers are therefore facing a reasonable number of periods each day.

We want our shock variable to evolve over time in a random fashion. The class of mathematical objects used to describe this are stochastic processes. The simplest stochastic process one can think of, and indeed the most important historically, is a Brownian motion process.

Unfortunately, Brownian processes are unbounded and cannot therefore be used to describing the dynamics of the electricity market in which demand shocks, denoted $\theta(t)$, are bounded: there are no days for which demand is null nor are there days for which demand tends towards infinity. The structure to be imposed on the dynamics of the shocks has to imply bounded shocks.

1.3.1 The stochastic process

A regular candidate for richer stochastic dynamics than a simple brownian process is an Ornstein–Uhlenbeck process. Unfortunately for us, such a process has unbounded support. We are going to use a richer set of stochastic processes: Itô processes.



The simplest Itô process one can consider that leads to bounded shocks is defined by the following stochastic differential equation (SDE) [Hertzler, 2003]:

$$d\theta(t) = -2\theta(t)dt + \sqrt{1 - \theta(t)^2}dB_t \quad (1.3)$$

with B_t a brownian and dX an infinitesimal variation of quantity X . Its extended version will happen to be very useful in the rest of this chapter, as it leads to a quadratic distribution of shocks, a property that will allow us to obtain closed form solutions.

Observe that this SDE is formed by a deterministic mean-returning term $-2\theta(t)dt$ and a bounded stochastic one $\sqrt{1 - \theta(t)^2}dB_t$. As $\theta(t)$ approaches -1 or 1 the stochastic term goes to 0, thus $\theta(t) \in [-1, 1]$. Without loss of generality we can restrain ourselves to this special case. Other bounded supports, $\theta \in [\underline{\theta}, \bar{\theta}]$, can be captured through renormalizations of θ .

Such a stochastic process has a distribution of probability $f(\theta)$ given by Fokker-Planck's equation, easily solved here. In the general case of an Itô process given by SDE 1.4, one obtains in 1.5 the generic Fokker-Planck equation for its distribution of probability $f(\theta, t)$. This equation allows, given an initial condition on the distribution of probability of the variable, to observe how this distribution evolves to reach the steady-state distribution, that is the limit distribution that any initial condition yields. If one knows the value of the stochastic variable at one point in time, one can use this equation to obtain the spread in its distribution over time.

$$d\theta = \mu(\theta, t)dt + \sigma(\theta, t)dB_t \quad (1.4)$$

$$\frac{\partial}{\partial t}f(\theta, t) = \frac{\partial}{\partial \theta}(\mu(\theta, t)f(\theta, t)) + \frac{1}{2} \frac{\partial^2}{\partial \theta^2}(\sigma(\theta, t)^2 f(\theta, t)) \quad (1.5)$$

Here, for SDE 1.3, this yields that $f(\theta) = \frac{3}{4}(1 - \theta^2)$ on $[-1, 1]$ and 0 elsewhere.

1.3.2 The ramping costs

In the rest of the chapter, we are going to consider quadratic ramping costs. More precisely we consider the costs induced by fluctuations in the production level. As described in the introduction, fluctuations imply increased wear and tear, whether the production is increasing or decreasing. In addition, these ramping costs are null in the absence of fluctuations. This means that they can be captured by a function $C_r(\cdot)$ verifying $C_r(0) = 0$, $C_r(\cdot) \geq 0$ and increasing in the absolute value of its argument. In the absence of more detailed knowledge about the actual shape of these ramping costs, it seems reasonable to consider a quadratic cost function, that is the first term in a Taylor expansion of the actual real ramping cost function.

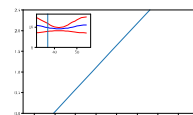
We cannot compute $\frac{d\theta}{dt}$ as it appears in Eq. 1.2, as a stochastic process, although everywhere continuous, is nowhere differentiable. The goal of this section is to express properly the maximization program of the producer that we presented rapidly in Eq. 1.2, and most importantly, to introduce properly how we can work in continuous time with a cost function which depends on fluctuations, which, importantly, are nowhere differentiable.

We are therefore going to first consider the discrete case of a random walk of timestep Δt which converges towards the Itô process 1.4, using the Euler-Maruyama approximation [Kloeden and Platen, 2011]⁴, a generalization of the Euler method to stochastic differential equations. We consider a Markov chain Y defined as follows:

$$\Delta Y_n = Y_{n+1} - Y_n = \mu(Y_n, n\Delta t)\Delta t + \sigma(Y_n, n\Delta t)\Delta B_n \quad (1.6)$$

where $\Delta B_n = B_{(n+1)\Delta t} - B_{n\Delta t}$. These ΔB_n are *i.i.d.* normal random variables of mean 0 and variance Δt . Note that as Δt is taken towards 0, this Markov chain converges

⁴This formula can be found on page 305. This book focuses on numerical approximations of continuous stochastic processes, which is the reverse of what we are doing here, but it is only in such numeric-centric books that this scheme is introduced. For a more general approach to stochastic differential equations, see [Øksendal, 2003]



towards its underlying stochastic process defined by eq.(1.4).

The ramping costs are taken as quadratic in the variation of the production, and also depend on a ramping cost parameter $\Gamma(\Delta t)$, that is the cost per unit of quadratic variation at horizon Δt , so we compute the following quantity:

$$\mathbb{E} \left[\frac{\Gamma(\Delta t)}{2} \cdot \left(\frac{Y_{n+1} - Y_n}{\Delta t} \right)^2 \middle| Y_n \right] = \frac{\Gamma(\Delta t)}{2} \cdot \frac{\sigma(Y_n, n\Delta t)^2}{\Delta t} \quad (1.7)$$

For this quantity to converge to a finite value when the Markov chain is taken towards its underlying stochastic process we have to consider that for small enough timescales, the ramping cost parameter $\Gamma(\Delta t)$ is linear in Δt , i.e. $\Gamma(\Delta t) = \gamma\Delta t + o(\Delta t)$. Mathematically, if $\Gamma(\Delta t)$ had a slower than linear relationship at small timescales, the ramping costs would diverge, and if it was faster they would converge to 0. A physical constraint, namely thermal inertia, ensures that the ramping cost parameter does actually behave in this way.⁵

Consider for now that the mean function μ and the variance function σ from eq. 1.4 do not depend on time explicitly and are therefore written $\mu(\theta)$ and $\sigma(\theta)$. Consider now a transformation $T(\cdot)$ that we apply to the Markov chain Y . Then:

$$\mathbb{E} \left[\frac{\Gamma(\Delta t)}{2} \cdot \left(\frac{T(Y_{n+1}) - T(Y_n)}{\Delta t} \right)^2 \middle| Y_n \right] = \mathbb{E} \left[\frac{\Gamma(\Delta t)}{2} \cdot \left(\frac{T(Y_{n+1}) - T(Y_n)}{Y_{n+1} - Y_n} \cdot \frac{Y_{n+1} - Y_n}{\Delta t} \right)^2 \middle| Y_n \right] \quad (1.8)$$

And in the limit where the markov process Y converges towards the Itô process θ of equation 1.4:

$$\lim_{\Delta t \rightarrow 0} \mathbb{E} \left[\frac{\Gamma(\Delta t)}{2} \cdot \left(\frac{T(Y_{n+1}) - T(Y_n)}{\Delta t} \right)^2 \middle| Y_n \right] = \frac{\gamma}{2} \cdot T'(\theta(t))^2 \cdot \sigma(\theta)^2 \quad (1.9)$$

⁵Ramping costs come from thermal fluctuations in the core of the plant. Therefore, we have to describe how temperature responds to fluctuations in production. Thermal inertia acts as a low pass filter, meaning that it smoothes out fluctuations on short timescales. Think about heating a saucepan full of water: although lighting the stove is almost instantancous, the temperature of the water being heated increases only progressively, in an exponential fashion that is therefore linear in time for short timescales.

We apply this result to the problem at hand, that is we evaluate the ramping costs in the case where the demand shocks are given by eq. 1.3:

$$\lim_{\Delta t \rightarrow 0} \mathbb{E} \left[\frac{\Gamma(\Delta t)}{2} \cdot \left(\frac{\Delta S_i(p(\theta(t)))}{\Delta t} \right)^2 \middle| \theta(t) \right] = \frac{\gamma}{2} \cdot S_i'(p(\theta(t)))^2 \dot{p}(\theta(t))^2 (1 - \theta^2) \quad (1.10)$$

with X' the derivative of quantity X with respect to its argument and \dot{X} its derivative with respect to θ . Note that we considered here that the variance term $\sigma(\theta) = 1 - \theta^2$ depends only on θ and not explicitly on t , which in turn implies that the strategy S_i does not depend explicitly on t either.

Let us consider the case where the strategy and the variance depend explicitly on time and are thus written $S_i(p(\theta(t), t), t)$ and $\sigma(\theta, t)$ respectively. By using a first order expansion as before, the ramping cost function can be approximated as follows:

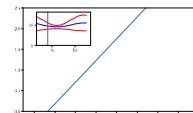
$$\begin{aligned} \lim_{\Delta t \rightarrow 0} \mathbb{E} \left[\frac{\Gamma(\Delta t)}{2} \left(\frac{\Delta S_i(p(\theta(t), t), t)}{\Delta t} \right)^2 \middle| \theta(t) \right] &= \lim_{\Delta t \rightarrow 0} \mathbb{E} \left[\frac{\gamma}{2} (\partial_1 S_i(p(\theta(t), t), t) \partial_1 p(\theta(t), t))^2 \frac{\Delta \theta^2}{\Delta t} + \mathcal{O}(\Delta t)) \right] \\ &= \frac{\gamma}{2} (\partial_1 S_i(p(\theta(t), t), t) \partial_1 p(\theta(t), t))^2 \sigma(\theta, t)^2 \end{aligned} \quad (1.11)$$

with $\partial_i X$ the partial derivative of quantity X with respect to its i^{th} argument. See Annex. 1.A for more details on this derivation.

Now, we can write down the instantaneous expected value of the profit of producer i if the demand shock is $\theta(t)$, $\pi_i^e(t, \theta(t))$, that is the profit that one expects to obtain when demand is at $\theta(t)$ given the expected value of the ramping costs:

$$\pi_i^e(t, \theta(t)) = p(\theta(t), t) S_i(p(\theta(t), t), t) - C_s(S_i(p(\theta(t), t), t)) - \frac{\gamma}{2} \partial_1 S_i(p(\theta(t), t), t)^2 \partial_1 p(\theta(t), t)^2 \sigma(\theta, t)^2 \quad (1.12)$$

Lastly, we have to write down the expected profit for a day's worth of submitted strategies. Let us consider that the chosen unit of time is the day. Therefore, the total expected profit Π_i^e writes:



$$\begin{aligned}
 \Pi_i^e &= \int_0^1 \mathbb{E}_{\theta(t)}[\pi_i^e(t, \theta(t))] dt \\
 &= \int_0^1 \int_{\underline{\theta}}^{\bar{\theta}} f(\theta, t) \left[p(\theta, t) S_i(p(\theta, t), t) - C_s(S_i(p(\theta, t), t)) \right. \\
 &\quad \left. - \frac{\gamma}{2} \partial_1 S_i(p(\theta, t), t)^2 \partial_1 p(\theta, t)^2 \sigma(\theta, t)^2 \right] d\theta dt \tag{1.13}
 \end{aligned}$$

We want to note that all the stochastic calculus presented here is very standard, our contribution is in noting the low-pass filter effect of the physical power plant on fluctuations which allow us to obtain convergent expressions.

1.3.3 Discussion of the approximations

We want a tractable mathematical formulation of the dynamic problem faced by producers on the electricity market. To achieve this, we seek to describe the discrete real-life problem by an approximated continuous one. We first use two technological facts: fluctuations in production are costly and these costs decrease linearly in time for short timescales. We then rely heavily on first order expansions of the different terms we have to compute.

1.3.4 The maximization program

Here, we consider that the dynamics of demand shocks are given by eq.(1.3), and that therefore $\sigma(\theta, t)^2 = \sigma(\theta)^2 = (1 - \theta^2)$.

We now introduce the different conditions that have to be satisfied by the various terms in this problem. First, on most electricity markets, schedules must be increasing, therefore here we take $S_i'(\cdot) \geq 0$. Second, the aggregate demand is non negative as consumers do not have production facilities at their disposal: $D(\theta(t), p(\theta(t))) = \sum_i S_i(p(\theta(t))) \geq 0$. Last, we consider that the shocks θ are ordered so that the demand is increasing in θ , i.e. $\frac{\partial D}{\partial \theta} \geq 0$, and that the price has to weakly increase with the shocks, i.e. $\dot{p} \geq 0$,

which guaranties that the supply function increases with shocks. Our initial stochastic maximization program can thus be rewritten as a regular optimal control problem:

$$\max_{S_i(p)} \int_{-1}^1 f(\theta) \left(p(\theta) S_i(p(\theta)) - C_s(S_i(p(\theta))) - \frac{\gamma}{2} (1 - \theta^2) (S_i'(p(\theta)) \dot{p}(\theta))^2 \right) d\theta \quad (1.14)$$

$$\begin{aligned} \text{s.t.} \quad & S_i'(\cdot) \geq 0 \\ & \dot{p} \geq 0 \end{aligned} \quad (1.15)$$

$$D(\cdot, \cdot) \geq 0 \quad (1.16)$$

The next section solves this problem for a monopoly.

1.4 The Monopoly

Let us consider that the aggregate demand is linear, that is:

$$D(\theta(t), p(\theta(t))) = a\theta(t) + b - p(\theta(t))$$

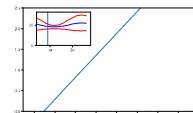
with a and b parameters taken to describe any bounded support of shocks given the stochastic dynamics introduced in the previous section for which $\theta \in [-1, 1]$. Here $(a\theta + b) \in [b - a, b + a]$.

In a monopoly situation we have $S = D(\theta(t), p(\theta(t)))$, therefore the constraints reduce to:

$$\dot{p}(\theta) \in [0, a], \text{ and } p(\theta) \leq a\theta + b$$

where \dot{X} corresponds to $\frac{dX}{d\theta}$.

Consider in addition that the static cost function is also quadratic: $C_s(S_i) = \frac{\lambda}{2} S_i^2$.



The maximization program is rewritten as:

$$\max_{p(\cdot)} \int_{-1}^1 f(\theta) \left(p(\theta)(a\theta + b - p(\theta)) - \frac{\lambda}{2}(a\theta + b - p(\theta))^2 - \frac{\gamma}{2}(1 - \theta^2) (a - \dot{p}(\theta))^2 \right) d\theta \quad (1.17)$$

$$\begin{aligned} s.t. \quad & \dot{p}(\theta) \in [0, a] \\ & p(\theta) \leq a\theta + b \end{aligned}$$

1.4.1 Results

Proposition 1.4.1 *The solution exists, is unique, and has the following form:*

$$\forall \theta \in [-1, 1] \quad p^*(\theta) = a \frac{4\gamma + 1 + \lambda}{4\gamma + 2 + \lambda} \theta + b \frac{1 + \lambda}{2 + \lambda} \quad (1.18)$$

The optimal schedule is parametrised by θ so that $S(p(\theta))$ is formed by the points of coordinate $(a\theta + b - p(\theta), p(\theta))$. Its equation is given by:

$$S^*(p) = \frac{1}{4\gamma + 1 + \lambda} \left(p + \frac{4\gamma}{2 + \lambda} b \right) \quad (1.19)$$

These expressions depend explicitly on our parametrization of the shocks by the parameters a and b .

Proof See annex 1.B. ■

Proposition 1.4.2 *When taking $\gamma \rightarrow 0$, the above solution converges towards the solution obtained in the Klemperer and Meyer framework, which for a monopoly is also unique.*

Proof See annex 1.C. ■

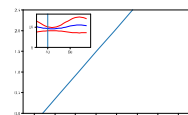
We present in Fig. 1.1(a) the results obtained for increasing values of the ramping cost parameter γ , starting at $\gamma = 0$ in black and moving progressively from black to blue to red to green.

As the ramping costs increase, a given change in quantity is more costly. To mitigate this effect out, the supplier bids a steeper supply curve, which in turn means that for a given demand shock, more of this shock is transferred to a change in price, which is not costly, than to a change in quantity, which is costly. Adding ramping costs narrows down the domain of attainable quantities produced, as a larger quantity domain implies larger incurred ramping costs.

Solutions are steeper than the traditional monopoly situation, bringing the schedules ever closer to a Cournot-like situation (fixed quantity, i.e. a vertical bid). In addition, the optimal supply schedules do not depend on a , the parameter determining the width of the possible shocks, but only on b which defines the average value of the shocks. This result is very strong, but is very much a consequence of our choice of specification: linear demand with additive shocks. In this case the increase in costs due to larger shocks is exactly counterbalanced by larger possible demand, so that the slope of the solution is unaffected. It is very likely that this result is not robust to other specifications.

In Fig. 1.1(b), we illustrate the impact of the specification of shocks on the optimal solution. First, if we consider an increase in a without changing b , that is a change in the amplitude of the possible shocks, without changing their expected value, as explained above the solution is essentially unaffected: a larger region of the solution is simply explored. This is illustrated by the red supply function changing to the rose one as a is increased. On the other hand, if we consider a fixed a but an increasing b , that is a constant amount of uncertainty, so to speak, but with larger expected values, the explored length is kept constant, but the optimal schedule is translated towards the north-east region of the plane. This is quite intuitive: more demand implies higher supplied quantities and higher prices. This is illustrated by the red supply function changing to the blue one.

Note that all schedules cross at a single point. These quadratic ramping costs imply



a symmetric deformation of the solution obtained in the case of an absence of ramping costs. The limit of extremely high ramping costs is a Cournot-like schedule, i.e. a vertical one, taken at this crossing point: any demand shock is too costly to be accommodated by changes in quantities, and yields only changes in price.

Lastly, our solutions exhibit organically the possibility for negative prices.

Proposition 1.4.3 *With $\gamma > 0$, there exists values of shocks for which the prices are negative. More precisely, there exist negative prices if the following condition on the parameters of the shocks holds:*

$$\frac{4\gamma + 2 + \lambda}{2 + \lambda} \frac{1 + \lambda}{4\gamma + 1 + \lambda} b < a < \frac{4\gamma + 2 + \lambda}{2 + \lambda} b$$

Proof We want the condition under which our solutions exhibit negative prices. First, our supply schedule needs to be positive. Second, for there to exist possible negative prices, one needs the smaller possible price, the one obtained for $\theta = -1$, to be negative.

This can be rewritten as conditions on the shocks, using the expressions from eq. 1.18 and eq. 1.19:

$$\begin{aligned} S^*(p(-1)) &> 0 \\ p(-1) + \frac{4\gamma}{2 + \lambda} b &> 0 \\ -a \frac{4\gamma + 1 + \lambda}{4\gamma + 2 + \lambda} + b \frac{1 + \lambda}{2 + \lambda} &> \frac{4\gamma}{2 + \lambda} b \\ a &< \frac{4\gamma + 2 + \lambda}{2 + \lambda} b \end{aligned}$$

and:

$$\begin{aligned} p(-1) &< 0 \\ -a \frac{4\gamma + 1 + \lambda}{4\gamma + 2 + \lambda} + b \frac{1 + \lambda}{2 + \lambda} &< 0 \\ a &> \frac{4\gamma + 2 + \lambda}{2 + \lambda} \frac{1 + \lambda}{4\gamma + 1 + \lambda} b \end{aligned}$$

■

We will discuss the existence of negative prices more in detail in the following sections.

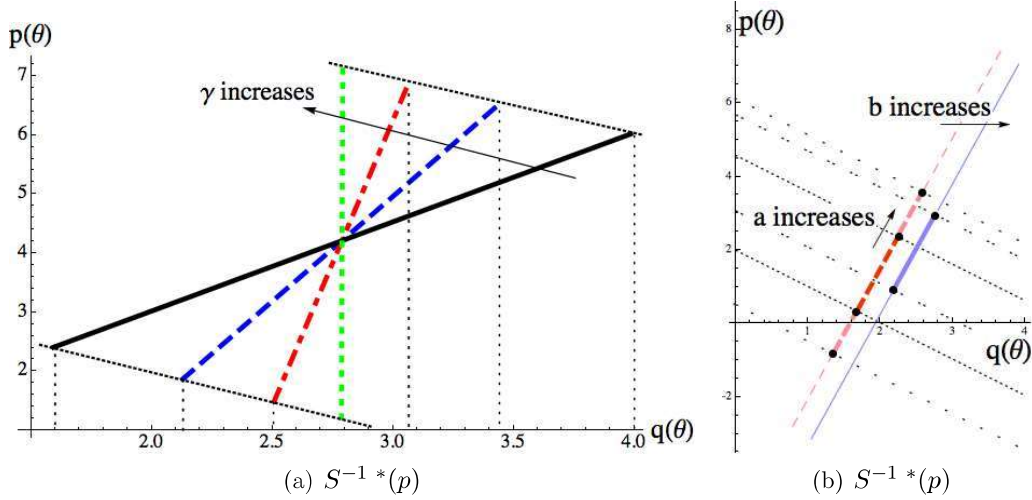


Figure 1.1: **(a)** Four optimal supply schedules are plotted. In black (full line) $\gamma = 0$. As γ increases we transition from the black curve to the blue curve (large dashes), then the red curve (mixed dashes) and then finally for $\gamma \rightarrow \infty$ to the green one (small dashes). The range of production is highlighted for each curve through the thin vertical dotted lines.

(b) The thin black dotted lines represent the extremal demand functions given a and b , i.e. $D(\underline{\theta}, p)$ and $D(\bar{\theta}, p)$. In red (dashed) the solution for a given value of b . As a increases, the solution widens from the thick deep red region to the thick light red one. In the case for which a is kept constant and b is increased the solution shifts from the dashed deep red region to the full thick blue one.

1.5 The Symmetric Oligopoly

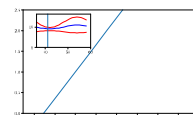
We keep the same linear demand specification as in the monopoly, therefore, with n competitors one has to consider the residual demand faced by each producer:

$$S(p(\theta)) = a\theta + b - (n - 1)S(p(\theta)) - p \tag{1.20}$$

$$S(p(\theta)) = \frac{a\theta + b - p}{n} \tag{1.21}$$

$$S'(p(\theta)) = \frac{a - \dot{p}}{n\dot{p}} \tag{1.22}$$

$$S''(p(\theta)) = -\frac{a\ddot{p}}{n\dot{p}^3} \tag{1.23}$$



For concision, we drop the explicit dependencies of the different functions on their arguments in the following equations; $f(\theta)$, $p(\theta)$ and $S(p(\theta))$ will be noted f , p and S respectively. The maximization program now writes:

$$\max_{p(\cdot)} \int_{-1}^1 f \left(p(a\theta + b - p - (n-1)S) - \frac{\lambda}{2}(a\theta + b - p - (n-1)S)^2 - \frac{\gamma}{2}(1-\theta^2)(a - \dot{p}(1 + (n-1)S'))^2 \right) d\theta \quad (1.24)$$

$$\begin{aligned} \text{s.t.} \quad & \dot{p} \in [0, a] \\ & p \leq a\theta + b \end{aligned}$$

with, as before, $\dot{X} = \frac{dX}{d\theta}$ and X' is the derivative of function X with respect to its argument.

Results

Proposition 1.5.1 *The solution exists, is unique, and has the following form:*

$$\forall \theta \in [-1, 1], p^*(\theta) = aK_1\theta + bK_2 \quad (1.25)$$

with

$$K_1 = \frac{n\sqrt{(4\gamma + \lambda + n)^2 - 4n + 4} - (4\gamma + \lambda + n)(n-2)}{2(4\gamma + \lambda + 2n)} \quad (1.26)$$

$$K_2 = \frac{\lambda(n-1) + K_1(\lambda + n)}{(\lambda + n)(n-1) + K_1(\lambda + 2n)} \quad (1.27)$$

and the supply schedule has the following expression:

$$S^*(p) = \frac{1}{n} \left(p \left(\frac{1}{K_1} - 1 \right) + b \left(1 - \frac{K_2}{K_1} \right) \right) \quad (1.28)$$

Proof See Annex 1.D. ■

Proposition 1.5.2 *The slope of the supply schedule is increasing with γ and the schedule is shifted to the right of the plane (q, p) as γ increases. This is to say that the schedule*

rotates around a point in the positive quadrant of the plane.

Proof See Annex 1.E. ■

We are now going to focus on the graphical representation of these solutions. As in the monopoly case we obtain unique solutions of increasing steepness in the ramping cost parameter γ . When the ramping costs increase, it becomes more and more costly to allow for a large domain of potential quantities to be produced.

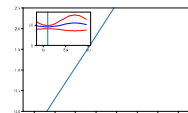
The black curve in Fig. 1.2 corresponds to the limit solution when $\gamma \rightarrow 0$, for which the problem gets closer to that of KM, i.e. no ramping costs. Note that as long as $\gamma \neq 0$ the solutions are unique. This contrasts with the case of $\gamma = 0$ which is the model presented in KM, for which there is a continuum of solutions. There is no smooth transition between our sets of solution: when considering ramping costs, there is a single Nash equilibria, even in the limit of small such costs.

Secondly, in their paper, Klemperer and Meyer show that in the limit of a diverging upper bound for their shocks, their continuum of solutions converges towards a unique solution. Our unique solution in the limit of small ramping costs is the same as that of KM in the limit of infinite support of demand shocks.

Proposition 1.5.3 *When $\gamma \rightarrow 0$, with $\gamma > 0$, the solution remains unique and converges towards the linear schedule available in KM's set of solutions, that is the same schedule selected with KM's selection rule obtained when considering an infinite support for the shocks.*

Proof It is straightforward to check that K_1 and K_2 have the same values as KM for $\gamma \rightarrow 0$.

More intuitively the argument is as follows. When $\gamma \rightarrow 0$, with $\gamma > 0$, we retain a unique solution although the problem itself converges towards that of KM. When $\gamma = 0$, we are back to the KM situation with a continuum, however we can come as close to 0 as we want



while maintaining a unique solution. We should therefore select an equilibrium present in KM's continuum. When KM take the limiting case of an infinite support of shocks, they select a unique equilibrium. In our case we can do the same thing by taking $a \rightarrow \infty$. In the limit, our solution being in their set which converges to a unique equilibrium, those two selected equilibria should be equal. Now note that our solution does not depend explicitly on a so that when the support is finite, we still select the same equilibria out of what is now a continuum of equilibria in KM's framework. ■

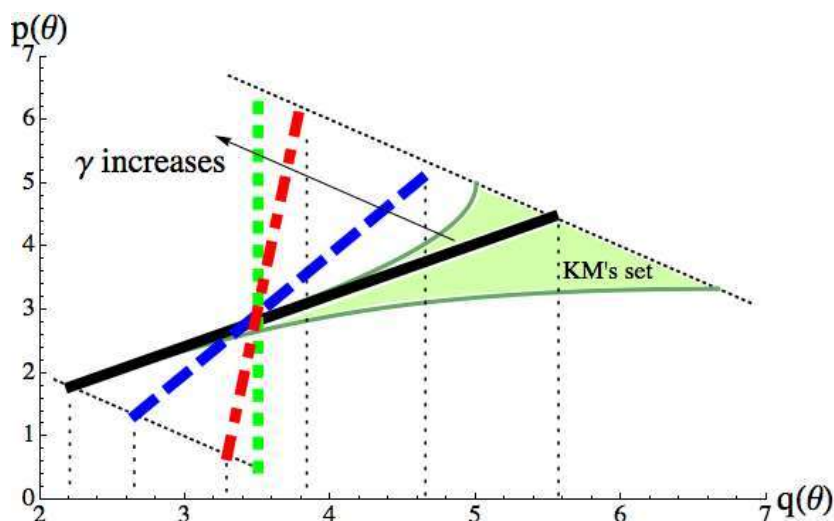


Figure 1.2: This graph plots $S^*(p)$ for different values of the ramping cost parameter, and compares them to the set of equilibria obtained in KM's framework. Four optimal supply schedules are plotted. The black curve (full line) corresponds to the case where $\gamma \rightarrow 0$. As before, as γ increases the optimal schedules get steeper and steeper until in the limit of $\gamma \rightarrow \infty$, the optimal schedule attains a vertical slope. In addition, we show the set of available equilibria in KM's model in light green, and the extremal demand schedules in dashed black.

Intuitively, as we take γ to 0 we come closer to the situation captured in KM, but as long as $\gamma > 0$, the producer still faces ramping costs, and therefore converges towards the only linear schedule available in KM's set, as shown in Fig. 1.2, in which we plot our solutions on top of KM's solution set in order to clarify the comparison.

Note that it isn't possible to transition smoothly from our model to that of KM, although they are obviously closely related. Indeed, $\forall \gamma > 0$, our model yields unique solutions, but for $\gamma = 0$ we return to KM's model for which there is a continuum of

equilibria. There is an intrinsic discontinuity between these two models, namely, the correspondence $\Gamma(\gamma)$ associating the set of equilibria to the symmetric oligopoly problem obtained for a given value of the ramping cost parameter γ is not lower hemicontinuous at $\gamma = 0$.

In addition to proposing a way to take into account dynamic technological constraints, our model provides a selection rule to choose from the continuum of equilibria described in KM's seminal work, i.e. the solutions' stability to ramping costs.

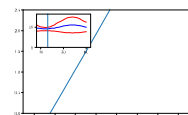
Proposition 1.5.4 *With $\gamma > 0$, there exists values of shocks for which the prices are negative. More precisely, there exist negative prices if the following condition on the parameters of the shocks holds:*

$$b \frac{K_2}{K_1} < a < b(1 - K_1)$$

Proof The method is exactly the same as that used in proving proposition 1.4.3, noting that $(K_1, K_2) \in (0, 1)^2$. ■

As in the monopoly case, we have the property that there exists situations in which our model can exhibit negative prices. This is interesting because negative prices are observed on the electricity market, and is often described as a way for non-flexible committed producers to subsidize consumption in order to avoid reducing production too much.

To sum up, we have here a model whose solutions depend on the distribution of shocks, therefore we are able to capture the interday variation of bids by assuming that the distribution of shocks varies from day to day. In this case, there exists only one symmetric equilibria each day, function of the distribution of shocks.



1.5.1 Discussion

This result sheds some light on one of the questions that the electricity market literature focuses on.

Accounting for ramping costs induces a collapse of the equilibria set from a continuum to a unique element.

Most of the tacit collusion concern that is present in the literature is based on the existence of a continuum of solutions [Bolle, 1992]. This continuum is thought as being conducive of tacit collusion because the electricity market entails repeated interactions between producers. In this case, producers can be feared to be able to learn to pick the most profitable Nash equilibria. Although a Nash equilibrium is not usually considered conducive to collusion, as each player's strategy is the best response to the other's and there is no profitable deviation, a multiplicity of Nash equilibrium lets open the possibility to pick and choose the most profitable one out of the available options, as compared to the one leading to the strongest competition.

Our result implies this pathway for tacit collusion is not available anymore. With only one Nash equilibrium at any given time no learning can bring about tacit collusion. This is a strong result about the structure of competition in our framework. The existence of ramping costs leads to a model in which no tacit collusion can exist, suggesting that the policy recommendations about such collusion stemming from the supply function equilibria literature might be strongly dependent on not taking into account ramping costs.

Our solutions are also not ex-post optimal contrary to the traditional results. As our solutions depend explicitly on the structure of the uncertainty around demand shocks, any additional information shifting the expected distribution of shocks would imply a different bid. Ex-post optimality is a very strong result, and, one could argue, more of a quirk from the usual models than its absence in ours.

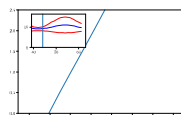
We are also able to account for negative prices which was impossible in the previous framework. Such negative prices are actually observed, although rarely, on the market: producers prefer to subsidize consumption instead of decreasing production by a lot. In our framework, if the ramping costs are large enough, and the demand shocks can reach a small enough value, our solutions can yield negative values: the equilibrium price might even be below the marginal cost of production, understood here as $\partial_q C$ which by definition does not capture our ramping costs.

In the next section we are going to present how to capture richer dynamics, and especially how the surface of bids should evolve with time when the producers have information about the anticipated variation of shocks during the day.

1.6 Dynamic Behavior of the Bids

The classical supply function equilibria models, as described before, yield a continuum of Nash equilibria, and each one of those equilibria is ex-post optimal. This a very strong result that we are going to take some time to describe and comment.

Consider for a moment that firms competing in supply schedules reach one of the many possible Nash equilibria under such a setup, and that they commit to their schedules. Now consider that the firms face a succession of demand shocks, and that this yields a succession of market outcomes. As the Nash equilibria are ex-post optimal, it means that given the strategies played by the other firms, no firm has any regrets concerning its strategy. Knowing about the realized demand shocks does not imply any willingness to change strategy as long as other firms keep their strategies fixed, and as long as the support of shocks is not reduced to a point (one could think of observed realizations of shocks as helping to narrow down the expected range of shocks without implying a pinpoint accuracy).



A corollary to this observation is that the distribution of anticipated shocks does not play any role in KM's paper, apart from its bounds. Knowing that the demand shocks are going to be drawn from distributions of high or low values does not affect the willingness to play a given strategy, as long as the support does not evolve. The little role that is played by information about shocks in KM's paper is even more counter-intuitive: to a certain extent, information about demand shocks gives rise to a larger continuum of solutions. Indeed, if one compares the equilibria available to firms for a given support $\{\theta\}_1 = [\underline{\theta}_1, \bar{\theta}_1]$, noted S^*_1 , to those obtained for a support strictly included in the first one $\{\theta\}_2 = [\underline{\theta}_2, \bar{\theta}_2] \subset \{\theta\}_1$, noted S^*_2 , then the set of equilibria will be larger in the second case, in the sense that $S^*_1 \upharpoonright_{\{\theta\}_2} \subset S^*_2$ (where $\upharpoonright_{\{\theta\}_2}$ denotes that the supply functions are restricted to values over $\{\theta\}_2$).

However, actual firms bidding on the electricity markets are known to be actively engaged in forecasting the future demand levels in order to build their strategies. Bids that we can observe on the electricity markets change from hour to hour even when demand does not vary enough to warrant a change of online plants, a consideration that could explain some of the supply schedules variations.

The general interpretation of KM's paper when applied to electricity markets is that for some unknown underlying process, strategies converge towards different equilibria of the set of available equilibria from hour to hour. One can note that the general intuition for strategies converging towards Nash equilibria in the first place is through either a high degree of sophistication on the part of firms, or through a more organic learning process. Neither of these two explanations can account for frequent switching from one Nash equilibrium to another, out of a myriad of available options, without considering some communication among firms. Furthermore, if such communication existed, it should be expected to yield the most profitable equilibria out of the available lot.

We think that this strand of argument trying to explain bids' dynamics in the light of the supply function equilibria framework is unsatisfying and we argue that forecasting demand becomes important for firms when one considers dynamic effects, that is effects that are history dependent, of which ramping costs which we model in this paper are an instance (one can think of start-up and shut-down costs as another instance of such dynamic effects).

The model described in the previous section doesn't account for these hourly dynamics. Here we present a way to capture these intraday variations, by considering bids that depend continuously on the time t . We will show that our results imply that firms are not oblivious to information about the distribution of shocks anymore, and more than that, that their strategies directly evolve with the evolution of their knowledge about uncertain future shocks.

1.6.1 The setup

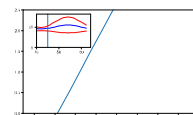
Previously, the SDE (stochastic differential equation) defining the dynamics of the problem was written as:

$$d\theta(t) = -2\theta(t)dt + \sqrt{1 - \theta(t)^2}dB_t$$

This specification implied a stochastic trajectory for the shocks, bounded by a constant envelope. That is to mean that, lacking any knowledge of the value of the shock at a point in time close to the period under consideration, the distribution of shocks does not depend on time.

To account for these intraday variations, we are going to define a richer SDE, a non-stationary one.

SDEs have been well studied and as a consequence there exists a number of families of SDEs satisfying numerous characteristics [Hertzer, 2003]. The goal here is to find one



SDE that will allow us to capture some of the dynamics of shocks and how this might influence strategies, while keeping it as simple as possible. Just as in the previous section, the first characteristic that we want is to consider SDEs that imply a bounded support of shocks. This restricts our possible choice to four families out of the classical ones: Generalised Beta I, Beta, Power, Uniform. We also consider that a desirable property is that the distribution reaches 0 continuously at the bounds of its support, because there is no boundary condition on the demand for electricity that would justify that one has a positive probability of reaching a given bound, but a zero probability of reaching an infinitesimally close value to this bound. This restricts us further to only two families: Generalised Beta I and Beta. For tractability reasons we will focus here on the Beta family of SDEs, and more precisely on one of the simplest Beta SDE. However, we want to note that this choice stems from our focus towards solving analytically the problem at hand and obtain closed form solutions. If one were to try and estimate the distribution of shocks anticipated by firms from market data one might want to try and find which of the Beta or Generalised Beta I SDEs might match the distribution of errors between the published day -1 estimates for demand and the observed quantities.

Define the evolving envelope of shocks by two functions, $(\underline{\theta}(t), \bar{\theta}(t))$, respectively the lower and upper bounds of the shocks. These two functions, although very easy to comprehend, are not the most useful way to define the boundary. Instead, we are going to use the average value of the shocks, and the half width of the envelope, $(\hat{\theta}(t), \omega(t))$. This means that $\underline{\theta}(t) = \hat{\theta}(t) - \omega(t)$ and $\bar{\theta}(t) = \hat{\theta}(t) + \omega(t)$. The only restriction we impose on the envelope is that we require it to be continuously differentiable, that is $(\hat{\theta}, \omega) \in \mathcal{C}^1(\mathbb{R})$.

Consider the following SDE which is the simplest Beta SDE that we can pick that still allows us to have a free choice of the bounds of shocks. We want the simplest possible form to make it possible to obtain closed form solutions, yet still account for free dynamics of the bounds. For readability, we drop the explicit dependency of the different functions

on time, that is $\theta(t)$, $\hat{\theta}(t)$ and $\omega(t)$ will be noted θ , $\hat{\theta}$ and ω :

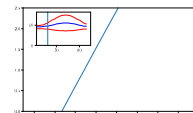
$$d\theta = \left[(\hat{\theta} - \omega - \theta) + \left(1 + \frac{1}{\omega} \frac{d\omega}{dt}\right) (\hat{\theta} + \omega - \theta) + \left(\frac{d\hat{\theta}}{dt} - \frac{d\omega}{dt}\right) \right] \cdot dt + \sqrt{\left(1 + \frac{1}{\omega} \frac{d\omega}{dt}\right) (\theta - \hat{\theta} + \omega)(\hat{\theta} + \omega - \theta)} \cdot dB_t \quad (1.29)$$

The distribution of the shocks can be obtained through Fokker-Planck's equation 1.5 and we obtain:

$$f(\theta, t) = \frac{3}{4\omega(t)^3} (\theta(t) - \hat{\theta}(t) + \omega(t)) (\hat{\theta}(t) + \omega(t) - \theta(t)) \quad (1.30)$$

In the following analysis, we are going to rely on the fact that the term $\left(1 + \frac{1}{\omega} \frac{d\omega}{dt}\right) > 0$. The justification for this inequality comes from the following remark: if one were to rescale time in the above equations, there wouldn't be any explicit change in the equilibrium distribution 1.30. The only effect that such a rescaling would play is in the variance of the Brownian term. In order to insure that our inequality is correct, one has to make sure that the variation of the envelope term occurs on longer timescales than the characteristic timescale of fluctuations in our problem, that is the timescale that fixes the rate at which information leaks out of the knowledge of the value of one shock at a given point in time. We are trying to capture the hourly changes in firms strategies when demand fluctuates at higher frequencies (think of the collection of individuals that choose to switch lights on or off at any given point in time in an entire country for instance). We therefore consider that this assumption is sound in this situation.

More formally, one can define τ a rescaling parameter allowing to change the rate at which the brownian process blurs information pertaining to an initial condition. We rescale time using this parameter, so that time t and the rescaled time t_r verify $t_r = \tau t$.



We can rewrite the above equations as:

$$d\theta = \left[(\hat{\theta} - \omega - \theta) + \left(1 + \frac{\tau}{\omega} \frac{d\omega}{dt_r} \right) (\hat{\theta} + \omega - \theta) + \tau \left(\frac{d\hat{\theta}}{dt_r} - \frac{d\omega}{dt_r} \right) \right] \cdot dt_r + \sqrt{\left(1 + \frac{\tau}{\omega} \frac{d\omega}{dt_r} \right) (\theta - \hat{\theta} + \omega)(\hat{\theta} + \omega - \theta)} \cdot dB_{t_r} \quad (1.31)$$

and

$$f(\theta, t_r) = \frac{3}{4\omega(t_r)^3} (\theta(t_r) - \hat{\theta}(t_r) + \omega(t_r)) (\hat{\theta}(t_r) + \omega(t_r) - \theta(t_r)) \quad (1.32)$$

By assumption, τ is small enough for the loss of information due to the stochastic nature of the process to be faster than the typical timescale of variation of strategies, therefore by hypothesis $\left(1 + \frac{\tau}{\omega} \frac{d\omega}{dt_r} \right) > 0$ is valid. We will drop this rescaled time index in the following sections as equations 1.30 and 1.32 are equal, it was just a temporary definition to justify the sign of the term that depends on the time derivative of the envelope. We will keep this τ parameter explicit however, in order to allow discussions differentiating effects related to the speed of variation of the envelope or to the relative timescales of this variation and the underlying stochastic process.

1.6.2 Results

Dynamics in the case of the Monopoly and of the oligopoly

We start by describing the dynamics of the monopoly case because the oligopoly case is not richer dynamically, but it is more complex to describe.

Our stochastic maximization program can thus be rewritten as a regular optimal control problem as in section 1.4, but taking into account the time dependency:

$$\max_{S_i(p,t)} \int_0^T \int_{\underline{\theta}(t)}^{\bar{\theta}(t)} f(\theta, t) \left(p(\theta, t) S_i(p(\theta, t), t) - C_s(S_i(p(\theta, t), t)) - \frac{\gamma}{2} \sigma(\theta, t)^2 (S'_i(p(\theta, t), t) \dot{p}(\theta, t))^2 \right) d\theta dt \quad (1.33)$$

$$\begin{aligned}
 s.t. \quad & S'_i(\cdot) \geq 0 \\
 & \dot{p} \geq 0
 \end{aligned} \tag{1.34}$$

$$D(\cdot, \cdot) \geq 0 \tag{1.35}$$

Proposition 1.6.1 *In the case of an envelope evolving with time, that is shocks belonging to the bounded support $[\hat{\theta}(t) - \omega(t), \hat{\theta}(t) + \omega(t)]$, there exists a unique optimal solution to the monopoly problem. It can be expressed as a surface in the price-quantity-time space:*

$$p^*(\theta(t), t) = \frac{4\gamma \left(1 + \frac{\tau}{\omega} \frac{d\omega}{dt}(t)\right) + 1 + \lambda}{4\gamma \left(1 + \frac{\tau}{\omega} \frac{d\omega}{dt}(t)\right) + 2 + \lambda} \cdot \theta(t) - \frac{1 + \lambda}{2 + \lambda} \cdot \hat{\theta}(t) \tag{1.36}$$

The corresponding optimal supply schedule writes as:

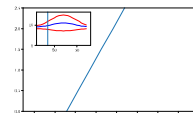
$$S^*(p, t) = \frac{1}{4\gamma \left(1 + \frac{\tau}{\omega} \frac{d\omega}{dt}(t)\right) + 1 + \lambda} \left(p(t) + \frac{4\gamma \left(1 + \frac{\tau}{\omega} \frac{d\omega}{dt}(t)\right)}{2 + \lambda} \cdot \hat{\theta}(t) \right) \tag{1.37}$$

$$\forall p(t) \in [p(\hat{\theta}(t) - \omega(t), t), p(\hat{\theta}(t) + \omega(t), t)]$$

Proof See Annex 1.G. ■

Note that if $\frac{d\omega}{dt} = 0$ equations 1.36 and 1.37 are equal to equations 1.18 and 1.19 respectively as expected. Note also that the solution is exactly the same as in the static monopoly case in which one replaces the ramping cost parameter γ by $\gamma \left(1 + \frac{\tau}{\omega} \frac{d\omega}{dt}(t)\right)$. This surprising fact, that our dynamic optimal strategy is simply a naive version of the static one with a specified dynamic stochastic process, can be understood as a consequence of the assumptions we have had to make in section 1.3.2.

That is because in section 1.3.2, in Annex. 1.A in which we develop the argument in more detail, and in section 1.6.1 we end up in effect making a scale separation argument: the ramping costs are completely driven by the very short-term fluctuations, whereas the evolution of these ramping costs is driven by the longer timescale at which our information about the demand shocks evolves over time. This means that we make



a version of what physicists call a quasi-static argument: because of this time-scale separation between what drives our ramping cost and our information about the shocks, we can effectively reason in two steps, first solving for the static situation, and then injecting naively the slow changes in the static results with confidence as to the validity of this approximation as long as the assumption about this separation of scale is verified.

The consequence of this is that we have a dynamic version of our static oligopoly of the same nature as for the monopoly above.

Proposition 1.6.2 *The solution exists, is unique, and has the following form:*

$$\forall \theta \in [-1, 1], p^*(\theta) = aK_1(t)\theta + bK_2(t) \quad (1.38)$$

with

$$K_1(t) = \frac{n\sqrt{(4\gamma(1 + \frac{\tau}{\omega}\frac{d\omega}{dt}) + \lambda + n)^2 - 4n + 4 - (4\gamma(1 + \frac{\tau}{\omega}\frac{d\omega}{dt}) + \lambda + n)(n - 2)}}{2(4\gamma(1 + \frac{\tau}{\omega}\frac{d\omega}{dt}) + \lambda + 2n)} \quad (1.39)$$

$$K_2(t) = \frac{\lambda(n - 1) + K_1(t)(\lambda + n)}{(\lambda + n)(n - 1) + K_1(t)(\lambda + 2n)} \quad (1.40)$$

and the supply schedule has the following expression:

$$S^*(p, t) = \frac{1}{n} \left(p \left(\frac{1}{K_1(t)} - 1 \right) + \hat{\theta} \left(1 - \frac{K_2(t)}{K_1(t)} \right) \right) \quad (1.41)$$

Proof See Annex 1.H. ■

Proposition 1.6.3 *The slope of the supply schedule is increasing with $\frac{d\omega}{dt}$ and the schedule is shifted to the right of the plane (q, p) as $\frac{d\omega}{dt}$ increases. This is to say that the schedule rotates around a point in the positive quadrant of the plane when the uncertainty increases over time.*

Proof See Annex 1.F. ■

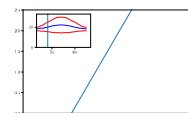
We remind the reader that at the bottom of all right pages, there is a small graph showcasing the optimal strategy for a given set of parameters of the model, in the plane quantity-price, with an insert illustrating the evolution of the support of demand shocks over time. The vertical line in the insert represents the point in time for which the strategy is presented in the actual graph. This forms a flipbook, which allows the reader to get a feeling for the evolution of strategies with the evolution of the support of shocks by flipping rapidly the pages and observing the graph becomes animated, just like a cartoon. This serves only an illustrative purpose.

1.6.3 Discussion

In both situations, the optimal supply schedule is shifted uniformly following the expected shock $\hat{\theta}(t)$, which is a rather intuitive result: if on average demand shifts upwards, the producers want to extract more profit and shift their supply curve accordingly, but there is no reason to change slope.

What is less trivial is the way the slope behaves. Let us focus on the monopoly result for a start. The slope is affected as if the ramping cost parameter was fluctuating with the relative change in the width of the bounds of the shocks (term in $\frac{1}{\omega} \frac{d\omega}{dt}$). The transition between a low uncertainty region to a higher uncertainty one behaves as if during the transient regime the ramping cost parameter had a higher value, implying a higher slope.

The optimal supply schedule depends on the relative rate of change of the width $\frac{1}{\omega} \frac{d\omega}{dt}$ and on the average shock $\hat{\theta}$. More precisely, with a constant width, the optimal supply schedule varies according to variations in the expected average value of the shocks. This is quite standard, if demand is higher, the price and quantities both increase, and here this increase occurs with a constant slope. The behavior of the supply schedule when the width varies is less trivial.



Remember that when describing the slope of the schedule, we are considering the plane (*quantity, price*) while the schedule as defined by $S^*(p)$ represents the same curve but in the plane (*price, quantity*). An increase in width is equivalent to a higher ramping cost parameter while a decrease in width is equivalent to a lower ramping cost parameter. These results are illustrated in Fig. 1.3.

To understand the economic intuition behind this result, consider first an increase in the width of the envelope on the graphic on the right, the uncertainty level given by the orange line. At this point in time, the uncertainty is increasing, therefore the shocks are going to be larger, the variations in demand too, and to face this increase, the slope is larger so as to reduce this expected increase in ramping costs. In the case of the green line, the uncertainty is decreasing, the ramping costs incurred are expected to decrease, thus this constraint being relaxed the slope can reduce to extract more profit through more variations in quantity than if the slope had remained high.

One can contrast this behaviour to the one described on the left hand side of the figure, where the uncertainty is constant, but the average shock is not. This implies only a vertical translation of the curve, without changes in the slope. Here the slope is fixed by a given level of uncertainty, to exploit an increased demand, the supplier simply increases its prices, but it doesn't need to hedge against increased variation by encouraging or discouraging variations in quantity by playing with the slope.

All of this reasoning applies to the dynamic oligopoly result as well as the effect can be understood in the same way as for the monopoly: changes in the width of the shock's bounds behave as if there was an effective dynamic cost that was higher than the baseline when information about the shocks is lost, and lower than the baseline when information is gained.

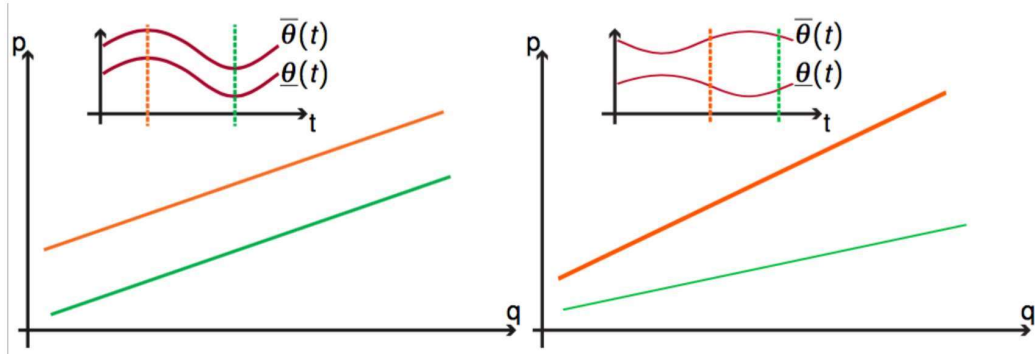


Figure 1.3: On the left, this graph plots an envelope of constant width $\omega(t)$ but varying average. The insert represents this envelope, while the graph itself represents the supply curves associated with the points in time represented by the green and orange dotted lines on the insert. On the right, this graph plots an envelope of constant average but varying width $\omega(t)$.

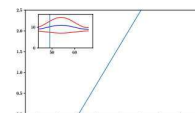
1.7 Limits

This section aims at discussing whether or not one can consider that the mapping of these results on the real world is a set of non-zero measure, to put it bluntly.

Further avenues of research would be to generalize our results to larger classes of demand functions. One could also solve the static case for different SDE's in order to test the sensitivity of our results on the underlying "mechanics" of the stochastic process. This has also been pursued without conclusive results: solving the optimization problem becomes quickly extremely difficult, as the second order differential equations exhibit poles, and divergences are difficult to cope with in optimization problems.

The nature of these avenues of research is testament to the fact that our results are obtained for a very narrow setting, one chosen to obtain closed form formulas. However, although a healthy dose a skepticism as to the applicability of the closed form formulas is therefore warranted, I would like to argue that the results hint towards at least one more general takeaway message, namely the collapse of the set of equilibria.

This result stems from the nature of the mathematical problem and not from the way we set up the problem in order to maximize our chances of closed-form success per se.



Therefore, I think it hints towards possible more general results. The problem is the complexity of the maths as soon as one deviates from the simplest version of the problem presented here.

