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# Supply function equilibria on the electricity market

Alexis Bergès

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ÉCOLE DES PONTS ET CHAUSSÉES

# SUPPLY FUNCTION EQUILIBRIA ON THE ELECTRICITY MARKET

**THÈSE**

pour l'obtention du titre de Docteur en Sciences Économiques

par

Alexis Bergès

19 juin 2018

**Jury**

| | |
|---|---|
| Philippe Choné, ENSAE Paris Tech | *Président* |
| Laurent Lamy, ENPC Paris Tech | *Suffragant* |
| David Martimort, PSE et EHESS | *Directeur de thèse* |
| Jean-Christophe Poudou, Université de Montpellier | *Rapporteur* |
| Jérôme Pouyet, Université de Cergy-Pontoise et ESSEC | *Rapporteur* |

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# Summary

## Chapter 1:

The first chapter focuses on what the introduction of ramping costs in a theoretical framework brings to the table. Ramping costs represent the fact that electricity suppliers incur costs when their production varies over time. Our main contribution is to build and justify how these ramping costs can be tackled theoretically. First, we note that going to a continuous time description of the problem allows us to bring to the literature about supply function equilibria powerful mathematical tools mostly used in option pricing, that is stochastic dynamics: we want to model ramping costs, i.e. costs associated to the variation in production, while retaining the key ingredient brought by [Klemperer and Meyer, 1989], i.e. the uncertainty, through the use of brownians, and more precisely, Itō processes. In so doing we face the issue that one cannot derive a brownian, and bring our second contribution, a physical argument about how power plants function that effectively operates as a low pass filter on our stochastic processes, and allow us to continue to build a tractable model of ramping costs under uncertainty. Third, we find in the literature a specification of Itō processes that allows the model to remain tractable.

From these technical contributions we obtain our economic contributions in having a rich tractable model that yields results that contrast strongly with past results from the literature. First, in the specific case of linear demand and linear costs we obtain a unique Nash equilibria, which contrasts with the usual continuum of Nash equilibria

in the supply function equilibria literature. Second, our solutions are not ex-post optimal, meaning that gathering information about the expected future evolution of demand yields different optimal strategies for suppliers, which in turn means that producers in our framework have a motive for submitting different supply functions from one time step to the next. Third, we have closed form solutions which yield specific predictions about the evolution of bids under uncertainty, namely that when uncertainty increase, suppliers submit steeper supply schedules in order to transmit more of these shocks to changes in price and not quantities, which are costly due to the existence of ramping costs. Finally, and less importantly, our framework justifies the existence of negative prices [1] by producers being willing to pay consumers to consume more in order to avoid facing large variations in production, in contrast to everywhere positive schedules in the case of the supply function equilibria literature. These results open the door to models being able to differentiate between day-ahead and intraday markets and therefore to offer a framework in which their interactions might be possible.

## Chapter 2:

In the second chapter our main focus is on analyzing our data, on building a way to describe it, and on building proxies for the uncertainty that producers face about the residual demand they have to anticipate when bidding on the day-ahead market.

First, we note that aggregate supply functions on the day ahead market cannot be well captured by parametric functions. Therefore, we devise a way to describe them non-parametrically: we note that although they cannot be captured parametrically, they still have a rough S shape, and therefore four main parts, two extremal sections, and two interior ones separated by the inflection point of the curve in its middle section. We define the transition points between these sections as the points of maximal absolute

[1] Note that such negative prices happen, a few hours a year for example in France or Germany, for example in 2017 there were 146 such hours, spread over 24 days in Germany [EPEX, 2018a]

value for the derivative and second derivative of the supply schedules. This definition relies on kernel density estimates and is therefore non-parametric. We observe that by using 5 such points, we are able to capture about 98% of the intrinsic variability of the supply schedules, and stop there although our method can be used to define more non-parametric points. This method allows us to define points that we consider comparable across auctions, that allow use to perform cross-sectional analysis of our data in the third chapter.

Second, we build proxies for the amount of weather uncertainty that producers face and variables that capture information that suppliers have before bidding and should therefore be controlled for. For the information available to suppliers, we note that the effect of weather on the demand, and more importantly temperature, is well understood and that we need to control for it. To do so we build an effective temperature for France, as an average of the localised temperature weighted by the population of the spatial region considered, in order to capture the overall effect temperature has on heating.[2] The rest of our focus is on building a proxy for the uncertainty concerning renewable production. To do so we analyze spatialized wind and sunlight data, and study it's spatial structure. We argue that spatial autocorrelation is a proxy for the uncertainty associated with weather forecasts, noting that if this data displays more spatial gradients, it is likely to be of a lesser quality due to the numerical nature of the weather simulations used to predict the weather, and therefore more uncertain.

Our contribution in the second chapter is to provide a non parametric way to define comparable points across auctions, and a measure of the uncertainty associated with weather forecasts.

[2]France has a high level of electric heating overall, which means that demand for electricity is quite sensitive to temperature.

## Chapter 3:

In this empirical chapter, we study the impact that uncertainty about the demand plays on the shape of the aggregate supply functions bidden by suppliers on the French electricity market. We segment our analysis to different parts of the supply functions in order to show how the overall shape changes with respect to our explanatory variables. We test some of the predictions from our first chapter, mainly that the supply function should see its slope increase when uncertainty increases.

We note that the main uncertainty is about the shape of the demand schedules itself. Therefore, we consider data available to the producers and regress the demand schedules on these variables. Next, we study the residuals of these regressions, and more specifically note that they are heteroskedastic. We leverage this, regressing the square of these residuals on our variables, in order to predict the expected amplitude of the residuals, that is the amplitude of the uncertainty of the demand schedule regression.

We then study the effect of our different proxies for uncertainty on the slope of the supply schedules, and note that if our proxies about the weather uncertainty (through the channel of renewable production) have the expected effect, the results are less clear cut for our residuals on the demand schedules. As we are working with full-blown schedules in the quantity-price plane, we perform our residual analysis both on the prices and the quantities. We therefore obtain estimates for the uncertainty pertaining to the position of a given point of our demand schedule either in price or in quantity. In our theoretical framework, we make the strong assumptions that demand schedules are linear, and that demand shocks are additive, i.e. they do not impact the slope of the demand schedules. These assumptions yield that we cannot differentiate between shocks in price or quantity, and that they should have effects in the same direction: more uncertainty implying steeper supply curves to reduce the amount of fluctuation in production. However, we observe that the effects of price and quantity uncertainty as estimated by our residuals' method yield opposite effects. Both of these assumptions, although required to obtain

closed form results, are clearly not satisfied by our data, and we think that this is a clear path for improvement of the model.

The contribution of the third chapter is to provide a way to estimate the uncertainty about the demand schedules faced by suppliers, and to estimate how this uncertainty affects the shape of the supply schedules at different points along its overall length, i.e. we provide a framework to describe how the functional form of schedules is affected by estimates of the uncertainty faced by suppliers.

# Table of Contents - short


**Remerciements—Acknowledgements** **i**

**Summary** **v**

**General Introduction** **1**

**Chapter 1: Dynamics of the Electricity Day-Ahead Market : Supply Function Equilibria and Ramping Costs** **33**

**Appendix** **77**

**Chapter 2: Methodological Tools for Non-Parametric Functional data Evaluation and Weather Data Usage** **87**

**Appendix** **121**

**Chapter 3: Investigating the Impact of Uncertainty on Firms with Dynamic Costs: A Case Study of the French Electricity Market** **129**

**Appendix** **183**

**General conclusion** **193**

**References** **203**

**Code Annex** **209**

**List of Figures** **269**

**Table of Contents - Long** **273**

# General Introduction

# General Introduction

## The electricity markets and their modelization over time

### Public utility pricing

The interest for modelling the electricity markets can be traced back to the reference work by Marcel Boiteux, vice president in charge of economic studies at Electricité de France, at the outset of the second world war. The question at the time was mainly that of public utility pricing: in the context of a public monopoly, which price should the consumers face in order to allow the producers to recover their costs.

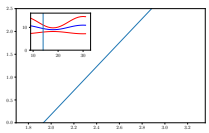
There are two main concerns that electricity producers have to face: the uncertainty of demand and the cyclicity of demand, for a commodity that essentially cannot be stored.[3]

The first question is addressed in [Boiteux, 1951]. In this paper, Boiteux considers a constant expected demand with fluctuations. The goal is to find the correct marginal pricing so that consumers internalize the additional cost that an uncertain demand entails for the producer. With a certain probability that demand is above its expected value by a given amount, how much more reserve capacity has to be kept in order to insure an accepted failure probability.[4]

The second question is addressed in [Boiteux, 1960]. Contrary to the previous situation, demand is now considered to change over time in a deterministic and cyclical fashion. The question is to price electricity in order for consumers to be sensitive to the additional investment cost implied by higher demand peaks.

[3]Electricity can be stored in hydroelectric dams, but the total energy stored is not enough to stabilize completely the demand faced by the other generation units, and only a fraction of the hydroelectric storage capacities can be actively replenished: the pumped storage facilities, which have two lakes and can therefore pump from the lower lake to the upper one on demand to store more electricity that that naturally stored in a lake that would be naturally replenished by a river.

[4]In the context of electricity, as production has to match demand at every point in time, every national grid is built with the notion of an acceptable probability of mismatch which translates in curtailments



These contributions have sparked a larger literature on the question of the pricing of economically non-storable commodities whose demand varies periodically, first in [Brown and Johnson, 1969] which studies the impact of stochastic demand on expected welfare. We refer the interested reader to the following review [Crew et al., 1995].

This literature has been mainly interested in questions of optimal pricing when the agent choosing the pricing tries to maximize the consumer's welfare, that is in the case of public monopolies.

### Regulatory evolution

The previous litterature took as an assumption the fact that these commodities were produced by public monopolies. Network utilities, such as gas, telecoms and electricity were thought to require to be organised as vertically integrated monopolies.

This view started to change in the 80s, with pressure to create competition. In 1984, access to gas pipelines was opened to competition in the USA and in 1990 Britain privatised electricity, separating generation and transmission. It was indeed thought that the natural monopoly emerged from the network, and that by separating generation from the network, generation could be opened to competition.

The overall argument for liberalization is that private competition is considered a safer road towards efficiency than regulation of a monopoly. In a situation of perfect competition, actors would be strongly incentivized for efficiency gains, and these gains would be transferred to consumers [Schmidt, 1996]. As perfect competition is a very rare situation, a new branch of the literature started to coalesce around the questions of modeling competition in the case of electricity markets [Newbery, 1997].

Although this liberalization movement is empirically considered to bring at least mod-

est medium-term efficiency [Fabrizio et al., 2007], it has been somewhat slowed down after the California crisis in the early 2000s [Jamasb and Pollitt, 2005], which mainly concentrated on wholesale electricity markets. Because of very little price responsiveness of demand as well as interactions with forward contracts, there was very high fluctuations in price as well as shortages [Borenstein, 2002]. In Europe, the European Commission has pushed with success for the continuation of the program of liberalization and integration, and wholesale markets for electricity are now ubiquitous, without further instances of failure as in California.

### The markets for electricity

The way the markets for electricity are organised stem from two main characteristics:

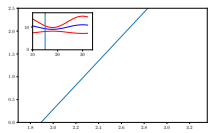
- The market has to reflect the changing demand for electricity.
- The form of the bids has to allow them to cope with the uncertain nature of demand at the time of bidding.

These ingredients have pushed for the creation of hourly or half-hourly markets, where suppliers are asked to submit supply schedules for a set number of bids (generally every 24 hours, that is 24 or 48 supply schedules once a day depending on whether the bids are hourly or half-hourly). These supply schedules take the form of a set of monotonous price quantity pairs, that can be considered as forming step functions[5] or linear functions by parts.[6]

In the 1980s, a theoretical push was made to model competition in supply functions. The first occurrences of this approach can be found in [Grossman, 1981] and [Hart, 1982]. They consider situations where producers compete in supply curves when facing a given demand curve. The main result is that such problems can be solved and one can obtain specifications for optimal strategies in supply functions, but that there exists a very large

[5] as in the case of the England and Wales pool in the 1990s

[6] Where price-quantity pairs are considered to be joined by lines instead of steps, which is the case for the French electricity day-ahead market, as well as the UK day-ahead market (half-hourly). Both of these markets are exchanged through EPEX Spot as of 2017.



multiplicity of equilibria in this setting.

Around the same time, [Klemperer and Meyer, 1986] introduce a setting in which firms choose endogenously to compete either in quantity or prices. This too yields a large multiplicity of outcomes, but the key insight comes from the fact that this multiplicity is drastically reduced when uncertainty is introduced.

This insight brings along the seminal paper [Klemperer and Meyer, 1989] which studies supply function competition under uncertainty. In this paper, it is shown that although there is still a continuum of equilibria, this continuum has a structure that can be studied when suppliers face an uncertain demand. In the rest of this thesis, we denote supply function equilibria as SFE.

The setting introduced by Klemperer and Meyer is then rapidly put to use in the context of electricity markets, where [Green and Newbery, 1992] studies the competition in the British spot market through the SFE framework.

This use of SFE sparks some debate as to whether a smooth function approximation can or not capture the correct effects in markets which are largely at the time asking bidders to submit step functions: [von der Fehr and Harbord, 1993] argue that step functions of finite length are different to continuous functions.[7] In addition, there is empirical evidence that strategies predicted by SFE and actual observed strategies are significantly different, see [Willems et al., 2009] and [Willems et al., 2009]. These results question whether the SFE is the correct approach that only needs to be perfected, for example by using functions that are affine by parts and not only affine [Baldick et al., 2004], or a framework that is not adapted to describing these markets.

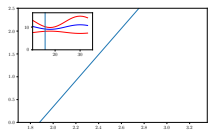
However, this approach is still considered relevant by a number of authors, although

[7] This debate is largely obsolete now that most of the market rules imply bids that are linear by parts and not step functions anymore.

the multiplicity of equilibria makes it difficult to obtain clear results. In addition, the solutions are not exactly easily usable, these functions being defined as solutions to a differential equation, therefore without an analytical formula. To overcome this issue, a number of authors either consider competition in simpler settings, for example Cournot competition settings applied to the electricity market in the case of [Borenstein and Bushnell, 1999], or choose to restrict themselves to one special solution out of the continuum of possible solutions that come out of the SFE framework: the supply function that is the unique linear solution out of this continuum. In so doing these authors pick arbitrarily one solution with a functional form and then use it to further analyze some economic questions. For example, [Green, 1996] focuses on the linear supply solution out of the SFE multiple equilibria in order to have analytical tractable forms and study the effect of three different policies on competition, where [Hobbs et al., 2000] is able to model transmission constraints with an affine supply function.

We also want to note that day-ahead markets do not exist in a vacuum, and in fact electricity can be traded through forward contracts, on the day-ahead market, as well as on the intraday markets. Capacity markets on which guaranteed online capacity is traded for also exist. All these markets interact with one another, and part of the literature focuses on modelling these interactions. Generally the SFE considered are simplified to be able to perform such analysis, for example to linear functions [Green, 1999] to study the interaction with forward markets, or to linear asymmetric function [Anderson and Hu, 2012] for the same purpose. Generally speaking, these papers focus on the interaction between day-ahead markets and forward contracts because the SFE framework does not allow differentiating between day-ahead and intraday markets.

Overall, we refer the interested reader to the review by [Ventosa et al., 2005] for a more detailed overview. In this thesis we rely heavily on the work by Klemperer and Meyer, and comment and contrast their results to ours. In order to make this easier to follow, we summarize in the following section the results of their paper that will be used



in this thesis.

## Klemperer and Meyer 1989

Consider a setting in which firms bid supply functions while facing an uncertain demand.

Let $D(p, \theta)$ be the demand function as a function of demand shock $\theta$. Consider that for all $(p, \theta)$, $-\infty < D_p < 0$, $D_{pp} \leq 0$ and $D_\theta > 0$.

All firms are considered to be facing the same cost function $C(\cdot)$, with $C'(q) > 0$ and $0 < C''(q) < \infty$ for all $q > 0$.

The timing is such that suppliers have to bid simultaneously a supply function prior to the realization of demand shock $\theta$ being known. Consider for now two firms $i$ and $j$ with $S^k(p)$ the supply function of supplier $k$ and that these supply functions are twice differentiable. After this shock is known, every firm produces quantity $S^k(p^*(\theta))$ at price $p^*(\theta)$, such that $D(p^*(\theta)) = S^i(p^*(\theta)) + S^j(p^*(\theta))$.

Firm $i$'s residual demand is given by the total demand from which the supply of firm $j$ is subtracted, $D(p, \theta) - S^j(p)$. As $\theta$ is considered a scalar, the set of profit-maximizing points for every possible shock $\theta$ define a curve. If there is a unique intersection between $i$'s supply curve and every possible demand curve, then such a supply curve is ex-post optimal, meaning that it is pointwise optimal for every realization of the shock $\theta$.

Given the assumption that supply curves indeed behave in this way, then maximizing the expected profit for the distribution of shocks can be abstracted away from the distribution of shocks, and $i$'s optimal supply curve solves for every shock $\theta$ the following program:

$$\max_p p\left(D(p, \theta) - S^j(p)\right) - C\left(D(p, \theta) - S^j(p)\right) \tag{1}$$

which F.O.C writes:

$$D(p,\theta) - S^j(p) + \big(p - C'\left(D(p,\theta) - S^j(p)\right)\big)\left(D_p(p,\theta) - S^{j\prime}(p)\right) = 0 \tag{2}$$

with eq 1 being strictly concave in $p$ (we refer the reader to the original paper for more justifications), then eq 2 defines the unique profit maximizing $p^*(\theta)$ for every $\theta$, which parametrizes the optimal supply function.

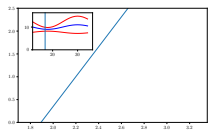
Consider that $D_{\theta p} = 0$, that is that $\theta$ is an additive shock, and that we focus on symmetric equilibria, which allows us to drop the firms' superscripts. In addition, consider the fact that eq 2 has to hold for every shock, it can therefore be rewritten as:

$$S'(p) = \frac{S(p)}{p - C'(S(p))} + D_p(p) = f(p,S) \tag{3}$$

This differential equation defines the supply function equilibria, the role of uncertainty being to ensure that this equation has to hold for every shock, therefore for every possible price. However, we can see that this differential equation is not accompanied by an initial condition. Therefore, there exists many admissible solutions to this equation.

Supply functions are therefore bounded by possible values of their slope, namely that the functions have slopes bounded between 0 and $+\infty$. By solving the differential equation, one can define the locus of points for which the solutions have slopes equal to these bounds and thus obtain a region of admissible solutions, in the context of our problem:

Therefore, the admissible set of solutions is defined by the upper bound of the demand shocks $\overline{\theta}$, in that if the solutions cross the slope boundaries before reaching the maximal shock, they cannot be accepted as solutions to the problem which constrains the solutions more strictly than the differential equation alone. In figure 1 the demand associated with the upper bound of the shocks $D(p,\overline{\theta})$ is represented in orange, and solutions A and B to the differential equation are not solutions to the problem as they reach the boundaries



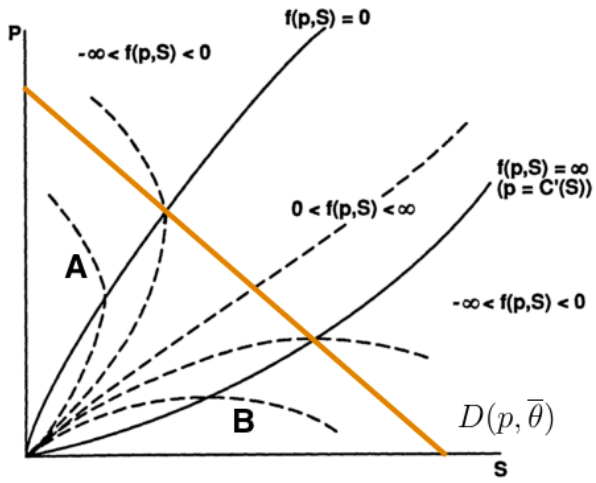



Figure 1: This graph is adapted from the original paper by Klemperer and Meyer and illustrates the admissible region of solutions to the differential equation so as to verify the constraints on the slope of the supply function.

for smaller values of shocks.

The last result we will review here, is that in the case of an unbounded support of shocks, the set of equilibria is at its smallest, as it means that solutions have to have a positive finite slope for every value of the shocks, and not only for a segment of the real line.

In some cases, for example for linear demand schedules, this set can collapse to a unique solution.

## The case for ramping costs

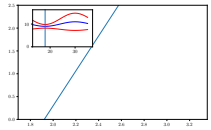
This framework models the costs as depending only on the quantity produced. In the context of electricity generation, an important type of costs that cannot be captured in such a specification of the cost function is that of ramping costs. These ramping costs refer to the fact that making production change over time induces specific costs.

To explain how such costs can arise, consider a thermal power plant (fossil fuels, nu-

clear, etc.), and more precisely its core. Physically, to produce a given level of electricity, one has to maintain the core at a given temperature. To increase production, the temperature of the core has to increase. This means that when production is increased some fuel has to be lost to simply increase the temperature, this energy expenditure is not attached to any additional production.

This issue of ramping costs is at the heart of the choice of "quick" gas power plants to match sudden peaks in demand, where nuclear plants are more generally used for low frequency adjustments. Therefore these ramping costs are important technically on the electricity market. They are important enough for the project of European Power Exchanges named "Price Coupling of Regions" (PCR), which aims to develop a single price coupling solution to be used to calculate electricity prices across Europe, to consider the possibility to use load gradient orders, that is orders that condition their availability on the change in production from one hour to the next. However, at the moment of writing, PCR is still very much a work in progress [EPEX, 2018c].

Some papers have tried to estimate their values empirically, [Wolak, 2007] and more recently [Reguant, 2011]. There is also a strand of literature concerned with ramping costs, looking at the optimal price that allows to maximize the overall social welfare [Tanaka, 2006], that is, which price schedule allows to maximize the consumer welfare from which the production costs are subtracted. This literature does not use game-theoretical frameworks, but concerns itself with the best price signal to use in order to limit the ramping costs incurred due to varying demand, while still considering that the trajectory of demand is known. To our knowledge, there is no game-theoretical framework that has been brought to take ramping costs into account, and describe their effects on optimal strategies for the agents bidding on the market.



## Contribution

This thesis focuses on the question of these ramping costs. In the first chapter, I tackle this question in a theoretical framework which yields predictions on the change of shape in supply functions over time as a function of the underlying uncertainty about demand shocks. The second chapter then introduces methods to study the shape of supply functions as observed on the French electricity market for data from 2011 to 2013, as well as methods to estimate the uncertainty contributed by the weather. The third chapter applies these methods to test these theoretical predictions on actual market data. The second and third chapters have been co-written with Henri de Belsunce, who finished his PhD in 2015 at the Munich-based Max Planck Institute for Innovation and Competition, under the supervision of Prof. Dr. Klaus M. Schmidt [Belsunce, 2015].

The first chapter focuses on what the introduction of ramping costs in a theoretical framework brings to the table. Our main contribution is to build and justify how these ramping costs can be tackled theoretically. First, we note that going to a continuous time description of the problem allows us to bring to the literature about supply function equilibria powerful mathematical tools mostly used in option pricing, that is stochastic dynamics: we want to model ramping costs, i.e. costs associated to the variation in production, while retaining the key ingredient brought by [Klemperer and Meyer, 1989], i.e. the uncertainty, through the use of brownians, and more precisely, Itō processes. In so doing we face the issue that one cannot derive a brownian, and bring our second contribution, a physical argument about how power plants function that effectively operates as a low pass filter on our stochastic processes, and allow us to continue to build a tractable model of ramping costs under uncertainty. Third, we find in the literature a specification of Itō processes that allows the model to remain tractable.

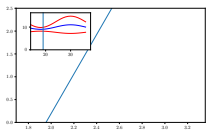
From these technical contributions we obtain our economic contributions in having a rich tractable model that yields results that contrast strongly with past results from the literature. First, our solutions are unique, which contrasts with the usual continuum

of Nash equilibria in the supply function equilibria literature. Second, our solutions are not ex-post optimal, meaning that gathering information about the expected future evolution of demand yields different optimal strategies for suppliers, which in turn means that producers in our framework have a motive for submitting different supply functions from one time step to the next. Third, we have closed form solutions which yield specific predictions about the evolution of bids under uncertainty, namely that when uncertainty increase, suppliers submit steeper supply schedules in order to transmit more of these shocks to changes in price and not quantities, which are costly due to the existence of ramping costs. Finally, and less importantly, our framework justifies the existence of negative prices [8] by producers being willing to pay consumers to consume more in order to avoid facing large variations in production, in contrast to everywhere positive schedules in the case of the supply function equilibria litterature. These results open the door to models being able to differentiate between day-ahead and intraday markets and therefore to offer a framework in which their interactions might be possible.

At the bottom of all right pages, there is a small graph showcasing the optimal strategy for a given set of parameters of the model, in the plane quantity-price, with an insert illustrating the evolution of the support of demand shocks over time. The vertical line in the insert represents the point in time for which the strategy is presented in the actual graph. This forms a flipbook, which allows the reader to get a feeling for the evolution of strategies with the evolution of the support of shocks by flipping rapidly the pages and observing the graph becomes animated, just like a cartoon. This serves only an illustrative purpose.

In the rest of the thesis, the goal is to test our predictions on data from the French day-ahead market. In so doing, as our theoretical predictions are mainly about the effect that the amount of uncertainty has on the slope of the optimal supply schedule, we separate the issue of building proxies for this uncertainty in our second chapter and the

[8] Note that such negative prices happen, a few hours a year for example in France or Germany, for example in 2017 there were 146 such hours, spread over 24 days in Germany [EPEX, 2018a]



actual analysis of the evolution of bids on these proxies in the third chapter.

In the second chapter our main focus is on analyzing our data, on building a way to describe it, and on building proxies for the uncertainty that producers face about the residual demand they have to anticipate when bidding on the day-ahead market.

First, we note that aggregate supply functions on the day ahead market cannot be well captured by parametric functions. Therefore, we devise a way to describe them non-parametrically: we note that although they cannot be captured parametrically, they still have a rough S shape, and therefore four main parts, two extremal sections, and two interior ones separated by the inflection point of the curve in its middle section. We define the transition points between these sections as the points of maximal absolute value for the derivative and second derivative of the supply schedules. This definition relies on kernel density estimates and is therefore non-parametric. We observe that by using 5 such points, we are able to capture about 98% of the intrinsic variability of the supply schedules, and stop there although our method can be used to define more points. This method allows us to define points that we consider comparable across auctions, that allow use to perform cross-sectional analysis of our data in the third chapter.

Second, we build proxies for the amount of weather uncertainty that producers face and variables that capture information that suppliers have before bidding and should therefore be controlled for. For the information available to suppliers, we note that the effect of weather on the demand, and more importantly temperature, is well understood and that we need to control for it. To do so we build an effective temperature for France, as an average of the localised temperature weighted by the population of the spatial region considered, in order to capture the overall effect temperature has on heating.[9] The rest of our focus is on building a proxy for the uncertainty concerning renewable production. To do so we analyze spatialized wind and sunlight data, and study it's spatial structure. We

[9]France has a high level of electric heating overall, which means that demand for electricity is quite sensitive to temperature.

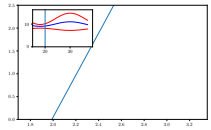
argue that spatial autocorrelation is a proxy for the uncertainty associated with weather forecasts, noting that if this data displays more spatial gradients, it is likely to be of a lesser quality due to the numerical nature of the weather simulations used to predict the weather, and therefore more uncertain.

Our contribution in the second chapter is to provide a non-parametric way to define comparable points across auctions, and a measure of the uncertainty associated with weather forecasts.

In the third chapter, we focus on building the main proxy for the uncertainty faced by producers, and then on analyzing how the bids evolve relative to these proxies.

We note that the main uncertainty is about the shape of the demand schedules itself. Therefore, we consider data available to the producers and regress the demand schedules on these variables. Next, we study the residuals of these regressions, and more specifically note that they are heteroskedastic. We leverage this, regressing the square of these residuals on our variables, in order to predict the expected amplitude of the residuals, that is the amplitude of the uncertainty of the demand schedule regression.

We then study the effect of our different proxies for uncertainty on the slope of the supply schedules, and note that if our proxies about the weather uncertainty (through the channel of renewable production) have the expected effect, the results are less clear cut for our residuals on the demand schedules. As we are working with full-blown schedules in the quantity-price plane, we perform our residual analysis both on the prices and the quantities. We therefore obtain estimates for the uncertainty pertaining to the position of a given point of our demand schedule either in price or in quantity. In our theoretical framework, we make the strong assumptions that demand schedules are linear, and that demand shocks are additive, i.e. they do not impact the slope of the demand schedules. These assumptions yield that we cannot differentiate between shocks in price or quan-



tity, and that they should have effects in the same direction: more uncertainty implying steeper supply curves to reduce the amount of fluctuation in production. However, we observe that the effects of price and quantity uncertainty as estimated by our residuals' method yield opposite effects. Both of these assumptions, although required to obtain closed form results, are clearly not satisfied by our data, and we think that this is a clear path for improvement of the model.

The contribution of the third chapter is to provide a way to estimate the uncertainty about the demand schedules faced by suppliers, and to estimate how this uncertainty affects the shape of the supply schedules at different points along its overall length, i.e. we provide a framework to describe how the functional form of schedules is affected by estimates of the uncertainty faced by suppliers.

# Introduction Générale

## Évolution de la modélisation des marchés de l'électricité

### Tarification des services publics

L'intérêt pour la modélisation des marchés de l'électricité remonte aux travaux de référence de Marcel Boiteux, vice-président en charge des études économiques d'EDF au sortir de la Seconde Guerre Mondiale. La question principale à l'époque est celle de la tarification d'un service public : dans le contexte d'un monopole d'état, à quel prix les consommateurs devraient-ils faire face afin que les producteurs recouvrent leurs coûts.

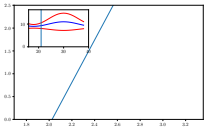
Les producteurs d'électricité font face à deux contraintes particulières : le caractère incertain de la demande ainsi que sa périodicité, le tout pour un bien qui ne peut essentiellement pas être stocké.[10]

La première contrainte est traitée dans [Boiteux, 1951]. Dans cet article, Boiteux considère une demande constante en moyenne, mais sujette à des fluctuations. L'objectif est de trouver la tarification marginale permettant au consommateur d'internaliser le coût supplémentaire qu'une demande incertaine fait peser sur le producteur. Étant donné une certaine probabilité que la demande soit au-dessus de sa valeur espérée d'une certaine quantité, il s'agit de trouver quelle capacité de reserve doit être maintenue en ligne pour garantir une probabilité cible de défaillance. [11]

La seconde contrainte est traitée dans [Boiteux, 1960]. Contrairement à la situation précédente, la demande est ici considérée comme périodique et déterministe. L'objectif

[10] Il est possible de stocker de l'électricité grâce à des barrages, mais l'énergie totale ainsi stockable n'est pas suffisante pour complètement stabiliser la demande à laquelle les moyens de production font face. Par ailleurs, seule une fraction de l'énergie ainsi stockée est renouvelable volontairement : les stations de transfert d'énergie par pompage disposent de deux lacs ce qui permet de pomper de l'eau d'un lac situé en aval vers un lac en amont et ainsi de reconstruire les réserves plus rapidement qu'en attendant que les affluents naturels du lac amont ne le remplissent.

[11] Dans le contexte de l'électricité, comme la production doit être égale à la demande à chaque instant, chaque réseau national est dimensionné avec un niveau de probabilité de défaillance acceptable.



est de trouver la tarification permettant de transmettre aux consommateurs le coût d'investissement supplémentaire associé à une demande présentant des pics plus élevés.

Ces contributions nourrissent une large littérature sur la question de la tarification de biens non-stockables faisant face à une demande cyclique, tout d'abord par [Brown and Johnson, 1969] qui étudie l'impact d'une demande stochastique sur le bien-être social espéré. Nous renvoyons la lectrice intéressée à la revue de littérature suivante : [Crew et al., 1995].

Cette branche de la littérature se concentre principalement sur la question de la tarification optimale lorsque l'agent fixant le prix a pour objectif de maximiser le bien-être des consommateurs, dans le cas de monopoles d'état.

### Évolutions de la régulation

Cette branche de la littérature prend pour hypothèse que ces biens sont produits par des monopoles d'état. Il est alors admis que les services de réseau, comme le gaz, l'électricité ou les télécoms doivent être organisés sous la forme de monopoles verticalement intégrés.

Cette position évolue dans les années quatre-vingt, avec l'ouverture à la compétition de ces monopoles d'état. En 1984, l'accès aux gazoducs est ouvert à la compétition aux États-Unis et en 1990, la Grande-Bretagne privatise la fourniture d'électricité, en séparant production et transmission. Il est alors considéré que la condition de monopole naturel est liée au réseau, et qu'en séparant production et transmission, la production peut s'ouvrir à la compétition.

L'argument général en faveur de la libéralisation est que la compétition privée est considérée comme une voie plus sûre vers l'efficacité économique que la régulation d'un monopole. Dans une situation de compétition parfaite, les agents seraient ainsi fortement

incités à rechercher des gains d'efficacité, et ces gains seraient transmis aux consommateurs [Schmidt, 1996]. La compétition parfaite étant une situation rare, une nouvelle branche de la littérature émerge autour de la question de la modélisation de la compétition dans le marché de l'électricité [Newbery, 1997].

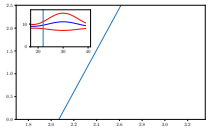
Bien que ce mouvement de libéralisation soit considéré comme permettant à minima des gains d'efficacité modestes à moyen terme [Fabrizio et al., 2007], il ralentit après la crise californienne du début des années 2000 [Jamasb and Pollitt, 2005], qui se concentre principalement sur les marchés de gros de l'électricité. De très grandes fluctuations de prix ainsi que des pénuries ont lieu, principalement induites par une très faible elasticité-prix de la demande, ainsi que l'interaction entre le marché de gros et les contrats à terme [Borenstein, 2002]. En Europe, la Commission européenne pousse avec succès pour la poursuite du programme de libéralisation et d'intégration européenne, et les marchés de gros de l'électricité sont maintenant répandus sur le continent, sans que ne se produise de défaillances semblables à celles observées en Californie.

### Les marchés de l'électricité

L'organisation des marchés de l'électricité est très fortement induite par deux caractéristiques importantes :

- Le marché se doit de refléter les changements rapides de demande.
- La forme des enchères doit leur permettre de se satisfaire de la nature incertaine de la demande au moment de l'enchère.

Ces ingrédients sont à la source de la construction de marchés horaires voire même demi-horaires, au sein desquels les producteurs doivent soumettre des courbes d'offre pour un nombre déterminé d'enchères (en général une fois par jour, soit 24 ou 48 courbes d'offre à la fois selon que l'enchère est horaire ou demi-horaire). Ces courbes d'offres prennent la forme d'un ensemble de paires de prix et de quantités monotone, qui peuvent



définir des fonctions constantes par morceaux [12] ou des fonctions linéaires par morceaux. [13]

Dans les années quatre-vingt, une impulsion théorique cherche à modéliser ces marchés sous forme d'équilibres en courbes d'offres. Cette approche est introduite en premier lieu par [Grossman, 1981] and [Hart, 1982]. Ils considèrent une situation où des producteurs rivalisent via des courbes d'offres en faisant face à une courbe de demande connue et donnée. Le principal résultat de cette approche est que ces modèles peuvent être résolus, que les stratégies optimales en courbe d'offre peuvent être spécifiées, mais qu'il existe une forte multiplicité d'équilibre dans ce contexte.

Peu de temps après, [Klemperer and Meyer, 1986] introduisent un modèle dans lequel les producteurs choisissent d'entrer en concurrence en prix ou en quantité de façon endogène. Cette approche donne également lieu à une grande multiplicité d'équilibre, mais le résultat clef est que cette multiplicité est drastiquement réduite lorsque de l'incertitude est introduite dans le modèle.

Ce résultat inspire le papier fondateur [Klemperer and Meyer, 1989] qui étudie une situation de compétition en courbes d'offres face à une demande incertaine. Dans cet article, bien qu'il y ait toujours un continuum d'équilibres, ce continuum possède une structure qui peut être étudiée lorsque la demande est incertaine. Dans le reste de cette thèse, nous appelons les équilibres en courbes d'offre des SFE (supply function equilibria).

Le modèle général proposé par Klemperer et Meyer est rapidement mis à profit pour décrire les marchés de l'électricité, la compétition sur le marché spot anglais est ainsi étudiée avec des SFE par [Green and Newbery, 1992].

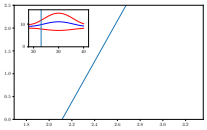
[12] Comme dans le cas du marché de gros d'Angleterre et du Pays de Galles dans les années quatre-vingt-dix

[13] Les paires prix quantité sont considérées comme étant reliées par des linéaires, comme dans le cas du marché day-ahead français, mais aussi le marché anglais actuel (demi-horaire). Ces deux marchés spot sont gérés par la bourse EPEX Spot.

Cet usage des SFE déclenche des débats sur la validité qu'il y a à décrire le marché de l'électricité, à l'époque encore principalement caractérisé par des enchères constantes par morceaux, avec des fonctions continues et dérivables : [von der Fehr and Harbord, 1993] présente un argument montrant qu'une compétition via des constantes par morceaux de tailles finies exhibe des comportements fondamentalement différents du cas de fonctions continues. [14] Des résultats empiriques montrent également que les stratégies prédites par cette classe de modèles diffèrent de façon significative des stratégies effectivement observées sur les marchés, voir [Willems et al., 2009] et [Willems et al., 2009]. Ces résultats interrogent sur la validité de l'approche SFE pour décrire les marchés de l'électricité, plus précisément la question est de savoir si les modèles de SFE sont valides, mais attendent d'être perfectionnés, par exemple en les modélisant explicitement sous la forme de fonctions affines par morceaux [Baldick et al., 2004], ou si cette approche est fondamentalement incapable de capturer les stratégies observées.

Cette approche est toutefois encore considérée comme pertinente par nombre d'auteurs, bien que la multiplicité des équilibres complique l'analyse des résultats théoriques. Par ailleurs, les solutions ne sont pas directement exploitables, étant définies implicitement comme solution d'équations différentielles, et donc sans forme analytique dans la majorité des cas. Pour dépasser ces limites, la littérature cherche à décrire la compétition dans des cadres plus simples, par exemple dans des modèles de compétition de Cournot appliquée au cas des marchés de l'électricité dans le cas de [Borenstein and Bushnell, 1999], ou choisissent de se restreindre à une solution particulière parmi le continuum de solutions obtenu dans un contexte de SFE: l'unique solution linéaire du lot. Ces auteurs choisissent donc arbitrairement une solution tractable et s'en servent pour pousser le raisonnement économique plus loin qu'habituellement possible en conservant le continuum de solutions. À titre d'exemple, [Green, 1996] se focalise sur la courbe d'offre linéaire parmi le continuum obtenu dans le cadre des SFE et se sert de son expression analytique pour étudier

[14] Ce débat est principalement obsolète maintenant que la plupart des marchés sont passés à des linéaires par morceaux et plus des constantes par morceaux.



les effets de trois politiques d'encadrement de la compétition, quand [Hobbs et al., 2000] fait de même pour étudier les contraintes de transmission.

Par ailleurs, il est important de noter que les marchés day-ahead n'existent pas hors-sol, et que l'électricité peut être échangée sur ces marchés, mais aussi par des contrats à terme ou encore sur les marchés intraday. Il existe aussi des marchés de capacité ou une garantie de capacité disponible à une certaine date s'échange. Tous ces marchés interagissent les uns avec les autres, et une partie de la littérature s'attache à décrire ces interactions. Les SFE utilisés à cette fin sont généralement simplifiés, par exemple en ne considérant que les équilibres linéaires pour étudier l'interaction entre marché day-ahead et contrats à terme [Green, 1999], ou encore en considérant des équilibres asymétriques linéaires dans le même but [Anderson and Hu, 2012]. Ces articles se concentrent généralement sur les interactions entre contrats à terme et marché day-ahead, car les SFE ne sont pas en mesure de distinguer le marché day-ahead du marché intraday.

Nous renvoyons la lectrice intéressée vers la revue de littérature [Ventosa et al., 2005] pour une vue d'ensemble plus détaillés. Dans cette thèse, nous nous appuyons fortement sur le travail de Klemperer et Meyer, et contrastons souvent nos résultats aux leurs. Pour faciliter la lecture au lecteur qui ne serait pas familier avec leurs travaux, nous résumons ci-après leurs résultats sur lesquels nous nous appuyons.

### Klemperer et Meyer 1989

Soit un marché sur lequel des producteurs enchérissent des courbes d'offre tout en faisant face à une demande incertaine.

Soit $D(p, \theta)$ la courbe de demande comme fonction du prix $p$ et du choc $\theta$. Considérons que pour tout $(p, \theta)$, $-\infty < D_p < 0$, $D_{pp} \leq 0$ et $D_\theta > 0$.

Les producteurs font face à la même fonction de coût $C(\cdot)$, avec $C'(q) > 0$ et

$0 < C''(q) < \infty$ pour tout $q > 0$.

Les producteurs soumettent leurs offres en même temps avant la réalisation du choc de demande $\theta$. Considérons pour l'instant deux producteurs $i$ et $j$ avec $S^k(p)$ la courbe d'offre du producteur $k$ différentiable deux fois. Une fois le choc connu, chaque producteur produit la quantité $S^k(p^*(\theta))$ au prix $p^*(\theta)$, tel que $D(p^*(\theta)) = S^i(p^*(\theta)) + S^j(p^*(\theta))$.

La demande résiduelle du producteur $i$ est donnée par la demande totale à laquelle la production du producteur $j$ est soustraite, $D(p, \theta) - S^j(p)$. Comme $\theta$ est un scalaire, l'ensemble de points maximisant le profit pour chaque choc $\theta$ possible définit une courbe. Si il existe une intersection unique entre la courbe d'offre de $i$ et toute courbe de demande possible, alors cette courbe d'offre est ex-post optimale, c'est à dire qu'elle est optimale point à point pour chaque réalisation possible de $\theta$.

Sous l'hypothèse que les courbes d'offres se comportent effectivement de cette manière, la maximisation du profit espéré devient indépendante de la distribution des chocs de demande, et la courbe d'offre optimale de $i$ résout pour chaque choc $\theta$ le programme de maximisation suivant :

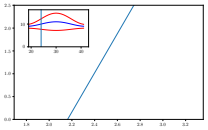
$$\max_p p\left(D(p,\theta) - S^j(p)\right) - C\left(D(p,\theta) - S^j(p)\right) \tag{4}$$

dont la condition du premier ordre s'écrit :

$$D(p,\theta) - S^j(p) + \left(p - C'\left(D(p,\theta) - S^j(p)\right)\right)\left(D_p(p,\theta) - S^{j\prime}(p)\right) = 0 \tag{5}$$

Comme l'équation 4 est strictement concave en $p$ (nous renvoyons le lecteur vers le papier original pour plus de détails), l'équation 5 définit un unique prix $p^*(\theta)$ maximisant le profit pour chaque choc $\theta$, qui paramétrise la courbe d'offre optimale.