The question then becomes one of the method to employ to obtain those results. There is one tool that might prove useful: numerical simulations. One can solve the differential equations involved here numerically, check ex-post whether they satisfy the other conditions, and in so doing provide boundaries around possible solutions. If the unicity is a characteristic that is indeed more general than our model here, there is 0 probability of finding such a solution by the method proposed, quite literally. However providing such bounds, although not demonstrating the existence of a solution, could provide circumstantial evidence towards such a result.

More generally, I think numerical methods as a guide for theoretical results have not been exploited to the fullest of their potential.

1.8 Concluding Remarks

In this chapter we have introduced a supply function equilibria model of ramping costs under uncertainty.

By introducing technological constraints previously neglected we are able to take into account the effects of the dynamics of demand shocks on the supply function framework. We restrict ourselves to linear demand. The optimal supply schedules obtained are unique. This is a striking result when compared to traditional multiplicity of equilibria. Although we do not solve the model in the case of a general demand function, we think that our results make a strong case for the reduction of the set of equilibria, in our case to a unique equilibrium, when taking into account dynamic effects, that is strategies that are history dependent.

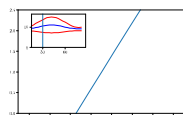
We introduce a mathematical toolbox that was absent from this literature in the past, and notably classes of stochastic differential equations that can be used to pick and choose processes yielding specific closed form distributions of probability of shocks at equilibrium.

Our methodology further introduces the notion of time-scale separation to our problem, which allows to transcribe quite simply static solutions to the case of dynamic envelopes of shocks, as long as the static case is solved for the same functional form of stochastic processes. In our case we focus our study to quadratic distributions, which we then extend to cope with any functional form for the time dependency of the envelope of shocks.

Our results are congruent with the economic intuition one can have about ramping costs: when they increase, the slope of the supply schedule increases in order to reduce the range of variation in production for a given range of variation of demand shocks.

Although mathematically more demanding than the traditional model by Klemperer and Meyer, we consider that this new model, while conceptually sparing (we only add ramping costs) allows for a richer, more realistic description of the electricity market, and opens new research avenues. It yields precise and testable predictions on the dynamics of the electricity market with tractable functional forms, at least in the linear demand case.

In addition, by explicitly modeling the dynamics, our work opens the possibility to explore interactions between intraday and day-ahead markets, markets that were indistinguishable in the previous framework, which explains why the analysis of the interaction between different ways to trade electricity focused on day-ahead markets and forward contracts: if solutions are ex-post optimal, there is no need to create a second type of spot market with a shorter time horizon, the bids of the previous day should suffice.



Further avenues of research would be to generalize our results to larger classes of demand functions. One could also solve the static case for different SDE's in order to test the sensitivity of our results on the underlying "mechanics" of the stochastic process. This has also been pursued without conclusive results: solving the optimization problem becomes quickly extremely difficult, as the second order differential equations exhibit poles, and divergences are difficult to cope with in optimization problems.

Finally, and more generally, we think that this concept of ramping costs, the fact that change is costly, is ubiquitous and could fuel interesting research into the dynamics of a large range of markets. Such avenues have been pursued in the case of stochastic optimal control, that is, instantaneous reactions to stochastic shocks. Here we are describing a market on which agents are forced to optimize in advance, so that they have to react to continuous changes in the anticipated shocks, but not the shocks themselves, which can be understood as stochastic optimization with periodic commitment.

The goal of the second and third chapter is to test these theoretical predictions on French day-ahead data. The next chapter focuses on building methods to be able to perform such tests in the third chapter. As the actual supply schedules are not linear, we need to be able to define their local slopes and to define points that can be compared to one another across schedules. We also describe how we build the different controls that need to come into play to estimate properly the impact of uncertainty on the slope, but more importantly we introduce one class of uncertainty estimates associated with weather. With these elements in place, we have a way to compare schedules to one another, and to estimate uncertainty for a given schedule, and therefore look into the effect of uncertainty in the third chapter.

Appendix

Appendix 1.A Proof of Equation 1.11

We are here going to detail how we obtain the result in equation 1.11 on which the proofs of our dynamic results rely heavily. Recall that we are computing the continuous time limit of our ramping cost term which can be quite simply defined in the case of discrete dynamics but for which one has to work a bit more in order to cope with the non differentiable nature of stochastic processes.

We are therefore going to first consider the discrete case of a random walk of timestep Δt which converges towards the Itô process 1.4, using the Euler-Maruyama approximation, a generalization of the Euler method to stochastic differential equations. We consider a Markov chain Y defined as follows:

$$\Delta Y_n = Y_{n+1} - Y_n = \mu(Y_n, n\Delta t)\Delta t + \sigma(Y_n, n\Delta t)\Delta B_n \quad (1.A.1)$$

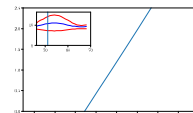
We want to derive the following:

$$\lim_{\Delta t \rightarrow 0} \mathbb{E} \left[\frac{\Gamma(\Delta t)}{2} \left(\frac{\Delta S_i(p(\theta(t), t), t)}{\Delta t} \right)^2 \middle| \theta(t) \right] = \frac{\gamma}{2} (\partial_1 S(p(\theta(t), t), t) \partial_1 p(\theta(t), t))^2 \sigma(\theta, t)^2 \quad (1.A.2)$$

Let us first compute the first order expansion of $\Delta S_i(p(Y_n, n\Delta t), n\Delta t)$, by assuming that both S_i and p are continuously differentiable with respect to their arguments:

$$\Delta S_i(p(Y_n, n\Delta t), n\Delta t) = \frac{\Delta S_i}{\Delta p} \left(\frac{\Delta p}{\Delta Y} \frac{\Delta Y}{\Delta t} \Delta t + \frac{\Delta p}{\Delta t} \Delta t \right) + \frac{\Delta S_i}{\Delta t} \Delta t + \mathcal{O}(\Delta t^2) \quad (1.A.3)$$

Using our differentiability assumption, note that the terms that do not depend on ΔY scale with Δt , and that the term depending on ΔY cannot be grouped in the same way,



due to its stochastic nature, therefore:

$$\frac{\Delta S_i(p(Y_n, n\Delta t), n\Delta t)}{\Delta t} = \frac{\Delta S_i}{\Delta p} \frac{\Delta p}{\Delta Y} \frac{\Delta Y}{\Delta t} + \mathcal{O}(1) \quad (1.A.4)$$

$$\left(\frac{\Delta S_i(p(Y_n, n\Delta t), n\Delta t)}{\Delta t} \right)^2 = \left(\frac{\Delta S_i}{\Delta p} \frac{\Delta p}{\Delta Y} \frac{\Delta Y}{\Delta t} \right)^2 + C \cdot \frac{\Delta S_i}{\Delta p} \frac{\Delta p}{\Delta Y} \frac{\Delta Y}{\Delta t} + \mathcal{O}(1) \quad (1.A.5)$$

with C a term that does not depend on ΔY or Δt .

Now by considering the specification of our stochastic process we know that $\mathbb{E} \left[\frac{\Delta Y}{\Delta t} | Y_n \right] = \mu(Y_n, n\Delta t)$ and that $\mathbb{E} \left[\frac{\Delta Y^2}{\Delta t} | Y_n \right] = \mu(Y_n, n\Delta t)^2 + \frac{\sigma(Y_n, n\Delta t)^2}{\Delta t}$. Using the fact that $\Gamma(\Delta t) = \gamma\Delta t + o(\Delta t)$ we obtain the result of equation 1.11.

Appendix 1.B Proof of Proposition 1.4.1

Define the following Hamiltonian:

$$H(p(\theta), \dot{p}(\theta), \mu(\theta), \theta) = f(\theta) \left(p(\theta)(a\theta + b - p(\theta)) - \frac{\lambda}{2}(a\theta + b - p(\theta))^2 - \frac{\gamma}{2}(1 - \theta^2)(a - u(\theta))^2 \right) + \mu(\theta)u(\theta) \quad (1.B.1)$$

where $u(\theta)$ is the control variable defined through the following equation of motion: $u(\theta) = \dot{p}(\theta)$, $u(\theta) \in [0, a]$. We do not consider the non-negative demand constraint and will check ex-post that our solution verifies this condition.

Now note that:

$$\forall \theta \in (-1, 1), \quad \frac{\partial^2 H}{\partial p^2} = -(2 + \lambda)f(\theta) < 0 \quad (1.B.2)$$

$$\frac{\partial^2 H}{\partial u^2} = -\gamma(1 - \theta^2)f(\theta) < 0 \quad (1.B.3)$$

The Hamiltonian is therefore strictly concave in $p(\theta)$ and $u(\theta)$. Let $(p^*(\theta), u^*(\theta))$ be an admissible pair to the problem, that is a pair such that $u^*(\theta) = \dot{p}^*(\theta)$. If there exists a

continuous and piecewise continuously differentiable function $\mu(\theta)$ such that:

$$\dot{\mu}(\theta) = -\frac{\partial H^*}{\partial p} \tag{1.B.4}$$

$$\mu(-1) = \mu(1) = 0 \quad \text{in order for prices to be free at the boundaries} \tag{1.B.5}$$

$$\forall(\theta, u) \in [-1, 1] \times [0, a], \quad \frac{\partial H^*}{\partial u} (u^*(\theta) - u) \geq 0 \tag{1.B.6}$$

with $\frac{\partial H^*}{\partial u} = \frac{\partial H}{\partial u}(p^*(\theta), u^*(\theta), \mu(\theta), \theta)$, then the Mangasarian sufficiency theorem ensures that $(p^*(\theta), u^*(\theta))$ is the optimal solution [Seierstad and Sydsaeter, 1987, p.105]. Let us check that eq. 1.18 defines the optimal solution.

Equation 1.B.4 defines $\mu(\theta)$ up to a constant. Through direct integration we obtain:

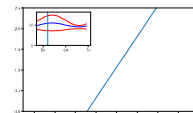
$$\mu(\theta) = 3a \left((2 + \lambda) \frac{4\gamma + 1 + \lambda}{4\gamma + 2 + \lambda} - 1 - \lambda \right) (2\theta^2 - \theta^4) + const.$$

This expression is symmetric in θ therefore by choosing the adequate value for the constant, we ensure that eq. 1.B.5 is satisfied. The slope of the proposed p^* is in $[0, a]$ therefore eq. 1.B.6 requires $\frac{\partial H}{\partial u}$ to be null.

$$\begin{aligned} \forall \theta \in [-1, 1], \quad \frac{\partial H}{\partial u} = 0 &\implies \frac{d}{d\theta} \frac{\partial H}{\partial u} = 0 \\ \text{i.e. } \dot{u}(\theta) &= -\frac{4\theta}{1 - \theta^2} (a - u(\theta)) - \frac{(1 + \lambda)(a\theta + b)}{\gamma(1 - \theta^2)} + \frac{(2 + \lambda)p(\theta)}{\gamma(1 - \theta^2)} \end{aligned} \tag{1.B.7}$$

It is straightforward to see that the proposed solution satisfies this differential equation, thus we know that $\frac{\partial H}{\partial u}$ is a constant and as $\mu(-1) = 0$ it is in fact null. Lastly, we see that $p^*(\theta) \leq a\theta + b$.

The proposed $p^*(\theta)$ therefore defines the unique optimal supply function, i.e. the parametrized curve $(a\theta + b - p^*(\theta), p^*(\theta))$.



Appendix 1.C Proof of Proposition 1.4.2

Consider the following profit:

$$p \cdot D(\theta, p) - C(D(\theta, p))$$

Now take a linear demand schedule $D(\theta, p) = a \cdot \theta + b - p$ and a quadratic cost function $C(D(\theta, p)) = \frac{\lambda}{2} D(\theta, p)^2 = \frac{\lambda}{2} (a \cdot \theta + b - p)^2$. The F.O.C. with respect to $p(\theta)$ writes:

$$D(\theta, p) + p \cdot \partial_p D(\theta, p) - C'(D(\theta, p)) \partial_p D(\theta, p) = 0 \quad (1.C.1)$$

$$C'(D(\theta, p)) - \frac{D(\theta, p)}{\partial_p D(\theta, p)} = p \quad (1.C.2)$$

$$(1 + \lambda)(a \cdot \theta + b - p) = p \quad (1.C.3)$$

$$p = \frac{1 + \lambda}{2 + \lambda} (a \cdot \theta + b) \quad (1.C.4)$$

$$(1.C.5)$$

This result is the same as that of proposition 1.4.1 with $\gamma \rightarrow 0$.

Appendix 1.D Proof of Proposition 1.5.1

As for eq. 1.24, for the sake of concision, we do not write the explicit dependencies of the different functions on θ , thus $f(\theta)$, $p(\theta)$, $u(\theta)$, $\mu(\theta)$ and $S(p(\theta))$ will be written as f , p , u , μ and S respectively. Define the following Hamiltonian:

$$H(p, u, \mu, \theta) = f \left(p(a\theta + b - p - (n-1)S) - \frac{\lambda}{2} (a\theta + b - p - (n-1)S)^2 - \frac{\gamma}{2} (1 - \theta^2) (a - u(1 + (n-1)S'))^2 \right) + \mu u \quad (1.D.1)$$

where u is the control variable defined through the following equation of motion: $u = \dot{p}$, $u \in [0, a]$. We do not consider the non-negative demand constraint and will check ex-post that our solution verifies this condition.

If a symmetric equilibria exists, eqs. 1.20 through 1.23 imply that the regular conditions for an admissible pair to be optimal write:

$$u = \dot{p} \in [0, a] \tag{1.D.2}$$

$$\partial_u H < 0 \implies u = 0 \tag{1.D.3}$$

$$\partial_u H > 0 \implies u = a \tag{1.D.4}$$

$$\begin{aligned} \partial_u H = 0 &\implies u \in [0, a] \text{ and} \\ \ddot{p} &= -\frac{4\theta(a - \dot{p})}{1 - \theta^2} - \frac{\lambda(a\theta + b - p)}{\gamma(1 - \theta^2)} - n \frac{\dot{p}(a\theta + b - 2p) - a(n - 1)p}{\gamma(1 - \theta^2)(a(n - 1) + \dot{p})} \end{aligned} \tag{1.D.5}$$

$$\dot{\mu} = -\partial_p H \tag{1.D.6}$$

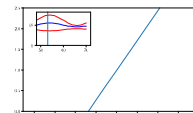
$$\mu(-1) = \mu(1) = 0 \tag{1.D.7}$$

It is easy to check that $(K_1, K_2) \in (0, 1)$ and that the solution 1.25 solves eq. 1.D.5 subject to the boundary conditions 1.D.7. The supply schedule is therefore also linear, with equation:

$$S(p) = \frac{1}{n} \left(p \left(\frac{1}{K_1} - 1 \right) + b \left(1 - \frac{K_2}{K_1} \right) \right) \tag{1.D.8}$$

We can now use the Mangasarian theorem to obtain that our admissible pair is indeed solution, $H(p, u, \mu, \theta)$ being concave in (p, u) for linear supply schedules. However the Mangasarian cannot yield that this solution is unique because for a symmetric equilibria, if supply schedules are modified, the hamiltonian changes alongside and we are faced with a new maximization program.

To obtain that the solution is unique we are going to show explicitly that no other candidate solution exists.



First, note that:

$$\begin{aligned} \dot{\mu} = & -f \left(\frac{a\theta + b - 2p}{n} - a \frac{(n-1)p}{n\dot{p}} \right. \\ & \left. + \lambda \frac{a\theta + b - p}{n} \cdot \frac{a(n-1) + \dot{p}}{n\dot{p}} - \gamma(1-\theta^2)(n-1) \frac{a - \dot{p}}{n} \cdot \frac{a\ddot{p}}{n\dot{p}^2} \right) \end{aligned} \quad (1.D.9)$$

If (p^*, u^*) maximises the program then the maximum principle implies that there exists a continuous and piecewise continuously differentiable function μ , as shown in [Seierstad and Sydsaeter, 1997, Theorem 2 p.85]. This combined with the above equation implies that $\dot{p} \neq 0$ a.e.

Assume now a solution of the form $\forall \theta \in [-1, 1]$, $p = a\theta + \beta$, by injecting this expression in eq. 1.D.9 there is no β such that the boundary conditions 1.D.7 are verified.

In addition:

$$\forall \theta \in (-1, 1), \quad \frac{\partial^2 H}{\partial u^2} = -f\gamma(1-\theta^2)(1+(n-1)S')^2 < 0 \quad (1.D.10)$$

The Hamiltonian is therefore strictly concave in u and $[0, a]$ is convex. These two properties yield that u^* is continuous, as shown in [Seierstad and Sydsaeter, 1987, Note 2.b. p.86]. We have proved the following result:

Lemma 1.D.1 *For any symmetric equilibrium $\exists A \subseteq [-1, 1]$ s.t. A is the union of segments of $[-1, 1]$ and $\forall \theta \in A$, $\partial_u H = 0$*

Assume the following hypothesis is true, $H_1 : \exists \theta_c \in (-1, 1)$ s.t. $[-1, \theta_c] \subseteq A$, then knowing that $\dot{p} \in \mathcal{C}^0([-1, 1], [0, a])$ we can rewrite differential equation 1.D.5 around the value $\theta = -1$ by defining $\theta = -1 + \epsilon$ with $\epsilon = o(1)$:

$$\frac{d^2 p}{d\epsilon^2} = \frac{C}{\epsilon} + o(1) \text{ with } C \neq 0 \text{ if } p(\theta) \neq aK_1\theta + bK_2 \quad (1.D.11)$$

This means that locally around -1 , any solution to eq. 1.D.5 but solution 1.25 diverges. Hypothesis H_1 is therefore wrong and $\exists \theta_c \in (-1, 1)$ s.t. $\forall \theta \in [-1, \theta_c]$, $\exists \beta$ s.t. $p(\theta) = a\theta + \beta$.

At θ_c we have $\partial_u H = 0$ and as \dot{p} is continuous, $\dot{p}(\theta_c) = a$. For the solution to be interior we need $\ddot{p}(\theta_c) \leq 0$.

$$\partial_{\dot{p}} H(p, \dot{p}, \mu, \theta_c) = 0 \Leftrightarrow \mu(\theta_c) = 0 \quad (1.D.12)$$

$$\ddot{p}(\theta_c) \leq 0 \Leftrightarrow b(1 + \lambda) - \beta(n + 1 + \lambda) \geq na\theta \quad (1.D.13)$$

Straightforward computations show that both conditions are mutually exclusive, therefore there doesn't exist another candidate symmetric equilibria, and our solution is unique.

Lastly, to compute the optimal supply function, we inverse the optimal price in order to get the shock as a function of the price at the equilibrium, and we inject this expression in Eq. 1.21.

Appendix 1.E Proof of proposition 1.5.2

We want to prove that the slope of the supply schedules increases as the ramping cost parameter increases. As a reminder:

$$K_1 = \frac{n\sqrt{(4\gamma + \lambda + n)^2 - 4n + 4} - (4\gamma + \lambda + n)(n - 2)}{2(4\gamma + \lambda + 2n)} \quad (1.E.1)$$

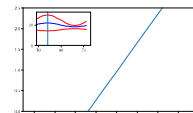
$$K_2 = \frac{\lambda(n - 1) + K_1(\lambda + n)}{(\lambda + n)(n - 1) + K_1(\lambda + 2n)} \quad (1.E.2)$$

and the supply schedule has the following expression:

$$S^*(p) = \frac{1}{n} \left(p \left(\frac{1}{K_1} - 1 \right) + b \left(1 - \frac{K_2}{K_1} \right) \right) \quad (1.E.3)$$

Let us study how K_1 varies with γ . Note first that if one defines $G = 4\gamma + \lambda + n$, then

$$\frac{\partial K_1}{\partial \gamma} = \frac{\partial K_1}{\partial G} \frac{\partial G}{\partial \gamma} = 2 \frac{\partial K_1}{\partial G}.$$



therefore the sign of $\frac{\partial K_1}{\partial \gamma}$ is given by that of:

$$\frac{\partial K_1}{\partial G} = \frac{\partial}{\partial G} \left[\frac{n\sqrt{G^2 - 4n + 4} - G(n - 2)}{2(G + n)} \right] \quad (1.E.4)$$

$$= \frac{(\sqrt{G^2 - 4n + 4})((G + n)(nG - (n - 2)\sqrt{G^2 - 4n + 4}) - (n\sqrt{G^2 - 4n + 4} - G(n - 2)))}{4(G + n)^2} \quad (1.E.5)$$

$$= \frac{(\sqrt{G^2 - 4n + 4})(n^2G + 4n^2 - 4n - n(n - 2)\sqrt{G^2 - 4n + 4})}{4(G + n)^2} \quad (1.E.6)$$

$$= \frac{(\sqrt{G^2 - 4n + 4})(2G + 4 + (n - 2)(G + 4 - \sqrt{(G + 4)^2 - 8G - 4n - 12}))}{4(G + n)^2} > 0 \quad (1.E.7)$$

Therefore $\frac{\partial S^*(p)}{\partial \gamma} < 0$ which implies that schedules see their slope increase with γ in the plane (q, p) .

We can perform the same type of computation for the ratio $\frac{K_2}{K_1}$, using the fact that $\partial_\gamma K_1 > 0$:

$$\frac{\partial K_2/K_1}{\partial \gamma} = - \frac{\partial_\gamma K_1 (K_1^2(\lambda + 2n)(\lambda + n) + 2K_1(\lambda + 2n)(n - 1)\lambda + \lambda(\lambda + n)(n - 1)^2)}{K_1^2((\lambda + n)(n - 1) + K_1(\lambda + 2n))^2} < 0 \quad (1.E.8)$$

This implies that the schedule is shifted to the right in the plane (q, p) when ramping costs increase.

Appendix 1.F Proof of proposition 1.6.3

The proof is exactly the same as that detailed in annex 1.E, by replacing γ by $\gamma(1 + \frac{\tau}{\omega} \frac{d\omega}{dt})$ and noting that under our assumptions, $1 + \frac{\tau}{\omega} \frac{d\omega}{dt} > 0$.

Appendix 1.G Proof of proposition 1.6.1

Define the following Hamiltonian:

$$H(p(\theta, t), \dot{p}(\theta, t), \mu(\theta, t), \theta, t) = f(\theta, t) \left(p(\theta, t)(\theta - p(\theta, t)) - \frac{\lambda}{2}(\theta - p(\theta, t))^2 - \frac{\gamma}{2}\sigma(\theta, t)^2(1 - u(\theta, t))^2 \right) + \mu(\theta, t)u(\theta, t) \quad (1.G.1)$$

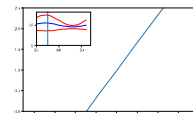
where $u(\theta, t)$ is the control variables defined through the following equation of motion: $u(\theta, t) = \dot{p}(\theta, t)$, $u(\theta, t) \in [0, 1]$.

Note that the methods used previously generalize to multi-dimensional problems, and that here, our problem depends on θ and t instead of only θ as in the case of the static monopoly problem.

Further note that the problem does not depend on the time derivative of $p(\theta, t)$. This means that what would be a general Euler-Lagrange formulation expressed as $\frac{\partial \mathcal{L}}{\partial p} - \frac{\partial}{\partial \theta} \frac{\partial \mathcal{L}}{\partial \dot{p}} - \frac{\partial}{\partial t} \frac{\partial \mathcal{L}}{\partial \theta_t}$, which is the equation that has to be solved for interior solutions, reduces to $\frac{\partial \mathcal{L}}{\partial p} - \frac{\partial}{\partial \theta} \frac{\partial \mathcal{L}}{\partial \dot{p}}$, where $\mathcal{L}(t, \theta, p, \dot{p}) = H(p, \dot{p}, 0, \theta, t)$. This is the exact same problem as before, with the only addition that our parameters now vary with t , but the partial differential equation is the same one as before.

Therefore the problem can be solved exactly as before by replacing the variance term by its new dynamic version, that is that it is as if the ramping cost parameter γ was replaced by $\gamma \cdot \left(1 + \frac{\tau}{\omega} \frac{d\omega}{dt}\right)$ in the static solution.

This can be seen by noting that $\sigma(\theta, t)^2 = \left(1 + \frac{\tau}{\omega} \frac{d\omega}{dt}\right) (\theta - \hat{\theta} + \omega)(\hat{\theta} + \omega - \theta)$ which has to fall back to the static case in the limit, therefore we see that we simply get an additional $\left(1 + \frac{\tau}{\omega} \frac{d\omega}{dt}\right)$ term that appears on the ramping cost term, that is that multiplies γ .



Appendix 1.H Proof of proposition 1.6.2

The exact same reasoning as the one in Annex 1.G applies here and we only have to take our static oligopoly result and replace γ by $\gamma \cdot \left(1 + \frac{\tau}{\omega} \frac{d\omega}{dt}\right)$ to obtain the dynamic results.

Chapter 2

Methodological Tools for

Non-Parametric Functional data

Evaluation and Weather Data Usage

1

¹Joint work with Henri de Belsunce.

The weather data was obtained through a research convention with Météo-France - ref. DIRIC/13/024

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Keywords: Functional, Electricity, Auctions, Day-ahead, Weather, Uncertainty, Dynamics, Regression

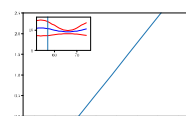
2.1 Introduction

In the previous chapter, we have introduced a model of supply function equilibria under uncertainty that takes ramping costs into account, and we derived solutions that depend on the information the firms have about the future demand at the time of bidding. Here, we will focus on introducing tools that will allow us to perform an empirical analysis of the French day ahead market and test these theoretical predictions, which will be the focus of our third chapter.

In this short chapter we develop a methodology to analyze data of two specific formats. The focus lies on the methodological details as well as evaluating the performance of our technique. The aim is first to extract points of interests from functional data in order to be able to compare functions to one another across bids, and second to describe a methodology that will allow us take into account the uncertainty related to the weather. The economic interpretation is secondary in this chapter. Chapter 3 will use the methodology developed here for a case study of the French electricity market.

2.2 Point Selection on Functional Data: a Non-Parametric Approach

Reduced form models often rely on exploiting market outcomes for their analysis, i.e. equilibrium prices and quantities, in order to identify the determinants of firm behaviour and test predictions of the theory. On a few markets, sufficient information is available to get around the problem of using endogenous equilibrium data. For example on the government bond markets, both the full aggregate demand and supply functions are observed. This market is of a specific type, it is a divisible goods auction (also called multi-unit or share auctions). These are auctions where multiple units of a good are sold in a single auction. The exact quantity is not predetermined but endogenous and depends on the



price. Furthermore, the auction format is more complex than for indivisible, single unit auctions and most notably requires that bidders simultaneously submit full bid functions for the goods, i.e. multiple price-quantity combinations at which each bidder is willing to buy or sell the goods. The market price and quantity are determined by the intersection of the aggregate demand and supply functions.

The aggregate bid functions are very rich in information and the reduced form models can be adapted to use this data. However, the literature on exploiting functional data is limited. This idea has been applied to investigate the determinants of demand bid functions in French government bond auctions [Préget and Waelbroeck, 2005]. They rely on the propositions first put forward in [Boukai and Landsberger, 1998] and [Berg et al., 1999], who identified that aggregate bid functions in divisible goods auctions follow an S-shaped curve that can be estimated by a logistic function. The fluctuations across auctions are claimed to be due to random shocks on the parameters of the estimated logistic function. The methodology is applied in [Özcan, 2004] to investigate the revenue superiority of the discriminatory price auction format over the uniform price auction format for the Turkish government bonds market.

More generally, their methodology consists of a two-stage regression. The first stage summarises the (presumably parametric) functional data of the aggregated demand function as parameters of an estimated smooth logistic function. The second stage reuses the information (concentrated in the estimated parameters) for cross-sectional analyses.

Although the auction mechanism is identical to that of the Treasury market and data availability is comparable, the logistic function approach does not suit the context of the electricity market due to the strong heterogeneity of the bid functions and their deviations from such logistic shapes, as can be seen in the example of Figure 2.2.1.

The heterogeneity arises from the fact that the bid functions for the electricity auc-

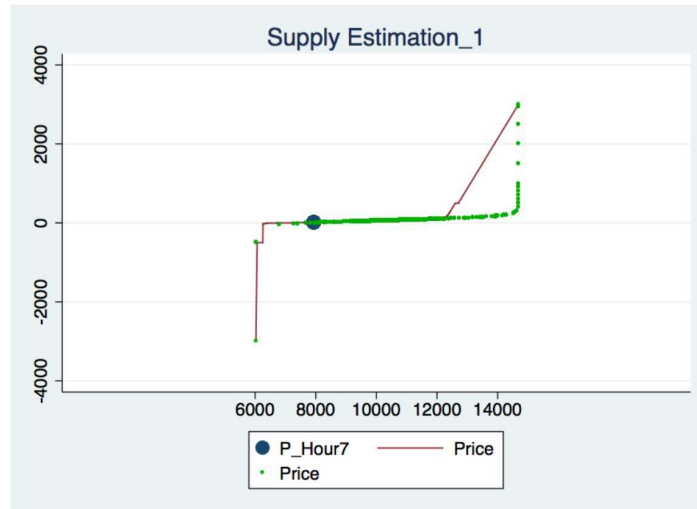
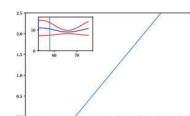


Figure 2.2.1: Example of an asymmetric aggregate supply function. The x axis is the quantity in MWh, the y axis is the price in €. In red is the actual aggregate function, in green is an estimated logistic function showcasing the large discrepancies that can arise with this parametric approach. The blue point is the market outcome.

tions are much richer since we have multiple, strategic players on both the demand and the supply side of the market (unlike the market of government bonds, where the supply is monopolistically determined by the Treasury itself). Furthermore, supplier bidding is strongly influenced by the underlying cost of the production technologies. The observed data is consequently less homogeneous and the fitting of the logistic model not convincing. Furthermore, the economic interpretation of the logistic function parameters is very difficult and reducing the whole bid function to two parameters of interest discards a lot of the original information of the bid functions. Finally, we are uncomfortable with the strong assumption of smooth underlying functions and want to circumvent the problems of fitting these.

Instead, we develop a non-parametric, functional data analysis approach to select comparable data points from the original bid functions. In our case, this selection of points will yield 4 regions for every curves. Each region can be thought of as linear. These selected points are comparable across repetitions of the market (i.e. auctions for different hourly contracts) and can then be used to run a cross-sectional reduced form



model. The interest of this approach is threefold. First, it aims to use as much of the original information as possible without distorting it into parameters of a logistic function. What we mean by distortion is the example displayed in Fig. 2.2.1, where one can see that the fitted logistic function in green is very far from the data (in the sense that the integral of the absolute value of the difference of the two curves is very large) because the underlying data simply does not have the proper shape. Also, information about different parts of the bid function does not influence one another, contrary to a parametrized form in which one tries to fit a specified function to data. This implies that the error between the functional form and the data at any point of the curve influences the fitted parameters, therefore “mixing” information from the whole curve into the choice of a given value for the parameter. Second, our approach can be extended to as many points as necessary. The cross-sectional analyses are then conditioned on the type of comparable points selected. Third, while our analysis provides support for an underlying tri-linear or S-shaped functional form, we do not need to assume a specific functional form nor impose overly simplistic assumptions, such as symmetry of the functional forms, to ensure convergence of the estimator.

Here we present the methodology of our point selection and apply it to data from the French electricity market. For now, we ignore specificities of the market for the sake of concentrating on the methodology. We briefly introduce the data and the market in section 2.2.1. For a full explanation of the data and the market, we refer the reader to chapter 3. In section 2.2.2, we explain the point selection algorithm. In section 2.2.3 we discuss the results of the methodology. Section 2.2.5 concludes.

2.2.1 Information about our data

Our methodology is general and can be applied to any market where the structure of data observations is similar. Here, we present and discuss the performance of the methodology on data from the electricity market. For the purposes of this chapter, we will focus only on the statistical properties of the data, not on the economic interpretation.

We apply our methodology to data from a divisible goods auction. In this auction, each buyer and seller submits a full individual bid function, i.e. a demand or a supply function, which consists of 2 to 256 monotone price-quantity combinations. The final bid function consists of these explicitly submitted bid points and all linearly interpolated points between them.

The market is cleared by computing the intersection of the aggregate demand and aggregate supply functions, which are each obtained by summing up all individual bid functions for the demand and supply side of the market respectively. In a uniform pricing format, the determined equilibrium price is applied to all units sold in that auction.

2.2.2 Point selection algorithm

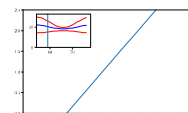
To briefly fix ideas, let's assume that we are interested in a regression à la:

$$S' = \alpha + \beta \mathbf{X} + \epsilon$$

where S' is the steepness of the bid function, \mathbf{X} the stacked vector of exogenous variables (not specified further here), α the regression constant, β the stacked vector of regression coefficients and ϵ the error term.

The information S' is drawn from the bid functions of the electricity market, and varies along the different points of the bids.

For comparability, we require that a chosen point k from a supply function must be comparable to the k^{th} point from the supply functions of another auction. The same goes for chosen points of the demand functions. The reason for this assumption is that comparing those points across auction allows us to describe how the functions, that is the aggregate strategies, change shape when our independent variables vary. Note that



we do not impose comparability between a pair k of points from a supply and a demand function of the same auction.

Non-parametric technique to compare bid functions

Consider two demand functions (as shown in figure 2.2.2). We have to identify "features"

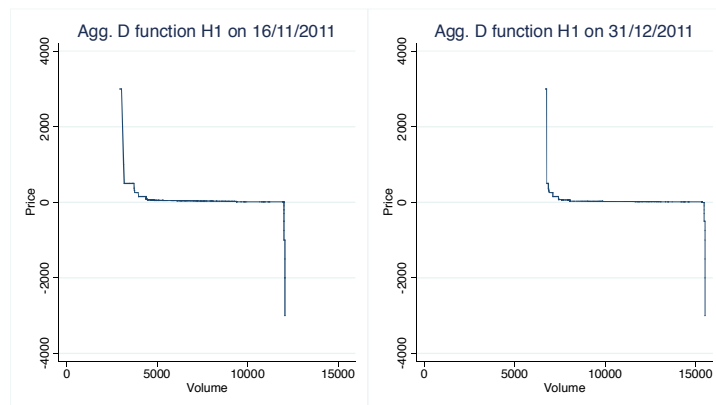


Figure 2.2.2: Comparison of two aggregate demand functions for the same hour

of the different functions in order to determine which points can be compared to one another. We aim to reproduce the type of analysis that the brain performs automatically when faced with such curve: we clearly identify three regions of different slopes, where the central region is less steep than the left and right regions.

To recognise these features, we perform two successive kernel density analyses.² For details on the bandwidth and kernel selection as well as algorithm specificities, see appendix 2.A.1. This allows us to access estimates of the absolute values of the first and second derivatives of the demand functions as shown in graphs B and C of figure 2.2.3.

We are therefore able to identify the regions of very high curvature, which define the transition between the three characteristic regions of these functions. We assume that these maxima can be compared across different auctions. This hypothesis is commonly made in functional data analysis and known under the method of landmark registration

²Bandwidth in the first estimation = 45, bandwidth in the second estimation = 2, kernels: epanechnikov.

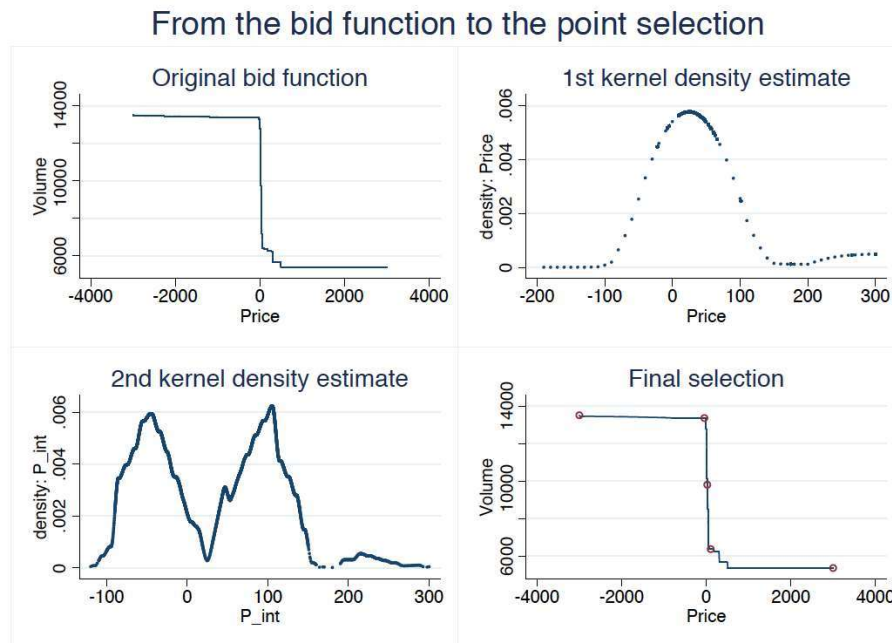


Figure 2.2.3: Steps of the point selection process

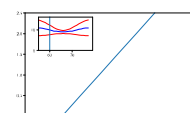
Top left (A): The full original aggregate demand bid function for hour 8 on 15.01.2011 in the quantity - price dimension. Top right (B): Kernel density estimates of the first derivative, zoomed on the relevant price range. Bottom left (C): Zoomed kernel density estimates of the second derivative. Bottom right (D): The full original bid function with the $K = 5$ selected points.

[Silverman and Ramsay, 2005]. This has been applied in [Wölfing, 2013], chapter 4, to day-ahead electricity data, in order to identify the effect of fuel price shocks on supply curves. However, this landmark registration was applied in a parametric form: the regions of high steepness were identified as any part of the curve above 90€/MWh.

We can develop this method further and define intermediary points³ that can again be compared to one another. This method allows to define as many points as needed, for computational reasons we limit ourselves to $K = 5$ points.⁴

³As an example, we could extract those points corresponding to half the density value of the maximum density of the second order derivative. The four points selected, one for each monotone portion of the graph of second derivative estimates, would then correspond to those where the curvature of the function is halved. Together with the maximum, the additional point would contain information on the speed (radius of the curvature) at which the function changes.

⁴The point selection algorithm took 2 weeks runtime to complete its task of selecting 5 points per function. Defining intermediary points would have taken disproportionately more time since many sorting and interpolation steps are necessary for each intermediate point.



Graph D of figure 2.2.3 visualises an original demand bid function and the selected points that we retain as an informative summary of the original curve. Once this work is done we are left with $K = 5$ points per observed aggregate function, those points being defined in such a way that they can be compared from one auction to another.

In our setting, the selected points are the two end-points of the curves (where bidding is imposed by the auction rules at the minimum ($k = 1$) and maximum ($k = 5$) Price), the point corresponding to what can be thought of as the point of inflection (determined by the maximum of the first derivative, ($k = 3$) in the plane (p, q)) and the points separating the regions of high and low elasticity in price (determined by the maximum second derivatives to the left ($k = 2$) and right ($k = 4$) of the POI).

We described the technique here for the case of a demand function. The information measured at these points can thereby be compared across demand bid functions of different auctions. The method is used analogously for selecting comparable points on the supply function. We are hence able to extract slopes at these selected supply bid points, which are again comparable across auctions.

2.2.3 Results of the point selection methodology

Precision of point selection

We have selected $K = 5$ types of comparable points for each of the 37500 demand and supply functions present in our dataset. This section details the results of the point selection methodology and presents evidence why the point selection algorithm has produced comparable points reliably.

The graphs in figure 2.2.4 show the local density of selected points in the price - quantity space for the demand (left) and supply (right) curves. The fact that the groups of

data points are disjoint from one another indicates that the points selected are distinctly different across groups.

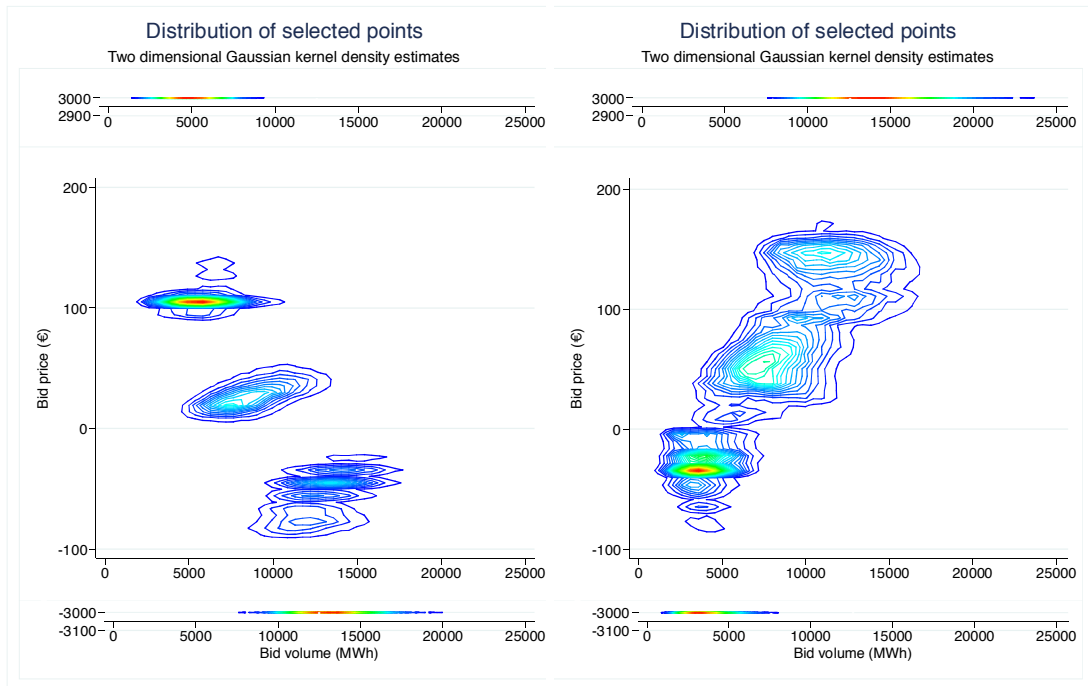
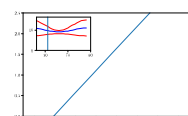


Figure 2.2.4: Heat map on selected, comparable demand and supply points

Note: Please note the discontinuity in the scale of the y-axis. The three separate graphs are arranged to be understood as a single one. The warmer the colours of the heat map, the higher the frequency of selected price-quantity pairs. The colour legend is omitted for brevity. Density changes between contours are of the order of 10^{-4} .

In figure 2.2.4, selected points of type $k = 1$ manifest at the bottom of the graph with prices fixed at $-3000\text{€}/\text{MWh}$. Similarly, $k = 5$ points appear at the top of the graph with prices fixed at $+3000\text{€}/\text{MWh}$. The three distinct groups of data points refer to points of type $k = 4$, $k = 3$ and $k = 2$, respectively, when reading the zoomed, center part of the graph from top to bottom.

We note that the point selection for the demand curves has produced groups of points that are more distinct (and thus more robustly attributed to a certain type k) then for the supply function.



Our methodology only relies on assuming that the first derivative is uni-modal and that sufficient variation exists in the data to distinctly identify the regions of different slope. Overall, this is strong evidence that the algorithm is able to distinctly differentiate between points of different types.

2.2.4 Observations of bidding frictions

Distinct point selection is further supported by the evidence in figure 2.2.5. These graphs show the distribution in the price-quantity space of the selected points separately for the demand and supply function. Distinct clouds are an indication that selected points are different across types k .

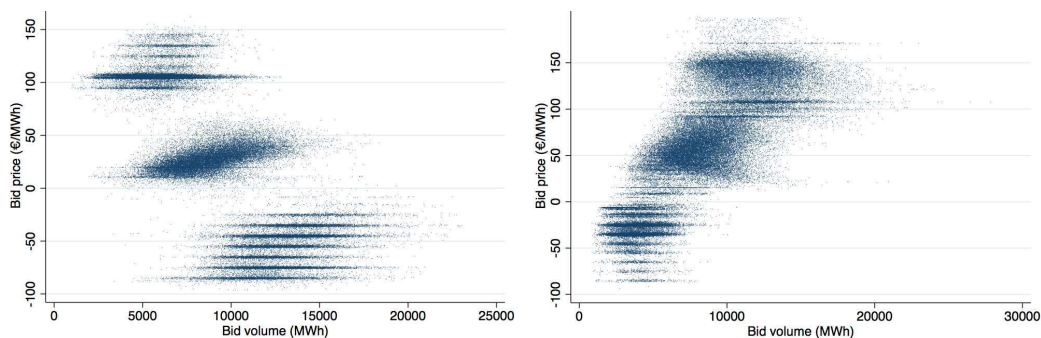


Figure 2.2.5: Distribution of selected demand (left) and supply (right) points

However, a feature of the graphs is striking: patterns (horizontal lines) seem to exist for the selected points of type⁵ $k = 2$ and $k = 4$. Many selected points accumulate at certain prices of regular intervals of $10\text{€}/\text{MWh}$, i.e. there seem to be focal price points for the bidders at the curvature points of the bid functions. The pattern is present for selected points of both the supply and demand functions, although the selected points from the supply function exhibit this pattern slightly less.

The points following the pattern (types $k = 2, 4$) represent the points of maximum

⁵Types $k = 1$ and $k = 5$ do not exhibit variation in price, because bidding at the extreme prices of $\pm 3000\text{€}/\text{MWh}$ is imposed by the auction rules. We thus neglect their analysis here.

curvature of the aggregate bid functions, i.e. the region where the aggregate bid function transitions from a price elastic center portion to the price inelastic extremities of the bid function.

Without prioritizing any explanation⁶, we acknowledge the existence of bid point patterns in the values (i.e. prices and quantities) of selected points.

We are, however, interested in S' , the slope at each selected point - an information measured at the selected point. We therefore investigate whether the values of the first derivative at the selected points display a pattern. Figure 2.2.6 shows the histograms of slopes of supply functions for the points $k = 2, 3$ and 4. No pattern in the values of the derivatives is apparent.

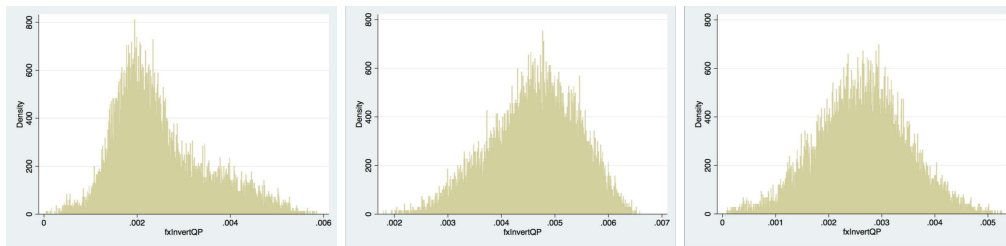


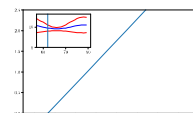
Figure 2.2.6: Histogram of slopes per point type

Note: Histograms of extracted slopes at points of type $k = 2$ (left), $k = 3$ (middle) and $k = 4$ (right).

Although values of the selected points are possibly biased due to focal price points, we do not observe patterns in the variable of interest (i.e. the first derivatives of the selected points) and deem the methodology adequate for our purposes.

Finally, we emphasize that the observed patterns are not caused by the point selection

⁶We do not investigate the origins of bidding frictions in this section, we focus purely on the methodology. For the electricity market, a few possible explanations are that (1) bid functions are driven by marginal costs consideration towards the extremes of the bid curve, (2) bidders bid coarsely since they have used up much of their bid point allowance (256 points) on the center portion of the curve, (3) bidders spend less effort on adequately bidding at extremes since the likelihood of the market outcome occurring at the extremes is much lower.



mechanism since the algorithm can only choose between explicitly bid points or linearly interpolated points, that could be part of a market equilibrium under the reigning price setting algorithm. The pattern arises from many horizontal steps occurring at the same prices in different auctions.

Value of selected points (determining K)

We remind the reader that the aim is to recover points that summarize well the behaviour of the full aggregate bid functions in different auctions. Our technique allows us to extract representative and comparable points across bid functions of different auctions. From the selected points, we can also go back to infer the original bid function from which the points were selected. In order to evaluate the utility of our methodology, we investigate the added benefit of an additional point in our point selection.

By selecting $K = 5$ points per curve, rather than fewer points per curve, we are able to significantly reduce the degrees of freedom for inferring the original bid function. In other words, our information (as captured by the selected points) about the original bid function is more precise.

In order to investigate the marginal gain of information for additional points, we first define the mean registered curve. Consider a set of curves that each has N registered points. Take the average coordinates of every point across curves. Rescale linearly every curve by parts so that the registered points fall on their average.⁷ Define the mean registered curve as the averaged rescaled curves. Now, separate the data into two groups: curves that are above or below this average curve. Take the averages of these two groups: this defines a measure of the variability of the curves around the total average which is able to capture asymmetries between the two groups.

⁷We rescale all points between the reference points by a vector obtained as a linear combination of the displacement vectors of the closest reference points, of which weights are obtained as the inverse of the distance of the considered point to the enclosing reference points.

Now that these quantities are defined, we can display how much information is captured by the successive addition of registration points for $K = 0$ to $K = 5$ points. We look at the decrease in uncertainty achieved by including an additional point, obtained using our technique. Figure 2.2.7 shows the mean registered curves (red lines) and the mean inferior or superior curves (pink shaded interval above and below the mean registered curves) as a function of the number of reference points.

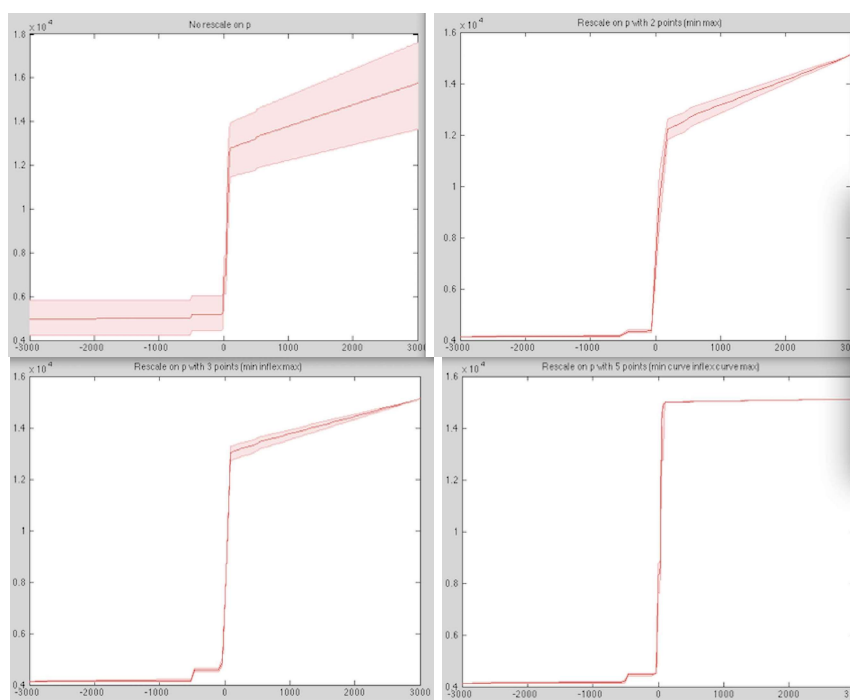
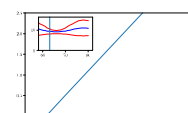


Figure 2.2.7: Error bars as a function of the number of extracted points

Note: The graphs represent the master curve with the error interval for inferring the original bid function, conditional on the number of extracted reference points (RP). Top left (A): Computed without any RP. Top right (B): Computed using 2 RP. Bottom left (C): Computed using 3 RP. Bottom right (D): Computed using 5 RP.

We can see that as we include an increasing number of points the shaded areas shrink: this is a measure of how much of the information contained in the raw curves is captured by the registration points. We see that at 5 registration points, the shaded area is very small, so much so that one can consider that by registering these 5 points, we capture a so-called "master curve": most of the information about the curves is contained in those 5 points.



More quantitatively: without any reference point, inferred bid functions would lie in the interval shown in graph A of figure 2.2.7. With two reference points (namely the minimum and the maximum quantity), the uncertainty is reduced as shown by the smaller error interval in graph B. Graph C adds a third point (the inflection point) and Graph D adds another two points (the two points of maximum curvatures). Figure 2.2.7 shows clearly that with an increasing number of reference points, we obtain a more precise information about the original bid function. We quantify the informational gain by measuring the pink shaded area in each graph A to D. The result is shown in figure 2.2.8 and reveals decreasing marginal information for each additional point. By selecting $K = 5$ points, we are able to reduce the shaded area by a factor of about 50 when compared to figure A (see figure 2.2.8). We see this insight as support for using $K = 5$ points for further work.

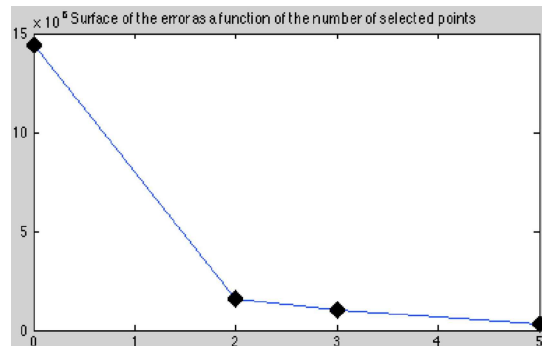


Figure 2.2.8: Proxy for degrees of freedom on master curve

Note: The graph plots a proxy for the number of degrees of freedom for the inference of the original bid function on the number of reference points. Specifically, it plots the size of the pink shaded area in figure 2.2.7 against the number of points.

While the graphs in figure 2.2.7 are displayed on inverted axes and rescaled units, we show the final master curve and uncertainty interval on the original axes and units in figure 2.2.9.

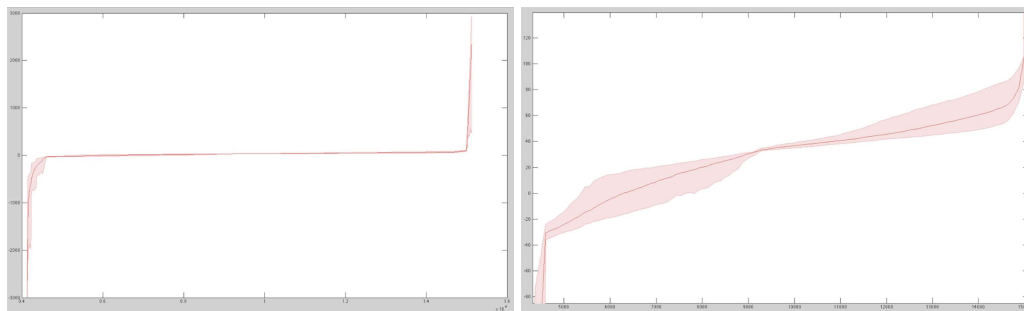


Figure 2.2.9: Overall (left) and zoomed (right) Mastercurve with confidence interval

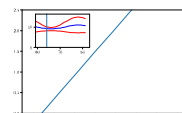
Note: Master curve in the quantity - price dimension.

2.2.5 Discussion

In this section, we have developed an alternative technique to run a cross-section reduced form model on data generated by a market that keeps track of the full aggregate demand and/or supply functions. While we apply it to aggregate demand functions, the methodology is fit for the analysis of aggregate supply functions and individual bid functions of either market side.

The methodology is inspired by the techniques used in the literature on Treasury auctions, but has been set up from scratch to allow treatment of more heterogenous data. Furthermore, the hard assumption of an underlying logistic function is relaxed, and our non-parametric point selection avoids the storing of bid function information in the form of estimated function parameters, which are difficult to interpret.

Smoothing of the original bid functions is a component in both the traditional logistic function approach and our comparable point selection methodology. The smoothing enables the user to abstract of small bid function particularities and imprecision, e.g. steps in the function. However, in the traditional approach, the reduction of plus 1000 bid points into very few parameters resulted in the blurring of “local” bid function information from all parts of the function at once. Our non-parametric approach allows specifically to control the extent to which one smoothes the underlying data through the amount of registration points considered.



The results of the comparable point selection are encouraging. We show that each type of point is distinctly chosen and that patterns of the original bid functions do not influence the quality of derivative information extracted at the selected points. We acknowledge the existence of bidding frictions in the original data and highlight this observation for further work.

2.3 Methodology to Aggregate Geographically Dispersed Information on a National Level

The theoretical results of chapter 1 indicate that a key ingredient in explaining the dynamics of the bids submitted by suppliers on the electricity market is the uncertainty about demand shocks.

Energy demand addressed to the electricity markets depends on temperature (through the heating of buildings), on wind speed (through the production of wind turbines which reduces the net demand) and on luminosity (through the production of solar panels which reduces the net demand). However, these three weather variables vary in space, whereas the market is at the national level. We introduce here the methodology with which we estimate the associated uncertainty.

We have two types of meteorological data: observations and forecasts. We use both types of data to estimate the underlying uncertainty. The methodologies for each differ slightly.

2.3.1 Dealing with meteorological data

Interpolation methodology on weather observations

Observations are obtained from MétéoFrance for three parameters of particular interest: temperature, wind speed and light intensity. These observations take the form of tables of hourly observations for a given set of weather stations. Each parameter is observed on a different set of stations.

Due to their hourly nature, the analysis of the electricity market's sensitivity to weather requires a very high number of observations. Therefore, we select between one and two stations per Département⁸, a French administrative unit of roughly 6000 km^2 , i.e. of a typical lengthscale of about 75 km . We have 161 stations for temperature, 113 stations for wind speed and 106 for light intensity, as shown in Fig 2.3.1.

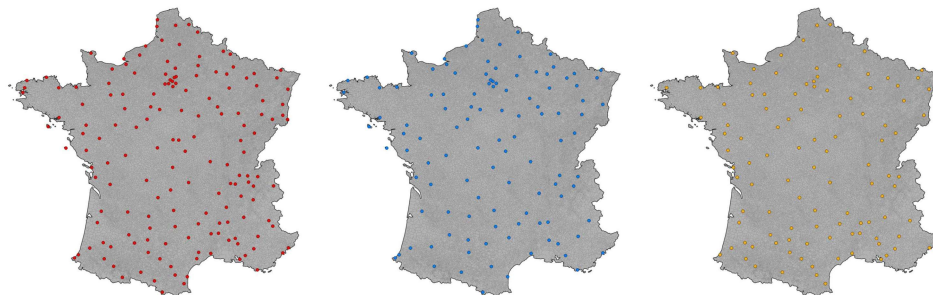
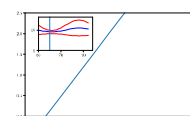


Figure 2.3.1: Stations for which we have hourly data. Left: temperature, center: wind speed, right: light intensity.

For each hour, we select the corresponding observations and interpolate them in order to reconstruct the weather on the entire French territory. An interpolation consists on inferring the value of a variable at query points using a reference data set of known values. One very important underlying assumption of interpolation methods is that of the continuity of the process underlying the data generation. The easiest interpolation method is the linear interpolation: think about a dataset of hourly observations with

⁸There are 95 Département in France



one missing value; to reconstruct the missing value, take the average of the value of the preceding and following hour. There are numerous methods of interpolation, even more so when the data is spatial in nature, all revolving around two main steps. First, given a query point at which one would like to infer the value of the variable, there needs to be a selection rule to know which of the points from the reference data set should be used (in our example the preceding and following values). Second, once these points are selected, one needs a weighting function to know their relative importance in order to obtain the interpolated value (in our example it is a simple averaging, that is weights of 0.5).

We use the natural neighbor interpolation method, well known for its good balance between speed and accuracy. In short, in this case, the first step makes use of the Voronoi tessellation algorithm⁹, one is able to define the natural neighbors of a point for which one seeks an interpolated value. These natural neighbors are used in the second step, which performs the actual interpolation as a weighted average of the values of these natural neighbors using a ratio of surfaces as weights (see Fig 2.3.2 for more details).

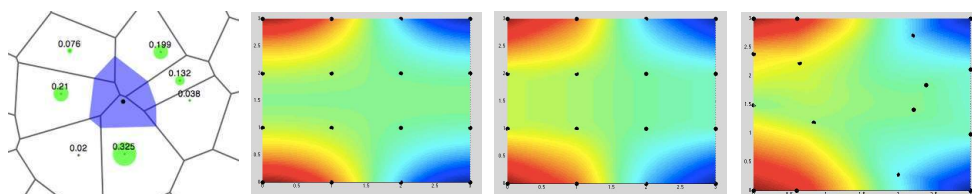


Figure 2.3.2: Left: Voronoi’s algorithm is applied once on the reference points highlighted in green to obtain the white surfaces, and a second time on the same points to which is added the query point in the center to obtain the new blue cell. The green circles, which represent the interpolating weights, are generated using the ratio of the shaded area to that of the cell area of the surrounding points. Center left: example of a reference surface (color mapped) to be reconstructed through a natural neighbor interpolation. Center right: interpolated surface with a reference set of 16 evenly organized points, represented in black. Right: interpolated surface with a reference set of 16 unevenly organized points, represented in black. From 16 points one is able to reconstruct the color mapped surfaces which are approximations of the reference one, represented in the center left image.

⁹The Voronoi tessellation algorithm takes a collection of points $\{p_k\}$ in the plane, and then partitions the plane as regions ”belonging” to each point, called cells. A Voronoi cell associated with a given point p_k is defined as the collection of every point in the plane whose distance to p_k is less than or equal to its distance to any other p_{-k} . Each such cell is a convex polygon.

Image transformation to recover weather forecasts

Forecasts are obtained from the Global Forecast System (GFS), and come in the form of colormaps, as shown in Fig 2.3.3. We are going to illustrate our methodology on temperature data, but the same exact approach is performed on wind speed data. The general idea is that the pointwise precision is low ($2^{\circ}C$ per color) but that the overall map contains quite a lot of information through the topology of the colored regions. We describe below how to extract this information.

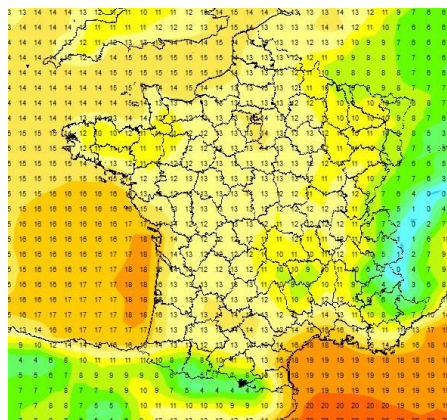
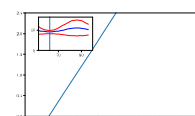


Figure 2.3.3: Temperature forecast from a simulation run by the GFS at 6 a.m. on the 3rd of November 2011, for a forecast at 22 p.m.

First: image cleaning To extract the relevant data we first clean the color map from its irrelevant information, namely the temperature in numbers and the administrative borders. Note that this step introduces a small amount of high spatial frequency noise, see Fig 2.3.4 left and center left.

Second: removal of redundant information A lot of information is lost from the actual GFS simulations by using a color map representation, as temperature is described as a discontinuous variable: each color has a precision of $2^{\circ}C$. In order to correct for this, we leverage the fact that all the information contained in this color map, that is the color at each pixel, is actually contained in a smaller set of points. Consider the value at the boundaries between different color regions: by knowing that the interior of a constant



color region has a constant value, one is able to represent all the information contained in the original image by keeping only track of the values at the boundaries. To recognise those boundaries, we perform image analysis, more precisely we use edge recognition methods based on finding high gradient regions, thus obtaining Fig 2.3.4 center right.

Third: surface fitting Once we represent the information in this denser form we can perform the last step, which consists in fitting a surface to our newly defined dataset, i.e. the temperature values at the boundaries, which take the form of (x, y, T) triplets. We could perform an interpolation, but these methods are not well suited to reference sets having so much structure. Here, data points lie on curves representing iso-temperatures, so that along such a curve there is a lot of data points, whereas the information is very sparse along the direction of the gradient. In addition the first step introduced some spatial noise which we want to correct to some extent: we allow our fitted surface to take different values than our data points, so as to smoothen out this noise. We define the rigidity of our fitted surface, i.e. a penalty associated to fast changes in the value of the surface, and therefore reduce the importance of the high-frequency noise introduced in the first step. The end result is presented in Fig 2.3.4 right.

It is key to understand that this image is displayed using a colormap close to the one in the original picture to facilitate comparison but that its underlying data is continuous whereas the original image describes temperature by bins of $2^{\circ}C$. It can therefore be used to query the value at any given point, and these values will change continuously in space instead of discrete jumps in the raw format.

Autocorrelation lengthscale

We use this dataset to build measures of the weather uncertainty. To do so we measure the auto-correlation lengthscale of our three weather variables of interest: temperature, wind speed and light intensity. This lengthscale measures how much are the weather variables correlated spatially. Our hypothesis is that the auto-correlation lengthscale is

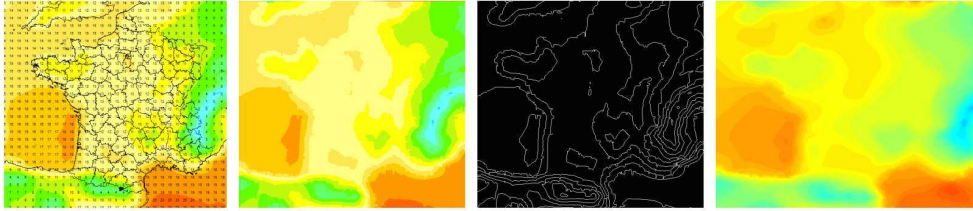


Figure 2.3.4: Left: reference image. Center left: borders and numbers are removed. Center right: edge recognition. Right: final fitted surface.

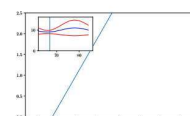
inversely proportional to uncertainty about the variable we are interested in. When it is small, the variable is less spatially correlated, which we interpret as being more difficult to forecast. Conversely, when the auto-correlation lengthscale is large, the variable is very correlated spatially, that is that the informational content of one datapoint is higher for the prospect of using it for the evaluation of a national effect.

More precisely, the argument is as follows:

First, renewable production is built by aggregating the forecast of all individual renewable sources. This means knowing the position and capacity of every renewable source, querying weather forecasts for all of these points, modeling the renewable's response to the forecasted weather and adding the forecasted productions.

Second, we note that weather is spatially correlated, which means that the closer two points are, the closer the values for a given weather variable (the air temperature at your left hand is very close to that at your left hand, but less so across the city, and even less so across the country). This correlation roughly follows an exponential law: the difference between the values of a weather variable between two points behaves in a linear fashion for small distances and saturates at large distances.¹⁰ The transition between those two

¹⁰Intuitively, the characteristic lengthscale of autocorrelation represents the distance required between two geographical points on a map of weather forecasts to observe a decorrelation of half of its maximum value. For example on the wind speeds prediction, a characteristic length of 80 km means that if we observe two very distant points (say 1000km) to have a difference in wind speeds of, on average, 50km/h (this being the maximum difference, we are in the saturated regime), then we will observe, on average, wind speed differences of 25km/h for points distant from each other by 80km.



regimes is given by a characteristic lengthscale, a bit less than 200km on average.

Third, we observe that the average distance between production points is large enough that the relevant regime of autocorrelation is the saturated part.¹¹

Fourth, we note that there are two main channels through which the overall uncertainty about renewable production is related to the weather. There is an issue of error averaging, which means that if the weather becomes very spatially uncorrelated, one can expect errors to cancel out relative to a given bias in the forecast. This channel would tend to imply that more spatial variations imply a smaller uncertainty about production. There is also the issue that weather forecasts are numerical simulations and that the mesh size for such simulations, typically 5km for the high precision ARPEGE model of Météo France, implies that the errors are higher as the simulated phenomena have higher gradients. This means in our case that the uncertainty about the forecast increases as the weather becomes more spatially uncorrelated.

Fifth, these two effects are of opposite signs, but our third point is an argument for considering that the averaging of errors is smaller than the simulation errors. Therefore, we expect our uncertainty to increase as the spatial autocorrelation decreases (i.e. more spatial variation).

This can be summed up with the following hand-waving argument: when there is more spatial variations, the weather is more messy, therefore more difficult to predict.

To understand what the autocorrelation lengthscale captures, take two points on a plane and a spatially correlated bounded variable. If those points are infinitely distant, the value of the variable at these points should be uncorrelated. That is that the abso-

¹¹For N production points, we compute the $N(N-1)/2$ pairs of points, consider their distances and compute the average of these distances weighted by the production capacity at every point. In the case of the wind, we have an average distance of 459 km, in the case of the photovoltaic production we have an average distance of 499km.

lute difference between the variable taken at those two points should have a given average value. Conversely, two points infinitely close should have the same value, i.e. a zero absolute difference between the variable taken at those two points. The question is how fast is the transition between those two limit cases. First, we define the average absolute difference between two points when distant of a given value. Second we extract a typical lengthscale.

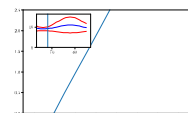
To define the average absolute difference between two points when distant of a given value, we consider at a given point in time every possible pair of points in our dataset. For a given pair we compute its distance and its absolute difference in value (in black in Fig.2.3.5). For 100 datapoints we obtain 4950 pairs. We then use a kernel smoother in order to obtain the average non-parametric autocorrelation function (in blue in Fig.2.3.5).

To recover a typical lengthscale, we make the parametric assumption that the autocorrelation is exponential in nature. We fit an exponential function through our smoothed data (in red in Fig.2.3.5), and recover the exponential decay parameter as our lengthscale (in green in Fig.2.3.5). We perform this operation for every hour in our dataset and every weather variable. The results are time series for the characteristic lengthscale of the weather parameters.

2.3.2 Aggregation of local information

Wind1DA *Wind speed (average speed in km/h):* Wind speeds influence the productivity of wind turbines, which are a source of unreliable electricity generation. In general, renewable technologies benefit from a feed-in guarantee by the state. That is, regardless of the trading outcome on all markets, renewable energies will be the first to be fed into the power grid at a guaranteed price.

Consequently, the electricity production of renewable technologies represents a production shock for all actors on the market. The production shock means that the demand



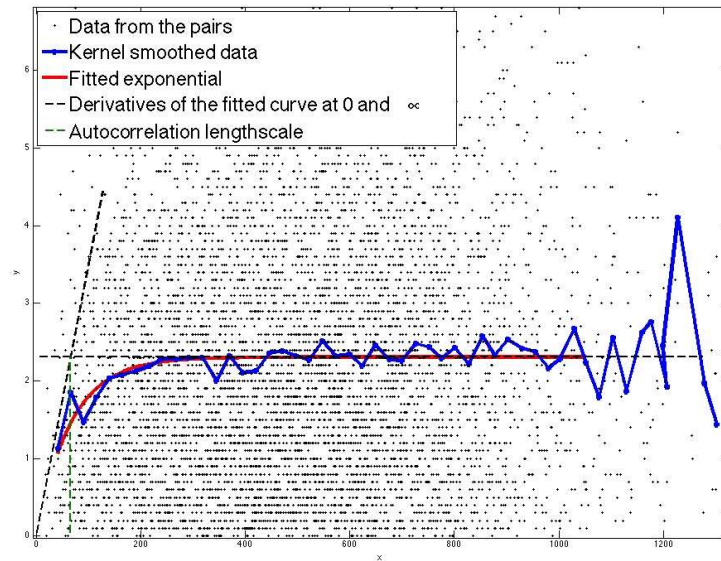


Figure 2.3.5: Auto-correlation lengthscale computation. In black are the points obtained from all the pairs from our original data, that is absolute wind speed differences as a function of the distance between the two points. In blue is the kernel smoothed function from those points. In red is the exponential fit. In black are the derivatives of the fit at 0 and ∞ . In green is the recovered auto-correlation lengthscale. The unit for the lengthscale is in km.

to be served by traditional electricity producing firms is reduced by the amount that is serviced by the electricity gained from renewable sources.

In the case of wind turbines, the average speed of the wind per hour allows to proxy for the size of the production shock due to the electricity generation from wind energy.

We use hourly windspeed forecast in the form of color maps from the Global Forecast System (GFS), giving the speed by bin of 5 km/h at 10m above ground, and the location and production capacity of the wind turbines present on the French territory, given by the SOeS (service d'observations et d'études statistiques - observations and study department) a department of the French environment ministry.

We consider that all turbines in France are of the same type, that is that they have the same response curve and height.

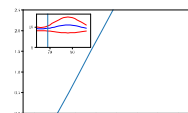
A typical response curve is represented in Fig. 2.3.6. It has three main characteristics: the wind speed at which the turbine starts to produce electricity, called the cut-in speed, the speed at which the turbine reaches its rated output, called the rated output speed, and the speed at which the turbine has to stop to avoid damage, called the cut-out speed. We use data publicly available¹² to obtain a rough estimate of the French average wind turbine characteristics. We use a cut-in speed of 2.5 m/s, a rated output speed of 14 m/s, and reduce arbitrarily the cut-out speed from an estimate of 24 m/s to 20 m/s to account for the fact that a turbine is shut down not when the average speed is too high but when the maximal speed becomes dangerous for the turbine.

Wind speed also increases with height, and turbines are typically between 60 and 80m high. We therefore apply a multiplier to the reconstructed wind speed at 10m.

We seek to reconstruct the French wind energy production from meteorological data. The two adjusted values, the cut-out speed and the speed multiplier, are adjusted by hand to obtain reasonable fits. The reason for this is that the reconstruction of wind speed and aggregate production is computationally intensive, therefore we cannot perform a full-blown estimation. We choose these values with a precision of roughly 10% with respect to their admissible range of values.

We obtain a reconstruction of wind production from day-ahead wind speed forecasts that we compare to actual observed production and to day-ahead wind production forecast computed by RTE, the French grid operator as shown in Fig.2.3.7. We stress here that our aim is two-fold: to link wind turbines' production to weather data and to use forecast data as the market actors only possess this information when bidding. We do not aim at producing better forecasts than the grid operator, the figure is only displayed to show that our methodology produces reasonable estimates (we obtain a correlation co-

¹²<http://www.thewindpower.net>



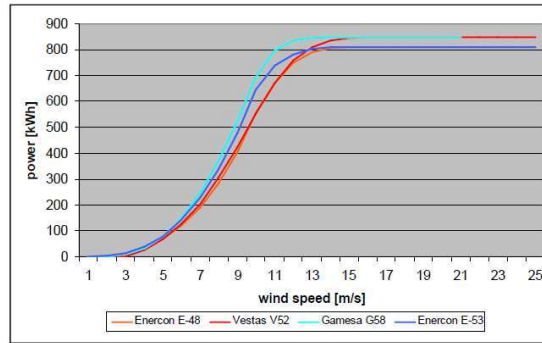


Figure 2.3.6: Typical response curves of different wind turbines

efficient between our forecast and the observation of 0.85 where the grid operator obtains 0.97).

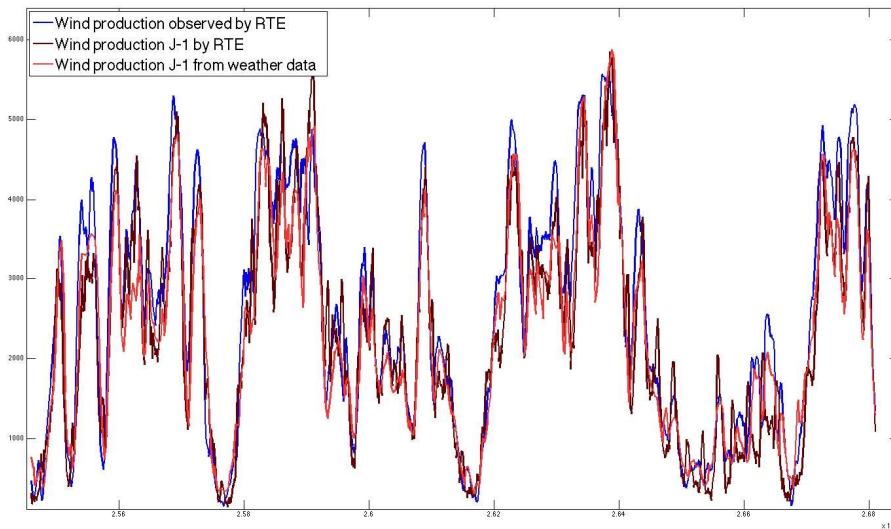


Figure 2.3.7: All curves are hourly production data. The origin of the hours is the first of January 2011, and the production is in MWh. In blue: the observed wind production. In dark red: the day-ahead predictions from the grid operator. In light red: the day-ahead predictions from weather data.

Tempeff15 We focus on the effect of temperature on the demand of electricity first. In France, a high percentage of the population heats their housing with electricity, therefore cold waves have a high impact on electricity consumption: 2300MW of additional power consumption for every drop of $1^{\circ}C$ below $15^{\circ}C$, as shown in Fig.2.3.8 sourced from [RTE, 2014], the French grid operator.

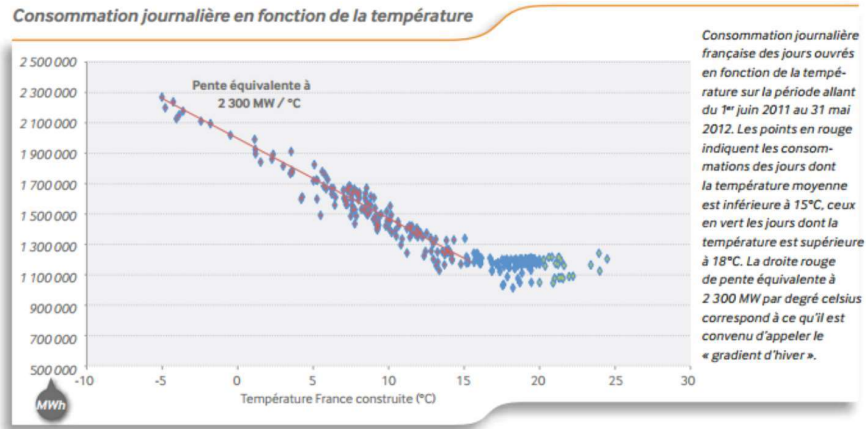
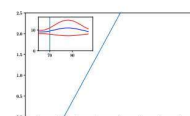


Figure 2.3.8: Daily electricity consumption in France as a function of the temperature, [RTE, 2012]

We apply this information to our observed meteorological data in order to build an effective temperature for France aimed at capturing its effect on consumption. To do so, we reconstruct temperature data for every French *commune*, the smallest administrative unit in France (there are around 36000 of those). We consider population as being a good proxy for potential heat consumption, therefore we apply it as a weight to the *commune* temperature. Lastly, we consider that temperatures saturate at 15°C. This allows us to build an effective temperature taking into account where the population is located and the nonlinearity of heat start up which in turn allows us to account at the country-level for the local impact of temperature on the electricity consumption.

Solar Light intensity (in $W.m^{-2}$) impacts the electricity market through multiple channels. The most obvious one is the associated electric production from photovoltaic panels. But there is another channel through which lighting can be seen as impacting electricity consumption: more sunlight decreases artificial light usage. In France, annually, the electric consumption that can be attributed to lighting represents roughly 50 TWh where solar production is roughly 4 TWh.¹³

¹³These estimates are computed by the authors based on numbers coming from [Bertoldi and Atanasiu, 2007], INSEE and EDF



We have photovoltaic production data, which in itself is a blackbox. As we aim to link meteorological data to consumption, we first want to validate the quality of our meteorological data. To do so we reconstruct the photovoltaic production from weather data. We know what are the hourly luminosity conditions on the French territory but also where is installed the photovoltaic production capacity. The SOeS (statistical observation and study department), a branch of government, publishes each year a file containing the installed capacity of renewable energy sources per communes, a French administrative unit with a typical size of roughly 3 *km*.

We use observed luminosity data from MétéoFrance, as there is no hourly forecast of luminosity, and assume a sigmoid response from photovoltaic panels to light intensity with a saturation towards high light intensity, that is approximately a linear response up to a certain threshold. The results are shown in Fig.2.3.9.

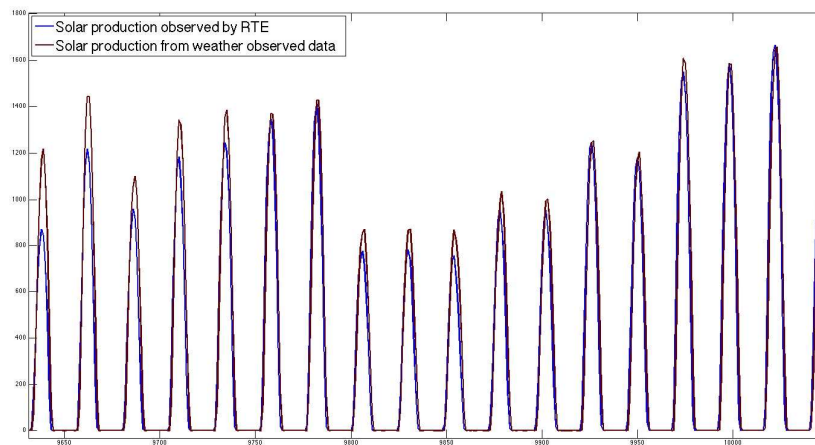


Figure 2.3.9: Hourly solar production in MWh. The time origin is the first of January 2011. In blue: observed production by RTE. In dark red: reconstructed production from observed weather data.

We observe that solar production is much more regular than wind production, therefore it is not possible to build a proxy for lighting consumption that would allow us to decorrelate the effects from production and lighting. We therefore stick to this proxy to

capture the net effect of both channels.

Other controls

Tempeff We also build an effective temperature that does not account for the non-linearity at 15°C following the same methodology otherwise as a control.

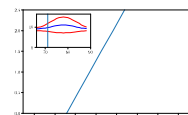
Roll_Temp H Variable capturing seasonal trends by using the rolling average temperature on effective temperature (Tempeff15) over the last H hours.

Roll $_{avgTH}$ Variable capturing seasonal trends by using the rolling average temperature on temperature Tempeff (no kink) over the last H hours, i.e. the last $H/24$ days.

suncycle Variable capturing intraday seasonality by measuring the intensity of sunlight as a percentage of the maximum daily observation. Midday is defined at the maximum sun intensity every day, i.e. $\text{Midday} = \max(\text{Solar})$. Thus, $\text{suncycle}_H = \text{Solar}_H / \text{Midday}$.

deltasun Variable computed to proxy for dusk and dawn. It is computed as the absolute difference between $\text{suncycle}_H - \text{suncycle}_{H-1}$.

SolarRest Solar represents estimates of solar production. Therefore, it is highly collinear to the daily suncycle variable since solar production is light dependent. SolarRest is the residual from a regression of Solar on suncycle and captures the unexplained part of solar production on top of pure light intensity considerations. Table 2.1 gives the results of the regression.



(1)		
	Solar	SE
suncycle	1,500***	3.903
Constant	0.876**	0.383
Observations	150,959	
R^2	0.702	
*** p<0.01, ** p<0.05, * p<0.1		

Table 2.1: Regression of Solar on suncycle

RteBlackBox RTE, the French grid operator gives day ahead predictions of the total hourly consumption, which are available at the time of bidding. This variable is called PrevConsoH.

We do not have access to the exact definition of the index and it is thus a black box. However, it is available to the firms at the time of bidding and we want to include it in the demand estimations.

At the same time, it is evident that this variable uses much of the information that we explicitly control for in the regressions, the variables defined above, therefore in addition to the possibility that we might not have all the variables that go into building this prediction for the hourly consumption, collinearity is an issue. In order to have correct coefficient estimates, we adopt an instrumental variable approach by regressing the RTE prediction on our exogenous factors, extracting the residuals and only including the unexplainable component of the RTE prediction in the demand estimation in the form of a separate variable called RteBlackBox.

Formally, RteBlackBox is equal to the predicted residuals (u) of the following regres-

sion, where X stands for the vector of explanatory variables: Tempeff15, Roll_Temp24 , Roll_Temp240, suncycle, morning, deltasun and EWH.

$$\text{PrevConsoH} = a + bX + u \tag{2.3.1}$$

In table 2.2 we give the output of regression 2.3.1 in column 1, which is strong support that our prepared data for exogenous variables is of very high quality. We highlight the significance of all explanatory variables at the 1% level and the R^2 statistic of 85.3%.

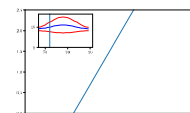
	(1)	(2)
	PrevConsoH	PrevConsoH
Tempeff15	-682.6***	
Roll_Temp24	-802.0***	
Roll_Temp240	-1,175***	
SolarRest	-0.860***	-0.345***
suncycle	7,849***	7,418***
morning	-4,759***	-4,398***
deltasun	10,108***	9,010***
EWH	-1,245***	-1,254***
Tempeff		-301.4***
Roll_avgT24		-687.3***
Roll_avgT240		-918.2***
Constant	77,701***	76,651***
Observations	146,909	146,909
R^2	0.853	0.816

*** p<0.01, ** p<0.05, * p<0.1

Table 2.2: "Black box" regression on RTE predicted consumption

Note: The dependent variable PrevConsoH is the day-ahead prediction by RTE of the total consumption in France.

We highlight that the comparison of columns 1 and 2 gives very strong support to our adjusted measure of effective temperature (Tempeff15 instead of Tempeff), which takes into account the demand behaviour as a function of the temperature. Temperatures above 15°C are considered not to impact demand behaviour [RTE, 2014].



2.4 Conclusion

In this methodological chapter we present the different methods that we developed to study in the next chapter the impact that uncertainty about demand shocks can have on suppliers' bids.

We want to be able to describe how bids change shape as a function of a number of regressors. To do so we apply functional data analysis to the bids and argue that the landmark registration technique allows us to compare important features across bids.

Finally, as we are interested in the impact of uncertainty about demand shocks, we note that weather is an important source of uncertainty and introduce a number of metrics, based on the intrinsic structure of weather data or on its relationship to the processes at play when considering consumption or production of electricity.

In the next chapter we will therefore be able to focus on the econometric analysis of our data. More specifically, now that we have defined points that allow us to compare schedules to one another, and that we have defined proxies for the weather uncertainty, we can measure how the slope of the schedules is impacted by the level of uncertainty, and if it follows the predictions of the theoretical model presented in the first chapter.

Appendix

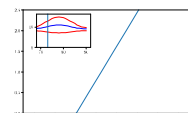
Appendix 2.A Technical Details

2.A.1 Using the kernel density estimation (KDE) in our setting

In order to estimate the first and second derivatives of the bid functions, we use a kernel density estimation. The estimator is essentially a smooth version of a histogram and counts the number of points in moving intervals (called a window) of predefined width along a dimension of the data. In our case, it counts bid points per price interval. In addition, the KDE assigns a weight to each observation based on the distance from the observation to the center of the window. The weighing function is called the kernel.

The observed bid functions are each a multitude of price-quantity combinations. However, a naive kernel density estimation on the observed points of the bid function would be useless since the number of points per price interval does not vary much with the slope of the curve.

The supply and demand functions, although defined by discrete points, whose number changes from bid to bid, are continuous functions. That is that between two successive points, the function is considered to be linear. Strictly speaking, we can therefore define a constant value of the first derivative, and we cannot define values for the second derivative. In order to circumvent this problem we want to smooth our data, which defines functions that are not twice differentiable, by using a kernel density estimate. However, this estimate needs to measure the “density of function”, so to speak, and not the density of points: if the function has two successive but distant points, a naive kernel would count no points in between them although our function is actually comprised of a segment of a given length in this region. What such an estimate should instead measure is the arc length of the function represented by the points we have, that is the summed length of



all segments present in the window of the kernel.¹⁴

This quantity can be computed exactly, however it does not play well with the regular tools for kernel density estimates in stata, so we resolve to approximate it by adding a large number of linearly interpolated points at the unit cent level (corresponding to the minimum bidding unit). The kernel density estimation is then able to estimate the absolute value of the slope of the function by simply counting the points in an interval since the number of points per price interval of constant width varies proportionally with the slope of the function over that interval. This effectively returns the estimates for the absolute values of the first and second derivatives of a smoothed version of our supply and demand functions.

Hard choices in the code of the KDE

A few specific choices have been made in the code and are detailed here.

Kernel choice: First, we use the default Epanechnikov kernel for simplicity. It is generally considered that the kernel choice has significantly less impact than the choice of the bandwidth. The use of the kernel is to weigh more the observations close to the centre of the moving window. The performance of a kernel is judged on the trade-off between variance and bias. The used Epanechnikov kernel is optimally efficient. However,

¹⁴Consider a continuously differentiable function f :

$$\begin{aligned} f: [a, b] \subset \mathbb{R} &\rightarrow \mathbb{R} \\ x &\mapsto y = f(x) \end{aligned}$$

Then the following parametrization defines the points of the graph of this function:

$$\begin{aligned} g: [a, b] \subset \mathbb{R} &\rightarrow \mathbb{R} \\ t &\mapsto (t, f(t)) \end{aligned}$$

The arc length of the graph of function f is then:

$$\begin{aligned} L(g) &= \int_a^b \|g'(t)\| dt \\ &= \int_a^b \sqrt{1 + (f'(t))^2} dt \end{aligned}$$

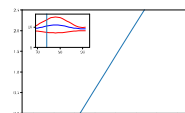
even simplistic kernel functions, such as the rectangular, have a relative efficiency of 93%. Thus, kernel choice is not important and other factors may influence the decision, such as computational effort [Salgado-Ugarte et al., 1994, Silverman, 1986].

Bandwidth choice: Second, we hard code the bandwidth selection for computational reasons. The bandwidth of the kernel (and thus the width of the price interval over which points are counted) is determined on the basis of a trade-off between smoothing the original bid function and mixing up information of different parts of the bid function. By smoothing the original bid function, we obtain estimates of the information that our KDE measures (i.e. points in the interval and thereby the slope) that are less sensitive to local specificities of the bid functions. The larger the selected bandwidth, the larger the interval over which points are counted and the stronger the smoothing of the estimates. However, as the width of the interval increases, we mix up more information of a selected point of interest with the information of its neighbouring points. Therefore, in setting the bandwidth we aim to achieve smoothed estimates with a reasonable compromise between respecting local curve information, while not being fragile to steps in the bid function.

For estimates of the first order derivative, these considerations are minor and we could use the default bandwidth, optimal for a Gaussian distribution, to extract the point of maximum slope from the distribution. However, one reason we slightly increase it is to ensure that the distribution of the first derivatives is uni-modal.¹⁵ Furthermore, the selection of the bandwidth in the first stage density estimation impacts both the precision and speed of the second stage estimation. A better smoothing in the first stage gives a large advantage in the second stage estimation¹⁶, thus we have a further incentive to increase the bandwidth.

¹⁵Uni-modal at the point of inflection in the price-quantity dimension. The smoothing ensures that the selected point is not mistaken due to steps in the bid function that have a very large slope locally, but which is not representative of the neighbouring portion of the bid function.

¹⁶The gain in computation in the second stage arises from the fact that a stronger smoothing in the first stage produces a more homogenous dataset for the second stage estimation. By more homogenous, we mean that fewer monotone regions of the graph of first derivatives must be interpolated at the unit cent level to ensure that our algorithm works correctly.



For the second derivative the trade-off is more critical: We want to obtain a reasonably broad smoothing to obtain a meaningful selection of points that is not driven by random noise. On the other hand, a large bandwidth reduces the importance of local information of a part of the curve as a consequence of which, selected points (points $k = 2$ and $k = 4$) are pushed towards the point of inflection ($k = 3$). This is due to the maximum point of the first derivative gaining more weight in the second derivative's estimation. The fact that first derivative estimates are already smoothed rather strongly, we can choose a narrow bandwidth in the second stage KDE.

In the end, we select a rather broad bandwidth of 45 units in the first estimation. This gains robustness of the point selection mechanism against noise in the data and estimation speed in the second stage. The bandwidth in the second stage is set more narrowly at a level of 2 units to keep as much information as possible from the first stage estimation and allow sufficient variation to select the k points.

To support our choice, we illustrate the impact of different bandwidths on the first and second stage estimation in figures 2.A.1 and 2.A.2. Our choice is based on an adequate point selection and the fastest runtime.

In these graphs, the top row shows the first stage KDE, over the whole function on the left and zoomed on the right. The large bandwidth in figure 2.A.1 shows the impact of smoothing on the estimates of the first derivative as compared to figure 2.A.2. The second row in both graphs shows the second stage KDE in two versions: Using a wide kernel bandwidth on the left and a tight bandwidth on the right. Again, we disclose the result as seen over the whole function (left) and zoomed on the central price range (right).

The third row details the original demand function with the final point selection given the bandwidth selection as given by the two rows above. Regardless of the first stage bandwidth, we see that a large bandwidth in the second stage KDE easily distorts the point selection. Selected points of type $k = 2, 4$ are either too centred or too wide as a

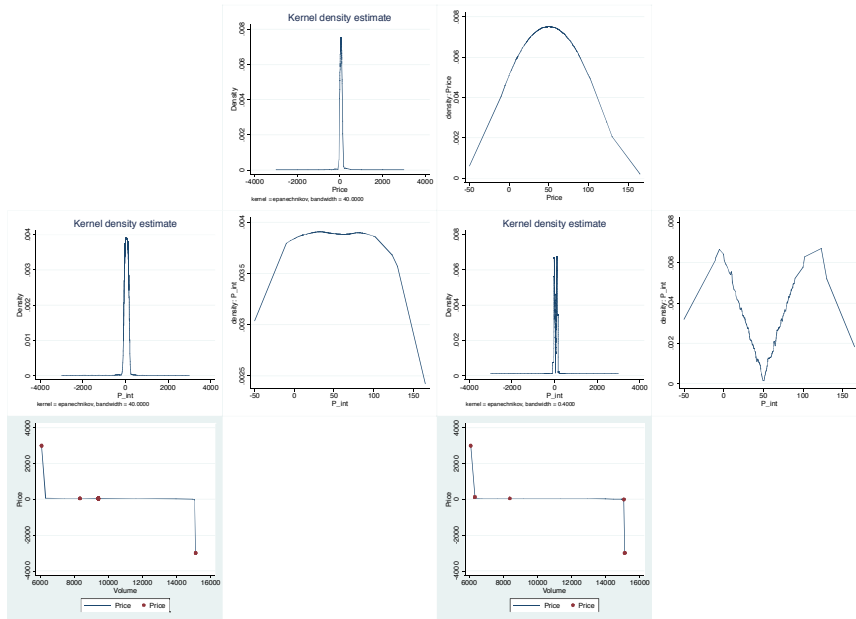


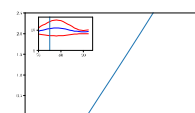
Figure 2.A.1: Comparison of bandwidths: Large bandwidth in first stage

Note: Large bandwidth in first stage (top row), large bandwidth in second stage (second row left), small bandwidth in second stage (second row right), Resulting selection of points for large bandwidth in stage one and two (bottom row left, A) and selection of points for large bandwidth in stage one and small bandwidth in stage two (bottom row right, B).

result of the second derivatives being smoothed excessively and not precisely representing the local specificities of the curve.

The right hand side of both figures show that a tighter bandwidth on the KDE can easily mistake large slope changes due to steps in the bid functions as the appropriate points of maximum curvature of the full bid function and thereby make an error. Therefore, we apply a sensitive second stage KDE on rather smooth estimates of the first derivatives, which yields an adequate point selection in our setting (figure 2.A.1B).

The bandwidth selection received much attention in this work in order to obtain a reasonable selection of points based on local information of the curves, while achieving a satisfying robustness to noise in the bid function. We are aware that this subjective setting of the bandwidth is not without consequence for our work. However for computational



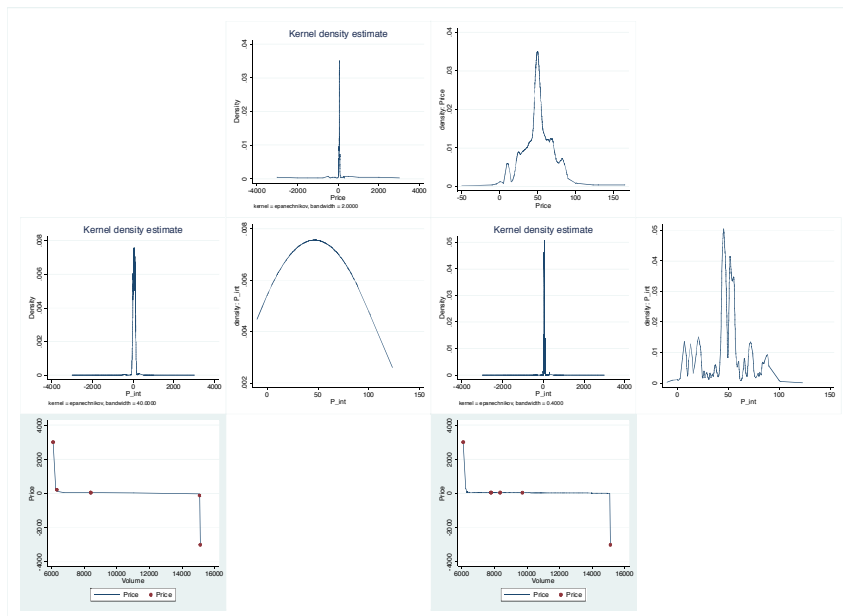


Figure 2.A.2: Comparison of bandwidths: Small bandwidth in first stage

Note: Small bandwidth in first stage (top row), large bandwidth in second stage (second row left), small bandwidth in second stage (second row right), Resulting selection of points for large bandwidth in stage one and small bandwidth in stage two (bottom row left, C) and selection of points for small bandwidth in stage one and two (bottom row right, D).

reasons¹⁷, we do not run a full robustness test on this choice ex-post.

2.A.2 Outlier detection and removal

In some rare cases, our point selection mechanism does not work. This is the case when curves have very small number of points at a kink and it is thus very difficult to detect their curvature.

As a result, the selected points are then quasi in-differentiable from the next selected point type, i.e. a point of type $k = 2$ is almost identical to the selected point $k = 3$. The code is unable to select the right points due to a data lack on the original curve (second derivative on a constant slope up to POI is zero).

We screen for adjacent points that display quasi no variation in volumes. Figure 2.A.3

¹⁷The point selection algorithm ran for more than two weeks in the current setting.

shows a histogram of volumes differences over 2 selected points (from $k = 2$ to $k = 4$) and reveals a positive mass point at zero, indicating outliers that do not display any volume variation between points of the same bid function. We use the histogram to identify and drop those outliers from our dataset.

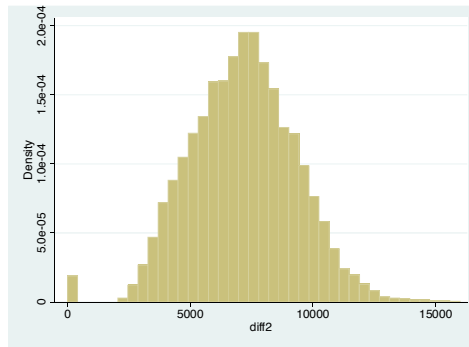
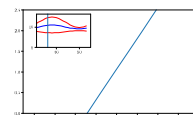


Figure 2.A.3: Histogram of volume variation between points

Note: The histogram shows the volume difference between points $k = 2$ and $k = 4$ of the same bid functions.



Chapter 3

Investigating the Impact of Uncertainty on Firms with Dynamic Costs: A Case Study of the French Electricity Market ¹

¹Joint work with Henri de Belsunce.

The weather data was obtained through a research convention with Météo-France - ref. DIRIC/13/024
JEL Classification Numbers: C10, C57, L94, Q41

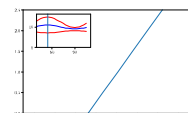
3.1 Introduction

In the last chapter, we have given some attention to a methodology that allows us to use functional data for reduced form analysis. In this chapter, we focus on the economic questions that can be asked using such a methodology. Specifically, we focus on an investigation of the effect of uncertainty on the behavior of electricity producers, and testing the predictions of our first chapter, by leveraging the results of our second chapter that allow us to compare schedules to one another.

There exists a consensus that dynamic costs, also referred to as ramping or adjustment costs, are important on the electricity market.² These are the costs incurred by a producer when production varies. The importance of uncertainty for the expectation of dynamic costs is shown in chapter 1. Uncertainty itself on the electricity market as well as estimates for the value of ramping costs have been studied empirically by [Wolak, 2007], in the case of step functions. We focus on two sources of uncertainty for traditional electricity suppliers, namely uncertainty about the realization of the market demand and uncertainty from the inherently unpredictable meteorological situation (which affects renewables generation), mainly because those are the two main sources of uncertainty in the span of time covered by our data (2011-2013). There is one blind spot in our analysis: we do not have data about the interconnected countries, which themselves affect the French market and therefore introduce another important of uncertainty. Not taking this effect into account essentially introduces noise in our data and means that we need more data to infer the significance of an effect compared to a case where we would be able to control for it. We propose a methodology to measure this uncertainty and its impact on firm strategies on the electricity market.

Electricity as a market is very important in and of itself (\$2 trillion in worldwide sales in 2010). It is also a crucial input for many industries; power outages induce very large

² [Anderson and Xu, 2005], [Hobbs, 2001], [Hortacsu and Puller, 2008], [Reguant, 2011], [Sewalt and De Jong, 2003].



costs to society ([LaCommare and Eto, 2004], [Reichl et al., 2013]). The electricity market is, however, quite different from the markets for other commodities in a few respects. First, electricity cannot be efficiently stored. As a consequence, electricity markets are high frequency (prices can update down to 15-min intervals) and firm strategies are purer as they are free of stock management considerations.

Second and in addition to non-storability, a generation surplus cannot be disposed of freely.³ Thus, generation of electricity must always be matched with consumption in real time (modulo a small tolerance). This represents a hard constraint on the market⁴ and forces suppliers to be reactive. However, this reactivity is costly as plant operators incur dynamic costs when adjusting production and the larger the adjustment made, the larger the cost. Hence, suppliers face a trade-off between cheap generation of electricity and costly reactivity to the demand realization. Indeed, no single generation technology exists that satisfies both cheap generation and sufficient reactivity to allow production fluctuations at a reasonable price. Existing generation techniques are either cheap and unresponsive, e.g. nuclear plants, or expensive and flexible, e.g. gas turbines.

Interestingly, we also observe negative prices. In France for example, during the weekend of the 15th June 2013, the price per MWh dropped to -200€ . This contrasts to the yearly average of approx. $45\text{€}/\text{MWh}$ and is generally understood as a sign that subsidizing consumption temporarily is cheaper for a supplier than shutting down a plant [EPEX, 2014].⁵ The increase of the share of renewable generation in the energy mix contributes to the occurrence of negative prices on the market. The intermittency of renewables causes large residual demand shocks [EPEX, 2014]. The unreliability of renewable generation also means that more flexible plants (i.e. plants with lower dynamic costs) are required to provide rapid responses to fluctuations in production from renew-

³The common assumption of free disposal as made in standard microeconomics is violated.

⁴Mismatches between consumption and generation ultimately result in power outages.

⁵“Negative prices are a price signal on the power wholesale market that occurs when a high inflexible power generation meets low demand. Inflexible power sources can’t be shut down and restarted in a quick and cost-efficient manner. Renewables do count in, as they are dependent from external factors (wind, sun).”

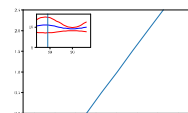
ables [REN21, 2013].

Furthermore, uncertainty arises from the fact that renewable production is a local and dispersed production, but feeds into a national market with a single price. When meteorological conditions change, the geographic production profile also changes. This further complicates the predictability of renewables generation and contributes to the uncertainty that electricity producers face when playing on the electricity market [Meibom et al., 2009].

This paper explores the effect that the absolute level of uncertainty about residual demand has on players' strategies on the electricity market. In the light of the existence of dynamic costs, which are inherent to the production technologies, uncertainty is costly to suppliers as shown in chapter 1. Thus when faced with uncertainty, we expect that electricity producers smooth production volume over time in order to minimize dynamic costs. In a single market interaction with a symmetric oligopoly and linear demand functions, this translates to playing a steeper supply function when uncertainty is high. The detailed intuition behind the predictions tested is given in section 3.1.2.

We show that uncertainty does impact supplier strategies. However, this prediction and result only apply locally to the central, flat and linear part of the supply bid function. Towards the high and low volume extremities of the bid functions when capacity constraints start to matter, bid functions are steeper and the effect of uncertainty vanishes. Furthermore, we observe results that indicate that demand-side bidding is also impacted by uncertainty.