Considérons que $D_{\theta p} = 0$, c'est à dire que $\theta$ est un choc additif, et concentrons nous



sur les équilibres symétriques, ce qui nous permet de ne plus faire attention aux exposants $i$ et $j$ caractérisant la productrice. Sachant que l'eq. 5 doit être vérifiée pour tout choc, elle peut se réécrire :

$$S'(p) = \frac{S(p)}{p - C'(S(p))} + D_p(p) = f(p, S) \tag{6}$$

Cette équation différentielle définit l'équilibre en courbes d'offres, l'incertitude ayant pour conséquence que cette équation doit être vérifiée pour tout choc, et donc pour tout prix possible. Cette équation ne s'accompagne toutefois pas d'une condition initiale. Il existe donc une multiplicité de solutions admissibles.

Les courbes d'offres sont bornées par les possibles valeurs de leur pente, qui doit être comprise entre 0 et $+\infty$. Il est possible de définir le lieu des points pour lesquels les solutions de l'équation différentielle ont pour pente ces valeurs extrémales, qui définit donc la région des solutions admissibles dans notre contexte :

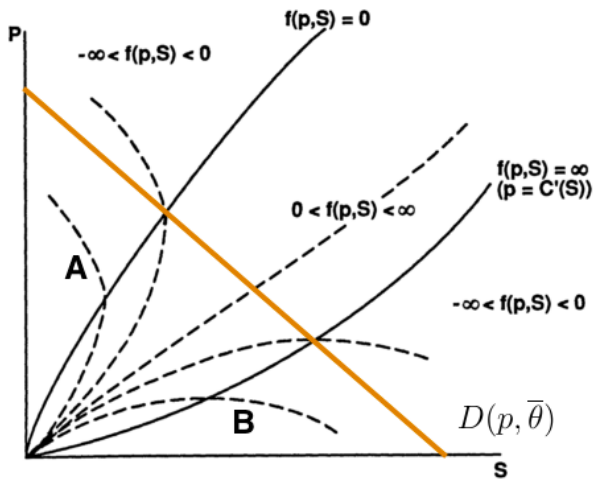



Figure 2: Ce graphique est adapté du papier original de Klemperer et Meyer et illustre la région de solutions de l'équation différentielle admissibles dans le cadre de notre problème, c'est-à-dire vérifiant les contraintes sur leur pente.

L'ensemble des solutions admissibles est donc définit par la borne supérieure de nos chocs de demande $\overline{\theta}$, en cela que si une solution de l'équation différentielle devait traverser les frontières définies ci-dessus pour un choc inférieur au choc maximal, elle ne serait pas pour autant solution de notre problème qui contraint plus fortement les solutions que

l'équation différentielle seule. Dans la figure 2, la demande associée avec le choc maximal $D(p, \overline{\theta})$ est représenté en orange et les solutions A et B de l'équation différentielle ne sont pas solutions de notre problème, car elles franchissent nos frontières pour des valeurs de chocs inférieures à ce maximum.

Le dernier résultat que nous évoquerons ici est que dans le cas d'un support de choc infini, l'ensemble d'équilibres symétriques est le plus petit possible, car il faut dans ce cas que toute solution ait une pente positive et finie pour tout choc positif et plus seulement pour un segment de la droite des réels.

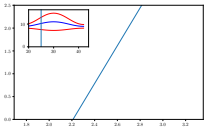
Dans certains cas, par exemple pour des courbes de demande linéaires, cet ensemble peut converger vers une solution unique.

## Les coûts de variation

Ce modèle choisit de considérer des coûts ne dépendant que de la quantité produite. Dans le contexte de la production d'électricité, il existe un type de coûts qui ne peut pas se modéliser ainsi, les coûts de variation. Ces coûts sont induits lorsque la production varie dans le temps.

Afin d'expliquer ces coûts, considérons une centrale thermique (fossile, nucléaire, etc.) et plus précisément son réacteur. Physiquement, pour produire une certaine puissance, il faut maintenir le réacteur à une température donnée. Afin d'accroître la production, la température du réacteur doit augmenter. Cela implique que lorsque la production augmente, du combustible doit être perdu afin de simplement réchauffer le réacteur, cette dépense énergétique n'étant pas associée à une production d'énergie.

L'existence de ces coûts est au cœur du choix d'utiliser des centrales gaz "rapides" afin de suivre une hausse soudaine de la demande, là où les centrales nucléaires sont plus



généralement utilisées afin de suivre les changements basse fréquence de la demande. Ces coûts sont suffisamment importants pour que le projet des bourses européennes d'électricité EPEX intitulé "Price Coupling of Regions" (PCR), qui cherche à développer un unique mécanisme de couplage des prix européens de l'électricité, considère la possibilité de soumettre des ordres conditionnés sur les gradients de charge, c'est-à-dire des ordres dont la disponibilité serait conditionnée sur le respect de valeurs maximales de variation de la production d'une enchère à l'autre. Le projet PCR n'est toutefois qu'à l'était d'ébauche au moment de l'écriture de ces lignes [EPEX, 2018c].

Des articles se sont attachés à estimer la valeur de ces coûts de variation empiriquement, [Wolak, 2007] et plus récemment [Reguant, 2011]. Il existe également une branche de la littérature s'intéressant à ces coûts à travers le prisme de leur impact sur le prix optimal maximisant le bien-être social [Tanaka, 2006], c'est-à-dire s'intéressant à la chronique temporelle de prix permettant de maximiser l'utilité des consommateurs à laquelle sont soustraits les coûts de variation. Ces travaux ne s'inscrivent pas dans une approche de théorie des jeux, mais cherchent à trouver le signal prix permettant de limiter les coûts de variation induits par une demande variant de façon déterministe. À notre connaissance il n'existe pas de modèle de ces coûts dans un contexte de théorie des jeux permettant de décrire leur impact sur les stratégies optimales des producteurs jouant sur le marché de l'électricité.

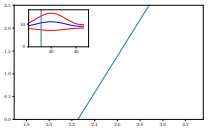
## Contribution

Cette thèse se concentre sur la question des coûts de variation. Dans le premier chapitre, cette question est abordée dans un modèle théorique produisant des prédictions sur l'évolution de la forme des courbes d'offres optimales dans le temps en fonction de la dynamique sous-jacente des chocs de demande. Le deuxième chapitre introduit ensuite des techniques permettant d'étudier empiriquement la forme des courbes d'offre observées sur le marché de l'électricité français entre 2011 et 2013, ainsi que des méthodes permettant d'estimer l'incertitude sur la demande associée à la météo. Le troisième chapitre

applique ces méthodes afin de tester empiriquement les prédictions théoriques du premier chapitre. Ces deux derniers chapitres sont issus d'une collaboration avec Henri de Belsunce, qui a soutenu son doctorat en 2015 au Max Planck Institute for Innovation and Competition de Munich, sous la supervision de Prof. Klaus M. Schmidt.

Le premier chapitre se concentre sur ce que la prise en compte des coûts de variation apporte dans un contexte théorique. La contribution principale est de construire et de justifier comment tenir compte théoriquement de ces coûts. En premier lieu, nous passons à une description du problème en temps continu afin d'apporter à la littérature sur les SFE des outils mathématiques puissants surtout utilisés en finance, à savoir la dynamique stochastique : nous cherchons en effet à modéliser les coûts de variation, en conservant l'ingrédient clef introduit par [Klemperer and Meyer, 1989], i.e. l'incertitude, grâce à l'utilisation de browniens, plus précisémment des processus d'Itō. Ce faisant, nous faisons face au caractère non-dérivable des processus stochastiques, et apportons notre deuxième contribution, un argument physique concernant la mode de fonctionnement des centrales de production qui opèrent effectivement comme des filtres passe-bas sur nos processus stochastiques et qui nous permet de continuer à construire un modèle tractable de coûts de variation avec incertitude. Troisièmement, nous trouvons une spécification d'un processus d'Itō nous permettant de conserver une forme analytique.

De ces contributions techniques, nous obtenons nos contributions économiques grâce à un modèle riche et tractable qui propose des résultats contrastant fortement avec les résultats passés de la littérature. Tout d'abord, nos solutions sont uniques, contrairement aux continuums d'équilibres de Nash habituels dans la littérature des SFE. Deuxièmement, nos solutions ne sont pas ex-post optimales, c'est-à-dire qu'acquérir de l'information sur l'évolution attendue de la demande induit des stratégies optimales différentes pour les producteurs, ce qui a pour conséquence que les producteurs dans notre modèle ont une justification pour soumettre des enchères variant dans le temps. Troisièmement, nous obtenons des formes analytiques faisant des prédictions précises sur



l'évolution des stratégies avec la dynamique des chocs de demande, à savoir que lorsque l'incertitude sur la demande augmente, les producteurs soumettent des courbes d'offre de plus en plus pentues afin de transmettre une plus grande part des chocs de demande aux prix plutôt qu'aux quantités dont les variations sont coûteuses. Enfin, notre modèle justifie l'existence de prix négatifs[15] avec des producteurs étant prêts à subventionner la demande pour ne pas avoir à faire face de forts coûts de variation, ce qui contraste avec les courbes d'offre partout positives dans les modèles de SFE. Ces résultats ouvrent la porte à des modèles capables de différencier entre marchés day-ahead et intraday et donc potentiellement de modéliser leurs interactions.

Au bas des pages de droite de la thèse, il y a de petits graphiques présentant la stratégie optimale pour un certain jeu de paramètres de notre modèle, dans le plan quantité-prix, avec un insert illustrant l'évolution dans le temps du support des chocs de demande, dans le plan temps-choc. La ligne verticale dans l'insert représente le point dans le temps pour lequel la solution optimale est représentée dans le reste du graph. Ceci forme un folioscope, ce qui permet au lecteur de se rendre compte de la dynamique de l'évolution des stratégies optimales en fonction de l'évolution du support des chocs en feuilletant rapidement la thèse et en animant ainsi ce graphique à la manière d'un dessin animé. Ceci ne remplit qu'un rôle d'illustration.

Dans le reste de la thèse, l'objectif est de tester ces prédictions sur des données issues du marché day-ahead français. Comme ces prédictions portent avant tout sur l'impact de l'incertitude sur la forme des courbes d'offre, nous séparons en deux chapitres la contruction de proxies pour cette incertitude dans le deuxième chapitre et l'analyse de l'évolution des enchères en fonction de cette incertitude dans le troisième chapitre.

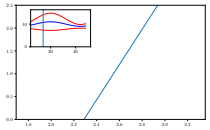
Dans le second chapitre nous nous concentrons sur l'analyse de nos données, sur le

[15] De tels prix négatifs s'observent sur les marchés de l'électricité quelques heures par an en moyenne pour l'Allemagne et la France, par exemple en 2017 il y a eu 146 heures en Allemagne pour lesquelles les prix étaient négatifs, répartis sur 24 jours [EPEX, 2018a]

développement d'une méthodologie permettant d'utiliser des données dont chaque observation est une fonction, et sur la construction de proxies pour l'incertitude à laquelle les producteurs font face vis-à-vis de la demande résiduelle qu'ils se doivent d'anticiper lorsqu'ils jouent leurs enchères sur le marché day-ahead.

Nous notons d'abord que les courbes d'offre agrégées sur le marché day-ahead ne sont pas bien capturées par des fonctions paramétriques. Nous construisons une approche non-paramétrique pour les décrire : nous notons que malgré leur mauvaise approximation par des fonctions paramétriques, elles sont néanmoins grossièrement en forme de S, et donc constituées de quatre grandes parties, les deux sections extrémales, et deux sections intérieures séparées par le point d'inflexion de la fonction. Nous définissons les points de transition entre ces sections comme étant les points pour lesquels les valeurs absolues des premières et secondes dérivées de nos courbes d'offres sont maximales. Cette définition repose sur des estimations de densité de kernels, et est donc bien non paramétrique. Nous remarquons qu'en utilisant 5 de ces points, nous sommes en mesure de capturer environ 98% de la variabilité intrinsèque des courbes d'offre, et nous contentons donc de ces 5 points bien que notre méthode peut servir à définir plus de points. Cette méthode nous permet de caractériser des points que nous considérons comparables d'une enchère à l'autre et nous permet de réaliser l'analyse de nos données dans le chapitre 3.

Nous construisons ensuite des proxies pour le degré d'incertitude associée à la météo à laquelle les producteurs doivent faire face ainsi que des variables capturant l'information disponible aux producteurs avant les enchères et que nous devons contrôler. La météo étant connue au moment d'enchérir et ayant une influence sur la demande résiduelle, il nous faut en tenir compte. Néanmoins la météo influence la demande par des effets locaux là où le marché a lieu au niveau national, il nous faut donc construire des proxies nationaux des variables météo localisées sur le territoire français dont nous disposons. Nous construisons notamment un proxy national de température basé sur une moyenne de la météo localisée pondérée par la population afin de capturer l'effet de la température



sur la demande à travers le chauffage notamment. [16] Le reste de notre travail consiste à construire des proxies pour l'incertitude concernant la production renouvelable qui vient réduire la demande résiduelle. Pour ce faire, nous analysons la structure spatiale de nos données de vent et de rayonnement. Nous défendons le fait que l'autocorrélation spatiale de ces variables est un proxy de l'incertitude, en notant que plus les gradients spatiaux sont élevés, moins la prédiction météo sur laquelle se basent les producteurs au moment de jouer sur le marché sera de qualité.

Dans ce second chapitre nous développons une méthode non-paramétrique permettant de comparer les observations fonctionnelles dont nous disposons les unes aux autres, ainsi que des proxies pour l'incertitude associée à la météo.

Dans le troisième chapitre, nous nous concentrons sur la construction du proxy principal pour l'incertitude à laquelle les producteurs font face, l'incertitude sur la courbe de demande agrégée, et nous attachons ensuite à décrire comment les enchères évoluent avec nos différentes sources d'incertitude.

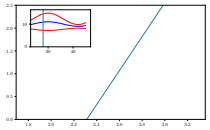
Nous considérons les informations accessibles aux producteurs au moment d'enchérir et régressons les courbes de demandes sur ces variables. Nous étudions ensuite les résidus de cette régression et notons qu'ils sont hétéroskédastiques. Nous nous servons de cette propriété en régressant le carré de ces résidus sur nos variables explicatives, afin de prédire l'amplitude attendue de ces résidus, c'est-à-dire l'amplitude de l'incertitude sur la courbe de demande au moment d'enchérir.

Nous étudions ensuite l'effet de nos différentes sources d'incertitude sur la pente de nos courbes d'offre, et notons que nos proxies pour l'incertitude météo se comportent comme attendus selon nos résultats théoriques obtenus dans le premier chapitre, alors que les résultats sur l'incertitude associée aux courbes de demande sont moins tranchés.

[16] Une large proportion du chauffage en France est électrique, ce qui induit que la demande électrique en France est assez météo-sensible.

Comme nos courbes de demande sont des fonctions complètes dans le plan prix-quantité, nous réalisons l'analyse de nos résidus tant en prix qu'en quantité. Nous obtenons donc des estimations de l'incertitude associée à la position d'un point de nos courbes de demande en prix et en quantité. Dans notre modèle théorique, nous faisons des hypothèses fortes sur la forme de la demande, que nous supposons linéaire, et nous supposons que les chocs de demande sont additifs, i.e., ils ne changent pas la pente de la demande. Ces hypothèses ont pour conséquence qu'il est impossible de différencier l'impact d'une incertitude en prix ou en quantité, et que ces deux incertitudes devraient avoir des effets de même signe : plus d'incertitude induit une pente plus grande de la courbe d'offre pour réduire la variation de la production. Nous observons néanmoins que les effets estimés de l'incertitude sur la quantité et le prix sont de signe opposés. Ces deux hypothèses, bien que nécessaires pour obtenir des solutions analytiques, sont évidemment violées par nos données, et nous pensons qu'il s'agit d'une piste importante d'amélioration du modèle.

La contribution du dernier chapitre est de proposer une méthodologie estimant l'incertitude directe sur la courbe de demande et d'estimer comment cette incertitude impacte la forme des courbes d'offre en différents points de son graph, i.e. nous proposons une approche permettant d'estimer comment les courbes d'offre se déforment sous l'effet de l'incertitude.



# Chapter 1

# Dynamics of the Electricity Day-Ahead Market : Supply Function Equilibria and Ramping Costs [1]

[1] JEL Classification Numbers: C62, C72, D21, D43, D84, L94, Q41
Keywords: Supply function, Equilibria, Unicity, Oligopoly, Electricity, Auctions, Day-ahead, Uncertainty, Dynamics

## 1.1 Introduction

In this chapter, we introduce ramping costs to the theoretical framework of supply function equilibria.

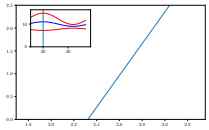
Supply function equilibria are used as a theoretical approach to describe the electricity market on which suppliers bid actual functions contrary to most markets, where one's assumptions about demand and supply curves never translate to agents actually bidding on these objects. The most striking results of this literature are that there exists many (in fact a continuum) of Nash equilibria, that all of those equilibria are ex-post optimal, and that they exhibit always positive prices.

The ex-post optimality implies that once an equilibrium is reached, this equilibrium shouldn't change from auction to auction, given that the cost structure remains constant, even when new information is gathered. However, the observation of the hourly bids on the day-ahead electricity market shows that bids indeed do change from hour to hour. There are many reasons for which the bids might change from one another.

The first is that power plants are brought online or offline to face varying levels of demand for electricity. In so doing, the cost structure of the suppliers changes, which can justify changes in bids. The second is that changes in production are costly in and of themselves, that is, there exists ramping costs associated to the production of electricity. This effect, although technically well supported, also sees support from the existence of negative prices from time to time on the electricity market, on days of high production and low demand. These cases show that subsidizing consumption is less costly than not producing for suppliers.

In this chapter, we propose a theoretical framework to account for these ramping costs.

We choose to model the discrete time bidding as a continuous time process. This



allows us to bring to the literature about supply function equilibria powerful mathematical tools mostly used in option pricing, that is stochastic dynamics: we want to model ramping costs, i.e. costs associated to the variation in production, while retaining the key ingredient brought by [Klemperer and Meyer, 1989], i.e. the uncertainty, through the use of brownians, and more precisely, Itō processes. These tools are the same introduced in the recent literature on dynamic games, see for example [Sannikov et al., 2016]. However, where the focus of this literature is to revisit classical results of repeated games in the context of a time-continuous framework as well as to describe real-world cases more appropriately captured by continuous time models (for example trading), our focus is to be able to capture the effects of ramping costs on the electricity day-ahead market, a market which is discrete in nature.

We obtain a rich and tractable model that yields results that contrast strongly with past results from the literature. First, in the specific case of linear demand and linear costs we obtain a unique Nash equilibria, which contrasts with the usual continuum of Nash equilibria in the supply function equilibria literature. Second, our solutions are not ex-post optimal, meaning that gathering information about the expected future evolution of demand yields different optimal strategies for suppliers, which in turn means that producers in our framework have a motive for submitting different supply functions from one time step to the next. Third, we have closed form solutions which yield specific predictions about the evolution of bids under uncertainty, namely that when uncertainty increase, suppliers submit steeper supply schedules in order to transmit more of these shocks to changes in price and not quantities, which are costly due to the existence of ramping costs. Finally, and less importantly, our framework justifies the existence of negative prices [2] by producers being willing to pay consumers to consume more in order to avoid facing large variations in production, in contrast to everywhere positive schedules in the case of the supply function equilibria literature.

[2]Note that such negative prices happen, a few hours a year for example in France or Germany, for example in 2017 there were 146 such hours, spread over 24 days in Germany [EPEX, 2018a]

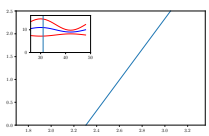
At the bottom of all right pages, there is a small graph showcasing the optimal strategy for a given set of parameters of the model, in the plane quantity-price, with an insert illustrating the evolution of the support of demand shocks over time. The vertical line in the insert represents the point in time for which the strategy is presented in the actual graph. This forms a flipbook, which allows the reader to get a feeling for the evolution of strategies with the evolution of the support of shocks by flipping rapidly the pages and observing the graph becomes animated, just like a cartoon. This serves only an illustrative purpose.

### 1.1.1 Litterature review

The electricity markets flourished in Europe during the 1990s during the wave of privatization. The argument for their creation was one of competition, that was supposed to bring lower prices to the end consumer of electricity.

An important specificity to the economics of electricity is that electricity cannot be stored in large amounts, which in turn implies that at every moment production and consumption have to match. This means that in order to have a working electric grid, that is one that can produce electricity at higher levels during the winter and lower levels in summer, one has to have production units ready to be turned on if the demand is high enough, but turned off otherwise. This, in turn, means that although their existence is required, it is difficult to see how marginal cost pricing can cover their investment costs, which has been a long running argument in the litterature [Boiteux, 1960]. For this reason, from the very beginning, the issue of the market design was deemed to be crucial to insure that the wished for outcome of the privatization wave came to fruition [Green, 1991].

Most countries having opened the production of electricity to competition have implemented day-ahead markets. As said above, the production and the consumption have



to match constantly. The very short term matching is done by automating tiny adjustments around what a producer is already producing in order to match the fluctuating consumption. To plan which plant should be online at which hour of the day however, the day-ahead markets come in. The idea is that producers and big consumers of electricity (either for themselves, or as aggregators of the individual consumptions) are asked to bid demand or supply functions respectively. The market operator then aggregates the demand and supply curves, which yields an equilibrium giving the price and quantities to be produced for each producer.

There has been an active literature trying to model and measure the market power of oligopolists on these newly created markets [Green and Newbery, 1992, Newbery, 1998, Green, 1999]. The models have mainly been based on Klemperer and Meyer 1989's Econometrica founding paper about supply function equilibria [Klemperer and Meyer, 1989] (henceforth known as KM).

This paper builds upon previous results about competition in supply schedules without uncertainty [Grossman, 1981], which yielded a very high multiplicity of equilibria. KM add a key ingredient: uncertainty about the demand schedule facing the suppliers. This addition greatly reduces the multiplicity, and adds more structure to it, although in this framework there is still a continuum of Nash equilibria, which are always pinned between Cournot and Bertrand outcomes.

Groundbreaking and fertile, the original model by KM studied how demand uncertainty collapses dramatically the set of available supply function equilibria to a well-defined continuum when contrasted to the case of competition in supply schedules without uncertainty [Grossman, 1981]. These equilibria are always pinned between Cournot and Bertrand outcomes. This continuum collapses further to a single Nash equilibria by considering an infinite support of demand shocks, that is an unbounded distribution of shocks. All of these equilibria are ex-post optimal, meaning that changes in anticipated

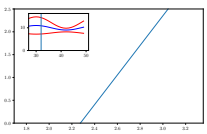
demand shocks do not impact the actual solutions, but only the parts of the solutions that are actually explored as shocks realize, a very strong result.

The electricity markets literature has embraced this framework because it is considered to capture some of the structure at play in the electricity markets: the producers do not know what demand they are going to face when they choose their supply schedule, the demand side is considered much less sophisticated than the supply side, and their demand schedules can therefore be considered to some extent as being exogenous. Some have argued that the schedules submitted in the real markets are discrete and that this discrete nature makes their modelling as continuously differentiable schedules is both incorrect and yields different results from discrete ones [von der Fehr and Harbord, 1993]. However, recent results suggest that with a sufficient number of steps both approaches converge [Holmberg et al., 2008], and indeed we see that recent implementations of the market rules increase the number of steps allowed for a single bid, and consider that these points are linearly joined instead of stepwise.

One of the most striking aspects of the supply function equilibria approaches is, as was alluded to above, the multiplicity of Nash equilibria. This result has been generally viewed as the source of the danger of tacit collusion in electricity markets: if there is a continuum of nash equilibria, repeated interactions are feared to be conducive to a convergence of bidding strategies towards the most profitable equilibria [Bolle, 1992].

Furthermore, these models abstract away some of the details of the actual markets, reason for which authors which try and evaluate the market power of producers on the electricity markets view their endeavour as painting the situation with an optimistic brush [Green and Newbery, 1992].

Here we will tackle the points raised in the last two paragraphs to some extent. We propose to consider a technical reality of the operating of power plants: their cost struc-



ture is history-dependant, more precisely, producing a quantity $q_1$ does not entail the same cost if the previous quantity produced was already $q_1$ or if the previous quantity was different from it. Raising or decreasing production in and of itself imply costs. By introducing these costs we aim to produce a model capable of capturing more precisely the competition that arises in the electricity markets, and in so doing we will show that the continuum of equilibria characteristic of supply function equilibria under uncertainty collapses to unique equilibria, which in turn allows us to comment on the question of tacit collusion.

### 1.1.2 The day-ahead markets

On the electricity day-ahead markets, producers are generally required to submit supply schedules once a day for all the auctions taking place during the next day. The APX (England) and the EPEX (Austria, France, Germany and Switzerland) markets allow hourly auctions [APX, 2017, EPEX, 2015], and EPEX allows for bids comprising up to 256 price quantity combinations, effectively approximating smooth supply functions. Producers can submit different supply schedules for each individual auction, but every bid must be placed at the same time one day in advance for each block of 24 hours. Customers go through the same process and submit their demand schedules, then the market operator matches supply and demand for each auction. Producers thus have to submit schedules facing uncertain demand, which is the reason for the popularity of supply function equilibria approaches to the electricity market.

However, on this market, bids change from auction to auction. From the point of view of KM's model, this should happen only through a coordination of agents agreeing to collectively swap from one Nash equilibrium to another in the available continuum. Describing these dynamics, however, is increasingly important as the energetic mix is bound to include an increasing fraction of renewables. Power production can be separated in two classes: dispatchable and non-dispatchable technologies. Nuclear, coal,

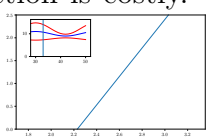
land-fill gas or hydroelectric power generation are mainly dispatchable as one can actually choose their level of production whereas the two rising stars of renewable energy, namely wind and solar, are non-dispatchable: they react to weather conditions. Having these technologies in the mix introduces uncertainty on the production side, which comes down to dispatchable units facing a more uncertain residual demand [Boyle, 2007]. In this paper we want to explore how to model these dynamics.

Electricity production faces very specific technological constraints. These constraints, generally labelled as ramping costs, vary across production technologies and have yet to be captured in a model. We propose to do so by introducing a multivariate cost function, depending as always on the quantity produced, but also on the rate at which production varies: $C(S, \frac{dS}{dt})$. We call this class of cost functions dynamic cost functions. We detail which form these costs take, and its shape in section 1.3.2.

All power plants face maintenance costs. However, part of these maintenance costs is induced by the dynamics of production, and can be seen as ramping costs. More precisely, whatever the production technology, fluctuations in production are costly. Indeed, they imply fluctuations in the temperature of the core of the power plant, thus dilation and contraction cycles of the different parts, which cause wear and tear. The industry is aware of these effects [GE, 2015], as well as the literature, although it focuses mainly on other types of effects, such as the impact on shutdowns due to maintenance [Rothwell et al., 1995], some B2B companies even specialize in minimizing the related long term costs. For example, Wartsila Power Plants, a supplier of power plants and tools to forecast long term costs, explains in a white paper [Arima, 2012]:

> Increased variability in net load demand means that dispatchable generating units have to ramp considerably more steeply and deeper than traditionally, thus increasing wear and tear to components.

We are going to model these ramping costs through a dynamic cost function, increasing in the absolute value of its second argument: any change in production is costly. This



paper will focus on the implications of considering this type of ramping costs. Other types of ramping costs exist, for example startup costs, but they will not be studied in this paper.

These effects cannot be captured by traditional cost functions depending on the level of production alone. One needs to take into account the actual path leading to a given quantity produced, and more precisely in the instantaneous variation, not the whole history. This implies that we need to impose structure on the dynamics of the system while retaining uncertainty, the key ingredient of KM's paper. To do so, we use stochastic dynamics.

This seemingly small addition to KM's framework has a lot of implications on the results obtained. The solutions are not ex-post optimal anymore, allowing to account more satisfactorily for the dynamics of optimal supply schedules, and our solutions are unique, even for bounded demand shocks. We also define a novel selection rule to choose from KM's continuum of equilibria. Finally these results open the possibility to distinguish intraday and day-ahead markets.

In section 1.2 we will present a heuristic approach to get the intuition of the model. Then, in section 1.3 we will introduce the mathematical tools needed to use stochastic dynamics in this context, in section 1.4 and section 1.5 we will solve the monopoly and the symmetric oligopoly cases while considering that producers have information about the overall distribution of shocks during the day, but do not have information about differences in the shocks at different dates. Finally in section 1.6 we will discuss the dynamic variation of the optimal bids, while sections 1.7 and 1.8 will respectively cover some limits of this approach and conclude the paper.

## 1.2 Heuristic Description of the Model

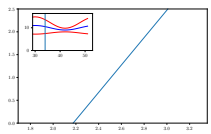
In this section the essence of the model is presented before introducing the proper mathematical tools needed to treat this problem rigorously in the next section. It is thought of as an overview of the mathematical methods that are going to be used, as a way to give a sense of the intent of the modelling choices.

As in KM's setup, the aim is to model an oligopoly facing uncertain demand, taken as exogenous. Before the demand shocks are realized, each firm needs to commit on a strategy. Firms also incur costs that not only depend on the level of production but also on the evolution of the production given its anterior level produced.

More formally, the producer, as in KM, faces uncertain demand, $D(\theta, p)$, with $\theta$ a stochastic shock to the demand and $p$ the price. We add to that both ramping costs and uncertain dynamics of demand. As we want to keep the key ingredient of KM, the introduction of uncertainty, but take into account the dynamics of this uncertainty, of these demand shocks, we need to add more structure.

Consider the following notation, where $\theta(t)$ denotes the value of the stochastic shock at time $t$, whereas $\Theta$ denotes the family of all available time trajectories of our demand shocks.

In the real market, bidders submit a finite number of bids once a day, and face the ramping costs inter-period, that is, when production has to be adjusted to reach the subsequent market outcome. The first bit of structure we introduce is that we are going to assume that time is continuous. The second is that ramping costs are incurred continuously and can be thought of as costs depending on the variation of production over time. Finally, we consider that bidders are allowed to submit a different supply schedule for every point in time between 0 and $T$. This amounts to being asked to submit a surface of strategy in the price-quantity-time space for the next day.



The producer maximises her expected profits, and we consider the simplest case here in which the distribution of shocks is static, that is that the distribution of probability of shocks does not depend on time, and the producer is asked by the market operator to submit the same supply schedule for every point in time a day in advance. In an oligopoly, the program maximised by producer $i$ is, therefore:

$$\max_{S_i(p)} \mathbb{E}_\Theta \left[ \int_0^T \left( p(\theta(t))S_i(p(\theta(t))) - C\left( S_i(p(\theta(t))), \frac{dS_i(p(\theta(t)))}{dt} \right) \right) dt \right] \tag{1.1}$$

with $p(\theta(t))$ the price given the demand shock $\theta(t)$ at date $t$, $S_i(\cdot)$ the supply schedule of producer $i$ and $C(\cdot,\cdot)$ the dynamic cost function. Note that the price depends on $t$ only through $\theta(t)$, i.e. a given level of demand shock implies a given price.

The goal of this section is to provide a first run through of the model, therefore we will not describe here the conditions that must be verified by the different terms of the model. We will simply assume that the dynamic cost function is additively separable between a static and a ramping term, $C(S_i, \frac{dS_i}{dt}) = C_s(S_i) + C_r(\frac{dS_i}{dt})$, and that the demand shocks $\theta$ are bounded in $[\underline{\theta}, \overline{\theta}]$. Lastly we require the ramping term $C_r(\cdot) = \frac{\gamma}{2}(\cdot)^k$ for clarity, and $k \geq 2$ an integer. We distribute the expectation operator and write that $\frac{dS_i}{dt} = \frac{dS_i}{dp}\frac{dp}{d\theta}\frac{d\theta}{dt} = S_i' \cdot \dot{p} \cdot \frac{d\theta}{dt}$, with $X'$ the derivative of univariate function $X$ with respect to its argument, $\dot{X} = \frac{dX}{d\theta}$.

With this setup, by distributing the expectation operator over all possible trajectories of shocks, we are able to rewrite the problem without having time $t$ appear explicitly. This point is crucial, as it is what will let us use mathematical tools that will yield our unicity results. The maximization program can indeed be written as follows:

$$\max_{S_i(p)} T \int_{\underline{\theta}}^{\overline{\theta}} f(\theta) \left( p(\theta)S_i(p(\theta)) - C_s(S_i(p(\theta))) - \frac{\gamma}{2}\left(S_i' \cdot \dot{p}\right)^k \mathbb{E}_\Theta \left[ \left(\frac{d\theta}{dt}\right)^k \middle| \theta \right] \right) d\theta \tag{1.2}$$

with $f(\theta)$ the distribution of shocks, and $\gamma$ the ramping cost parameter capturing the magnitude of the ramping costs. The expected value on the trajectory of shocks of any of the terms above that only depends on $\theta(t)$, that is the value of the shock at a point in time, can be rewritten simply as an integral over the possible values of the shock.

We are left with $\mathbb{E}_\Theta\left[\left(\frac{d\theta}{dt}\right)^k\middle|\theta\right]$ as the only term that depends on the trajectory of shocks. Take for granted that this term can only depend on $\theta$ for now, this result will be justified properly in the next section.

Note now that producer $i$ faces a residual demand so that $S_i(p(\theta(t))) = D(\theta, p(\theta(t))) - S_{-i}(p(\theta(t)))$ which depends only on $\theta$ and $p$, $t$ does not intervene directly, with $S_{-i}$ the aggregate supply schedule of all the other producers, taken as given by producer $i$. This implies that the integrand in eq. 1.2 depends only on three variables: $\theta$, $p$ and $\dot{p}$. The maximization program is therefore equivalent to an Euler-Lagrange problem, a very well described mathematical object: $\max_p \int \mathcal{L}(\theta, p, \dot{p})d\theta$.[3] The information obtained from taking the first-order condition of an Euler-Lagrange problem yields a second order dif-

[3]The Euler–Lagrange equation is an equation satisfied by a function $p$ of a real argument $\theta$, which is a stationary point of the functional:

$$S(q) = \int_a^b \mathcal{L}(\theta, p(\theta), \dot{p}(\theta))dt$$

where:

1. $p$ is the function to be found:

$$\begin{aligned} p\colon [a,b] \subset \mathbb{R} &\to X \\ \theta &\mapsto x = p(\theta) \end{aligned}$$

   s.t. $p$ is differentiable

2. $\dot{p}$ is the derivative of $p$ w.r.t. $\theta$:

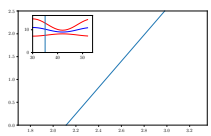
$$\begin{aligned} \dot{p}\colon [a,b] &\to T_{p(\theta)}X \\ \theta &\mapsto v = \dot{p}(\theta) \end{aligned}$$

   $T_{p(\theta)}X$ denotes the tangent space to $X$ at the point $p(\theta)$.

3. $\mathcal{L}$ is a real-valued function with continuous first derivatives:

$$\begin{aligned} \mathcal{L}\colon [a,b] \times TX &\to \mathbb{R} \\ (\theta, x, v) &\mapsto \mathcal{L}(\theta, x, v) \end{aligned}$$

   $T_{p(\theta)}X$ denotes the tangent space to $X$ at the point $p(\theta)$.



ferential equation as well as two boundary conditions: $\frac{\partial \mathcal{L}}{\partial p} = \frac{d}{d\theta}\frac{\partial \mathcal{L}}{\partial \dot{p}}$ and $\frac{\partial \mathcal{L}}{\partial \dot{p}}\Big|_{\underline{\theta}} = \frac{\partial \mathcal{L}}{\partial \dot{p}}\Big|_{\overline{\theta}} = 0$. This is why we obtain unique solutions: if the boundary conditions are not verified, there exists profitable deviations.

In less mathematical term, taking ramping costs into account as specified above means that for a given level of shock, the producer not only cares about the optimal level of production for this shock, but also about the optimal slope of the supply schedule at this level of production. Effectively, this means that optimal levels of production cannot be chosen independently for different level of shocks as is the case in KM, thus shrinking the continuum of equilibria. The boundary conditions' argument explains why the continuum not only shrinks, but collapses to a unique equilibrium.

Note that if the ramping cost parameter $\gamma$ is taken equal to 0 we are back to KM's model: one doesn't care about the slope of the supply schedule anymore, and the problem comes down to a pointwise maximization which therefore yields ex-post optimal equilibria. We want to stress that this means that it is not sufficient to specify the dynamics of the shocks to obtain a supply function model that would react to these dynamics, one needs to take into account ramping costs.

The maximization program 1.2 is a heuristic description of the situation. We want to model the stochastic nature of demand and of its dynamics. We do this by using Itō processes, a class of stochastic processes built through brownians, to describe the stochastic trajectory of the demand shocks with respect to time. The difficulty is that brownians are everywhere continuous but nowhere differentiable, therefore the way program 1.2 is written, with a term in $\frac{d\theta}{dt}$, is a shortcut.

In the next section we introduce the stochastic dynamics properly without using the concept of a derivative.

## 1.3 Stochastic Dynamics

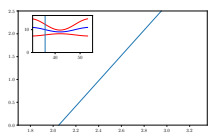
As described in the previous section, we consider that bidders submit surfaces, that is supply schedules for every point in time. The reason to describe a discrete dynamic market as a continuous one is that although discrete time is conceptually more easily understood, continuous time allows to use much more powerful mathematical tools and to obtain closed form solutions, which we think are crucial in gaining intuitive insights about these dynamics. Therefore, we consider that demand fluctuates continuously and that ramping costs are incurred instantaneously. This approximation would need to be tested, although it should be noted that day ahead markets operate with hourly or half-hourly periods and producers are therefore facing a reasonable number of periods each day.

We want our shock variable to evolve over time in a random fashion. The class of mathematical objects used to describe this are stochastic processes. The simplest stochastic process one can think of, and indeed the most important historically, is a Brownian motion process.

Unfortunately, Brownian processes are unbounded and cannot therefore be used to describing the dynamics of the electricity market in which demand shocks, denoted $\theta(t)$, are bounded: there are no days for which demand is null nor are there days for which demand tends towards infinity. The structure to be imposed on the dynamics of the shocks has to imply bounded shocks.

### 1.3.1 The stochastic process

A regular candidate for richer stochastic dynamics than a simple brownian process is an Ornstein–Uhlenbeck process. Unfortunately for us, such a process has unbounded support. We are going to use a richer set of stochastic processes: Itō processes.



The simplest Itō process one can consider that leads to bounded shocks is defined by the following stochastic differential equation (SDE) [Hertzler, 2003]:

$$d\theta(t) = -2\theta(t)dt + \sqrt{1-\theta(t)^2}dB_t \tag{1.3}$$

with $B_t$ a brownian and $dX$ an infinitesimal variation of quantity $X$. Its extended version will happen to be very useful in the rest of this chapter, as it leads to a quadratic distribution of shocks, a property that will allow us to obtain closed form solutions.

Observe that this SDE is formed by a deterministic mean-returning term $-2\theta(t)dt$ and a bounded stochastic one $\sqrt{1-\theta(t)^2}dB_t$. As $\theta(t)$ approaches $-1$ or 1 the stochastic term goes to 0, thus $\theta(t) \in [-1, 1]$. Without loss of generality we can restrain ourselves to this special case. Other bounded supports, $\theta \in [\underline{\theta}, \overline{\theta}]$, can be captured through renormalizations of $\theta$.

Such a stochastic process has a distribution of probability $f(\theta)$ given by Fokker-Planck's equation, easily solved here. In the general case of an Itō process given by SDE 1.4, one obtains in 1.5 the generic Fokker-Planck equation for its distribution of probability $f(\theta, t)$. This equation allows, given an initial condition on the distribution of probability of the variable, to observe how this distribution evolves to reach the steady-state distribution, that is the limit distribution that any initial condition yields. If one knows the value of the stochastic variable at one point in time, one can use this equation to obtain the spread in its distribution over time.

$$d\theta = \mu(\theta, t)dt + \sigma(\theta, t)dB_t \tag{1.4}$$

$$\frac{\partial}{\partial t}f(\theta, t) = \frac{\partial}{\partial \theta}(\mu(\theta, t)f(\theta, t)) + \frac{1}{2}\frac{\partial^2}{\partial \theta^2}(\sigma(\theta, t)^2 f(\theta, t)) \tag{1.5}$$

Here, for SDE 1.3, this yields that $f(\theta) = \frac{3}{4}(1-\theta^2)$ on $[-1, 1]$ and 0 elsewhere.

### 1.3.2 The ramping costs

In the rest of the chapter, we are going to consider quadratic ramping costs. More precisely we consider the costs induced by fluctuations in the production level. As described in the introduction, fluctuations imply increased wear and tear, whether the production is increasing or decreasing. In addition, these ramping costs are null in the absence of fluctuations. This means that they can be captured by a function $C_r(\cdot)$ verifying $C_r(0) = 0$, $C_r(\cdot) \geq 0$ and increasing in the absolute value of its argument. In the absence of more detailed knowledge about the actual shape of these ramping costs, it seems reasonable to consider a quadratic cost function, that is the first term in a Taylor expansion of the actual real ramping cost function.

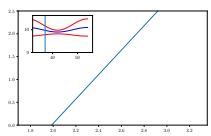
We cannot compute $\frac{d\theta}{dt}$ as it appears in Eq. 1.2, as a stochastic process, although everywhere continuous, is nowhere differentiable. The goal of this section is to express properly the maximization program of the producer that we presented rapidly in Eq. 1.2, and most importantly, to introduce properly how we can work in continuous time with a cost function which depends on fluctuations, which, importantly, are nowhere differentiable.

We are therefore going to first consider the discrete case of a random walk of timestep $\Delta t$ which converges towards the Itō process 1.4, using the Euler-Maruyama approximation [Kloeden and Platen, 2011][4], a generalization of the Euler method to stochastic differential equations. We consider a Markov chain $Y$ defined as follows:

$$\Delta Y_n = Y_{n+1} - Y_n = \mu(Y_n, n\Delta t)\Delta t + \sigma(Y_n, n\Delta t)\Delta B_n \tag{1.6}$$

where $\Delta B_n = B_{(n+1)\Delta t} - B_{n\Delta t}$. These $\Delta B_n$ are *i.i.d.* normal random variables of mean 0 and variance $\Delta t$. Note that as $\Delta t$ is taken towards 0, this Markov chain converges

[4]This formula can be found on page 305. This book focuses on numerical approximations of continuous stochastic processes, which is the reverse of what we are doing here, but it is only in such numeric-centric books that this scheme is introduced. For a more general approach to stochastic differential equations, see [Øksendal, 2003]



towards its underlying stochastic process defined by eq.(1.4).