We focus on the French one-day ahead market, EPEX Spot. This market is a divisible goods auction and particularly suited for our analysis as we observe data on the full aggregate bid functions for both supply and demand. We introduce the market's auction format and rules in section 3.2.



The dataset and its sources are presented in section 3.3. We develop our identification methodology in section 3.4. Our empirical strategy relies on the non-parametric, comparable point selection technique presented in chapter 3.3. We reuse the selected points of the previous chapter for our analysis here. We present and interpret the results in section 3.5. Finally, we discuss some overarching points in section 3.6 and conclude in section 3.7.

3.1.1 Literature review and contribution

There exists a literature on supply function equilibria initiated by [Klemperer and Meyer, 1989]. In traditional models, firms choose between quantities (Cournot) or prices (Bertrand) as their strategic quantities. In the intermediate case, firms choose a relationship between quantities and prices, namely a supply function. This is the focus of the supply function equilibrium models. A key ingredient of these models is uncertainty.

Supply function equilibrium models are very relevant for the analysis of electricity markets, since many electricity market designs allow firms to submit a price-volume function rather than a specific price or quantity. [Green and Newbery, 1992], [Newbery, 1998] and [Bolle, 1992] have used these models to analyze competition on the electricity markets.

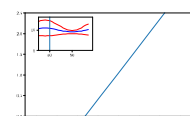
These papers have contributed to a broader investigation of the competition on the electricity markets, which has also been looked at from empirical perspectives [Wolfram, 1998, Borenstein et al., 2002]. While those initial papers have focused on the supply function equilibria of the market, they have abstracted from some technological specificities for the sake for simplification.

One such aspect that we are interested in and that has been the subject of research in recent years is the importance of dynamic costs for electricity production.

Our first chapter extends [Klemperer and Meyer, 1989] to derive predictions on firms facing dynamic costs in a supply function oligopoly under uncertainty. When varying production is costly, suppliers take these costs into consideration by submitting steeper functions when facing more uncertainty, in order to limit the range of variation in production. [Reguant, 2011] develops a model and an empirical strategy to measure dynamic costs on the Spanish one-day-ahead electricity market. She finds that “complex bids”, which allow firms to minimize dynamic costs by linking production in one time period to production in a subsequent time period, reduce the volatility and the level of prices on the market. Her work is also unique in terms of data availability. By using individual bid functions she is able to produce estimates of start-up and ramping costs per production technology.

In order to quantify dynamic costs on the Australian electricity market, [Wolak, 2007] derives a methodology to recover estimates of the parameters of parametric cost functions at the level of the production unit. His identification is based on the assumption that each profit maximizing supplier knows the distribution of shocks on the demand function when playing on the market. Uncertainty is thus an explicit ingredient of his paper and he captures two sources of uncertainty in a single index: (i) the uncertainty from not knowing the aggregate supply function served by all other suppliers and (ii) the uncertainty about the realization of the market demand. The recovered cost functions quantify the cost of varying output. Forward contracts are useful to avoid output variations. By comparing the observed level of forward contracting (assumed to be the profit maximizing choice for production variation) with the theoretical minimum cost production pattern, he does not find support for ramping costs.

We contribute to this literature by providing an empirical analysis of the French electricity market. Specifically, we look at the impact of uncertainty on supplier strategies and take this as evidence that dynamic costs matter. Our approach to separate out



the uncertainty from market demand expectations and predictability of renewables generation is novel. Both proxies for uncertainty used are new, uncertainty from market demand is inferred from the prediction errors that firms make in a demand estimation and uncertainty from renewable production is computed in a bottom-up approach from local weather forecasts. Instead of opting for a time series regression, we understand all hourly auctions as a cross-sectional dataset and control for the time of the day by using continuous transition variables for daytime periods. Similarly, we control for seasonality using continuous variables rather than dummies. Thereby, we are able to leverage our dataset and increase the sample size for each of our regressions and improve the precision of our estimates.

Furthermore, our work contributes to the empirical literature testing strategic behaviour of market participants. Generally, these studies focus on point-wise analyses for reasons of data availability. Not only does this cause endogeneity problems when the data used is equilibrium data, but also the analysis is restricted to an understanding of the usually observed outcomes of the market.

In our setting, we benefit from an interesting dataset in which we observe full aggregate bid functions of players. The functions describe the players' behaviour both in the region where the equilibrium is likely to occur as well as in regions that rarely have an impact on the equilibrium outcome. As such, they provide a much fuller description of the firms' strategies. The additional information contained in the full aggregate bid functions has been used extensively in theoretical work (notably in the supply function equilibria literature mentioned above). However, few papers exploit these full bid functions empirically.

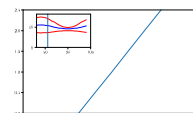
For the government bond market, [Préget and Waelbroeck, 2005] and [Özcan, 2004] use a parametric approach to this functional data for a description of the variation of bid functions with respect to exogenous factors and an investigation of the revenue superior-

ity of the uniform or discriminatory multi-unit auction mechanism, respectively. On the electricity market, [Wolfram, 1999] leaves the analysis of equilibrium data to investigate duopoly power of firms on the UK day-ahead spot market. Instead, she uses information from the whole aggregate supply function to investigate the impact of price caps for electricity producers. Using an analysis conditioned on 25 different demand levels, she shows that the introduction of price caps resulted in a counter-clockwise rotation of the aggregate supply function. She relates these results to produce a lower bound on the extent to which firms can increase their prices above marginal costs when regulatory pressure makes it advantageous to do so. Thereby, she contributes empirical evidence for the distorting effects of price caps.

Our work adds to this empirical literature using the information contained in the full bid functions by developing a non-parametric approach which allows to condition our analyses on multiple, representative points of the bid functions. The statistical ingredients rely on [Silverman and Ramsay, 2005] and are detailed in chapter 3.3. Thereby we are able to leverage our dataset, increase the sample size in individual regressions as well as obtain a fuller picture of the effects of exogenous variables on the behaviour of electricity producing firms. We emphasize that our approach allows to overcome structural restrictions underlying previous parametric approaches, e.g. the symmetry of the logistic function used in [Préget and Waelbroeck, 2005].

3.1.2 Theoretical prediction

We test the impact of uncertainty of supplier strategies by testing the prediction that suppliers bid steeper supply bid functions when faced with a larger uncertainty concerning the outcome of the (residual) demand realization, for which the traditional supply function equilibria framework provides no prediction at all, which means that our null hypothesis, that there is no effect of uncertainty of the slope, corresponds to the regular SFE framework.



In a discontinuous setting, where the supplier produces volume Q_H of electricity in hour H , we assume that he faces a cost function $C_i(\cdot)$ for each production plant i . This cost function depends on both marginal costs of production as well as the dynamic costs for changing production rapidly: $C_i((Q_H), (Q_H - Q_{H-1})^2)$. The larger the variation in production between hours, the larger the dynamic costs. Even when the expected residual demand is constant, there are still fluctuations in the production due to possible shocks to the residual demand. The larger the shocks, the larger the change in production and thus the larger the dynamic costs. Consequently, increased uncertainty (as represented by shocks on the demand function) translates into increased expected dynamic costs. We assume that the profit maximizing supplier knows the distribution of shocks on the demand function when choosing his supply function. In order to minimize these costs, the producer can choose a steeper supply function when uncertainty is high. We want to test this prediction.

We illustrate the intuition behind this prediction using a stylised case in figure 3.1.1. The graphs depict a situation in which a single, risk-neutral supplier bids a supply function to supply electricity in the hours 9 and 10 of the next day. For both hours, the supplier faces a constant expected residual demand function represented by $E(D)$. In a static optimization problem, the supplier would bid a supply function S_0 in both auctions.

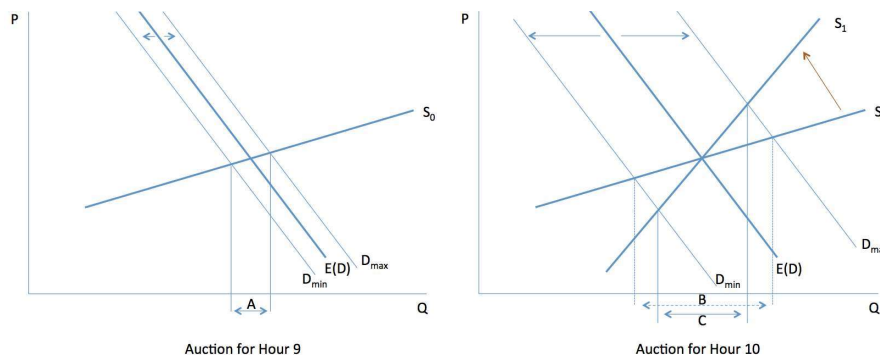


Figure 3.1.1: Illustrating the effect of increased uncertainty.

The uncertainty in the market is represented by the width of the envelope of shocks

that affect the residual demand function (represented by the arrows on $E(D)$). Thus, in each hour, the residual demand fluctuates between D_{min} and D_{max} , where the range between the extremal demands may vary from one hour to the next.

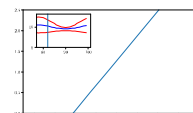
Before submitting a supply function to the market, the supplier estimates the distribution of probabilities of demand shocks that he will face. In hour 9, the supplier is able to rather precisely predict the realization of the demand function in the auction, i.e. it realises within a tight confidence interval. In hour 10, however, uncertainty in predicting the outcome of the demand realization has grown strongly as represented by the much wider confidence interval on the demand realization.

Given a fixed supply bid function S_0 , the possible range of quantities to be produced by the supplier when going from hour 9 to hour 10 has increased due to the increase in the size of the uncertainty (interval on the Q-axis has grown from length A for hour 9 to the dotted length B in hour 10).

Now, we assume that the supplier faces dynamic costs, i.e. it is costly for production to vary on top of any traditional marginal cost consideration and the larger the variation, the larger the cost. Then in the case of a fixed supply bid function (S_0 in both auctions), an increase in uncertainty implies an increase in expected dynamic costs.

The supplier's reaction to increased uncertainty is therefore to bid a steeper supply function S_1 in order to trade-off static optimality and dynamic effects. As a consequence, the range of volumes produced in equilibrium is reduced (the firm produces in the range C instead of B). When seen over time, these considerations lead to a smoother production as compared to a constant supply curve: demand shocks are absorbed through a higher price volatility and a lower production volatility.

If cautious behavior under high uncertainty is true for all firms on the market and



each firm has the same expectation of the probability distribution of the uncertainty, then the reaction of bidding a more price inelastic supply function to increased uncertainty should be observable on the aggregated supply function.

We emphasize that this prediction relies on linear demand and supply functions and does not incorporate capacity constraint considerations (both upper and lower bounds on the production volume of plants), which are also important on the market. Furthermore, we have outlined our prediction using a discrete time-setting. The continuous version of this analysis on dynamic costs is explored in detail in chapter 1.

The present paper tests this mechanism empirically and understands an increase in the slope of aggregate supply bid functions due to an increased level of uncertainty as evidence that firms minimize dynamic costs across auctions.

3.2 The EPEX Spot Market

3.2.1 General background

The EPEX Spot market is an auction market, which allows firms to trade electricity 12-36h ahead of delivery. It covers France, Germany with Austria and Switzerland. The volume traded on EPEX Spot represents 12%, 40% and 30% of the total electricity consumption in these countries respectively in 2013 [EPEX, 2014].

The EPEX Spot market has considerably gained in importance over time, and the daily trading volume has almost quadrupled since 2005, whereas the total electricity consumption has essentially remained constant. The graph in figure 3.2.1 shows these trends very clearly. Furthermore, it shows the significant volatility of the market trading volume (as indicated by the width of the grey-shaded confidence interval).

On the EPEX Spot market, the participants submit supply or demand bid functions

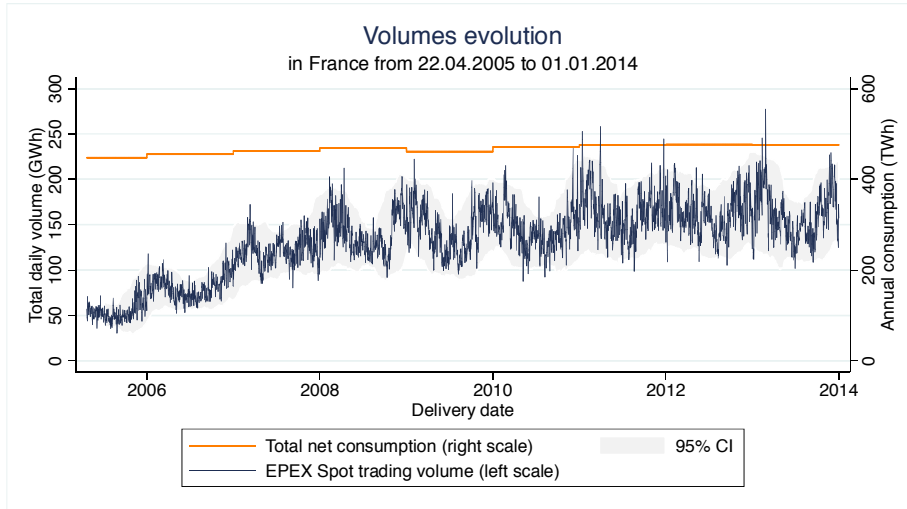
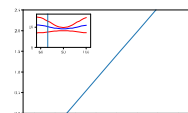


Figure 3.2.1: Traded volume plotted against total annual consumption

Note: Total consumption is netted of the electricity withdrawal at the level of the production unit. The 95% confidence interval is based on a 150-days moving window and assumes that volumes are normally distributed in the time window. GWh and TWh stand for giga and terawatt hours, respectively.

to be able to meet their next day’s supply commitment. This market is important, because it allows the firms to adjust their portfolio to the upcoming demand. The market matches business to business trades, where producers (the suppliers and transmission system operators) and industrial consumers may participate.

The EPEX Spot market settles in a three-headed market that firms use to achieve their desired power position: The long-term bilateral contracting market, the day-ahead market and the intra-day market. Energy cannot be stored, thus a precise power position must be achieved at each point in time. Firms thus face a trade-off between cheap up-front sourcing and costly uncertainty. The closer the market gets to the delivery of its power, the less uncertainty does the firm face in determining its power requirements (pushing firms to wait until the last minute to fill their energy position). However, the imperfect flexibility of the electricity production landscape cannot satisfy the whole demand short-term at a reasonable price, hence firms must anticipate their requirements in order to obtain cheaper power. Consequently, these three markets complement each other to allow firms to gather a power position at a reasonable price.



3.2.2 Auction rules and mechanism

We present the rules here that applied during the period for which we have data.

The EPEX Spot auction occurs daily, all year-round, and proceeds as follows: the order book closes every day at noon for contracts of the following day, results are published two hours later. Bids may be submitted 24/7 from 45 days prior until the closing of the books.

Tradable contracts exist for each hour of the day and firms submit an individual bid function for each of these hours, i.e. a separate, simultaneous auction is run for all hours of the following day and trading is specific for each of these hourly tranches.

The bid submission must be a supply function (or a demand function depending on the position of the firm) with at least 2 and at most 256 price/quantity combinations for single contract orders. The final bid function, thus, consists of the explicitly submitted points and all linearly interpolated points between them. The bid curves must be monotonically increasing for a supply function and vice versa for a demand function. Orders are transmitted via an online IT-platform and a redundant confirmation process aims to avoid erroneous bids. Bids are anonymous, and the final electricity distribution is done via the French distribution network controlled by RTE EDF Transport SA.

Prices are specified in €/MWh with two decimal digits and must range from -3000€/MWh to +3000€/MWh. Quantities are specified in whole MWh. In addition to single contract orders for an individual hour, bidders may submit block orders.

These are combined single contract orders with a minimum of two consecutive hours. The vital difference with multiple single contract orders is the "All-or-None" condition, namely that the executions of the individual contract orders forming the block are dependent on one another. That is for a block order covering hours 17 to 20, the quantity

demanded for the hour 17 is only awarded if the corresponding quantity is also awarded for the hours 18, 19 and 20. Each registered bidder account is limited to a maximum of 40 block orders per delivery day, each of which is limited in volume to 400 MWh (approx. equal to 0.25% of total daily volume traded on EPEX Spot).

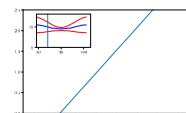
The price-quantity determining mechanism is a uniform price, multi-unit auction mechanism: the summed demand and supply curves are computed and the intersection of these gives the equilibrium price and quantity pair.⁶ The market clearing mechanism takes into account single and block orders simultaneously and hence solves the corresponding programme by an algorithm of full enumeration of possible solutions, where each partial solution is verified to provide real, compatible prices. The mechanism works under a time limit. In the case of a curtailment, i.e. a disequilibrium with disproportionate prices due to unmatched supply and demand or an abnormal price for a specific hourly contract, the system proceeds to a second price fixing.

Of particular interest is the clear distribution of information. Ex-ante bidding, firms in the market know the identities of the rival bidders they face (but neither their individual bid functions nor their results in past auctions), the history of aggregated equilibrium prices and quantities up to that day, their clients' past demand realizations and their individual long term contracting position. Upon the clearing of the market, the aggregated supply and demand bid functions, equilibrium quantity and the equilibrium price

6

The Auction takes place daily, after the Order Book has closed. The price corresponds to the Matching of Exchange Members' aggregate supply and demand curves of both Single Orders and Block Orders for each Contract. The price determined by the algorithm at the time of Auction is the price at which all Trades will be executed. For price determination purposes, the Exchange Member's interest is assumed to be linear between two price/quantity combinations. The price determination algorithm aims at optimising the total welfare, i.e. the seller surplus, the buyer surplus and the congestion rent including tariff rates. The algorithm determines the execution prices, the matched volumes and the net positions of each coupled market if applicable. It also returns the selection of blocks that will be executed and other complex Orders allowed in other Coupled Markets¹ if applicable. The presence of all-or-none Block Orders in the Order Book makes necessary the use of a specific search algorithm, in order to determine a market clearing price.

[EPEX, 2018b]



become common knowledge. Each bidding account is informed of the contracts it has been awarded, i.e. the individual quantities to be sold and bought through the system.

3.3 Our Data Explained

Auction market data

We have data from the French EPEX Spot market for the period 01.01.2011 to 30.06.2013. This is the latest period, where no significant changes in the auction rules have occurred and where data for all variables can be observed.

We observe the full aggregate bid functions for the day-ahead auctions of each hourly contract for both supply and demand. We understand the dataset as a cross-section rather than a time-series⁷ and focus on weekday trading contracts only. This sums up to about 31 500 observations.⁸ A single aggregate bid function is the sum of the individual bid functions, which are not available. We also observe the equilibrium price and quantity for each auction. Moreover, we observe the block bidding results at the equilibrium solution only. We ignore the blocked aspects and treat subsequent auctions as independent from one another.

The two graphs in figure 3.3.1 show the aggregate supply and demand bid functions for the same hour of the same day. For a glimpse at the variation of bid functions over time, see figure 3.3.2. The table 3.1 sheds some light on the raw data. For further details as well as the plotted distribution of realised market equilibria, refer to appendix 3.B.2.

Finally, we reuse the data output from chapter 2. Specifically, we reuse the specific points extracted from the aggregate demand and supply bid functions, which are comparable across auctions. Why these points are useful for our analysis is explained in the

⁷This is supported by the graph in figure 3.2.1, which shows a flat total consumption and average trading volume on EPEX Spot since 01.01.2011.

⁸31 500 observations \approx 2.5 years of hourly (*365 * 24) demand and supply (*2) functions for weekday trading (*5/7).

methodology section 3.4.

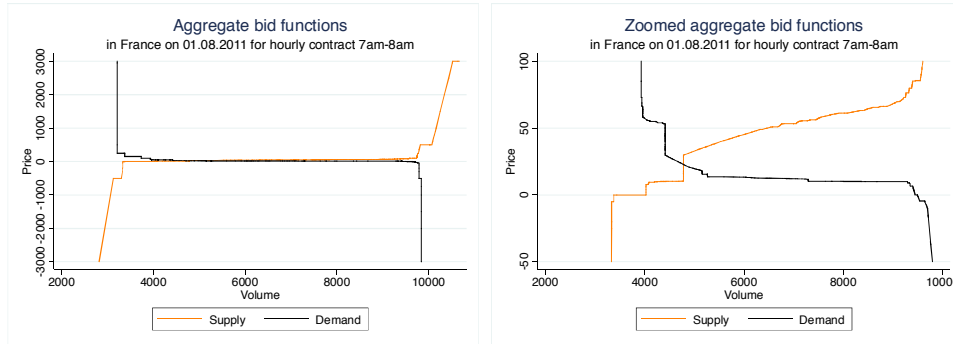


Figure 3.3.1: Example aggregate demand and supply bid functions

Note: The right-hand-side graph is a zoom of the left graph on for the price range $-50\text{€}/\text{MWh}$ to $+100\text{€}/\text{MWh}$.

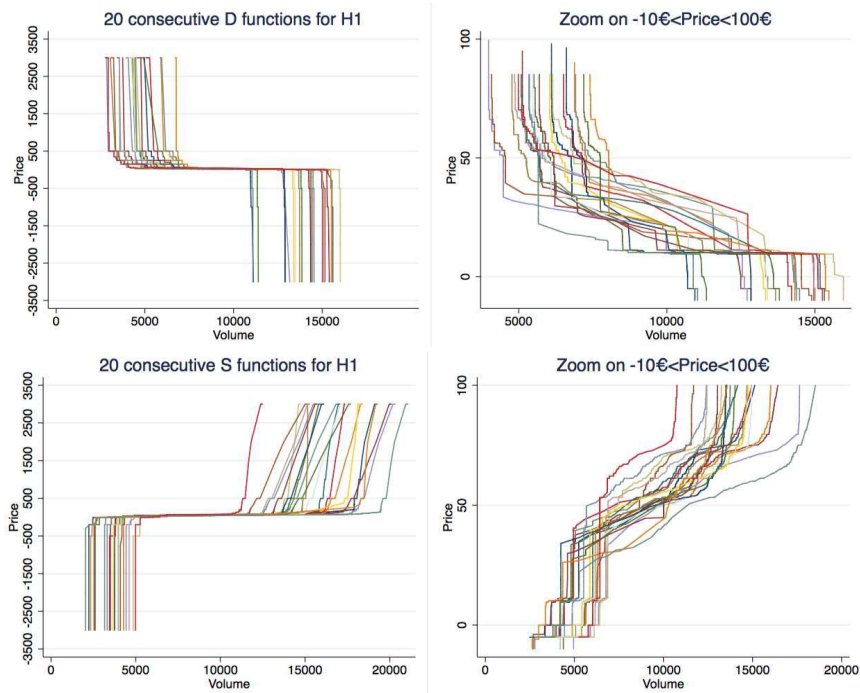
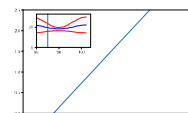


Figure 3.3.2: Aggregate bid functions for 20 consecutive days

Note: The graph shows 20 consecutive aggregate demand and supply functions for the contracts on hour 1 (between 12 am and 1 am) for the time period 11/12/2011 to 31/12/2011. The graph on the right is a zoom on the price elastic region of the curves on the left.



	Mean	Median	Std. Dev	Min	Max
Total daily volume	161,912	159,313	25,059	99,054	277,531
Average realised daily price ⁹	46.6	48.3	17.2	-39.0	381.2
Minimum demanded agg. volume ¹⁰	5,030	4,968	1,467	914	11301
Maximum demanded agg. volume	13,327	13,222	2,212	4,990	23,254
Minimum supplied agg. volume	3,721	3,526	1,344	618	10594
Maximum supplied agg. volume	14,390	14,142	3,051	6,580	35,356
Bid points per demand function	543	531	163	115	1,253
Bid points per supply function	640	632	143	184	1,283
Bidders per auction ¹¹	-	-	-	1	101

Table 3.1: Some descriptive statistics

Exogenous factors

Regarding weather statistics, we have hourly previsions for temperature, wind and cloudiness from the GFS (Global Forecast System) as well as hourly observations for these quantities and luminosity from MétéoFrance . The previsions from the GFS are in the form of weather maps that are outputted from simulations that run one-day ahead at 6 am. This is the weather information that market participants have access to when bidding on EPEX Spot.¹² The weather observations are in the form of tables for specific weather stations (between 100 and 200 depending on the specific parameter of interest).

Moreover, we have the location of the total installed capacity per generation type

⁹Average price is volume weighted over the 24 hourly contracts of the delivery day.

¹⁰Minimum and maximum volumes for both demand and supply refer to the aggregate volume bid on the market for a single hour contract at the extremal prices of +3000€/MWh or -3000€/MWh.

¹¹Due to the anonymity of the auction procedure, it is unknown which bidders submitted bids. Consequently, it cannot be deduced how many bid steps a typical bidder submits. Number of registered bidders for the French EPEX Spot market as of 01.10.2014.

¹²The next weather simulation run takes place at 12 noon and is therefore not being used by the bidders on the EPEX day-ahead market, as the deadline for submitting bids is precisely 12 noon.

(i.e. wind turbines, solar panels, etc.) at the level of the postcode, that is roughly a 3km precision. We obtain this data from the SOeS, a branch of the French government producing data on environmental issues at large.

Population data and data on the level of the domestic production from the manufacturing industry is obtained in monthly steps from the French National Institute of Statistics and Economic Studies (INSEE). From the same source, we obtain the spot prices for petrol and natural gas as well as the import prices at the border for coal, which we use as a proxy for the domestic prices. Prices for the European CO₂ emission certificates are taken from the Portuguese secondary market (SENDECO₂) for European Unit Allowances (EUA).¹³

As a very coarse proxy for generation from hydro power plants, we have the total weekly stock of water in domestic dams (in the form of the summed height of all dam water levels in France) from RTE the grid operator.

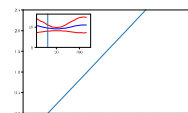
3.4 Methodology

We want to identify the impact that the level of uncertainty has on the price elasticity of the aggregate supply function. In data terms, this means that we aim to regress the slope of (aggregate) supply bid functions on a proxy corresponding to the uncertainty that existed at the time of bidding. The uncertainty may come from two different sources: (i) uncertainty about the realization of market demand and (ii) uncertainty on the generation from renewables. Both types of uncertainty affect the residual demand curve faced by each supplier.¹⁴

This regression is able to explain how supply firms adjust their bidding strategies to the expectation of demand shocks that they face. Statistical significance of the level of

¹³Each unit EUA permit allows one tonne of CO₂ emissions.

¹⁴Renewable generation benefits from a feed-in guarantee on the market and thus reduces the residual demand for all traditional electricity producers.



uncertainty on the slope of the supply function would be evidence that firms take the strategic considerations of dynamic costs into account.

First, in section 3.4.1 we show the final regression of interest. Sections 3.4.1 and 3.4.1 then detail the theory and empirics underlying the variables that feed into the final regression.

Some of the information used in our analysis is drawn from the bid functions of the EPEX Spot market. As introduced in section 3.3, we observe the full aggregate bid functions for both supply and demand, the shape of which (and thus the information that we aim to extract from them, e.g. their slope) varies differently at different points (recall the graphs in figure 3.3.2).

Generally speaking, a regression aims at quantifying the impact of some independent variables on a dependent one. The dependent variable is most frequently numerical, and the independent variables explain part of its value. Here the dependent variable is functional in nature, that is that we aim to describe how the supply function changes shape with respect to some independent variables. One observation is formed of one function coupled to the value of some independent variables. We therefore adopt a functional data analysis approach, which allows us to condition our analysis at specific points $k = 1, \dots, K$ of the functions. This approach allows us to define comparable points across auctions, that is different functions, in order to derive insights.

More precisely we want to quantify how uncertainty affects the strategy of bidders from one hour to the next. For this we cannot rely on a standard estimation of the overall demand or supply functions from market outcomes, we want to actually measure how the functions that we observe change shape.

The methodology to select comparable points across auctions is detailed in chapter 2.

This appendix also evaluates the results when applying the technique to our data from the EPEX Spot market. Figure 3.4.1 shows the selected points on an exemple of demand and supply curve.

The different types of points selected capture different information of the aggregate bid functions. The most important point is the one we label $k = 3$, which corresponds to the central part of the bids. This point is most relevant for equilibrium determination.¹⁵ The points $k = 2, 4$ are the points of maximum curvature and represent the transition points between the central (very price elastic) region and the outer (very price inelastic) regions of the bid function. Last, we have the points $k = 1, 5$ which are imposed by the auction rules and are the endpoints of the bid functions.

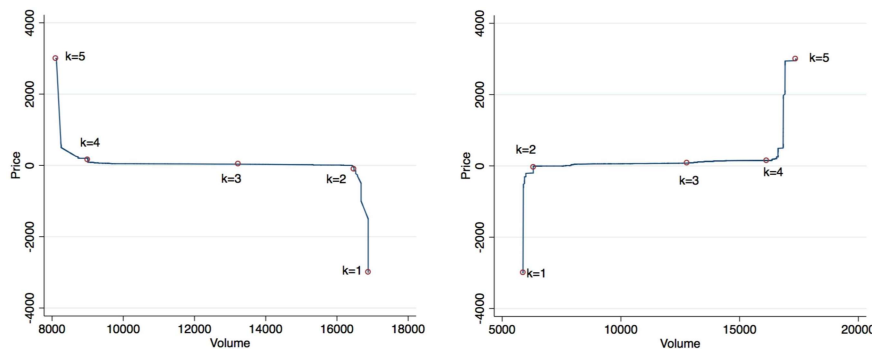


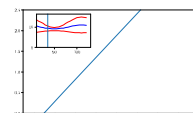
Figure 3.4.1: Selected points on original bid functions

Note: The demand function left, the supply function right, the graph superposes and names the points selected according to the methodology of section 3.4.

In chapter 2, we also detail the choice of setting $K = 5$ and show that this choice allows us to improve the precision of our analysis by a factor of 50 when it is conducted on the 5 points simultaneously.¹⁶

¹⁵See figure 3.B.2 for a glimpse at the distribution of equilibrium outcomes.

¹⁶We briefly mention that the evaluation of the point selection has revealed focal price points for the points $k = 2, 4$. These points are however rarely relevant for equilibrium determination.



3.4.1 Regression methodology

Identification

At each of these comparable points, we want to identify the effect of uncertainty on the slope of the supply function.

Defining $S'_{i,k}$ the slope of the supply function of auction i at point k in the quantity (X-axis) - price (Y-axis) dimension, \mathbf{X}^S being the vector of exogenous variables, $\text{PLU}_{i,k}^D$ being the proxy for the level of demand uncertainty, PLU_i^R being the proxy for the level of uncertainty from renewables, what we called the width of our possible shocks in the first chapter, α being the regression constant and ϵ being the error term, we estimate the following:

$$S'_{i,k} = \alpha_k^S + \beta_k^S \text{PLU}_{i,k}^D + \gamma_k^S \text{PLU}_i^R + \boldsymbol{\delta}_k^S \mathbf{X}_i^S + \epsilon_{i,k}^S \quad (3.4.1)$$

We are interested in the sign and magnitude of the coefficients β^S and γ^S , which identify the effects of the PLUs (PLU^D and PLU^R , respectively) on the shape of the supply bid function. From the predictions outlined in section 3.1.2, we expect a positive coefficient when uncertainty levels increase.¹⁷

Left-hand-side variables

We extract the slope of the aggregate supply function at any given point k from a kernel density estimation with a bandwidth of 45 units.¹⁸

Effectively, this is a smoothed version of the slope. This makes our slope estimates robust to steps in the bid function¹⁹, which in turn allows us to test the pre-

¹⁷Specifically, we want β^S to be positive, γ_1^S positive and γ_2^S negative. For details on γ^S , see section 3.4.1.

¹⁸The slope is a by-product of the point selection mechanism, and the bandwidth selection for the smoothing thus follows the same considerations as for the latter. The details of this choice are specified in chapter 2.

¹⁹In our data, we observe that bid functions are effectively step functions. On EPEX spot 256 price-

dictions from the theoretical paper. Steps in the bid functions mostly are much larger towards the extremities of the bid functions and probably arise from capacity constraints considerations. Working with smoothed slopes is in line with previous work à la [Préget and Waelbroeck, 2005] and [Özcan, 2004], who also apply reduced form models to aggregate bid function data.

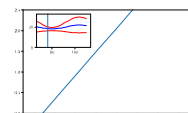
Right-hand-side variables

We are regressing an ex-post measure of the auction market (realised slope of the supply bid function) on ex-ante information that bidders have at the time of bidding, i.e. which is available at midday of the day ahead of delivery. We thus keep a strict separation of the ex-post and ex-ante information to the left and right hand side of equation 3.4.1, respectively. This separation allows us to circumvent endogeneity problems and validates the use of simple OLS regressions.

For this reason, we construct our PLUs on the basis of predicted uncertainty. However, for data availability reasons we cannot exclude endogeneity problems completely. For details, see the discussion in section 3.6.3.

In this subsection, we first outline how we generate the proxies for the level of market demand uncertainty (PLU^D) in section 3.4.1. Second, we construct the proxies for the level of uncertainty from renewables energies (PLU^R) in section 3.4.1. Third, we detail how the vector of exogenous variables (\mathbf{X}) is constructed in subsection 3.4.1.

Generating proxies for uncertainty from market demand (PLU^D) We construct a proxy for the level of the demand uncertainty (PLU^D) by using the residuals from a demand estimation on exogenous parameters as a measure of the uncertainty that quantity combinations are allowed per bidder. When additional bid points are costly, then stepwise bidding behaviour may be very different from a setting where continuous functions can be bid [Kastl, 2011]. Due to the fact that, on average, we do not observe that firms use up all available price-quantity combinations, the cost argument of an additional bid point seems weak. Hence, by smoothing the slope we approximate the unconstrained, continuous bid function.



bidders face in an auction. Specifically, our PLU^D is the expected squared level of the prediction errors that firms expect to make when anticipating the demand level of the day ahead. We assume that the ex-post prediction errors give a reasonable estimate of the uncertainty at the time of bidding.

The uncertainty proxy is obtained as detailed next in a three-step procedure. In the first step, we explain what kind of uncertainty our PLU^D refers to. The second step details the conceptual details of constructing the PLU^D . The third step computes the PLU^D .

In the first step, we focus the analysis to a fixed number K of comparable points across auctions by using the non-parametric point selection technique outlined in section 3.4. Each k^{th} point is defined by a price and a quantity, which we regress independently on the exogenous variables.

Let us call $P_{i,k}^D$ and $Q_{i,k}^D$ the price and quantity of point k of the realised demand function in auction i , \mathbf{X}_i^D the vector of exogenous variables relevant for the demand estimation.

$$P_{i,k}^D = \alpha_k^{D,P} + \beta_k^{D,P} \mathbf{X}_i^D + \epsilon_{i,k}^{D,P} \quad (3.4.2)$$

$$Q_{i,k}^D = \alpha_k^{D,Q} + \beta_k^{D,Q} \mathbf{X}_i^D + \epsilon_{i,k}^{D,Q} \quad (3.4.3)$$

In regressions 3.4.2 and 3.4.3, firms try to anticipate the realization of the demand using the exogenous information available. We consider that the producers are able to do such an analysis at the time of bidding.

The prediction errors $\epsilon_{i,k}^{D,J}$, $J = \{Q, P\}$ are a consequence of the stochastic nature of the demand and hence a manifestation of the uncertainty. We consider that more uncertainty will lead to larger prediction errors being made in equilibrium and adopt the square of the residuals $(\epsilon_{i,k}^{D,J})^2$ as our measure for the realised level of demand uncertainty.

In the second step, we recover the residuals from the demand estimation in regressions 3.4.2 and 3.4.3 and test for heteroskedasticity using [White, 1980], which is clearly confirmed (see tables 3.3 and 3.4).

Heteroskedasticity means here that the variation of error terms varies conditional on the levels of the exogenous factors: $E(\epsilon_i^2 | \mathbf{X}_i) = g(\mathbf{X}_i)$. However, they are still orthogonal: $E(\epsilon_i | \mathbf{X}_i) = 0$, thus ensuring that the prediction is unbiased, but not “best” in the sense of the best linear unbiased estimator (BLUE). Thus, heteroskedasticity results in inefficient regressions where the estimator is not minimum variance. Since we do not interpret regressions 3.4.2 and 3.4.3 for causality, but only for predictive purposes, we stick to the unbiased OLS.

The heteroskedasticity regression is given for $J = \{P, Q\}$ by

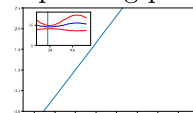
$$(\epsilon_{i,k}^{D,J})^2 = \alpha_k^{U,J} + \beta_k^{U,J} \mathbf{X}_i^D + \epsilon_k^{U,J} \quad (3.4.4)$$

In the third step, we compute the predicted $\text{PLU}_{i,k}^D$ that firms use when bidding in the auction as:

$$\underbrace{(\epsilon_{i,k}^{D,J})^2}_{\widehat{\text{PLU}}_{i,k}^D} = \alpha_k^{U,J} + \beta_k^{U,J} \mathbf{X}_i^D \quad (3.4.5)$$

The idea is that by experience, firms in the market know that their predictions are more or less accurate depending on the environmental conditions (in the sense of realizations of exogenous factors). In other words, firms can use the realizations of \mathbf{X}^D to infer the accuracy of their demand predictions. Technically speaking, they can use the heteroskedastic nature of the residuals to forecast the level of uncertainty that they face.

The PLU^D subs into regression 3.4.1. For simplicity, we do not include the uncertainty proxies $\text{PLU}_{i,k}^D$ measured at all $K = 5$ points in regression 3.4.1 simultaneously, but only a single $\text{PLU}_{i,k}^D$ at a time. Therefore in the final regression 3.4.1, we regress the slope at a point of the supply function on the $\text{PLU}_{i,k}^D$ estimated at the corresponding point on



the demand function. The pairing is done in the quantity dimension. This means that the slope of the supply function at point $k = 2$ is regressed on the uncertainty measured at point $k = 4$ of the demand function (recall the labelling of the points as given in figure 3.4.1). We indicate this quantity pairing in the index k^{-1} of the PLU:

$$\text{PLU}_{i,k}^D = \widehat{\text{PLU}}_{i,k^{-1}}^D \quad (3.4.6)$$

An increase in PLU_i^D corresponds to an increase in the uncertainty about the market demand realization. We thus expect β^S to be positive in regression 3.4.1.

Generating proxy for uncertainty from renewable energies (PLU^R) We have already referred to the statement that the intermittency of renewables causes large residual demand shocks [EPEX, 2014]. Suppliers are thus wary of the expected production of renewables generation.

Given that renewable generation is an exogenous source of supply and is completely injected on the network without being bidden for (fixed feed-in tariff), it affects the residual demand curve for each supplier, but does not enter the PLU^D , which captures the uncertainty on market demand only.

In predicting the generation from renewables, we assume that suppliers are able to infer renewables generation from meteorological forecasts.²⁰

When forecasting the residual demand shocks due to generation from renewables, we consider that suppliers have an idea of the precision of their estimate based on the “look” of the meteorological forecasts that they have. By look, we mean the geographical heterogeneity or homogeneity of the forecasts, i.e. if when looking at a weather map, one sees a lot of spatial variations or not.

²⁰We specify the technique in chapter 2 and use it to construct our controls in section 3.4.1.

We consider that this notion of geographical heterogeneity of the forecasts correlates to the uncertainty associated with the prediction of renewable production. The argument is as follows:

First, renewable production is built by aggregating the forecast of all individual renewable sources. This means knowing the position and capacity of every renewable source, querying weather forecasts for all of these points, modeling the renewable's response to the forecasted weather and adding the forecasted productions.

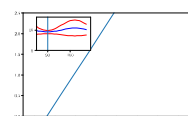
Second, we note that weather is spatially correlated, which means that the closer two points are, the closer the values for a given weather variable (the air temperature at your left hand is very close to that at your left hand, but less so across the city, and even less so across the country). This correlation roughly follows an exponential law: the difference between the values of a weather variable between two points behaves in a linear fashion for small distances and saturates at large distances.²¹ The transition between those two regimes is given by a characteristic lengthscale, a bit less than 200km on average.

Third, we observe that the average distance between production points is large enough that the relevant regime of autocorrelation is the saturated part.²²

Fourth, we note that there are two main channels through which the overall uncertainty about renewable production is related to the weather. There is an issue of error averaging, which means that if the weather becomes very spatially uncorrelated, one can expect errors to cancel out relative to a given bias in the forecast. This channel would

²¹Intuitively, the characteristic lengthscale of autocorrelation represents the distance required between two geographical points on a map of weather forecasts to observe a decorrelation of half of its maximum value. For example on the wind speeds prediction, a characteristic length of 80 km means that if we observe two very distant points (say 1000km) to have a difference in wind speeds of, on average, 50km/h (this being the maximum difference, we are in the saturated regime), then we will observe, on average, wind speed differences of 25km/h for points distant from each other by 80km.

²²For N production points, we compute the $N(N-1)/2$ pairs of points, consider their distances and compute the average of these distances weighted by the production capacity at every point. In the case of the wind, we have an average distance of 459 km, in the case of the photovoltaic production we have an average distance of 499km.



tend to imply that more spatial variations imply a smaller uncertainty about production. There is also the issue that weather forecasts are numerical simulations and that the mesh size for such simulations, typically 5km for the high precision ARPEGE model of Météo France, implies that the errors are higher as the simulated phenomena have higher gradients. This means in our case that the uncertainty about the forecast increases as the weather becomes more spatially uncorrelated.

Fifth, these two effects are of opposite signs, but our third point is an argument for considering that the averaging of errors is smaller than the simulation errors. Therefore, we expect our uncertainty to increase as the spatial autocorrelation decreases (i.e. more spatial variation).

This can be summed up with the following hand-waving argument: when there is more spatial variations, the weather is more messy, therefore more difficult to predict.

We compute this characteristic lengthscale (L) as described in chapter 2. Our PLU^R is defined as the two proxies

$$PLU_{1,m}^R = \frac{1}{L_m}, \quad \text{where } m = \{\text{Wind, Solar, Temperature}\} \quad (3.4.7)$$

$$\text{and } PLU_{2,m}^R = \left(\frac{1}{L_m}\right)^2 \quad (3.4.8)$$

As explained above, we expect firms to face less uncertainty in predicting weather conditions when the lengthscale of autocorrelation L is longer since the overall weather conditions will be more homogenous. A longer length L (less uncertainty), will yield a smaller PLU^R and we expect a flattening of the supply curve. I.e. we expect a positive coefficient γ_1^S on the $PLU_{1,m}^R$ variables in the final slope regression.

However, we also expect the effect of L on the slope to be attenuated, if not coun-

terbalanced, by the squared term.²³ This means that for very small L , we expect an additional effect, that of the summation of errors, to become significant and reduce the uncertainty, or at least its rate of increase: we thus expect a negative coefficient γ_2^S on the squared PLU^R term in the final slope regression (equation 3.4.1).

Controls This section details the exogenous variables, which we use for our study. The stacked vector of exogenous variables is not identical for the supply and demand regressions of equations 3.4.1 and 3.4.2.

The vector \mathbf{X}^D for the demand equation includes the variables: Tempeff15, Roll_Temp24, Roll_Temp240, suncycle, morning, deltasun, EWH, SolarRest, RteBlackBox.

For the supply regression we include in \mathbf{X}^S the following variables²⁴: Coal, Brent, Gas, IT2, EUA, Wind1DA, Hydro.

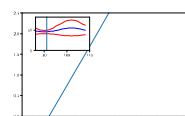
Table 3.2 gives a brief overview of the controls used. Details on the computation of some variables are given in the appendix (see links in table). The last column indicates the frequency with which we observe the variable in question.

Name	Explanation	Unit	Frequency
Wind1DA	The day-ahead predicted electricity volume generated from wind turbines. Details on p. 111.	MWh	Hourly
Solar	The electricity volume generated from photovoltaic sources. Details p. 115	MWh	Hourly

Continued on next page...

²³We expect the effects of L on the slope to be of the shape of a Laffer curve.

²⁴We do not include the variables used for the demand estimation as they indirectly feed into the final regression via the PLU^D .



... table 3.2 continued

Name	Explanation	Unit	Frequency
Tempeff15	Effective predicted temperature in France (with a cutoff point at 15°C to reflect demand patterns), aggregated on a national level. Details on p. 114.	°C	Hourly
Roll.Temp24	Mean of <i>Tempeff15</i> over the last 24 consecutive hours.	°C	Hourly
Roll.Temp240	Mean of <i>Tempeff15</i> over the last 240 consecutive hours.	°C	Hourly
suncycle	Luminosity as a percentage of maximum luminosity of the day. <i>Midday</i> defined as <i>suncycle</i> =1. Details on p. 117.	%	Hourly
morning	Indicator variable for hours before <i>Midday</i> .	{0, 1}	Hourly
deltasun	Absolute value of the change in <i>suncycle</i> . Details on p. 117.	[0, 1]	Hourly
EWH	Indicator variable for hours between 10 pm and 4 am.	{0, 1}	Hourly
SolarRest	The unexplained component of photovoltaic generation. Specifically, the residuals from a regression of <i>Solar</i> on <i>suncycle</i> . Details on p. 117.	MWh	Hourly

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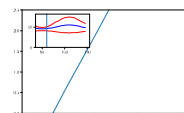
... table 3.2 continued

Name	Explanation	Unit	Frequency
RteBlackBox	The unexplained component of the day ahead prediction of total consumption in France issued by the grid operator (RTE). Specifically, the residuals from a consumption estimation. Details on p. 118.	MWh	Hourly
Coal	Average coal import prices at the French border.	€/ton	Monthly
Brent	Average of spot prices for crude oil on the London based stock exchange.	\$/bl	Monthly
Gas	Average of closing prices for natural gas at 1 month on the London market (NBP).	£/Therm	Monthly
IT2	Interaction term between gas and demand: <i>Gas</i> weighted by an hourly index for the demand level ²⁵	£/Therm	Hourly
EUA	Price of CO ² emissions.	€/ton	Daily
Hydro	Sum of dam level heights on a national level.	%	Weekly

Table 3.2: Overview of exogenous variables.

The rationale for the included variables is the following:

²⁵Gas turbines generate electricity using natural gas as a fuel. We thus proxy for its input price using a Gas variable for which we take the closing price for natural gas at 1 month on the London market (NBP). Electricity generation from gas is expensive and flexible. In general gas plants are only called upon to provide peak load electricity generation in moments of high demand. We, therefore, compute an interaction term between Gas and an index for the hourly level of the demand. The index acts as a weight on the gas price. The weight is computed as the percentage demand level as compared to the maximum demand level observed in our dataset.



First, Wind1DA and Solar control for the expected level of renewables generation²⁶ on the day ahead market. These are computed using a novel bottom-up methodology described in the appendix 2.3.1.

Second, Tempeff15 controls for the demand patterns as a function of the temperature.²⁷ Tempeff15 includes a cut-off at 15°C in order to take into account the demand pattern as a function of temperature according to [RTE, 2014]. Table 2.2 reveals the improved fit over a simple temperature variable without respecting the demand cut-off (Tempeff).

Third, Roll_Temp24 and Roll_Temp240 capture the demand seasonality via the temperature. The former gives the daily average temperature, while the latter captures the average temperature over the last 10 days. The demand cut-off at 15°C for Tempeff15 is respected for these means. Including these as seasonality controls allows to get away from using dummy variables for the seasonality. In short, avoiding dummies yields more transparency of the results as we do not have the problem of interpreting the dummies, which are often black boxes.²⁸

Fourth, we use the four variables suncycle, morning, deltasun and EWH collectively to continuously control for the time of the day. The reasoning is again the ability to get away from using dummies and being able to interpret the results. Figure 3.4.3 shows how the controls describe the daily patterns continuously.

Fifth, SolarRest and RteBlackBox are the residual information gained from the variables Solar and the day ahead consumption prediction of RTE (PrevConsoH) over other variables included in \mathbf{X}^D or \mathbf{X}^S , respectively.²⁹

²⁶For data availability reasons, Solar is computed on realised luminosity values rather than forecasts of luminosity.

²⁷Note that electric heating is widely spread in France. It is used in 32% of principal residences (INSEE, RP2011 exploitation principale).

²⁸See section 3.6.2 for a full discussion on the advantage of avoiding dummies.

²⁹E.g. Solar is strongly correlated with suncycle, thus SolarRest is the residual from a regression of

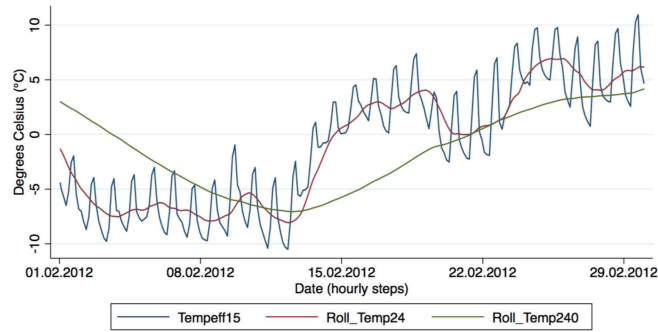


Figure 3.4.2: Temperature based seasonality controls

Note: The graph shows the evolution of the temperature based controls for seasonality for the month of February 2012. The graph shows the lagged nature of the rolling average temperature controls.

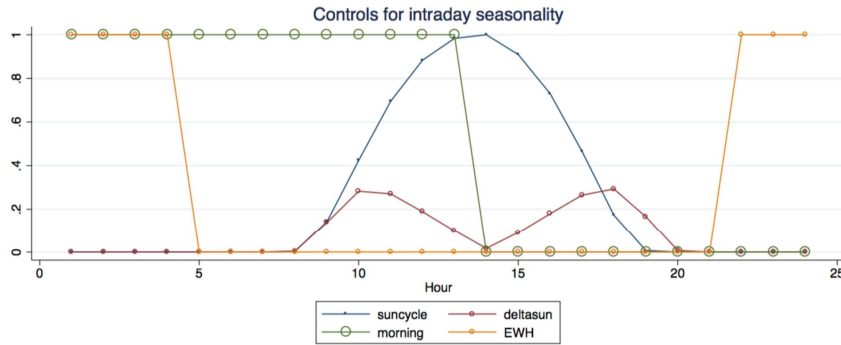


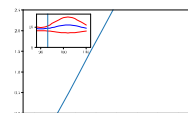
Figure 3.4.3: Continuous controls for daily patterns

Note: With the exception of EWH, all intraday seasonality controls (suncycle, morning, deltasun) are determined endogenously by the prevalent luminosity as captured by Solar.

Sixth, Coal, Brent, Gas, IT2 and EUA are rough proxies for the input prices for electricity suppliers. Hydro is used as a crude proxy for dam operator’s ability to generate short term electricity using hydro reserves.

We briefly emphasize that novel methodologies have been used to compute all variables derived from weather forecasts or observations. When tracing back the shape of aggregate bid functions on exogenous factors in the second stage estimation, we use aggregated statistics (at the national level) for the exogenous variables. We thus use an

the former on the latter. RteBlackBox is computed as the residuals from regressing PrevConsoH on Tempeff15, Roll_Temp24, Roll_Temp240, suncycle, morning, deltasun and EWH. See appendix 2.3.2 and 2.3.2 for details.



aggregation methodology to summarize local information (collected at the level of the individual postcodes) in order to generate an aggregate statistic at the national level.

The general methodology for the aggregation is explained using the example of Solar and as follows: We observe the value of a weather parameter (e.g. luminosity) every hour at known weather stations in France. We apply an interpolation technique in order to obtain parameter values for all possible geographic locations in France. At any local point, we can thus infer the electricity volume generated by using the information of the locally installed capacity (of solar panels) and the renewable energy available (i.e. sunlight inferred by the inverse of nebulosity). We then take the sum of all solar generated electricity per hour in France and use this as our aggregate statistic at the national level in our regression analyses. We used forecast data wherever possible in order to approximate the level of information that bidders have at the time of bidding and circumvent endogeneity problems. For cases where forecast data was not available, e.g. Solar, realised weather data was used.

Extensions and robustness checks

In order to test the robustness of our results and circumvent some drawbacks of the baseline model, we use a few alternative specifications of our empirical model.

Bootstrapping standard errors

The set-up of our empirical analysis relies on stochastic variables, e.g. PLU^D , which are computed in the first stage of our identification. The assumption made for an OLS regression of normally distributed residuals is a very strong one (particularly with the forecast variable) and one which can flaw the precision of estimates in the second stage regression. We therefore bootstrap the standard errors of the final regression by using random sampling with replacement at each stage of the analysis, i.e. for both the PLU computation and the final slope regression with 300 repetitions.

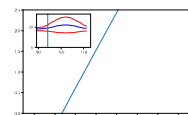
Bootstrapping allows us to non-parametrically approximate the distribution of the forecast PLUs and thus enables us to correct the standard errors of our coefficient estimates.

Kernel based uncertainty forecasts (PLU^D) The PLU^D computed as described in section 3.4.1 is noisy since we assume a linear forecast model to be valid for any combination of realizations of exogenous parameters, i.e. the same model applies winter and summer, day and night. While the results are as desired for the baseline PLU^D, a bootstrapping of the standard errors indicates that the first stage forecast is too imprecise for effects of a satisfactory significance level.

We therefore develop an extension of the uncertainty prediction model in which we use the idea of demand forecasts (equation 3.4.5) only locally, i.e. for a limited range of variation in the exogenous parameters. In other words, we estimate the PLU^D corresponding to an auction only in the neighbourhood of this auction, i.e. over all auctions that occurred in similar conditions. By conditions, we mean realizations of exogenous parameters, and the neighbourhood refers to the concept of measuring the similarity of these realizations by means of a range. The next step explains how this is done formally.

We consider that firms predict the level of the uncertainty by comparing it with the level of uncertainty in past³⁰ auctions of similar exogenous conditions. The methodology is analogous to the computation of the baseline PLU^D. The suppliers forecast the precision (squared residuals) of their demand estimation as before, but only on a subsample of the data. The subsample is defined as all observations which lie within a distance b_{X_e} of the observation of interest with respect to each control variable X_e , $\forall e = \{1, \dots, E\}$. Effectively, this is a multi-variate kernel regression and subsequent forecast with a rectangular (also called “boxcar”) weighting function. Observations within the kernel window are given equal weight, while observations outside the kernel window are given zero weight.

³⁰For data availability reasons, we pool all (past and future) auctions for the computation of this PLU. This introduces some endogeneity. For a discussion of this choice, please see section 3.6.3.



We set the bandwidth b_{X_e} with respect to each variable equal to $\frac{1}{3}$ of the range of that variable.³¹

At any arbitrary observation (auction) with the realization $\tilde{\mathbf{X}}$ for the stacked vector of exogenous variables (X_e), the simple weight function is

$$W(\mathbf{X}) = \prod_e W(X_e), \quad \text{where } W(X_e) = \begin{cases} 1, & \text{if } |\tilde{X}_e - X_e| \leq b_{X_e} \\ 0, & \text{otherwise.} \end{cases} \quad (3.4.9)$$

and the subsample based regressions are then

$$P_k^D(\mathbf{X}) = \alpha_{k,\tilde{\mathbf{X}}}^{D,P} + \beta_{k,\tilde{\mathbf{X}}}^{D,P} W(\mathbf{X}) + \epsilon_{k,\tilde{\mathbf{X}}}^{D,P} \quad (3.4.10)$$

$$Q_k^D(\mathbf{X}) = \alpha_{k,\tilde{\mathbf{X}}}^{D,Q} + \beta_{k,\tilde{\mathbf{X}}}^{D,Q} W(\mathbf{X}) + \epsilon_{k,\tilde{\mathbf{X}}}^{D,Q} \quad (3.4.11)$$

and the local uncertainty regressions and forecasts $\forall J = \{P, Q\}$ are given by

$$(\epsilon_{k,\tilde{\mathbf{X}}}^{D,J})^2 = \alpha_{k,\tilde{\mathbf{X}}}^{U,J} + \beta_{k,\tilde{\mathbf{X}}}^{U,J} W(\mathbf{X}) + \epsilon_{k,\tilde{\mathbf{X}}}^{U,J} \quad (3.4.12)$$

$$\underbrace{\widehat{(\epsilon_{k,\tilde{\mathbf{X}}}^{D,J})^2}}_{\widehat{\text{PLU}}_{k,\tilde{\mathbf{X}}}^D} = \alpha_{k,\tilde{\mathbf{X}}}^{U,J} + \beta_{k,\tilde{\mathbf{X}}}^{U,J} \tilde{\mathbf{X}} \quad (3.4.13)$$

When firms infer the upcoming uncertainty by looking at the uncertainty in past auctions, the precision of their estimate depends on the number of comparable auctions available, i.e. the sample size. Given that the sample size varies greatly across auctions, we use a sample-size-weighted OLS regression in the final estimation of equation 3.4.1. Finally, we bootstrap the standard errors on the kernel-based PLUs using 50 repetitions.³²

³¹See appendix 3.B.1 for details. Column 2 of table 3.14 indicates the choice of b_{X_e} for each exogenous variable considered.

³²For computational reasons, we only bootstrap the kernel based PLUs for the point of inflection ($k = 3$). We choose only 50 repetitions for the same reason. Given the size of our dataset, we consider it acceptable. The general criterion for convergence is that each observation is selected at least once in the bootstrapping exercise.

3.5 Results

We first present the results for the demand estimation in both the Price and Volume dimension since this step is identical for all PLU specifications. We then present the results of the final regression in the baseline and alternative specifications.

3.5.1 Demand estimation

Table 3.3 gives the results for the demand estimation on volumes (equation 3.4.3). Table 3.4 shows the results for the demand estimation on prices (equation 3.4.2).

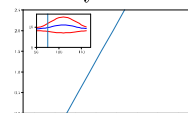
These tables are interesting for two reasons. First, they provide the basis for our computation of the PLU^D. Second and the reason why we disclose them in such detail, they are already a result in themselves.

It is comforting to see that all variables used are significant and, more importantly, of the expected sign. This significance provides support for our specification of the demand estimation. For the interpretation here, we focus on the effects at the point of inflection³³ ($k = 3$).

First, looking at the volume effects of the exogenous variables: All variables included in the regression are highly significant at the 1% level. All temperature statistics (Temp_eff15, Roll_temp24, Roll_temp240) bear coefficients with a negative sign and confirm that electricity demand falls with increasing ambient temperature. All daytime controls show up the expected sign as well: suncycle and deltasun have positive coefficients. This is sensible as electricity demand is higher during the day than at night (proxied for by suncycle) and rush or activity hours (proxied for by deltasun) in the morning and evening are also characterised by increased demand. The variables morning and EWH have coefficients of a plausible negative sign. The morning as controlled for by our indicator variable³⁴

³³As mentioned, the point of inflection is the centre point of the bid curves and the most relevant for equilibrium determination.

³⁴The morning is defined as the hours before midday, which occurs when luminosity is at its daily



is shorter than the afternoon and evening together, thus total electricity consumption is lower as well. EWH stands for the deep night between 10 pm and 4 am and thus also corresponds to low demand periods. SolarRest controls for selfgeneration to cover own consumption and has a plausible negative coefficient. RteBlackBox, on the other hand, has a very sensible positive coefficient and confirms that actual demand is higher when the grid operator expects it to be the case.

The analysis of the price effects of these controls on demand functions is in line with the analysis of volume effects. This is coherent since for a linear downwards sloping demand curve, a left shift (volume decrease) is synonymous for a downwards shift (price decrease) of the curve. We consider that at the point $k = 3$, the demand functions are locally linear. We note the only exception for the coefficient of SolarRest which has a positive price effect, while a negative volume effect.³⁵

Second, these tables already give a descriptive analysis of the effects of exogenous variables on the shape of the demand bid function: We now compare all coefficients for a specific variable on the $K = 5$ different points on the demand function (we read the table horizontally and compare sign changes across columns).

In table 3.3, we observe for each row at most a single sign change across the coefficients for the different points. Furthermore (and with few exceptions), the magnitudes of the coefficients generally increase or decrease monotonically along a row. This is very convincing as it suggests that exogenous variables have a monotone effect on the shape of the bid function. We thus only observe one-directional shifts (e.g. a unilateral left shift) or two-directional shifts (extension or contraction) in the volume dimension induced by the variation in exogenous variables. While the unilateral effects are explained analogously.

³⁵We emphasize in the construction of our variable (appendix 2.3.2) that it is not possible to build a proxy for lighting consumption that would allow us to decorrelate the effects from photovoltaic production and lighting consumption. We therefore stick to the SolarRest proxy, which aims to capture the effect of Solar which is not captured by suncycle.

gously to our point specific interpretation on the point $k = 3$ above, we do not have a story to tell about two-directional effects.

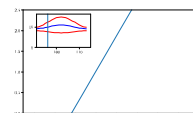
Tempeff15 results in a contraction of the bid function in terms of volumes (right shifts on low volume points, $k = 5, 4$ and left shifts on high volume points $k = 3, 2, 1$). Roll_Temp24 has the opposite effect and results in a volume extension of the curve. Roll_Temp240 induces a pure left shift of the whole function.³⁶ For the intraday seasonality controls, the results are very clear. While suncycle results in an extension of the demand function³⁷, all other intraday controls (morning, deltasun, EWH) have unilateral effects. When the indicators morning and EWH are positive, we observe volume decreases at all points and thus a left shift of the function. Higher values of deltasun induces volume increases at all points of the bid function.

Finally, we have SolarRest which induces an expansion of the curve and RteBlackBox which has a unilateral right shifting effect on the aggregate demand bid function.

The price variation of the demand bid function yields interesting results, too. Given that the prices of points $k = 1, 5$ are fixed, we only observe effects for the interior points. We thus focus on the effects on the points $k = 4, 3, 2$ only (called the “central demand function” here). Again, we only observe at most a single sign change across columns for any exogenous variable. Both Tempeff15 and Roll_Temp240 lead to an extension of the central demand function (we are now looking at vertical variation of the bid function as shown in fig. 3.4.1), while Roll_Temp24 causes a unilateral downwards shift. For intraday seasonality controls, we see that suncycle and deltasun have a contracting effect on the central demand function and morning a unilaterally negative effect. EWH leads to an expansion of the central demand function. SolarRest and RteBlackBox indicate an

³⁶Excluding interaction effects, we note that the net effect of a simultaneous 1°C increase for all three temperature variables results in a net left shift of the function. In the price dimension (table 3.4) we observe a net downwards shift. Both effects suggest that electricity demand decreases with the prevailing temperature.

³⁷Combined with the observed price effects from table 3.4, this suggests that demand is more price elastic during the day.



extension of the central demand function in the price dimension.

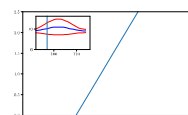
	$k = 5$ Volume	$k = 4$ Volume	$k = 3$ Volume	$k = 2$ Volume	$k = 1$ Volume
Tempeff15	50.72*** (9.942)	38.58*** (10.13)	-130.3*** (10.94)	-189.3*** (13.32)	-204.0*** (13.20)
Roll_Temp24	-63.57*** (11.78)	-67.13*** (12.06)	-48.87*** (13.14)	19.76 (15.83)	34.16** (15.76)
Roll_Temp240	-60.15*** (6.655)	-68.38*** (6.867)	-78.49*** (7.450)	-78.44*** (10.05)	-87.38*** (10.00)
suncycle	-894.0*** (44.27)	-652.1*** (45.50)	508.2*** (48.52)	1,351*** (56.36)	1,400*** (55.73)
morning	-101.2*** (27.52)	-220.3*** (28.33)	-814.8*** (30.44)	-872.2*** (37.71)	-885.8*** (37.28)
deltasun	2,659*** (153.5)	2,850*** (158.5)	3,201*** (166.1)	1,721*** (197.8)	1,821*** (196.5)
EWH	-803.1*** (30.74)	-833.1*** (31.91)	-782.7*** (33.15)	-354.7*** (42.09)	-322.8*** (41.78)
SolarRest	-0.595*** (0.0282)	-0.363*** (0.0305)	-0.145*** (0.0342)	-0.0137 (0.0418)	0.246*** (0.0407)
RteBlackBox	-0.00259 (0.00235)	0.0127*** (0.00243)	0.105*** (0.00255)	0.107*** (0.00316)	0.0979*** (0.00317)
Constant	6,054*** (33.71)	7,086*** (35.04)	11,446*** (37.15)	15,215*** (48.68)	15,502*** (48.27)
Observations	14,691	14,691	14,691	14,690	14,691
R^2	0.201	0.219	0.478	0.344	0.346
White	548.6	524.9	407.9	961.8	944.8

Robust standard errors in parentheses

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 3.3: Estimation results for demand volumes

Note: The estimated constants of this table or the left graph of fig. 3.4.1 indicate to which portion of the demand function the types of points $k = 1, \dots, 5$ refer.



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	$k = 5$ Price	$k = 4$ Price	$k = 3$ Price	$k = 2$ Price	$k = 1$ Price
Tempeff15	0 (0)	4.675*** (1.523)	-0.969*** (0.0599)	-1.308*** (0.0980)	0 (0)
Roll_Temp24	0 (0)	-10.07*** (2.233)	-0.124* (0.0713)	-0.0470 (0.116)	0 (0)
Roll_Temp240	0 (0)	4.250*** (1.147)	-0.0901** (0.0404)	-0.353*** (0.0607)	0 (0)
suncycle	0 (0)	-10.98** (5.020)	6.870*** (0.258)	11.60*** (0.445)	0 (0)
morning	0 (0)	-0.226 (4.133)	-5.748*** (0.173)	-9.009*** (0.285)	0 (0)
deltasun	0 (0)	-16.54 (19.16)	10.60*** (0.881)	18.72*** (1.497)	0 (0)
EWH	0 (0)	5.136 (4.448)	-1.756*** (0.192)	-3.014*** (0.302)	0 (0)
SolarRest	0 (0)	0.000532 (0.00307)	0.00192*** (0.000193)	0.00253*** (0.000326)	0 (0)
RteBlackBox	0 (0)	9.91e-05 (0.000301)	0.000906*** (1.47e-05)	0.00147*** (2.26e-05)	0 (0)
Constant	3,000 (0)	131.3*** (4.210)	39.45*** (0.217)	-39.43*** (0.319)	-3,000 (0)
Observations	14,691	14,691	14,691	14,690	14,691
R^2		0.005	0.463	0.420	
White		138.2	640.9	761.2	

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 3.4: Estimation results for demand prices

Note: The estimated constants of this table or the left graph of fig. 3.4.1 indicate to which portion of the demand function the types of points $k = 1, \dots, 5$ refer.

Overall, we take away a solid R^2 with coefficients of the correct sign. We furthermore have disclosed the White statistic which unanimously confirms heteroskedasticity in these regressions. The significance levels have been measured using robust standard errors.

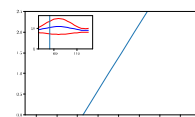
We point to the fact that the explanatory power of our demand estimations is highest for the point of inflection, in line with our expectations. Points of maximum curvature $k = 2, 4$ reveal lower R^2 statistics. This is likely due to the underlying data patterns that arise from bidding frictions, e.g. focal price points. For these points, it is thus not surprising that we do not observe convincing demand estimates - we note in particular the lack of explanatory power for the demand estimation in the price dimension for points of type $k = 4$.

3.5.2 Final regression

For the final regression, we first lay the focus on the point of inflection ($k = 3$) for a detailed interpretation of our results. We choose the point $k = 3$, because this type of point is the most relevant for equilibrium determination. We then disclose the results for all other points $k \neq 3$ to give an overview of the effects of uncertainty on the whole aggregate supply bid function.

Each result table has four (three³⁸) columns to show the results for different estimators and two specifications of the PLU^D . All other variables remain unchanged across the columns. In the tables, column 1 refers to the baseline specification of the $PLU^{D,J}$, where standard errors are calculated using the Huber-White sandwich estimator. Column 2 reports the results for the baseline model using bootstrapped standard errors with 300 repetitions. Column 3 reports the results for the regression on the kernel based $PLU_{\hat{X}}^{D,J}$, using the sample size of each kernel as weights in the regression. Column 4 reports the re-

³⁸For computational reasons, we do not run the bootstrapping of the kernel based PLU^D for the points $k \neq 3$, thus we only have three columns for these tables.



sults of the kernel based model using bootstrapped standard errors using 50 repetitions.³⁹

Regarding notation: In the results tables, PLUvRvar‘m’ stands for $PLU_{1,m}^R$ with ‘m’ being replaced by the initial of the variable in question (W, S and T, respectively). $PLU_{2,m}^R$ is indicated by the extension “sq”. PLUvDvar‘J’ stands for $PLU^{D,J}$ with $J = \{P, Q\}$ representing the dimension in which the demand uncertainty is measured. The kernel based $PLU_{\bar{X}}^D$ are given by PLUvDvarK‘J’ in the tables. To facilitate the reading of the tables, we adopt this notation for the discussion of the results.

For the point of inflection ($k = 3$), the results are shown in table 3.5. Regarding uncertainty from renewables production, only that of wind has a significant and robust impact. PLUvRvarW has a positive effect (significant at the 1% level) on the slope in all specifications. PLUvRvarWsq has a negative effect on the slope in all specifications, however this second effect is not robust to bootstrapping the standard errors. The signs of the estimated coefficients are in line with our expectations. To show this, we recall that both versions of the PLUvRvarW are based on the inverse of the characteristic length-scale L_W of autocorrelation of the wind speed measurements. Thus, when L_W increases (it represents a decrease in the uncertainty since wind speeds are homogenous over longer distances), the PLU decreases (corresponding to a decrease in uncertainty).

While an increase in the PLUvRvarW leads to an increase in the slope of the supply function, the effect is attenuated by the squared term PLUvRvarWsq for very small and large L_W .⁴⁰ The estimated coefficient for the latter is negative and suggests that for very short L_W (i.e. very heterogenous wind speeds over the country), prediction errors cancel out. For very long L_W (i.e. very homogenous wind speed profile), the marginal impact

³⁹Coefficients vary slightly ($< \pm 20\%$, no sign change), because the bootstrapping loop includes the kernel-based prediction of the uncertainty and thus varies the kernel sample sizes, which are used as weights in the final regression. Furthermore, the estimator has probably not yet fully converged with 50 repetitions, however for computational reasons we stick to this choice.

⁴⁰By looking at the variation of our data, we see that the negative effect of the PLUvRvarWsq term merely attenuates, rather than overrides, the positive effect of the PLUvRvarW term on the slope since in our dataset we very rarely observe PLUvRvarW values sufficiently large to exceed the maximum of the Laffer curve of the impact on the slope.

of L_W on the level of uncertainty decreases.

With respect to the uncertainty from temperature forecasts, the results are insignificant (although of the anticipated sign). We expect the impact of temperature uncertainty goes via the demand response, which we account for in our proxy for the uncertainty from demand realization (PLUvD). Similarly, uncertainty from Solar production is attributed no effect. This is not surprising as generation from solar is only a fraction of that generated from wind power and thus negligible. Furthermore, we are unable to disentangle the effect of solar generation from the reduced demand effect from high luminosity (which result in low demand for lighting). We do not find evidence for a direct response from suppliers to uncertainty in temperature or solar predictions.

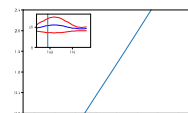
Uncertainty from the realization of market demand has a negative and significant effect when proxied for by price-based PLUvDvarP (see table 3.5) as opposed to a positive and significant effect when proxied for by a volume-based PLUvDvarQ (see table 3.5). The positive effect on PLUvDvarQ is in line with our prediction made in section 3.1.2. This result supports the theory that firms take uncertainty when bidding into account and consequently adjust their bidding strategy in order to minimize dynamic costs.

However, our theory produces a prediction for volume based uncertainty only. We include the uncertainty proxy for price PLUvDvarP as a control and its effect seems rather robust. The effects of PLUvD in either the price or volume dimension are robust to the exclusion of the other.⁴¹ We try to explain the opposing signs for the coefficients of the two proxies in section 3.6.1.⁴²

Furthermore, table 3.5 gives support to our extension using kernel based PLUvDs.

⁴¹Results available from the authors.

⁴²The net effect cannot be precisely computed as the conversion of the PLUvD from the price dimension to the quantity dimension is not possible. We approximate the comparison however, by including both PLU^D simultaneously in the regression. All PLUvD are rescaled by their respective means to allow some degree of comparison.



Column 2 shows that the effects of the baseline PLUvD are not significant when bootstrapped. Our alternative is to use a more elaborate uncertainty prediction model. These kernel based PLUvD are more sophisticated in two respects: (i) the forecasting model is only applied locally, that is auctions are only compared to similar auctions and (ii) the obtained forecast is weighted by the sample size used for its prediction. Thereby, we control for the confidence of the firms in making those predictions. The results of the weighted regression are given in column 3. The results using the more elaborate prediction model are in line with those from the baseline regression, while being more accurate as indicated by the improved explanatory power of our model (we see a 16.5% increase of the R^2 from columns 1-2 to columns 3-4). Finally, the results of our kernel based model are more precise as indicated by the higher significance level for the PLUvDvarKP and PLUvDvarKQ, which are now also robust to a bootstrap (column 4).

Finally, we explicitly include the controls for the levels of the input prices of electricity producers (\mathbf{X}^S). We do not interpret these coefficients since there are no ex-ante expectations of their levels to affect the slope of the supply bid function. We briefly mention that intraday seasonality controls as well as other demand related variables are not included in this regression to avoid multicollinearity problems with the PLUvD, which are themselves computed as a linear combination of the demand control variables (\mathbf{X}^D).

Overall, we take away a goodness of fit of $\geq 20\%$ for our empirical model as well as the robust positive coefficients for both the demand based uncertainty proxy (PLUvDvarQ) and the weather based uncertainty proxies (PLUvRvarW and PLUvRvarWsq). We note the puzzling result for the PLUvDvarP.

For the other points ($k = 1, 2, 4, 5$), the results are given in tables 3.6, 3.7, 3.8 and 3.9, respectively.⁴³ We comment on the effects over all points collectively in order to give

⁴³Variables marked “(omitted)” are drop due to perfect collinearity.

an overview of the full bid function behaviour.

The specification of the proxies for the uncertainty from renewables as well as of the controls does not vary across columns, we thus focus on column 2 for these (in order to take bootstrapped standard errors into account). While we observe in table 3.5 a convincing effect for the uncertainty from wind predictions on the slope of the point of inflection ($k = 3$), we cannot observe this effects on the other points of the bid function.⁴⁴ No other proxy for the uncertainty from renewables has a significant effect on the slope at any point.

The proxies for uncertainty from market demand produce opposing effect depending on the prediction model. PLUvDvarP has a negative and significant effect on all points (with the exception of points $k = 1$ and $k = 5$ of course, which do not exhibit variation in prices due to the auction rules). PLUvDvarQ has a positive effect, when significant⁴⁵, on all points ($k = 2 - 5$), but not on $k = 1$.

On the remaining controls, we do not observe a clear pattern on the effects at the different points. We run the analysis without these controls and note that the signs of all significant variables remain unchanged.⁴⁶

3.6 Discussion

In this section we reflect on the results and use the opportunity to address a few issues, drawbacks as well as qualities of the research conducted. We first discuss the findings of the paper and their internal and external validity. We briefly review the design of the empirical strategy and lend particular focus to how we deal with the issue of endogeneity.

⁴⁴We note the exception of a negative effect for PLUvRvarW on the slopes at points $k = 2$ and 5 (significant at the 5% level).

⁴⁵For both the bootstrapped baseline results (col. 2) and the weighted kernel based specification (col. 3).

⁴⁶Results available from the authors.

3.6.1 Findings

In this paper, we investigate whether uncertainty affects supplier bidding as predicted by the theory. We find that uncertainty from weather forecasts indeed affects the suppliers' bid function as expected. The aggregate supply function steepens when the level of uncertainty increases. We take this as evidence that firms take dynamic cost considerations into account and adjust their behaviour when facing increased expected dynamic costs.

We also find significant results for the effect of the level of uncertainty about the realization of market demand on the suppliers' behaviour. However, we observe a strong discrepancy between the effect for uncertainty as measured on price volatility and the effect of uncertainty as measured on volume volatility. While the former sees itself attributed with a negative effect, the latter sees itself attributed with a positive effect on the slope of the aggregate supply function. These opposing signs are robust in all specifications and seem to be of too much importance to be neglected.

The two proxies in question ($PLU^{D,P}$ and $PLU^{D,Q}$) are two variables designed to measure the same information, namely the prediction error of the demand function. As such, they are identical with respect to the set-up, computation as well as point at which they are extracted. They only differ with respect to the dimension in which the variation of the demand function is measured, the former in the price dimension and the latter in the volume dimension.

A theory using linear functions would predict that these measures of the shifts of the demand line are identical and interchangeable (modulo a translation by the slope). Also our data, i.e. the observed bid functions, suggests that, at least locally at the point $k = 3$, the bid functions are linear.⁴⁷ Furthermore, our demand estimation models for both price and volume variation⁴⁸ indicate that the prediction model used works well in both dimension. In particular at $k = 3$, significance and equal signs on coefficients for

⁴⁷Recall the graph in figure 3.3.2.

⁴⁸Precisely look at columns 3 of tables 3.3 and 3.4.

all terms included as well as similar explanatory power⁴⁹ in both regressions confirms the similar nature of the two proxies.

Our recovered $PLU^{D,P}$ and $PLU^{D,Q}$ are, as expected, collinear.⁵⁰ While OLS remains unbiased in the presence of collinearity between two regressors, its precision is reduced. We correct for the collinearity by dropping one proxy or the other, but the individual results remain unchanged - the coefficients of the two proxies keep opposite signs.

Assuming that our empirical strategy is valid to test the relationship of interest, a possible reason for our intriguing observations could be that the slope of the demand function, which relates $PLU^{D,Q}$ and $PLU^{D,P}$, is endogenous on the uncertainty. Uncertain demand does not only unilaterally shift the demand function in one dimension (either P or Q), but also affects the shape and thus the slope of the curve. This effect is not accounted for in our research design and could drive the opposing results for both proxies. The endogeneity of the slope of the demand curve could be accounted for in our model by extracting the residuals from a regression of $PLU^{D,P}$ on $PLU^{D,Q}$ in an analysis to see if endogeneity exists and then reusing the residuals to control for slope effects of the demand curve in the final regression. We leave this avenue for further research.

Without having resolved the empirical discrepancy in the results, the stark contrast between the two could also hint at the fact that we need new theories to explain both demand and supplier bidding behaviour on the electricity market. This calls for new theoretical models to better explain the shape of aggregate bid functions, which are S-shaped overall. Special attention in these models should be placed on the effect of uncertainty and its importance for bidders via the link of dynamic costs.

Finally, our analysis relies strongly on the analysis of the point of inflection ($k = 3$), but the functional analysis is important, too. While results on the whole bid function

⁴⁹ R^2 of 0.463 for the price and 0.478 for the volume regression.

⁵⁰Not perfectly, but with a correlation coefficient of 0,62

are broadly speaking in line with the point-specific analysis on the point of inflection, the significance of the results is weaker and the results less clear. Furthermore, we often observe varying effects on low and high volume points.⁵¹ We conclude that the impacts of variations in exogenous factors on the shape of the bid functions are not uniform. Non-linear effects are neither predicted by our linear theory nor have been shown in previous studies (with the exception of [Wolfram, 1999]). Our results hint at more intricate mechanisms which drive the shape of these bid functions.

3.6.2 Internal and external validity

We believe that the work is credible due to many aspects of the research design.

First, our set-up is based on rather intuitive relations which we test exclusively using simple OLS regressions. These regressions are econometrically unbiased given the data impurities that we observe. To guarantee precision of our estimates, we use bootstrapping techniques.

Second, considerable effort has gone into the treatment of the information that goes into the right hand side of our regressions. We do not only refer to the final PLUs used, but also point at the precise use of our controls. See, for example, the treatment of the variable `RteBlackBox` (details see page 118), which proxies for the information contained in the day ahead demand estimates (`PrevConsoH`) given out by the grid operator RTE. In order to extract the marginal information of the `PrevConsoH` estimate, which is not explained by other controls variables that we include in our analysis, we compute the residuals from a regression of `PrevConsoH` on our other controls, e.g. daytime controls such as `suncycle`. These residuals (called `RteBlackBox`) enable us to achieve a more sophisticated understanding of our regression output.⁵²

⁵¹We refer specifically to the strengthening or weakening effects of exogenous variables on different points as shown in demand level estimation tables 3.3 and 3.4 as well as in the slope regressions tables 3.6 - 3.9.

⁵²See, for example, the regression output of the demand estimation in tables 3.3 and 3.4.

We also emphasize the aspect that we understand our dataset as a cross-sectional dataset rather than a time-series. While we do segment our dataset into weekday and weekend days and only run our analysis on the former, there is not reason why demand on a Tuesday afternoon should not be comparable to demand on a Thursday afternoon. We therefore ignore weekday dummies to increase our sample size. Furthermore, we avoid the use of dummy variables to control for the hour of the contracts in our regressions in order to further increase the sample size. However, we cannot compare electricity consumption between 4 am and 4 pm within a day. Neither can we compare two 4 pm hours of a day in winter and another in the summer. Using dummies would first restrict our sample size, plus make our interpretation more difficult since the dummy variable aggregates the effect over all conditions that change between samples. We use a bottom up approach that allows us to circumvent the sample size restriction and interpretation difficulties from daytime or seasonality dummies. Instead, we use continuous variables to control for the daytime and season by means of short and longer term temperature averages or other weather characteristics such as luminosity, which generates controls like deltasun .⁵³

Finally, we point at the empirical framework that allows us to run reduced form regressions on multiple regions of bid functions to better understand functional responses of those bids to variation in exogenous factors. We use 5 points for our analysis and refer to chapter 2 for the full details on this choice and the evaluation of the point selection. With hindsight, we feel that an additional two points would have been useful to better understand functional behaviour of the part of the bid functions, which is more relevant in equilibrium, i.e. on the centre part.⁵⁴ We note the computational demands of more points.

The methodology developed for our exercise on data from the French electricity mar-

⁵³See section 3.4.1 for full details on our set of control variables for both demand and supply.

⁵⁴For that we would recommend the points representing half of the maximum curvature between the current points $k = 2, 4$ and $k = 3$.

ket has applications in other domains. This is valid for the non-parametric point selection mechanism (section 3.4), the mechanism to aggregate local geographic data to a national level (chapter 2) as well as the identification strategy based on purely ex-ante data.

In particular, we note that the possibility to run reduced form estimation strategies for the analysis of markets which make access to functional data available. This includes all markets which use a multi-unit, uniform (or discriminatory) auction mechanism.

3.6.3 Endogeneity

The set-up of this work is specifically aimed at circumventing problems of endogeneity. For that sake, we keep a strict separation of ex-post and ex-ante information to the left and right hand sides, respectively, of any regression.

To achieve this separation of ex-ante and ex-post information, both newly developed methodologies are highly useful. The point selection methodology from section 3.4 allows us to extract proxies for the level of uncertainty about the realization of market demand, which are unaffected by the equilibrium interaction with the market supply. The weather data treatment methodology from chapter 2 enables us to base our proxies for the level of uncertainty from renewables on measures of the expected homogeneity of weather forecasts. Both methodologies allow us to recover ex-ante information on the prevailing uncertainty that firms have at their hands at the time of bidding. The information contained in all other controls used is also available at the time of bidding.

However for data availability reasons, we are not able to keep this strict separation at all times in practice and revert to using ex-post data to compute some variables that should ideally be computed on ex-ante information only. This is the case twice in this work: (i) we use observed weather data to compute the variable Solar⁵⁵ and (ii) we use

⁵⁵Contrary to Wind1DA and Tempeff15, which we are able to compute purely on forecast data.

the pooled data over all auctions for the demand estimation and subsequent uncertainty forecast of equations 3.4.2 - 3.4.5.

In both cases, we do not believe that this choice compromise our results. For the case of Solar, we use realised luminosity instead of forecast data. This is as if weather forecasts were perfectly accurate. Given that solar production only accounts for a small fraction for of total electricity generation and that we extract the very informative component of the Solar variable by using the variable suncycle (which is arguably very well predictable), we do not see the use of ex-post data as problematic.

For the case of the PLU^D computation, we run the demand estimation pooled over all observed auctions (i.e. past and future) and say that firms have this level of information when bidding in each auction of our sample. We do so because we do not have the necessary data before 01.01.2011 and thus cannot calibrate our forecasting model on a “learning” dataset. Instead, we assume that demand patterns conditional on the explanatory variables has remained constant over our 2.5 years time period of analysis. The estimation based on pooled data then yields, on average, the same insights as an analysis conducted purely on past data.

We could test robustness of our pooled approach by investigating the effect of a restriction on using only past data in the demand estimation. A learning effect could arise from more precise estimations of demand functions. However, due to the long experience of most firms on the market in reality, this learning effect would be artificial and not represent a real insight. We therefore accept the possibility of a (small) endogeneity concern in this paper and further work could fully circumvent this issue by extending the database appropriately.

3.7 Conclusion

This paper is a sophisticated proof of concept of our methodology applied to the electricity market. We observe that bidders take uncertainty from renewables generation as well as uncertainty from demand realization into account. The results indicate that electricity suppliers react to an increased level of uncertainty by bidding more volume elastically (steeper supply functions in the dimension Q (x-axis) - P (y-axis)) in order to minimize expected dynamic costs, which increase with the uncertainty. The results also indicate that not only supplier bidding is affected by uncertainty, but that the level of uncertainty also impacts bidding from the demand side of the market.

Future empirical work should focus on investigating the endogeneity of the demand function on uncertainty as well as better understand frictions in the bidding (e.g. focal price points). Concurrently, the results also call for more advanced theoretical work on the shape of bid functions of players, in particular to explain non-linear shapes. This is also suggested by our bid functional analysis which hints at non-unilateral effects of exogenous variables on the shape of the functions. The economic insight hidden in full bid functions is vast and a better understanding of these could be applied to address important welfare questions.⁵⁶

⁵⁶Such an application, which the authors currently focus on is the question of the optimal choice of the geographic installation of renewable electricity generation units (solar panels and wind turbines) with respect to minimizing the intermittency of renewables generation. A clear understanding of the effects of uncertainty on the market is vital to close the analysis on organizational questions of the market. This is outside of the focus of this paper

Appendix

Appendix 3.A Summary Statistics of Selected Points

	Mean	Median	StdDev	Min	Max
Prices for $k = 1$	-3,000.0	-3,000.0	0	-3,000	-3,000
Prices for $k = 2$	-56.7	-55.0	19	-97	70
Prices for $k = 3$	27.6	26.8	11	-27	93
Prices for $k = 4$	120.2	105.4	193	-11	2,999
Prices for $k = 5$	3,000.0	3,000.0	0	3,000	3,000

Table 3.10: Prices of selected demand points

	Mean	Median	Std. Dev	Min	Max
Volumes for $k = 1$	13,328	13,222	2,213	4,990	23,254
Volumes for $k = 2$	12,919	12,824	2,238	3,321	23,001
Volumes for $k = 3$	8,779	8,664	2,028	1,958	18,335
Volumes for $k = 4$	5,777	5,730	1,558	987	12,773
Volumes for $k = 5$	5,031	4,968	1,467	914	11,301

Table 3.11: Volumes of selected demand points

	Mean	Median	Std. Dev.	Min	Max
Prices for $k = 1$	-3,000.0	-3,000.0	0	-3,000	-3,000
Prices for $k = 2$	-30.3	-25.0	219	-2,999	439
Prices for $k = 3$	61.3	58.6	24	11	526
Prices for $k = 4$	133.9	136.3	32	36	626
Prices for $k = 5$	3,000.0	3,000.0	0	3,000	3,000

Table 3.12: Prices of selected supply points

	Mean	Median	Std. Dev.	Min	Max
Volumes for $k = 1$	3,721.7	3,526.0	1,344	618	10,594
Volumes for $k = 2$	4,432.8	4,226.0	1,602	844	11,765
Volumes for $k = 3$	8,467.2	8,365.5	1,814	3,431	20,932
Volumes for $k = 4$	11,849.5	11,717.7	2,411	3,641	27,810
Volumes for $k = 5$	14,390.6	14,142.0	3,052	6,580	35,356

Table 3.13: Volumes of selected supply points

Appendix 3.B Computational Details and Descriptives

3.B.1 Hard choices in the PLU computation

In computing the multi-variate kernel based prediction of the uncertainty for a given auction, we select auctions of a sufficient degree of similarity. We base the forecast equation 3.4.5 on this subsample dataset. We thereby consider that firms use the forecasting equation only *locally* in the neighbourhood of the auction of interest.

In order to define the size of the neighbourhood of an auction, we have to explicitly specify the width of the kernel window used in selecting the respective subsamples.

The trade-off involved is that we want to have small kernels for a precise computation of the PLU, while we want large kernels to make sure that we have a sufficient sample size in each kernel in order to derive meaningful statistics.

We choose to use a constant kernel window length with respect to each conditioning variable. We set the length of the window for each variable equal to $\frac{1}{3}$ of the variation of that variable. E.g. for Tempeff15, we observe a range of values from -10°C to 14°C . The subsample used to compute the PLU^D corresponding to a specific observation will

consist of all observations that are within a range of $\pm 4^\circ\text{C}$ of that observation for Temp-eff15. The same logic is applied to selecting the neighbourhood with respect to all other conditioning variables.

Table 3.14 gives descriptive statistics about the conditioning variables for the kernel and the explicit choice m , which determines the length of the kernel window for a variable X_e using the formula $b_{X_e} = \frac{2}{m_{X_e}}$.

3.B.2 Descriptive Statistics

On realised market equilibria

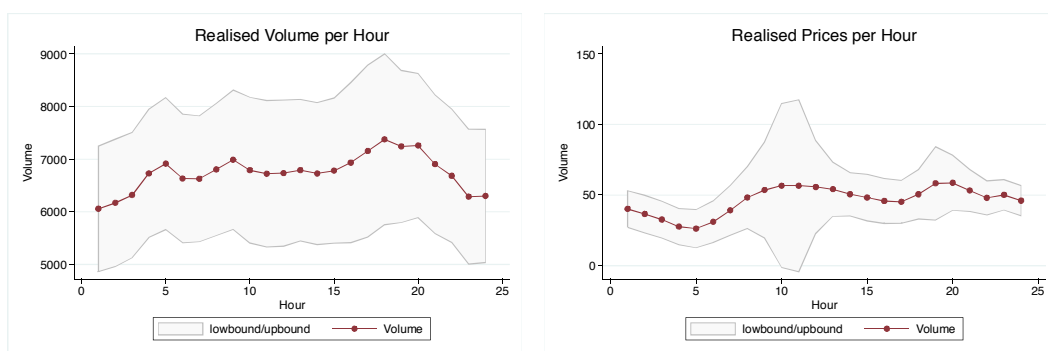


Figure 3.B.1: Plotted average realised Volume (left) and Price (right) per Hour with 95% confidence intervals.

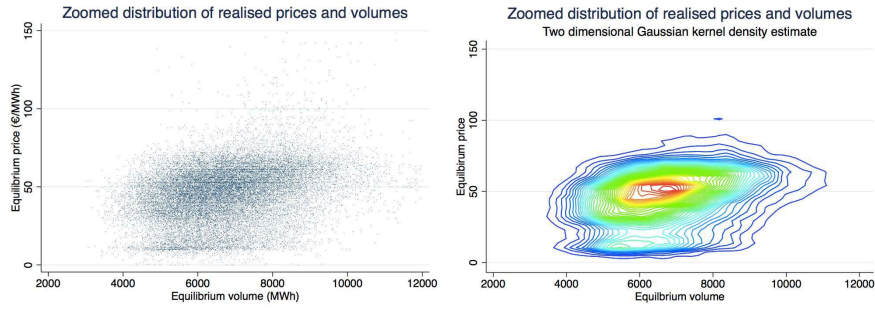


Figure 3.B.2: Distribution of observed market equilibria

Note: The warmer the colours of the heat map, the higher the frequency of realised price-quantity schedules. The colour legend is omitted for brevity, density changes between contours are of the order of 10^{-4} .

On player bid functions

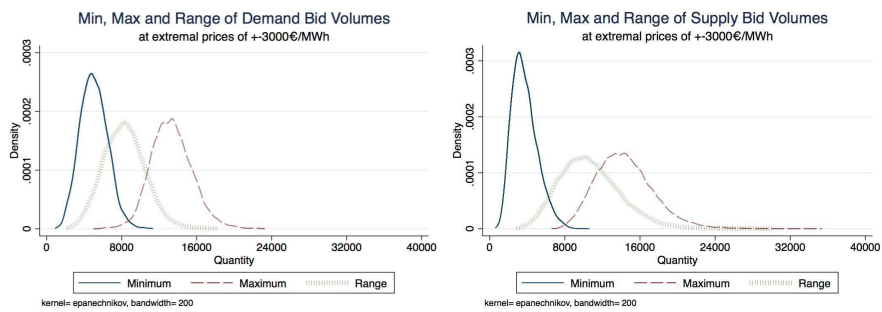


Figure 3.B.3: Distribution of minimum and maximum production volumes (and corresponding range) bid in an hourly auction.

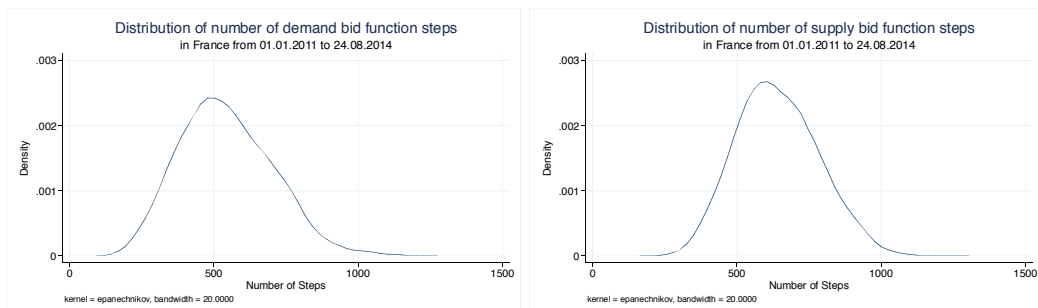


Figure 3.B.4: Distribution of number of bid function steps

On exogenous factors

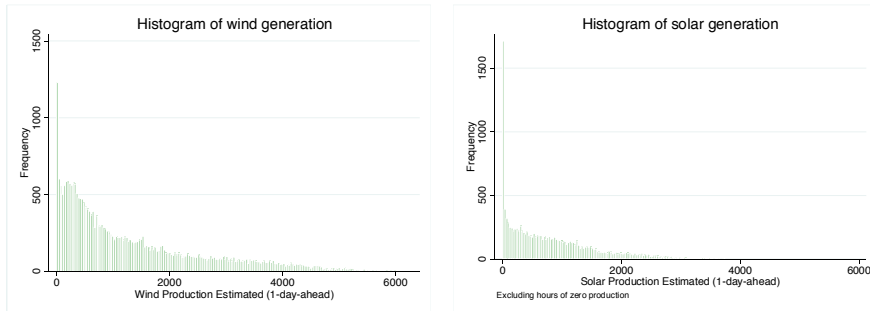


Figure 3.B.5: Histogram of predicted wind (left) and predicted solar (right) generation

For $k=3$ (Point of inflection)

	(1) fxInvertQP	(2) fxInvertQP	(3) fxInvertQP	(4) fxInvertQP
PLUvRvarT	0.000882 (0.00152)	0.000882 (0.00415)	0.00374** (0.00155)	0.00508 (0.00354)
PLUvRvarTsq	-0.000529 (0.000584)	-0.000529 (0.168)	-0.00161*** (0.000603)	-0.00215 (0.183)
PLUvRvarW	0.00790*** (0.00123)	0.00790*** (0.00257)	0.00647*** (0.00121)	0.00574*** (0.00207)
PLUvRvarWsq	-0.00235*** (0.000373)	-0.00235 (0.0644)	-0.00192*** (0.000370)	-0.00170 (0.0479)
PLUvRvarS	-5.20e-10 (2.68e-09)	-5.20e-10 (3.58e-08)	-2.28e-09 (3.16e-09)	-2.23e-09 (3.69e-08)
PLUvRvarSsq	0 (0)	0 (0)	0 (0)	0 (0)
Coal	6.90e-06*** (4.35e-07)	6.90e-06*** (4.64e-07)	5.18e-06*** (4.39e-07)	6.29e-06*** (6.87e-07)
Brent	-2.36e-05*** (1.51e-06)	-2.36e-05*** (1.96e-06)	-1.18e-05*** (1.53e-06)	-1.40e-05*** (2.01e-06)
Gas	-2.82e-07 (1.89e-06)	-2.82e-07 (9.41e-06)	1.37e-05*** (1.67e-06)	1.36e-05*** (2.46e-06)
IT2	-2.71e-05*** (2.17e-06)	-2.71e-05 (1.80e-05)	-1.73e-05*** (1.34e-06)	-1.99e-05*** (1.69e-06)
EUA	7.20e-05*** (2.31e-06)	7.20e-05*** (4.49e-06)	2.62e-05*** (3.34e-06)	2.71e-05*** (6.84e-06)
Wind1DA	1.04e-07*** (6.45e-09)	1.04e-07*** (1.03e-08)	1.18e-07*** (6.51e-09)	1.25e-07*** (7.63e-09)
Hydro	-7.55e-06*** (8.33e-07)	-7.55e-06*** (2.24e-06)	-4.08e-06*** (8.61e-07)	-5.88e-06*** (1.11e-06)
PLUvDvarP	-0.000219*** (4.57e-05)	-0.000219 (0.000203)		
PLUvDvarQ	0.000567*** (9.44e-05)	0.000567 (0.000585)		
PLUvDvarKP			-0.000600*** (2.69e-05)	-0.000462*** (4.24e-05)
PLUvDvarKQ			0.000151*** (3.39e-05)	0.000170** (6.80e-05)
Constant	0.00651*** (0.000208)	0.00651*** (0.000789)	0.00513*** (0.000195)	0.00538*** (0.000257)
Observations	11,702	11,702	11,702	11,702
R^2	0.200	0.200	0.233	0.234

(Standard errors in parentheses)

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 3.5: Regressions of the slope on PLU^R and PLU^D and PLU^D at $k = 3$
Note: Standard errors are reported in parenthesis. Column 1 refers to the baseline specification. Column 2 reports bootstrapped results for the baseline model. Column 3 reports the results for the (weighted) regression on the kernel based $PLU_{\bar{X}}^D$. Column 4 reports bootstrapped results of the model in column 3.

For k=1 (Left extremal point)

	(1)	(2)	(3)
	fxInvertQP	fxInvertQP	fxInvertQP
PLUvRvarT	-4.14e-05***	-4.14e-05	0.000277*
PLUvRvarTsq	1.56e-05***	1.56e-05	-0.0122
PLUvRvarW	-6.04e-06	-6.04e-06	-0.000138
PLUvRvarWsq	1.71e-06	1.71e-06	0.00738
PLUvRvarS	0	0	-5.70e-05
PLUvRvarSsq	-0	-0	0.00172
Coal	-8.54e-09***	-8.54e-09***	(omitted)
Brent	8.64e-08***	8.64e-08***	(omitted)
Gas	-6.20e-08***	-6.20e-08***	(omitted)
IT2	4.95e-08***	4.95e-08***	3.50e-08
EUA	-3.14e-08***	-3.14e-08***	4.43e-06***
Wind1DA	-3.38e-10***	-3.38e-10***	2.48e-10
Hydro	4.69e-08***	4.69e-08***	(omitted)
PLUvDvarQ	-3.87e-06***	-3.87e-06***	
PLUvDvarKQ			-7.21e-10***
Constant	2.11e-06***	2.11e-06**	-6.00e-05***
Observations	11,702	11,702	50
R^2	0.152	0.152	0.681

Standard errors available from the authors

*** p<0.01, ** p<0.05, * p<0.1

Table 3.6: Regressions of slope on PLU^R and PLU^D and PLU^D at $k = 1$

For k=2 (Left point of maximum curvature)

	(1)	(2)	(3)
	fxInvertQP	fxInvertQP	fxInvertQP
PLUvRvarT	-0.00252	-0.00252	0.292
PLUvRvarTsq	0.00106	0.00106	-17.27
PLUvRvarW	-0.00549***	-0.00549**	0.339
PLUvRvarWsq	0.00158***	0.00158	-21.86
PLUvRvarS	-6.82e-10	-6.82e-10	0.0669
PLUvRvarSsq	0	0	-1.968
Coal	2.36e-06***	2.36e-06***	(omitted)
Brent	-1.86e-05***	-1.86e-05***	(omitted)
Gas	-8.94e-06***	-8.94e-06***	(omitted)
IT2	1.98e-05***	1.98e-05***	6.92e-05
EUA	8.69e-05***	8.69e-05***	-0.000439
Wind1DA	6.13e-09	6.13e-09	6.70e-07
Hydro	-5.82e-06***	-5.82e-06***	(omitted)
PLUvDvarP	-4.81e-05***	-4.81e-05***	
PLUvDvarQ	0.000442***	0.000442***	
PLUvDvarKP			-9.62e-07*
PLUvDvarKQ			-4.27e-07
Constant	0.00319***	0.00319***	0.00279
Observations	11,702	11,702	50
R^2	0.158	0.158	0.414

Standard errors available from the authors

For k=4 (Right point of maximum curvature)

	(1)	(2)	(3)
	fxInvertQP	fxInvertQP	fxInvertQP
PLUvRvarT	-0.00442***	-0.00442	0.000559
PLUvRvarTsqr	0.00149**	0.00149	-0.000368
PLUvRvarW	-0.000137	-0.000137	-0.00205
PLUvRvarWsqr	0.000173	0.000173	0.000739*
PLUvRvarS	2.59e-09	2.59e-09	2.40e-09
PLUvRvarSsq	-0	-0	-0
Coal	2.22e-07	2.22e-07	1.48e-06***
Brent	-7.46e-06***	-7.46e-06***	-1.30e-05***
Gas	9.04e-06***	9.04e-06***	2.04e-05***
IT2	-1.96e-05***	-1.96e-05***	-2.61e-05***
EUA	4.71e-05***	4.71e-05***	3.19e-05***
Wind1DA	1.64e-08**	1.64e-08**	1.50e-08**
Hydro	-8.73e-06***	-8.73e-06***	-1.33e-05***
PLUvDvarP	-0.000212***	-0.000212***	
PLUvDvarQ	0.000110	0.000110**	
PLUvDvarKP			-0.000163***
PLUvDvarKQ			4.08e-05
Constant	0.00370***	0.00370***	0.00406***
Observations	11,701	11,701	11,701
R ²	0.086	0.086	0.117

Standard errors available from the authors

*** p<0.01, ** p<0.05, * p<0.1

Table 3.8: Regressions of slope on PLU^R and PLU^D and PLU^D at $k = 4$

For k=5 (Right extremal point)

	(1)	(2)	(3)
	fxInvertQP	fxInvertQP	fxInvertQP
PLUvRvarT	-0.000252	-0.000252	0.000734***
PLUvRvarTsqr	9.10e-05	9.10e-05	-0.000280***
PLUvRvarW	-0.000555***	-0.000555**	-0.000545***
PLUvRvarWsqr	0.000169***	0.000169	0.000163***
PLUvRvarS	-4.17e-10	-4.17e-10	-3.07e-10
PLUvRvarSsq	0	0	0
Coal	-8.70e-07***	-8.70e-07***	-4.90e-07***
Brent	1.72e-06***	1.72e-06***	4.90e-07**
Gas	4.53e-06***	4.53e-06***	2.96e-06***
IT2	2.23e-06***	2.23e-06***	2.47e-06***
EUA	2.89e-06***	2.89e-06***	8.35e-06***
Wind1DA	-5.41e-10	-5.41e-10	3.49e-09***
Hydro	1.78e-06***	1.78e-06***	1.19e-06***
PLUvDvarQ	4.29e-05***	4.29e-05***	
PLUvDvarKQ			5.56e-05***
Constant	-0.000494***	-0.000494***	-0.000351***
Observations	11,702	11,702	11,702
R ²	0.128	0.128	0.131

Standard errors available from the authors

X_e	m	Mean	Median	Std. dev.	Min	Max
Tempeff15	6	7.7	8	5	-10	14
Roll_Temp24	6	7.7	9	4	-8	14
Roll_Temp240	1	7.6	8	4	-7	13
suncycle	6	0.3	0	0	0	1
morning	6	0.5	1	0	0	1
deltasun	6	0.1	0.1	0	0	0.4
EWH	6	0.3	0	0	0	1
SolarRest	6	5.4	-1	364	-1,337	2,241
RteBlackBox	6	-0.0	37	4,755	-16,966	18,209

Table 3.14: Variables used in the kernel based PLU^D computation

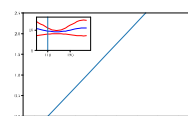
Note: For the $PLUv51$, we have excluded the variable Roll_Temp240 from the conditioning in order to increase the size of each subsample used for the calculation of the observation specific PLU^D . Version 52 also conditions on the variable Roll_Temp240 using $m = 6$.