The ramping costs are taken as quadratic in the variation of the production, and also depend on a ramping cost parameter $\Gamma(\Delta t)$, that is the cost per unit of quadratic variation at horizon $\Delta t$, so we compute the following quantity:

$$\mathbb{E}\left[\frac{\Gamma(\Delta t)}{2}\cdot\left(\frac{Y_{n+1}-Y_n}{\Delta t}\right)^2\middle|Y_n\right]=\frac{\Gamma(\Delta t)}{2}\cdot\frac{\sigma(Y_n,n\Delta t)^2}{\Delta t} \tag{1.7}$$

For this quantity to converge to a finite value when the Markov chain is taken towards its underlying stochastic process we have to consider that for small enough timescales, the ramping cost parameter $\Gamma(\Delta t)$ is linear in $\Delta t$, i.e. $\Gamma(\Delta t)=\gamma\Delta t+o(\Delta t)$. Mathematically, if $\Gamma(\Delta t)$ had a slower than linear relationship at small timescales, the ramping costs would diverge, and if it was faster they would converge to 0. A physical constraint, namely thermal inertia, ensures that the ramping cost parameter does actually behave in this way.[5]

Consider for now that the mean function $\mu$ and the variance function $\sigma$ from eq. 1.4 do not depend on time explicitly and are therefore written $\mu(\theta)$ and $\sigma(\theta)$. Consider now a transformation $T(\cdot)$ that we apply to the Markov chain $Y$. Then:

$$\mathbb{E}\left[\frac{\Gamma(\Delta t)}{2}\cdot\left(\frac{T(Y_{n+1})-T(Y_n)}{\Delta t}\right)^2\middle|Y_n\right]=\mathbb{E}\left[\frac{\Gamma(\Delta t)}{2}\cdot\left(\frac{T(Y_{n+1})-T(Y_n)}{Y_{n+1}-Y_n}\cdot\frac{Y_{n+1}-Y_n}{\Delta t}\right)^2\middle|Y_n\right] \tag{1.8}$$

And in the limit where the markov process $Y$ converges towards the Itō process $\theta$ of equation 1.4:

$$\lim_{\Delta t\to 0}\mathbb{E}\left[\frac{\Gamma(\Delta t)}{2}\cdot\left(\frac{T(Y_{n+1})-T(Y_n)}{\Delta t}\right)^2\middle|Y_n\right]=\frac{\gamma}{2}\cdot T'(\theta(t))^2\cdot\sigma(\theta)^2 \tag{1.9}$$

[5]Ramping costs come from thermal fluctuations in the core of the plant. Therefore, we have to describe how temperature responds to fluctuations in production. Thermal inertia acts as a low pass filter, meaning that it smoothes out fluctuations on short timescales. Think about heating a saucepan full of water: although lighting the stove is almost instantaneous, the temperature of the water being heated increases only progressively, in an exponential fashion that is therefore linear in time for short timescales.

We apply this result to the problem at hand, that is we evaluate the ramping costs in the case where the demand shocks are given by eq. 1.3:

$$\lim_{\Delta t \to 0} \mathbb{E}\left[\frac{\Gamma(\Delta t)}{2} \cdot \left(\frac{\Delta S_i(p(\theta(t)))}{\Delta t}\right)^2 \middle| \theta(t)\right] = \frac{\gamma}{2} \cdot S_i'(p(\theta(t)))^2 \dot{p}(\theta(t))^2 (1-\theta^2) \tag{1.10}$$

with $X'$ the derivative of quantity X with respect to its argument and $\dot{X}$ its derivative with respect to $\theta$. Note that we considered here that the variance term $\sigma(\theta) = 1 - \theta^2$ depends only on $\theta$ and not explicitly on $t$, which in turn implies that the strategy $S_i$ does not depend explicitly on $t$ either.

Let us consider the case where the strategy and the variance depend explicitly on time and are thus written $S_i(p(\theta(t), t), t)$ and $\sigma(\theta, t)$ respectively. By using a first order expansion as before, the ramping cost function can be approximated as follows:

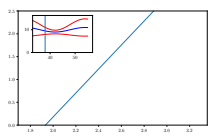
$$\begin{aligned}\lim_{\Delta t \to 0} \mathbb{E}\left[\frac{\Gamma(\Delta t)}{2} \left(\frac{\Delta S_i(p(\theta(t), t), t)}{\Delta t}\right)^2 \middle| \theta(t)\right] &= \lim_{\Delta t \to 0} \mathbb{E}\left[\frac{\gamma}{2} (\partial_1 S(p(\theta(t), t), t) \partial_1 p(\theta(t), t))^2 \frac{\Delta \theta^2}{\Delta t} + \mathcal{O}(\Delta t)\right] \\ &= \frac{\gamma}{2} (\partial_1 S(p(\theta(t), t), t) \partial_1 p(\theta(t), t))^2 \sigma(\theta, t)^2 \end{aligned} \tag{1.11}$$

with $\partial_i X$ the partial derivative of quantity $X$ with respect to its $i^{th}$ argument. See Annex. 1.A for more details on this derivation.

Now, we can write down the instantaneous expected value of the profit of producer $i$ if the demand shock is $\theta(t)$, $\pi_i^e(t, \theta(t))$, that is the profit that one expects to obtain when demand is at $\theta(t)$ given the expected value of the ramping costs:

$$\pi_i^e(t, \theta(t)) = p(\theta(t), t) S_i(p(\theta(t), t), t) - C_s(S_i(p(\theta(t), t), t)) - \frac{\gamma}{2} \partial_1 S_i(p(\theta(t), t), t)^2 \partial_1 p(\theta(t), t)^2 \sigma(\theta, t)^2 \tag{1.12}$$

Lastly, we have to write down the expected profit for a day's worth of submitted strategies. Let us consider that the chosen unit of time is the day. Therefore, the total expected profit $\Pi_i^e$ writes:



$$\begin{aligned}\Pi_i^e &= \int_0^1 \mathbb{E}_{\theta(t)}[\pi_i^e(t,\theta(t))]dt \\ &= \int_0^1 \int_{\underline{\theta}}^{\overline{\theta}} f(\theta,t)\Big[p(\theta,t)S_i(p(\theta,t),t) - C_s(S_i(p(\theta),t)) \\ &\quad -\frac{\gamma}{2}\partial_1 S_i(p(\theta,t),t)^2\partial_1 p(\theta,t)^2\sigma(\theta,t)^2\Big]\, d\theta dt \end{aligned} \tag{1.13}$$

We want to note that all the stochastic calculus presented here is very standard, our contribution is in noting the low-pass filter effect of the physical power plant on fluctuations which allow us to obtain convergent expressions.

### 1.3.3 Discussion of the approximations

We want a tractable mathematical formulation of the dynamic problem faced by producers on the electricity market. To achieve this, we seek to describe the discrete real-life problem by an approximated continuous one. We first use two technological facts: fluctuations in production are costly and these costs decrease linearly in time for short timescales. We then rely heavily on first order expansions of the different terms we have to compute.

### 1.3.4 The maximization program

Here, we consider that the dynamics of demand shocks are given by eq.(1.3), and that therefore $\sigma(\theta,t)^2 = \sigma(\theta)^2 = (1-\theta^2)$.

We now introduce the different conditions that have to be satisfied by the various terms in this problem. First, on most electricity markets, schedules must be increasing, therefore here we take $S_i'(\cdot) \geq 0$. Second, the aggregate demand is non negative as consumers do not have production facilities at their disposal: $D(\theta(t), p(\theta(t))) = \sum_i S_i(p(\theta(t))) \geq 0$. Last, we consider that the shocks $\theta$ are ordered so that the demand is increasing in $\theta$, i.e. $\frac{\partial D}{\partial \theta} \geq 0$, and that the price has to weakly increase with the shocks, i.e. $\dot{p} \geq 0$,

which guaranties that the supply function increases with shocks. Our initial stochastic maximization program can thus be rewritten as a regular optimal control problem:

$$\max_{S_i(p)} \int_{-1}^{1} f(\theta) \left( p(\theta) S_i(p(\theta)) - C_s(S_i(p(\theta))) - \frac{\gamma}{2}(1-\theta^2) \left( S_i'(p(\theta))\dot{p}(\theta) \right)^2 \right) d\theta \tag{1.14}$$

$$s.t. \qquad S_i'(\cdot) \geq 0$$

$$\dot{p} \geq 0 \tag{1.15}$$

$$D(\cdot,\cdot) \geq 0$$

$$\tag{1.16}$$

The next section solves this problem for a monopoly.

## 1.4 The Monopoly

Let us consider that the aggregate demand is linear, that is:

$$D(\theta(t), p(\theta(t))) = a\theta(t) + b - p(\theta(t))$$

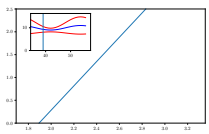
with $a$ and $b$ parameters taken to describe any bounded support of shocks given the stochastic dynamics introduced in the previous section for which $\theta \in [-1, 1]$. Here $(a\theta + b) \in [b - a, b + a]$.

In a monopoly situation we have $S = D(\theta(t), p(\theta(t)))$, therefore the constraints reduce to:

$$\dot{p}(\theta) \in [0, a], \text{ and } p(\theta) \leq a\theta + b$$

where $\dot{X}$ corresponds to $\frac{dX}{d\theta}$.

Consider in addition that the static cost function is also quadratic: $C_s(S_i) = \frac{\lambda}{2} S_i^2$.



The maximization program is rewritten as:

$$\max_{p(\cdot)} \int_{-1}^{1} f(\theta) \left( p(\theta)(a\theta + b - p(\theta)) - \frac{\lambda}{2}(a\theta + b - p(\theta))^2 - \frac{\gamma}{2}(1-\theta^2)\left(a - \dot{p}(\theta)\right)^2 \right) d\theta \tag{1.17}$$

$$\begin{aligned} s.t. \qquad & \dot{p}(\theta) \in [0, a] \\ & p(\theta) \leq a\theta + b \end{aligned}$$

### 1.4.1 Results

**Proposition 1.4.1** *The solution exists, is unique, and has the following form:*

$$\forall \theta \in [-1, 1] \; p^*(\theta) = a\frac{4\gamma + 1 + \lambda}{4\gamma + 2 + \lambda}\theta + b\frac{1+\lambda}{2+\lambda} \tag{1.18}$$

*The optimal schedule is parametrised by $\theta$ so that $S(p(\theta))$ is formed by the points of coordinate $(a\theta + b - p(\theta), p(\theta))$. Its equation is given by:*

$$S^*(p) = \frac{1}{4\gamma + 1 + \lambda}\left(p + \frac{4\gamma}{2+\lambda}b\right) \tag{1.19}$$

*These expressions depend explicitly on our parametrization of the shocks by the parameters $a$ and $b$.*

**Proof** See annex 1.B. ■

**Proposition 1.4.2** *When taking $\gamma \to 0$, the above solution converges towards the solution obtained in the Klemperer and Meyer framework, which for a monopoly is also unique.*

**Proof** See annex 1.C. ■

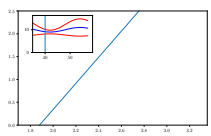
We present in Fig. 1.1(a) the results obtained for increasing values of the ramping cost parameter $\gamma$, starting at $\gamma = 0$ in black and moving progressively from black to blue to red to green.

As the ramping costs increase, a given change in quantity is more costly. To mitigate this effect out, the supplier bids a steeper supply curve, which in turn means that for a given demand shock, more of this shock is transferred to a change in price, which is not costly, than to a change in quantity, which is costly. Adding ramping costs narrows down the domain of attainable quantities produced, as a larger quantity domain implies larger incurred ramping costs.

Solutions are steeper than the traditional monopoly situation, bringing the schedules ever closer to a Cournot-like situation (fixed quantity, i.e. a vertical bid). In addition, the optimal supply schedules do not depend on $a$, the parameter determining the width of the possible shocks, but only on $b$ which defines the average value of the shocks. This result is very strong, but is very much a consequence of our choice of specification: linear demand with additive shocks. In this case the increase in costs due to larger shocks is exactly counterbalanced by larger possible demand, so that the slope of the solution is unaffected. It is very likely that this result is not robust to other specifications.

In Fig. 1.1(b), we illustrate the impact of the specification of shocks on the optimal solution. First, if we consider an increase in $a$ without changing $b$, that is a change in the amplitude of the possible shocks, without changing their expected value, as explained above the solution is essentially unaffected: a larger region of the solution is simply explored. This is illustrated by the red supply function changing to the rose one as a is increased. On the other hand, if we consider a fixed $a$ but an increasing $b$, that is a constant amount of uncertainty, so to speak, but with larger expected values, the explored length is kept constant, but the optimal schedule is translated towards the north-east region of the plane. This is quite intuitive: more demand implies higher supplied quantities and higher prices. This is illustrated by the red supply function changing to the blue one.

Note that all schedules cross at a single point. These quadratic ramping costs imply



a symmetric deformation of the solution obtained in the case of an absence of ramping costs. The limit of extremely high ramping costs is a Cournot-like schedule, i.e. a vertical one, taken at this crossing point: any demand shock is too costly to be accommodated by changes in quantities, and yields only changes in price.

Lastly, our solutions exhibit organically the possibility for negative prices.

**Proposition 1.4.3** *With $\gamma > 0$, their exists values of shocks for which the prices are negative. More precisely, there exist negative prices if the following condition on the parameters of the shocks holds:*

$$\frac{4\gamma+2+\lambda}{2+\lambda}\frac{1+\lambda}{4\gamma+1+\lambda}b < a < \frac{4\gamma+2+\lambda}{2+\lambda}b$$

**Proof** We want the condition under which our solutions exhibit negative prices. First, our supply schedule needs to be positive. Second, for there to exist possible negative prices, one needs the smaller possible price, the one obtained for $\theta = -1$, to be negative.

This can be rewritten as conditions on the shocks, using the expressions from eq. 1.18 and eq. 1.19:

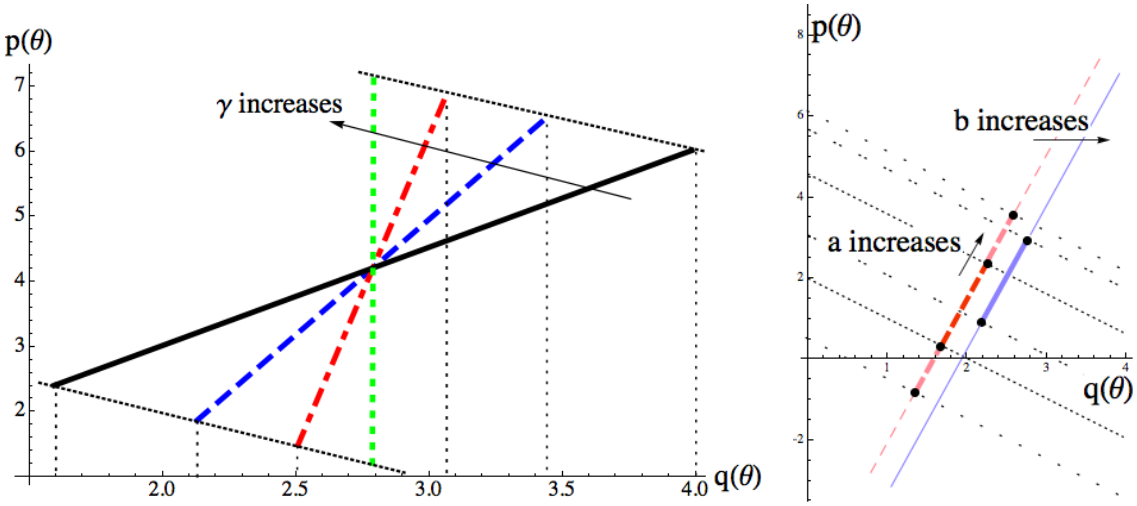
$$\begin{aligned} S * (p(-1)) &> 0 \\ p(-1) + \frac{4\gamma}{2+\lambda}b &> 0 \\ -a\frac{4\gamma+1+\lambda}{4\gamma+2+\lambda} + b\frac{1+\lambda}{2+\lambda} &> \frac{4\gamma}{2+\lambda}b \\ a &< \frac{4\gamma+2+\lambda}{2+\lambda}b \end{aligned}$$

and:

$$\begin{aligned} p(-1) &< 0 \\ -a\frac{4\gamma+1+\lambda}{4\gamma+2+\lambda} + b\frac{1+\lambda}{2+\lambda} &< 0 \\ a &> \frac{4\gamma+2+\lambda}{2+\lambda}\frac{1+\lambda}{4\gamma+1+\lambda}b \end{aligned}$$

■

We will discuss the existence of negative prices more in detail in the following sections.




(a) $S^{-1}{}^{*}(p)$ (b) $S^{-1}{}^{*}(p)$

Figure 1.1: **(a)** Four optimal supply schedules are plotted. In black (full line) $\gamma = 0$. As $\gamma$ increases we transition from the black curve to the blue curve (large dashes), then the red curve (mixed dashes) and then finally for $\gamma \to \infty$ to the green one (small dashes). The range of production is highlighted for each curve through the thin vertical dotted lines.

**(b)** The thin black dotted lines represent the extremal demand functions given $a$ and $b$, i.e. $D(\underline{\theta}, p)$ and $D(\overline{\theta}, p)$. In red (dashed) the solution for a given value of $b$. As $a$ increases, the solution widens from the thick deep red region to the thick light red one. In the case for which $a$ is kept constant and $b$ is increased the solution shifts from the dashed deep red region to the full thick blue one.

## 1.5 The Symmetric Oligopoly

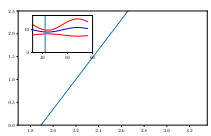
We keep the same linear demand specification as in the monopoly, therefore, with $n$ competitors one has to consider the residual demand faced by each producer:

$$S(p(\theta)) = a\theta + b - (n-1)S(p(\theta)) - p \tag{1.20}$$

$$S(p(\theta)) = \frac{a\theta + b - p}{n} \tag{1.21}$$

$$S'(p(\theta)) = \frac{a - \dot{p}}{n\dot{p}} \tag{1.22}$$

$$S''(p(\theta)) = -\frac{a\ddot{p}}{n\dot{p}^3} \tag{1.23}$$



For concision, we drop the explicit dependencies of the different functions on their arguments in the following equations; $f(\theta)$, $p(\theta)$ and $S(p(\theta))$ will be noted $f$, $p$ and $S$ respectively. The maximization program now writes:

$$\max_{p(\cdot)} \int_{-1}^{1} f\left(p(a\theta+b-p-(n-1)S)-\frac{\lambda}{2}(a\theta+b-p-(n-1)S)^2-\frac{\gamma}{2}(1-\theta^2)\left(a-\dot{p}(1+(n-1)S')\right)^2\right)d\theta \tag{1.24}$$

$$s.t. \qquad \dot{p} \in [0, a]$$

$$p \leq a\theta + b$$

with, as before, $\dot{X} = \frac{dX}{d\theta}$ and $X'$ is the derivative of function $X$ with respect to its argument.

## Results

**Proposition 1.5.1** *The solution exists, is unique, and has the following form:*

$$\forall \theta \in [-1, 1],\ p^*(\theta) = aK_1\theta + bK_2 \tag{1.25}$$

*with*

$$K_1 = \frac{n\sqrt{(4\gamma+\lambda+n)^2-4n+4}-(4\gamma+\lambda+n)(n-2)}{2(4\gamma+\lambda+2n)} \tag{1.26}$$

$$K_2 = \frac{\lambda(n-1)+K_1(\lambda+n)}{(\lambda+n)(n-1)+K_1(\lambda+2n)} \tag{1.27}$$

*and the supply schedule has the following expression:*

$$S^*(p) = \frac{1}{n}\left(p\left(\frac{1}{K_1}-1\right)+b\left(1-\frac{K_2}{K_1}\right)\right) \tag{1.28}$$

**Proof** See Annex 1.D. ■

**Proposition 1.5.2** *The slope of the supply schedule is increasing with $\gamma$ and the schedule is shifted to the right of the plane $(q, p)$ as $\gamma$ increases. This is to say that the schedule*

*rotates around a point in the positive quadrant of the plane.*

**Proof** See Annex 1.E. ■

We are now going to focus on the graphical representation of these solutions. As in the monopoly case we obtain unique solutions of increasing steepness in the ramping cost parameter $\gamma$. When the ramping costs increase, it becomes more and more costly to allow for a large domain of potential quantities to be produced.

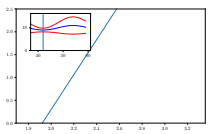
The black curve in Fig. 1.2 corresponds to the limit solution when $\gamma \to 0$, for which the problem gets closer to that of KM, i.e. no ramping costs. Note that as long as $\gamma \neq 0$ the solutions are unique. This contrasts with the case of $\gamma = 0$ which is the model presented in KM, for which there is a continuum of solutions. There is no smooth transition between our sets of solution: when considering ramping costs, there is a single Nash equilibria, even in the limit of small such costs.

Secondly, in their paper, Klemperer and Meyer show that in the limit of a diverging upper bound for their shocks, their continuum of solutions converges towards a unique solution. Our unique solution in the limit of small ramping costs is the same as that ok KM in the limit of infinite support of demand shocks.

**Proposition 1.5.3** *When $\gamma \to 0$, with $\gamma > 0$, the solution remains unique and converges towards the linear schedule available in KM's set of solutions, that is the same schedule selected with KM's selection rule obtained when considering an infinite support for the shocks.*

**Proof** It is straightforward to check that $K_1$ and $K_2$ have the same values as KM for $\gamma \to 0$.

More intuitively the argument is as follows. When $\gamma \to 0$, with $\gamma > 0$, we retain a unique solution although the problem itself converges towards that of KM. When $\gamma = 0$, we are back to the KM situation with a continuum, however we can come as close to 0 as we want



while maintaining a unique solution. We should therefore select an equilibrium present in KM's continuum. When KM take the limiting case of an infinite support of shocks, they select a unique equilibrium. In our case we can do the same thing by taking $a \to \infty$. In the limit, our solution being in their set which converges to a unique equilibrium, those two selected equilibria should be equal. Now note that our solution does not depend explicitly on $a$ so that when the support is finite, we still select the same equilibria out of what is now a continuum of equilibria in KM's framework. ■

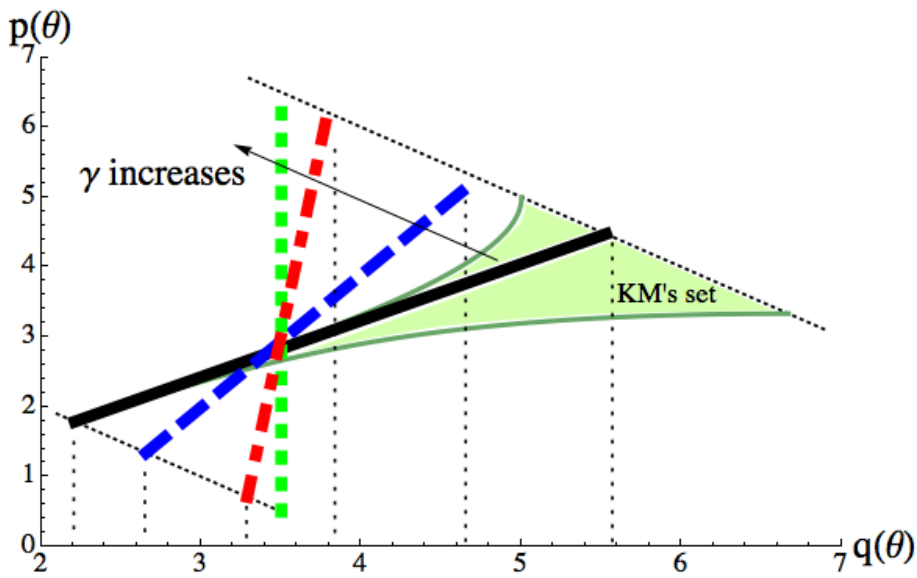



Figure 1.2: This graph plots $S^*(p)$ for different values of the ramping cost parameter, and compares them to the set of equilibria obtained in KM's framework. Four optimal supply schedules are plotted. The black curve (full line) corresponds to the case where $\gamma \to 0$. As before, as $\gamma$ increases the optimal schedules get steeper and steeper until in the limit of $\gamma \to \infty$, the optimal schedule attains a vertical slope. In addition, we show the set of available equilibria in KM's model in light green, and the extremal demand schedules in dashed black.

Intuitively, as we take $\gamma$ to 0 we come closer to the situation captured in KM, but as long as $\gamma > 0$, the producer still faces ramping costs, and therefore converges towards the only linear schedule available in KM's set, as shown in Fig. 1.2, in which we plot our solutions on top of KM's solution set in order to clarify the comparison.

Note that it isn't possible to transition smoothly from our model to that of KM, although they are obviously closely related. Indeed, $\forall \gamma > 0$, our model yields unique solutions, but for $\gamma = 0$ we return to KM's model for which there is a continuum of

equilibria. There is an intrinsic discontinuity between these two models, namely, the correspondence $\Gamma(\gamma)$ associating the set of equilibria to the symmetric oligopoly problem obtained for a given value of the ramping cost parameter $\gamma$ is not lower hemicontinuous at $\gamma = 0$.

In addition to proposing a way to take into account dynamic technological constraints, our model provides a selection rule to choose from the continuum of equilibria described in KM's seminal work, i.e. the solutions' stability to ramping costs.

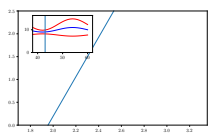
**Proposition 1.5.4** *With $\gamma > 0$, their exists values of shocks for which the prices are negative. More precisely, there exist negative prices if the following condition on the parameters of the shocks holds:*

$$b\frac{K_2}{K_1} < a < b(1 - K_1)$$

**Proof** The method is exactly the same as that used in prooving proposition 1.4.3, noting that $(K_1, K_2) \in (0, 1)^2$. ■

As in the monopoly case, we have the property that there exists situations in which our model can exhibit negative prices. This is interesting because negative prices are observed on the electricity market, and is often described as a way for non-flexible committed producers to subsidize consumption in order to avoid reducing production too much.

To sum up, we have here a model whose solutions depend on the distribution of shocks, therefore we are able to capture the interday variation of bids by assuming that the distribution of shocks varies from day to day. In this case, there exists only one symmetric equilibria each day, function of the distribution of shocks.



### 1.5.1 Discussion

This result sheds some light on one of the questions that the electricity market literature focuses on.

Accounting for ramping costs induces a collapse of the equilibria set from a continuum to a unique element.

Most of the tacit collusion concern that is present in the literature is based on the existence of a continuum of solutions [Bolle, 1992]. This continuum is thought as being conducive of tacit collusion because the electricity market entails repeated interactions between producers. In this case, producers can be feared to be able to learn to pick the most profitable Nash equilibria. Although a Nash equilibrium is not usually considered conducive to collusion, as each player's strategy is the best response to the other's and there is no profitable deviation, a multiplicity of Nash equilibrium lets open the possibility to pick and choose the most profitable one out of the available options, as compared to the one leading to the strongest competition.

Our result implies this pathway for tacit collusion is not available anymore. With only one Nash equilibrium at any given time no learning can bring about tacit collusion. This is a strong result about the structure of competition in our framework. The existence of ramping costs leads to a model in which no tacit collusion can exist, suggesting that the policy recommendations about such collusion stemming from the supply function equilibria literature might be strongly dependent on not taking into account ramping costs.

Our solutions are also not ex-post optimal contrary to the traditional results. As our solutions depend explicitly on the structure of the uncertainty around demand shocks, any additional information shifting the expected distribution of shocks would imply a different bid. Ex-post optimality is a very strong result, and, one could argue, more of a quirk from the usual models than its absence in ours.

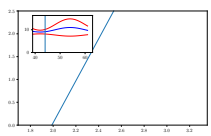
We are also able to account for negative prices which was impossible in the previous framework. Such negative prices are actually observed, although rarely, on the market: producers prefer to subsidize consumption instead of decreasing production by a lot. In our framework, if the ramping costs are large enough, and the demand shocks can reach a small enough value, our solutions can yield negative values: the equilibrium price might even be below the marginal cost of production, understood here as $\partial_q C$ which by definition does not capture our ramping costs.

In the next section we are going to present how to capture richer dynamics, and especially how the surface of bids should evolve with time when the producers have information about the anticipated variation of shocks during the day.

## 1.6 Dynamic Behavior of the Bids

The classical supply function equilibria models, as described before, yield a continuum of Nash equilibria, and each one of those equilibria is ex-post optimal. This a very strong result that we are going to take some time to describe and comment.

Consider for a moment that firms competing in supply schedules reach one of the many possible Nash equilibria under such a setup, and that they commit to their schedules. Now consider that the firms face a succession of demand shocks, and that this yields a succession of market outcomes. As the Nash equilibria are ex-post optimal, it means that given the strategies played by the other firms, no firm has any regrets concerning its strategy. Knowing about the realized demand shocks does not imply any willingness to change strategy as long as other firms keep their strategies fixed, and as long as the support of shocks is not reduced to a point (one could think of observed realizations of shocks as helping to narrow down the expected range of shocks without implying a pinpoint accuracy).



A corollary to this observation is that the distribution of anticipated shocks does not play any role in KM's paper, apart from its bounds. Knowing that the demand shocks are going to be drawn from distributions of high or low values does not affect the willingness to play a given strategy, as long as the support does not evolve. The little role that is played by information about shocks in KM's paper is even more counter-intuitive: to a certain extent, information about demand shocks gives rise to a larger continuum of solutions. Indeed, if one compares the equilibria available to firms for a given support $\{\theta\}_1 = [\underline{\theta}_1, \overline{\theta}_1]$, noted $S^*{}_1$, to those obtained for a support strictly included in the first one $\{\theta\}_2 = [\underline{\theta}_2, \overline{\theta}_2] \subset \{\theta\}_1$, noted $S^*{}_2$, then the set of equilibria will be larger in the second case, in the sense that $S^*{}_1 \restriction_{\{\theta\}_2} \subset S^*{}_2$ (where $\restriction_{\{\theta\}_2}$ denotes that the supply functions are restricted to values over $\{\theta\}_2$).

However, actual firms bidding on the electricity markets are known to be actively engaged in forecasting the future demand levels in order to build their strategies. Bids that we can observe on the electricity markets change from hour to hour even when demand does not vary enough to warrant a change of online plants, a consideration that could explain some of the supply schedules variations.

The general interpretation of KM's paper when applied to electricity markets is that for some unknown underlying process, strategies converge towards different equilibria of the set of available equilibria from hour to hour. One can note that the general intuition for strategies converging towards Nash equilibria in the first place is through either a high degree of sophistication on the part of firms, or through a more organic learning process. Neither of these two explanations can account for frequent switching from one Nash equilibrium to another, out of a myriad of available options, without considering some communication among firms. Furthermore, if such communication existed, it should be expected to yield the most profitable equilibria out of the available lot.

We think that this strand of argument trying to explain bids' dynamics in the light of the supply function equilibria framework is unsatisfying and we argue that forecasting demand becomes important for firms when one considers dynamic effects, that is effects that are history dependent, of which ramping costs which we model in this paper are an instance (one can think of start-up and shut-down costs as another instance of such dynamic effects).

The model described in the previous section doesn't account for these hourly dynamics. Here we present a way to capture these intraday variations, by considering bids that depend continuously on the time $t$. We will show that our results imply that firms are not oblivious to information about the distribution of shocks anymore, and more than that, that their strategies directly evolve with the evolution of their knowledge about uncertain future shocks.

### 1.6.1 The setup

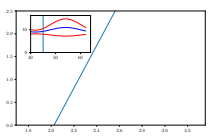
Previously, the SDE (stochastic differential equation) defining the dynamics of the problem was written as:

$$d\theta(t) = -2\theta(t)dt + \sqrt{1-\theta(t)^2}dB_t$$

This specification implied a stochastic trajectory for the shocks, bounded by a constant envelope. That is to mean that, lacking any knowledge of the value of the shock at a point in time close to the period under consideration, the distribution of shocks does not depend on time.

To account for these intraday variations, we are going to define a richer SDE, a non-stationary one.

SDEs have been well studied and as a consequence there exists a number of families of SDEs satisfying numerous characteristics [Hertzler, 2003]. The goal here is to find one



SDE that will allow us to capture some of the dynamics of shocks and how this might influence strategies, while keeping it as simple as possible. Just as in the previous section, the first characteristic that we want is to consider SDEs that imply a bounded support of shocks. This restricts our possible choice to four families out of the classical ones: Generalised Beta I, Beta, Power, Uniform. We also consider that a desirable property is that the distribution reaches 0 continuously at the bounds of its support, because there is no boundary condition on the demand for electricity that would justify that one has a positive probability of reaching a given bound, but a zero probability of reaching an infinitesimally close value to this bound. This restricts us further to only two families: Generalised Beta I and Beta. For tractability reasons we will focus here on the Beta family of SDEs, and more precisely on one of the simplest Beta SDE. However, we want to note that this choice stems from our focus towards solving analytically the problem at hand and obtain closed form solutions. If one were to try and estimate the distribution of shocks anticipated by firms from market data one might want to try and find which of the Beta or Generalised Beta I SDEs might match the distribution of errors between the published day -1 estimates for demand and the observed quantities.

Define the evolving envelope of shocks by two functions, $(\underline{\theta}(t), \overline{\theta}(t))$, respectively the lower and upper bounds of the shocks. These two functions, although very easy to comprehend, are not the most useful way to define the boundary. Instead, we are going to use the average value of the shocks, and the half width of the envelope, $(\hat{\theta}(t), \omega(t))$. This means that $\underline{\theta}(t) = \hat{\theta}(t) - \omega(t)$ and $\overline{\theta}(t) = \hat{\theta}(t) + \omega(t)$. The only restriction we impose on the envelope is that we require it to be continuously differentiable, that is $(\hat{\theta}, \omega) \in \mathcal{C}^1(\mathbb{R})$.

Consider the following SDE which is the simplest Beta SDE that we can pick that still allows us to have a free choice of the bounds of shocks. We want the simplest possible form to make it possible to obtain closed form solutions, yet still account for free dynamics of the bounds. For readability, we drop the explicit dependency of the different functions

on time, that is $\theta(t)$, $\hat{\theta}(t)$ and $\omega(t)$ will be noted $\theta$, $\hat{\theta}$ and $\omega$:

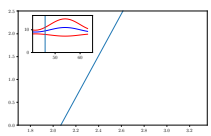
$$\begin{aligned} d\theta = &\left[(\hat{\theta} - \omega - \theta) + \left(1 + \frac{1}{\omega}\frac{d\omega}{dt}\right)(\hat{\theta} + \omega - \theta) + \left(\frac{d\hat{\theta}}{dt} - \frac{d\omega}{dt}\right)\right] \cdot dt \\ &+ \sqrt{\left(1 + \frac{1}{\omega}\frac{d\omega}{dt}\right)(\theta - \hat{\theta} + \omega)(\hat{\theta} + \omega - \theta)} \cdot dB_t \end{aligned} \tag{1.29}$$

The distribution of the shocks can be obtained through Fokker-Planck's equation 1.5 and we obtain:

$$f(\theta, t) = \frac{3}{4\omega(t)^3}(\theta(t) - \hat{\theta}(t) + \omega(t))(\hat{\theta}(t) + \omega(t) - \theta(t)) \tag{1.30}$$

In the following analysis, we are going to rely on the fact that the term $\left(1 + \frac{1}{\omega}\frac{d\omega}{dt}\right) > 0$. The justification for this inequality comes from the following remark: if one were to rescale time in the above equations, there wouldn't be any explicit change in the equilibrium distribution 1.30. The only effect that such a rescaling would play is in the variance of the Brownian term. In order to insure that our inequality is correct, one has to make sure that the variation of the envelope term occurs on longer timescales than the characteristic timescale of fluctuations in our problem, that is the timescale that fixes the rate at which information leaks out of the knowledge of the value of one shock at a given point in time. We are trying to capture the hourly changes in firms strategies when demand fluctuates at higher frequencies (think of the collection of individuals that choose to switch lights on or off at any given point in time in an entire country for instance). We therefore consider that this assumption is sound in this situation.

More formally, one can define $\tau$ a rescaling parameter allowing to change the rate at which the brownian process blurs information pertaining to an initial condition. We rescale time using this parameter, so that time $t$ and the rescaled time $t_r$ verify $t_r = \tau t$.



We can rewrite the above equations as:

$$d\theta = \left[ (\hat{\theta} - \omega - \theta) + \left(1 + \frac{\tau}{\omega}\frac{d\omega}{dt_r}\right)(\hat{\theta} + \omega - \theta) + \tau \left( \frac{d\hat{\theta}}{dt_r} - \frac{d\omega}{dt_r} \right) \right] \cdot dt_r \\ + \sqrt{\left(1 + \frac{\tau}{\omega}\frac{d\omega}{dt_r}\right)(\theta - \hat{\theta} + \omega)(\hat{\theta} + \omega - \theta)} \cdot dB_{t_r} \tag{1.31}$$

and

$$f(\theta, t_r) = \frac{3}{4\omega(t_r)^3}(\theta(t_r) - \hat{\theta}(t_r) + \omega(t_r))(\hat{\theta}(t_r) + \omega(t_r) - \theta(t_r)) \tag{1.32}$$

By assumption, $\tau$ is small enough for the loss of information due to the stochastic nature of the process to be faster than the typical timescale of variation of strategies, therefore by hypothesis $\left(1 + \frac{\tau}{\omega}\frac{d\omega}{dt_r}\right) > 0$ is valid. We will drop this rescaled time index in the following sections as equations 1.30 and 1.32 are equal, it was just a temporary definition to justify the sign of the term that depends on the time derivative of the envelope. We will keep this $\tau$ parameter explicit however, in order to allow discussions differentiating effects related to the speed of variation of the envelope or to the relative timescales of this variation and the underlying stochastic process.

### 1.6.2 Results

#### Dynamics in the case of the Monopoly and of the oligopoly

We start by describing the dynamics of the monopoly case because the oligopoly case is not richer dynamically, but it is more complex to describe.

Our stochastic maximization program can thus be rewritten as a regular optimal control problem as in section 1.4, but taking into account the time dependency:

$$\max_{S_i(p,t)} \int_0^T \int_{\underline{\theta}(t)}^{\bar{\theta}(t)} f(\theta, t) \left( p(\theta, t) S_i(p(\theta, t), t) - C_s(S_i(p(\theta, t), t)) - \frac{\gamma}{2}\sigma(\theta, t)^2 \left(S_i'(p(\theta, t), t)\dot{p}(\theta, t)\right)^2 \right) d\theta dt \tag{1.33}$$

$$s.t. \qquad S_i'(\cdot) \geq 0$$

$$\dot{p} \geq 0 \tag{1.34}$$

$$D(\cdot,\cdot) \geq 0$$

$$\tag{1.35}$$

**Proposition 1.6.1** *In the case of an envelope evolving with time, that is shocks belonging to the bounded support* $[\hat{\theta}(t) - \omega(t), \hat{\theta}(t) + \omega(t)]$*, there exists a unique optimal solution to the monopoly problem. It can be expressed as as surface in the price-quantity-time space:*

$$p^*(\theta(t), t) = \frac{4\gamma\left(1 + \frac{\tau}{\omega}\frac{d\omega}{dt}(t)\right) + 1 + \lambda}{4\gamma\left(1 + \frac{\tau}{\omega}\frac{d\omega}{dt}(t)\right) + 2 + \lambda} \cdot \theta(t) - \frac{1+\lambda}{2+\lambda} \cdot \hat{\theta}(t) \tag{1.36}$$

*The corresponding optimal supply schedule writes as:*

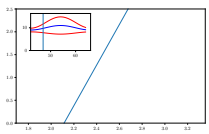
$$S^*(p, t) = \frac{1}{4\gamma\left(1 + \frac{\tau}{\omega}\frac{d\omega}{dt}(t)\right) + 1 + \lambda}\left(p(t) + \frac{4\gamma\left(1 + \frac{\tau}{\omega}\frac{d\omega}{dt}(t)\right)}{2+\lambda} \cdot \hat{\theta}(t)\right) \tag{1.37}$$

$\forall p(t) \in [p(\hat{\theta}(t) - \omega(t), t), p(\hat{\theta}(t) + \omega(t), t)]$

**Proof** See Annex 1.G. ■

Note that if $\frac{d\omega}{dt} = 0$ equations 1.36 and 1.37 are equal to equations 1.18 and 1.19 respectively as expected. Note also that the solution is exactly the same as in the static monopoly case in which one replaces the ramping cost parameter $\gamma$ by $\gamma\left(1 + \frac{\tau}{\omega}\frac{d\omega}{dt}(t)\right)$. This surprising fact, that our dynamic optimal strategy is simply a naive version of the static one with a specified dynamic stochastic process, can be understood as a consequence of the assumptions we have had to make in section 1.3.2.

That is because in section 1.3.2, in Annex. 1.A in which we develop the argument in more detail, and in section 1.6.1 we end up in effect making a scale separation argument: the ramping costs are completely driven by the very short-term fluctuations, whereas the evolution of these ramping costs is driven by the longer timescale at which our information about the demand shocks evolves over time. This means that we make



a version of what physicists call a quasi-static argument: because of this time-scale separation between what drives our ramping cost and our information about the shocks, we can effectively reason in two steps, first solving for the static situation, and then injecting naively the slow changes in the static results with confidence as to the validity of this approximation as long as the assumption about this separation of scale is verified.

The consequence of this is that we have a dynamic version of our static oligopoly of the same nature as for the monopoly above.

**Proposition 1.6.2** *The solution exists, is unique, and has the following form:*

$$\forall \theta \in [-1,1],\ p^*(\theta) = aK_1(t)\theta + bK_2(t) \tag{1.38}$$

*with*

$$K_1(t) = \frac{n\sqrt{\left(4\gamma\left(1+\frac{\tau}{\omega}\frac{d\omega}{dt}\right)+\lambda+n\right)^2 - 4n + 4} - \left(4\gamma\left(1+\frac{\tau}{\omega}\frac{d\omega}{dt}\right)+\lambda+n\right)(n-2)}{2\left(4\gamma\left(1+\frac{\tau}{\omega}\frac{d\omega}{dt}\right)+\lambda+2n\right)} \tag{1.39}$$

$$K_2(t) = \frac{\lambda(n-1) + K_1(t)(\lambda+n)}{(\lambda+n)(n-1) + K_1(t)(\lambda+2n)} \tag{1.40}$$

*and the supply schedule has the following expression:*

$$S^*(p,t) = \frac{1}{n}\left(p\left(\frac{1}{K_1(t)} - 1\right) + \hat{\theta}\left(1 - \frac{K_2(t)}{K_1(t)}\right)\right) \tag{1.41}$$

**Proof** See Annex 1.H.■

**Proposition 1.6.3** *The slope of the supply schedule is increasing with $\frac{d\omega}{dt}$ and the schedule is shifted to the right of the plane $(q,p)$ as $\frac{d\omega}{dt}$ increases. This is to say that the schedule rotates around a point in the positive quadrant of the plane when the uncertainty increases over time.*

**Proof** See Annex 1.F. ■

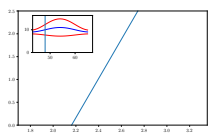
We remind the reader that at the bottom of all right pages, there is a small graph showcasing the optimal strategy for a given set of parameters of the model, in the plane quantity-price, with an insert illustrating the evolution of the support of demand shocks over time. The vertical line in the insert represents the point in time for which the strategy is presented in the actual graph. This forms a flipbook, which allows the reader to get a feeling for the evolution of strategies with the evolution of the support of shocks by flipping rapidly the pages and observing the graph becomes animated, just like a cartoon. This serves only an illustrative purpose.

### 1.6.3 Discussion

In both situations, the optimal supply schedule is shifted uniformly following the expected shock $\hat{\theta}(t)$, which is a rather intuitive result: if on average demand shifts upwards, the producers want to extract more profit and shift their supply curve accordingly, but there is no reason to change slope.

What is less trivial is the way the slope behaves. Let us focus on the monopoly result for a start. The slope is affected as if the ramping cost parameter was fluctuating with the relative change in the width of the bounds of the shocks (term in $\frac{1}{\omega}\frac{d\omega}{dt}$). The transition between a low uncertainty region to a higher uncertainty one behaves as if during the transient regime the ramping cost parameter had a higher value, implying a higher slope.

The optimal supply schedule depends on the relative rate of change of the width $\frac{1}{\omega}\frac{d\omega}{dt}$ and on the average shock $\hat{\theta}$. More precisely, with a constant width, the optimal supply schedule varies according to variations in the expected average value of the shocks. This is quite standard, if demand is higher, the price and quantities both increase, and here this increase occurs with a constant slope. The behavior of the supply schedule when the width varies is less trivial.



Remember that when describing the slope of the schedule, we are considering the plane $(quantity, price)$ while the schedule as defined by $S^*(p)$ represents the same curve but in the plane $(price, quantity)$. An increase in width is equivalent to a higher ramping cost parameter while a decrease in width is equivalent to a lower ramping cost parameter. These results are illustrated in Fig. 1.3.

To understand the economic intuition behind this result, consider first an increase in the width of the envelope on the graphic on the right, the uncertainty level given by the orange line. At this point in time, the uncertainty is increasing, therefore the shocks are going to be larger, the variations in demand too, and to face this increase, the slope is larger so as to reduce this expected increase in ramping costs. In the case of the green line, the uncertainty is decreasing, the ramping costs incurred are expected to decrease, thus this constraint being relaxed the slope can reduce to extract more profit through more variations in quantity than if the slope had remained high.

One can contrast this behaviour to the one described on the left hand side of the figure, where the uncertainty is constant, but the average shock is not. This implies only a vertical translation of the curve, without changes in the slope. Here the slope is fixed by a given level of uncertainty, to exploit an increased demand, the supplier simply increases its prices, but it doesn't need to hedge against increased variation by encouraging or discouraging variations in quantity by playing with the slope.

All of this reasoning applies to the dynamic oligopoly result as well as the effect can be understood in the same way as for the monopoly: changes in the width of the shock's bounds behave as if there was an effective dynamic cost that was higher than the baseline when information about the shocks is lost, and lower than the baseline when information is gained.

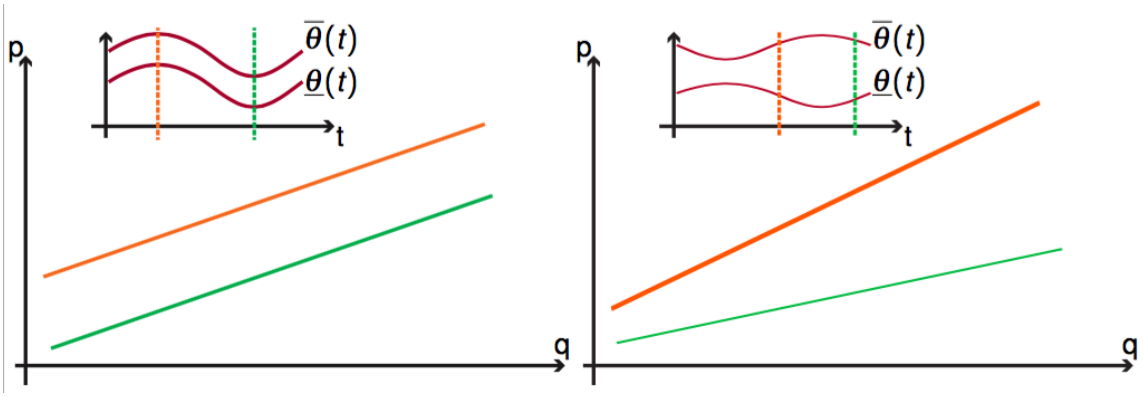



Figure 1.3: On the left, this graph plots an envelope of constant width $\omega(t)$ but varying average. The insert represents this envelope, while the graph itself represents the supply curves associated with the points in time represented by the green and orange dotted lines on the insert. On the right, this graph plots an envelope of constant average but varying width $\omega(t)$.

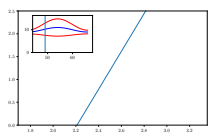
## 1.7 Limits

This section aims at discussing whether or not one can consider that the mapping of these results on the real world is a set of non-zero measure, to put it bluntly.

Further avenues of research would be to generalize our results to larger classes of demand functions. One could also solve the static case for different SDE's in order to test the sensitivity of our results on the underlying "mechanics" of the stochastic process. This has also been pursued without conclusive results: solving the optimization problem becomes quickly extremely difficult, as the second order differential equations exhibit poles, and divergences are difficult to cope with in optimization problems.

The nature of these avenues of research is testament to the fact that our results are obtained for a very narrow setting, one chosen to obtain closed form formulas. However, although a healthy dose a skepticism as to the applicability of the closed form formulas is therefore warranted, I would like to argue that the results hint towards at least one more general takeaway message, namely the collapse of the set of equilibria.

This result stems from the nature of the mathematical problem and not from the way we set up the problem in order to maximize our chances of closed-form success per se.



Therefore, I think it hints towards possible more general results. The problem is the complexity of the maths as soon as one deviates from the simplest version of the problem presented here.

The question then becomes one of the method to employ to obtain those results. There is one tool that might prove useful: numerical simulations. One can solve the differential equations involved here numerically, check ex-post whether they satisfy the other conditions, and in so doing provide boundaries around possible solutions. If the unicity is a characteristic that is indeed more general than our model here, there is 0 probability of finding such a solution by the method proposed, quite literally. However providing such bounds, although not demonstrating the existence of a solution, could provide circumstantial evidence towards such a result.

More generally, I think numerical methods as a guide for theoretical results have not been exploited to the fullest of their potential.

## 1.8 Concluding Remarks

In this chapter we have introduced a supply function equilibria model of ramping costs under uncertainty.

By introducing technological constraints previously neglected we are able to take into account the effects of the dynamics of demand shocks on the supply function framework. We restrict ourselves to linear demand. The optimal supply schedules obtained are unique. This is a striking result when compared to traditional multiplicity of equilibria. Although we do not solve the model in the case of a general demand function, we think that our results make a strong case for the reduction of the set of equilibria, in our case to a unique equilibrium, when taking into account dynamic effects, that is strategies that are history dependent.

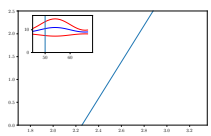
We introduce a mathematical toolbox that was absent from this literature in the past, and notably classes of stochastic differential equations that can be used to pick and choose processes yielding specific closed form distributions of probability of shocks at equilibrium.

Our methodology further introduces the notion of time-scale separation to our problem, which allows to transcribe quite simply static solutions to the case of dynamic envelopes of shocks, as long as the static case is solved for the same functional form of stochastic processes. In our case we focus our study to quadratic distributions, which we then extend to cope with any functional form for the time dependency of the envelope of shocks.

Our results are congruent with the economic intuition one can have about ramping costs: when they increase, the slope of the supply schedule increases in order to reduce the range of variation in production for a given range of variation of demand shocks.

Although mathematically more demanding than the traditional model by Klemperer and Meyer, we consider that this new model, while conceptually sparing (we only add ramping costs) allows for a richer, more realistic description of the electricity market, and opens new research avenues. It yields precise and testable predictions on the dynamics of the electricity market with tractable functional forms, at least in the linear demand case.

In addition, by explicitly modeling the dynamics, our work opens the possibility to explore interactions between intraday and day-ahead markets, markets that were indistinguishable in the previous framework, which explains why the analysis of the interaction between different ways to trade electricity focused on day-ahead markets and forward contracts: if solutions are ex-post optimal, there is no need to create a second type of spot market with a shorter time horizon, the bids of the previous day should suffice.



Further avenues of research would be to generalize our results to larger classes of demand functions. One could also solve the static case for different SDE's in order to test the sensitivity of our results on the underlying "mechanics" of the stochastic process. This has also been pursued without conclusive results: solving the optimization problem becomes quickly extremely difficult, as the second order differential equations exhibit poles, and divergences are difficult to cope with in optimization problems.

Finally, and more generally, we think that this concept of ramping costs, the fact that change is costly, is ubiquitous and could fuel interesting research into the dynamics of a large range of markets. Such avenues have been pursued in the case of stochastic optimal control, that is, instantaneous reactions to stochastic shocks. Here we are describing a market on which agents are forced to optimize in advance, so that they have to react to continuous changes in the anticipated shocks, but not the shocks themselves, which can be understood as stochastic optimization with periodic commitment.

The goal of the second and third chapter is to test these theoretical predictions on French day-ahead data. The next chapter focuses on building methods to be able to perform such tests in the third chapter. As the actual supply schedules are not linear, we need to be able to define their local slopes and to define points that can be compared to one another across schedules. We also describe how we build the different controls that need to come into play to estimate properly the impact of uncertainty on the slope, but more importantly we introduce one class of uncertainty estimates associated with weather. With these elements in place, we have a way to compare schedules to one another, and to estimate uncertainty for a given schedule, and therefore look into the effect of uncertainty in the third chapter.

# Appendix

## Appendix 1.A Proof of Equation 1.11

We are here going to detail how we obtain the result in equation 1.11 on which the proofs of our dynamic results rely heavily. Recall that we are computing the continuous time limit of our ramping cost term which can be quite simply defined in the case of discrete dynamics but for which one has to work a bit more in order to cope with the non differentiable nature of stochastic processes.

We are therefore going to first consider the discrete case of a random walk of timestep $\Delta t$ which converges towards the Itō process 1.4, using the Euler-Maruyama approximation, a generalization of the Euler method to stochastic differential equations. We consider a Markov chain $Y$ defined as follows:

$$\Delta Y_n = Y_{n+1} - Y_n = \mu(Y_n, n\Delta t)\Delta t + \sigma(Y_n, n\Delta t)\Delta B_n \tag{1.A.1}$$

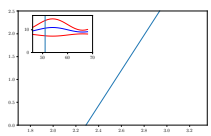
We want to derive the following:

$$\lim_{\Delta t \to 0} \mathbb{E}\left[\frac{\Gamma(\Delta t)}{2}\left(\frac{\Delta S_i(p(\theta(t),t),t)}{\Delta t}\right)^2 \middle| \theta(t)\right] = \frac{\gamma}{2}(\partial_1 S(p(\theta(t),t),t)\partial_1 p(\theta(t),t))^2\sigma(\theta,t)^2 \tag{1.A.2}$$

Let us first compute the first order expansion of $\Delta S_i(p(Y_n, n\Delta t), n\Delta t)$, by assuming that both $S_i$ and $p$ are continuously differentiable with respect to their arguments:

$$\Delta S_i(p(Y_n, n\Delta t), n\Delta t) = \frac{\Delta S_i}{\Delta p}\left(\frac{\Delta p}{\Delta Y}\frac{\Delta Y}{\Delta t}\Delta t + \frac{\Delta p}{\Delta t}\Delta t\right) + \frac{\Delta S_i}{\Delta t}\Delta t + \mathcal{O}(\Delta t^2) \tag{1.A.3}$$

Using our differentiability assumption, note that the terms that do not depend on $\Delta Y$ scale with $\Delta t$, and that the term depending on $\Delta Y$ cannot be grouped in the same way,



due to its stochastic nature, therefore:

$$\frac{\Delta S_i(p(Y_n, n\Delta t), n\Delta t)}{\Delta t} = \frac{\Delta S_i}{\Delta p}\frac{\Delta p}{\Delta Y}\frac{\Delta Y}{\Delta t} + \mathcal{O}(1) \tag{1.A.4}$$

$$\left(\frac{\Delta S_i(p(Y_n, n\Delta t), n\Delta t)}{\Delta t}\right)^2 = \left(\frac{\Delta S_i}{\Delta p}\frac{\Delta p}{\Delta Y}\frac{\Delta Y}{\Delta t}\right)^2 + C \cdot \frac{\Delta S_i}{\Delta p}\frac{\Delta p}{\Delta Y}\frac{\Delta Y}{\Delta t} + \mathcal{O}(1) \tag{1.A.5}$$

with $C$ a term that does not depend on $\Delta Y$ or $\Delta t$.

Now by considering the specification of our stochastic process we know that $\mathbb{E}\left[\frac{\Delta Y}{\Delta t}|Y_n\right] = \mu(Y_n, n\Delta t)$ and that $\mathbb{E}\left[\frac{\Delta Y}{\Delta t}^2\middle| Y_n\right] = \mu(Y_n, n\Delta t)^2 + \frac{\sigma(Y_n, n\Delta t)^2}{\Delta t}$. Using the fact that $\Gamma(\Delta t) = \gamma\Delta t + o(\Delta t)$ we obtain the result of equation 1.11.

# Appendix 1.B Proof of Proposition 1.4.1

Define the following Hamiltonian:

$$\begin{aligned} H(p(\theta), \dot{p}(\theta), \mu(\theta), \theta) = f(\theta)\Bigg(&p(\theta)(a\theta + b - p(\theta)) - \frac{\lambda}{2}(a\theta + b - p(\theta))^2 \\ &-\frac{\gamma}{2}(1-\theta^2)\left(a - u(\theta)\right)^2\Bigg) + \mu(\theta)u(\theta) \end{aligned} \tag{1.B.1}$$

where $u(\theta)$ is the control variable defined through the following equation of motion: $u(\theta) = \dot{p}(\theta)$, $u(\theta) \in [0, a]$. We do not consider the non-negative demand constraint and will check ex-post that our solution verifies this condition.

Now note that:

$$\forall\theta \in (-1, 1), \quad \frac{\partial^2 H}{\partial p^2} = -(2+\lambda)f(\theta) < 0 \tag{1.B.2}$$

$$\frac{\partial^2 H}{\partial u^2} = -\gamma(1-\theta^2)f(\theta) < 0 \tag{1.B.3}$$

The Hamiltonian is therefore strictly concave in $p(\theta)$ and $u(\theta)$. Let $(p^*(\theta), u^*(\theta))$ be an admissible pair to the problem, that is a pair such that $u^*(\theta) = \dot{p}^*(\theta)$. If there exists a

continuous and piecewise continuously differentiable function $\mu(\theta)$ such that:

$$\dot{\mu}(\theta) = -\frac{\partial H^*}{\partial p} \tag{1.B.4}$$

$$\mu(-1) = \mu(1) = 0 \qquad \text{in order for prices to be free at the boundaries} \tag{1.B.5}$$

$$\forall(\theta, u) \in [-1, 1] \times [0, a],\ \frac{\partial H^*}{\partial u}(u^*(\theta) - u) \geq 0 \tag{1.B.6}$$

with $\frac{\partial H^*}{\partial u} = \frac{\partial H}{\partial u}(p^*(\theta), u^*(\theta), \mu(\theta), \theta)$, then the Mangasarian sufficiency theorem ensures that $(p^*(\theta), u^*(\theta))$ is the optimal solution [Seierstad and Sydsaeter, 1987, p.105]. Let us check that eq. 1.18 defines the optimal solution.

Equation 1.B.4 defines $\mu(\theta)$ up to a constant. Through direct integration we obtain:

$$\mu(\theta) = 3a\left((2+\lambda)\frac{4\gamma+1+\lambda}{4\gamma+2+\lambda} - 1 - \lambda\right)(2\theta^2 - \theta^4) + const.$$

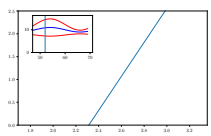
This expression is symmetric in $\theta$ therefore by choosing the adequate value for the constant, we ensure that eq. 1.B.5 is satisfied. The slope of the proposed $p^*$ is in $[0, a]$ therefore eq. 1.B.6 requires $\frac{\partial H}{\partial u}$ to be null.

$$\forall\theta \in [-1, 1],\ \frac{\partial H}{\partial u} = 0 \implies \frac{d}{d\theta}\frac{\partial H}{\partial u} = 0$$

$$\text{i.e. } \dot{u}(\theta) = -\frac{4\theta}{1-\theta^2}(a - u(\theta)) - \frac{(1+\lambda)(a\theta+b)}{\gamma(1-\theta^2)} + \frac{(2+\lambda)p(\theta)}{\gamma(1-\theta^2)} \tag{1.B.7}$$

It is straightforward to see that the proposed solution satisfies this differential equation, thus we know that $\frac{\partial H}{\partial u}$ is a constant and as $\mu(-1) = 0$ it is in fact null. Lastly, we see that $p^*(\theta) \leq a\theta + b$.

The proposed $p^*(\theta)$ therefore defines the unique optimal supply function, i.e. the parametrized curve $(a\theta + b - p^*(\theta), p^*(\theta))$.



# Appendix 1.C Proof of Proposition 1.4.2

Consider the following profit:

$$p \cdot D(\theta, p) - C(D(\theta, p))$$

Now take a linear demand schedule $D(\theta, p) = a \cdot \theta + b - p$ and a quadratic cost function $C(D(\theta, p)) = \frac{\lambda}{2} D(\theta, p)^2 = \frac{\lambda}{2}(a \cdot \theta + b - p)^2$. The F.O.C. with respect to $p(\theta)$ writes:

$$\begin{aligned}
D(\theta, p) + p \cdot \partial_p D(\theta, p) - C'(D(\theta, p))\partial_p D(\theta, p) &= 0 && (1.C.1)\\
C'(D(\theta, p)) - \frac{D(\theta, p)}{\partial_p D(\theta, p)} &= p && (1.C.2)\\
(1 + \lambda)(a \cdot \theta + b - p) &= p && (1.C.3)\\
p &= \frac{1 + \lambda}{2 + \lambda}(a \cdot \theta + b) && (1.C.4)\\
& && (1.C.5)
\end{aligned}$$

This result is the same as that of proposition 1.4.1 with $\gamma \to 0$.

# Appendix 1.D Proof of Proposition 1.5.1

As for eq. 1.24, for the sake of concision, we do not write the explicit depencies of the different functions on $\theta$, thus $f(\theta)$, $p(\theta)$, $u(\theta)$, $\mu(\theta)$ and $S(p(\theta))$ will be written as $f$, $p$, $u$, $\mu$ and $S$ respectively. Define the following Hamiltonian:

$$\begin{aligned}
H(p, u, \mu, \theta) = f\Bigg(p(a\theta + b - p - (n-1)S) - \frac{\lambda}{2}(a\theta + b - p - (n-1)S)^2 \\
-\frac{\gamma}{2}(1 - \theta^2)\left(a - u(1 + (n-1)S')\right)^2\Bigg) + \mu u
\end{aligned} \tag{1.D.1}$$

where $u$ is the control variable defined through the following equation of motion: $u = \dot{p}$, $u \in [0, a]$. We do not consider the non-negative demand constraint and will check ex-post that our solution verifies this condition.

If a symmetric equilibria exists, eqs. 1.20 through 1.23 imply that the regular conditions for an admissible pair to be optimal write:

$$u = \dot{p} \in [0, a] \tag{1.D.2}$$

$$\partial_u H < 0 \implies u = 0 \tag{1.D.3}$$

$$\partial_u H > 0 \implies u = a \tag{1.D.4}$$

$$\begin{aligned} &\partial_u H = 0 \implies u \in [0, a] \text{ and} \\ &\ddot{p} = -\frac{4\theta(a - \dot{p})}{1 - \theta^2} - \frac{\lambda(a\theta + b - p)}{\gamma(1 - \theta^2)} - n\frac{\dot{p}(a\theta + b - 2p) - a(n-1)p}{\gamma(1 - \theta^2)(a(n-1) + \dot{p})} \end{aligned} \tag{1.D.5}$$

$$\dot{\mu} = -\partial_p H \tag{1.D.6}$$

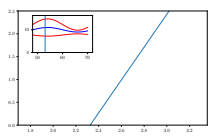
$$\mu(-1) = \mu(1) = 0 \tag{1.D.7}$$

It is easy to check that $(K_1, K_2) \in (0, 1)$ and that the solution 1.25 solves eq. 1.D.5 subject to the boundary conditions 1.D.7. The supply schedule is therefore also linear, with equation:

$$S(p) = \frac{1}{n}\left(p\left(\frac{1}{K_1} - 1\right) + b\left(1 - \frac{K_2}{K_1}\right)\right) \tag{1.D.8}$$

We can now use the Mangasarian theorem to obtain that our admissible pair is indeed solution, $H(p, u, \mu, \theta)$ being concave in $(p, u)$ for linear supply schedules. However the Mangasarian cannot yield that this solution is unique because for a symmetric equilibria, if supply schedules are modified, the hamiltonian changes alongside and we are faced with a new maximization program.

To obtain that the solution is unique we are going to show explicitly that no other candidate solution exists.



First, note that:

$$\begin{aligned}\dot{\mu} = -f\Bigg(&\frac{a\theta + b - 2p}{n} - a\frac{(n-1)p}{n\dot{p}} \\ &+\lambda\frac{a\theta + b - p}{n}\cdot\frac{a(n-1)+\dot{p}}{n\dot{p}} - \gamma(1-\theta^2)(n-1)\frac{a-\dot{p}}{n}\cdot\frac{a\ddot{p}}{n\dot{p}^2}\Bigg)\end{aligned} \tag{1.D.9}$$

If $(p^*, u^*)$ maximises the program then the maximum principle implies that there exists a continuous and piecewise continuously differentiable function $\mu$, as shown in [Seierstad and Sydsaeter, 19 Theorem 2 p.85]. This combined with the above equation implies that $\dot{p} \neq 0$ a.e.

Assume now a solution of the form $\forall\theta \in [-1, 1],\ p = a\theta + \beta$, by injecting this expression in eq. 1.D.9 there is no $\beta$ such that the boundary conditions 1.D.7 are verified.

In addition:

$$\forall\theta \in (-1, 1),\ \frac{\partial^2 H}{\partial u^2} = -f\gamma(1-\theta^2)(1+(n-1)S')^2 < 0 \tag{1.D.10}$$

The Hamiltonian is therefore strictly concave in $u$ and $[0, a]$ is convex. These two properties yield that $u^*$ is continuous, as shown in [Seierstad and Sydsaeter, 1987, Note 2.b. p.86]. We have proved the following result:

**Lemma 1.D.1** *For any symmetric equilibrium $\exists A \subseteq [-1, 1]$ s.t. $A$ is the union of segments of $[-1, 1]$ and $\forall\theta \in A$, $\partial_u H = 0$*

Assume the following hypothesis is true, $H_1 : \exists\theta_c \in (-1, 1)$ s.t. $[-1, \theta_c] \subseteq A$, then knowing that $\dot{p} \in \mathcal{C}^0([-1, 1], [0, a])$ we can rewrite differential equation 1.D.5 around the value $\theta = -1$ by defining $\theta = -1 + \epsilon$ with $\epsilon = o(1)$:

$$\frac{d^2p}{d\epsilon^2} = \frac{C}{\epsilon} + o(1) \text{ with } C \neq 0 \text{ if } p(\theta) \neq aK_1\theta + bK_2 \tag{1.D.11}$$

This means that locally around $-1$, any solution to eq. 1.D.5 but solution 1.25 diverges. Hypothesis $H_1$ is therefore wrong and $\exists\theta_c \in (-1, 1)$ s.t. $\forall\theta \in [-1, \theta_c]$, $\exists\beta$ s.t. $p(\theta) = a\theta + \beta$.

At $\theta_c$ we have $\partial_u H = 0$ and as $\dot{p}$ is continuous, $\dot{p}(\theta_c) = a$. For the solution to be interior we need $\ddot{p}(\theta_c) \leq 0$.

$$\partial_{\dot{p}} H(p, \dot{p}, \mu, \theta_c) = 0 \Leftrightarrow \mu(\theta_c) = 0 \tag{1.D.12}$$

$$\ddot{p}(\theta_c) \leq 0 \Leftrightarrow b(1+\lambda) - \beta(n+1+\lambda) \geq na\theta \tag{1.D.13}$$

Straightforward computations show that both conditions are mutually exclusive, therefore there doesn't exist another candidate symmetric equilibria, and our solution is unique.

Lastly, to compute the optimal supply function, we inverse the optimal price in order to get the shock as a function of the price at the equilibrium, and we inject this expression in Eq. 1.21.

## Appendix 1.E Proof of proposition 1.5.2

We want to prove that the slope of the supply schedules increases as the ramping cost parameter increases. As a reminder:

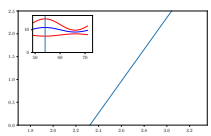
$$K_1 = \frac{n\sqrt{(4\gamma+\lambda+n)^2 - 4n + 4} - (4\gamma+\lambda+n)(n-2)}{2(4\gamma+\lambda+2n)} \tag{1.E.1}$$

$$K_2 = \frac{\lambda(n-1) + K_1(\lambda+n)}{(\lambda+n)(n-1) + K_1(\lambda+2n)} \tag{1.E.2}$$

and the supply schedule has the following expression:

$$S^*(p) = \frac{1}{n}\left(p\left(\frac{1}{K_1} - 1\right) + b\left(1 - \frac{K_2}{K_1}\right)\right) \tag{1.E.3}$$

Let us study how $K_1$ varies with $\gamma$. Note first that if one defines $G = 4\gamma + \lambda + n$, then $\frac{\partial K_1}{\partial \gamma} = \frac{\partial K_1}{\partial G}\frac{\partial G}{\partial \gamma} = 2\frac{\partial K_1}{\partial G}$.



therefore the sign of $\frac{\partial K_1}{\partial \gamma}$ is given by that of:

$$\frac{\partial K_1}{\partial G} = \frac{\partial}{\partial G}\left[\frac{n\sqrt{G^2-4n+4}-G(n-2)}{2(G+n)}\right] \tag{1.E.4}$$

$$= \frac{(\sqrt{G^2-4n+4})((G+n)(nG-(n-2)\sqrt{G^2-4n+4})-(n\sqrt{G^2-4n+4}-G(n-2)))}{4(G+n)^2} \tag{1.E.5}$$

$$= \frac{(\sqrt{G^2-4n+4})(n^2G+4n^2-4n-n(n-2)\sqrt{G^2-4n+4})}{4(G+n)^2} \tag{1.E.6}$$

$$= \frac{(\sqrt{G^2-4n+4})(2G+4+(n-2)(G+4-\sqrt{(G+4)^2-8G-4n-12}))}{4(G+n)^2} > 0 \tag{1.E.7}$$

Therefore $\frac{\partial S^*(p)}{\partial \gamma} < 0$ which implies that schedules see their slope increase with $\gamma$ in the plane $(q, p)$.

We can perform the same type of computation for the ratio $\frac{K_2}{K_1}$, using the fact that $\partial_\gamma K_1 > 0$:

$$\frac{\partial K_2/K_1}{\partial \gamma} = -\frac{\partial_\gamma K_1(K_1^2(\lambda+2n)(\lambda+n)+2K_1(\lambda+2n)(n-1)\lambda+\lambda(\lambda+n)(n-1)^2)}{K_1^2((\lambda+n)(n-1)+K_1(\lambda+2n))^2} < 0 \tag{1.E.8}$$

This implies that the schedule is shifted to the right in the plane $(q, p)$ when ramping costs increase.

# Appendix 1.F Proof of proposition 1.6.3

The proof is exactly the same as that detailed in annex 1.E, by replacing $\gamma$ by $\gamma(1+\frac{\tau}{\omega}\frac{d\omega}{dt})$ and noting that under our assumptions, $1+\frac{\tau}{\omega}\frac{d\omega}{dt} > 0$.

# Appendix 1.G Proof of proposition 1.6.1

Define the following Hamiltonian:

$$\begin{aligned} H(p(\theta,t),\dot{p}(\theta,t),\mu(\theta,t),\theta,t) = f(\theta,t)\Big(p(\theta,t)(\theta-p(\theta,t)) - \frac{\lambda}{2}(\theta-p(\theta,t))^2 \\ -\frac{\gamma}{2}\sigma(\theta,t)^2\left(1-u(\theta,t)\right)^2\Big) + \mu(\theta,t)u(\theta,t) \end{aligned} \tag{1.G.1}$$

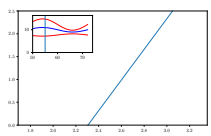
where $u(\theta,t)$ is the control variables defined through the following equation of motion: $u(\theta,t) = \dot{p}(\theta,t)$, $u(\theta,t) \in [0,1]$.

Note that the methods used previsouly generalize to multi-dimensional problems, and that here, our problem depends on $\theta$ and $t$ instead of only $\theta$ as in the case of the static monopoly problem.

Further note that the problem does not depend on the time derivative of $p(\theta,t)$. This means that what would be a general Euler-Lagrange formulation expressed as $\frac{\partial\mathcal{L}}{\partial p} - \frac{\partial}{\partial\theta}\frac{\partial\mathcal{L}}{\partial\dot{p}} - \frac{\partial}{\partial t}\frac{\partial\mathcal{L}}{\partial\partial_t p}$, which is the equation that has to be solved for interior solutions, reduces to $\frac{\partial\mathcal{L}}{\partial p} - \frac{\partial}{\partial\theta}\frac{\partial\mathcal{L}}{\partial\dot{p}}$, where $\mathcal{L}(t,\theta,p,\dot{p}) = H(p,\dot{p},0,\theta,t)$. This is the exact same problem as before, with the only addition that our parameters now vary with $t$, but the partial differential equation is the same one as before.

Therefore the problem can be solved exactly as before by replacing the variance term by its new dynamic version, that is that it is as if the ramping cost parameter $\gamma$ was replaced by $\gamma\cdot\left(1+\frac{\tau}{\omega}\frac{d\omega}{dt}\right)$ in the static solution.

This can be seen by noting that $\sigma(\theta,t)^2 = \left(1+\frac{\tau}{\omega}\frac{d\omega}{dt}\right)(\theta-\hat{\theta}+\omega)(\hat{\theta}+\omega-\theta)$ which has to fall back to the static case in the limit, therefore we see that we simply get an additional $\left(1+\frac{\tau}{\omega}\frac{d\omega}{dt}\right)$ term that appears on the ramping cost term, that is that multiplies $\gamma$.



# Appendix 1.H Proof of proposition 1.6.2

The exact same reasoning as the one in Annex 1.G applies here and we only have to take our static oligopoly result and replace $\gamma$ by $\gamma \cdot \left(1 + \frac{\tau}{\omega}\frac{d\omega}{dt}\right)$ to obtain the dynamic results.

# Chapter 2

# Methodological Tools for Non-Parametric Functional data Evaluation and Weather Data Usage

[1]

[1] Joint work with Henri de Belsunce.

The weather data was obtained through a research convention with Météo-France - ref. DIRIC/13/024
JEL Classification Numbers: C10, C57, C81, L94, Q41
Keywords: Functional, Electricity, Auctions, Day-ahead, Weather, Uncertainty, Dynamics, Regression

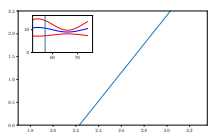
## 2.1 Introduction

In the previous chapter, we have introduced a model of supply function equilibria under uncertainty that takes ramping costs into account, and we derived solutions that depend on the information the firms have about the future demand at the time of bidding. Here, we will focus on introducing tools that will allow us to perform an empirical analysis of the French day ahead market and test these theoretical predictions, which will be the focus of our third chapter.

In this short chapter we develop a methodology to analyze data of two specific formats. The focus lies on the methodological details as well as evaluating the performance of our technique. The aim is first to extract points of interests from functional data in order to be able to compare functions to one another across bids, and second to describe a methodology that will allow us take into account the uncertainty related to the weather. The economic interpretation is secondary in this chapter. Chapter 3 will use the methodology developed here for a case study of the French electricity market.

## 2.2 Point Selection on Functional Data: a Non-Parametric Approach

Reduced form models often rely on exploiting market outcomes for their analysis, i.e. equilibrium prices and quantities, in order to identify the determinants of firm behaviour and test predictions of the theory. On a few markets, sufficient information is available to get around the problem of using endogenous equilibrium data. For example on the government bond markets, both the full aggregate demand and supply functions are observed. This market is of a specific type, it is a divisible goods auction (also called multi-unit or share auctions). These are auctions where multiple units of a good are sold in a single auction. The exact quantity is not predetermined but endogenous and depends on the



price. Furthermore, the auction format is more complex than for indivisible, single unit auctions and most notably requires that bidders simultaneously submit full bid functions for the goods, i.e. multiple price-quantity combinations at which each bidder is willing to buy or sell the goods. The market price and quantity are determined by the intersection of the aggregate demand and supply functions.

The aggregate bid functions are very rich in information and the reduced form models can be adapted to use this data. However, the literature on exploiting functional data is limited. This idea has been applied to investigate the determinants of demand bid functions in French government bond auctions [Préget and Waelbroeck, 2005]. They rely on the propositions first put forward in [Boukai and Landsberger, 1998] and [Berg et al., 1999], who identified that aggregate bid functions in divisible goods auctions follow an S-shaped curve that can be estimated by a logistic function. The fluctuations across auctions are claimed to be due to random shocks on the parameters of the estimated logistic function. The methodology is applied in [Özcan, 2004] to investigate the revenue superiority of the discriminatory price auction format over the uniform price auction format for the Turkish government bonds market.

More generally, their methodology consists of a two-stage regression. The first stage summarises the (presumably parametric) functional data of the aggregated demand function as parameters of an estimated smooth logistic function. The second stage reuses the information (concentrated in the estimated parameters) for cross-sectional analyses.

Although the auction mechanism is identical to that of the Treasury market and data availability is comparable, the logistic function approach does not suit the context of the electricity market due to the strong heterogeneity of the bid functions and their deviations from such logistic shapes, as can be seen in the example of Figure 2.2.1.

The heterogeneity arises from the fact that the bid functions for the electricity auc-

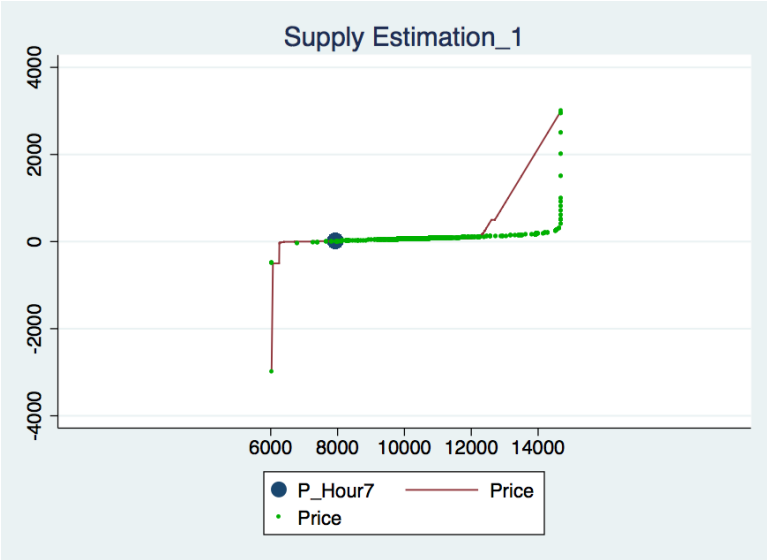
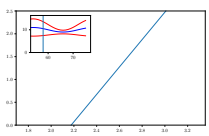



Figure 2.2.1: Example of an asymetric aggregate supply function. The x axis is the quantity in MWh, the y axis if the price in €. In red is the actual aggregate function, in green is an estimated logistic function showcasing the large discrepancies that can arise with this parametric approach. The blue point is the market outcome.

tions are much richer since we have multiple, strategic players on both the demand and the supply side of the market (unlike the market of government bonds, where the supply is monopolistically determined by the Treasury itself). Furthermore, supplier bidding is strongly influenced by the underlying cost of the production technologies. The observed data is consequently less homogeneous and the fitting of the logistic model not convincing. Furthermore, the economic interpretation of the logistic function parameters is very difficult and reducing the whole bid function to two parameters of interest discards a lot of the original information of the bid functions. Finally, we are uncomfortable with the strong assumption of smooth underlying functions and want to circumvent the problems of fitting these.

Instead, we develop a non-parametric, functional data analysis approach to select comparable data points from the original bid functions. In our case, this selection of points will yield 4 regions for every curves. Each region can be thought of as linear. These selected points are comparable across repetitions of the market (i.e. auctions for different hourly contracts) and can then be used to run a cross-sectional reduced form



model. The interest of this approach is threefold. First, it aims to use as much of the original information as possible without distorting it into parameters of a logistic function. What we mean by distortion is the example displayed in Fig. 2.2.1, where one can see that the fitted logistic function in green is very far from the data (in the sense that the integral of the absolute value of the difference of the two curves is very large) because the underlying data simply does not have the proper shape. Also, information about different parts of the bid function does not influence one another, contrary to a parametrized form in which one tries to fit a specified function to data. This implies that the error between the functional form and the data at any point of the curve influences the fitted parameters, therefore "mixing" information from the whole curve into the choice of a given value for the parameter. Second, our approach can be extended to as many points as necessary. The cross-sectional analyses are then conditioned on the type of comparable points selected. Third, while our analysis provides support for an underlying tri-linear or S-shaped functional form, we do not need to assume a specific functional form nor impose overly simplistic assumptions, such as symmetry of the functional forms, to ensure convergence of the estimator.

Here we present the methodology of our point selection and apply it to data from the French electricity market. For now, we ignore specificities of the market for the sake of concentrating on the methodology. We briefly introduce the data and the market in section 2.2.1. For a full explanation of the data and the market, we refer the reader to chapter 3. In section 2.2.2, we explain the point selection algorithm. In section 2.2.3 we discuss the results of the methodology. Section 2.2.5 concludes.

### 2.2.1 Information about our data

Our methodology is general and can be applied to any market where the structure of data observations is similar. Here, we present and discuss the performance of the methodology on data from the electricity market. For the purposes of this chapter, we will focus only on the statistical properties of the data, not on the economic interpretation.

We apply our methodology to data from a divisible goods auction. In this auction, each buyer and seller submits a full individual bid function, i.e. a demand or a supply function, which consists of 2 to 256 monotone price-quantity combinations. The final bid function consists of these explicitly submitted bid points and all linearly interpolated points between them.

The market is cleared by computing the intersection of the aggregate demand and aggregate supply functions, which are each obtained by summing up all individual bid functions for the demand and supply side of the market respectively. In a uniform pricing format, the determined equilibrium price is applied to all units sold in that auction.

### 2.2.2 Point selection algorithm

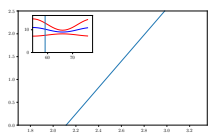
To briefly fix ideas, let's assume that we are interested in a regression à la:

$$S' = \alpha + \boldsymbol{\beta}\boldsymbol{X} + \epsilon$$

where $S'$ is the steepness of the bid function, $\boldsymbol{X}$ the stacked vector of exogenous variables (not specified further here), $\alpha$ the regression constant, $\boldsymbol{\beta}$ the stacked vector of regression coefficients and $\epsilon$ the error term.

The information $S'$ is drawn from the bid functions of the electricity market, and varies along the different points of the bids.

For comparability, we require that a chosen point $k$ from a supply function must be comparable to the $k^{\text{th}}$ point from the supply functions of another auction. The same goes for chosen points of the demand functions. The reason for this assumption is that comparing those points across auction allows us to describe how the functions, that is the aggregate strategies, change shape when our independent variables vary. Note that



we do not impose comparability between a pair $k$ of points from a supply and a demand function of the same auction.

### Non-parametric technique to compare bid functions

Consider two demand functions (as shown in figure 2.2.2). We have to identify "features"

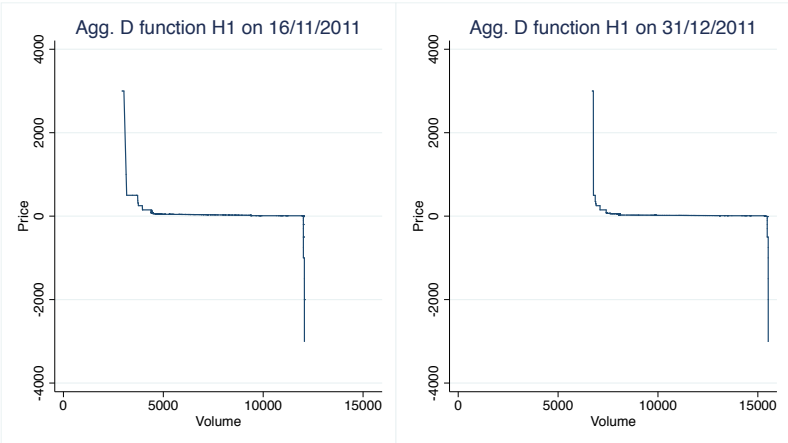



Figure 2.2.2: Comparison of two aggregate demand functions for the same hour

of the different functions in order to determine which points can be compared to one another. We aim to reproduce the type of analysis that the brain performs automatically when faced with such curve: we clearly identify three regions of different slopes, where the central region is less steep than the left and right regions.

To recognise these features, we perform two successive kernel density analyses.[2] For details on the bandwidth and kernel selection as well as algorithm specificities, see appendix 2.A.1. This allows us to access estimates of the absolute values of the first and second derivatives of the demand functions as shown in graphs B and C of figure 2.2.3.

We are therefore able to identify the regions of very high curvature, which define the transition between the three characteristic regions of these functions. We assume that these maxima can be compared across different auctions. This hypothesis is commonly made in functional data analysis and known under the method of landmark registration

[2]Bandwidth in the first estimation = 45, bandwidth in the second estimation = 2, kernels: epanechnikov.

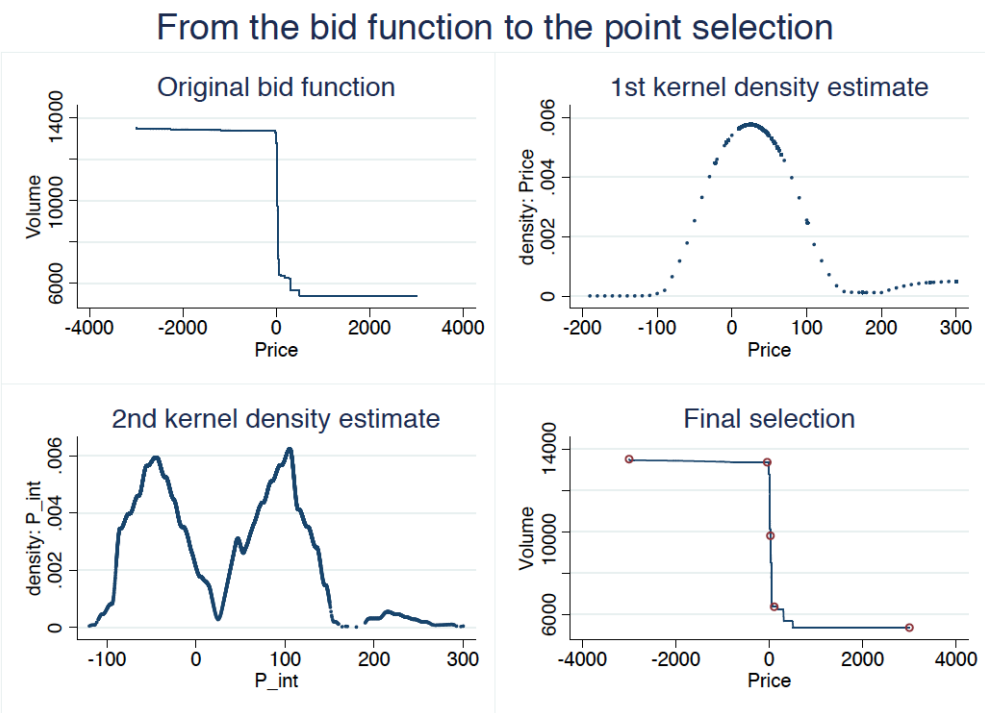



Figure 2.2.3: Steps of the point selection process

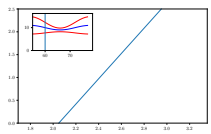
Top left (A): The full original aggregate demand bid function for hour 8 on 15.01.2011 in the quantity - price dimension. Top right (B): Kernel density estimates of the first derivative, zoomed on the relevant price range. Bottom left (C): Zoomed kernel density estimates of the second derivative. Bottom right (D): The full original bid function with the $K = 5$ selected points.