General conclusion

This thesis focused on the effect of ramping costs on the electricity market at a theoretical level, and then on the empirical analysis of market data to test the theoretical predictions.

The first chapter focused on what the introduction of ramping costs in a theoretical framework brings to the table. Ramping costs represent the fact that electricity suppliers incur costs when their production varies over time. Our main contribution has been to build and justify how these ramping costs can be tackled theoretically. First, we noted that going to a continuous time description of the problem allowed us to bring to the literature about supply function equilibria powerful mathematical tools mostly used in option pricing, that is stochastic dynamics: we want to model ramping costs, i.e. costs associated to the variation in production, while retaining the key ingredient brought by [Klemperer and Meyer, 1989], i.e. the uncertainty, through the use of brownians, and more precisely, Itô processes. In so doing we faced the issue that one cannot derive a brownian, and brought our second contribution, a physical argument about how power plants function that effectively operates as a low pass filter on our stochastic processes, and allowed us to continue to build a tractable model of ramping costs under uncertainty. Third, we found in the literature a specification of Itô processes that allowed the model to remain tractable.

From these technical contributions we obtained our economic contributions in having a rich tractable model that yields results that contrast strongly with past results from the literature. First, in the specific case of linear demand and linear costs we obtained a unique Nash equilibrium, which contrasts with the usual continuum of Nash equilibria in the supply function equilibria literature. Second, our solutions were not ex-post optimal, meaning that gathering information about the expected future evolution of demand yielded different optimal strategies for suppliers, which in turn meant that producers in our framework have a motive for submitting different supply functions from one time step to the next. Third, we have closed form solutions which yield specific predictions



about the evolution of bids under uncertainty, namely that when uncertainty increase, suppliers submit steeper supply schedules in order to transmit more of these shocks to changes in price and not quantities, which are costly due to the existence of ramping costs. Finally, and less importantly, our framework justified the existence of negative prices⁵⁷ by producers being willing to pay consumers to consume more in order to avoid facing large variations in production, in contrast to everywhere positive schedules in the case of the supply function equilibria literature. These results open the door to models being able to differentiate between day-ahead and intraday markets and therefore to offer a framework in which their interactions might be possible.

In the second chapter our main focus was on analyzing our data, on building a way to describe it, and on building proxies for the uncertainty that producers face about the residual demand they have to anticipate when bidding on the day-ahead market.

First, we noted that aggregate supply functions on the day ahead market cannot be well captured by parametric functions. Therefore, we devised a way to describe them non-parametrically: we noted that although they cannot be captured parametrically, they still have a rough S shape, and therefore four main parts, two extremal sections, and two interior ones separated by the inflection point of the curve in its middle section. We defined the transition points between these sections as the points of maximal absolute value for the derivative and second derivative of the supply schedules. This definition relied on kernel density estimates, and was therefore non-parametric. We observed that by using 5 such points, we were able to capture about 98% of the intrinsic variability of the supply schedules, and stopped there although our method can be used to define more non-parametric points. This method allowed us to define points that we considered comparable across auctions, that allowed us to perform cross-sectional analysis of our data in the third chapter.

⁵⁷Note that such negative prices happen, a few hours a year for example in France or Germany, for example in 2017 there were 146 such hours, spread over 24 days in Germany [EPEX, 2018a]

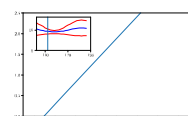
Second, we built proxies for the amount of weather uncertainty that producers face and variables that capture information that suppliers have before bidding and should therefore be controlled for. For the information available to suppliers, we noted that the effect of weather on the demand and more importantly temperature, was well understood and that we needed to control for it. To do so we built an effective temperature for France, as an average of the localised temperature weighted by the population of the spatial region considered, in order to capture the overall effect temperature has on heating.⁵⁸ The rest of our focus was on building a proxy for the uncertainty concerning renewable production. To do so we analyzed spatialized wind and sunlight data and studied its spatial structure. We argued that spatial autocorrelation is a proxy for the uncertainty associated with weather forecasts, noting that if this data displays more spatial gradients, it was likely to be of a lesser quality due to the numerical nature of the weather simulations used to predict the weather, and therefore more uncertain.

Our contribution in the second chapter was to provide a non-parametric way to define comparable points across auctions, and a measure of the uncertainty associated with weather forecasts.

In the final chapter, we studied the impact that uncertainty about the demand plays on the shape of the aggregate supply functions suppliers bid on the French electricity market. We segmented our analysis to different parts of the supply functions in order to show how the overall shape changed with respect to our explanatory variables. We tested some of the predictions from our first chapter, mainly that the supply function should see its slope increase when uncertainty increases.

We noted that the main uncertainty is about the shape of the demand schedules itself. Therefore, we considered data available to the producers and regress the demand schedules on these variables. Next, we studied the residuals of these regressions, and

⁵⁸France has a high level of electric heating overall, which means that demand for electricity is quite sensitive to temperature.



more specifically noted that they are heteroskedastic. We leveraged this, regressing the square of these residuals on our variables, in order to predict the expected amplitude of the residuals, that is the amplitude of the uncertainty of the demand schedule regression.

We then studied the effect of our different proxies for uncertainty on the slope of the supply schedules and noted that if our proxies about the weather uncertainty (through the channel of renewable production) have the expected effect, the results are less clear cut for our residuals on the demand schedules. As we are working with full-blown schedules in the quantity-price plane, we performed our residual analysis both on the prices and the quantities. We therefore obtained estimates for the uncertainty pertaining to the position of a given point of our demand schedule either in price or in quantity. In our theoretical framework, we made the strong assumptions that demand schedules are linear, and that demand shocks are additive, i.e. they do not impact the slope of the demand schedules. These assumptions yielded that we cannot differentiate between shocks in price or quantity, and that they should have effects in the same direction: more uncertainty implying steeper supply curves to reduce the amount of fluctuation in production. However, we observed that the effects of price and quantity uncertainty as estimated by our residuals' method yield opposite effects. Both of these assumptions, although required to obtain closed form results, are clearly not satisfied by our data, and we think that this is a clear path for improvement of the model.

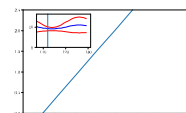
The contribution of the third chapter is to provide a way to estimate the uncertainty about the demand schedules faced by suppliers, and to estimate how this uncertainty affects the shape of the supply schedules at different points along its overall length, i.e. we provide a framework to describe how the functional form of schedules is affected by estimates of the uncertainty faced by suppliers.

Avenues of research

The work presented in this thesis opens new possible avenues of research, that we will outline here.

Theoretical model

- Generalize the functional forms of the demand: we developed our model in the context of linear demand functions, and finding either general results, for example for positive and decreasing demand functions, would lend more support to our results. It would already be interesting to find whether these results hold for other specific functional forms for the demand functions. The issue is that the second order differential equations do not belong to solved for classes of equations in the cases that were tested in the course of this thesis (power demand functions for example). It is therefore unlikely that analytical results can be obtained, however numerical approaches could prove useful in this context.
- Study the impact of other stochastic processes: our results hold in the case of stochastic shocks leading to an equilibrium distribution of a quadratic form. The processes that we use to obtain our results are part of a larger class of processes, which can exhibit richer characteristics, for example assymetric distributions. As previously, the analytical nature of our results relies partly on the specific choice of stochastic process we made, therefore analytical results are unlikely, but numerical approaches could shed light on the effect of skewed distributions.
- Study how a time discrete approach converges towards our continuous time one: the derivation is doable in the case of discrete states for demand shock. A toy model not presented in this thesis was derived in the case of a two period two valued shocks model. Although the results are consistent with those of our continuous model, the expressions derived analytically are already horribly tedious. It is therefore once again a strand of analysis that could profit from a numerical approach.



- The interaction between intraday and day-ahead markets: knowing that when one bids on the day-ahead market, it will still be possible to adjust one's position tomorrow on the intraday market is bound to impact the strategies of the suppliers. Trying to tackle this problem, if challenging, could prove very interesting. Here are key ingredients that should be taken into account:

First, intraday bids can be submitted anytime during the day with an expiry date attached. Therefore, there is tension between, on the one hand, the will to start and correct the outcome of the day-ahead market as soon as new information enters about the demand shocks, so as to increase the likelihood for another agent to buy the intraday bid, and on the other hand, the will to wait and see as information enters to be as precise as possible on the submitted bid to correct the outcome of the day-ahead market, but therefore decreasing the likelihood to find a buyer.

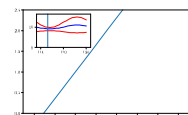
Second, the ramping costs associated with changing production are incurred only after the net of the day-ahead market and intraday market is fixed.

- Implement the actual market clearing algorithm and study it numerically.

Empirical analysis

- Take into account the block orders: these orders impact the bids and should be accounted for.
- Study in more detail the overall function without restriction to only 5 points, which should be doable with the increase in computational power
- Leverage the difference between weather prediction data and observations for more accurate weather uncertainty.
- Take into account the uncertainty associated with international interconnexions.

- Analyze the individual submitted points on the aggregate supply schedules to find whether it is possible to attach some of them to specific power plants with any certainty.



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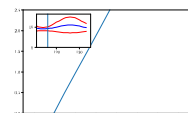
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Code Annex

Autocorrelation lengthscale

Overall, the Matlab code was written by Alexis Bergès, whereas the Stata code was written by Henri de Belsunce. All ideas, architecture choices, methods were co-developed with equal contributions from both

This first bit of code builds the autocorrelation lengthscales introduced in chapter 2.

It takes data from Météofrance, that comes as files containing a row per observation, each row containing the timestamp of the observation, as well as the code of the station, and the observation itself, to which was added the latitude and longitude of the current station. For a given year, only data pertaining to this year is kept in memory, to avoid running out of RAM.

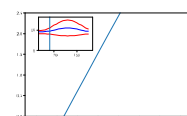
Once this first step is done, and for every timestamp, all pairs of stations are taken, and the difference of value in the observation is taken, as well as the distance between the stations.

Once this treated data is generated, an exponential is fitted, and its coefficient is saved as the autocorrelation lengthscale for this given variable, and this given timestamp. The code to generate a graph of the cloud of points and the fitted function is also included.

```

1 clear all
2
3 filenamemaster='~/Desktop/Google Drive/Encheres Elec/For Matlab/Exogenous_treated_03.2014/Vent/
  ExoVent.numeric.csv';
4 M = csvread(filenamemaster);
5
6 %%Keep only year of interest
7 yyyy=2011;
8 for i=1:size(M,1)
9     if mod(i,10000)==0
10        disp('y of interest'), disp(i)
11    end
12    if M(i,5)==yyyy&&(i==1||M(i-1,5)<yyyy)
13        yi=i;
14    end
15    if M(i,5)==yyyy&&(i==size(M,1)||M(i+1,5)>yyyy)

```




```

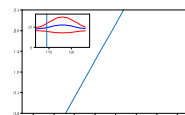
16         yf=i ;
17     end
18 end
19 Myyyy=M(yi:yf,:) ;
20 % clear M
21
22
23 kini=1;
24 kend=0;
25 lautocor=[];
26 for k=1:(size(Myyyy,1)-1)
27     if mod(k,1000)==0
28         k
29     end
30     if Myyyy(k,8)~=Myyyy(k+1,8 )
31         kend=k;
32         autocor=[];
33         for i=kini:(kend-1)
34             for j=(i+1):kend
35                 ertemp=abs(Myyyy(i,1)-Myyyy(j,1));
36                 disttemp=sqrt((Myyyy(i,9)-Myyyy(j,9))^2+(Myyyy(i,10)-Myyyy(j,10))^2);
37                 autocortemp=cat(2,disttemp,ertemp);
38                 autocor=cat(1,autocor,autocortemp);
39             end
40         end
41         binranges=0:0.004:max(autocor(:,1));
42         [bincounts,ind]=histc(autocor(:,1),binranges);
43         autocor=cat(2,autocor,ind);
44         A=sortrows(autocor,3);
45         rini=1;
46         rend=0;
47         meanstd=[];
48         for i=1:size(bincounts,1)
49             if bincounts(i)>0
50                 rend=rend+bincounts(i);
51                 m1=mean(A(rini:rend,2));
52                 m2=mean(A(rini:rend,1));
53                 meantemp=cat(2,bincounts(i),m1,m2);
54                 meanstd=cat(1,meanstd,meantemp);
55                 rini=rend+1;
56             end
57         end
58
59         y=meanstd(1:round(4/5*size(meanstd,1)),2);
60         x=meanstd(1:round(4/5*size(meanstd,1)),3);
61 %         y=meanstd(:,2);
62 %         x=meanstd(:,3);
63 %         figure , plot(meanstd(2:size(meanstd,1),3),meanstd(2:size(meanstd,1),2))
64 %         g = fittype('a*(1-exp(-x/b))','dependent',{'y'},'independent',{'x'},...
65 %         'coefficients',{'a','b'});
66 %         myfit=fit(x,y,g,'Lower',[0,0],'Upper',[10,0.3],'Startpoint',[4.5 0.04]);
67 %         coefffit=coeffvalues(myfit);
68 %         figure , plot(myfit,meanstd(2:size(meanstd,1),3),meanstd(2:size(meanstd,1),2))
69         kini=k+1;
70         lautocortemp=cat(2,coefffit(2),Myyyy(k,5),Myyyy(k,6),Myyyy(k,7),Myyyy(k,8));
71         lautocor=cat(1,lautocor,lautocortemp);
72     end
73 end
74
75 %%%Convert lautocor to string to be stata friendly

```

```

76 dateseff=[];
77 for i=1:size(lautocor,1)
78     if lautocor(i,4)<10
79         dd=['0' num2str(lautocor(i,4))];
80     else dd=num2str(lautocor(i,4));
81     end
82     if lautocor(i,3)<10
83         mm=['0' num2str(lautocor(i,3))];
84     else mm=num2str(lautocor(i,3));
85     end
86     if lautocor(i,5)<10
87         hh=['0' num2str(lautocor(i,5))];
88     else hh=num2str(lautocor(i,5));
89     end
90     dated=[dd '/' mm '/' num2str(lautocor(i,2))];
91     datesefftemp={dated, hh, num2str(lautocor(i,1))};
92     dateseff=cat(1, dateseff, datesefftemp);
93 end
94
95 %%Prepare the data to be written to .txt file
96 dateseff=dateseff;
97 fid = fopen(['\Users/alexisberges/Desktop/Google Drive/Encheres Elec/For Matlab/lautocor_' num2str(
98     yyyy) '.txt'], 'w');
99 fprintf(fid, '%s, %s, %s\n', dateseff{:,:});
100 fclose(fid);
101
102 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% graph lautocor well
103 houra=[0:M(:,8)];
104 hourb=[M(:,8);0];
105 idxchange=find(houra-hourb);
106
107 lautocor=[];
108 hourlook=470;
109 kini=idxchange(hourlook);
110 kend=idxchange(hourlook+1)-1;
111 autocor=[];
112 k=1;
113 for i=kini:(kend-1)
114     for j=(i+1):kend
115         ertemp=abs(M(i,1)-M(j,1));
116         disttemp=sqrt((M(i,9)-M(j,9))^2+(M(i,10)-M(j,10))^2)*6371;
117         autocortemp=cat(2, disttemp, ertemp);
118         autocor=cat(1, autocor, autocortemp);
119     end
120 end
121 binranges=0:0.004*6371:max(autocor(:,1));
122 [bincounts, ind]=histc(autocor(:,1), binranges);
123 autocor=cat(2, autocor, ind);
124 A=sortrows(autocor,3);
125 rini=1;
126 rend=0;
127 meanstd=[];
128 for i=1:size(bincounts,1)
129     if bincounts(i)>0
130         rend=rend+bincounts(i);
131         m1=mean(A(rini:rend,2));
132         m2=mean(A(rini:rend,1));
133         meantemp=cat(2, bincounts(i), m1, m2);
134         meanstd=cat(1, meanstd, meantemp);

```



```

135         rini=rend+1;
136     end
137 end
138
139 y=meanstd(1:round(4/5*size(meanstd,1)),2);
140 x=meanstd(1:round(4/5*size(meanstd,1)),3);
141 %     y=meanstd(:,2);
142 %     x=meanstd(:,3);
143 %     figure, plot(meanstd(2:size(meanstd,1),3),meanstd(2:size(meanstd,1),2),'-','LineWidth',2)
144 g = fittype('a*(1-exp(-x/b))','dependent',{'y'},'independent',{'x'},...
145     'coefficients',{'a','b'});
146 myfit=fit(x,y,g,'Lower',[0,0],'Upper',[10,0.3*6371],'Startpoint',[4.5 0.04*6371]);
147 coefffit=coeffvalues(myfit);
148     figure
149     plot1=plot(myfit,'r');
150     set(plot1,'LineWidth',4)
151     hold on
152     plot2=plot(meanstd(2:size(meanstd,1),3),meanstd(2:size(meanstd,1),2),'b+');
153     set(plot2,'LineWidth',4)
154     hold on
155     plot3=plot(autocor(:,1),autocor(:,2),'ko','MarkerSize',2,'MarkerFaceColor',[0,0,0]);
156     line1=line([0, 0.02*6371], [0,coefffit(1)/coefffit(2)*0.02*6371],'Color',[0 0 0],'LineStyle','--','LineWidth',2);
157     line2=line([0, 0.2*6371], [coefffit(1),coefffit(1)],'Color',[0 0 0],'LineStyle','--','LineWidth',2);
158     line3=line([coefffit(2), coefffit(2)], [0,coefffit(1)],'Color',[0 0.5 0],'LineStyle','--','LineWidth',2);
159     plotleg=legend([plot3 plot2 plot1 line1 line3],
160         {'Data from the pairs' 'Kernel smoothed data' 'Fitted exponential'
161         'Derivatives of the fitted curve at 0 and \infty' 'Autocorrelation lengthscale'
162         },
163         'Location','northwest');
164     set(plotleg,'FontSize',24);
165     kini=k+1;
166     lautocortemp=cat(2,coefffit(2),M(k,5),M(k,6),M(k,7),M(k,8));
167     lautocor=cat(1,lautocor,lautocortemp);

```

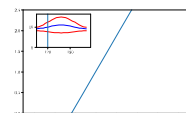
Regressions

The next bit of code is the general do file for the regressions of chapter 3.

```

1 * COMPUTE COMPARABLE TABLES FOR SQ-RESIDUALS.
2
3 *** VERSION FEBRUARY 2015
4
5 * Final Do-file
6
7 /* Notes:
8 ----- MAKE SURE NORMAL VERSION RUNS LAST TO ENSURE CORRECT TABLES FOR DEMAND ESTIMATION
9 */
10
11 macro drop _all
12 global YearINIT = 2011
13 global YearFIN = 2013
14 global HourINIT = 1
15 global HourFIN = 24
16 global vINPUT= "v38"
17 global vOUTPUT= "v38"
18
19 global dofiledirectoryorig= Path to directory containing do files
20 global LATEXPATH = Path to directory containing latex for article
21 global CLOUDPATH = Path to directory containing data
22
23 global focusk = 5
24
25 *****
26 * Data Prep
27 *****
28
29 use "%{CLOUDPATH}v38/Temp_data/DataReady_Y${YearINIT}-Y${YearFIN}.dta", clear
30 * manually merged Roll_avgT24 to dataset.
31 * manually merged winddiffonly to dataset
32
33 destring YYYY MM DD, replace
34 drop Year Month
35 gsort YYYY MM DD Hour SalePurchase select
36 order Date Hour Price Volume SalePurchase select
37
38
39 rename COALIMPORTPRICEEURpTON Coal
40 rename EST_PRICE_ELEC_EXPORT EExportPrice /*in EUR*/
41 rename BRENT_LDN_AVG Brent
42 rename GAS.SPOT.GBPpTHERM Gas
43 rename LcWind2011 LcWind
44
45 label var Roll_Temp24 "Roll\_Temp24"
46 label var Roll_Temp240 "Roll\_Temp240"
47 label var Roll_Temp720 "Roll\_Temp720"
48 label var Roll_avgT24 "Roll\_avgT24"
49 label var Roll_avgT240 "Roll\_avgT240"
50
51 * dataprep:
52 gen IT2 = (PrevConsoH / 99400) * Gas /*PrevConsoH not included in supply, 99400 is max of
53 PrevConsoH */

```



```

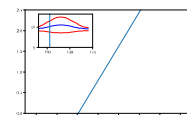
53     gen EWH = 0
54     replace EWH = 1   if Hour<=4
55     replace EWH = 1   if Hour>=22
56     gen dfaT15 = Roll_Temp24 -Roll_Temp240
57
58
59     * generate slope of opposite function (on Demand, add fx of supply function)
60     gsort select Datestata Hour SalePurchase
61     capture drop errorindic
62     gen errorindic = 1 if SalePurchase[_n]==SalePurchase[_n+1]
63     drop if errorindic == 1 & SalePurchase=="Purchase"
64     drop errorindic
65     capture drop fxInvertPQ_viaP fxInvertQP_viaP
66     capture drop Price_S_viaP Volume_S_viaP
67     gen fxInvertPQ_viaP = 1/( fx[_n+1]) if SalePurchase=="Purchase"
68     gen fxInvertQP_viaP = fx[_n+1] if SalePurchase=="Purchase"
69     gen Price_S_viaP = Price[_n+1] if SalePurchase=="Purchase"
70     gen Volume_S_viaP = Volume[_n+1] if SalePurchase=="Purchase"
71     gsort SalePurchase select Datestata Hour
72     gen fxInvertQP = fxInvertQP_viaP
73     gen fxInvertPQ = fxInvertPQ_viaP
74
75     * make points comparable by volume (not by price!)
76     ***** remove to do other dimension + must rename fxinvertQP
77     gen selectQ = select
78     capture drop lowpk
79     capture drop selectviaP
80     gen lowpk = 1 if select <5 & SalePurchase=="Purchase"
81     replace selectQ =9   if selectQ ==1 & SalePurchase=="Purchase" & lowpk == 1
82     replace selectQ =7   if selectQ ==3 & SalePurchase=="Purchase" & lowpk == 1
83     replace selectQ =3   if selectQ ==7 & SalePurchase=="Purchase" & lowpk == .
84     replace selectQ =1   if selectQ ==9 & SalePurchase=="Purchase" & lowpk == .
85     rename select selectviaP
86     rename selectQ select
87
88     * generate slope of opposite function (on Demand, add fx of supply function)
89     gsort select Datestata Hour SalePurchase
90     capture drop errorindic
91     gen errorindic = 1 if SalePurchase[_n]==SalePurchase[_n+1]
92     drop if errorindic == 1 & SalePurchase=="Purchase"
93     drop errorindic
94     capture drop fxInvertPQ_viaQ fxInvertQP_viaQ
95     capture drop Price_S_viaQ Volume_S_viaQ
96     gen fxInvertPQ_viaQ = 1/( fx[_n+1]) if SalePurchase=="Purchase"
97     gen fxInvertQP_viaQ = fx[_n+1] if SalePurchase=="Purchase"
98     gen Price_S_viaQ = Price[_n+1] if SalePurchase=="Purchase"
99     gen Volume_S_viaQ = Volume[_n+1] if SalePurchase=="Purchase"
100    gsort SalePurchase select Datestata Hour
101    capture drop fxInvertQP fxInvertPQ
102    gen fxInvertQP = fxInvertQP_viaQ
103    gen fxInvertPQ = fxInvertPQ_viaQ
104    drop fxInvertPQ_viaP fxInvertQP_viaP
105    order Date Hour Price Volume SalePurchase select selectviaP
106
107
108
109 *****
110 * Solar1DA prediction
111 *****
112 capture drop IT1

```

```

113     capture drop SolarRest
114     gen IT1 = suncycle * Roll_avgT240 /*Interaction term 1 = suncycle * average
      temp no cutoff at 15 = proxy for sunangle - decided against as no
      change in ry of rte black box and easier interpretation*/
115     global solarestimationvariables " suncycle"
116
117     reg Solar1DA $solarestimationvariables , robust
118     est store Black_3
119
120     capture drop SolarBlackBox
121     capture drop blackepsilon
122     capture mat drop blackalpha
123     mat blackmat = e(b)
124     mat li blackmat
125     scalar blackalpha = blackmat[1,3]
126     di blackalpha
127
128     predict blackepsilon if e(sample), residuals
129     gen SolarRest = blackepsilon /*+ blackalpha*/
130     drop blackepsilon
131
132     btoutreg2 [Black_3] using "${LATEXPATH}SolarBlack.tex", replace tex(frag pretty )
      /*stats(coef Var se)*/ label(proper) level(95) sideways noparen
133
134
135     *drop irrelevant OBS and variables
136     drop _est*
137     drop if SolarRest==.
138
139
140
141     *****
142     * Black box prediction RTE
143     *****
144     global blackestimationvariables1 " Tempeff15 Roll_Temp24 Roll_Temp240
      SolarRest suncycle morning deltasun EWH"
145     global blackestimationvariables2 "Tempeff Roll_avgT24 Roll_avgT240
      SolarRest suncycle morning deltasun EWH "
146     global blackestimationvariables3 " Tempeff15 Roll_Temp24 Roll_Temp240
      SolarRest suncycle morning deltasun IT1 EWH CZlag EExportPlag"
147     global blackestimationvariables4 " Tempeff15 Roll_Temp24 Roll_Temp240
      SolarRest suncycle morning deltasun IT1 "
148
149     *** INTERPRETATION : coeff on tempeff15 much larger than tempeff -> positive for us
150
151     reg PrevConsoH $blackestimationvariables1 , robust
152     est store Black_1
153
154     capture drop RteBlackBox
155     capture drop blackepsilon
156     capture mat drop blackalpha
157     mat blackmat = e(b)
158     mat li blackmat
159     scalar blackalpha = blackmat[1,9]
160     di blackalpha
161
162     predict blackepsilon if e(sample), residuals
163     gen RteBlackBox = blackepsilon /*+ blackalpha*/
164     drop blackepsilon

```



```

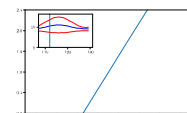
165
166         reg PrevConsoH $blackestimationvariables2, robust
167         est store Black_2
168
169         reg PrevConsoH $blackestimationvariables3, robust
170         est store Black_3
171
172         reg PrevConsoH $blackestimationvariables4, robust
173         est store Black_4
174
175         btoutreg2 [Black_1 Black_2 ] using "${LATEXPATH}Black1.tex", replace tex(frag
176         pretty ) stats(coef) label(proper) level(95) sideways noparen
177         btoutreg2 [Black_1 Black_2 Black_3 Black_4] using "${LATEXPATH}Black2.tex", replace
178         tex(frag pretty ) stats(coef) label(proper) level(95) sideways noparen
179
180         *drop irrelevant OBS and variables
181         drop _est*
182         drop if RteBlackBox==.
183
184         *save
185         save "${CLOUDPATH}v38/Temp_data/Pre4and5.dta", replace
186
187 *****
188 * Generate PLU using forecast model
189 *****
190
191         *****
192         * DEMAND ESTIMATION + generate residuals for PLUD
193         *****
194
195         * DEMAND ESTIMATION (no CZlag)
196         local version "52"
197         global runversionD `version'
198         global demandestimationvariables `version' "Tempeff15 Roll_Temp24 Roll_Temp240
199         suncycle morning deltasun EWH SolarRest RteBlackBox "
200
201         * run do file for demand estimation
202         do "${CLOUDPATH}v38/DoFiles/107_Eqn4demand.do"
203
204
205         *****
206         * PLUvD : predicted uncertainty directly using regression
207         *****
208
209         * Only predict uncertainty for demand function
210         * Cannot do so for supply function since mixed with ex-post own bidding strategy.
211
212         global DEV ${demandestimationvariables${runversionD}}
213
214         capture drop PLUvDvarP
215         capture drop PLUvDvarQ
216         gen PLUvDvarP =.
217         gen PLUvDvarQ =.
218
219         forvalues k = 1(2)9{
220         foreach s in /*Sell*/ Purchase{
221         di "Point " `k' " Curve " "`s'"
222         reg sqresVolume ${DEV} if select=='k' & SalePurchase=="`s'"

```

```

222         predict PLUvDvartmpQ'k'a's' if e(sample), xb
223         est store predictuncQa'k'a's'
224         replace PLUvDvarQ = PLUvDvartmpQ'k'a's' if e(sample)
225         drop PLUvDvartmpQ'k'a's'
226     }
227 }
228
229 forvalues k = 1(2)9{
230     foreach s in /*Sell*/ Purchase{
231         di "Point " 'k' " Curve " "'s'"
232         reg sqresPrice ${DEV} if select=='k' & SalePurchase=="'s'"
233         predict PLUvDvartmpP'k'a's' if e(sample), xb
234         est store predictuncPa'k'a's'
235         replace PLUvDvarP = PLUvDvartmpP'k'a's' if e(sample)
236         drop PLUvDvartmpP'k'a's'
237     }
238 }
239
240 capture drop PLUvDvarPabs
241 capture drop PLUvDvarQabs
242 gen PLUvDvarPabs =.
243 gen PLUvDvarQabs =.
244
245 forvalues k = 1(2)9{
246     foreach s in /*Sell*/ Purchase{
247         di "Point " 'k' " Curve " "'s'"
248         reg absresVolume ${DEV} if select=='k' & SalePurchase=="'s'"
249         predict PLUvDvartmpQ'k'a's' if e(sample), xb
250         est store predictuncQa'k'a's'ABS
251         replace PLUvDvarQabs = PLUvDvartmpQ'k'a's' if e(sample)
252         drop PLUvDvartmpQ'k'a's'
253     }
254 }
255
256 forvalues k = 1(2)9{
257     foreach s in /*Sell*/ Purchase{
258         di "Point " 'k' " Curve " "'s'"
259         reg absresPrice ${DEV} if select=='k' & SalePurchase=="'s'"
260         predict PLUvDvartmpP'k'a's' if e(sample), xb
261         est store predictuncPa'k'a's'ABS
262         replace PLUvDvarPabs = PLUvDvartmpP'k'a's' if e(sample)
263         drop PLUvDvartmpP'k'a's'
264     }
265 }
266
267
268 * (generating) tables on PLUvD
269 btoutreg2 [predictuncPa5aPurchase predictuncPa5aPurchaseABS /*predictuncPa5aSell*/]
270     using "${LATEXPATH}predictunc1.tex", replace tex(frag pretty ) stats(coef)
271     label(proper) level(95)
272
273 btoutreg2 [predictuncQa5aPurchase predictuncQa5aPurchaseABS /*predictuncQa5aSell*/]
274     using "${LATEXPATH}predictunc2.tex", replace tex(frag pretty ) stats(coef)
275     label(proper) level(95)
276
277 * drop table data
278 drop _est*

```

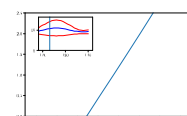



```

278
279 * take sqrt of squared PLUs to get other order of magnitudes
280     gen PLUvDrtP = sqrt(PLUvDvarP)
281     gen PLUvDrtQ = sqrt(PLUvDvarQ)
282
283
284 *****
285 * PLUvR: LongueurCorrel Temp:
286 *****
287
288     gen PLUvRvarT = 1 / LcTemp
289     gen PLUvRvarW = 1 / LcWind
290     gen PLUvRvarS = 1 / LcSolar
291
292 * add u-shaped term
293     gen PLUvRvarTsq = 1/(LcTemp * LcTemp)
294     gen PLUvRvarWsqr = 1/(LcWind * LcWind)
295     gen PLUvRvarSsq = 1/( LcSolar * LcSolar)
296
297
298     /* table of extracted slopes
299     capture mat drop M
300     local variables "1 3 5 7 9"
301     local FUNC "r(mean) r(p50) r(sd) r(min) r(max)"
302     local i=0
303     foreach var of local variables{
304         local i='i'+1
305     }
306     local j=0
307     foreach var of local FUNC{
308         local j='j'+1
309     }
310     di `j' " " `i'
311     mat M = J(`i',5,.)
312
313     local c=1
314     foreach FF of local FUNC{
315         local r=1
316         foreach VV of local variables{
317             su /* fxInvertPQ */ /* fxInvertQP if select=='VV', detail
318             mat M[`,r',`c']= `FF'
319             local r='r'+1
320         }
321         local c='c'+1
322     }
323     mat rownames M = `variables'
324     mat colnames M = Mean Median StdDev Min Max
325     mat li M
326
327     btouttable using "${LATEXPATH}extractedslopes", replace mat(M) asis nobox
328         caption("Estimated slopes of the supply function per point k") format
329         (%9.4fc %9.4fc %9.4fc %9.4fc %9.4fc %9.4fc) */
330
331 *save
332     save "${CLOUDPATH}v38/Temp_data/Finaldataset.dta", replace
333
334 *****
335 * generate kernel based PLUs

```

```
336 *****
337             use "${CLOUDPATH}v38/Temp_data/Finaldataset.dta", clear
338
339 ***** generate kernel PLUS
340             * do "${CLOUDPATH}v38/DoFiles/107_kernelbucketreg.do"
341 *****
342
343 ***** dataset manipulations to obtain final dataset
344             do "${CLOUDPATH}v38/DoFiles/107_PrepkernelPLUdata.do"
345 *****
346
347 *save
348             save "${CLOUDPATH}v38/Temp_data/Finalrundataset.dta", replace
349
350
351
352
353
354
355
356
357
358
359
360
361
362
363
364
365
366
367
368
369
370
371
372
373
374
375
376 *****
377 * Columns based on kernel PLUvD
378 *****
379             * do "${CLOUDPATH}v38/DoFiles/107_BootstrapKernel2702.do"
380
381
382
383
384
385
386
387
388
389
390
391
392
393
394
395
```



```

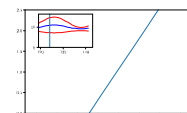
396
397
398
399 *****
400 * START OF BASELINE RESULTS + BOOTSTRAP
401 * ROBUSTNESS
402 *****
403
404 ***** BASELINE
405
406
407 global focusk = 5
408
409
410 *****
411 * Some regressions: ----- on point k only -----
412 *****
413
414 use "${CLOUDPATH}v38/Temp_data/Finalrundataset.dta", clear
415
416 *****
417 * FOCUS POINT
418 *****
419     local k=${focusk}
420     keep if select=='k' & SalePurchase=="Purchase"
421
422 * MACROS:
423     global PLUsR "PLUvRvarT PLUvRvarTsq PLUvRvarW PLUvRvarWsq PLUvRvarS PLUvRvarSsq"
424     global SupCo "Coal Brent Gas IT2 EUA WindIDA Hydro"
425
426 ***** Program bootstrap of baseline
427     capture program drop my2slsforbaselinebootstrap
428     program my2slsforbaselinebootstrap
429         local k=${focusk}
430         capture drop PLU_P_boot PLU_Q_boot
431         capture drop PLUvDvarP'k'resc PLUvDvarQ'k'resc
432         capture noisily reg sqresPrice Tempeff15 Roll_Temp24 Roll_Temp240 suncycle morning
433             deltasun EWH SolarRest RteBlackBox if select =='k' & SalePurchase=="Purchase",
434             robust
435         predict PLU_P_boot, xb
436         su PLU_P_boot, meanonly
437         scalar tmpP = r(mean)
438         capture noisily gen PLUvDvarP'k'resc = PLU_P_boot / tmpP
439         reg sqresVolume Tempeff15 Roll_Temp24 Roll_Temp240 suncycle morning deltasun EWH
440             SolarRest RteBlackBox if select =='k' & SalePurchase=="Purchase", robust
441         predict PLU_Q_boot, xb
442         su PLU_Q_boot, meanonly
443         scalar tmpQ = r(mean)
444         gen PLUvDvarQ'k'resc = PLU_Q_boot / tmpQ
445         if 'k'==1 | 'k'==9 {
446             reg fxInvertQP PLUvRvarT PLUvRvarTsq PLUvRvarW PLUvRvarWsq PLUvRvarS PLUvRvarSsq
447                 Coal Brent Gas IT2 EUA WindIDA Hydro /*PLUvDvarP'k'resc*/ PLUvDvarQ'k'resc
448                 if select =='k' & SalePurchase=="Purchase" , robust
449         }
450         else {
451             reg fxInvertQP PLUvRvarT PLUvRvarTsq PLUvRvarW PLUvRvarWsq PLUvRvarS PLUvRvarSsq
452                 Coal Brent Gas IT2 EUA WindIDA Hydro PLUvDvarP'k'resc PLUvDvarQ'k'resc if

```

```

447         select =='k' & SalePurchase=="Purchase" , robust
448     }
449     capture drop PLU_P_boot PLUvDvarP'k'resc PLU_Q_boot PLUvDvarQ'k'resc
450
451     end
452 *FIRST TABLE:
453
454     *col 1
455         local k=${focusk}
456         capture drop PLUvDvarP'k'resc
457         capture drop PLUvDvarQ'k'resc
458         su PLUvDvarP'k' , meanonly
459         scalar tmpresc = r(mean)
460         capture noisily gen PLUvDvarP'k'resc = PLUvDvarP'k' / tmpresc
461         su PLUvDvarQ'k' , meanonly
462         scalar tmpresc = r(mean)
463         gen PLUvDvarQ'k'resc = PLUvDvarQ'k' / tmpresc
464                 if 'k'==1 | 'k'==9 {
465         reg fxInvertQP    ${PLUSR} ${SupCo} /*PLUvDvarP'k'resc*/ PLUvDvarQ'k'resc
466         est store dlshort1_`k'
467                 }
468         else {
469         reg fxInvertQP    ${PLUSR} ${SupCo} PLUvDvarP'k'resc PLUvDvarQ'k'resc
470         est store dlshort1_`k'
471         }
472     *col 2
473         local k=${focusk}
474         bootstrap _b, reps(200) seed(12345): my2slsforbaselinebootstrap
475         est store bs_baseline_`k'
476         est save "${CLOUDPATH}v38/Temp_data/bs_baseline_`k'.ster" , replace
477     *col 3
478         local k=${focusk}
479         estimates use "${CLOUDPATH}v38/Temp_data/kernelweighthed_`k'.ster"
480         regress
481         estimates esample
482         estimates store kernel4_`k'
483     *col 4
484         local k=${focusk}
485         if 'k'==5{
486             estimates use "${CLOUDPATH}v38/Temp_data/kernelbootstrap_`k'.ster"
487             regress
488             estimates esample:
489             estimates store kernel5_`k'
490         }
491
492     * table main
493         local k=${focusk}
494         if 'k'==5{
495             btoutreg2 [dlshort1_`k' bs_baseline_`k' kernel4_`k' kernel5_`k'] using "${
496                 LATEXPATH}main1_`k'.tex" , replace tex(frag pretty ) stats(coef se)
497                 label(proper) level(95) title(For k=`k')
498             btoutreg2 [dlshort1_`k' bs_baseline_`k' kernel4_`k' kernel5_`k'] using "${
499                 LATEXPATH}mainNS1_`k'.tex" , replace tex(frag pretty ) stats(coef) label
500                 (proper) level(95) title(For k=`k')
501             btoutreg2 [dlshort1_`k' bs_baseline_`k' kernel4_`k' kernel5_`k'] using "${
502                 LATEXPATH}mainO1_`k'.tex" , replace tex(frag pretty ) stats(se) label(
503                 proper) level(95) title(For k=`k')
504         }
505         else{

```



```

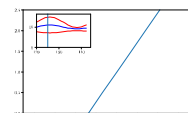
500      btoutreg2 [d1short1_`k' bs_baseline_`k' kernel4_`k' /*kernel5_`k'*/] using
          `${LATEXPATH}main1_`k'.tex", replace tex(frag pretty ) stats(coef se)
          label(proper) level(95) title(For k=`k')
501      btoutreg2 [d1short1_`k' bs_baseline_`k' kernel4_`k' /*kernel5_`k'*/] using
          `${LATEXPATH}mainNS1_`k'.tex", replace tex(frag pretty ) stats(coef)
          label(proper) level(95) title(For k=`k')
502      btoutreg2 [d1short1_`k' bs_baseline_`k' kernel4_`k' /*kernel5_`k'*/] using
          `${LATEXPATH}mainoS1_`k'.tex", replace tex(frag pretty ) stats(se)
          label(proper) level(95) title(For k=`k')
503    }
504
505    * table compare
506    forvalues k=1(2)9{
507      estimates use `${CLOUDPATH}v38/Temp_data/bs_baseline_`k'.ster"
508      regress
509      estimates esample:
510      estimates store bs_baseline_`k'
511    }
512
513      btoutreg2 [bs_baseline_1 bs_baseline_3 bs_baseline_5 bs_baseline_7
          bs_baseline_9] using `${LATEXPATH}compare_col2.tex", replace tex(frag
          pretty ) stats(coef se) label(proper) level(95) title(Comparison of
          col. 2)
514      btoutreg2 [bs_baseline_1 bs_baseline_3 bs_baseline_5 bs_baseline_7
          bs_baseline_9] using `${LATEXPATH}compareNS_col2.tex", replace tex(frag
          pretty ) stats(coef) label(proper) level(95) title(Comparison of col.
          2)
515
516    forvalues k=1(2)9{
517      estimates use `${CLOUDPATH}v38/Temp_data/kernelweighthed_`k'.ster"
518      regress
519      estimates esample
520      estimates store kernel4_`k'
521    }
522
523      btoutreg2 [ kernel4_1 kernel4_3 kernel4_5 kernel4_7 kernel4_9] using `${
          LATEXPATH}compare_col3.tex", replace tex(frag pretty ) stats(coef se)
          label(proper) level(95) title(Comparison of col. 3)
524      btoutreg2 [ kernel4_1 kernel4_3 kernel4_5 kernel4_7 kernel4_9] using `${
          LATEXPATH}compareNS_col3.tex", replace tex(frag pretty ) stats(coef)
          label(proper) level(95) title(Comparison of col. 3)
525
526
527
528
529
530      *****
531      * END OF BASELINE RESULTS + BOOTSTRAP
          ROBUSTNESS
532      *****
533
534
535      *****
536      * START OF DROPPING 1 PLUvDvarP or Q
          *****
537
538

```

```

539 global focusk = 5
540
541
542 *****
543 * Some regressions: ----- on point k only -----
544 *****
545
546         use "{CLOUDPATH}v38/Temp_data/Finalrundataset.dta", clear
547
548 *****
549 * FOCUS POINT
550 *****
551     local k=${focusk}
552     keep if select=='k' & SalePurchase=="Purchase"
553
554 * MACROS:
555     global PLUsR "PLUvRvarT PLUvRvarTsq PLUvRvarW PLUvRvarWsQ PLUvRvarS PLUvRvarSsq"
556     global SupCo "Coal Brent Gas IT2 EUA WindIDA Hydro"
557
558 ***** Program bootstrap of baseline
559     capture program drop my2slsforbaselinebootstrap
560     program my2slsforbaselinebootstrap
561         local k=${focusk}
562         capture drop PLU_P_boot PLU_Q_boot
563         capture drop PLUvDvarP'k'resc PLUvDvarQ'k'resc
564         capture noisily reg sqresPrice Tempeff15 Roll_Temp24 Roll_Temp240 suncycle morning
565             deltasun EWH SolarRest RteBlackBox if select =='k' & SalePurchase=="Purchase" ,
566             robust
567         predict PLU_P_boot, xb
568         su PLU_P_boot, meanonly
569         scalar tmpP = r(mean)
570         capture noisily gen PLUvDvarP'k'resc = PLU_P_boot / tmpP
571         reg sqresVolume Tempeff15 Roll_Temp24 Roll_Temp240 suncycle morning deltasun EWH
572             SolarRest RteBlackBox if select =='k' & SalePurchase=="Purchase" , robust
573         predict PLU_Q_boot, xb
574         su PLU_Q_boot, meanonly
575         scalar tmpQ = r(mean)
576         gen PLUvDvarQ'k'resc = PLU_Q_boot / tmpQ
577         if 'k'==1 | 'k'==9 {
578             reg fxInvertQP PLUvRvarT PLUvRvarTsQ PLUvRvarW PLUvRvarWsQ PLUvRvarS PLUvRvarSsq
579                 Coal Brent Gas IT2 EUA WindIDA Hydro /*PLUvDvarP'k'resc*/ PLUvDvarQ'k'resc
580                 if select =='k' & SalePurchase=="Purchase" , robust
581         }
582         else {
583             reg fxInvertQP PLUvRvarT PLUvRvarTsQ PLUvRvarW PLUvRvarWsQ PLUvRvarS PLUvRvarSsq
584                 Coal Brent Gas IT2 EUA WindIDA Hydro /*PLUvDvarP'k'resc*/ PLUvDvarQ'k'resc
585                 if select =='k' & SalePurchase=="Purchase" , robust
586         }
587         capture drop PLU_P_boot PLUvDvarP'k'resc PLU_Q_boot PLUvDvarQ'k'resc
588     end
589
590 *col 1
591     local k=${focusk}
592     capture drop PLUvDvarP'k'resc
593     capture drop PLUvDvarQ'k'resc
594     su PLUvDvarP'k', meanonly
595     scalar tmpresc = r(mean)
596     capture noisily gen PLUvDvarP'k'resc = PLUvDvarP'k' / tmpresc
597     su PLUvDvarQ'k', meanonly

```



```

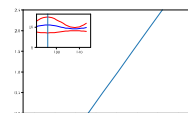
592         scalar tmpresc = r(mean)
593         gen PLUvDvarQ'k'resc = PLUvDvarQ'k' / tmpresc
594             if 'k'==1 | 'k'==9 {
595         reg fxInvertQP    ${PLUsR}  ${SupCo} /*PLUvDvarP'k'resc*/ PLUvDvarQ'k'resc
596         est store dlshort1_`k'
597             }
598         else{
599         reg fxInvertQP    ${PLUsR}  ${SupCo} /*PLUvDvarP'k'resc*/ PLUvDvarQ'k'resc
600         est store dlshort1_`k'
601         }
602     *col 2
603         local k=${focusk}
604         bootstrap _b, reps(200) seed(12345): my2slsforbaselinebootstrap
605         est store bs_baseline_`k'
606         est save "${CLOUDPATH}v38/Temp_data/bs_baseDROP_P_`k'.ster", replace
607     *col 3
608         local k=${focusk}
609         estimates use "${CLOUDPATH}v38/Temp_data/kernelweigDROP_P_`k'.ster"
610         regress
611         estimates esample
612         estimates store kernel4_`k'
613     *col 4
614 /*
615     local k=${focusk}
616     if 'k'==5{
617         estimates use "${CLOUDPATH}v38/Temp_data/kernelbootDROP_P_`k'.ster"
618         regress
619         estimates esample:
620         estimates store kernel5_`k'
621     }*/
622     * table main
623         local k=${focusk}
624         if 'k'==5{
625         btoutreg2 [dlshort1_`k' bs_baseline_`k' kernel4_`k' /*kernel5_`k'*/] using
626             "${LATEXPATH}main1DROP_P_`k'.tex", replace tex(frag pretty) stats(coef
627             se) label(proper) level(95) title(For k=`k')
628         btoutreg2 [dlshort1_`k' bs_baseline_`k' kernel4_`k' /*kernel5_`k'*/] using
629             "${LATEXPATH}mainNS1DROP_P_`k'.tex", replace tex(frag pretty) stats(
630             coef) label(proper) level(95) title(For k=`k')
631         btoutreg2 [dlshort1_`k' bs_baseline_`k' kernel4_`k' /*kernel5_`k'*/] using
632             "${LATEXPATH}mainoS1DROP_P_`k'.tex", replace tex(frag pretty) stats(se
633             ) label(proper) level(95) title(For k=`k')
634         }
635         else{
636         btoutreg2 [dlshort1_`k' bs_baseline_`k' kernel4_`k' /*kernel5_`k'*/] using
637             "${LATEXPATH}main1DROP_P_`k'.tex", replace tex(frag pretty) stats(coef
638             se) label(proper) level(95) title(For k=`k')
639         btoutreg2 [dlshort1_`k' bs_baseline_`k' kernel4_`k' /*kernel5_`k'*/] using
640             "${LATEXPATH}mainNS1DROP_P_`k'.tex", replace tex(frag pretty) stats(
641             coef) label(proper) level(95) title(For k=`k')
642         btoutreg2 [dlshort1_`k' bs_baseline_`k' kernel4_`k' /*kernel5_`k'*/] using
643             "${LATEXPATH}mainoS1DROP_P_`k'.tex", replace tex(frag pretty) stats(se
644             ) label(proper) level(95) title(For k=`k')
645         }
646     }
647
648     use "${CLOUDPATH}v38/Temp_data/Finalrundataset.dta", clear
649
650     *****
651     * FOCUS POINT

```

```

640 *****
641     local k=${focusk}
642     keep if select=='k' & SalePurchase=="Purchase"
643
644 * MACROS:
645     global PLUsR "PLUvRvarT PLUvRvarTsq PLUvRvarW PLUvRvarWsqr PLUvRvarS PLUvRvarSsq"
646     global SupCo "Coal Brent Gas IT2 EUA WindIDA Hydro"
647
648 ***** Program bootstrap of baseline
649     capture program drop my2sfsforbaselinebootstrap
650     program my2sfsforbaselinebootstrap
651         local k=${focusk}
652         capture drop PLU_P_boot PLU_Q_boot
653         capture drop PLUvDvarP'k'resc PLUvDvarQ'k'resc
654         capture noisily reg sqresPrice Tempeff15 Roll_Temp24 Roll_Temp240 suncycle morning
655             deltasun EWH SolarRest RteBlackBox if select == 'k' & SalePurchase=="Purchase" ,
656             robust
657         predict PLU_P_boot, xb
658         su PLU_P_boot, meanonly
659         scalar tmpP = r(mean)
660         capture noisily gen PLUvDvarP'k'resc = PLU_P_boot / tmpP
661         reg sqresVolume Tempeff15 Roll_Temp24 Roll_Temp240 suncycle morning deltasun EWH
662             SolarRest RteBlackBox if select == 'k' & SalePurchase=="Purchase" , robust
663         predict PLU_Q_boot, xb
664         su PLU_Q_boot, meanonly
665         scalar tmpQ = r(mean)
666         gen PLUvDvarQ'k'resc = PLU_Q_boot / tmpQ
667         if 'k'==1 | 'k'==9 {
668             reg fxInvertQP PLUvRvarT PLUvRvarTsq PLUvRvarW PLUvRvarWsqr PLUvRvarS PLUvRvarSsq
669                 Coal Brent Gas IT2 EUA WindIDA Hydro PLUvDvarP'k'resc /*PLUvDvarQ'k'resc*/
670                 if select == 'k' & SalePurchase=="Purchase" , robust
671         }
672         else{
673             reg fxInvertQP PLUvRvarT PLUvRvarTsq PLUvRvarW PLUvRvarWsqr PLUvRvarS PLUvRvarSsq
674                 Coal Brent Gas IT2 EUA WindIDA Hydro PLUvDvarP'k'resc /*PLUvDvarQ'k'resc*/
675                 if select == 'k' & SalePurchase=="Purchase" , robust
676         }
677         capture drop PLU_P_boot PLUvDvarP'k'resc PLU_Q_boot PLUvDvarQ'k'resc
678     end
679
680 *col 1
681     local k=${focusk}
682     capture drop PLUvDvarP'k'resc
683     capture drop PLUvDvarQ'k'resc
684     su PLUvDvarP'k', meanonly
685     scalar tmpresc = r(mean)
686     capture noisily gen PLUvDvarP'k'resc = PLUvDvarP'k' / tmpresc
687     su PLUvDvarQ'k', meanonly
688     scalar tmpresc = r(mean)
689     gen PLUvDvarQ'k'resc = PLUvDvarQ'k' / tmpresc
690     if 'k'==1 | 'k'==9 {
691         reg fxInvertQP ${PLUsR} ${SupCo} PLUvDvarP'k'resc /*PLUvDvarQ'k'resc */
692         est store dshort1_`k'
693     }
694     else{
695         reg fxInvertQP ${PLUsR} ${SupCo} PLUvDvarP'k'resc /*PLUvDvarQ'k'resc */
696         est store dshort1_`k'
697     }
698 *col 2