[Silverman and Ramsay, 2005]. This has been applied in [Wölfing, 2013], chapter 4, to day-ahead electricity data, in order to identify the effect of fuel price shocks on supply curves. However, this landmark registration was applied in a parametric form: the regions of high steepness were identified as any part of the curve above 90€/MWh.

We can develop this method further and define intermediary points[3] that can again be compared to one another. This method allows to define as many points as needed, for computational reasons we limit ourselves to $K = 5$ points.[4]

[3]As an example, we could extract those points corresponding to half the density value of the maximum density of the second order derivative. The four points selected, one for each monotone portion of the graph of second derivative estimates, would then correspond to those where the curvature of the function is halved. Together with the maximum, the additional point would contain information on the speed (radius of the curvature) at which the function changes.

[4]The point selection algorithm took 2 weeks runtime to complete its task of selecting 5 points per function. Defining intermediary points would have taken disproportionately more time since many sorting and interpolation steps are necessary for each intermediate point.



Graph D of figure 2.2.3 visualises an original demand bid function and the selected points that we retain as an informative summary of the original curve. Once this work is done we are left with $K = 5$ points per observed aggregate function, those points being defined in such a way that they can be compared from one auction to another.

In our setting, the selected points are the two end-points of the curves (where bidding is imposed by the auction rules at the minimum ($k = 1$) and maximum ($k = 5$) Price), the point corresponding to what can be thought of as the point of inflection (determined by the maximum of the first derivative, ($k = 3$) in the plane $(p, q)$) and the points separating the regions of high and low elasticity in price (determined by the maximum second derivatives to the left ($k = 2$) and right ($k = 4$) of the POI).

We described the technique here for the case of a demand function. The information measured at these points can thereby be compared across demand bid functions of different auctions. The method is used analogously for selecting comparable points on the supply function. We are hence able to extract slopes at these selected supply bid points, which are again comparable across auctions.

### 2.2.3 Results of the point selection methodology

#### Precision of point selection

We have selected $K = 5$ types of comparable points for each of the 37500 demand and supply functions present in our dataset. This section details the results of the point selection methodology and presents evidence why the point selection algorithm has produced comparable points reliably.

The graphs in figure 2.2.4 show the local density of selected points in the price - quantity space for the demand (left) and supply (right) curves. The fact that the groups of

data points are disjoint from one another indicates that the points selected are distinctly different across groups.

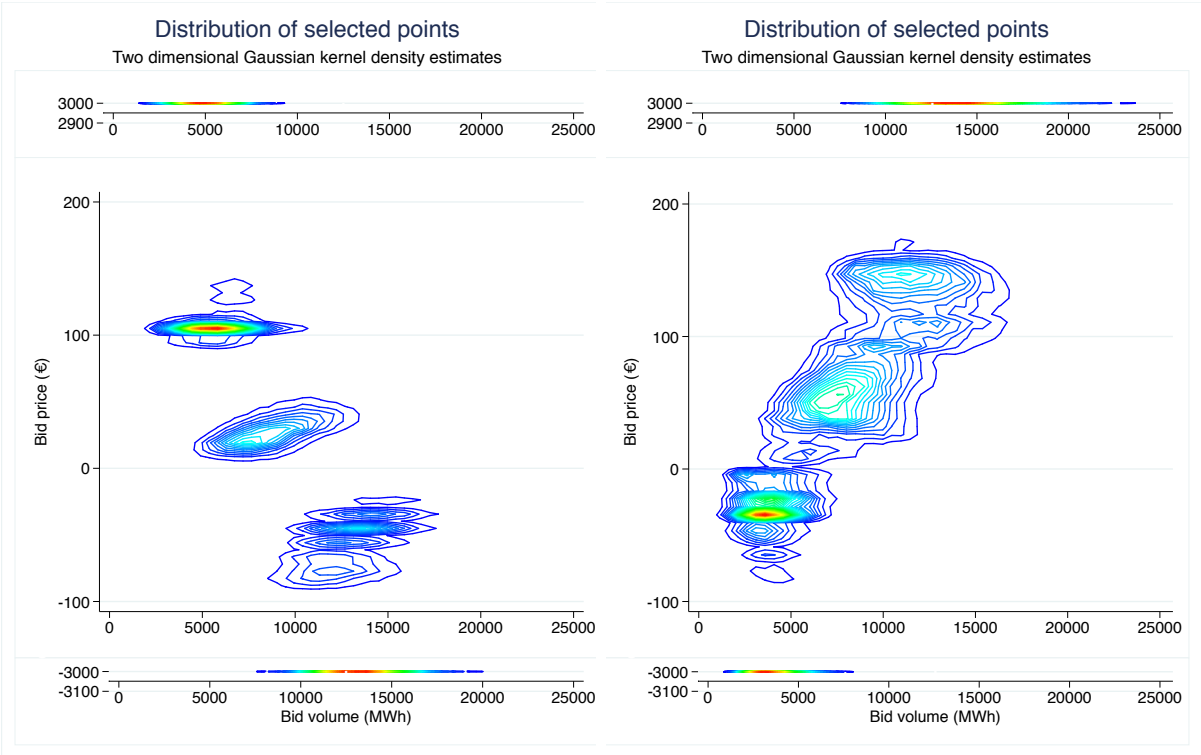
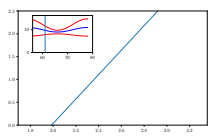



Figure 2.2.4: Heat map on selected, comparable demand and supply points

Note: Please note the discontinuity in the scale of the y-axis. The three seperate graphs are arranged to be understood as a single one. The warmer the colours of the heat map, the higher the frequency of selected price-quantity pairs. The colour legend is omitted for brevity. Density changes between contours are of the order of $10^{-4}$.

In figure 2.2.4, selected points of type $k = 1$ manifest at the bottom of the graph with prices fixed at $-3000$€/MWh. Similarly, $k = 5$ points appear at the top of the graph with prices fixed at $+3000$€/MWh. The three distinct groups of data points refer to points of type $k = 4$, $k = 3$ and $k = 2$, respectively, when reading the zoomed, center part of the graph from top to bottom.

We note that the point selection for the demand curves has produced groups of points that are more distinct (and thus more robustly attributed to a certain type $k$) then for the supply function.



Our methodology only relies on assuming that the first derivative is uni-modal and that sufficient variation exists in the data to distinctly identify the regions of different slope. Overall, this is strong evidence that the algorithm is able to distinctly differentiate between points of different types.

### 2.2.4 Observations of bidding frictions

Distinct point selection is further supported by the evidence in figure 2.2.5. These graphs show the distribution in the price-quantity space of the selected points separately for the demand and supply function. Distinct clouds are an indication that selected points are different across types $k$.

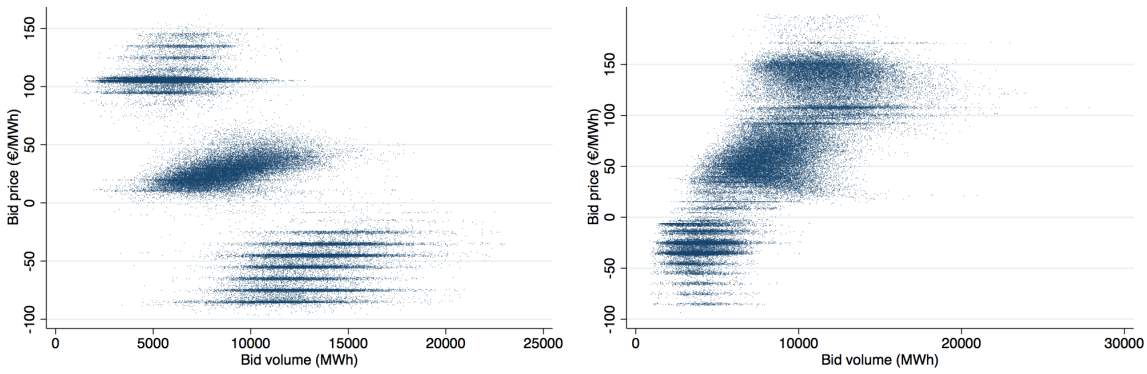



Figure 2.2.5: Distribution of selected demand (left) and supply (right) points

However, a feature of the graphs is striking: patterns (horizontal lines) seem to exist for the selected points of type[5] $k = 2$ and $k = 4$. Many selected points accumulate at certain prices of regular intervals of 10€/MWh, i.e. there seem to be focal price points for the bidders at the curvature points of the bid functions. The pattern is present for selected points of both the supply and demand functions, although the selected points from the supply function exhibit this pattern slightly less.

The points following the pattern (types $k = 2, 4$) represent the points of maximum

[5]Types $k = 1$ and $k = 5$ do not exhibit variation in price, because bidding at the extreme prices of +-3000€/MWh is imposed by the auction rules. We thus neglect their analysis here.

curvature of the aggregate bid functions, i.e. the region where the aggregate bid function transitions from a price elastic center portion to the price inelastic extremities of the bid function.

Without prioritizing any explanation[6], we acknowledge the existence of bid point patterns in the values (i.e. prices and quantities) of selected points.

We are, however, interested in $S'$, the slope at each selected point - an information measured at the selected point. We therefore investigate whether the values of the first derivative at the selected points display a pattern. Figure 2.2.6 shows the histograms of slopes of supply functions for the points $k = 2, 3$ and 4. No pattern in the values of the derivatives is apparent.

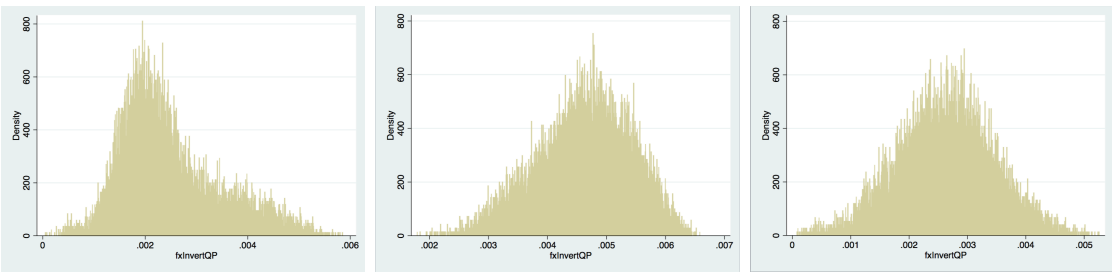


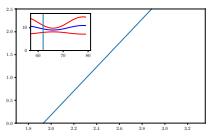

Figure 2.2.6: Histogram of slopes per point type

Note: Histograms of extracted slopes at points of type $k = 2$ (left), $k = 3$ (middle) and $k = 4$ (right).

Although values of the selected points are possibly biased due to focal price points, we do not observe patterns in the variable of interest (i.e. the first derivatives of the selected points) and deem the methodology adequate for our purposes.

Finally, we emphasize that the observed patterns are not caused by the point selection

[6]We do not investigate the origins of bidding frictions in this section, we focus purely on the methodology. For the electricity market, a few possible explanations are that (1) bid functions are driven by marginal costs consideration towards the extremes of the bid curve, (2) bidders bid coarsely since they have used up much of their bid point allowance (256 points) on the center portion of the curve, (3) bidders spend less effort on adequately bidding at extremes since the likelihood of the market outcome occurring at the extremes is much lower.



mechanism since the algorithm can only choose between explicitly bid points or linearly interpolated points, that could be part of a market equilibrium under the reigning price setting algorithm. The pattern arises from many horizontal steps occurring at the same prices in different auctions.

**Value of selected points (determining $K$)**

We remind the reader that the aim is to recover points that summarize well the behaviour of the full aggregate bid functions in different auctions. Our technique allows us to extract representative and comparable points across bid functions of different auctions. Form the selected points, we can also go back to infer the original bid function from which the points were selected. In order to evaluate the utility of our methodology, we investigate the added benefit of an additional point in our point selection.

By selecting $K = 5$ points per curve, rather than fewer points per curve, we are able to significantly reduce the degrees of freedom for inferring the original bid function. In other words, our information (as captured by the selected points) about the original bid function is more precise.

In order to investigate the marginal gain of information for additional points, we first define the mean registered curve. Consider a set of curves that each has N registered points. Take the average coordinates of every point across curves. Rescale linearly every curve by parts so that the registered points fall on their average.[7] Define the mean registered curve as the averaged rescaled curves. Now, separate the data into two groups: curves that are above or below this average curve. Take the averages of these two groups: this defines a measure of the variability of the curves around the total average which is able to capture asymmetries between the two groups.

[7]We rescale all points between the reference points by a vector obtained as a linear combination of the displacement vectors of the closest reference points, of which weights are obtained as the inverse of the distance of the considered point to the enclosing reference points.

Now that these quantities are defined, we can display how much information is captured by the successive addition of registration points for $K = 0$ to $K = 5$ points. We look at the decrease in uncertainty achieved by including an additional point, obtained using our technique. Figure 2.2.7 shows the mean registered curves (red lines) and the mean inferior or superior curves (pink shaded interval above and below the mean registered curves) as a function of the number of reference points.

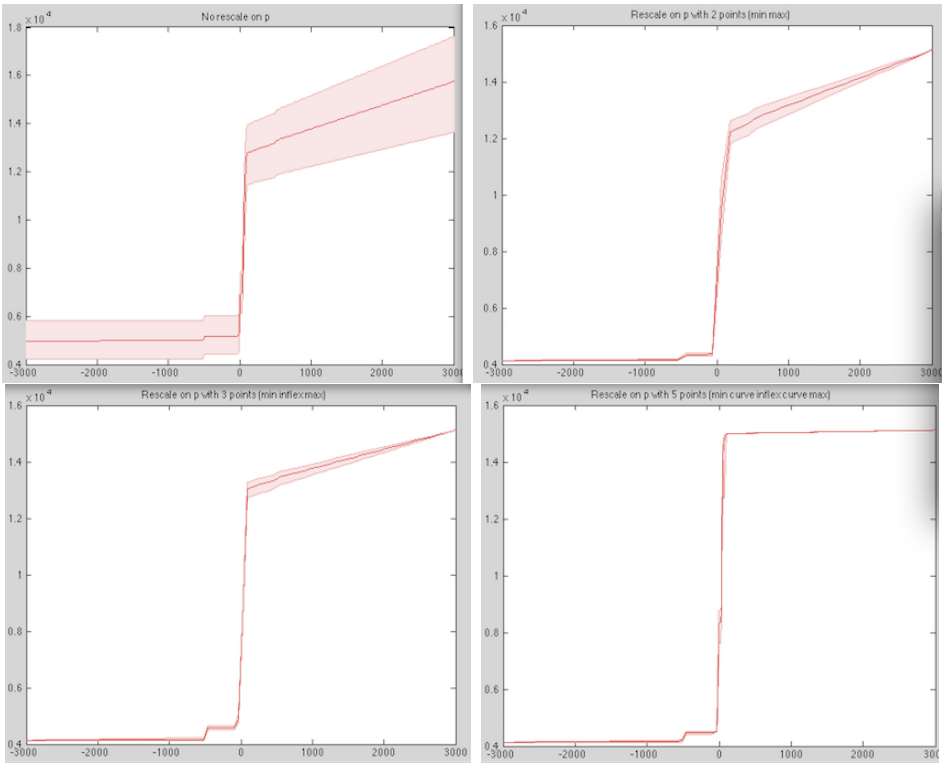
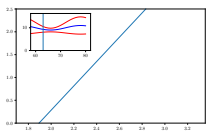



Figure 2.2.7: Error bars as a function of the number of extracted points

Note: The graphs represent the master curve with the error interval for inferring the original bid function, conditional on the number of extracted reference points (RP). Top left (A): Computed without any RP. Top right (B): Computed using 2 RP. Bottom left (C): Computed using 3 RP. Bottom right (D): Computed using 5 RP.

We can see that as we include an increasing number of points the shaded areas shrink: this is a measure of how much of the information contained in the raw curves is captured by the registration points. We see that at 5 registration points, the shaded area is very small, so much so that one can consider that by registering these 5 points, we capture a so-called "master curve": most of the information about the curves is contained in those 5 points.



More quantitatively: without any reference point, inferred bid functions would lie in the interval shown in graph A of figure 2.2.7. With two reference points (namely the minimum an the maximum quantity), the uncertainty is reduced as shown by the smaller error interval in graph B. Graph C adds a third point (the inflection point) and Graph D adds another two points (the two points of maximum curvatures). Figure 2.2.7 shows clearly that with an increasing number of reference points, we obtain a more precise information about the original bid function. We quantify the informational gain by measuring the pink shaded area in each graph A to D. The result is shown in figure 2.2.8 and reveals decreasing marginal information for each additional point. By selecting $K = 5$ points, we are able to reduce the shaded area by a factor of about 50 when compared to figure A (see figure 2.2.8). We see this insight as support for using $K = 5$ points for further work.

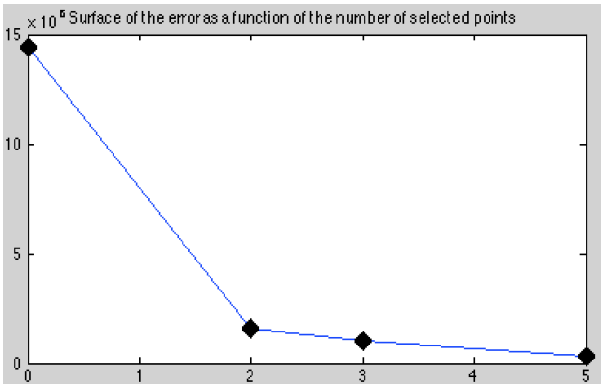



Figure 2.2.8: Proxy for degrees of freedom on master curve

Note: The graph plots a proxy for the number of degrees of freedom for the inference of the original bid function on the number of reference points. Specifically, it plots the size of the pink shaded area in figure 2.2.7 against the number of points.

While the graphs in figure 2.2.7 are displayed on inverted axes and rescaled units, we show the final master curve and uncertainty interval on the original axes and units in figure 2.2.9.

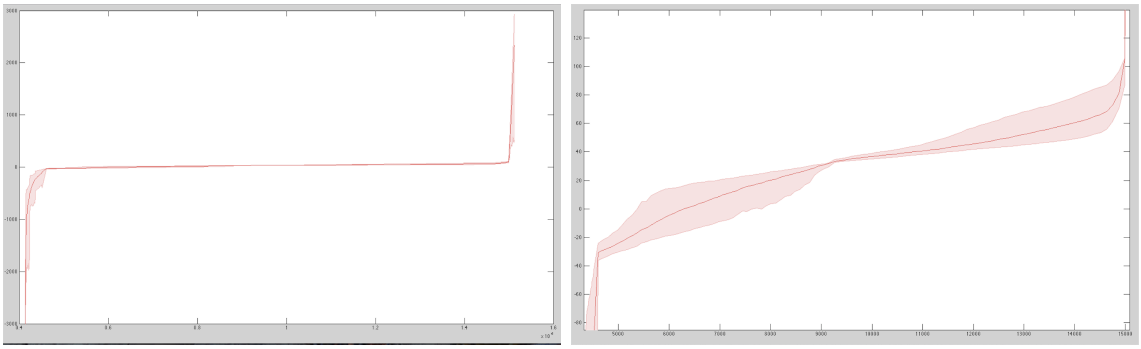


Figure 2.2.9: Overall (left) and zoomed (right) Mastercurve with confidence interval

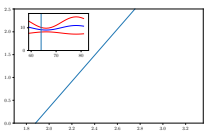
Note: Master curve in the quantity - price dimension.

### 2.2.5 Discussion

In this section, we have developed an alternative technique to run a cross-section reduced form model on data generated by a market that keeps track of the full aggregate demand and/or supply functions. While we apply it to aggregate demand functions, the methodology is fit for the analysis of aggregate supply functions and individual bid functions of either market side.

The methodology is inspired by the techniques used in the literature on Treasury auctions, but has been set up from scratch to allow treatment of more heterogenous data. Furthermore, the hard assumption of an underlying logistic function is relaxed, and our non-parametric point selection avoids the storing of bid function information in the form of estimated function parameters, which are difficult to interpret.

Smoothing of the original bid functions is a component in both the traditional logistic function approach and our comparable point selection methodology. The smoothing enables the user to abstract of small bid function particularities and imprecision, e.g. steps in the function. However, in the traditional approach, the reduction of plus 1000 bid points into very few parameters resulted in the blurring of "local" bid function information from all parts of the function at once. Our non-parametric approach allows specifically to control the extent to which one smoothes the underlying data through the amount of registration points considered.



The results of the comparable point selection are encouraging. We show that each type of point is distinctly chosen and that patterns of the original bid functions do not influence the quality of derivative information extracted at the selected points. We acknowledge the existence of bidding frictions in the original data and highlight this observation for further work.

## 2.3 Methodology to Aggregate Geographically Dispersed Information on a National Level

The theoretical results of chapter 1 indicate that a key ingredient in explaining the dynamics of the bids submitted by suppliers on the electricity market is the uncertainty about demand shocks.

Energy demand addressed to the electricity markets depends on temperature (through the heating of buildings), on wind speed (through the production of wind turbines which reduces the net demand) and on luminosity (through the production of solar panels which reduces the net demand). However, these three weather variables vary in space, whereas the market is at the national level. We introduce here the methodology with which we estimate the associated uncertainty.

We have two types of meteorological data: observations and forecasts. We use both types of data to estimate the underlying uncertainty. The methodologies for each differ slightly.

### 2.3.1 Dealing with meteorological data

#### Interpolation methodology on weather observations

Observations are obtained from MétéoFrance for three parameters of particular interest: temperature, wind speed and light intensity. These observations take the form of tables of hourly observations for a given set of weather stations. Each parameter is observed on a different set of stations.

Due to their hourly nature, the analysis of the electricity market's sensitivity to weather requires a very high number of observations. Therefore, we select between one and two stations per Département[8], a French administrative unit of roughly 6000 $km^2$, i.e. of a typical lengthscale of about 75 $km$. We have 161 stations for temperature, 113 stations for wind speed and 106 for light intensity, as shown in Fig 2.3.1.

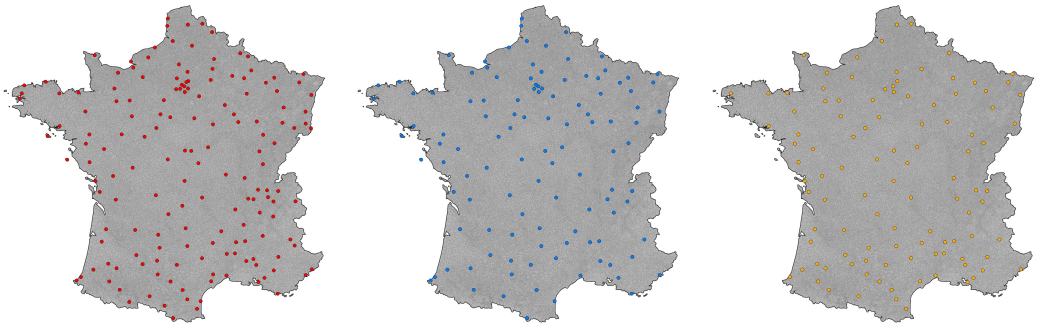
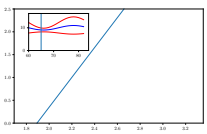


Figure 2.3.1: Stations for which we have hourly data. Left: temperature, center: wind speed, right: light intensity.

For each hour, we select the corresponding observations and interpolate them in order to reconstruct the weather on the entire French territory. An interpolation consists on inferring the value of a variable at query points using a reference data set of known values. One very important underlying assumption of interpolation methods is that of the continuity of the process underlying the data generation. The easiest interpolation method is the linear interpolation: think about a dataset of hourly observations with

[8]There are 95 Département in France



one missing value; to reconstruct the missing value, take the average of the value of the preceding and following hour. There are numerous methods of interpolation, even more so when the data is spatial in nature, all revolving around two main steps. First, given a query point at which one would like to infer the value of the variable, there needs to be a selection rule to know which of the points from the reference data set should be used (in our example the preceding and following values). Second, once these points are selected, one needs a weighting function to know their relative importance in order to obtain the interpolated value (in our example it is a simple averaging, that is weights of 0.5).

We use the natural neighbor interpolation method, well known for its good balance between speed and accuracy. In short, in this case, the first step makes use of the Voronoi tessellation algorithm[9], one is able to define the natural neighbors of a point for which one seeks an interpolated value. These natural neighbors are used in the second step, which performs the actual interpolation as a weighted average of the values of these natural neighbors using a ratio of surfaces as weights (see Fig 2.3.2 for more details).

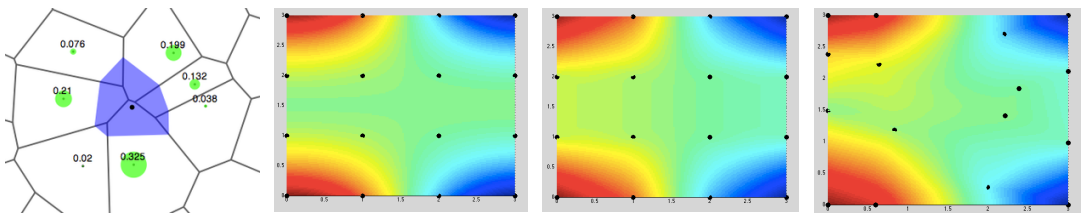



Figure 2.3.2: Left: Voronoi's algorithm is applied once on the reference points highlighted in green to obtain the white surfaces, and a second time on the same points to which is added the query point in the center to obtain the new blue cell. The green circles, which represent the interpolating weights, are generated using the ratio of the shaded area to that of the cell area of the surrounding points. Center left: example of a reference surface (color mapped) to be reconstructed through a natural neighbor interpolation. Center right: interpolated surface with a reference set of 16 evenly organized points, represented in black. Right: interpolated surface with a reference set of 16 unevenly organized points, represented in black. From 16 points one is able to reconstruct the color mapped surfaces which are approximations of the reference one, represented in the center left image.

[9]The Voronoi tessellation algorithm takes a collection of points $\{p_k\}$ in the plane, and then partitions the plane as regions "belonging" to each point, called cells. A Voronoi cell associated with a given point $p_k$ is defined as the collection of every point in the plane whose distance to $p_k$ is less than or equal to its distance to any other $p_{-k}$. Each such cell is a convex polygon.

### Image transformation to recover weather forecasts

Forecasts are obtained from the Global Forecast System (GFS), and come in the form of colormaps, as shown in Fig 2.3.3. We are going to illustrate our methodology on temperature data, but the same exact approach is performed on wind speed data. The general idea is that the pointwise precision is low ($2°C$ per color) but that the overall map contains quite a lot of information through the topology of the colored regions. We describe below how to extract this information.

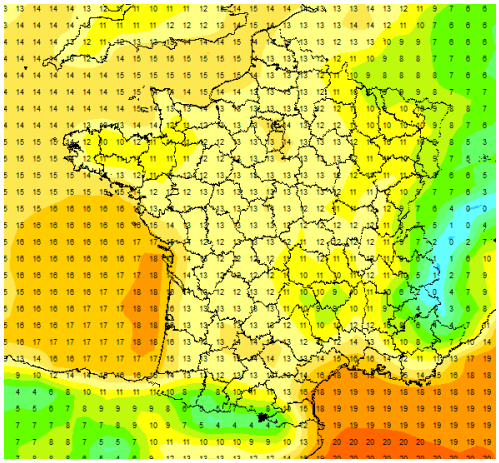
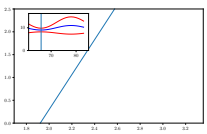


Figure 2.3.3: Temperature forecast from a simulation run by the GFS at 6 a.m. on the 3rd of November 2011, for a forecast at 22 p.m.

**First: image cleaning** To extract the relevant data we first clean the color map from its irrelevant information, namely the temperature in numbers and the administrative borders. Note that this step introduces a small amount of high spatial frequency noise, see Fig 2.3.4 left and center left.

**Second: removal of redundant information** A lot of information is lost from the actual GFS simulations by using a color map representation, as temperature is described as a discontinuous variable: each color has a precision of $2°C$. In order to correct for this, we leverage the fact that all the information contained in this color map, that is the color at each pixel, is actually contained in a smaller set of points. Consider the value at the boundaries between different color regions: by knowing that the interior of a constant



color region has a constant value, one is able to represent all the information contained in the original image by keeping only track of the values at the boundaries. To recognise those boundaries, we perform image analysis, more precisely we use edge recognition methods based on finding high gradient regions, thus obtaining Fig 2.3.4 center right.

**Third: surface fitting** Once we represent the information in this denser form we can perform the last step, which consists in fitting a surface to our newly defined dataset, i.e. the temperature values at the boundaries, which take the form of $(x, y, T)$ triplets. We could perform an interpolation, but these methods are not well suited to reference sets having so much structure. Here, data points lie on curves representing iso-temperatures, so that along such a curve there is a lot of data points, whereas the information is very sparse along the direction of the gradient. In addition the first step introduced some spatial noise which we want to correct to some extent: we allow our fitted surface to take different values than our data points, so as to smoothen out this noise. We define the rigidity of our fitted surface, i.e. a penalty associated to fast changes in the value of the surface, and therefore reduce the importance of the high-frequency noise introduced in the first step. The end result is presented in Fig 2.3.4 right.

It is key to understand that this image is displayed using a colormap close to the one in the original picture to facilitate comparison but that its underlying data is continuous whereas the original image describes temperature by bins of $2°C$. It can therefore be used to query the value at any given point, and these values will change continuously in space instead of discrete jumps in the raw format.

### Autocorrelation lengthscale

We use this dataset to build measures of the weather uncertainty. To do so we measure the auto-correlation lengthscale of our three weather variables of interest: temperature, wind speed and light intensity. This lengthscale measures how much are the weather variables correlated spatially. Our hypothesis is that the auto-correlation lengthscale is

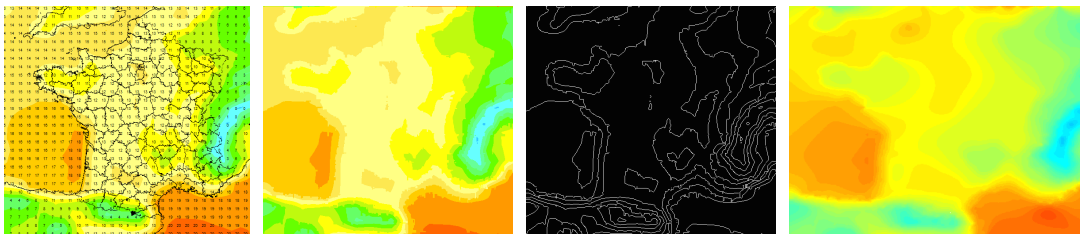


Figure 2.3.4: Left: reference image. Center left: borders and numbers are removed. Center right: edge recognition. Right: final fitted surface.

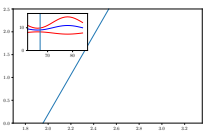
inversely proportional to uncertainty about the variable we are interested in. When it is small, the variable is less spatially correlated, which we interpret as being more difficult to forecast. Conversely, when the auto-correlation lengthscale is large, the variable is very correlated spatially, that is that the informational content of one datapoint is higher for the prospect of using it for the evaluation of a national effect.

More precisely, the argument is as follows:

**First**, renewable production is built by aggregating the forecast of all individual renewable sources. This means knowing the position and capacity of every renewable source, querying weather forecasts for all of these points, modeling the renewable's response to the forecasted weather and adding the forecasted productions.

**Second**, we note that weather is spatially correlated, which means that the closer two points are, the closer the values for a given weather variable (the air temperature at your left hand is very close to that at your left hand, but less so across the city, and even less so across the country). This correlation roughly follows an exponential law: the difference between the values of a weather variable between two points behaves in a linear fashion for small distances and saturates at large distances.[10] The transition between those two

[10]Intuitively, the characteristic lengthscale of autocorrelation represents the distance required between two geographical points on a map of weather forecasts to observe a decorrelation of half of its maximum value. For example on the wind speeds prediction, a characteristic length of 80 km means that if we observe two very distant points (say 1000km) to have a difference in wind speeds of, on average, 50km/h (this being the maximum difference, we are in the saturated regime), then we will observe, on average, wind speed differences of 25km/h for points distant from each other by 80km.



regimes is given by a characteristic lengthscale, a bit less than 200km on average.

**Third**, we observe that the average distance between production points is large enough that the relevant regime of autocorrelation is the saturated part.[11]

**Fourth**, we note that there are two main channels through which the overall uncertainty about renewable production is related to the weather. There is an issue of error averaging, which means that if the weather becomes very spatially uncorrelated, one can expect errors to cancel out relative to a given bias in the forecast. This channel would tend to imply that more spatial variations imply a smaller uncertainty about production. There is also the issue that weather forecasts are numerical simulations and that the mesh size for such simulations, typically 5km for the high precision ARPEGE model of Météo France, implies that the errors are higher as the simulated phenomenons have higher gradients. This means in our case that the uncertainty about the forecast increases as the weather becomes more spatially uncorrelated.

**Fifth**, these two effects are of opposite signs, but our third point is an argument for considering that the averaging of errors is smaller than the simulation errors. Therefore, we expect our uncertainty to increase as the spatial autocorrelation decreases (i.e. more spatial variation).

This can be summed up with the following hand-waving argument: when there is more spatial variations, the weather is more messy, therefore more difficult to predict.

To understand what the autocorrelation lengthscale captures, take two points on a plane and a spatially correlated bounded variable. If those points are infinitely distant, the value of the variable at these points should be uncorrelated. That is that the abso-

[11]For N production points, we compute the N(N-1)/2 pairs of points, consider their distances and compute the average of these distances weighted by the production capacity at every point. In the case of the wind, we have an average distance of 459 km, in the case of the photovoltaic production we have an average distance of 499km.

lute difference between the variable taken at those two points should have a given average value. Conversely, two points infinitely close should have the same value, i.e. a zero absolute difference between the variable taken at those two points. The question is how fast is the transition between those two limit cases. First, we define the average absolute difference between two points when distant of a given value. Second we extract a typical lengthscale.

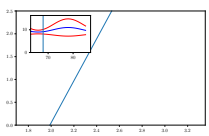
To define the average absolute difference between two points when distant of a given value, we consider at a given point in time every possible pair of points in our dataset. For a given pair we compute its distance and its absolute difference in value (in black in Fig.2.3.5). For 100 datapoints we obtain 4950 pairs. We then use a kernel smoother in order to obtain the average non-parametric autocorrelation function (in blue in Fig.2.3.5).

To recover a typical lengthscale, we make the parametric assumption that the autocorrelation is exponential in nature. We fit an exponential function through our smoothed data (in red in Fig.2.3.5), and recover the exponential decay parameter as our lengthscale (in green in Fig.2.3.5). We perform this operation for every hour in our dataset and every weather variable. The results are time series for the characteristic lengthscale of the weather parameters.

### 2.3.2 Aggregation of local information

**Wind1DA** *Wind speed (average speed in km/h):* Wind speeds influence the productivity of wind turbines, which are a source of unreliable electricity generation. In general, renewable technologies benefit from a feed-in guarantee by the state. That is, regardless of the trading outcome on all markets, renewable energies will be the first to be fed into the power grid at a guaranteed price.

Consequently, the electricity production of renewable technologies represents a production shock for all actors on the market. The production shock means that the demand



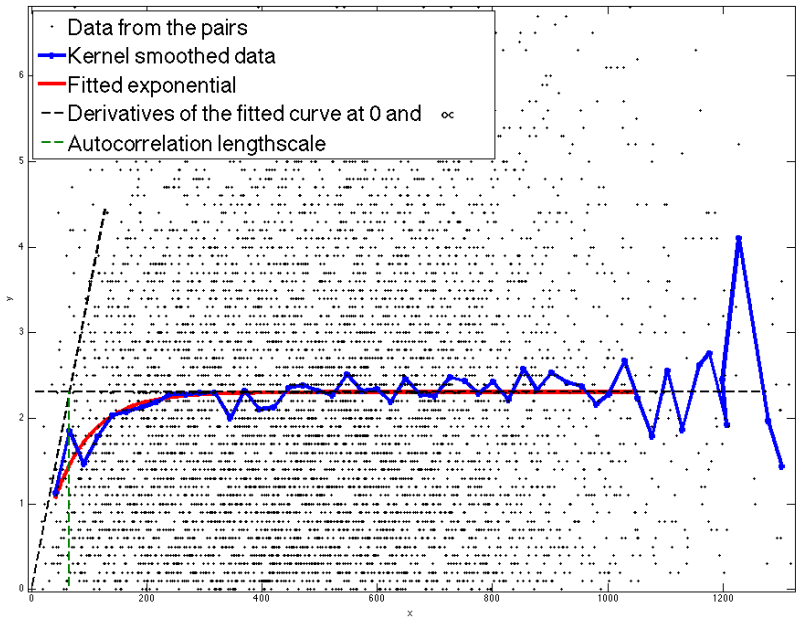



Figure 2.3.5: Auto-correlation lengthscale computation. In black are the points obtained from all the pairs from our original data, that is absolute wind speed differences as a function of the distance between the two points. In blue is the kernel smoothed function from those points. In red is the exponential fit. In black are the derivatives of the fit at 0 and $\infty$. In green is the recovered auto-correlation lengthscale. The unit for the lengthscale is in km.

to be served by traditional electricity producing firms is reduced by the amount that is serviced by the electricity gained from renewable sources.

In the case of wind turbines, the average speed of the wind per hour allows to proxy for the size of the production shock due to the electricity generation from wind energy.

We use hourly windspeed forecast in the form of color maps from the Global Forecast System (GFS), giving the speed by bin of 5 km/h at 10m above ground, and the location and production capacity of the wind turbines present on the French territory, given by the SOeS (service d'observations et d'études statistiques - observations and study department) a department of the French environment ministry.

We consider that all turbines in France are of the same type, that is that they have the same response curve and height.

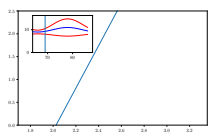
A typical response curve is represented in Fig. 2.3.6. It has three main characteritics: the wind speed at which the turbine starts to produce electricity, called the cut-in speed, the speed at which the turbine reaches its rated output, called the rated output speed, and the speed at which the turbine has to stop to avoid damage, called the cut-out speed. We use data publicly available[12] to obtain a rough estimate of the French average wind turbine characteristics. We use a cut-in speed of 2.5 m/s, a rated output speed of 14 m/s, and reduce arbitrarily the cut-out speed from an estimate of 24 m/s to 20 m/s to account for the fact that a turbine is shut down not when the average speed is too high but when the maximal speed becomes dangerous for the turbine.

Wind speed also increases with height, and turbines are typically between 60 and 80m high. We therefore apply a multiplier to the reconstructed wind speed at 10m.

We seek to reconstruct the French wind energy production from meteorological data. The two adjusted values, the cut-out speed and the speed multiplier, are adjusted by hand to obtain reasonable fits. The reason for this is that the reconstruction of wind speed and aggregate production is computationally intensive, therefore we cannot perform a full-blown estimation. We choose these values with a precision of roughly 10% with respect to their admissible range of values.

We obtain a reconstruction of wind production from day-ahead wind speed forecasts that we compare to actual observed production and to day-ahead wind production forecast computed by RTE, the French grid operator as shown in Fig.2.3.7. We stress here that our aim is two-fold: to link wind turbines' production to weather data and to use forecast data as the market actors only possess this information when bidding. We do not aim at producing better forecasts than the grid operator, the figure is only displayed to show that our methodology produces reasonable estimates (we obtain a correlation co-

[12]http://www.thewindpower.net



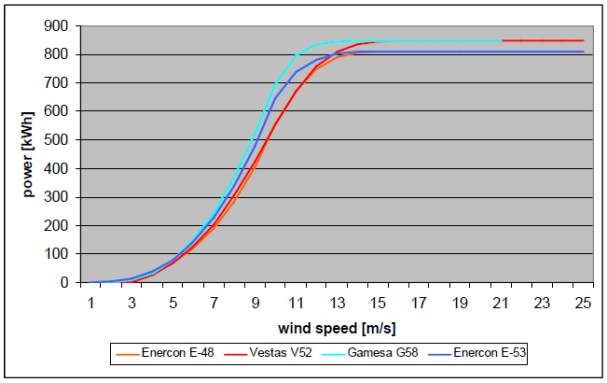



Figure 2.3.6: Typical response curves of different wind turbines

efficient between our forecast and the observation of 0.85 where the grid operator obtains 0.97).

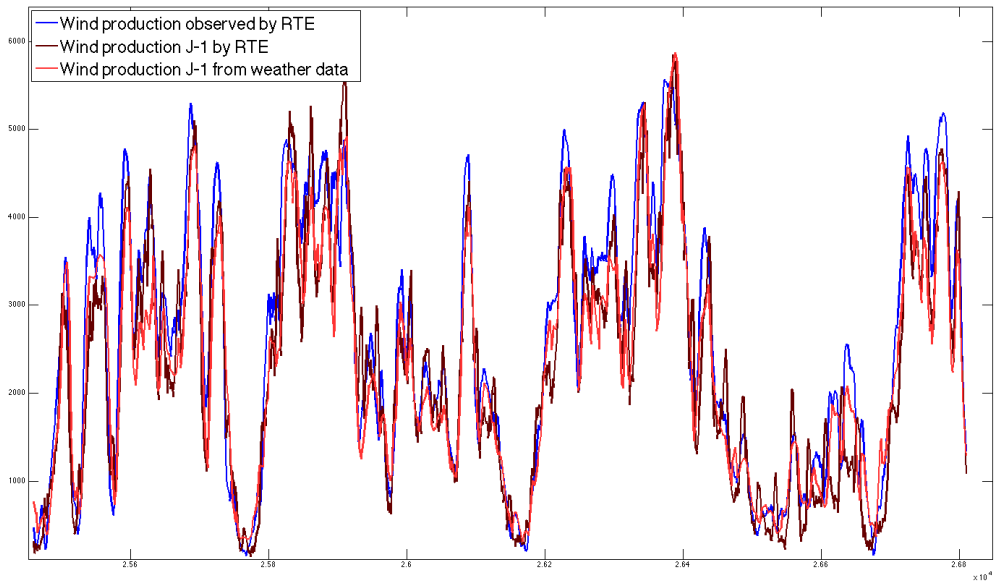



Figure 2.3.7: All curves are hourly production data. The origin of the hours is the first of January 2011, and the production is in MWh. In blue: the observed wind production. In dark red: the day-ahead predictions from the grid operator. In light red: the day-ahead predictions from weather data.

**Tempeff15** We focus on the effect of temperature on the demand of electricity first. In France, a high percentage of the population heats their housing with electricity, therefore cold waves have a high impact on electricity consumption: 2300MW of additional power consumption for every drop of $1^oC$ below $15^oC$, as shown in Fig.2.3.8 sourced from [RTE, 2014], the French grid operator.

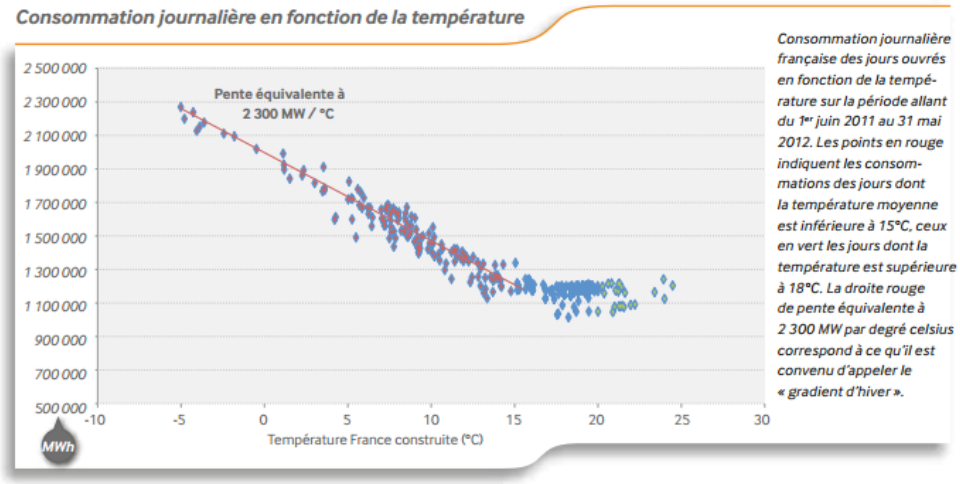
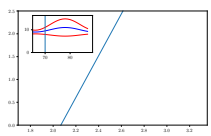



Figure 2.3.8: Daily electricity consumption in France as a function of the temperature, [RTE, 2012]

We apply this information to our observed meteorological data in order to build an effective temperature for France aimed at capturing its effect on consumption. To do so, we reconstruct temperature data for every French *commune*, the smallest administrative unit in France (there are around 36000 of those). We consider population as being a good proxy for potential heat consumption, therefore we apply it as a weight to the *commune* temperature. Lastly, we consider that temperatures saturate at 15°C. This allows us to build an effective temperature taking into account where the population is located and the nonlinearity of heat start up which in turn allows us to account at the country-level for the local impact of temperature on the electricity consumption.

**Solar** Light intensity (in $W.m^{-2}$) impacts the electricity market through multiple channels. The most obvious one is the associated electric production from photovoltaic panels. But there is another channel through which lighting can be seen as impacting electricity consumption: more sunlight decreases artificial light usage. In France, annually, the electric consumption that can be attributed to lighting represents roughly 50 TWh where solar production is roughly 4 TWh.[13]

[13]These estimates are computed by the authors based on numbers coming from [Bertoldi and Atanasiu, 2007], INSEE and EDF



We have photovoltaic production data, which in itself is a blackbox. As we aim to link meteorological data to consumption, we first want to validate the quality of our meteorological data. To do so we reconstruct the photovoltaic production from weather data. We know what are the hourly luminosity conditions on the French territory but also where is installed the photovoltaic production capacity. The SOeS (statistical observation and study department), a branch of government, publishes each year a file containing the installed capacity of renewable energy sources per communes, a French administrative unit with a typical size of roughly 3 $km$.

We use observed luminosity data from MétéoFrance, as there is no hourly forecast of luminosity, and assume a sigmoid response from photovoltaic panels to light intensity with a saturation towards high light intensity, that is approximately a linear response up to a certain threshold. The results are shown in Fig.2.3.9.

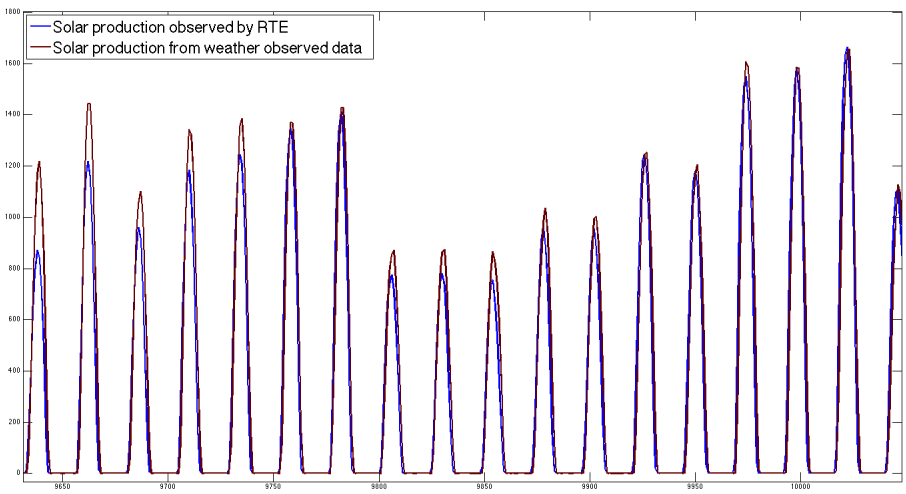



Figure 2.3.9: Hourly solar production in MWh. The time origin is the first of January 2011. In blue: observed production by RTE. In dark red: reconstructed production from observed weather data.

We observe that solar production is much more regular than wind production, therefore it is not possible to build a proxy for lighting consumption that would allow us to decorrelate the effects from production and lighting. We therefore stick to this proxy to

capture the net effect of both channels.

### Other controls

**Tempeff** We also build an effective temperature that does not account for the non-linearity at 15°C following the same methodolgy otherwise as a control.

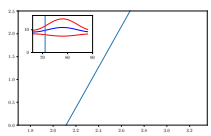
**Roll_Temp**$H$ Variable capturing seasonal trends by using the rolling average temperature on effective temperature (Tempeff15) over the last $H$ hours.

**Roll**$_{avgTH}$ Variable capturing seasonal trends by using the rolling average temperature on temperature Tempeff (no kink) over the last H hours, i.e. the last H/24 days.

**suncycle** Variable capturing intraday seasonality by measuring the intensity of sunlight as a percentage of the maximum daily observation. Midday is defined at the maximum sun intensity every day, i.e. Midday = max(Solar). Thus, suncycle$_H$ = Solar$_H$ / Midday.

**deltasun** Variable computed to proxy for dusk and dawn. It is computed as the absolute difference between suncycle$_H$ - suncycle$_{H-1}$.

**SolarRest** Solar represents estimates of solar production. Therefore, it is highly collinear to the daily suncycle variable since solar production is light dependent. SolarRest is the residual from a regression of Solar on suncycle and captures the unexplained part of solar production on top of pure light intensity considerations. Table 2.1 gives the results of the regression.



| | (1) | |
|---|---|---|
| | Solar | SE |
| suncycle | 1,500*** | 3.903 |
| Constant | 0.876** | 0.383 |
| Observations | 150,959 | |
| $R^2$ | 0.702 | |

*** p<0.01, ** p<0.05, * p<0.1

Table 2.1: Regression of Solar on suncycle

**RteBlackBox** RTE, the French grid operator gives day ahead predictions of the total hourly consumption, which are available at the time of bidding. This variable is called PrevConsoH.

We do not have access to the exact definition of the index and it is thus a black box. However, it is available to the firms at the time of bidding and we want to include it in the demand estimations.

At the same time, it is evident that this variable uses much of the information that we explicitly control for in the regressions, the variables defined above, therefore in addition to the possibility that we might not have all the variables that go into building this prediction for the hourly consumption, collinearity is an issue. In order to have correct coefficient estimates, we adopt an instrumental variable approach by regressing the RTE prediction on our exogenous factors, extracting the residuals and only including the unexplainable component of the RTE prediction in the demand estimation in the form of a separate variable called RteBlackBox.

Formally, RteBlackBox is equal to the predicted residuals ($u$) of the following regres-

sion, where $X$ stands for the vector of explanatory variables: Tempeff15, Roll_Temp24 , Roll_Temp240, suncycle, morning, deltasun and EWH.

$$\text{PrevConsoH} = a + bX + u \tag{2.3.1}$$

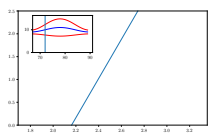
In table 2.2 we give the output of regression 2.3.1 in column 1, which is strong support that our prepared data for exogenous variables is of very high quality. We highlight the significance of all explanatory variables at the 1% level and the $R^2$ statistic of 85.3%.

| | (1)<br>PrevConsoH | (2)<br>PrevConsoH |
|---|---|---|
| Tempeff15 | -682.6*** | |
| Roll_Temp24 | -802.0*** | |
| Roll_Temp240 | -1,175*** | |
| SolarRest | -0.860*** | -0.345*** |
| suncycle | 7,849*** | 7,418*** |
| morning | -4,759*** | -4,398*** |
| deltasun | 10,108*** | 9,010*** |
| EWH | -1,245*** | -1,254*** |
| Tempeff | | -301.4*** |
| Roll_avgT24 | | -687.3*** |
| Roll_avgT240 | | -918.2*** |
| Constant | 77,701*** | 76,651*** |
| Observations | 146,909 | 146,909 |
| $R^2$ | 0.853 | 0.816 |

*** p<0.01, ** p<0.05, * p<0.1

Table 2.2: "Black box" regression on RTE predicted consumption
*Note*: The dependent variable PrevConsoH is the day-ahead prediction by RTE of the total consumption in France.

We highlight that the comparison of columns 1 and 2 gives very strong support to our adjusted measure of effective temperature (Tempeff15 instead of Tempeff), which takes into account the demand behaviour as a function of the temperature. Temperatures above 15$^o$C are considered not to impact demand behaviour [RTE, 2014].



## 2.4 Conclusion

In this methodological chapter we present the different methods that we developed to study in the next chapter the impact that uncertainty about demand shocks can have on suppliers' bids.

We want to be able to describe how bids change shape as a function of a number of regressors. To do so we apply functional data analysis to the bids and argue that the landmark registration technique allows us to compare important features across bids.

Finally, as we are interested in the impact of uncertainty about demand shocks, we note that weather is an important source of uncertainty and introduce a number of metrics, based on the intrinsic structure of weather data or on its relationship to the processes at play when considering consumption or production of electricity.

In the next chapter we will therefore be able to focus on the econometric analysis of our data. More specifically, now that we have defined points that allow us to compare schedules to one another, and that we have defined proxies for the weather uncertainty, we can measure how the slope of the schedules is impacted by the level of uncertainty, and if it follows the predictions of the theoretical model presented in the first chapter.

# Appendix

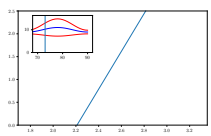
## Appendix 2.A Technical Details

### 2.A.1 Using the kernel density estimation (KDE) in our setting

In order to estimate the first and second derivatives of the bid functions, we use a kernel density estimation. The estimator is essentially a smooth version of a histogram and counts the number of points in moving intervals (called a window) of predefined width along a dimension of the data. In our case, it counts bid points per price interval. In addition, the KDE assigns a weight to each observation based on the distance from the observation to the center of the window. The weighing function is called the kernel.

The observed bid functions are each a multitude of price-quantity combinations. However, a naive kernel density estimation on the observed points of the bid function would be useless since the number of points per price interval does not vary much with the slope of the curve.

The supply and demand functions, although defined by discrete points, whose number changes from bid to bid, are continuous functions. That is that between to successive points, the function is considered to be linear. Strictly speaking, we can therefore define a constant value of the first derivative, and we cannot define values for the second derivative. In order to circumvent this problem we want to smooth our data, which defines functions that are not twice differentiable, by using a kernel density estimate. However, this estimate needs to measure the "density of function", so to speak, and not the density of points: if the function has two successive but distant points, a naive kernel would count no points in between them although our function is actually comprised of a segment of a given length in this region. What such an estimate should instead measure is the arc length of the function represented by the points we have, that is the summed length of



all segments present in the window of the kernel.[14]

This quantity can be computed exactly, however it does not play well with the regular tools for kernel density estimates in stata, so we resolve to approximate it by adding a large number of linearly interpolated points at the unit cent level (corresponding to the minimum bidding unit). The kernel density estimation is then able to estimate the absolute value of the slope of the function by simply counting the points in an interval since the number of points per price interval of constant width varies proportionally with the slope of the function over that interval. This effectively returns the estimates for the absolute values of the first and second derivatives of a smoothed version of our supply and demand functions.

### Hard choices in the code of the KDE

A few specific choices have been made in the code and are detailed here.

**Kernel choice:** First, we use the default Epanechnikov kernel for simplicity. It is generally considered that the kernel choice has significantly less impact than the choice of the bandwidth. The use of the kernel is to weigh more the observations close to the centre of the moving window. The performance of a kernel is judged on the trade-off between variance and bias. The used Epanechnikov kernel is optimally efficient. However,

[14] Consider a continuously differentiable function $f$:

$$\begin{aligned} f\colon [a,b] \subset \mathbb{R} &\to \mathbb{R} \\ x &\mapsto y = f(x) \end{aligned}$$

Then the following parametrization defines the points of the graph of this function:

$$\begin{aligned} g\colon [a,b] \subset \mathbb{R} &\to \mathbb{R} \\ t &\mapsto (t, f(t)) \end{aligned}$$

The arc length of the graph of function $f$ is then:

$$\begin{aligned} L(g) &= \int_a^b \|g'(t)\| dt \\ &= \int_a^b \sqrt{1 + (f'(t))^2} dt \end{aligned}$$

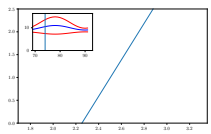
even simplistic kernel functions, such as the rectangular, have a relative efficiency of 93%. Thus, kernel choice is not important and other factors may influence the decision, such as computational effort [Salgado-Ugarte et al., 1994, Silverman, 1986].

**Bandwidth choice:** Second, we hard code the bandwidth selection for computational reasons. The bandwidth of the kernel (and thus the width of the price interval over which points are counted) is determined on the basis of a trade-off between smoothing the original bid function and mixing up information of different parts of the bid function. By smoothing the original bid function, we obtain estimates of the information that our KDE measures (i.e. points in the interval and thereby the slope) that are less sensitive to local specificities of the bid functions. The larger the selected bandwidth, the larger the interval over which points are counted and the stronger the smoothing of the estimates. However, as the width of the interval increases, we mix up more information of a selected point of interest with the information of its neighbouring points. Therefore, in setting the bandwidth we aim to achieve smoothed estimates with a reasonable compromise between respecting local curve information, while not being fragile to steps in the bid function.

For estimates of the first order derivative, these considerations are minor and we could use the default bandwidth, optimal for a Gaussian distribution, to extract the point of maximum slope from the distribution. However, one reason we slightly increase it is to ensure that the distribution of the first derivatives is uni-modal.[15] Furthermore, the selection of the bandwidth in the first stage density estimation impacts both the precision and speed of the second stage estimation. A better smoothing in the first stage gives a large advantage in the second stage estimation[16], thus we have a further incentive to increase the bandwidth.

[15]Uni-modal at the point of inflection in the price-quantity dimension. The smoothing ensures that the selected point is not mistaken due to steps in the bid function that have a very large slope locally, but which is not representative of the neighbouring portion of the bid function.

[16]The gain in computation in the second stage arises from the fact that a stronger smoothing in the first stage produces a more homogenous dataset for the second stage estimation. By more homogenous, we mean that fewer monotone regions of the graph of first derivatives must be interpolated at the unit cent level to ensure that our algorithm works correctly.



For the second derivative the trade-off is more critical: We want to obtain a reasonably broad smoothing to obtain a meaningful selection of points that is not driven by random noise. On the other hand, a large bandwidth reduces the importance of local information of a part of the curve as a consequence of which, selected points (points $k = 2$ and $k = 4$) are pushed towards the point of inflection ($k = 3$). This is due to the maximum point of the first derivative gaining more weight in the second derivative's estimation. The fact that first derivative estimates are already smoothed rather strongly, we can choose a narrow bandwidth in the second stage KDE.

In the end, we select a rather broad bandwidth of 45 units in the first estimation. This gains robustness of the point selection mechanism against noise in the data and estimation speed in the second stage. The bandwidth in the second stage is set more narrowly at a level of 2 units to keep as much information as possible from the first stage estimation and allow sufficient variation to select the $k$ points.

To support our choice, we illustrate the impact of different bandwidths on the first and second stage estimation in figures 2.A.1 and 2.A.2. Our choice is based on an adequate point selection and the fastest runtime.

In these graphs, the top row shows the first stage KDE, over the whole function on the left and zoomed on the right. The large bandwidth in figure 2.A.1 shows the impact of smoothing on the estimates of the first derivative as compared to figure 2.A.2. The second row in both graphs shows the second stage KDE in two versions: Using a wide kernel bandwidth on the left and a tight bandwidth on the right. Again, we disclose the result as seen over the whole function (left) and zoomed on the central price range (right).

The third row details the original demand function with the final point selection given the bandwidth selection as given by the two rows above. Regardless of the first stage bandwidth, we see that a large bandwidth in the second stage KDE easily distorts the point selection. Selected points of type $k = 2, 4$ are either two centred or too wide as a

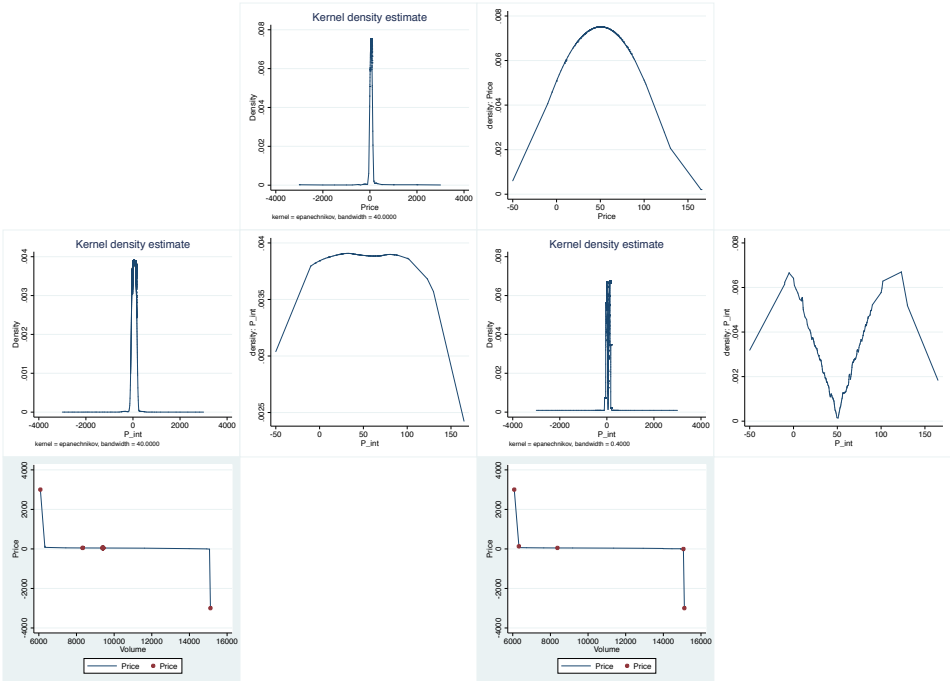


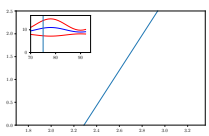

Figure 2.A.1: Comparison of bandwidths: Large bandwidth in first stage

*Note: Large bandwidth in first stage (top row), large bandwidth in second stage (second row left), small bandwidth in second stage (second row right), Resulting selection of points for large bandwidth in stage one and two (bottow row left, A) and selection of points for large bandwidth in stage one and small bandwidth in stage two (bottom row right, B).*

result of the second derivatives being smoothed excessively and not precisely representing the local specificities of the curve.

The right hand side of both figures show that a tighter bandwidth on the KDE can easily mistake large slope changes due to steps in the bid functions as the appropriate points of maximum curvature of the full bid function and thereby make an error. Therefore, we apply a sensitive second stage KDE on rather smooth estimates of the first derivatives, which yields an adequate point selection in our setting (figure 2.A.1B).

The bandwidth selection received much attention in this work in order to obtain a reasonable selection of points based on local information of the curves, while achieving a satisfying robustness to noise in the bid function. We are aware that this subjective setting of the bandwidth is not without consequence for our work. However for computational



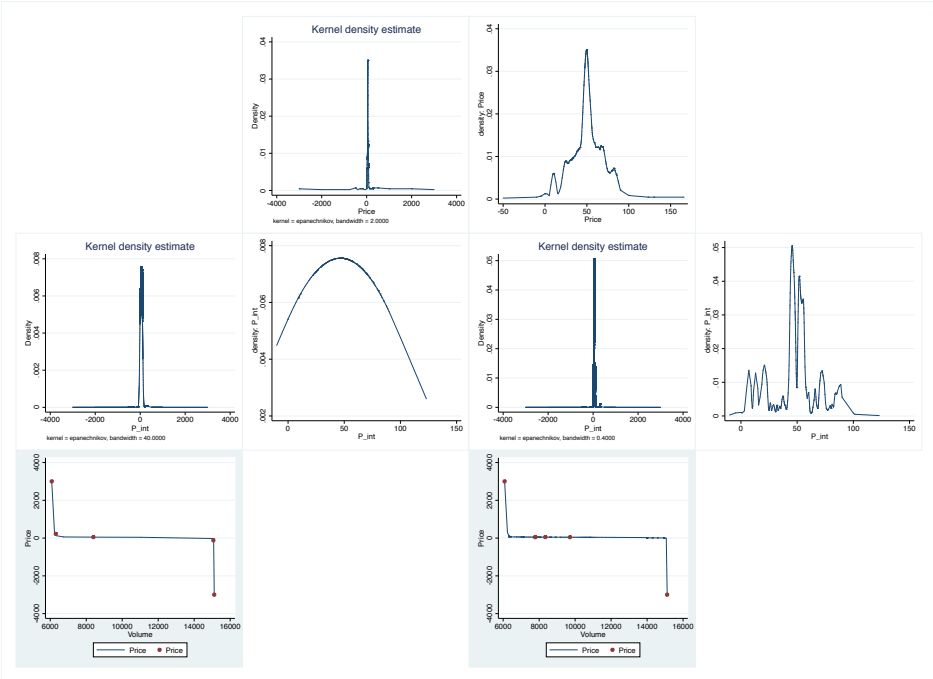



Figure 2.A.2: Comparison of bandwidths: Small bandwidth in first stage

*Note: Small bandwidth in first stage (top row), large bandwidth in second stage (second row left), small bandwidth in second stage (second row right), Resulting selection of points for large bandwidth in stage one and small bandwidth in stage two (bottom row left, C) and selection of points for small bandwidth in stage one and two (bottow row right, D).*

reasons[17], we do not run a full robustness test on this choice ex-post.

### 2.A.2 Outlier detection and removal

In some rare cases, our point selection mechanism does not work. This is the case when curves have very small number of points at a kink and it is thus very difficult to detect their curvature.

As a result, the selected points are then quasi in-differentiable from the next selected point type, i.e. a point of type $k = 2$ is almost identical to the selected point $k = 3$. The code is unable to select the right points due to a data lack on the original curve (second derivative on a constant slope up to POI is zero).

We screen for adjacent points that display quasi no variation in volumes. Figure 2.A.3

[17]The point selection algorithm ran for more than two weeks in the current setting.

shows a histogram of volumes differences over 2 selected points (from $k = 2$ to $k = 4$) and reveals a positive mass point at zero, indicating outliers that do not display any volume variation between points of the same bid function. We use the histogram to identify and drop those outliers from our dataset.

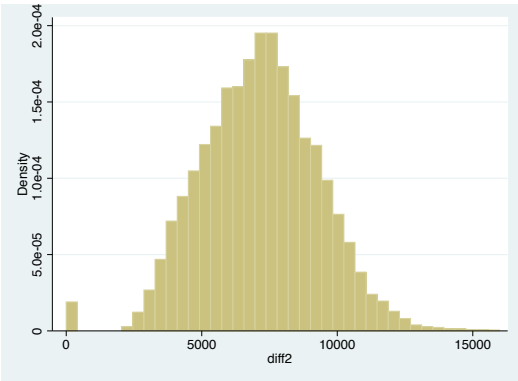
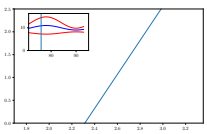



Figure 2.A.3: Histogram of volume variation between points

*Note: The histogram shows the volume difference between points $k = 2$ and $k = 4$ of the same bid functions.*



# Chapter 3

# Investigating the Impact of Uncertainty on Firms with Dynamic Costs: A Case Study of the French Electricity Market [1]

[1] Joint work with Henri de Belsunce.
The weather data was obtained through a research convention with Météo-France - ref. DIRIC/13/024
JEL Classification Numbers: C10, C57, L94, Q41

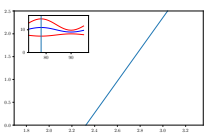
## 3.1 Introduction

In the last chapter, we have given some attention to a methodology that allows us to use functional data for reduced form analysis. In this chapter, we focus on the economic questions that can be asked using such a methodology. Specifically, we focus on an investigation of the effect of uncertainty on the behavior of electricity producers, and testing the predictions of our first chapter, by leveraging the results of our second chapter that allow us to compare schedules to one another.

There exists a consensus that dynamic costs, also referred to as ramping or adjustment costs, are important on the electricity market.[2] These are the costs incurred by a producer when production varies. The importance of uncertainty for the expectation of dynamic costs is shown in chapter 1. Uncertainty itself on the electricity market as well as estimates for the value of ramping costs have been studied empirically by [Wolak, 2007], in the case of step functions. We focus on two sources of uncertainty for traditional electricity suppliers, namely uncertainty about the realization of the market demand and uncertainty from the inherently unpredictable meteorological situation (which affects renewables generation), mainly because those are the two main sources of uncertainty in the span of time covered by our data (2011-2013). There is one blind spot in our analysis: we do not have data about the interconnected countries, which themselves affect the French market and therefore introduce another important of uncertainty. Not taking this effect into account essentially introduces noise in our data and means that we need more data to infer the significance of an effect compared to a case where we would be able to control for it. We propose a methodology to measure this uncertainty and its impact on firm strategies on the electricity market.

Electricity as a market is very important in and of itself ($2 trillion in worldwide sales in 2010). It is also a crucial input for many industries; power outages induce very large

[2] [Anderson and Xu, 2005], [Hobbs, 2001], [Hortacsu and Puller, 2008], [Reguant, 2011], [Sewalt and De Jong, 2003].



costs to society ([LaCommare and Eto, 2004], [Reichl et al., 2013]). The electricity market is, however, quite different from the markets for other commodities in a few respects. First, electricity cannot be efficiently stored. As a consequence, electricity markets are high frequency (prices can update down to 15-min intervals) and firm strategies are purer as they are free of stock management considerations.

Second and in addition to non-storability, a generation surplus cannot be disposed of freely.[3] Thus, generation of electricity must always be matched with consumption in real time (modulo a small tolerance). This represents a hard constraint on the market[4] and forces suppliers to be reactive. However, this reactivity is costly as plant operators incur dynamic costs when adjusting production and the larger the adjustment made, the larger the cost. Hence, suppliers face a trade-off between cheap generation of electricity and costly reactivity to the demand realization. Indeed, no single generation technology exists that satisfies both cheap generation and sufficient reactivity to allow production fluctuations at a reasonable price. Existing generation techniques are either cheap and unresponsive, e.g. nuclear plants, or expensive and flexible, e.g. gas turbines.

Interestingly, we also observe negative prices. In France for example, during the weekend of the 15$^{\text{th}}$ June 2013, the price per MWh dropped to $-200$ €. This contrasts to the yearly average of approx. 45 €/MWh and is generally understood as a sign that subsidizing consumption temporarily is cheaper for a supplier than shutting down a plant [EPEX, 2014].[5] The increase of the share of renewable generation in the energy mix contributes to the occurrence of negative prices on the market. The intermittency of renewables causes large residual demand shocks [EPEX, 2014]. The unreliability of renewable generation also means that more flexible plants (i.e. plants with lower dynamic costs) are required to provide rapid responses to fluctuations in production from renew-

[3] The common assumption of free disposal as made in standard microeconomics is violated.

[4] Mismatches between consumption and generation ultimately result in power outages.

[5] "Negative prices are a price signal on the power wholesale market that occurs when a high inflexible power generation meets low demand. Inflexible power sources can't be shut down and restarted in a quick and cost-efficient manner. Renewables do count in, as they are dependent from external factors (wind, sun)."

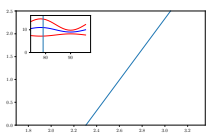
ables [REN21, 2013].

Furthermore, uncertainty arises from the fact that renewable production is a local and dispersed production, but feeds into a national market with a single price. When meteorological conditions change, the geographic production profile also changes. This further complicates the predictability of renewables generation and contributes to the uncertainty that electricity producers face when playing on the electricity market [Meibom et al., 2009].

This paper explores the effect that the absolute level of uncertainty about residual demand has on players' strategies on the electricity market. In the light of the existence of dynamic costs, which are inherent to the production technologies, uncertainty is costly to suppliers as shown in chapter 1. Thus when faced with uncertainty, we expect that electricity producers smooth production volume over time in order to minimize dynamic costs. In a single market interaction with a symmetric oligopoly and linear demand functions, this translates to playing a steeper supply function when uncertainty is high. The detailed intuition behind the predictions tested is given in section 3.1.2.

We show that uncertainty does impact supplier strategies. However, this prediction and result only apply locally to the central, flat and linear part of the supply bid function. Towards the high and low volume extremities of the bid functions when capacity constraints start to matter, bid functions are stepper and the effect of uncertainty vanishes. Furthermore, we observe results that indicate that demand-side bidding is also impacted by uncertainty.

We focus on the French one-day ahead market, EPEX Spot. This market is a divisible goods auction and particularly suited for our analysis as we observe data on the full aggregate bid functions for both supply and demand. We introduce the market's auction format and rules in section 3.2.



The dataset and its sources are presented in section 3.3. We develop our identification methodology in section 3.4. Our empirical strategy relies on the non-parametric, comparable point selection technique presented in chapter 3.3. We reuse the selected points of the previous chapter for our analysis here. We present and interpret the results in section 3.5. Finally, we discuss some overarching points in section 3.6 and conclude in section 3.7.

### 3.1.1 Literature review and contribution

There exists a literature on supply function equilibria initiated by [Klemperer and Meyer, 1989]. In traditional models, firms choose between quantities (Cournot) or prices (Bertrand) as their strategic quantities. In the intermediate case, firms choose a relationship between quantities and prices, namely a supply function. This is the focus of the supply function equilibrium models. A key ingredient of these models is uncertainty.

Supply function equilibrium models are very relevant for the analysis of electricity markets, since many electricity market designs allow firms to submit a price-volume function rather than a specific price or quantity. [Green and Newbery, 1992], [Newbery, 1998] and [Bolle, 1992] have used these models to analyze competition on the electricity markets.

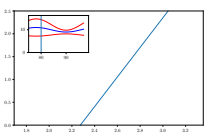
These papers have contributed to a broader investigation of the competition on the electricity markets, which has also been looked at from empirical perspectives [Wolfram, 1998, Borenstein et al., 2002]. While those initial papers have focused on the supply function equilibria of the market, they have abstracted from some technological specificities for the sake for simplification.

One such aspect that we are interested in and that has been the subject of research in recent years is the importance of dynamic costs for electricity production.

Our first chapter extends [Klemperer and Meyer, 1989] to derive predictions on firms facing dynamic costs in a supply function oligopoly under uncertainty. When varying production is costly, suppliers take these costs into consideration by submitting steeper functions when facing more uncertainty, in order to limit the range of variation in production. [Reguant, 2011] develops a model and an empirical strategy to measure dynamic costs on the Spanish one-day-ahead electricity market. She finds that "complex bids", which allow firms to minimize dynamic costs by linking production in one time period to production in a subsequent time period, reduce the volatility and the level of prices on the market. Her work is also unique in terms of data availability. By using individual bid functions she is able to produce estimates of start-up and ramping costs per production technology.

In order to quantify dynamic costs on the Australian electricity market, [Wolak, 2007] derives a methodology to recover estimates of the parameters of parametric cost functions at the level of the production unit. His identification is based on the assumption that each profit maximizing supplier knows the distribution of shocks on the demand function when playing on the market. Uncertainty is thus an explicit ingredient of his paper and he captures two sources of uncertainty in a single index: (i) the uncertainty from not knowing the aggregate supply function served by all other suppliers and (ii) the uncertainty about the realization of the market demand. The recovered cost functions quantify the cost of varying output. Forward contracts are useful to avoid output variations. By comparing the observed level of forward contracting (assumed to be the profit maximizing choice for production variation) with the theoretical minimum cost production pattern, he does not find support for ramping costs.

We contribute to this literature by providing an empirical analysis of the French electricity market. Specifically, we look at the impact of uncertainty on supplier strategies and take this as evidence that dynamic costs matter. Our approach to separate out



the uncertainty from market demand expectations and predictability of renewables generation is novel. Both proxies for uncertainty used are new, uncertainty from market demand is inferred from the prediction errors that firms make in a demand estimation and uncertainty from renewable production is computed in a bottom-up approach from local weather forecasts. Instead of opting for a time series regression, we understand all hourly auctions as a cross-sectional dataset and control for the time of the day by using continuous transition variables for daytime periods. Similarly, we control for seasonality using continuous variables rather than dummies. Thereby, we are able to leverage our dataset and increase the sample size for each of our regressions and improve the precision of our estimates.

Furthermore, our work contributes to the empirical literature testing strategic behaviour of market participants. Generally, these studies focus on point-wise analyses for reasons of data availability. Not only does this cause endogeneity problems when the data used is equilibrium data, but also the analysis is restricted to an understanding of the usually observed outcomes of the market.

In our setting, we benefit from an interesting dataset in which we observe full aggregate bid functions of players. The functions describe the players' behaviour both in the region where the equilibrium is likely to occur as well as in regions that rarely have an impact on the equilibrium outcome. As such, they provide a much fuller description of the firms' strategies. The additional information contained in the full aggregate bid functions has been used extensively in theoretical work (notably in the supply function equilibria literature mentioned above). However, few papers exploit these full bid functions empirically.

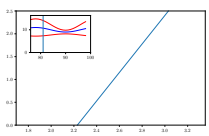
For the government bond market, [Préget and Waelbroeck, 2005] and [Özcan, 2004] use a parametric approach to this functional data for a description of the variation of bid functions with respect to exogenous factors and an investigation of the revenue superior-

ity of the uniform or discriminatory multi-unit auction mechanism, respectively. On the electricity market, [Wolfram, 1999] leaves the analysis of equilibrium data to investigate duopoly power of firms on the UK day-ahead spot market. Instead, she uses information from the whole aggregate supply function to investigate the impact of price caps for electricity producers. Using an analysis conditioned on 25 different demand levels, she shows that the introduction of price caps resulted in a counter-clockwise rotation of the aggregate supply function. She relates these results to produce a lower bound on the extent to which firms can increase their prices above marginal costs when regulatory pressure makes it advantageous to do so. Thereby, she contributes empirical evidence for the distorting effects of price caps.

Our work adds to this empirical literature using the information contained in the full bid functions by developing a non-parametric approach which allows to condition our analyses on multiple, representative points of the bid functions. The statistical ingredients rely on [Silverman and Ramsay, 2005] and are detailed in chapter 3.3. Thereby we are able to leverage our dataset, increase the sample size in individual regressions as well as obtain a fuller picture of the effects of exogenous variables on the behaviour of electricity producing firms. We emphasize that out approach allows to overcome structural restrictions underlying previous parametric approaches, e.g. the symmetry of the logistic function used in [Préget and Waelbroeck, 2005].

### 3.1.2 Theoretical prediction

We test the impact of uncertainty of supplier strategies by testing the prediction that suppliers bid steeper supply bid functions when faced with a larger uncertainty concerning the outcome of the (residual) demand realization, for which the traditional supply function equilibria framework provides no prediction at all, which means that our null hypothesis, that there is no effect of uncertainty of the slope, corresponds to the regular SFE framework.



In a discontinuous setting, where the supplier produces volume $Q_H$ of electricity in hour $H$, we assume that he faces a cost function $C_i(.)$ for each production plant $i$. This cost function depends on both marginal costs of production as well as the dynamic costs for changing production rapidly: $C_i\big((Q_H), (Q_H - Q_{H-1})^2\big)$. The larger the variation in production between hours, the larger the dynamic costs. Even when the expected residual demand is constant, there are still fluctuations in the production due to possible shocks to the residual demand. The larger the shocks, the larger the change in production and thus the larger the dynamic costs. Consequently, increased uncertainty (as represented by shocks on the demand function) translates into increased expected dynamic costs. We assume that the profit maximizing supplier knows the distribution of shocks on the demand function when choosing his supply function. In order to minimize these costs, the producer can choose a steeper supply function when uncertainty is high. We want to test this prediction.

We illustrate the intuition behind this prediction using a stylised case in figure 3.1.1. The graphs depict a situation in which a single, risk-neutral supplier bids a supply function to supply electricity in the hours 9 and 10 of the next day. For both hours, the supplier faces a constant expected residual demand function represented by $E(D)$. In a static optimization problem, the supplier would bid a supply function $S_0$ in both auctions.

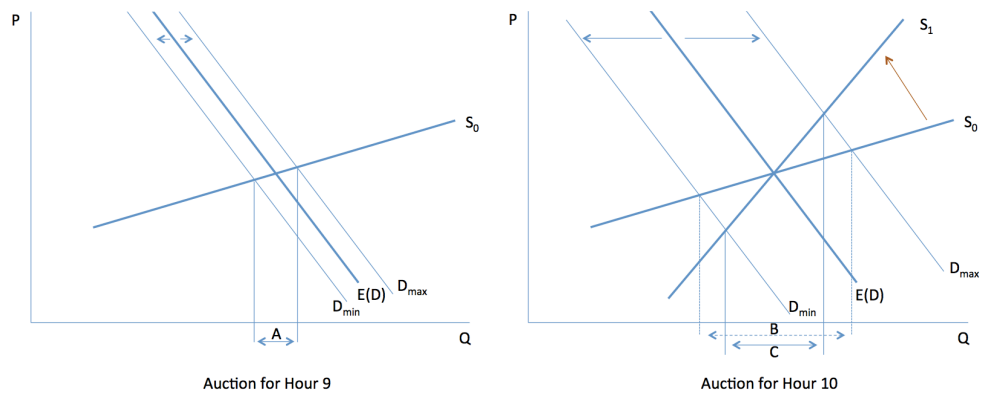



Figure 3.1.1: Illustrating the effect of increased uncertainty.

The uncertainty in the market is represented by the width of the envelope of shocks

that affect the residual demand function (represented by the arrows on $E(D)$). Thus, in each hour, the residual demand fluctuates between $D_{min}$ and $D_{max}$, where the range between the extremal demands may vary from one hour to the next.

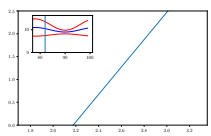
Before submitting a supply function to the market, the supplier estimates the distribution of probabilities of demand shocks that he will face. In hour 9, the supplier is able to rather precisely predict the realization of the demand function in the auction, i.e. it realises within a tight confidence interval. In hour 10, however, uncertainty in predicting the outcome of the demand realization has grown strongly as represented by the much wider confidence interval on the demand realization.

Given a fixed supply bid function $S_0$, the possible range of quantities to be produced by the supplier when going from hour 9 to hour 10 has increased due to the increase in the size of the uncertainty (interval on the Q-axis has grown from length $A$ for hour 9 to the dotted length $B$ in hour 10).

Now, we assume that the supplier faces dynamic costs, i.e. it is costly for production to vary on top of any traditional marginal cost consideration and the larger the variation, the larger the cost. Then in the case of a fixed supply bid function ($S_0$ in both auctions), an increase in uncertainty implies an increase in expected dynamic costs.

The supplier's reaction to increased uncertainty is therefore to bid a steeper supply function $S_1$ in order to trade-off static optimality and dynamic effects. As a consequence, the range of volumes produced in equilibrium is reduced (the firm produces in the range $C$ instead of $B$). When seen over time, these considerations lead to a smoother production as compared to a constant supply curve: demand shocks are absorbed through a higher price volatility and a lower production volatility.

If cautious behavior under high uncertainty is true for all firms on the market and



each firm has the same expectation of the probability distribution of the uncertainty, then the reaction of bidding a more price inelastic supply function to increased uncertainty should be observable on the aggregated supply function.

We emphasize that this prediction relies on linear demand and supply functions and does not incorporate capacity constraint considerations (both upper and lower bounds on the production volume of plants), which are also important on the market. Furthermore, we have outlined our prediction using a discrete time-setting. The continuous version of this analysis on dynamic costs is explored in detail in chapter 1.

The present paper tests this mechanism empirically and understands an increase in the slope of aggregate supply bid functions due to an increased level of uncertainty as evidence that firms minimize dynamic costs across auctions.

## 3.2 The EPEX Spot Market

### 3.2.1 General background

The EPEX Spot market is an auction market, which allows firms to trade electricity 12-36h ahead of delivery. It covers France, Germany with Austria and Switzerland. The volume traded on EPEX Spot represents 12%, 40% and 30% of the total electricity consumption in these countries respectively in 2013 [EPEX, 2014].

The EPEX Spot market has considerably gained in importance over time, and the daily trading volume has almost quadrupled since 2005, whereas the total electricity consumption has essentially remained constant. The graph in figure 3.2.1 shows these trends very clearly. Furthermore, it shows the significant volatility of the market trading volume (as indicated by the width of the grey-shaded confidence interval).

On the EPEX Spot market, the participants submit supply or demand bid functions

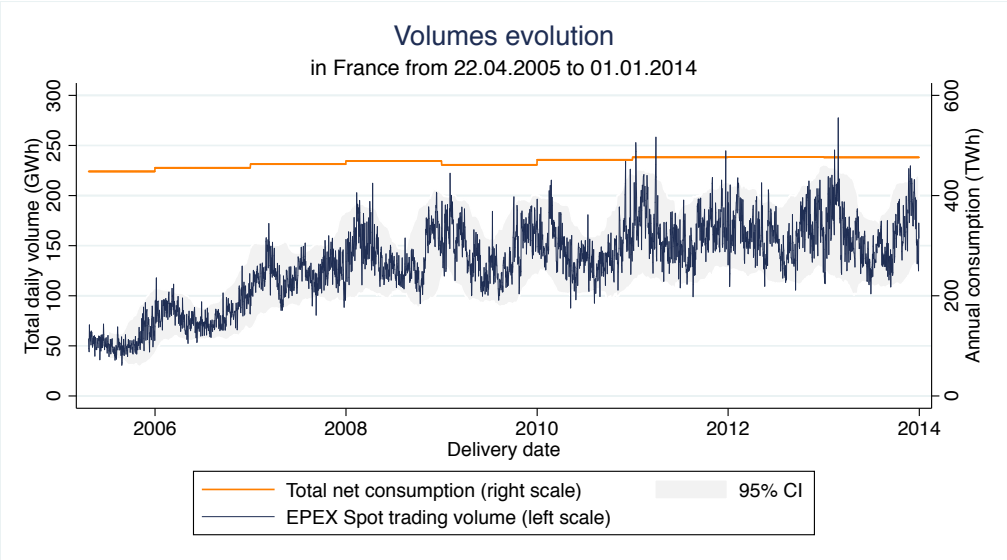
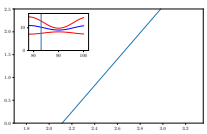



Figure 3.2.1: Traded volume plotted against total annual consumption

Note: Total consumption is netted of the electricity withdrawal at the level of the production unit. The 95% confidence interval is based on a 150-days moving window and assumes that volumes are normally distributed in the time window. GWh and TWh stand for giga and terawatt hours, respectively.

to be able to meet their next day's supply commitment. This market is important, because it allows the firms to adjust their portfolio to the upcoming demand. The market matches business to business trades, where producers (the suppliers and transmission system operators) and industrial consumers may participate.