```




```

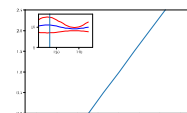
693         local k=${focusk}
694         bootstrap _b, reps(200) seed(12345): my2slsforbaselinebootstrap
695         est store bs_baseline_`k'
696         est save "${CLOUDPATH}v38/Temp_data/bs_baseDROP-Q_`k'.ster", replace
697     * col 3
698         local k=${focusk}
699         estimates use "${CLOUDPATH}v38/Temp_data/kernelweigDROP-Q_`k'.ster"
700         regress
701         estimates esample
702         estimates store kernel4_`k'
703     * col 4
704     /*
705     if `k'==5{
706         estimates use "${CLOUDPATH}v38/Temp_data/kernelbootDROP-Q_`k'.ster"
707         regress
708         estimates esample:
709         estimates store kernel5_`k'
710     }*/
711
712     * table main
713         local k=${focusk}
714         if `k'==5{
715             btoutreg2 [d1short1_`k' bs_baseline_`k' kernel4_`k' /*kernel5_`k'*/] using
716                 "${LATEXPATH}main1DROP-Q_`k'.tex", replace tex(frag pretty ) stats(coef
717                     se) label(proper) level(95) title(For k=`k')
718             btoutreg2 [d1short1_`k' bs_baseline_`k' kernel4_`k' /*kernel5_`k'*/] using
719                 "${LATEXPATH}mainNS1DROP-Q_`k'.tex", replace tex(frag pretty ) stats(
720                     coef) label(proper) level(95) title(For k=`k')
721             btoutreg2 [d1short1_`k' bs_baseline_`k' kernel4_`k' /*kernel5_`k'*/] using
722                 "${LATEXPATH}mainoS1DROP-Q_`k'.tex", replace tex(frag pretty ) stats(se
723                     ) label(proper) level(95) title(For k=`k')
724         }
725         else{
726             btoutreg2 [d1short1_`k' bs_baseline_`k' kernel4_`k' /*kernel5_`k'*/] using
727                 "${LATEXPATH}main1DROP-Q_`k'.tex", replace tex(frag pretty ) stats(coef
728                     se) label(proper) level(95) title(For k=`k')
729             btoutreg2 [d1short1_`k' bs_baseline_`k' kernel4_`k' /*kernel5_`k'*/] using
730                 "${LATEXPATH}mainNS1DROP-Q_`k'.tex", replace tex(frag pretty ) stats(
731                     coef) label(proper) level(95) title(For k=`k')
732             btoutreg2 [d1short1_`k' bs_baseline_`k' kernel4_`k' /*kernel5_`k'*/] using
733                 "${LATEXPATH}mainoS1DROP-Q_`k'.tex", replace tex(frag pretty ) stats(se
734                     ) label(proper) level(95) title(For k=`k')
735         }
736
737         *****
738
739         * END OF DROPPING 1 PLUvDvarP or Q
740         *****
741
742         *****
743
744         * REST NOT RELEVANT

```

```

738 |*****
739 |
740 |
741 |
742 |***** testing on combined PLU using new ones.
743 |
744 |
745 |global focusk = 5
746 |
747 |
748 |*****
749 |* Some regressions: ----- on point k only -----
750 |*****
751 |
752 |use "${CLOUDPATH}v38/Temp_data/Finalrundataset.dta", clear
753 |
754 |
755 |
756 |***
757 |* scalefactor to adjust fx to slope
758 |***
759 |capture drop group
760 |egen group= group(Datestatafrac)
761 |gsort group Datestata Hour SalePurchase select
762 |capture drop slopeDpost slopeDpre slopeDatk fxscalefactor
763 |gen slopeDpost =.
764 |gen slopeDpre =.
765 |gen slopeDatk =.
766 |gen fxscalefactor =.
767 |
768 |forvalues k= 1(2)9{
769 |    *note in p-q dimension!
770 |by group: replace slopeDpost = (Volume[_n+1] - Volume[_n])/(Price[_n+1]- Price[_n]) if select=='k'
771 |
772 |by group:    replace slopeDpre = (Volume[_n] - Volume[_n-1])/(Price[_n]- Price[_n-1]) if select
              =='k'
773 |by group:    replace slopeDatk = abs(slopeDpost[_n]+ slopeDpre[_n])/2 if select=='k'
774 |
775 |capture drop tmp1 tmp2
776 |egen tmp1 = mean(slopeDatk ) if select=='k'
777 |egen tmp2 = mean(fx) if select=='k'
778 |replace fxscalefactor = tmp1 / tmp2 if select=='k'
779 |drop tmp1 tmp2
780 |}
781 |capture drop fxscaled
782 |gen fxscaled = fx * fxscalefactor
783 |
784 |
785 |* scalefactor to adjust fx to slope
786 |capture drop group
787 |egen group= group(Datestatafrac)
788 |gsort group Datestata Hour SalePurchase select
789 |capture drop slopeDpostQP slopeDpreQP slopeDatkQP fxscalefactorQP
790 |gen slopeDpostQP =.
791 |gen slopeDpreQP =.
792 |gen slopeDatkQP =.
793 |gen fxscalefactorQP =.
794 |gen fxQP = (1/fx)
795 |

```



```

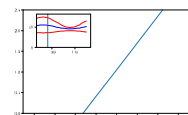
796 forvalues k= 1(2)9{
797     *note in q-p dimension!
798     by group: replace slopeDpostQP = (Price[_n+1]- Price[_n])/(Volume[_n+1] - Volume[_n]) if
       select=='k'
799     by group: replace slopeDpreQP = (Price[_n]- Price[_n-1])/(Volume[_n] - Volume[_n-1]) if
       select=='k'
800     by group: replace slopeDatkQP = abs(slopeDpostQP[_n]+ slopeDpreQP[_n])/2 if select=='k'
801     capture drop tmp1 tmp2
802     egen tmp1 = mean(slopeDatkQP ) if select=='k'
803     egen tmp2 = mean(fxQP) if select=='k'
804     replace fxscalefactorQP = tmp1 / tmp2 if select=='k'
805     drop tmp1 tmp2
806 }
807     capture drop fxscaledQP
808     gen fxscaledQP = fxQP * fxscalefactorQP
809     gen comparisonfx = 1/ fxscaled
810 * SCALING ONLY APPROPRIATE FOR K=5, otherwise too much mixing flat and vertical section.
811
812
813
814
815
816
817     local k=${focusk}
818     keep if select=='k' & SalePurchase=="Purchase"
819
820     *gen check1 = sqrt(PLUvDvarP) / (1 / fxscaled )
821     *gen check2= sqrt(PLUvDvarQ)
822     capture drop PLUvDcomb
823     capture noisily gen PLUvDcomb = (( sqrt(PLUvDvarP) / (1 / fxscaled ) ) + sqrt(PLUvDvarQ)
       )^2
824     capture drop PLUvDcombK
825     capture noisily gen PLUvDcombK = (( sqrt(PLUv51avarPsq) / (1 / fxscaled ) ) + sqrt(
       PLUv51avarQsq) )^2
826
827     *rescale combined
828     capture drop PLUvDvarC
829     su PLUvDcomb, meanonly
830     scalar tmpresc = r(mean)
831     capture noisily gen PLUvDvarC = PLUvDcomb / tmpresc
832
833     capture drop PLUvDvarCK
834     su PLUvDcombK, meanonly
835     scalar tmpresc = r(mean)
836     capture noisily gen PLUvDvarCK = PLUvDcomb / tmpresc
837
838
839 *** REGS
840     reg fxInvertQP PLUvRvarT PLUvRvarTsQ PLUvRvarW PLUvRvarWsQ PLUvRvarS PLUvRvarSsq Coal Brent
       Gas IT2 EUA WindIDA Hydro PLUvDvarC if select == 'k' & SalePurchase=="Purchase" ,
       robust
841     reg fxInvertQP PLUvRvarT PLUvRvarTsQ PLUvRvarW PLUvRvarWsQ PLUvRvarS PLUvRvarSsq Coal Brent
       Gas IT2 EUA WindIDA Hydro PLUvDvarC if select == 'k' & SalePurchase=="Purchase" ,
       vce(bootstrap, reps(300) seed(12345))
842
843
844     reg fxInvertQP PLUvRvarT PLUvRvarTsQ PLUvRvarW PLUvRvarWsQ PLUvRvarS PLUvRvarSsq Coal Brent
       Gas IT2 EUA WindIDA Hydro PLUvDvarCK if select == 'k' & SalePurchase=="Purchase" [
       aweight = PointsInBin51a]
845

```

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883
884
885
886
887
888
889
890
891 *****
892 * generate 1-dim PLUs:
893 *****
894     capture drop PLU.COMB*
895     capture noisily gen PLU.COMBa_Dresc = sqrt( (PLUvDvarPresc)^2 + (PLUvDvarQresc)^2 )
896     capture noisily gen PLU.COMBa_Dabsresc = sqrt( (PLUvDvarPabsresc)^2 + (PLUvDvarQabsresc)
897         ^2 )
898     capture noisily gen PLU.COMBa_Drtresc = sqrt( (PLUvDrtPresc)^2 + (PLUvDrtQresc)^2 )
899
900     /* version b: translation approach - not allowed for rescaled variables!!
901     capture noisily gen PLU.COMBb_D = ((PLUvDvarP)/(1 / fxscaled) ) + (PLUvDvarQ)
902     capture noisily gen PLU.COMBb_Dabs = ((PLUvDvarPabs)/(1 / fxscaled) ) + (PLUvDvarQabs)
903     capture noisily gen PLU.COMBb_Drt = ((PLUvDrtP)/(1 / fxscaled) ) + (PLUvDrtQ) */
904 /*     capture noisily gen PLU.COMBb_D = ((PLUvDvarP)/(fxscaledQP) ) + (PLUvDvarQ)

```



```

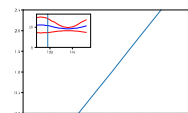
905     capture noisily gen PLU.COMBb.Dabs = ((PLUvDvarPabs) / (fxscaledQP) ) + (PLUvDvarQabs)
906     capture noisily gen PLU.COMBb.Drt = ((PLUvDrtP) / (fxscaledQP) ) + (PLUvDrtQ) */
907
908
909 ****
910 * defining variables
911 ****
912     local versionS "61"
913     global runversionS 'versionS'
914         global demandestimationvariables'version' "Tempeff15 Roll_Temp24
915             Roll_Temp240 suncycle morning deltasun EWH SolarRest RteBlackBox "
916     global supplyestimationvariables'versionS' "Coal Brent Gas IT2 EUA suncycle morning
917         deltasun EWH WindIDA SolarRest Hydro RteBlackBox"
918     global uncertaintyproxies'versionS' "PLUvDvarP PLUvDvarQ PLUvDvarPabs PLUvDvarQabs
919         PLUvDrtP PLUvDrtQ PLUvRvarT PLUvRvarW PLUvRvarS PLUvRvarTsq PLUvRvarWsq
920         PLUvRvarSsq PLUvDvarQresc PLUvDvarQabsresc PLUvDvarPresc PLUvDvarPabsresc
921         PLUvDrtPresc PLUvDrtQresc PLU.COMBa.Dresc PLU.COMBa.Dabsresc PLU.COMBa.Drtresc
922         PLU.COMBb.D PLU.COMBb.Dabs PLU.COMBb.Drt "
923     global PLUsD "PLUvDvarP PLUvDvarQ PLUvDvarPabs PLUvDvarQabs PLUvDrtP PLUvDrtQ
924         PLUvDvarPresc PLUvDvarQresc PLUvDvarPabsresc PLUvDvarQabsresc PLUvDrtPresc
925         PLUvDrtQresc PLU.COMBa.Dresc PLU.COMBa.Dabsresc PLU.COMBa.Drtresc PLU.COMBb.D
926         PLU.COMBb.Dabs PLU.COMBb.Drt "
927     global PLUsDP "PLUvDvarP'k' PLUvDvarPabs'k' PLUvDrtP'k' "
928     global PLUsDQ " PLUvDvarQ'k' PLUvDvarQabs'k' PLUvDrtQ'k' "
929     global PLUsR "PLUvRvarT PLUvRvarTsq PLUvRvarW PLUvRvarWsq PLUvRvarS PLUvRvarSsq"
930     global SEV ${supplyestimationvariables${runversionS}}
931     global UCP ${uncertaintyproxies${runversionS}}
932     di $SEV
933     di $UCP
934
935
936
937 ****reg on demand slope as cross check to interpretation from level functional regressions:
938     capture drop negfx
939     gen negfx = -fx
940     reg negfx $demandestimationvariables'version', robust
941     reg fx $demandestimationvariables'version', robust
942     est store demandslopepred${focusk}
943     btoutreg2 [demandslopepred${focusk} ] using "${LATEXPATH}demandslopepred${focusk}.
944         tex", replace tex( frag pretty ) stats(coef) label(proper) level(95) title(
945         Demand-slope-regression at k=${focusk})
946     drop _est*
947
948
949
950 *****
951 * REGRESSION 1 :
952
953     reg fxInvertQP ${PLUsR}, robust
954     est store onlyplus
955     * all plu_renouvelable are significant , only plu wind of correct sign
956     reg fxInvertQP ${PLUsR} $SEV, robust
957     est store onlyplur1
958     reg fxInvertQP ${PLUsR} Coal Brent Gas IT2 EUA WindIDA Hydro, robust
959     est store onlyplur2
960     reg fxInvertQP ${PLUsR} Coal Brent Gas IT2 EUA WindIDA Hydro $SEV, robust
961     est store onlyplur3
962     * when adding supply controls , only wind stay significant with correct sign , others non-sig. thats
963         good. :)
964
965     foreach UCP of global PLUsD{
966         reg fxInvertQP ${PLUsR} Coal Brent Gas IT2 EUA WindIDA Hydro 'UCP'

```

```

953         est store linreg1 'UCP'
954     }
955
956     foreach UCP1 of global PLUsDP{
957     foreach UCP2 of global PLUsDQ{
958     reg fxInvertQP    ${PLUsR} Coal Brent Gas IT2 EUA Wind1DA Hydro 'UCP1' 'UCP2'
959
960     est store 'UCP1' 'UCP2'
961     }
962     }
963
964
965
966     reg fxInvertQP    ${PLUsR} Coal Brent Gas IT2 EUA Wind1DA Hydro PLUvDvarP 'k'
967     resc    PLUvDvarQ 'k' resc
968     est store d1short1
969     *reg fxInvertQP    ${PLUsR} Coal Brent Gas IT2 EUA Wind1DA Hydro PLUvDvarP 'k'
970     resc    PLUvDvarQ 'k' resc , vce(bootstrap, rep(500))
971     *est store b1short1
972     reg fxInvertQP    ${PLUsR} Coal Brent Gas IT2 EUA Wind1DA Hydro PLUvDvarPabs
973     resc    PLUvDvarQabs 'k' resc
974     est store d1short2
975     reg fxInvertQP    ${PLUsR} Coal Brent Gas IT2 EUA Wind1DA Hydro PLUvDrtP 'k'
976     resc    PLUvDrtQ 'k' resc
977     est store d1short3
978
979     * (generating) regression output
980
981     /* Shows: PLU_temperature never significant
982     - wind1da : pos + sig (more wind, more uncertainty)
983     - plu wind: sig + positive effect only for PLUs_on_P (longer autocorrelation wind-> more
984     uncertainty)
985     - plu wind squared : very neg + sig only for PLUs_on_P (very short or long autocorrel = low
986     uncertainty, errors cancel out)
987     -PLU-solar never sig
988     - solar1da included in plusD
989     - HAVE EXCLUDED DAYTIME CONTROLS (but they are strongly included in PLUsD
990     - all input prices has sig effect: coal positive and all other negative (interpretation?)
991     - plu_D_on_P have negative, significant effects, plu_D_on_Q have positive effects, when very
992     significant
993     */
994
995     btoutreg2 [onlyplus onlyplur2 onlyplur1] using "${LATEXPATH}onlypluRs.tex",
996     replace tex(frag pretty ) stats(coef) label(proper) level(95) title(regression
997     for k='k')
998
999     btoutreg2 [/*linreg1PLUvDvarP linreg1PLUvDvarPabs linreg1PLUvDrtP*/
1000     linreg1PLUvDvarPresc linreg1PLUvDvarPabsresc linreg1PLUvDrtPresc ] using
1001     "${LATEXPATH}linregsummary1P_-'k'.tex", replace tex(frag pretty ) stats(coef)
1002     label(proper) level(95) title(regression for k='k')
1003
1004     btoutreg2 [/*linreg1PLUvDvarQ linreg1PLUvDvarQabs linreg1PLUvDrtQ */
1005     linreg1PLUvDvarQresc linreg1PLUvDvarQabsresc linreg1PLUvDrtQresc ] using "${
1006     LATEXPATH}linregsummary1Q_-'k'.tex", replace tex(frag pretty ) stats(coef) label
1007     (proper) level(95) title(regression for k='k')
1008
1009
1010     btoutreg2 [/*PLUvDvarP 'k' PLUvDvarQ 'k' PLUvDvarPabs 'k' PLUvDvarQabs 'k' PLUvDrtP 'k'
1011     PLUvDrtQ 'k' */ d1short1 d1short2 d1short3] using "${LATEXPATH}doubleg1_-'k'.
1012     tex", replace tex(frag pretty ) stats(coef) label(proper) level(95) title(
1013     regression for k='k')

```



```

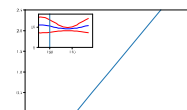
994
995
996
997
998
999
1000
1001
1002
1003
1004
1005
1006
1007
1008
1009
1010             *****
1011             * some robustness regressions to plu_d specifications
1012             *****
1013
1014 *****
1015 *****
1016
1017 /*
1018             *****KERNEL BASED PLU = v52
1019 */
1020
1021 use "${CLOUDPATH}v38/Temp-data/Finalrundataset.dta", clear
1022
1023
1024 * Generate 1 dimensional proxies for kernel based PLUs
1025 *****
1026
1027 *****
1028 * COMBINE P and Q uncertainty into single value on kernel based proxies.
1029 *****
1030             capture drop PLU_COMB*
1031             * version a: Hypothenuse approach
1032
1033             foreach versionD in "52a" "52b" {
1034                 foreach switch in /*"*/ "abs" {
1035                     forvalues k = 1(2)9 {
1036 capture noisily gen PLU_COMBa_v`versionD'_'switch'`k'`resc = sqrt( (PLUv`versionD'`varP'`switch'`k'
1037             resc)^2 + (PLUv`versionD'`varQ'`switch'`k'`resc)^2 )
1038             }
1039             }
1040
1041             * version b: translation approach - not correct conversion anymore after rescaling
1042             !!!!
1043             forvalues k = 1(2)9 {
1044 capture noisily gen PLU_COMBb_v52a`k' = ((PLUv52avarPabs`k') / ( 1 / fxscaled ) ) + (
1045             PLUv52avarQabs`k')
1046 capture noisily gen PLU_COMBb_v52b`k' = ((PLUv52bvarPabs`k') / ( 1 / fxscaled ) ) + (
1047             PLUv52bvarQabs`k')
1048             }
1049
1050             forvalues k = 1(2)9 {
1051 capture noisily gen PLU_COMBb_v52a`k' = ((PLUv52avarPabs`k') / ( fxscaledQP ) ) + (

```

```

1050         PLUv52avarQabs'k')
1051     capture noisily gen PLU.COMBb.v52b'k' = ((PLUv52bvarPabs'k') / ( fxscaledQP )) + (
1052         PLUv52bvarQabs'k')
1053     }
1054 */
1055 *****
1056 * FOCUS POINT
1057 *****
1058     local k=${focusk}
1059     keep if select=='k' & SalePurchase=="Purchase"
1060 */
1061     * general placeholders
1062     capture drop PLUvDvarQresc PLUvDvarQabsresc PLUvDvarPresc PLUvDvarPabsresc PLUvDrtPresc
1063         PLUvDrtQresc
1064     gen PLUvDvarQresc = PLUvDvarQ'k' resc
1065     gen PLUvDvarQabsresc = PLUvDvarQabs'k' resc
1066     gen PLUvDvarPresc = PLUvDvarP'k' resc
1067     gen PLUvDvarPabsresc = PLUvDvarPabs'k' resc
1068     gen PLUvDrtPresc = PLUvDrtP'k' resc
1069     gen PLUvDrtQresc = PLUvDrtQ'k' resc*/
1070 ****
1071 * defining variables
1072 ****
1073     local versionS "61"
1074     global runversionS 'versionS'
1075         global demandestimationvariables'version' "Tempeff15 Roll_Temp24
1076             Roll_Temp240 suncycle morning deltasun EWH SolarRest RteBlackBox "
1077     global supplyestimationvariables'versionS' "Coal Brent Gas IT2 EUA suncycle morning
1078         deltasun EWH WindIDA SolarRest Hydro RteBlackBox"
1079     *global uncertaintyproxies'versionS' "PLUvDvarP PLUvDvarQ PLUvDvarPabs PLUvDvarQabs
1080         PLUvDrtP PLUvDrtQ PLUvRvarT PLUvRvarW PLUvRvarS PLUvRvarTsqr PLUvRvarWsq
1081         PLUvRvarSsq PLUvDvarQresc PLUvDvarQabsresc PLUvDvarPresc PLUvDvarPabsresc
1082         PLUvDrtPresc PLUvDrtQresc PLU.COMBa_Dresc PLU.COMBa_Dabsresc PLU.COMBa_Drtresc
1083         PLU.COMBb_D PLU.COMBb_Dabs PLU.COMBb_Drt "
1084     *global PLUsD "PLUvDvarP PLUvDvarQ PLUvDvarPabs PLUvDvarQabs PLUvDrtP PLUvDrtQ
1085         PLUvDvarPresc PLUvDvarQresc PLUvDvarPabsresc PLUvDvarQabsresc PLUvDrtPresc
1086         PLUvDrtQresc PLU.COMBa_Dresc PLU.COMBa_Dabsresc PLU.COMBa_Drtresc PLU.COMBb_D
1087         PLU.COMBb_Dabs PLU.COMBb_Drt "
1088     global PLUsR "PLUvRvarT PLUvRvarTsqr PLUvRvarW PLUvRvarWsq PLUvRvarS PLUvRvarSsq"
1089     global PLUsROB "PLUv52avarPabs'k' PLUv52avarQabs'k' PLUv52bvarPabs'k'
1090         PLUv52bvarQabs'k' PLUv52avarPabs'k' resc PLUv52avarQabs'k' resc PLUv52bvarPabs'k' resc
1091         PLUv52bvarQabs'k' resc PLU.COMBa.v52a.abs'k' resc PLU.COMBa.v52b.abs'k' resc
1092         PLU.COMBb.v52a'k' PLU.COMBb.v52b'k'"
1093     global PLUsROBa "PLUv52avarPabs'k' PLUv52avarQabs'k' PLUv52avarPabs'k' resc
1094         PLUv52avarQabs'k' resc PLU.COMBa.v52a.abs'k' resc PLU.COMBb.v52a'k' "
1095     global PLUsROBb " PLUv52bvarPabs'k' PLUv52bvarQabs'k' PLUv52bvarPabs'k' resc
1096         PLUv52bvarQabs'k' resc PLU.COMBa.v52b.abs'k' resc PLU.COMBb.v52b'k'"
1097     global SEV ${supplyestimationvariables${runversionS}}
1098     global UCP ${uncertaintyproxies${runversionS}}
1099     di $SEV
1100     di ${PLUsROB}
1101
1102 *** first regression
1103 reg fxInvertQP ${PLUsR}, robust

```




```

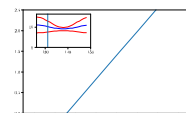
1093 est store onlyplus
1094 reg fxInvertQP Coal Brent Gas IT2 EUA WindIDA Hydro, robust
1095 est store onlycontrols
1096 * all plu_renouvelable are sigificant, only plu wind of correct sign
1097 reg fxInvertQP ${PLUsR} Coal Brent Gas IT2 EUA WindIDA Hydro, robust
1098 * when adding supply controls, only wind stay significant with correct sign, others non-sig. thats
    good. :)
1099
1100         foreach UCP of global PLUsROB{
1101             reg fxInvertQP  ${PLUsR} Coal Brent Gas IT2 EUA WindIDA Hydro /*${SEV}*/ '
                UCP'
1102             est store r1'UCP'
1103         }
1104
1105     * including weighting using Pointsperbin
1106     foreach UCP of global PLUsROBa{
1107         reg fxInvertQP  ${PLUsR} Coal Brent Gas IT2 EUA WindIDA Hydro /*${SEV}*/ '
                UCP' [aweight=PointsInBin52a]
1108         est store w1'UCP'
1109     }
1110     foreach UCP of global PLUsROBb{
1111         reg fxInvertQP  ${PLUsR} Coal Brent Gas IT2 EUA WindIDA Hydro /*${SEV}*/ '
                UCP' [aweight=PointsInBin52b]
1112         est store w1'UCP'
1113     }
1114
1115
1116 *simultaneous reg on PLU_P and PLU_Q
1117         reg fxInvertQP  ${PLUsR} Coal Brent Gas IT2 EUA WindIDA Hydro
                PLUv52avarPabs'k'resc PLUv52avarQabs'k'resc [aweight=PointsInBin52a]
1118         est store w2.52a
1119         reg fxInvertQP  ${PLUsR} Coal Brent Gas IT2 EUA WindIDA Hydro
                PLUv52bvarPabs'k'resc PLUv52bvarQabs'k'resc [aweight=PointsInBin52b]
1120         est store w2.52b
1121
1122
1123     * (generating) regression output
1124 /* Shows:
1125     - plu_r as before
1126     - PLU_temperature never significant
1127     - windida : pos + sig (more wind, more uncertainty)
1128     - plu wind: sig + positive effect only for PLUs_on_P (longer autocorrelation wind-> more
        uncertainty)
1129     - plu wind squared : very neg + sig only for PLUs_on_P (very short or long autocorrel = low
        uncertainty, errors cancel out)
1130     -PLU-solar never sig
1131     - solarida included in plusD
1132     - HAVE EXCLUDED DAYTIME CONTROLS (but they are strongly included in PLUsD
1133     - all input prices has sig effect: coal positive and all other negative (interpretation?)
1134     - plu_D_on_P have negative, significant effects, plu_D_on_Q have positive effects, when very
        significant
1135     *** PROMISING RESUTLS HERE ON ROBUSTNESS!
1136     - resc variables have nonsignificnat effect when combined, significant and pos for
        quantities plus when individual effect.
1137 */
1138     btoutreg2 [/*onlyplus*/ onlycontrols r1PLUv52avarPabs'k' r1PLUv52avarQabs'k'
                r1PLUv52bvarPabs'k' r1PLUv52bvarQabs'k' /* r1PLUv52avarPabs'k'resc
                r1PLUv52bvarPabs'k'resc r1PLUv52avarQabs'k'resc r1PLUv52bvarQabs'k'resc */
                r1PLU.COMBa.v52a.abs'k'resc r1PLU.COMBa.v52b.abs'k'resc r1PLU.COMBb.v52a'k'
                r1PLU.COMBb.v52b'k'] using "${LATEXPATh}r1_'.k'.tex", replace tex(frag pretty )

```

```

1139         stats(coef) label(proper) level(95) title(For k='k')
1140         btoutreg2 [/*onlyplus*/ onlycontrols w1PLUv52avarPabs'k' w1PLUv52avarQabs'k'
1141                 w1PLUv52bvarPabs'k' w1PLUv52bvarQabs'k' /* w1PLUv52avarPabs'k'resc
1142                 w1PLUv52bvarPabs'k'resc w1PLUv52avarQabs'k'resc w1PLUv52bvarQabs'k'resc */
1143                 w1PLU_COMBa_v52a_abs'k'resc w1PLU_COMBa_v52b_abs'k'resc w1PLU_COMBb_v52a'k'
1144                 w1PLU_COMBb_v52b'k'] using "${LATEXPATH}w1.'k'.tex", replace tex(frag pretty )
1145                 stats(coef) label(proper) level(95) title(For k='k')
1146
1147     *separate tables for individual P or Q
1148
1149     btoutreg2 [w1PLUv52avarPabs'k' w1PLUv52avarQabs'k' w1PLUv52bvarPabs'k' w1PLUv52bvarQabs'k']
1150     using "${LATEXPATH}w1a.'k'.tex", replace tex(frag pretty ) stats(coef) label(proper)
1151     level(95) title(For k='k')
1152
1153     btoutreg2 [w2_52a w2_52b w1PLU_COMBa_v52a_abs'k'resc w1PLU_COMBa_v52b_abs'k'resc /*
1154     w1PLU_COMBb_v52a'k' w1PLU_COMBb_v52b'k' */] using "${LATEXPATH}w1b.'k'.tex", replace
1155     tex(frag pretty ) stats(coef) label(proper) level(95) title(For k='k')
1156
1157     /* run individually if only running first part
1158     drop _cst*
1159 */
1160
1161
1162
1163
1164
1165
1166
1167
1168
1169
1170
1171
1172
1173
1174
1175
1176
1177
1178
1179
1180
1181     *****
1182     * some robustness regressions to plu_d specifications
1183     *****
1184
1185     *****
1186
1187     /*
1188     ***** kernel based plus. = v51

```



```

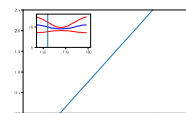
1189 */
1190
1191 * use "${CLOUDPATH}v38/Temp_data/FinalrundatasetK1.dta", clear
1192 * use "${CLOUDPATH}v38/Temp_data/Finalrundataset.dta", clear
1193
1194
1195 * Generate 1 dimensional proxies for kernel based PLUs
1196 *****
1197
1198     *****
1199     * COMBINE P and Q uncertainty into single value on kernel based proxies.
1200     *****
1201         capture drop PLUCOMB*
1202         * version a: Hypothenuse approach
1203
1204         foreach versionD in "51a" "51b" {
1205             foreach switch in /*"*/ "sq" {
1206                 forvalues k = 1(2)9 {
1207 capture noisily gen PLUCOMBa_v`versionD'_'switch'`k' resc = sqrt( (PLUv`versionD'varP`switch'`k'
1208 resc)^2 + (PLUv`versionD'varQ`switch'`k' resc)^2 )
1209             }
1210         }
1211
1212         * version b: translation approach – not correct conversion anymore after rescaling
1213         !!!!
1214         forvalues k = 1(2)9 {
1215 capture noisily gen PLUCOMBB_v51a`k' = ((PLUv51avarPsq`k') / (1 / fxscaled ) ) + (
1216 PLUv51avarQsq`k')
1217 capture noisily gen PLUCOMBB_v51b`k' = ((PLUv51bvarPsq`k') / (1 / fxscaled ) ) + (
1218 PLUv51bvarQsq`k')
1219 }
1220
1221 /*
1222         forvalues k = 1(2)9 {
1223 capture noisily gen PLUCOMBB_v51a`k' = ((PLUv51avarPsq`k') / ( fxscaledQP ) ) + (
1224 PLUv51avarQsq`k')
1225 capture noisily gen PLUCOMBB_v51b`k' = ((PLUv51bvarPsq`k') / ( fxscaledQP ) ) + (
1226 PLUv51bvarQsq`k')
1227 }
1228 */
1229
1230 *****
1231 * FOCUS POINT
1232 *****
1233 local k=${focusk}
1234 keep if select==`k' & SalePurchase=="Purchase"
1235
1236 /*
1237 * general placeholders
1238 capture drop PLUvDvarQresc PLUvDvarQsqresc PLUvDvarPresc PLUvDvarPsqresc PLUvDrtPresc
1239 PLUvDrtQresc
1240 gen PLUvDvarQresc = PLUvDvarQ`k' resc
1241 gen PLUvDvarQsqresc = PLUvDvarQsq`k' resc
1242 gen PLUvDvarPresc = PLUvDvarP`k' resc
1243 gen PLUvDvarPsqresc = PLUvDvarPsq`k' resc
1244 gen PLUvDrtPresc = PLUvDrtP`k' resc
1245 gen PLUvDrtQresc = PLUvDrtQ`k' resc*/
1246
1247
1248
1249
1250 ****

```

```

1242 * defining variables
1243 ****
1244     local versionS "61"
1245     global runversionS 'versionS'
1246         global demandestimationvariables 'version' "Tempeff15 Roll_Temp24
1247             Roll_Temp240 suncycle morning deltasun EWH SolarRest RteBlackBox "
1248 *global supplyestimationvariables 'versionS' "Coal Brent Gas IT2 EUA suncycle morning
1249     global supplyestimationvariables 'versionS' "Coal Brent Gas IT2 EUA suncycle morning
1250         deltasun EWH WindIDA SolarRest Hydro RteBlackBox"
1251 *global uncertaintyproxies 'versionS' "PLUvDvarP PLUvDvarQ PLUvDvarPsq PLUvDvarQsq
1252     PLUvDrtP PLUvDrtQ PLUvRvarT PLUvRvarW PLUvRvarS PLUvRvarTsq PLUvRvarWsqr
1253     PLUvRvarSsq PLUvDvarQresc PLUvDvarQsqresc PLUvDvarPresc PLUvDvarPsqresc
1254     PLUvDrtPresc PLUvDrtQresc PLU_COMBa_Dresc PLU_COMBa_Dsqresc PLU_COMBa_Drtresc
1255     PLU_COMBb_D PLU_COMBb_Dsq PLU_COMBb_Drt "
1256 *global PLUsD "PLUvDvarP PLUvDvarQ PLUvDvarPsq PLUvDvarQsq PLUvDrtP PLUvDrtQ
1257     PLUvDvarPresc PLUvDvarQresc PLUvDvarPsqresc PLUvDvarQsqresc PLUvDrtPresc
1258     PLUvDrtQresc PLU_COMBa_Dresc PLU_COMBa_Dsqresc PLU_COMBa_Drtresc PLU_COMBb_D
1259     PLU_COMBb_Dsq PLU_COMBb_Drt "
1260 global PLUsR "PLUvRvarT PLUvRvarTsqr PLUvRvarW PLUvRvarWsqr PLUvRvarS PLUvRvarSsq"
1261 global PLUsROB "PLUv51avarPsq'k' PLUv51avarQsq'k' PLUv51bvarPsq'k' PLUv51bvarQsq'k'
1262     PLUv51avarPsq'k'resc PLUv51bvarPsq'k'resc PLUv51avarQsq'k'resc PLUv51bvarQsq'k'
1263     resc PLU_COMBa_v51a_sq'k'resc PLU_COMBa_v51b_sq'k'resc PLU_COMBb_v51a'k'
1264     PLU_COMBb_v51b'k'"
1265 global PLUsROBa "PLUv51avarPsq'k' PLUv51avarQsq'k' PLUv51avarPsq'k'resc
1266     PLUv51avarQsq'k'resc PLU_COMBa_v51a_sq'k'resc PLU_COMBb_v51a'k'"
1267 global PLUsROBb " PLUv51bvarPsq'k' PLUv51bvarQsq'k' PLUv51bvarPsq'k'resc
1268     PLUv51bvarQsq'k'resc PLU_COMBa_v51b_sq'k'resc PLU_COMBb_v51b'k'"
1269 global SEV ${supplyestimationvariables${runversionS}}
1270 global UCP ${uncertaintyproxies${runversionS}}
1271 di $SEV
1272 di ${PLUsROB}
1273
1274 *** first regression
1275
1276 reg fxInvertQP ${PLUsR}, robust
1277 est store onlyplus
1278 reg fxInvertQP Coal Brent Gas IT2 EUA WindIDA Hydro, robust
1279 est store onlycontrols
1280 * all plu_renouvelable are sigigfcant , only plu wind of correct sign
1281 reg fxInvertQP ${PLUsR} Coal Brent Gas IT2 EUA WindIDA Hydro, robust
1282 * when adding supply controls , only wind stay significant with correct sign, others non-sig. thats
1283     good. :)
1284
1285     foreach UCP of global PLUsROB{
1286         reg fxInvertQP ${PLUsR} Coal Brent Gas IT2 EUA WindIDA Hydro /*${SEV}*/ '
1287             UCP'
1288         est store r1'UCP'
1289     }
1290
1291     * including weighting using Pointspersbin
1292     foreach UCP of global PLUsROBa{
1293         reg fxInvertQP ${PLUsR} Coal Brent Gas IT2 EUA WindIDA Hydro /*${SEV}*/ '
1294             UCP' [aweight=PointsInBin51a]
1295         est store w1'UCP'
1296     }
1297
1298     foreach UCP of global PLUsROBb{
1299         reg fxInvertQP ${PLUsR} Coal Brent Gas IT2 EUA WindIDA Hydro /*${SEV}*/ '
1300             UCP' [aweight=PointsInBin51b]
1301         est store w1'UCP'

```

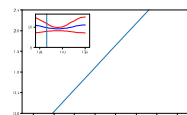


```

1284         }
1285
1286
1287 *simultaneous reg on PLU_P and PLU-Q
1288         reg fxInvertQP   ${PLUsR} Coal Brent Gas IT2 EUA WindIDA Hydro
1289             PLUv51avarPsq'k'resc   PLUv51avarQsq'k'resc [aweight=PointsInBinv51a]
1290             est store w2_51a
1291         reg fxInvertQP   ${PLUsR} Coal Brent Gas IT2 EUA WindIDA Hydro
1292             PLUv51bvarPsq'k'resc   PLUv51bvarQsq'k'resc [aweight=PointsInBinv51b]
1293             est store w2_51b
1294
1295         * (generating) regression output
1296 /* Shows:
1297 - plu_r as before
1298 - PLU_temperature never significant
1299 - windlda : pos + sig (more wind, more uncertainty)
1300 - plu wind: sig + positive effect only for PLUs_on_P (longer autocorrelation wind-> more
1301   uncertainty)
1302 - plu wind squared : very neg + sig only for PLUs_on_P (very short or long autocorrel = low
1303   uncertainty, errors cancel out)
1304 -PLU-solar never sig
1305 - solarlda included in plusD
1306 - HAVE EXCLUDED DAYTIME CONTROLS (but they are strongly included in PLUsD
1307 - all input prices has sig effect: coal positive and all other negative (interpretation?)
1308 - plu_D_on_P have negative, significant effects, plu_D_on_Q have positive effects, when very
1309   significant
1310 *** PROMISING RESUTLS HERE ON ROBUSTNESS!
1311 - resc variables have nonsignificnat effect when combined, significant and pos for
1312   quantities plus when individual effect.
1313 */
1314 btoutreg2 [/*onlyplus*/ onlycontrols r1PLUv51avarPsq'k' r1PLUv51avarQsq'k'
1315   r1PLUv51bvarPsq'k' r1PLUv51bvarQsq'k' /* r1PLUv51avarPsq'k'resc r1PLUv51bvarPsq
1316   'k'resc r1PLUv51avarQsq'k'resc r1PLUv51bvarQsq'k'resc */ r1PLU_COMBa.v51a.sq'k'
1317   resc r1PLU_COMBa.v51b.sq'k'resc r1PLU_COMBb.v51a'k' r1PLU_COMBb.v51b'k'] using
1318   "${LATEXPATH}k1.'k'.tex", replace tex(frag pretty ) stats(coef) label(proper)
1319   level(95) title(For k='k')
1320 btoutreg2 [/*onlyplus*/ onlycontrols w1PLUv51avarPsq'k' w1PLUv51avarQsq'k'
1321   w1PLUv51bvarPsq'k' w1PLUv51bvarQsq'k' /* w1PLUv51avarPsq'k'resc w1PLUv51bvarPsq
1322   'k'resc w1PLUv51avarQsq'k'resc w1PLUv51bvarQsq'k'resc */ w1PLU_COMBa.v51a.sq'k'
1323   resc w1PLU_COMBa.v51b.sq'k'resc w1PLU_COMBb.v51a'k' w1PLU_COMBb.v51b'k'] using
1324   "${LATEXPATH}k2.'k'.tex", replace tex(frag pretty ) stats(coef) label(proper)
1325   level(95) title(For k='k')
1326
1327 *separate tables for individual P or Q
1328
1329 btoutreg2 [w1PLUv51avarPsq'k' w1PLUv51avarQsq'k' w1PLUv51bvarPsq'k' w1PLUv51bvarQsq'k']
1330   using "${LATEXPATH}k2a.'k'.tex", replace tex(frag pretty ) stats(coef) label(proper)
1331   level(95) title(For k='k')
1332
1333 btoutreg2 [w2_51a w2_51b w1PLU_COMBa.v51a.sq'k'resc w1PLU_COMBa.v51b.sq'k'resc /*
1334   w1PLU_COMBb.v51a'k' w1PLU_COMBb.v51b'k' */] using "${LATEXPATH}k2b.'k'.tex", replace
1335   tex(frag pretty ) stats(coef) label(proper) level(95) title(For k='k')
1336
1337 btoutreg2 [w2_51a w2_51b w2_52a w2_52b ] using "${LATEXPATH}k5152.'k'.tex", replace tex(
1338   frag pretty ) stats(coef) label(proper) level(95) title(For k='k')
1339
1340
1341
1342 /* run individually if only running first part

```

```
1323         drop _est*
1324     */
1325
1326
1327
1328
1329
1330
1331
1332
1333
1334
1335
1336
1337
1338
1339
1340
1341
1342
1343
1344
1345
1346
1347
1348
1349
1350
1351
1352
1353
1354
1355
1356
1357
1358
1359
1360
1361
1362
1363
1364     ***** alternative pairing - not relevant for k=${focus}
1365
1366
1367
1368
1369
1370
1371
1372
1373
1374
1375
1376
1377
1378
1379
1380
1381
1382
```

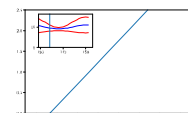


```
1383
1384
1385 *** BASELINE RESULTS:
1386
1387     local k=${focusk}
1388         btoutreg2 [d1short1 bs_baseline 'k' w2_51a w2_51b] using "${LATEXPATH}
1389             comparableregs-'k'.tex", replace tex( frag pretty ) stats(coef) label(proper)
1390             level(95) title(regression for k='k')
1391
1392
1393
1394
1395
1396
1397
1398
1399
1400
1401
1402
1403
1404
1405
1406
1407
1408
1409
1410
1411
1412
1413
1414
1415
1416
1417
1418
1419
1420
1421
1422
1423 * OLD CODE (to delete)
1424
1425
1426
1427 ** test in single step: ----- NOT CORRECT CODE; SINCE INCLUDE ALL EXOGENOUS IN PLU PREDICTION.
1428 /*ivregress 2sls fxInvertQP PLUvRvarT PLUvRvarTsq PLUvRvarW PLUvRvarWsq PLUvRvarS PLUvRvarSsq Coal
1429     Brent Gas IT2 EUA WindIDA Hydro (sqresPrice sqresVolume = Tempeff15 Roll_Temp24 Roll_Temp240
1430     suncycle morning deltasun EWH SolarRest RteBlackBox) if select ==5 & SalePurchase=="Purchase"
1431     , robust first
1432
1433 ivregress 2sls fxInvertQP PLUvRvarT PLUvRvarTsq PLUvRvarW PLUvRvarWsq PLUvRvarS PLUvRvarSsq Coal
1434     Brent Gas IT2 EUA WindIDA Hydro (sqresPrice sqresVolume = Tempeff15 Roll_Temp24 Roll_Temp240
1435     suncycle morning deltasun EWH SolarRest RteBlackBox) if select ==5 & SalePurchase=="Purchase"
1436     , vce(bootstrap, rep(200))
1437
1438
1439
1440
1441
1442
1443 ***** bootstrap example 1
1444 /*capture drop yhat PLU_boot
```

```

1435 capture program drop my2slsforboot
1436 program my2slsforboot
1437     reg sqresVolume Tempeff15 Roll_Temp24 Roll_Temp240 suncycle morning deltasun EWH SolarRest
        RteBlackBox if select ==5 & SalePurchase=="Purchase" , robust
1438     predict PLU_boot, xb
1439     reg fxInvertQP PLUvRvarT PLUvRvarTsq PLUvRvarW PLUvRvarWsq PLUvRvarS PLUvRvarSsq Coal Brent
        Gas IT2 EUA Wind1DA Hydro PLU_boot if select ==5 & SalePurchase=="Purchase" ,
        robust
1440     drop yhat PLU_boot
1441 end
1442     bootstrap _b[PLU_boot] _se[PLU_boot], reps(50) seed(10): my2slsforboot
1443     bootstrap , bca reps(50) seed(10): my2slsforboot
1444 di _se[PLU_boot]
1445
1446
1447
1448 ***** bootstrap example 2
1449 capture drop PLU_boot
1450 capture drop volhat
1451 capture program drop my2slsforboot
1452 program my2slsforboot
1453     reg Volume Tempeff15 Roll_Temp24 Roll_Temp240 suncycle morning deltasun EWH SolarRest
        RteBlackBox if select ==5 & SalePurchase=="Purchase" , robust
1454     predict yhat, xb
1455     gen PLU_boot = (Volume - yhat)^2
1456     reg fxInvertQP PLUvRvarT PLUvRvarTsq PLUvRvarW PLUvRvarWsq PLUvRvarS PLUvRvarSsq Coal Brent
        Gas IT2 EUA Wind1DA Hydro PLU_boot if select ==5 & SalePurchase=="Purchase" ,
        robust
1457     drop yhat PLU_boot
1458 end
1459     bootstrap _b[PLU_boot] _se[PLU_boot], reps(50) seed(10): my2slsforboot
1460     bootstrap , bca reps(50) seed(10): my2slsforboot
1461 di _se[PLU_boot]
1462
1463 */
1464
1465
1466
1467 ***
1468 * generate table of kernel variables
1469 ***
1470 /*     capture mat drop M
1471     mat M = J(9,6,.)
1472     local variables " ${demandestimationvariables'version'} "
1473     local FUNC " r(mean) r(p50) r(sd) r(min) r(max)"
1474     local c=2
1475     foreach FF of local FUNC{
1476         local r=1
1477         foreach VV of local variables{
1478             su 'VV', detail
1479             mat M['r','c']= 'FF'
1480             local r='r'+1
1481         }
1482         local c='c'+1
1483     }
1484     mat rownames M = 'variables'
1485     mat colnames M = NumberBin Mean Median StdDev Min Max
1486     mat li M
1487     forvalues mm = 1/9{
1488         mat M['mm', 1] = 'mm'

```



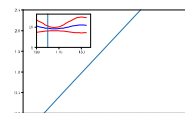

```
1489     }
1490     mat li M
1491 */
1492 *      Table for Variables used in the kernel based PLUS^D$ computation:
1493 *      btouttable using "$LATEXPATH}multikernel", replace mat(M) asis nobox format(%9.0fc
      %9.1fc %9.0fc %9.0fc %9.0fc %9.0fc) longtable
```

Next is the script named 107_Eqn4demand.do and called in the general file.

```

1
2
3
4 * pull in do-file: needs to draw in following global macros:
5
6         global DEV ${demandestimationvariables${runversionD}}
7
8
9 * open base file:
10        use "${CLOUDPATH}v38/Temp_data/Pre4and5.dta", clear
11
12
13 *****
14 * 1st step: demand estimation
15 *****
16
17
18 * initiate variables
19
20        capture drop absres*
21        capture drop sqres*
22        capture drop normalres*
23        capture drop stdres*
24        capture drop tmp
25
26        foreach m in Price Volume{
27            gen absres `m'=.
28            gen sqres `m'=.
29            gen normalres `m'=.
30            gen stdres `m'=.
31        }
32
33
34        *****
35        * eqn 4 : DEMAND
36        *****
37
38 * open loop for measure of uncertainty
39        foreach m in Price Volume{
40
41            * open loop for points and marketside
42            forvalues i=9(-2)1{
43                foreach k in Purchase{
44                    forvalues XXX=1/1{
45                        /*Irrelevant in this setting, left for copy
46                        convenience*/
47                        di "Next: " "'k' " 'i' " " 'XXX'
48
49                        * reg la: retrieve absolute prediction errors
50
51                        reg `m' ${DEV} if select ==`i' & SalePurchase=="`k'" , robust
52
53                        est store DE_`m'_`k'_`i'
54
55                        * White test for heteroskedasticity
56
57                        estat imtest , white

```



```

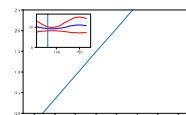
56         capture mat drop whitestat
57             mat whitestat = r(chi2_h)
58             di whitestat[1,1]
59             global a'm'a'k'a'i' = whitestat[1,1]
60         di "next global"
61         di ${a'm'a'k'a'i'}
62             if whitestat[1,1] == . {
63                 global a'm'a'k'a'i' = 99999999
64             }
65         di "next global"
66         di ${a'm'a'k'a'i'}
67
68         * predict errors
69         predict tmp if e(sample), residuals
70
71         * gen deviations of residuals
72         replace absres'm' = abs(tmp) if e(sample)
73         replace sqres'm' = tmp*tmp if e(sample) /*consistent with white*/
74         replace normalres'm' = tmp if e(sample)
75
76         * gen Stdev of residuals /*over all residuals of that regression, thus
77             single value for all -> add only to */
78         tabstat tmp if e(sample), stat(sd) save
79         mat tmpstdev = r(StatTotal)
80         di tmpstdev[1,1]
81         replace stdres'm' = tmpstdev[1,1] if e(sample)
82
83         drop tmp
84     }
85 }
86
87
88
89
90
91
92
93
94     *****
95     * generate tables for demand estimation, incl white test
96     *****
97
98     *****
99     * Tables for k=1...5
100    *****
101
102
103    foreach m in Price{
104        forvalues i=9/9{
105            foreach k in Purchase {
106                btoutreg2 [DE_`m'_'k'_'i'] using "${LATEXPATH}PriceDEPur${runversionD}.tex"
107                    , replace tex(frag pretty landscape) label(proper) addstat(White, ${a'
108                        m'a'k'a'i'}) )
109            }
110        }
111    }
112    foreach m in Price{
113        forvalues i=7(-2)1{
114            foreach k in Purchase {

```

```

113         btoutreg2 [DE_`m'_'k'_'i'] using "${LATEXPATH}PriceDEPur${runversionD}.tex"
           , append tex(frag pretty landscape) label(proper) addstat(White, ${a'm'
           'a'k'a'i'}) )
114     }
115 }
116 }
117
118
119     foreach m in Volume{
120     forvalues i=9/9{
121     foreach k in Purchase {
122         btoutreg2 [DE_`m'_'k'_'i'] using "${LATEXPATH}VolDEPur${runversionD}.tex" ,
           replace tex(frag pretty landscape) label(proper) addstat(White, ${a'm'
           a'k'a'i'}) )
123     }
124     }
125     }
126     foreach m in Volume{
127     forvalues i=7(-2)1{
128     foreach k in Purchase {
129         btoutreg2 [DE_`m'_'k'_'i'] using "${LATEXPATH}VolDEPur${runversionD}.tex" ,
           append tex(frag pretty landscape) label(proper) addstat(White, ${a'm'a
           'k'a'i'}) )
130     }
131     }
132     }
133
134
135
136
137     *****
138     * generate tables for heteroskedasticity test
139     *****
140
141     * can do later, already incl above
142     * if so, then create matrix with inputs, then export.
143
144
145     drop _est*
146
147
148 * save
149     save "${CLOUDPATH}v38/Temp_data/PreKernel_D-${runversionD}.dta", replace

```



Next is the script named `107_kernelbucketreg.do` and called in the general file.