The EPEX Spot market settles in a three-headed market that firms use to achieve their desired power position: The long-term bilateral contracting market, the day-ahead market and the intra-day market. Energy cannot be stored, thus a precise power position must be achieved at each point in time. Firms thus face a trade-off between cheap up-front sourcing and costly uncertainty. The closer the market gets to the delivery of its power, the less uncertainty does the firm face in determining its power requirements (pushing firms to wait until the last minute to fill their energy position). However, the imperfect flexibility of the electricity production landscape cannot satisfy the whole demand short-term at a reasonable price, hence firms must anticipate their requirements in order to obtain cheaper power. Consequently, these three markets complement each other to allow firms to gather a power position at a reasonable price.



### 3.2.2 Auction rules and mechanism

We present the rules here that applied during the period for which we have data.

The EPEX Spot auction occurs daily, all year-round, and proceeds as follows: the order book closes every day at noon for contracts of the following day, results are published two hours later. Bids may be submitted 24/7 from 45 days prior until the closing of the books.

Tradable contracts exist for each hour of the day and firms submit an individual bid function for each of these hours, i.e. a separate, simultaneous auction is run for all hours of the following day and trading is specific for each of these hourly tranches.

The bid submission must be a supply function (or a demand function depending on the position of the firm) with at least 2 and at most 256 price/quantity combinations for single contract orders. The final bid function, thus, consists of the explicitly submitted points and all linearly interpolated points between them. The bid curves must be monotonically increasing for a supply function and vice versa for a demand function. Orders are transmitted via an online IT-platform and a redundant confirmation process aims to avoid erroneous bids. Bids are anonymous, and the final electricity distribution is done via the French distribution network controlled by RTE EDF Transport SA.

Prices are specified in €/MWh with two decimal digits and must range from -3000€/MWh to +3000€/MWh. Quantities are specified in whole MWh. In addition to single contract orders for an individual hour, bidders may submit block orders.

These are combined single contract orders with a minimum of two consecutive hours. The vital difference with multiple single contract orders is the "All-or-None" condition, namely that the executions of the individual contract orders forming the block are dependent on one another. That is for a block order covering hours 17 to 20, the quantity

demanded for the hour 17 is only awarded if the corresponding quantity is also awarded for the hours 18, 19 and 20. Each registered bidder account is limited to a maximum of 40 block orders per delivery day, each of which is limited in volume to 400 MWh (approx. equal to 0.25% of total daily volume traded on EPEX Spot).

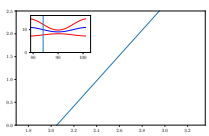
The price-quantity determining mechanism is a uniform price, multi-unit auction mechanism: the summed demand and supply curves are computed and the intersection of these gives the equilibrium price and quantity pair.[6] The market clearing mechanism takes into account single and block orders simultaneously and hence solves the corresponding programme by an algorithm of full enumeration of possible solutions, where each partial solution is verified to provide real, compatible prices. The mechanism works under a time limit. In the case of a curtailment, i.e. a disequilibrium with disproportionate prices due to unmatched supply and demand or an abnormal price for a specific hourly contract, the system proceeds to a second price fixing.

Of particular interest is the clear distribution of information. Ex-ante bidding, firms in the market know the identities of the rival bidders they face (but neither their individual bid functions nor their results in past auctions), the history of aggregated equilibrium prices and quantities up to that day, their clients' past demand realizations and their individual long term contracting position. Upon the clearing of the market, the aggregated supply and demand bid functions, equilibrium quantity and the equilibrium price

6

> The Auction takes place daily, after the Order Book has closed. The price corresponds to the Matching of Exchange Members' aggregate supply and demand curves of both Single Orders and Block Orders for each Contract. The price determined by the algorithm at the time of Auction is the price at which all Trades will be executed. For price determination purposes, the Exchange Member's interest is assumed to be linear between two price/quantity combinations. The price determination algorithm aims at optimising the total welfare, i.e. the seller surplus, the buyer surplus and the congestion rent including tariff rates. The algorithm determines the execution prices, the matched volumes and the net positions of each coupled market if applicable. It also returns the selection of blocks that will be executed and other complex Orders allowed in other Coupled Markets1 if applicable. The presence of all-or-none Block Orders in the Order Book makes necessary the use of a specific search algorithm, in order to determine a market clearing price.

[EPEX, 2018b]



become common knowledge. Each bidding account is informed of the contracts it has been awarded, i.e. the individual quantities to be sold and bought through the system.

## 3.3 Our Data Explained

### Auction market data

We have data from the French EPEX Spot market for the period 01.01.2011 to 30.06.2013. This is the latest period, where no significant changes in the auction rules have occurred and where data for all variables can be observed.

We observe the full aggregate bid functions for the day-ahead auctions of each hourly contract for both supply and demand. We understand the dataset as a cross-section rather than a time-series[7] and focus on weekday trading contracts only. This sums up to about 31 500 observations.[8] A single aggregate bid function is the sum of the individual bid functions, which are not available. We also observe the equilibrium price and quantity for each auction. Moreover, we observe the block bidding results at the equilibrium solution only. We ignore the blocked aspects and treat subsequent auctions as independent from one another.

The two graphs in figure 3.3.1 show the aggregate supply and demand bid functions for the same hour of the same day. For a glimpse at the variation of bid functions over time, see figure 3.3.2. The table 3.1 sheds some light on the raw data. For further details as well as the plotted distribution of realised market equilibria, refer to appendix 3.B.2.

Finally, we reuse the data output from chapter 2. Specifically, we reuse the specific points extracted from the aggregate demand and supply bid functions, which are comparable across auctions. Why these points are useful for our analysis is explained in the

[7]This is supported by the graph in figure 3.2.1, which shows a flat total consumption and average trading volume on EPEX Spot since 01.01.2011.

[8]31 500 observations $\approx$ 2.5 years of hourly ($*365*24$) demand and supply ($*2$) functions for weekday trading ($*5/7$).

methodology section 3.4.

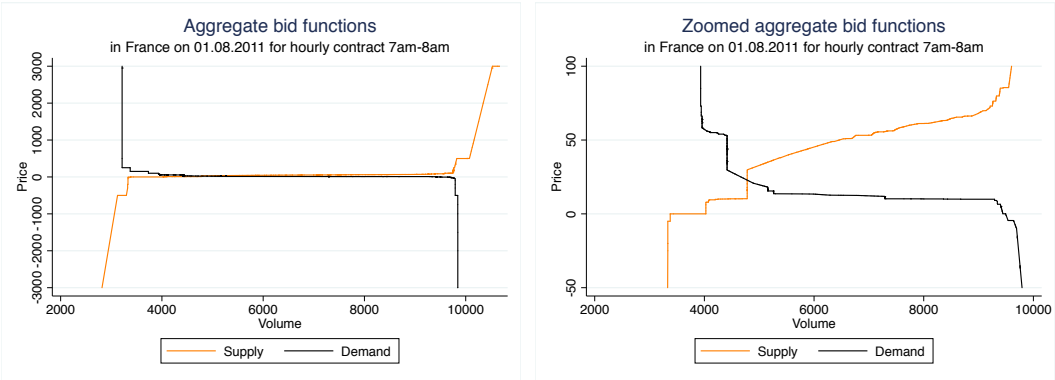



Figure 3.3.1: Example aggregate demand and supply bid functions

Note: The right-hand-side graph is a zoom of the left graph on for the price range −50 €/MWh to +100€/MWh.

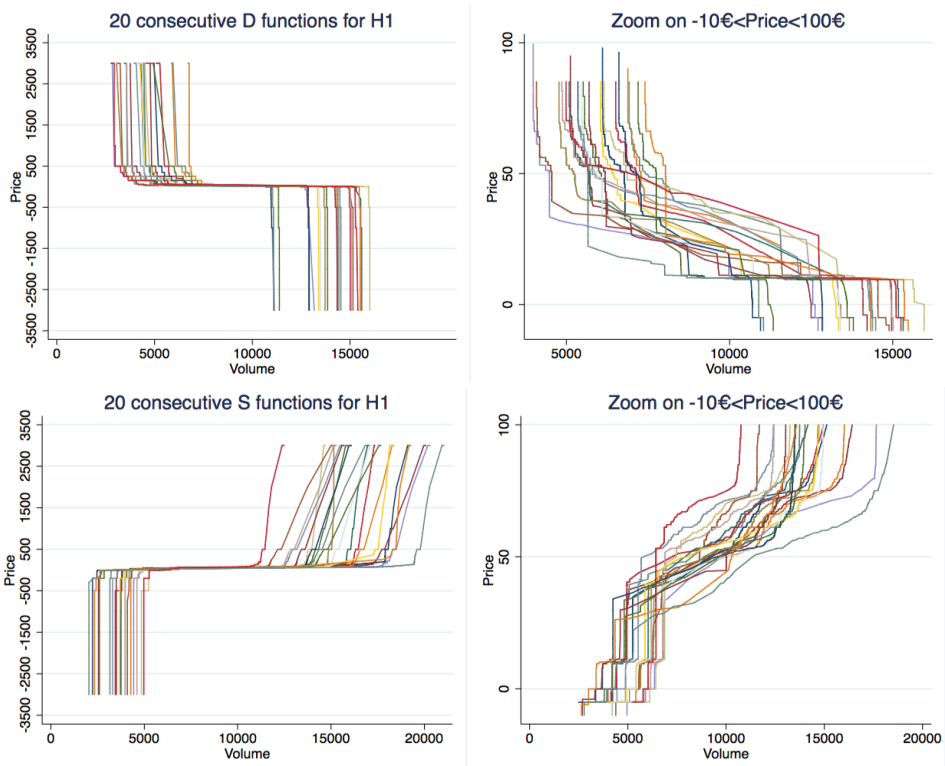
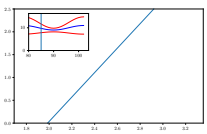



Figure 3.3.2: Aggregate bid functions for 20 consecutive days

Note: The graph shows 20 consecutive aggregate demand and supply functions for the contracts on hour 1 (between 12 am and 1 am) for the time period 11/12/2011 to 31/12/2011. The graph on the right is a zoom on the price elastic region of the curves on the left.



| | Mean | Median | Std. Dev | Min | Max |
|---|---|---|---|---|---|
| Total daily volume | 161,912 | 159,313 | 25,059 | 99,054 | 277,531 |
| Average realised daily price[9] | 46.6 | 48.3 | 17.2 | -39.0 | 381.2 |
| Minimum demanded agg. volume[10] | 5,030 | 4,968 | 1,467 | 914 | 11301 |
| Maximum demanded agg. volume | 13,327 | 13,222 | 2,212 | 4,990 | 23,254 |
| Minimum supplied agg. volume | 3,721 | 3,526 | 1,344 | 618 | 10594 |
| Maximum supplied agg. volume | 14,390 | 14,142 | 3,051 | 6,580 | 35,356 |
| Bid points per demand function | 543 | 531 | 163 | 115 | 1,253 |
| Bid points per supply function | 640 | 632 | 143 | 184 | 1,283 |
| Bidders per auction[11] | - | - | - | 1 | 101 |

Table 3.1: Some descriptive statistics

## Exogenous factors

Regarding weather statistics, we have hourly previsions for temperature, wind and cloudiness from the GFS (Global Forecast System) as well as hourly observations for these quantities and luminosity from MétéoFrance . The previsions from the GFS are in the form of weather maps that are outputted from simulations that run one-day ahead at 6 am. This is the weather information that market participants have access to when bidding on EPEX Spot.[12] The weather observations are in the form of tables for specific weather stations (between 100 and 200 depending on the specific parameter of interest).

Moreover, we have the location of the total installed capacity per generation type

[9] Average price is volume weighted over the 24 hourly contracts of the delivery day.

[10] Minimum and maximum volumes for both demand and supply refer to the aggregate volume bid on the market for a single hour contract at the extremal prices of +3000€/MWh or −3000€/MWh.

[11] Due to the anonymity of the auction procedure, it is unknown which bidders submitted bids. Consequently, it cannot be deduced how many bid steps a typical bidder submits. Number of registered bidders for the French EPEX Spot market as of 01.10.2014.

[12] The next weather simulation run takes place at 12 noon and is therefore not being used by the bidders on the EPEX day-ahead market, as the deadline for submitting bids is precisely 12 noon.

(i.e. wind turbines, solar panels, etc.) at the level of the postcode, that is roughly a 3km precision. We obtain this data from the SOeS, a branch of the French government producing data on environmental issues at large.

Population data and data on the level of the domestic production from the manufacturing industry is obtained in monthly steps from the French National Institute of Statistics and Economic Studies (INSEE). From the same source, we obtain the spot prices for petrol and natural gas as well as the import prices at the border for coal, which we use as a proxy for the domestic prices. Prices for the European CO2 emission certificates are taken from the Portuguese secondary market (SENDECO$_2$) for European Unit Allowances (EUA).[13]

As a very coarse proxy for generation from hydro power plants, we have the total weekly stock of water in domestic dams (in the form of the summed height of all dam water levels in France) from RTE the grid operator.

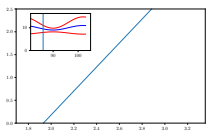
## 3.4 Methodology

We want to identify the impact that the level of uncertainty has on the price elasticity of the aggregate supply function. In data terms, this means that we aim to regress the slope of (aggregate) supply bid functions on a proxy corresponding to the uncertainty that existed at the time of bidding. The uncertainty may come from two different sources: (i) uncertainty about the realization of market demand and (ii) uncertainty on the generation from renewables. Both types of uncertainty affect the residual demand curve faced by each supplier.[14]

This regression is able to explain how supply firms adjust their bidding strategies to the expectation of demand shocks that they face. Statistical significance of the level of

[13]Each unit EUA permit allows one tonne of CO2 emissions.

[14]Renewable generation benefits from a feed-in guarantee on the market and thus reduces the residual demand for all traditional electricity producers.



uncertainty on the slope of the supply function would be evidence that firms take the strategic considerations of dynamic costs into account.

First, in section 3.4.1 we show the final regression of interest. Sections 3.4.1 and 3.4.1 then detail the theory and empirics underlying the variables that feed into the final regression.

Some of the information used in our analysis is drawn from the bid functions of the EPEX Spot market. As introduced in section 3.3, we observe the full aggregate bid functions for both supply and demand, the shape of which (and thus the information that we aim to extract from them, e.g. their slope) varies differently at different points (recall the graphs in figure 3.3.2).

Generally speaking, a regression aims at quantifying the impact of some independent variables on a dependent one. The dependent variable is most frequently numerical, and the independent variables explain part of its value. Here the dependent variable is functional in nature, that is that we aim to describe how the supply function changes shape with respect to some independent variables. One observation is formed of one function coupled to the value of some independent variables. We therefore adopt a functional data analysis approach, which allows us to condition our analysis at specific points $k = 1, .., K$ of the functions. This approach allows us to define comparable points across auctions, that is different functions, in order to derive insights.

More precisely we want to quantify how uncertainty affects the strategy of bidders from one hour to the next. For this we cannot rely on a standard estimation of the overall demand or supply functions from market outcomes, we want to actually measure how the functions that we observe change shape.

The methodology to select comparable points across auctions is detailed in chapter 2.

This appendix also evaluates the results when applying the technique to our data from the EPEX Spot market. Figure 3.4.1 shows the selected points on an exemple of demand and supply curve.

The different types of points selected capture different information of the aggregate bid functions. The most important point is the one we label $k = 3$, which corresponds to the central part of the bids. This point is most relevant for equilibrium determination.[15] The points $k = 2, 4$ are the points of maximum curvature and represent the transition points between the central (very price elastic) region and the outer (very price inelastic) regions of the bid function. Last, we have the points $k = 1, 5$ which are imposed by the auction rules and are the endpoints of the bid functions.

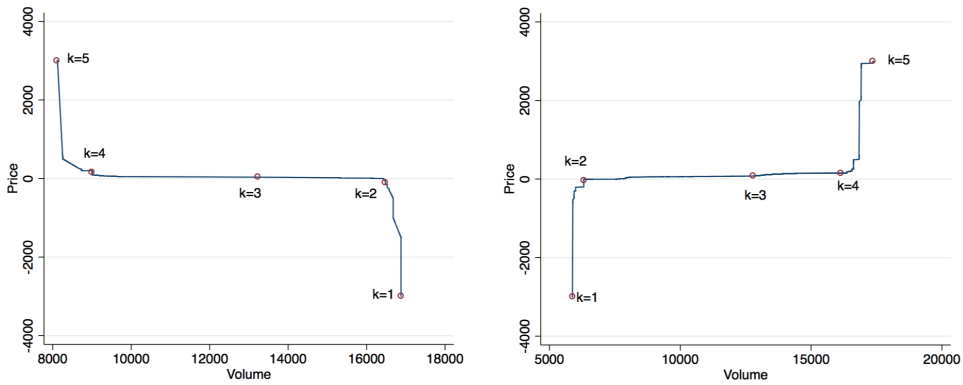
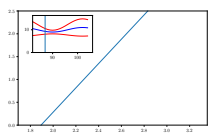



Figure 3.4.1: Selected points on original bid functions
*Note:* The demand function left, the supply function right, the graph superposes and names the points selected according to the methodology of section 3.4.

In chapter 2, we also detail the choice of setting $K = 5$ and show that this choice allows us to improve the precision of our analysis by a factor of 50 when it is conducted on the 5 points simultaneously.[16]

[15] See figure 3.B.2 for a glimpse at the distribution of equilibrium outcomes.
[16] We briefly mention that the evaluation of the point selection has revealed focal price points for the points $k = 2, 4$. These points are however rarely relevant for equilibrium determination.



### 3.4.1 Regression methodology

#### Identification

At each of these comparable points, we want to identify the effect of uncertainty on the slope of the supply function.

Defining $S'_{i,k}$ the slope of the supply function of auction $i$ at point $k$ in the quantity (X-axis) - price (Y-axis) dimension, $\boldsymbol{X^S}$ being the vector of exogenous variables, $\text{PLU}^D_{i,k}$ being the proxy for the level of demand uncertainty, $\text{PLU}^R_i$ being the proxy for the level of uncertainty from renewables, what we called the width of our possible shocks in the first chapter, $\alpha$ being the regression constant and $\epsilon$ being the error term, we estimate the following:

$$S'_{i,k} = \alpha^S_k + \beta^S_k \text{PLU}^D_{i,k} + \gamma^S_k \text{PLU}^R_i + \boldsymbol{\delta^S_k X^S_i} + \epsilon^S_{i,k} \tag{3.4.1}$$

We are interested in the sign and magnitude of the coefficients $\beta^S$ and $\gamma^S$, which identify the effects of the PLUs ($\text{PLU}^D$ and $\text{PLU}^R$, respectively) on the shape of the supply bid function. From the predictions outlined in section 3.1.2, we expect a positive coefficient when uncertainty levels increase.[17]

#### Left-hand-side variables

We extract the slope of the aggregate supply function at any given point $k$ from a kernel density estimation with a bandwidth of 45 units.[18]

Effectively, this is a smoothed version of the slope. This makes our slope estimates robust to steps in the bid function[19], which in turn allows us to test the pre-

[17]Specifically, we want $\beta^S$ to be positive, $\gamma^S_1$ positive and $\gamma^S_2$ negative. For details on $\gamma^S$, see section 3.4.1.

[18]The slope is a by-product of the point selection mechanism, and the bandwidth selection for the smoothing thus follows the same considerations as for the latter. The details of this choice are specified in chapter 2.

[19]In our data, we observe that bid functions are effectively step functions. On EPEX spot 256 price-

dictions from the theoretical paper. Steps in the bid functions mostly are much larger towards the extremities of the bid functions and probably arise from capacity constraints considerations. Working with smoothed slopes is in line with previous work à la [Préget and Waelbroeck, 2005] and [Özcan, 2004], who also apply reduced form models to aggregate bid function data.

**Right-hand-side variables**

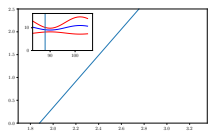
We are regressing an ex-post measure of the auction market (realised slope of the supply bid function) on ex-ante information that bidders have at the time of bidding, i.e. which is available at midday of the day ahead of delivery. We thus keep a strict separation of the ex-post and ex-ante information to the left and right hand side of equation 3.4.1, respectively. This separation allows us to circumvent endogeneity problems and validates the use of simple OLS regressions.

For this reason, we construct our PLUs on the basis of predicted uncertainty. However, for data availability reasons we cannot exclude endogeneity problems completely. For details, see the discussion in section 3.6.3.

In this subsection, we first outline how we generate the proxies for the level of market demand uncertainty ($\text{PLU}^D$) in section 3.4.1. Second, we construct the proxies for the level of uncertainty from renewables energies ($\text{PLU}^R$) in section 3.4.1. Third, we detail how the vector of exogenous variables ($\boldsymbol{X}$) is constructed in subsection 3.4.1.

**Generating proxies for uncertainty from market demand ($\text{PLU}^D$)** We construct a proxy for the level of the demand uncertainty ($\text{PLU}^D$) by using the residuals from a demand estimation on exogenous parameters as a measure of the uncertainty that

quantity combinations are allowed per bidder. When additional bid points are costly, then stepwise bidding behaviour may be very different from a setting where continuous functions can be bid [Kastl, 2011]. Due to the fact that, on average, we do not observe that firms use up all available price-quantity combinations, the cost argument of an additional bid point seems weak. Hence, by smoothing the slope we approximate the unconstrained, continuous bid function.



bidders face in an auction. Specifically, our PLU$^D$ is the expected squared level of the prediction errors that firms expect to make when anticipating the demand level of the day ahead. We assume that the ex-post prediction errors give a reasonable estimate of the uncertainty at the time of bidding.

The uncertainty proxy is obtained as detailed next in a three-step procedure. In the first step, we explain what kind of uncertainty our PLU$^D$ refers to. The second step details the conceptual details of constructing the PLU$^D$. The third step computes the PLU$^D$.

**In the first step,** we focus the analysis to a fixed number $K$ of comparable points across auctions by using the non-parametric point selection technique outlined in section 3.4. Each k$^{th}$ point is defined by a price and a quantity, which we regress independently on the exogenous variables.

Let us call $P^D_{i,k}$ and $Q^D_{i,k}$ the price and quantity of point $k$ of the realised demand function in auction $i$, $\boldsymbol{X_i^D}$ the vector of exogenous variables relevant for the demand estimation.

$$P^D_{i,k} = \alpha^{D,P}_k + \boldsymbol{\beta_k^{D,P} X_i^D} + \epsilon^{D,P}_{i,k} \tag{3.4.2}$$

$$Q^D_{i,k} = \alpha^{D,Q}_k + \boldsymbol{\beta_k^{D,Q} X_i^D} + \epsilon^{D,Q}_{i,k} \tag{3.4.3}$$

In regressions 3.4.2 and 3.4.3, firms try to anticipate the realization of the demand using the exogenous information available. We consider that the producers are able to do such an analysis at the time of bidding.

The prediction errors $\epsilon^{D,J}_{i,k}$, $J = \{Q, P\}$ are a consequence of the stochastic nature of the demand and hence a manifestation of the uncertainty. We consider that more uncertainty will lead to larger prediction errors being made in equilibrium and adopt the square of the residuals $\left(\epsilon^{D,J}_{i,k}\right)^2$ as our measure for the realised level of demand uncertainty.

**In the second step,** we recover the residuals from the demand estimation in regressions 3.4.2 and 3.4.3 and test for heteroskedasticity using [White, 1980], which is clearly confirmed (see tables 3.3 and 3.4).

Heteroskedasticity means here that the variation of error terms varies conditional on the levels of the exogenous factors: $E(\epsilon_i^2|\boldsymbol{X_i}) = g(\boldsymbol{X_i})$. However, they are still orthogonal: $E(\epsilon_i|\boldsymbol{X_i}) = 0$, thus ensuring that the prediction is unbiased, but not "best" in the sense of the best linear unbiased estimator (BLUE). Thus, heteroskedasticity results in inefficient regressions where the estimator is not minimum variance. Since we do not interpret regressions 3.4.2 and 3.4.3 for causality, but only for predictive purposes, we stick to the unbiased OLS.

The heteroskedasticity regression is given for $J = \{P, Q\}$ by

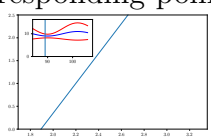
$$\left(\epsilon_{i,k}^{D,J}\right)^2 = \alpha_k^{U,J} + \beta_k^{U,J}\boldsymbol{X_i^D} + \epsilon_k^{U,J} \tag{3.4.4}$$

**In the third step,** we compute the predicted $\text{PLU}_{i,k}^D$ that firms use when bidding in the auction as:

$$\underbrace{\widehat{\left(\epsilon_{i,k}^{D,J}\right)^2}}_{\widehat{\text{PLU}}_{i,k}^D} = \alpha_k^{U,J} + \beta_k^{U,J}\boldsymbol{X_i^D} \tag{3.4.5}$$

The idea is that by experience, firms in the market know that their predictions are more or less accurate depending on the environmental conditions (in the sense of realizations of exogenous factors). In other words, firms can use the realizations of $\boldsymbol{X^D}$ to infer the accuracy of their demand predictions. Technically speaking, they can use the heteroskedastic nature of the residuals to forecast the level of uncertainty that they face.

The $\text{PLU}^D$ subs into regression 3.4.1. For simplicity, we do not include the uncertainty proxies $\text{PLU}_{i,k}^D$ measured at all $K = 5$ points in regression 3.4.1 simultaneously, but only a single $\text{PLU}_{i,k}^D$ at a time. Therefore in the final regression 3.4.1, we regress the slope at a point of the supply function on the $\text{PLU}_{\cdot,\cdot}^D$ estimated at the corresponding point on



the demand function. The pairing is done in the quantity dimension. This means that the slope of the supply function at point $k = 2$ is regressed on the uncertainty measured at point $k = 4$ of the demand function (recall the labelling of the points as given in figure 3.4.1). We indicate this quantity paring in the index $k^{-1}$ of the PLU:

$$\mathrm{PLU}^D_{i,k} = \widehat{\mathrm{PLU}}^D_{i,k^{-1}} \tag{3.4.6}$$

An increase in $\mathrm{PLU}^D_i$ corresponds to an increase in the uncertainty about the market demand realization. We thus expect $\beta^S$ to be positive in regression 3.4.1.

**Generating proxy for uncertainty from renewable energies ($\mathbf{PLU}^R$)** We have already referred to the statement that the intermittency of renewables causes large residual demand shocks [EPEX, 2014]. Suppliers are thus wary of the expected production of renewables generation.

Given that renewable generation is an exogenous source of supply and is completely injected on the network without being bidden for (fixed feed-in tariff), it affects the residual demand curve for each supplier, but does not enter the $\mathrm{PLU}^D$, which captures the uncertainty on market demand only.

In predicting the generation from renewables, we assume that suppliers are able to infer renewables generation from meteorological forecasts.[20]

When forecasting the residual demand shocks due to generation from renewables, we consider that suppliers have an idea of the precision of their estimate based on the "look" of the meteorological forecasts that they have. By look, we mean the geographical heterogeneity or homogeneity of the forecasts, i.e. if when looking at a weather map, one sees a lot of spatial variations or not.

[20] We specify the technique in chapter 2 and use it to construct our controls in section 3.4.1.

We consider that this notion of geographical heterogeneity of the forecasts correlates to the uncertainty associated with the prediction of renewable production. The argument is as follows:

**First**, renewable production is built by aggregating the forecast of all individual renewable sources. This means knowing the position and capacity of every renewable source, querying weather forecasts for all of these points, modeling the renewable's response to the forecasted weather and adding the forecasted productions.

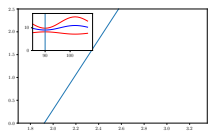
**Second**, we note that weather is spatially correlated, which means that the closer two points are, the closer the values for a given weather variable (the air temperature at your left hand is very close to that at your left hand, but less so across the city, and even less so across the country). This correlation roughly follows an exponential law: the difference between the values of a weather variable between two points behaves in a linear fashion for small distances and saturates at large distances.[21] The transition between those two regimes is given by a characteristic lengthscale, a bit less than 200km on average.

**Third**, we observe that the average distance between production points is large enough that the relevant regime of autocorrelation is the saturated part.[22]

**Fourth**, we note that there are two main channels through which the overall uncertainty about renewable production is related to the weather. There is an issue of error averaging, which means that if the weather becomes very spatially uncorrelated, one can expect errors to cancel out relative to a given bias in the forecast. This channel would

[21] Intuitively, the characteristic lengthscale of autocorrelation represents the distance required between two geographical points on a map of weather forecasts to observe a decorrelation of half of its maximum value. For example on the wind speeds prediction, a characteristic length of 80 km means that if we observe two very distant points (say 1000km) to have a difference in wind speeds of, on average, 50km/h (this being the maximum difference, we are in the saturated regime), then we will observe, on average, wind speed differences of 25km/h for points distant from each other by 80km.

[22] For N production points, we compute the N(N-1)/2 pairs of points, consider their distances and compute the average of these distances weighted by the production capacity at every point. In the case of the wind, we have an average distance of 459 km, in the case of the photovoltaic production we have an average distance of 499km.



tend to imply that more spatial variations imply a smaller uncertainty about production. There is also the issue that weather forecasts are numerical simulations and that the mesh size for such simulations, typically 5km for the high precision ARPEGE model of Météo France, implies that the errors are higher as the simulated phenomenons have higher gradients. This means in our case that the uncertainty about the forecast increases as the weather becomes more spatially uncorrelated.

**Fifth**, these two effects are of opposite signs, but our third point is an argument for considering that the averaging of errors is smaller than the simulation errors. Therefore, we expect our uncertainty to increase as the spatial autocorrelation decreases (i.e. more spatial variation).

This can be summed up with the following hand-waving argument: when there is more spatial variations, the weather is more messy, therefore more difficult to predict.

We compute this characteristic lengthscale ($L$) as described in chapter 2. Our $\text{PLU}^R$ is defined as the two proxies

$$\text{PLU}^R_{1,m} = \frac{1}{L_m}, \qquad \text{where } m = \{\text{Wind, Solar, Temperature}\} \tag{3.4.7}$$

$$\text{and} \quad \text{PLU}^R_{2,m} = \left(\frac{1}{L_m}\right)^2 \tag{3.4.8}$$

As explained above, we expect firms to face less uncertainty in predicting weather conditions when the lengthscale of autocorrelation $L$ is longer since the overall weather conditions will be more homogenous. A longer length $L$ (less uncertainty), will yield a smaller $\text{PLU}^R$ and we expect a flattening of the supply curve. I.e. we expect a positive coefficient $\gamma_1^S$ on the $\text{PLU}^R_{1,m}$ variables in the final slope regression.

However, we also expect the effect of $L$ on the slope to be attenuated, if not coun-

terbalanced, by the squared term.[23] This means that for very small $L$, we expect an additional effect, that of the summation of errors, to become significant and reduce the uncertainty, or at least its rate of increase: we thus expect a negative coefficient $\gamma_2^S$ on the squared PLU$^R$ term in the final slope regression (equation 3.4.1).

**Controls** This section details the exogenous variables, which we use for our study. The stacked vector of exogenous variables is not identical for the supply and demand regressions of equations 3.4.1 and 3.4.2.

The vector $\boldsymbol{X^D}$ for the demand equation includes the variables: Tempeff15, Roll_Temp24, Roll_Temp240, suncycle, morning, deltasun, EWH, SolarRest, RteBlackBox.

For the supply regression we include in $\boldsymbol{X^S}$ the following variables[24]: Coal, Brent, Gas, IT2, EUA, Wind1DA, Hydro.

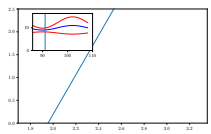
Table 3.2 gives a brief overview of the controls used. Details on the computation of some variables are given in the appendix (see links in table). The last column indicates the frequency with which we observe the variable in question.

| Name | Explanation | Unit | Frequency |
|---|---|---|---|
| Wind1DA | The day-ahead predicted electricity volume generated from wind turbines. Details on p. 111. | MWh | Hourly |
| Solar | The electricity volume generated from photovoltaic sources. Details p. 115 | MWh | Hourly |

*Continued on next page...*

[23] We expect the effects of $L$ on the slope to be of the shape of a Laffer curve.

[24] We do not include the variables used for the demand estimation as they indirectly feed into the final regression via the PLU$^D$.



*... table 3.2 continued*

| Name | Explanation | Unit | Frequency |
|---|---|---|---|
| Tempeff15 | Effective predicted temperature in France (with a cutoff point at 15$^o$C to reflect demand patterns), aggregated on a national level. Details on p. 114. | $^o$C | Hourly |
| Roll_Temp24 | Mean of *Tempeff15* over the last 24 consecutive hours. | $^o$C | Hourly |
| Roll_Temp240 | Mean of *Tempeff15* over the last 240 consecutive hours. | $^o$C | Hourly |
| suncycle | Luminosity as a percentage of maximum luminosity of the day. *Midday* defined as *suncycle*=1. Details on p. 117. | % | Hourly |
| morning | Indicator variable for hours before *Midday*. | $\{0, 1\}$ | Hourly |
| deltasun | Absolute value of the change in *suncycle*. Details on p. 117. | $[0, 1]$ | Hourly |
| EWH | Indicator variable for hours between 10 pm and 4 am. | $\{0, 1\}$ | Hourly |
| SolarRest | The unexplained component of photovoltaic generation. Specifically, the residuals from a regression of *Solar* on *suncycle*. Details on p. 117. | MWh | Hourly |

*Continued on next page...*

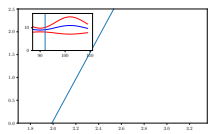
*... table 3.2 continued*

| Name | Explanation | Unit | Frequency |
|---|---|---|---|
| RteBlackBox | The unexplained component of the day ahead prediction of total consumption in France issued by the grid operator (RTE). Specifically, the residuals from a consumption estimation. Details on p. 118. | MWh | Hourly |
| Coal | Average coal import prices at the French border. | €/ton | Monthly |
| Brent | Average of spot prices for crude oil on the London based stock exchange. | $/bl | Monthly |
| Gas | Average of closing prices for natural gas at 1 month on the London market (NBP). | £/Therm | Monthly |
| IT2 | Interaction term between gas and demand: *Gas* weighted by an hourly index for the demand level [25] | £/Therm | Hourly |
| EUA | Price of $CO^2$ emissions. | €/ton | Daily |
| Hydro | Sum of dam level heights on a national level. | % | Weekly |

Table 3.2: Overview of exogenous variables.

The rationale for the included variables is the following:

[25] Gas turbines generate electricity using natural gas as a fuel. We thus proxy for its input price using a Gas variable for which we take the closing price for natural gas at 1 month on the London market (NBP). Electricity generation from gas is expensive and flexible. In general gas plants are only called upon to provide peak load electricity generation in moments of high demand. We, therefore, compute an interaction term between Gas and an index for the hourly level of the demand. The index acts as a weight on the gas price. The weight is computed as the percentage demand level as compared to the maximum demand level observed in our dataset.



First, Wind1DA and Solar control for the expected level of renewables generation[26] on the day ahead market. These are computed using a novel bottom-up methodology described in the appendix 2.3.1.

Second, Tempeff15 controls for the demand patterns as a function of the temperature.[27] Tempeff15 includes a cut-off at 15$^o$C in order to take into account the demand pattern as a function of temperature according to [RTE, 2014]. Table 2.2 reveals the improved fit over a simple temperature variable without respecting the demand cut-off (Tempeff).

Third, Roll_Temp24 and Roll_Temp240 capture the demand seasonality via the temperature. The former gives the daily average temperature, while the latter captures the average temperature over the last 10 days. The demand cut-off at 15$^o$C for Tempeff15 is respected for these means. Including these as seasonality controls allows to get away from using dummy variables for the seasonality. In short, avoiding dummies yields more transparency of the results as we do not have the problem of interpreting the dummies, which are often black boxes.[28]

Fourth, we use the four variables suncycle, morning, deltasun and EWH collectively to continuously control for the time of the day. The reasoning is again the ability to get away from using dummies and being able to interpret the results. Figure 3.4.3 shows how the controls describe the daily patterns continuously.

Fifth, SolarRest and RteBlackBox are the residual information gained from the variables Solar and the day ahead consumption prediction of RTE (PrevConsoH) over other variables included in $\boldsymbol{X^D}$ or $\boldsymbol{X^S}$, respectively.[29]

[26]For data availability reasons, Solar is computed on realised luminosity values rather than forecasts of luminosity.

[27]Note that electric heating is widely spread in France. It is used in 32% of principal residences (INSEE, RP2011 exploitation principale).

[28]See section 3.6.2 for a full discussion on the advantage of avoiding dummies.

[29]E.g. Solar is strongly correlated with suncycle, thus SolarRest is the residual from a regression of

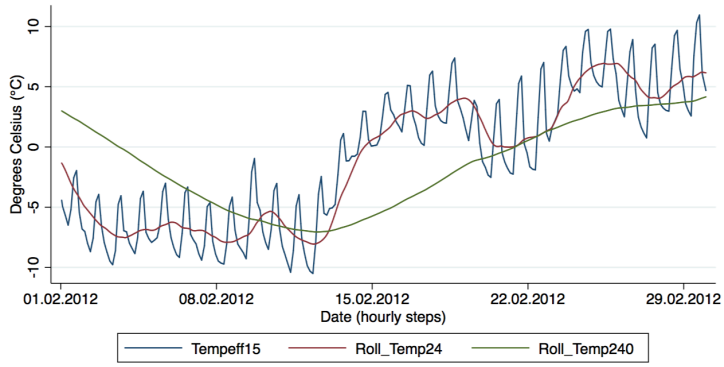



Figure 3.4.2: Temperature based seasonality controls

*Note:* The graph shows the evolution of the temperature based controls for seasonality for the month of February 2012. The graph shows the lagged nature of the rolling average temperature controls.

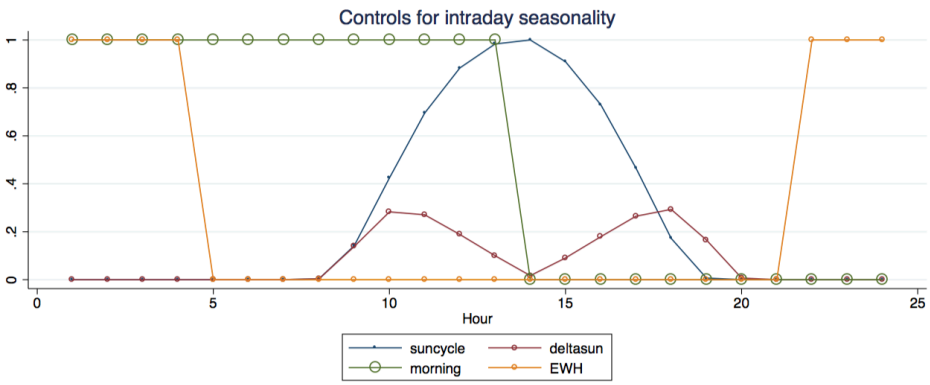


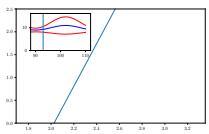

Figure 3.4.3: Continuous controls for daily patterns

*Note:* With the exception of EWH, all intraday seasonality controls (suncycle, morning, deltasun) are determined endogenously by the prevalent luminosity as captured by Solar.

Sixth, Coal, Brent, Gas, IT2 and EUA are rough proxies for the input prices for electricity suppliers. Hydro is used as a crude proxy for dam operator's ability to generate short term electricity using hydro reserves.

We briefly emphasize that novel methodologies have been used to compute all variables derived from weather forecasts or observations. When tracing back the shape of aggregate bid functions on exogenous factors in the second stage estimation, we use aggregated statistics (at the national level) for the exogenous variables. We thus use an

the former on the latter. RteBlackBox is computed as the residuals from regressing PrevConsoH on Tempeff15, Roll_Temp24, Roll_Temp240, suncycle, morning, deltasun and EWH. See appendix 2.3.2 and 2.3.2 for details.



aggregation methodology to summarize local information (collected at the level of the individual postcodes) in order to generate an aggregate statistic at the national level.

The general methodology for the aggregation is explained using the example of Solar and as follows: We observe the value of a weather parameter (e.g. luminosity) every hour at known weather stations in France. We apply an interpolation technique in order to obtain parameter values for all possible geographic locations in France. At any local point, we can thus infer the electricity volume generated by using the information of the locally installed capacity (of solar panels) and the renewable energy available (i.e. sunlight inferred by the inverse of nebulosity). We then take the sum of all solar generated electricity per hour in France and use this as our aggregate statistic at the national level in our regression analyses. We used forecast data wherever possible in order to approximate the level of information that bidders have at the time of bidding and circumvent endogeneity problems. For cases where forecast data was not available, e.g. Solar, realised weather data was used.

**Extensions and robustness checks**

In order to test the robustness of our results and circumvent some drawbacks of the baseline model, we use a few alternative specifications of our empirical model.

**Bootstrapping standard errors**

The set-up of our empirical analysis relies on stochastic variables, e.g. $\text{PLU}^D$, which are computed in the first stage of our identification. The assumption made for an OLS regression of normally distributed residuals is a very strong one (particularly with the forecast variable) and one which can flaw the precision of estimates in the second stage regression. We therefore bootstrap the standard errors of the final regression by using random sampling with replacement at each stage of the analysis, i.e. for both the PLU computation and the final slope regression with 300 repetitions.

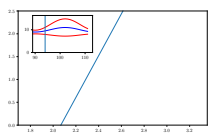
Bootstrapping allows us to non-parametrically approximate the distribution of the forecast PLUs and thus enables us to correct the standard errors of our coefficient estimates.

**Kernel based uncertainty forecasts (PLU$^D$)** The PLU$^D$ computed as described in section 3.4.1 is noisy since we assume a linear forecast model to be valid for any combination of realizations of exogenous paarameters, i.e. the same model applies winter and summer, day and night. While the results are as desired for the baseline PLU$^D$, a bootstrapping of the standard errors indicates that the first stage forecast is too imprecise for effects of a satisfactory significance level.

We therefore develop an extension of the uncertainty prediction model in which we use the idea of demand forecasts (equation 3.4.5) only locally, i.e. for a limited range of variation in the exogenous parameters. In other words, we estimate the PLU$^D$ corresponding to an auction only in the neighbourhood of this auction, i.e. over all auctions that occurred in similar conditions. By conditions, we mean realizations of exogenous parameters, and the neighbourhood refers to the concept of measuring the similarity of these realizations by means of a range. The next step explains how this is done formally.