```

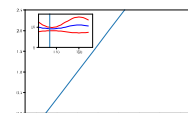
1 *DO file for robustness - 23.02
2
3 *using bucket specific linear regression.
4
5 * checking results with kernel based plus.
6     global dofiledirectoryorig= Path to directory containing do files
7     global LATEXPATH = Path to directory containing latex for article
8     global CLOUDPATH = Path to directory containing data
9
10 *****
11 * version
12     local binsetting "a"
13     global runversionD 'binsetting'
14     global demandestimationvariables 'binsetting' "Tempeff15 Roll_Temp24 Roll_Temp240
15         suncycle morning deltasun EWH SolarRest RteBlackBox "
16 *****
17
18 * Define macros
19     local sensitivity 0
20     local endofdata = _N
21     local REP1 SolarRest
22     local Numbin'REP1' 6*(1-'sensitivity')
23     local REP2 deltasun
24     local Numbin'REP2' 6*(1-'sensitivity')
25     local REP3 Tempeff15
26     local Numbin'REP3' 6*(1-'sensitivity')
27     local REP4 Roll_Temp24
28     local Numbin'REP4' 6*(1-'sensitivity')
29     local REP5 Roll_Temp240
30     local Numbin'REP5' 1*(1-'sensitivity')
31     local REP6 suncycle
32     local Numbin'REP6' 6*(1-'sensitivity')
33     local REP7 morning
34     local Numbin'REP7' 6*(1-'sensitivity')
35     local REP8 EWH
36     local Numbin'REP8' 6*(1-'sensitivity')
37     local REP9 RteBlackBox
38     local Numbin'REP9' 6*(1-'sensitivity')
39     global variablesusedkernel "${demandestimationvariables 'binsetting'}"
40
41     *****
42     * Execution PLU
43     *****
44
45     * open saving loop for speeding up computation
46     forvalues k = 1(2)9{
47     foreach s in Purchase{
48     use "${CLOUDPATH}v38/Temp_data/Pre4and5.dta", clear
49         keep if select=='k' & SalePurchase == "s"
50         local endofdata = _N
51
52     * run
53     quietly{
54         gsort SalePurchase select Datestata Hour
55

```

```

56      * gen variables to fill
57      foreach m in Price Volume{
58          capture drop Kabsres `m'
59          capture drop Ksqres `m'
60          capture drop PLUvKDvarabsres `m'
61          capture drop PLUvKDvarsqres `m'
62          capture drop Ksamplesizeabsres `m'
63          capture drop Ksamplesizesqres `m'
64          gen Kabsres `m'=0
65          gen Ksqres `m'=0
66          gen PLUvKDvarabsres `m'=0
67          gen PLUvKDvarsqres `m'=0
68          gen Ksamplesizeabsres `m'=0
69          gen Ksamplesizesqres `m'=0
70
71
72
73      forvalues obs= 1/'endofdata'{
74          * su Tempeff if _n==`obs'           /*crosscheck*/
75          * su select if _n==`obs'           /*crosscheck*/
76          local CompPoint = select['obs']
77          local CompFunc = "SalePurchase['obs']"
78          * di `CompFunc'                     /*crosscheck*/
79
80      foreach controfactor in $variablesusedkernel {
81          * find bincentre per controfactor for given observation
82          local Bincentre_`controfactor' = `controfactor'['obs']
83          * find binwidth per control factor
84          su `controfactor', meanonly
85          local topendrange_`controfactor' = r(max)
86          local lowendrange_`controfactor' = r(min)
87          local binwidth_`controfactor' = (r(max) - r(min))/ `='Numbin'
88              controfactor''''
89
90          di "----- `controfactor'"
91          di `='Numbin' controfactor''''
92          di "Max: " `topenrange_`controfactor''
93          di "Min: " `lowendrange_`controfactor''
94          di "Binwidth: " `binwidth_`controfactor''
95          di "Bincentre from current obs: " `Bincentre_`controfactor''
96      }
97
98      capture reg `m' ${{demandestimationvariables`version'}} ///
99      if select == `CompPoint' & SalePurchase== `CompFunc' ///
100      & `REP1'<= `='Bincentre_`REP1''+ `binwidth_`REP1'' & `REP1'>=`='Bincentre_`
101          REP1''- `binwidth_`REP1'' ///
102      & `REP2'<= `='Bincentre_`REP2''+ `binwidth_`REP2'' & `REP2'>=`='Bincentre_`
103          REP2''- `binwidth_`REP2'' ///
104      & `REP3'<= `='Bincentre_`REP3''+ `binwidth_`REP3'' & `REP3'>=`='Bincentre_`
105          REP3''- `binwidth_`REP3'' ///
106      & `REP4'<= `='Bincentre_`REP4''+ `binwidth_`REP4'' & `REP4'>=`='Bincentre_`
107          REP4''- `binwidth_`REP4'' ///
108      & `REP5'<= `='Bincentre_`REP5''+ `binwidth_`REP5'' & `REP5'>=`='Bincentre_`
109          REP5''- `binwidth_`REP5'' ///
110      & `REP6'<= `='Bincentre_`REP6''+ `binwidth_`REP6'' & `REP6'>=`='Bincentre_`
111          REP6''- `binwidth_`REP6'' ///
112      & `REP7'<= `='Bincentre_`REP7''+ `binwidth_`REP7'' & `REP7'>=`='Bincentre_`
113          REP7''- `binwidth_`REP7'' ///
114      & `REP8'<= `='Bincentre_`REP8''+ `binwidth_`REP8'' & `REP8'>=`='Bincentre_`
115          REP8''- `binwidth_`REP8'' ///

```



```

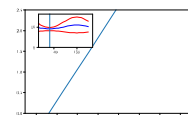
107 & 'REP9' <= '=' Bincentre_'REP9'+ 'binwidth' REP9'' & 'REP9' >= '=' Bincentre_'
      REP9'- 'binwidth' REP9'' ///
108 , robust
109
110 *predict residuals locally
111 capture predict tmp'm' if e(sample), residuals
112
113 * gen deviations of residuals
114 capture replace Kabsres'm' = abs(tmp) if _n=='obs'
115 capture replace Ksqrres'm' = tmp*tmp if _n=='obs' /*consistent with white*/
116 capture drop tmp'm'
117
118 foreach g in "absres" "sqrres" {
119   capture reg K'g'm' ${demandestimationvariables'version'} ///
120   if select == 'CompPoint' & SalePurchase== 'CompFunc' ///
121   & 'REP1' <= '=' Bincentre_'REP1'+ 'binwidth' REP1'' & 'REP1' >= '=' Bincentre_'
      REP1'- 'binwidth' REP1'' ///
122   & 'REP2' <= '=' Bincentre_'REP2'+ 'binwidth' REP2'' & 'REP2' >= '=' Bincentre_'
      REP2'- 'binwidth' REP2'' ///
123   & 'REP3' <= '=' Bincentre_'REP3'+ 'binwidth' REP3'' & 'REP3' >= '=' Bincentre_'
      REP3'- 'binwidth' REP3'' ///
124   & 'REP4' <= '=' Bincentre_'REP4'+ 'binwidth' REP4'' & 'REP4' >= '=' Bincentre_'
      REP4'- 'binwidth' REP4'' ///
125   & 'REP5' <= '=' Bincentre_'REP5'+ 'binwidth' REP5'' & 'REP5' >= '=' Bincentre_'
      REP5'- 'binwidth' REP5'' ///
126   & 'REP6' <= '=' Bincentre_'REP6'+ 'binwidth' REP6'' & 'REP6' >= '=' Bincentre_'
      REP6'- 'binwidth' REP6'' ///
127   & 'REP7' <= '=' Bincentre_'REP7'+ 'binwidth' REP7'' & 'REP7' >= '=' Bincentre_'
      REP7'- 'binwidth' REP7'' ///
128   & 'REP8' <= '=' Bincentre_'REP8'+ 'binwidth' REP8'' & 'REP8' >= '=' Bincentre_'
      REP8'- 'binwidth' REP8'' ///
129   & 'REP9' <= '=' Bincentre_'REP9'+ 'binwidth' REP9'' & 'REP9' >= '=' Bincentre_'
      REP9'- 'binwidth' REP9'' ///
130 , robust
131   capture predict PLUvKDtmp'g'm' if _n=='obs', xb
132   capture scalar sizetmp = e(N)
133   capture replace Ksamplesize'g'm' = sizetmp if _n=='obs'
134   capture replace PLUvKDvar'g'm' = PLUvKDtmp'g'm' if _n=='obs'
135   capture drop PLUvKDtmp'g'm'
136 }
137
138 di 'obs'
139                                     } /*close observations loop*/
140   } /* closes quietly*/
141 } /*close 'm'*/
142
143 * gen variables to fill
144   capture drop PLUv51'binsetting'var1
145   capture drop PLUv51'binsetting'var2
146   capture drop PLUv52'binsetting'var1
147   capture drop PLUv52'binsetting'var2
148   capture drop PointsInBinv51'binsetting'
149   capture drop PointsInBinv52'binsetting'
150   gen PLUv51'binsetting'var1 =.
151   gen PLUv51'binsetting'var2 =.
152   gen PLUv52'binsetting'var1 =.
153   gen PLUv52'binsetting'var2 =.
154   gen PointsInBinv51'binsetting' =.
155   gen PointsInBinv52'binsetting' =.
156 * generate PLUs

```

```

157         replace PLUv51'binsetting'var1 = PLUvKDvarsqresPrice
158         replace PLUv51'binsetting'var2 = PLUvKDvarsqresVolume
159         replace PLUv52'binsetting'var1 = PLUvKDvarabsresPrice
160         replace PLUv52'binsetting'var2 = PLUvKDvarabsresVolume
161         replace PointsInBinv51'binsetting' = KsamplesizesqresVolume
162         replace PointsInBinv52'binsetting' = KsamplesizeabsresVolume
163         drop Ksamplesize*
164
165
166
167         * close saving loop for speeding up computation
168         keep Date Hour SalePurchase select PLUv51'
            binsetting'var1 PLUv51'binsetting'var2 PLUv52'
            binsetting'var1 PLUv52'binsetting'var2
            PointsInBinv51'binsetting' PointsInBinv52'
            binsetting' KabsresPrice KsqresPrice
            KabsresVolume KsqresVolume PLUvKDvarabsresPrice
            PLUvKDvarsqresPrice PLUvKDvarabsresVolume
            PLUvKDvarsqresVolume
169         save "${CLOUDPATH}v38/Temp_data/KERNEL'binsetting'
            buck'k'and's'.dta", replace
170     }
171 }
172
173
174 *****
175 * version
176     local binsetting "b"
177     global runversionD 'binsetting'
178     global demandestimationvariables'binsetting' "Tempeff15 Roll_Temp24 Roll_Temp240
            suncycle morning deltasun EWH SolarRest RteBlackBox "
179
180 *****
181
182 * Define macros
183     local sensitivity 0
184     local endofdata = _N
185     local REP1 SolarRest
186     local Numbin'REP1' 6*(1-'sensitivity')
187     local REP2 deltasun
188     local Numbin'REP2' 6*(1-'sensitivity')
189     local REP3 Tempeff15
190     local Numbin'REP3' 6*(1-'sensitivity')
191     local REP4 Roll_Temp24
192     local Numbin'REP4' 6*(1-'sensitivity')
193     local REP5 Roll_Temp240
194     local Numbin'REP5' 6*(1-'sensitivity')
195     local REP6 suncycle
196     local Numbin'REP6' 6*(1-'sensitivity')
197     local REP7 morning
198     local Numbin'REP7' 6*(1-'sensitivity')
199     local REP8 EWH
200     local Numbin'REP8' 6*(1-'sensitivity')
201     local REP9 RteBlackBox
202     local Numbin'REP9' 6*(1-'sensitivity')
203     global variablesusedkernel "${demandestimationvariables'binsetting'}"
204
205     *****
206     * Execution PLU
207     *****

```




```

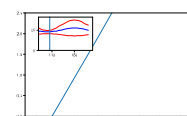
208
209         * open saving loop for speeding up computation
210         forvalues k = 1(2)9{
211         foreach s in Purchase{
212         use "${CLOUDPATH}v38/Temp.data/Pre4and5.dta", clear
213             keep if select==k' & SalePurchase == "'s'"
214             local endofdata = _N
215
216     * run
217         quietly{
218             gsort SalePurchase select Datestata Hour
219
220         * gen variables to fill
221         foreach m in Price Volume{
222             capture drop Kabsres 'm'
223             capture drop Ksqres 'm'
224             capture drop PLUvKDvarsres 'm'
225             capture drop PLUvKDvarsqres 'm'
226             capture drop Ksamplesizeabsres 'm'
227             capture drop Ksamplesizesqres 'm'
228             gen Kabsres 'm'=0
229             gen Ksqres 'm'=0
230             gen PLUvKDvarsres 'm'=0
231             gen PLUvKDvarsqres 'm'=0
232             gen Ksamplesizeabsres 'm'=0
233             gen Ksamplesizesqres 'm'=0
234
235
236
237         forvalues obs= 1/'endofdata'{
238         * su Tempeff if _n=='obs'                               /*crosscheck*/
239         * su select if _n=='obs'                               /*crosscheck*/
240         local CompPoint = select ['obs']
241         local CompFunc = "SalePurchase['obs']"
242         * di 'CompFunc'                                       /*crosscheck*/
243
244     foreach controfactor in $variablesusedkernel {
245         * find bincentre per controfactor for given observation
246         local Bincentre_'controfactor' = 'controfactor'['obs']
247         * find binwidth per control factor
248         su 'controfactor', meanonly
249         local topendrange'controfactor' = r(max)
250         local lowendrange'controfactor' = r(min)
251         local binwidth'controfactor' = (r(max) - r(min))/ '='Numbin'
252             controfactor''
253
254         di "-----" 'controfactor'
255         di '='Numbin'controfactor''
256         di "Max: " 'topenrange'controfactor''
257         di "Min: " 'lowendrange'controfactor''
258         di "Binwidth: " 'binwidth'controfactor''
259         di "Bincentre from current obs: " 'Bincentre_'controfactor''
260     }
261
262     capture reg 'm' ${demandestimationvariables'version'} ///
263     if select == 'CompPoint' & SalePurchase== 'CompFunc' ///
264     & 'REP1'<= '='Bincentre_'REP1'+ 'binwidth'REP1'' & 'REP1'>='Bincentre_'
265         REP1'- 'binwidth'REP1'' ///
266     & 'REP2'<= '='Bincentre_'REP2'+ 'binwidth'REP2'' & 'REP2'>='Bincentre_'
267         REP2'- 'binwidth'REP2'' ///

```

```

265 & 'REP3' <= '=' Bincentre_'REP3'+ 'binwidth' REP3'' & 'REP3' >= '=' Bincentre_'
      REP3'- 'binwidth' REP3'' ///
266 & 'REP4' <= '=' Bincentre_'REP4'+ 'binwidth' REP4'' & 'REP4' >= '=' Bincentre_'
      REP4'- 'binwidth' REP4'' ///
267 & 'REP5' <= '=' Bincentre_'REP5'+ 'binwidth' REP5'' & 'REP5' >= '=' Bincentre_'
      REP5'- 'binwidth' REP5'' ///
268 & 'REP6' <= '=' Bincentre_'REP6'+ 'binwidth' REP6'' & 'REP6' >= '=' Bincentre_'
      REP6'- 'binwidth' REP6'' ///
269 & 'REP7' <= '=' Bincentre_'REP7'+ 'binwidth' REP7'' & 'REP7' >= '=' Bincentre_'
      REP7'- 'binwidth' REP7'' ///
270 & 'REP8' <= '=' Bincentre_'REP8'+ 'binwidth' REP8'' & 'REP8' >= '=' Bincentre_'
      REP8'- 'binwidth' REP8'' ///
271 & 'REP9' <= '=' Bincentre_'REP9'+ 'binwidth' REP9'' & 'REP9' >= '=' Bincentre_'
      REP9'- 'binwidth' REP9'' ///
272 , robust
273
274 *predict residuals locally
275 capture predict tmp'm' if e(sample), residuals
276
277 * gen deviations of residuals
278 capture replace Kabsres'm' = abs(tmp) if _n== 'obs'
279 capture replace Ksqres'm' = tmp*tmp if _n== 'obs' /*consistent with white*/
280 capture drop tmp'm'
281
282 foreach g in "absres" "sqres" {
283   capture reg K'g'm' ${demandestimationvariables' version'} ///
284   if select == 'CompPoint' & SalePurchase == 'CompFunc' ///
285   & 'REP1' <= '=' Bincentre_'REP1'+ 'binwidth' REP1'' & 'REP1' >= '=' Bincentre_'
      REP1'- 'binwidth' REP1'' ///
286   & 'REP2' <= '=' Bincentre_'REP2'+ 'binwidth' REP2'' & 'REP2' >= '=' Bincentre_'
      REP2'- 'binwidth' REP2'' ///
287   & 'REP3' <= '=' Bincentre_'REP3'+ 'binwidth' REP3'' & 'REP3' >= '=' Bincentre_'
      REP3'- 'binwidth' REP3'' ///
288   & 'REP4' <= '=' Bincentre_'REP4'+ 'binwidth' REP4'' & 'REP4' >= '=' Bincentre_'
      REP4'- 'binwidth' REP4'' ///
289   & 'REP5' <= '=' Bincentre_'REP5'+ 'binwidth' REP5'' & 'REP5' >= '=' Bincentre_'
      REP5'- 'binwidth' REP5'' ///
290   & 'REP6' <= '=' Bincentre_'REP6'+ 'binwidth' REP6'' & 'REP6' >= '=' Bincentre_'
      REP6'- 'binwidth' REP6'' ///
291   & 'REP7' <= '=' Bincentre_'REP7'+ 'binwidth' REP7'' & 'REP7' >= '=' Bincentre_'
      REP7'- 'binwidth' REP7'' ///
292   & 'REP8' <= '=' Bincentre_'REP8'+ 'binwidth' REP8'' & 'REP8' >= '=' Bincentre_'
      REP8'- 'binwidth' REP8'' ///
293   & 'REP9' <= '=' Bincentre_'REP9'+ 'binwidth' REP9'' & 'REP9' >= '=' Bincentre_'
      REP9'- 'binwidth' REP9'' ///
294   , robust
295   capture predict PLUvKDtmp'g'm' if _n== 'obs', xb
296   capture scalar sizetmp = e(N)
297   capture replace Ksamplesize'g'm' = sizetmp if _n== 'obs'
298   capture replace PLUvKDvar'g'm' = PLUvKDtmp'g'm' if _n== 'obs'
299   capture drop PLUvKDtmp'g'm'
300 }
301
302 di 'obs'
303                                     } /*close observations loop*/
304   } /* closes quietly*/
305 } /*close 'm'*/
306
307 * gen variables to fill
308 capture drop PLUv51'binsetting'var1

```



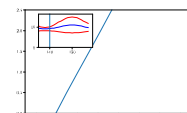
```
309         capture drop PLUv51'binsetting'var2
310         capture drop PLUv52'binsetting'var1
311         capture drop PLUv52'binsetting'var2
312         capture drop PointsInBinv51'binsetting'
313         capture drop PointsInBinv52'binsetting'
314         gen PLUv51'binsetting'var1 =.
315         gen PLUv51'binsetting'var2 =.
316         gen PLUv52'binsetting'var1 =.
317         gen PLUv52'binsetting'var2 =.
318         gen PointsInBinv51'binsetting' =.
319         gen PointsInBinv52'binsetting' =.
320     * generate PLUs
321         replace PLUv51'binsetting'var1 = PLUvKDvarsqresPrice
322         replace PLUv51'binsetting'var2 = PLUvKDvarsqresVolume
323         replace PLUv52'binsetting'var1 = PLUvKDvarabsresPrice
324         replace PLUv52'binsetting'var2 = PLUvKDvarabsresVolume
325         replace PointsInBinv51'binsetting' = KsamplesizesqresVolume
326         replace PointsInBinv52'binsetting' = KsamplesizeabsresVolume
327         drop Ksamplesize*
328
329
330
331     * close saving loop for speeding up computation
332         keep Date Hour SalePurchase select PLUv51'
            binsetting'var1 PLUv51'binsetting'var2 PLUv52'
            binsetting'var1 PLUv52'binsetting'var2
            PointsInBinv51'binsetting' PointsInBinv52'
            binsetting' KabsresPrice KsqresPrice
            KabsresVolume KsqresVolume PLUvKDvarabsresPrice
            PLUvKDvarsqresPrice PLUvKDvarabsresVolume
            PLUvKDvarsqresVolume
333         save "${CLOUDPATH}v38/Temp.data/KERNEL'binsetting'
            buck'k'and's'.dta", replace
334     }
335 }
```

Next is the script named 107_PrepkernelPLUdata.do and called in the general file.

```

1 *PrepkernelPLUdata
2
3
4     global dofiledirectoryorig= Path to directory containing do files
5     global LATEXPATH = Path to directory containing latex for article
6     global CLOUDPATH = Path to directory containing data
7
8
9     ****
10    * Append kernel PLUs
11    ****
12
13            * on sq residuals
14    foreach binsetting in "a" "b" {
15        use "${CLOUDPATH}v38/Temp_data/KERNEL'binsetting' bucklandPurchase.dta",
16            clear
17        *append
18        forvalues k = 3(2)9 {
19            foreach s in Purchase {
20                append using "${CLOUDPATH}v38/Temp_data/Kernel'binsetting' buck'k'
21                    andPurchase.dta"
22            }
23        }
24
25        duplicates list SalePurchase select Date Hour
26        duplicates drop SalePurchase select Date Hour, force
27
28    * - 51a. = sqres + 8 bins.
29    * - 51b. = sqres + 9 bins.
30    * - 52a. = absres + 8 bins.
31    * - 52b. = absres + 9 bins.
32
33            capture rename PLUv51avar1 PLUv51avarPsq
34            capture rename PLUv51avar2 PLUv51avarQsq
35            capture rename PLUv52avar1 PLUv52avarPabs
36            capture rename PLUv52avar2 PLUv52avarQabs
37
38            capture rename PLUv51bvar1 PLUv51bvarPsq
39            capture rename PLUv51bvar2 PLUv51bvarQsq
40            capture rename PLUv52bvar1 PLUv52bvarPabs
41            capture rename PLUv52bvar2 PLUv52bvarQabs
42
43            drop PLUKD*
44            save "${CLOUDPATH}v38/Temp_data/PLUKernel'binsetting'.dta", replace
45        }
46
47    ****
48    * Merge PLUs with dataset
49    ****
50
51    use "${CLOUDPATH}v38/Temp_data/Finaldataset.dta", clear
52    duplicates list SalePurchase select Date Hour
53    duplicates drop SalePurchase select Date Hour, force
54    foreach binsetting in "a" "b" {
55        /* here add other versions */
56        merge 1:1 SalePurchase select Date Hour using "${CLOUDPATH}v38/Temp_data/PLUKernel'

```



```

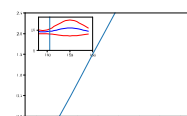
55         binsetting '.dta', nogenerate
56     }
57 *save
58     save "${CLOUDPATH}v38/Temp_data/Int1.dta", replace
59
60
61 ****
62 * transpose uncertainty of k points.
63 ****
64
65     use "${CLOUDPATH}v38/Temp_data/Int1.dta", clear
66     keep Date SalePurchase select Hour Datestata PLU*
67     reshape wide PLU* , i(SalePurchase Datestata Hour) j(select)
68     order _all, sequential
69     gsort Datestata Hour SalePurchase
70     duplicates list Date Hour SalePurchase
71     duplicates drop Date Hour SalePurchase, force
72     save "${CLOUDPATH}v38/Temp_data/Int2.dta", replace
73
74     use "${CLOUDPATH}v38/Temp_data/Int1.dta", clear
75     merge m:1 Date Hour SalePurchase using "${CLOUDPATH}v38/Temp_data/Int2.dta",
76         nogenerate
77
78 * drop constant plu_r
79 drop PLUvRvarS1 PLUvRvarS3 PLUvRvarS5 PLUvRvarS7 PLUvRvarS9 PLUvRvarSsq1
80     PLUvRvarSsq3 PLUvRvarSsq5 PLUvRvarSsq7 PLUvRvarSsq9 PLUvRvarT1 PLUvRvarT3
81     PLUvRvarT5 PLUvRvarT7 PLUvRvarT9 PLUvRvarTsq1 PLUvRvarTsq3 PLUvRvarTsq5
82     PLUvRvarTsq7 PLUvRvarTsq9 PLUvRvarW1 PLUvRvarW3 PLUvRvarW5 PLUvRvarW7
83     PLUvRvarW9 PLUvRvarWsq1 PLUvRvarWsq3 PLUvRvarWsq5 PLUvRvarWsq7 PLUvRvarWsq9
84
85     gsort Datestata Hour SalePurchase select
86     order Date Datestata Hour SalePurchase select , first
87     order PLU* Poin*, last
88     save "${CLOUDPATH}v38/Temp_data/Int3.dta", replace
89
90 ****
91 * table of Points in bin and PLU and PDU
92 ****
93
94     forvalues k = 5/5{
95         foreach versionD in "51a" "51b"{
96             foreach versionS in ""{
97                 foreach switch in /*"abs"*/ "sq"{
98
99                     capture mat drop M'versionD'
100                    local variables "PLUv'versionD'varP'switch''k' PLUv'versionD'varQ'
101                        switch''k' PointsInBinv'versionD' "
102                    local FUNC "r(mean) r(p50) r(sd) r(min) r(max)"
103                    local i=0
104                    foreach var of local variables{
105                        local i='i'+1
106                    }
107                    local j=0
108                    foreach var of local FUNC{
109                        local j='j'+1
110                    }
111                    di 'j' " " 'i'

```

```

108         mat M'versionD' = J('i',5,..)
109
110         local c=1
111         foreach FF of local FUNC{
112             local r=1
113             foreach VV of local variables{
114                 su 'VV', detail
115                 mat M'versionD'['r','c']= 'FF'
116                 local r='r'+1
117             }
118             local c='c'+1
119         }
120         mat rownames M'versionD' = 'variables'
121         mat colnames M'versionD' = Mean Median StdDev Min Max
122         mat li M'versionD'
123     }
124 }
125 }
126
127         capture mat drop Mtogether
128         mat Mtogether = J(6,5,..)
129         mat Mtogether = M51a \ M51b
130         mat li Mtogether
131
132 * Summary Statistics of PLUs and PDUs:
133     btouttable using "${LATEXPATH}suPDUPLU", replace mat(Mtogether) label asis nobox
134         format(%9.1fc %9.0fc %9.0fc %9.0fc %9.0fc) footnote(Proxies based on multi-
135         variate kernels) longtable caption(Summary statistics of kernel based PLUS`DS
136         for k='k')
137     }
138
139     *****
140     * all variables of interest in purchase obs.
141     *****
142
143     drop if SalePurchase=="Sell"
144
145
146
147
148
149
150
151
152
153
154
155
156
157
158
159
160
161
162
163
164

```

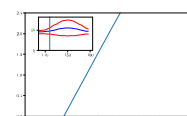


```
165
166
167
168
169
170
171
172
173
174
175
176
177
178
179
180
181
182
183 * OLD CODE
184 /*
185
186     ***
187     * scalefactor to adjust fx to slope
188     ***
189 /*     capture drop group
190         egen group= group(Datestatafrac)
191     gsort group Datestata Hour SalePurchase select
192     capture drop slopeDpost slopeDpre slopeDatk fxscalefactor
193         gen slopeDpost =.
194         gen slopeDpre =.
195         gen slopeDatk =.
196         gen fxscalefactor =.
197
198     forvalues k= 1(2)9{
199         *note in p-q dimension!
200     by group: replace slopeDpost = (Volume[_n+1] - Volume[_n])/(Price[_n+1]- Price[_n]) if select=='k'
201
202     by group:         replace slopeDpre = (Volume[_n] - Volume[_n-1])/(Price[_n]- Price[_n-1]) if select
203         =='k'
204
205     by group:         replace slopeDatk = abs(slopeDpost[_n]+ slopeDpre[_n])/2 if select=='k'
206
207     capture drop tmp1 tmp2
208     egen tmp1 = mean(slopeDatk ) if select=='k'
209     egen tmp2 = mean(fx) if select=='k'
210     replace fxscalefactor = tmp1 / tmp2 if select=='k'
211     drop tmp1 tmp2
212     }
213     capture drop fxscaled
214     gen fxscaled = fx * fxscalefactor
215
216     * scalefactor to adjust fx to slope
217     capture drop group
218         egen group= group(Datestatafrac)
219         gsort group Datestata Hour SalePurchase select
220     capture drop slopeDpostQP slopeDpreQP slopeDatkQP fxscalefactorQP
221         gen slopeDpostQP =.
222         gen slopeDpreQP =.
223         gen slopeDatkQP =.
224         gen fxscalefactorQP =.
```

```

224             gen fxQP = (1/fx)
225
226 forvalues k= 1(2)9{
227     *note in q-p dimension!
228     by group: replace slopeDpostQP = (Price[_n+1]- Price[_n])/(Volume[_n+1] - Volume[_n]) if
                select=='k'
229     by group: replace slopeDpreQP = (Price[_n]- Price[_n-1])/(Volume[_n] - Volume[_n-1]) if
                select=='k'
230     by group: replace slopeDatkQP = abs(slopeDpostQP[_n]+ slopeDpreQP[_n])/2 if select=='k'
231     capture drop tmp1 tmp2
232     egen tmp1 = mean(slopeDatkQP ) if select=='k'
233     egen tmp2 = mean(fxQP) if select=='k'
234     replace   fxscalefactorQP = tmp1 / tmp2 if select=='k'
235     drop tmp1 tmp2
236 }
237 capture drop fxscaledQP
238 gen fxscaledQP = fxQP * fxscalefactorQP
239 gen comparisonfx = 1/ fxscaled
240 * SCALING ONLY APPROPRIATE FOR K=5. otherwise too much mixing flat and vertical section.
241 */
242
243
244
245 /*
246     *****
247     * rescale variables
248     *****
249         foreach versionD in "D" "51a" "51b"{
250             foreach switch in "" "abs"{
251                 foreach switch2 in "var" "rt"{
252                     foreach dim in "P" {
253                         forvalues k = 3(2)7{
254                             capture noisily su PLUv'`versionD'`switch2'`dim'`switch'`k', detail
255                             capture noisily scalar meanPLUv'`versionD'`switch2'`dim'`switch'`k' = r(mean
                )
256                             capture noisily di  meanPLUv'`versionD'`switch2'`dim'`switch'`k'
257                             capture noisily gen PLUv'`versionD'`switch2'`dim'`switch'`k' resc = PLUv'
                `versionD'`switch2'`dim'`switch'`k' / meanPLUv'`versionD'`switch2'`dim'`
                switch'`k'
258             }
259         }
260     }
261 }
262 }
263
264     foreach versionD in "D" "52a" "52b"{
265         foreach switch in "" "abs"{
266             foreach switch2 in "var" "rt"{
267                 foreach dim in "P" {
268                     forvalues k = 1(8)9{
269                         capture noisily su PLUv'`versionD'`switch2'`dim'`switch'`k', detail
270                         capture noisily scalar meanPLUv'`versionD'`switch2'`dim'`switch'`k' = r(mean
                )
271                         capture noisily di  meanPLUv'`versionD'`switch2'`dim'`switch'`k'
272                         capture noisily gen PLUv'`versionD'`switch2'`dim'`switch'`k' resc = PLUv'
                `versionD'`switch2'`dim'`switch'`k'
273             }
274         }
275     }
276 }

```



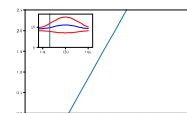

```
277     }
278
279     foreach versionD in "D" "52a" "52b"{
280     foreach switch in "" "abs"{
281     foreach switch2 in "var" "rt"{
282     foreach dim in "Q" {
283     forvalues k = 1(2)9{
284     capture noisily su PLUv'`versionD'`switch2'`dim'`switch'`k', detail
285     capture noisily scalar meanPLUv'`versionD'`switch2'`dim'`switch'`k' = r(mean
286     )
287     capture noisily di meanPLUv'`versionD'`switch2'`dim'`switch'`k'
288     capture noisily gen PLUv'`versionD'`switch2'`dim'`switch'`k' resc = PLUv'
289     versionD'`switch2'`dim'`switch'`k' / meanPLUv'`versionD'`switch2'`dim'`
290     switch'`k'
291     }
292     }
293     }
294     }
295     }
296     */
297
298     order Date Datestata Hour SalePurchase select , first
299     order PLU* Poin*, last
```

Next is the script named `107_BootstrapKernel2702.do` and called in the general file.

```

1  ** kernel based PLU
2
3
4
5  * Notes:
6  * - 51a. = sqres + 8 bins.
7  * - 51b. = sqres + 9 bins.
8  * - 52a. = absres + 8 bins.
9  * - 52b. = absres + 9 bins.
10
11
12     global dofiledirectoryorig= Path to directory containing do files
13     global LATEXPATh = Path to directory containing latex for article
14     global CLOUDPATH = Path to directory containing data
15
16     global focusk = 5
17     keep if select== ${focusk}
18     global VVV = 51
19     global APP = "sq"
20         *global VVV = 52
21         *global APP = "abs"
22
23     * drop observations that will not be used for final reg anyway. (dropped 2989 obs) = last 6
24       months approx.
25     drop if PLUvRvarW==.
26     drop if WindIDA==.
27
28
29
30
31 ***** (Col. 3)
32 * for comparison, without bootstrap but weighted:
33 *****
34
35         use "${CLOUDPATH}v38/Temp_data/Finalrundataset.dta", clear
36         local binsetting "a"
37         local k = ${focusk}
38         local VVV = ${VVV}
39         local APP = "${APP}"
40         keep if select== 'k'
41
42     *gen rescaled PLU
43     capture drop PLU_P_boot PLU_P_resc PLU_Q_boot PLU_Q_resc
44     gen PLU_P_boot = PLUv'VVV' binsetting varP 'APP'
45         su PLU_P_boot, meanonly
46         scalar tmpP = r(mean)
47         gen PLU_P_resc = PLU_P_boot / tmpP
48     gen PLU_Q_boot = PLUv'VVV' binsetting varQ 'APP'
49         su PLU_Q_boot, meanonly
50         scalar tmpQ = r(mean)
51         gen PLU_Q_resc = PLU_Q_boot / tmpQ
52
53     if 'k'==1 | 'k'==9 {
54         local k = ${focusk}
55         reg fxInvertQP PLUvRvarT PLUvRvarTsQ PLUvRvarW PLUvRvarWsQ

```



```

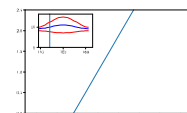
56         PLUvRvarS PLUvRvarSsq Coal Brent Gas IT2 EUA Wind1DA Hydro /*
57         PLU_P_resc*/ PLU_Q_resc    if select =='k' & SalePurchase=="
58         Purchase", robust
59     est store kernel3_ 'k'
60     reg fxInvertQP    PLUvRvarT PLUvRvarTsqr PLUvRvarW PLUvRvarWsq
61     PLUvRvarS PLUvRvarSsq Coal Brent Gas IT2 EUA Wind1DA Hydro /*
62     PLU_P_resc*/ PLU_Q_resc    if select =='k' & SalePurchase=="
63     Purchase" [aweight=PointsInBin51 'binsetting']
64     est store kernel4_ 'k'
65     est save "%{CLOUDPATH}v38/Temp_data/kernelweighthed_ 'k'.ster",
66     replace
67 }
68 else{
69     local k = ${focusk}
70     reg fxInvertQP    PLUvRvarT PLUvRvarTsqr PLUvRvarW PLUvRvarWsq
71     PLUvRvarS PLUvRvarSsq Coal Brent Gas IT2 EUA Wind1DA Hydro
72     PLU_P_resc PLU_Q_resc    if select =='k' & SalePurchase=="
73     Purchase", robust
74     est store kernel3_ 'k'
75     reg fxInvertQP    PLUvRvarT PLUvRvarTsqr PLUvRvarW PLUvRvarWsq
76     PLUvRvarS PLUvRvarSsq Coal Brent Gas IT2 EUA Wind1DA Hydro
77     PLU_P_resc PLU_Q_resc    if select =='k' & SalePurchase=="
78     Purchase" [aweight=PointsInBin51 'binsetting']
79     est store kernel4_ 'k'
80     est save "%{CLOUDPATH}v38/Temp_data/kernelweighthed_ 'k'.ster",
81     replace
82 }
83
84 ***** NOW SAME BUT DROPPING 1 PLUvD (COL.3)
85
86     use "%{CLOUDPATH}v38/Temp_data/Finalrundataset.dta", clear
87     local binsetting "a"
88     local k = ${focusk}
89     local VVV = ${VVV}
90     local APP = "${APP}"
91     keep if select== 'k'
92
93     *gen rescaled PLU
94     capture drop PLU_P_boot PLU_P_resc PLU_Q_boot PLU_Q_resc
95     gen PLU_P_boot = PLUv'VVV' 'binsetting' varP 'APP'
96     su PLU_P_boot, meanonly
97     scalar tmpP = r(mean)
98     gen PLU_P_resc = PLU_P_boot / tmpP
99     gen PLU_Q_boot = PLUv'VVV' 'binsetting' varQ 'APP'
100    su PLU_Q_boot, meanonly
101    scalar tmpQ = r(mean)
102    gen PLU_Q_resc = PLU_Q_boot / tmpQ
103
104    if 'k'==1 | 'k'==9 {
105        local k = ${focusk}
106        reg fxInvertQP    PLUvRvarT PLUvRvarTsqr PLUvRvarW PLUvRvarWsq
107        PLUvRvarS PLUvRvarSsq Coal Brent Gas IT2 EUA Wind1DA Hydro /*
108        PLU_P_resc*/ PLU_Q_resc    if select =='k' & SalePurchase=="
109        Purchase", robust
110        est store kernel3_ 'k'
111        reg fxInvertQP    PLUvRvarT PLUvRvarTsqr PLUvRvarW PLUvRvarWsq
112        PLUvRvarS PLUvRvarSsq Coal Brent Gas IT2 EUA Wind1DA Hydro /*
113        PLU_P_resc*/ PLU_Q_resc    if select =='k' & SalePurchase=="
114        Purchase" [aweight=PointsInBin51 'binsetting']

```

```

96         est store kernel4_`k'
97         est save "${CLOUDPATH}v38/Temp_data/kernelweigDROP_P_`k'.ster",
           replace
98     }
99     else{
100         local k = ${focusk}
101         reg fxInvertQP  PLUvRvarT PLUvRvarTsqr PLUvRvarW PLUvRvarWsqr
           PLUvRvarS PLUvRvarSsq Coal Brent Gas IT2 EUA Wind1DA Hydro /*
           PLU_P_resc*/ PLU_Q_resc    if select ==`k' & SalePurchase=="
           Purchase", robust
102         est store kernel3_`k'
103         reg fxInvertQP  PLUvRvarT PLUvRvarTsqr PLUvRvarW PLUvRvarWsqr
           PLUvRvarS PLUvRvarSsq Coal Brent Gas IT2 EUA Wind1DA Hydro /*
           PLU_P_resc*/ PLU_Q_resc    if select ==`k' & SalePurchase=="
           Purchase" [aweight=PointsInBin51`binsetting']
104         est store kernel4_`k'
105         est save "${CLOUDPATH}v38/Temp_data/kernelweigDROP_P_`k'.ster",
           replace
106         reg fxInvertQP  PLUvRvarT PLUvRvarTsqr PLUvRvarW PLUvRvarWsqr
           PLUvRvarS PLUvRvarSsq Coal Brent Gas IT2 EUA Wind1DA Hydro
           PLU_P_resc /*PLU_Q_resc*/    if select ==`k' & SalePurchase=="
           "Purchase", robust
107         est store kernel3_`k'
108         reg fxInvertQP  PLUvRvarT PLUvRvarTsqr PLUvRvarW PLUvRvarWsqr
           PLUvRvarS PLUvRvarSsq Coal Brent Gas IT2 EUA Wind1DA Hydro
           PLU_P_resc /*PLU_Q_resc*/    if select ==`k' & SalePurchase=="
           Purchase" [aweight=PointsInBin51`binsetting']
109         est store kernel4_`k'
110         est save "${CLOUDPATH}v38/Temp_data/kernelweigDROP_Q_`k'.ster",
           replace
111     }
112
113
114
115
116
117
118
119
120
121
122
123
124
125
126
127
128
129 *****
130 * for bootstrapping of kernel based PLUvD
131
132         ***** bootstrap of kernel based equation 4 forecasts
133 *****
134
135         use "${CLOUDPATH}v38/Temp_data/Finalrundataset.dta", clear
136         global focusk = 5
137         keep if select== ${focusk}
138         * drop observations with missing values (dropped 2989 obs) = last 6 months approx.
139         drop if PLUvRvarW==.
140         drop if Wind1DA==.

```



```

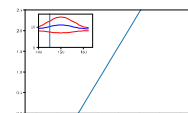
141
142     capture drop   PLU_P_boot
143     capture drop   PLU_P_resc
144     capture drop   PLU_Q_boot
145     capture drop   PLU_Q_resc
146
147
148
149 ***** START PROG
150     capture program drop my2slsforbootkernel
151     program my2slsforbootkernel
152
153 * version
154     local binsetting "a"
155     local k = ${focusk}
156     global runversionD 'binsetting'
157     global demandestimationvariables 'binsetting' "Tempeff15 Roll_Temp24 Roll_Temp240
158         suncycle morning deltasun EWH SolarRest RteBlackBox "
159 *****
160
161 * Define macros
162     local sensitivity 0
163     local endofdata = _N
164     local REP1 SolarRest
165     local Numbin'REP1' 6*(1-'sensitivity')
166     local REP2 deltasun
167     local Numbin'REP2' 6*(1-'sensitivity')
168     local REP3 Tempeff15
169     local Numbin'REP3' 6*(1-'sensitivity')
170     local REP4 Roll_Temp24
171     local Numbin'REP4' 6*(1-'sensitivity')
172     local REP5 Roll_Temp240
173     local Numbin'REP5' 1*(1-'sensitivity')
174     local REP6 suncycle
175     local Numbin'REP6' 6*(1-'sensitivity')
176     local REP7 morning
177     local Numbin'REP7' 6*(1-'sensitivity')
178     local REP8 EWH
179     local Numbin'REP8' 6*(1-'sensitivity')
180     local REP9 RteBlackBox
181     local Numbin'REP9' 6*(1-'sensitivity')
182     global variablesusedkernel "${demandestimationvariables 'binsetting'}"
183
184
185 ***** stage 1 -> bucket specific reg. 1
186 local endofdata = _N
187     quietly{
188         gsort SalePurchase select Datestata Hour
189
190         * gen variables to fill
191         foreach m in Price Volume{
192             capture drop   Kabsres 'm'
193             capture drop   Ksqres 'm'
194             capture drop   PLUvKDvarabsres 'm'
195             capture drop   PLUvKDvarsqres 'm'
196             capture drop   Ksamplesizeabsres 'm'
197             capture drop   Ksamplesizesqres 'm'
198             gen Kabsres 'm'=0
199             gen Ksqres 'm'=0

```

```

200         gen PLUvKDvarsres 'm'=0
201         gen PLUvKDvarsqres 'm'=0
202         gen Ksamplesizeabsres 'm'=0
203         gen Ksamplesizeqres 'm'=0
204
205
206         forvalues obs= 1/'endofdata' {
207             * su Tempeff if _n=='obs'           /*crosscheck*/
208             * su select if _n=='obs'           /*crosscheck*/
209             local CompPoint = select ['obs']
210             local CompFunc = "SalePurchase['obs']"
211             * di 'CompFunc'                     /*crosscheck*/
212
213         foreach controlfactor in $variablesusedkernel {
214             * find bincentre per controlfactor for given observation
215             local Bincentre_`controlfactor' = `controlfactor'['obs']
216             * find binwidth per control factor
217             su `controlfactor', meanonly
218             local topendrange`controlfactor' = r(max)
219             local lowendrange`controlfactor' = r(min)
220             local binwidth`controlfactor' = (r(max) - r(min))/ `='Numbin'
221             controlfactor''
222
223             di "-----" `controlfactor'
224             di `='Numbin'`controlfactor' ''
225             di "Max: " `topendrange`controlfactor' ''
226             di "Min: " `lowendrange`controlfactor' ''
227             di "Binwidth: " `binwidth`controlfactor' ''
228             di "Bincentre from current obs: " `Bincentre_`controlfactor' ''
229         }
230
231         capture reg `m' ${demandestimationvariables`version'} ///
232         if select == `CompPoint' & SalePurchase== `CompFunc' ///
233         & `REP1'<= `='Bincentre_`REP1'+ `binwidth`REP1'' & `REP1'>=`='Bincentre_`
234             REP1'- `binwidth`REP1'' ///
235         & `REP2'<= `='Bincentre_`REP2'+ `binwidth`REP2'' & `REP2'>=`='Bincentre_`
236             REP2'- `binwidth`REP2'' ///
237         & `REP3'<= `='Bincentre_`REP3'+ `binwidth`REP3'' & `REP3'>=`='Bincentre_`
238             REP3'- `binwidth`REP3'' ///
239         & `REP4'<= `='Bincentre_`REP4'+ `binwidth`REP4'' & `REP4'>=`='Bincentre_`
240             REP4'- `binwidth`REP4'' ///
241         & `REP5'<= `='Bincentre_`REP5'+ `binwidth`REP5'' & `REP5'>=`='Bincentre_`
242             REP5'- `binwidth`REP5'' ///
243         & `REP6'<= `='Bincentre_`REP6'+ `binwidth`REP6'' & `REP6'>=`='Bincentre_`
244             REP6'- `binwidth`REP6'' ///
245         & `REP7'<= `='Bincentre_`REP7'+ `binwidth`REP7'' & `REP7'>=`='Bincentre_`
246             REP7'- `binwidth`REP7'' ///
247         & `REP8'<= `='Bincentre_`REP8'+ `binwidth`REP8'' & `REP8'>=`='Bincentre_`
248             REP8'- `binwidth`REP8'' ///
249         & `REP9'<= `='Bincentre_`REP9'+ `binwidth`REP9'' & `REP9'>=`='Bincentre_`
250             REP9'- `binwidth`REP9'' ///
251         , robust
252
253         *predict residuals locally
254         predict tmp`m' if e(sample), residuals
255
256         * gen deviations of residuals
257         replace Kabsres `m' = abs(tmp) if _n=='obs'
258         replace Ksqres `m' = tmp*tmp if _n=='obs' /*consistent with white*/
259         capture drop tmp`m'

```



```

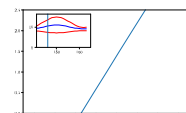
250
251
252 ***** stage 2 -> bucket specific reg. 2 (
      heteroskedasticity)
253     foreach g in /*"absres"*/ "sqres" {
254         capture reg K'g'm' ${demandestimationvariables'version'} ///
255         if select == 'CompPoint' & SalePurchase== 'CompFunc' ///
256         & 'REP1'<= '=' Bincentre_'REP1'+ 'binwidth' REP1'' & 'REP1'>='=' Bincentre_'
                REP1'- 'binwidth' REP1'' ///
257         & 'REP2'<= '=' Bincentre_'REP2'+ 'binwidth' REP2'' & 'REP2'>='=' Bincentre_'
                REP2'- 'binwidth' REP2'' ///
258         & 'REP3'<= '=' Bincentre_'REP3'+ 'binwidth' REP3'' & 'REP3'>='=' Bincentre_'
                REP3'- 'binwidth' REP3'' ///
259         & 'REP4'<= '=' Bincentre_'REP4'+ 'binwidth' REP4'' & 'REP4'>='=' Bincentre_'
                REP4'- 'binwidth' REP4'' ///
260         & 'REP5'<= '=' Bincentre_'REP5'+ 'binwidth' REP5'' & 'REP5'>='=' Bincentre_'
                REP5'- 'binwidth' REP5'' ///
261         & 'REP6'<= '=' Bincentre_'REP6'+ 'binwidth' REP6'' & 'REP6'>='=' Bincentre_'
                REP6'- 'binwidth' REP6'' ///
262         & 'REP7'<= '=' Bincentre_'REP7'+ 'binwidth' REP7'' & 'REP7'>='=' Bincentre_'
                REP7'- 'binwidth' REP7'' ///
263         & 'REP8'<= '=' Bincentre_'REP8'+ 'binwidth' REP8'' & 'REP8'>='=' Bincentre_'
                REP8'- 'binwidth' REP8'' ///
264         & 'REP9'<= '=' Bincentre_'REP9'+ 'binwidth' REP9'' & 'REP9'>='=' Bincentre_'
                REP9'- 'binwidth' REP9'' ///
265     , robust
266     predict PLUvKDtmp'g'm' if _n==obs', xb
267     scalar sizetmp = e(N)
268     replace Ksamplesize'g'm' = sizetmp if _n==obs'
269     replace PLUvKDvar'g'm' = PLUvKDtmp'g'm' if _n==obs'
270     capture drop PLUvKDtmp'g'm'
271     }
272
273     di 'obs'
274                                     } /*close observations loop*/
275     } /* closes quietly*/
276 } /*close 'm'*/
277
278 * gen variables to fill
279     capture drop PLUv51'binsetting'var1
280     capture drop PLUv51'binsetting'var2
281     capture drop PLUv52'binsetting'var1
282     capture drop PLUv52'binsetting'var2
283     capture drop PointsInBinv51'binsetting'
284     capture drop PointsInBinv52'binsetting'
285     gen PLUv51'binsetting'var1 =.
286     gen PLUv51'binsetting'var2 =.
287     gen PLUv52'binsetting'var1 =.
288     gen PLUv52'binsetting'var2 =.
289     gen PointsInBinv51'binsetting' =.
290     gen PointsInBinv52'binsetting' =.
291
292 * generate PLUs
293     replace PLUv51'binsetting'var1 = PLUvKDvarsqresPrice
294     replace PLUv51'binsetting'var2 = PLUvKDvarsqresVolume
295     *replace PLUv52'binsetting'var1 = PLUvKDvarabsresPrice
296     *replace PLUv52'binsetting'var2 = PLUvKDvarabsresVolume
297     replace PointsInBinv51'binsetting' = KsamplesizeqresVolume
298     *replace PointsInBinv52'binsetting' = KsamplesizeabsresVolume
299     drop Ksamplesize*

```

```

300 gen PLU_P_boot = PLUv51'binsetting'var1
301     su PLU_P_boot, meanonly
302     scalar tmpP = r(mean)
303     gen PLU_P_resc = PLU_P_boot / tmpP
304 gen PLU_Q_boot = PLUv51'binsetting'var2
305     su PLU_Q_boot, meanonly
306     scalar tmpQ = r(mean)
307     gen PLU_Q_resc = PLU_Q_boot / tmpQ
308
309
310 ***** stage 3 -> final reg
311
312
313     reg fxInvertQP   PLUvRvarT PLUvRvarTsq PLUvRvarW PLUvRvarWsqr PLUvRvarS PLUvRvarSsq
314     Coal Brent Gas IT2 EUA Wind1DA Hydro PLU_P_resc PLU_Q_resc   if select =='k'
315     & SalePurchase=="Purchase" [aweight=PointsInBinv51'binsetting']
316
317     drop PLU_P_boot PLU_P_resc PLU_Q_boot PLU_Q_resc
318     drop PointsInBinv51'binsetting'
319
320 ***** END PROG
321
322 end
323
324 ***** Run once and save dataset (not inside program)
325
326     *run prog
327     my2slsforbootkernel
328
329     local k= ${focusk}
330     save "${CLOUDPATH}v38/Temp_data/Finaldataset_withKERNELpluDa'k'.dta", replace
331
332 ***** Bootstrap (approx. 15h for 50 reps)
333
334     bootstrap _b, reps(50) seed(12345): my2slsforbootkernel
335
336     est save "${CLOUDPATH}v38/Temp_data/kernelbootstrap'k'.ster", replace
337
338
339
340
341
342
343
344
345
346
347
348
349
350
351
352
353
354
355
356
357

```




```
358
359
360
361 *OLD CODE:
362 /*      * gen variables to fill
363         capture drop PLUv51'binsetting'var1
364         capture drop PLUv51'binsetting'var2
365         capture drop PLUv52'binsetting'var1
366         capture drop PLUv52'binsetting'var2
367         capture drop PointsInBinv51'binsetting'
368         capture drop PointsInBinv52'binsetting'
369         gen PLUv51'binsetting'var1 =.
370         gen PLUv51'binsetting'var2 =.
371         gen PLUv52'binsetting'var1 =.
372         gen PLUv52'binsetting'var2 =.
373         gen PointsInBinv51'binsetting' =.
374         gen PointsInBinv52'binsetting' =.
375     * generate PLUs
376     capture {
377         replace PLUv51'binsetting'var1 = PLUvKDvarsqresPrice
378         replace PLUv51'binsetting'var2 = PLUvKDvarsqresVolume
379         replace PLUv52'binsetting'var1 = PLUvKDvarabsresPrice
380         replace PLUv52'binsetting'var2 = PLUvKDvarabsresVolume
381         replace PointsInBinv51'binsetting' = Ksample sizesqresVolume
382         replace PointsInBinv52'binsetting' = Ksample sizeabsresVolume
383         drop Ksample size*
384     }
```

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