We consider that firms predict the level of the uncertainty by comparing it with the level of uncertainty in past[30] auctions of similar exogenous conditions. The methodology is analogous to the computation of the baseline PLU$^D$. The suppliers forecast the precision (squared residuals) of their demand estimation as before, but only on a subsample of the data. The subsample is defined as all observations which lie within a distance $b_{X_e}$ of the observation of interest with respect to each control variable $X_e$, $\forall e = \{1, .., E\}$. Effectively, this is a multi-variate kernel regression and subsequent forecast with a rectangular (also called "boxcar") weighting function. Observations within the kernel window are given equal weight, while observations outside the kernel window are given zero weight.

[30]For data availability reasons, we pool all (past and future) auctions for the computation of this PLU. This introduces some endogeneity. For a discussion of this choice, please see section 3.6.3.



We set the bandwidth $b_{X_e}$ with respect to each variable equal to $\frac{1}{3}$ of the range of that variable.[31]

At any arbitrary observation (auction) with the realization $\tilde{\boldsymbol{X}}$ for the stacked vector of exogenous variables ($X_e$), the simple weight function is

$$W(\boldsymbol{X}) = \prod_e W(X_e)\ , \qquad \text{where } W(X_e) = \begin{cases} 1, & \text{if } |\tilde{X}_e - X_e| \leq b_{X_e} \\ 0, & \text{otherwise.} \end{cases} \tag{3.4.9}$$

and the subsample based regressions are then

$$P_k^D(\boldsymbol{X}) = \alpha_{k,\tilde{\boldsymbol{X}}}^{D,P} + \beta_{k,\tilde{\boldsymbol{X}}}^{D,P} W(\boldsymbol{X}) + \epsilon_{k,\tilde{\boldsymbol{X}}}^{D,P} \tag{3.4.10}$$

$$Q_k^D(\boldsymbol{X}) = \alpha_{k,\tilde{\boldsymbol{X}}}^{D,Q} + \beta_{k,\tilde{\boldsymbol{X}}}^{D,Q} W(\boldsymbol{X}) + \epsilon_{k,\tilde{\boldsymbol{X}}}^{D,Q} \tag{3.4.11}$$

and the local uncertainty regressions and forecasts $\forall J = \{P, Q\}$ are given by

$$\left(\epsilon_{k,\tilde{\boldsymbol{X}}}^{D,J}\right)^2 = \alpha_{k,\tilde{\boldsymbol{X}}}^{U,J} + \beta_{k,\tilde{\boldsymbol{X}}}^{U,J} W(\boldsymbol{X}) + \epsilon_{k,\tilde{\boldsymbol{X}}}^{U,J} \tag{3.4.12}$$

$$\underbrace{\widehat{\left(\epsilon_{k,\tilde{\boldsymbol{X}}}^{D,J}\right)^2}}_{\widehat{\mathrm{PLU}}_{k,\tilde{\boldsymbol{X}}}^{D}} = \alpha_{k,\tilde{\boldsymbol{X}}}^{U,J} + \beta_{k,\tilde{\boldsymbol{X}}}^{U,J} \tilde{\boldsymbol{X}} \tag{3.4.13}$$

When firms infer the upcoming uncertainty by looking at the uncertainty in past auctions, the precision of their estimate depends on the number of comparable auctions available, i.e. the sample size. Given that the sample size varies greatly across auctions, we use a sample-size-weighted OLS regression in the final estimation of equation 3.4.1. Finally, we bootstrap the standard errors on the kernel-based PLUs using 50 repetitions.[32]

[31] See appendix 3.B.1 for details. Column 2 of table 3.14 indicates the choice of $b_{X_e}$ for each exogenous variable considered.

[32] For computational reasons, we only bootstrap the kernel based PLUs for the point of inflection ($k = 3$). We choose only 50 repetitions for the same reason. Given the size of our dataset, we consider it acceptable. The general criterion for convergence is that each observation is selected at least once in the bootstrapping exercise.

## 3.5 Results

We first present the results for the demand estimation in both the Price and Volume dimension since this step is identical for all PLU specifications. We then present the results of the final regression in the baseline and alternative specifications.

### 3.5.1 Demand estimation

Table 3.3 gives the results for the demand estimation on volumes (equation 3.4.3). Table 3.4 shows the results for the demand estimation on prices (equation 3.4.2).

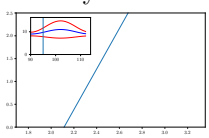
These tables are interesting for two reasons. First, they provide the basis for our computation of the PLU$^D$. Second and the reason why we disclose them in such detail, they are already a result in themselves.

It is comforting to see that all variables used are significant and, more importantly, of the expected sign. This significance provides support for our specification of the demand estimation. For the interpretation here, we focus on the effects at the point of inflection[33] ($k = 3$).

First, looking at the volume effects of the exogenous variables: All variables included in the regression are highly significant at the 1% level. All temperature statistics (Temp-eff15, Roll_temp24, Roll_temp240) bear coefficients with a negative sign and confirm that electricity demand falls with increasing ambient temperature. All daytime controls show up the expected sign as well: suncycle and deltasun have positive coefficients. This is sensible as electricity demand is higher during the day than at night (proxied for by suncycle) and rush or activity hours (proxied for by deltasun) in the morning and evening are also characterised by increased demand. The variables morning and EWH have coefficients of a plausible negative sign. The morning as controlled for by our indicator variable[34]

[33] As mentioned, the point of inflection is the centre point of the bid curves and the most relevant for equilibrium determination.

[34] The morning is defined as the hours before midday, which occurs when luminosity is at its daily



is shorter than the afternoon and evening together, thus total electricity consumption is lower as well. EWH stands for the deep night between 10 pm and 4 am and thus also corresponds to low demand periods. SolarRest controls for selfgeneration to cover own consumption and has a plausible negative coefficient. RteBlackBox, on the other hand, has a very sensible positive coefficient and confirms that actual demand is higher when the grid operator expects it to be the case.

The analysis of the price effects of these controls on demand functions is in line with the analysis of volume effects. This is coherent since for a linear downwards sloping demand curve, a left shift (volume decrease) is synonymous for a downwards shift (price decrease) of the curve. We consider that at the point $k = 3$, the demand functions are locally linear. We note the only exception for the coefficient of SolarRest which has a positive price effect, while a negative volume effect.[35]

Second, these tables already give a descriptive analysis of the effects of exogenous variables on the shape of the demand bid function: We now compare all coefficients for a specific variable on the $K = 5$ different points on the demand function (we read the table horizontally and compare sign changes across columns).

In table 3.3, we observe for each row at most a single sign change across the coefficients for the different points. Furthermore (and with few exceptions), the magnitudes of the coefficients generally increase or decrease monotonically along a row. This is very convincing as it suggests that exogenous variables have a monotone effect on the shape of the bid function. We thus only observe one-directional shifts (e.g. a unilateral left shift) or two-directional shifts (extension or contraction) in the volume dimension induced by the variation in exogenous variables. While the unilateral effects are explained analo-

maximum.

[35] We emphasize in the construction of our variable (appendix 2.3.2) that it is not possible to build a proxy for lighting consumption that would allow us to decorrelate the effects from photovoltaic production and lighting consumption. We therefore stick to the SolarRest proxy, which aims to capture the effect of Solar which is not captured by suncycle.

gously to our point specific interpretation on the point $k = 3$ above, we do not have a story to tell about two-directional effects.

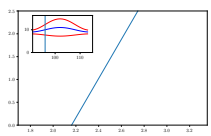
Tempeff15 results in a contraction of the bid function in terms of volumes (right shifts on low volume points, $k = 5, 4$ and left shifts on high volume points $k = 3, 2, 1$). Roll_Temp24 has the opposite effect and results in a volume extension of the curve. Roll_Temp240 induces a pure left shift of the whole function.[36] For the intraday seasonality controls, the results are very clear. While suncycle results in an extension of the demand function[37], all other intraday controls (morning, deltasun, EWH) have unilateral effects. When the indicators morning and EWH are positive, we observe volume decreases at all points and thus a left shift of the function. Higher values of deltasun induces volume increases at all points of the bid function.

Finally, we have SolarRest which induces an expansion of the curve and RteBlackBox which has a unilateral right shifting effect on the aggregate demand bid function.

The price variation of the demand bid function yields interesting results, too. Given that the prices of points $k = 1, 5$ are fixed, we only observe effects for the interior points. We thus focus on the effects on the points $k = 4, 3, 2$ only (called the "central demand function" here). Again, we only observe at most a single sign change across columns for any exogenous variable. Both Tempeff15 and Roll_Temp240 lead to an extension of the central demand function (we are now looking at vertical variation of the bid function as shown in fig. 3.4.1), while Roll_Temp24 causes a unilateral downwards shift. For intraday seasonality controls, we see that suncycle and deltasun have a contracting effect on the central demand function and morning a unilaterally negative effect. EWH leads to an expansion of the central demand function. SolarRest and RteBlackBox indicate an

[36] Excluding interaction effects, we note that the net effect of a simultaneous $1^o$C increase for all three temperature variables results in a net left shift of the function. In the price dimension (table 3.4) we observe a net downwards shift. Both effects suggest that electricity demand decreases with the prevailing temperature.

[37] Combined with the observed price effects from table 3.4, this suggests that demand is more price elastic during the day.



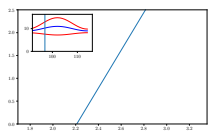
extension of the central demand function in the price dimension.

| | $k = 5$ Volume | $k = 4$ Volume | $k = 3$ Volume | $k = 2$ Volume | $k = 1$ Volume |
|---|---|---|---|---|---|
| Tempeff15 | 50.72*** | 38.58*** | -130.3*** | -189.3*** | -204.0*** |
| | (9.942) | (10.13) | (10.94) | (13.32) | (13.20) |
| Roll_Temp24 | -63.57*** | -67.13*** | -48.87*** | 19.76 | 34.16** |
| | (11.78) | (12.06) | (13.14) | (15.83) | (15.76) |
| Roll_Temp240 | -60.15*** | -68.38*** | -78.49*** | -78.44*** | -87.38*** |
| | (6.655) | (6.867) | (7.450) | (10.05) | (10.00) |
| suncycle | -894.0*** | -652.1*** | 508.2*** | 1,351*** | 1,400*** |
| | (44.27) | (45.50) | (48.52) | (56.36) | (55.73) |
| morning | -101.2*** | -220.3*** | -814.8*** | -872.2*** | -885.8*** |
| | (27.52) | (28.33) | (30.44) | (37.71) | (37.28) |
| deltasun | 2,659*** | 2,850*** | 3,201*** | 1,721*** | 1,821*** |
| | (153.5) | (158.5) | (166.1) | (197.8) | (196.5) |
| EWH | -803.1*** | -833.1*** | -782.7*** | -354.7*** | -322.8*** |
| | (30.74) | (31.91) | (33.15) | (42.09) | (41.78) |
| SolarRest | -0.595*** | -0.363*** | -0.145*** | -0.0137 | 0.246*** |
| | (0.0282) | (0.0305) | (0.0342) | (0.0418) | (0.0407) |
| RteBlackBox | -0.00259 | 0.0127*** | 0.105*** | 0.107*** | 0.0979*** |
| | (0.00235) | (0.00243) | (0.00255) | (0.00316) | (0.00317) |
| Constant | 6,054*** | 7,086*** | 11,446*** | 15,215*** | 15,502*** |
| | (33.71) | (35.04) | (37.15) | (48.68) | (48.27) |
| Observations | 14,691 | 14,691 | 14,691 | 14,690 | 14,691 |
| $R^2$ | 0.201 | 0.219 | 0.478 | 0.344 | 0.346 |
| White | 548.6 | 524.9 | 407.9 | 961.8 | 944.8 |

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 3.3: Estimation results for demand volumes

*Note*: The estimated constants of this table or the left graph of fig. 3.4.1 indicate to which portion of the demand function the types of points $k = 1, .., 5$ refer.



| | $k=5$ Price | $k=4$ Price | $k=3$ Price | $k=2$ Price | $k=1$ Price |
|---|---|---|---|---|---|
| Tempeff15 | 0 | 4.675*** | -0.969*** | -1.308*** | 0 |
| | (0) | (1.523) | (0.0599) | (0.0980) | (0) |
| Roll_Temp24 | 0 | -10.07*** | -0.124* | -0.0470 | 0 |
| | (0) | (2.233) | (0.0713) | (0.116) | (0) |
| Roll_Temp240 | 0 | 4.250*** | -0.0901** | -0.353*** | 0 |
| | (0) | (1.147) | (0.0404) | (0.0607) | (0) |
| suncycle | 0 | -10.98** | 6.870*** | 11.60*** | 0 |
| | (0) | (5.020) | (0.258) | (0.445) | (0) |
| morning | 0 | -0.226 | -5.748*** | -9.009*** | 0 |
| | (0) | (4.133) | (0.173) | (0.285) | (0) |
| deltasun | 0 | -16.54 | 10.60*** | 18.72*** | 0 |
| | (0) | (19.16) | (0.881) | (1.497) | (0) |
| EWH | 0 | 5.136 | -1.756*** | -3.014*** | 0 |
| | (0) | (4.448) | (0.192) | (0.302) | (0) |
| SolarRest | 0 | 0.000532 | 0.00192*** | 0.00253*** | 0 |
| | (0) | (0.00307) | (0.000193) | (0.000326) | (0) |
| RteBlackBox | 0 | 9.91e-05 | 0.000906*** | 0.00147*** | 0 |
| | (0) | (0.000301) | (1.47e-05) | (2.26e-05) | (0) |
| Constant | 3,000 | 131.3*** | 39.45*** | -39.43*** | -3,000 |
| | (0) | (4.210) | (0.217) | (0.319) | (0) |
| Observations | 14,691 | 14,691 | 14,691 | 14,690 | 14,691 |
| $R^2$ | | 0.005 | 0.463 | 0.420 | |
| White | | 138.2 | 640.9 | 761.2 | |

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 3.4: Estimation results for demand prices

*Note*: The estimated constants of this table or the left graph of fig. 3.4.1 indicate to which portion of the demand function the types of points $k = 1, .., 5$ refer.

Overall, we take away a solid $R^2$ with coefficients of the correct sign. We furthermore have disclosed the White statistic which unanimously confirms heteroskedasticity in these regressions. The significance levels have been measured using robust standard errors.

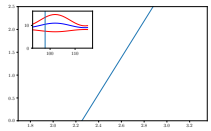
We point to the fact that the explanatory power of our demand estimations is highest for the point of inflection, in line with our expectations. Points of maximum curvature $k = 2, 4$ reveal lower $R^2$ statistics. This is likely due to the underlying data patterns that arise from bidding frictions, e.g. focal price points. For these points, it is thus not surprising that we do not observe convincing demand estimates - we note in particular the lack of explanatory power for the demand estimation in the price dimension for points of type $k = 4$.

### 3.5.2 Final regression

For the final regression, we first lay the focus on the point of inflection ($k = 3$) for a detailed interpretation of our results. We choose the point $k = 3$, because this type of point is the most relevant for equilibrium determination. We then disclose the results for all other points $k \neq 3$ to give an overview of the effects of uncertainty on the whole aggregate supply bid function.

Each result table has four (three[38]) columns to show the results for different estimators and two specifications of the $\text{PLU}^D$. All other variables remain unchanged across the columns. In the tables, column 1 refers to the baseline specification of the $\text{PLU}^{D,J}$, where standard errors are calculated using the Huber-White sandwich estimator. Column 2 reports the results for the baseline model using bootstrapped standard errors with 300 repetitions. Column 3 reports the results for the regression on the kernel based $\text{PLU}^{D,J}_{\tilde{X}}$, using the sample size of each kernel as weights in the regression. Column 4 reports the re-

[38]For computational reasons, we do not run the bootstrapping of the kernel based $\text{PLU}^D$ for the points $k \neq 3$, thus we only have three columns for these tables.



sults of the kernel based model using bootstrapped standard errors using 50 repetitions.[39]

*Regarding notation:* In the results tables, PLUvRvar'm' stands for $\text{PLU}^R_{1,m}$ with 'm' being replaced by the initial of the variable in question (W, S and T, respectively). $\text{PLU}^R_{2,m}$ is indicated by the extension "sq". PLUvDvar'J' stands for $\text{PLU}^{D,J}$ with $J = \{P, Q\}$ representing the dimension in which the demand uncertainty is measured. The kernel based $\text{PLU}^D_{\tilde{X}}$ are given by PLUvDvarK'J' in the tables. To facilitate the reading of the tables, we adopt this notation for the discussion of the results.

**For the point of inflection ($k = 3$),** the results are shown in table 3.5. Regarding uncertainty from renewables production, only that of wind has a significant and robust impact. PLUvRvarW has a positive effect (significant at the 1% level) on the slope in all specifications. PLUvRvarWsq has a negative effect on the slope in all specifications, however this second effect is not robust to bootstrapping the standard errors. The signs of the estimated coefficients are in line with our expectations. To show this, we recall that both versions of the PLUvRvarW are based on the inverse of the characteristic length-scale $L_W$ of autocorrelation of the wind speed measurements. Thus, when $L_W$ increases (it represents a decrease in the uncertainty since wind speeds are homogenous over longer distances), the PLU decreases (corresponding to a decrease in uncertainty).

While an increase in the PLUvRvarW leads to an increase in the slope of the supply function, the effect is attenuated by the squared term PLUvRvarWsq for very small and large $L_W$.[40] The estimated coefficient for the latter is negative and suggests that for very short $L_W$ (i.e. very heterogenous wind speeds over the country), prediction errors cancel out. For very long $L_W$ (i.e. very homogenous wind speed profile), the marginal impact

[39] Coefficients vary slightly ($< \pm 20\%$, no sign change), because the bootstrapping loop includes the kernel-based prediction of the uncertainty and thus varies the kernel sample sizes, which are used as weights in the final regression. Furthermore, the estimator has probably not yet fully converged with 50 repetitions, however for computational reasons we stick to this choice.

[40] By looking at the variation of our data, we see that the negative effect of the PLUvRvarWsq term merely attenuates, rather than overrides, the positive effect of the PLUvRvarW term on the slope since in our dataset we very rarely observe PLUvRvarW values sufficiently large to exceed the maximum of the Laffer curve of the impact on the slope.

of $L_W$ on the level of uncertainty decreases.

With respect to the uncertainty from temperature forecasts, the results are insignificant (although of the anticipated sign). We expect the impact of temperature uncertainty goes via the demand response, which we account for in our proxy for the uncertainty from demand realization (PLUvD). Similarly, uncertainty from Solar production is attributed no effect. This is not surprising as generation from solar is only a fraction of that generated from wind power and thus negligible. Furthermore, we are unable to disentangle the effect of solar generation from the reduced demand effect from high luminosity (which result in low demand for lighting). We do not find evidence for a direct response from suppliers to uncertainty in temperature or solar predictions.

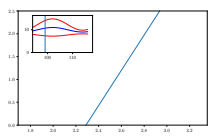
Uncertainty from the realization of market demand has a negative and significant effect when proxied for by price-based PLUvDvarP (see table 3.5) as opposed to a positive and significant effect when proxied for by a volume-based PLUvDvarQ (see table 3.5). The positive effect on PLUvDvarQ is in line with our prediction made in section 3.1.2. This result supports the theory that firms take uncertainty when bidding into account and consequently adjust their bidding strategy in order to minimize dynamic costs.

However, our theory produces a prediction for volume based uncertainty only. We include the uncertainty proxy for price PLUvDvarP as a control and its effect seems rather robust. The effects of PLUvD in either the price or volume dimension are robust to the exclusion of the other.[41] We try to explain the opposing signs for the coefficients of the two proxies in section 3.6.1.[42]

Furthermore, table 3.5 gives support to our extension using kernel based PLUvDs.

[41] Results available from the authors.

[42] The net effect cannot be precisely computed as the conversion of the PLUvD from the price dimension to the quantity dimension is not possible. We approximate the comparison however, by including both $\text{PLU}^D$ simultaneously in the regression. All PLUvD are rescaled by their respective means to allow some degree of comparison.



Column 2 shows that the effects of the baseline PLUvD are not significant when bootstrapped. Our alternative is to use a more elaborate uncertainty prediction model. These kernel based PLUvD are more sophisticated in two respects: (i) the forecasting model is only applied locally, that is auctions are only compared to similar auctions and (ii) the obtained forecast is weighted by the sample size used for its prediction. Thereby, we control for the confidence of the firms in making those predictions. The results of the weighted regression are given in column 3. The results using the more elaborate prediction model are in line with those from the baseline regression, while being more accurate as indicated by the improved explanatory power of our model (we see a 16.5% increase of the $R^2$ from columns 1-2 to columns 3-4). Finally, the results of our kernel based model are more precise as indicated by the higher significance level for the PLUvDvarKP and PLUvDvarKQ, which are now also robust to a bootstrap (column 4).

Finally, we explicitly include the controls for the levels of the input prices of electricity producers ($\boldsymbol{X^S}$). We do not interpret these coefficients since there are no ex-ante expectations of their levels to affect the slope of the supply bid function. We briefly mention that intraday seasonality controls as well as other demand related variables are not included in this regression to avoid multicollinearity problems with the PLUvD, which are themselves computed as a linear combination of the demand control variables ($\boldsymbol{X^D}$).

Overall, we take away a goodness of fit of $\geq 20\%$ for our empirical model as well as the robust positive coefficients for both the demand based uncertainty proxy (PLUvDvarQ) and the weather based uncertainty proxies (PLUvRvarW and PLUvRvarWsq). We note the puzzling result for the PLUvDvarP.

**For the other points ($k = 1, 2, 4, 5$),** the results are given in tables 3.6, 3.7, 3.8 and 3.9, respectively.[43] We comment on the effects over all points collectively in order to give

[43] Variables marked "(omitted)" are drop due to perfect collinearity.

an overview of the full bid function behaviour.

The specification of the proxies for the uncertainty from renewables as well as of the controls does not vary across columns, we thus focus on column 2 for these (in order to take bootstrapped standard errors into account). While we observe in table 3.5 a convincing effect for the uncertainty from wind predictions on the slope of the point of inflection ($k = 3$), we cannot observe this effects on the other points of the bid function.[44] No other proxy for the uncertainty from renewables has a significant effect on the slope at any point.

The proxies for uncertainty from market demand produce opposing effect depending on the prediction model. PLUvDvarP has a negative and significant effect on all points (with the exception of points $k = 1$ and $k = 5$ of course, which do not exhibit variation in prices due to the auction rules). PLUvDvarQ has a positive effect, when significant[45], on all points ($k = 2 - 5$), but not on $k = 1$.

On the remaining controls, we do not observe a clear pattern on the effects at the different points. We run the analysis without these controls and note that the signs of all significant variables remain unchanged.[46]

## 3.6 Discussion

In this section we reflect on the results and use the opportunity to address a few issues, drawbacks as well as qualities of the research conducted. We first discuss the findings of the paper and their internal and external validity. We briefly review the design of the empirical strategy and lend particular focus to how we deal with the issue of endogeneity.

[44] We note the exception of a negative effect for PLUvRvarW on the slopes at points $k = 2$ and 5 (significant at the 5% level).

[45] For both the bootstrapped baseline results (col. 2) and the weighted kernel based specification (col. 3).

[46] Results available from the authors.

### 3.6.1 Findings

In this paper, we investigate whether uncertainty affects supplier bidding as predicted by the theory. We find that uncertainty from weather forecasts indeed affects the suppliers' bid function as expected. The aggregate supply function steepens when the level of uncertainty increases. We take this as evidence that firms take dynamic cost considerations into account and adjust their behaviour when facing increased expected dynamic costs.

We also find significant results for the effect of the level of uncertainty about the realization of market demand on the suppliers' behaviour. However, we observe a strong discrepancy between the effect for uncertainty as measured on price volatility and the effect of uncertainty as measured on volume volatility. While the former sees itself attributed with a negative effect, the latter sees itself attributed with a positive effect on the slope of the aggregate supply function. These opposing signs are robust in all specifications and seem to be of too much importance to be neglected.

The two proxies in question ($\text{PLU}^{D,P}$ and $\text{PLU}^{D,Q}$) are two variables designed to measure the same information, namely the prediction error of the demand function. As such, they are identical with respect to the set-up, computation as well as point at which they are extracted. They only differ with respect to the dimension in which the variation of the demand function is measured, the former in the price dimension and the latter in the volume dimension.

A theory using linear functions would predict that these measures of the shifts of the demand line are identical and interchangeable (modulo a translation by the slope). Also our data, i.e. the observed bid functions, suggests that, at least locally at the point $k = 3$, the bid functions are linear.[47] Furthermore, our demand estimation models for both price and volume variation[48] indicate that the prediction model used works well in both dimension. In particular at $k = 3$, significance and equal signs on coefficients for

[47] Recall the graph in figure 3.3.2.
[48] Precisely look at columns 3 of tables 3.3 and 3.4.

all terms included as well as similar explanatory power[49] in both regressions confirms the similar nature of the two proxies.

Our recovered $\text{PLU}^{D,P}$ and $\text{PLU}^{D,Q}$ are, as expected, collinear.[50] While OLS remains unbiased in the presence of collinearity between two regressors, its precision is reduced. We correct for the collinearity by dropping one proxy or the other, but the individual results remain unchanged - the coefficients of the two proxies keep opposite signs.

Assuming that our empirical strategy is valid to test the relationship of interest, a possible reason for our intriguing observations could be that the slope of the demand function, which relates $\text{PLU}^{D,Q}$ and $\text{PLU}^{D,P}$, is endogenous on the uncertainty. Uncertain demand does not only unilaterally shift the demand function in one dimension (either P or Q), but also affects the shape and thus the slope of the curve. This effect is not accounted for in our research design and could drive the opposing results for both proxies. The endogeneity of the slope of the demand curve could be accounted for in our model by extracting the residuals from a regression of $\text{PLU}^{D,P}$ on $\text{PLU}^{D,Q}$ in an analysis to see if endogeneity exists and then reusing the residuals to control for slope effects of the demand curve in the final regression. We leave this avenue for further research.

Without having resolved the empirical discrepancy in the results, the stark contrast between the two could also hint at the fact that we need new theories to explain both demand and supplier bidding behaviour on the electricity market. This calls for new theoretical models to better explain the shape of aggregate bid functions, which are S-shaped overall. Special attention in these models should be placed on the effect of uncertainty and its importance for bidders via the link of dynamic costs.

Finally, our analysis relies strongly on the analysis of the point of inflection ($k = 3$), but the functional analysis is important, too. While results on the whole bid function

[49] $R^2$ of 0.463 for the price and 0.478 for the volume regression.
[50] Not perfectly, but with a correlation coefficient of 0,62

are broadly speaking in line with the point-specific analysis on the point of inflection, the significance of the results is weaker and the results less clear. Furthermore, we often observe varying effects on low and high volume points.[51] We conclude that the impacts of variations in exogenous factors on the shape of the bid functions are not uniform. Non-linear effects are neither predicted by our linear theory nor have been shown in previous studies (with the exception of [Wolfram, 1999]). Our results hint at more intricate mechanisms which drive the shape of these bid functions.

### 3.6.2 Internal and external validity

We believe that the work is credible due to many aspects of the research design.

First, our set-up is based on rather intuitive relations which we test exclusively using simple OLS regressions. These regressions are econometrically unbiased given the data impurities that we observe. To guarantee precision of our estimates, we use bootstrapping techniques.

Second, considerable effort has gone into the treatment of the information that goes into the right hand side of our regressions. We do not only refer to the final PLUs used, but also point at the precise use of our controls. See, for example, the treatment of the variable RteBlackBox (details see page 118), which proxies for the information contained in the day ahead demand estimates (PrevConsoH) given out by the grid operator RTE. In order to extract the marginal information of the PrevConsoH estimate, which is not explained by other controls variables that we include in our analysis, we compute the residuals from a regression of PrevConsoH on our other controls, e.g. daytime controls such as suncycle. These residuals (called RteBlackBox) enable us to achieve a more sophisticated understanding of our regression output.[52]

[51] We refer specifically to the strengthening or weakening effects of exogenous variables on different points a shown in demand level estimation tables 3.3 and 3.4 as well as in the slope regressions tables 3.6 - 3.9.

[52] See, for example, the regression output of the demand estimation in tables 3.3 and 3.4.

We also emphasize the aspect that we understand our dataset as a cross-sectional dataset rather than a time-series. While we do segment our dataset into weekday and weekend days and only run our analysis on the former, there is not reason why demand on a Tuesday afternoon should not be comparable to demand on a Thursday afternoon. We therefore ignore weekday dummies to increase our sample size. Furthermore, we avoid the use of dummy variables to control for the hour of the contracts in our regressions in order to further increase the sample size. However, we cannot compare electricity consumption between 4 am and 4 pm within a day. Neither can we compare two 4 pm hours of a day in winter and another in the summer. Using dummies would first restrict our sample size, plus make our interpretation more difficult since the dummy variable aggregates the effect over all conditions that change between samples. We use a bottom up approach that allows us to circumvent the sample size restriction and interpretation difficulties from daytime or seasonality dummies. Instead, we use continuous variables to control for the daytime and season by means of short and longer term temperature averages or other weather characteristics such as luminosity, which generates controls like deltasun.[53]

Finally, we point at the empirical framework that allows us to run reduced form regressions on multiple regions of bid functions to better understand functional responses of those bids to variation in exogenous factors. We use 5 points for our analysis and refer to chapter 2 for the full details on this choice and the evaluation of the point selection. With hindsight, we feel that an additional two points would have been useful to better understand functional behaviour of the part of the bid functions, which is more relevant in equilibrium, i.e. on the centre part.[54] We note the computational demands of more points.

The methodology developed for our exercise on data from the French electricity mar-

[53]See section 3.4.1 for full details on our set of control variables for both demand and supply.

[54]For that we would recommend the points representing half of the maximum curvature between the current points $k = 2, 4$ and $k = 3$.

ket has applications in other domains. This is valid for the non-parametric point selection mechanism (section 3.4), the mechanism to aggregate local geographic data to a national level (chapter 2) as well as the identification strategy based on purely ex-ante data.

In particular, we note that the possibility to run reduced form estimation strategies for the analysis of markets which make access to functional data available. This includes all markets which use a multi-unit, uniform (or discriminatory) auction mechanism.

### 3.6.3 Endogeneity

The set-up of this work is specifically aimed at circumventing problems of endogeneity. For that sake, we keep a strict separation of ex-post and ex-ante information to the left and right hand sides, respectively, of any regression.

To achieve this separation of ex-ante and ex-post information, both newly developed methodologies are highly useful. The point selection methodology from section 3.4 allows us to extract proxies for the level of uncertainty about the realization of market demand, which are unaffected by the equilibrium interaction with the market supply. The weather data treatment methodology from chapter 2 enables us to base our proxies for the level of uncertainty from renewables on measures of the expected homogeneity of weather forecasts. Both methodologies allow us to recover ex-ante information on the prevailing uncertainty that firms have at their hands at the time of bidding. The information contained in all other controls used is also available at the time of bidding.

However for data availability reasons, we are not able to keep this strict separation at all times in practice and revert to using ex-post data to compute some variables that should ideally be computed on ex-ante information only. This is the case twice in this work: (i) we use observed weather data to compute the variable Solar[55] and (ii) we use

[55]Contrary to Wind1DA and Tempeff15, which we are able to compute purely on forecast data.

the pooled data over all auctions for the demand estimation and subsequent uncertainty forecast of equations 3.4.2 - 3.4.5.

In both cases, we do not believe that this choice compromise our results. For the case of Solar, we use realised luminosity instead of forecast data. This is as if weather forecasts were perfectly accurate. Given that solar production only accounts for a small fraction for of total electricity generation and that we extract the very informative component of the Solar variable by using the variable suncycle (which is arguably very well predictable), we do not see the use of ex-post data as problematic.

For the case of the $\text{PLU}^D$ computation, we run the demand estimation pooled over all observed auctions (i.e. past and future) and say that firms have this level of information when bidding in each auction of our sample. We do so because we do not have the necessary data before 01.01.2011 and thus cannot calibrate our forecasting model on a "learning" dataset. Instead, we assume that demand patterns conditional on the explanatory variables has remained constant over our 2.5 years time period of analysis. The estimation based on pooled data then yields, on average, the same insights as an analysis conducted purely on past data.

We could test robustness of our pooled approach by investigating the effect of a restriction on using only past data in the demand estimation. A learning effect could arise from more precise estimations of demand functions. However, due to the long experience of most firms on the market in reality, this learning effect would be artificial and not represent a real insight. We therefore accept the possibility of a (small) endogeneity concern in this paper and further work could fully circumvent this issue by extending the database appropriately.

## 3.7 Conclusion

This paper is a sophisticated proof of concept of our methodology applied to the electricity market. We observe that bidders take uncertainty from renewables generation as well as uncertainty from demand realization into account. The results indicate that electricity suppliers react to an increased level of uncertainty by bidding more volume elastically (steeper supply functions in the dimension Q (x-axis) - P (y-axis)) in order to minimize expected dynamic costs, which increase with the uncertainty. The results also indicate that not only supplier bidding is affected by uncertainty, but that the level of uncertainty also impacts bidding from the demand side of the market.

Future empirical work should focus on investigating the endogeneity of the demand function on uncertainty as well as better understand frictions in the bidding (e.g. focal price points). Concurrently, the results also call for more advanced theoretical work on the shape of bid functions of players, in particular to explain non-linear shapes. This is also suggested by our bid functional analysis which hints at non-unilateral effects of exogenous variables on the shape of the functions. The economic insight hidden in full bid functions is vast and a better understanding of these could be applied to address important welfare questions.[56]

[56]Such an application, which the authors currently focus on is the question of the optimal choice of the geographic installation of renewable electricity generation units (solar panels and wind turbines) with respect to minimizing the intermittency of renewables generation. A clear understanding of the effects of uncertainty on the market is vital to close the analysis on organizational questions of the market. This is outside of the focus of this paper

# Appendix

## Appendix 3.A Summary Statistics of Selected Points

| | Mean | Median | StdDev | Min | Max |
|---|---|---|---|---|---|
| Prices for $k = 1$ | -3,000.0 | -3,000.0 | 0 | -3,000 | -3,000 |
| Prices for $k = 2$ | -56.7 | -55.0 | 19 | -97 | 70 |
| Prices for $k = 3$ | 27.6 | 26.8 | 11 | -27 | 93 |
| Prices for $k = 4$ | 120.2 | 105.4 | 193 | -11 | 2,999 |
| Prices for $k = 5$ | 3,000.0 | 3,000.0 | 0 | 3,000 | 3,000 |

Table 3.10: Prices of selected demand points

| | Mean | Median | Std. Dev | Min | Max |
|---|---|---|---|---|---|
| Volumes for $k = 1$ | 13,328 | 13,222 | 2,213 | 4,990 | 23,254 |
| Volumes for $k = 2$ | 12,919 | 12,824 | 2,238 | 3,321 | 23,001 |
| Volumes for $k = 3$ | 8,779 | 8,664 | 2,028 | 1,958 | 18,335 |
| Volumes for $k = 4$ | 5,777 | 5,730 | 1,558 | 987 | 12,773 |
| Volumes for $k = 5$ | 5,031 | 4,968 | 1,467 | 914 | 11,301 |

Table 3.11: Volumes of selected demand points

| | Mean | Median | Std. Dev. | Min | Max |
|---|---|---|---|---|---|
| Prices for $k = 1$ | -3,000.0 | -3,000.0 | 0 | -3,000 | -3,000 |
| Prices for $k = 2$ | -30.3 | -25.0 | 219 | -2,999 | 439 |
| Prices for $k = 3$ | 61.3 | 58.6 | 24 | 11 | 526 |
| Prices for $k = 4$ | 133.9 | 136.3 | 32 | 36 | 626 |
| Prices for $k = 5$ | 3,000.0 | 3,000.0 | 0 | 3,000 | 3,000 |

Table 3.12: Prices of selected supply points

| | Mean | Median | Std. Dev. | Min | Max |
|---|---|---|---|---|---|
| Volumes for $k = 1$ | 3,721.7 | 3,526.0 | 1,344 | 618 | 10,594 |
| Volumes for $k = 2$ | 4,432.8 | 4,226.0 | 1,602 | 844 | 11,765 |
| Volumes for $k = 3$ | 8,467.2 | 8,365.5 | 1,814 | 3,431 | 20,932 |
| Volumes for $k = 4$ | 11,849.5 | 11,717.7 | 2,411 | 3,641 | 27,810 |
| Volumes for $k = 5$ | 14,390.6 | 14,142.0 | 3,052 | 6,580 | 35,356 |

Table 3.13: Volumes of selected supply points

# Appendix 3.B Computational Details and Descriptives

## 3.B.1 Hard choices in the PLU computation

In computing the multi-variate kernel based prediction of the uncertainty for a given auction, we select auctions of a sufficient degree of similarity. We base the forecast equation 3.4.5 on this subsample dataset. We thereby consider that firms use the forecasting equation only *locally* in the neighbourhood of the auction of interest.

In order to define the size of the neighbourhood of an auction, we have to explicitly specify the width of the kernel window used in selecting the respective subsamples.

The trade-off involved is that we want to have small kernels for a precise computation of the PLU, while we want large kernels to make sure that we have a sufficient sample size in each kernel in order to derive meaningful statistics.

We choose to use a constant kernel window length with respect to each conditioning variable. We set the length of the window for each variable equal to $\frac{1}{3}$ of the variation of that variable. E.g. for Tempeff15, we observe a range of values from $-10^o$C to $14^o$C. The subsample used to compute the PLU$^D$ corresponding to a specific observation will

consist of all observations that are within a range of $\pm 4^o$C of that observation for Temp-eff15. The same logic is applied to selecting the neighbourhood with respect to all other conditioning variables.

Table 3.14 gives descriptive statistics about the conditioning variables for the kernel and the explicit choice $m$, which determines the length of the kernel window for a variable $X_e$ using the formula $b_{X_e} = \frac{2}{m_{X_e}}$.

## 3.B.2 Descriptive Statistics

### On realised market equilibria

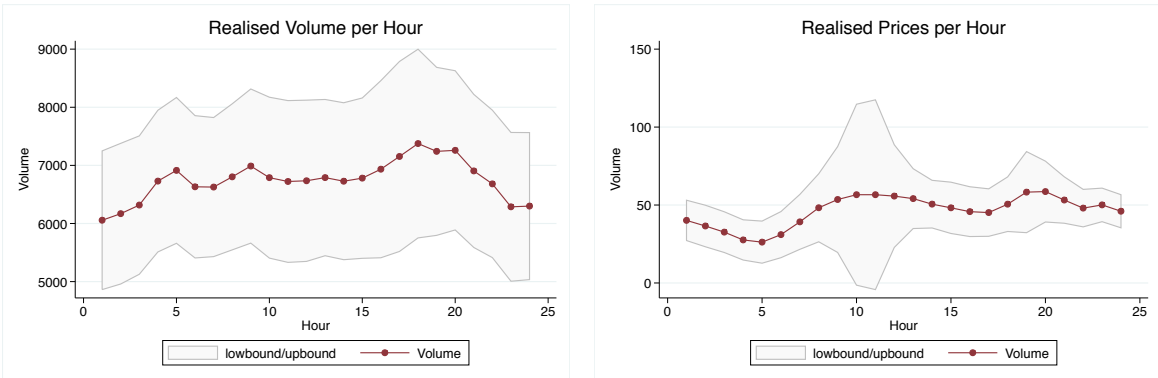



Figure 3.B.1: Plotted average realised Volume (left) and Price (right) per Hour with 95% confidence intervals.

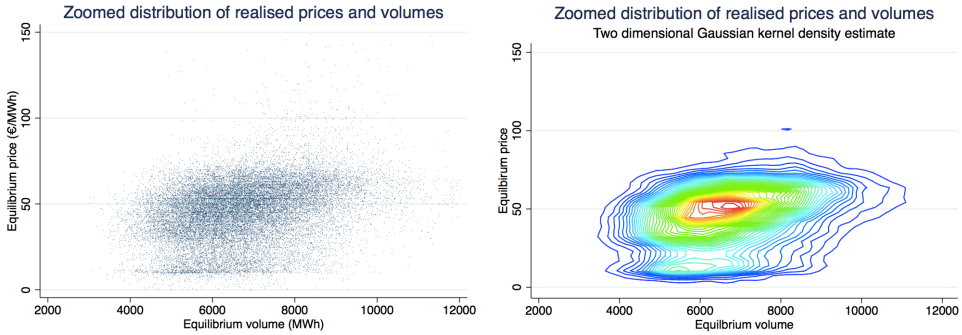



Figure 3.B.2: Distribution of observed market equilibria

Note: The warmer the colours of the heat map, the higher the frequency of realised price-quantity schedules. The colour legend is omitted for brevity, density changes between contours are of the order of $10^{-4}$.

## On player bid functions

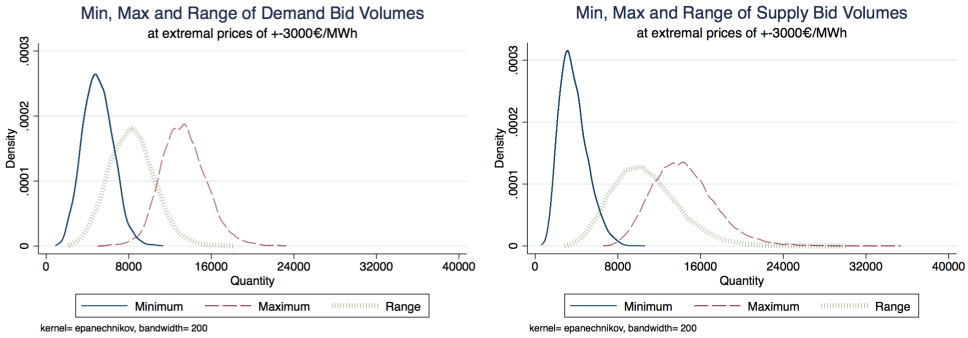



Figure 3.B.3: Distribution of minimum and maximum production volumes (and corresponding range) bid in an hourly auction.

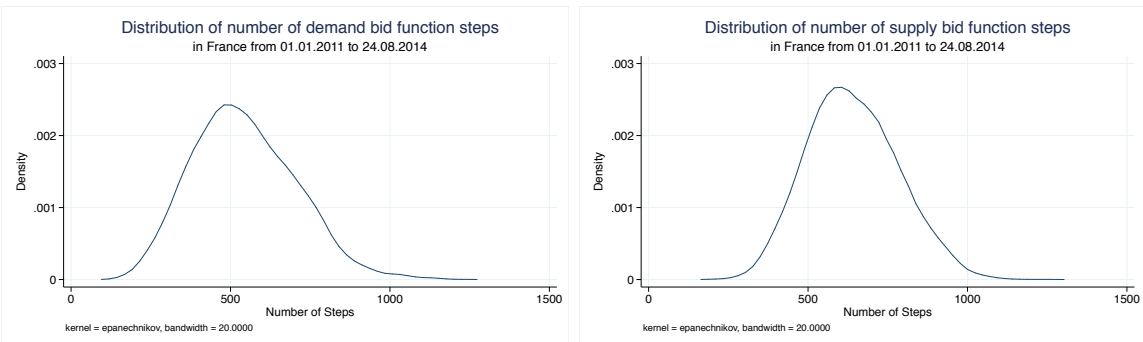



Figure 3.B.4: Distribution of number of bid function steps

## On exogenous factors

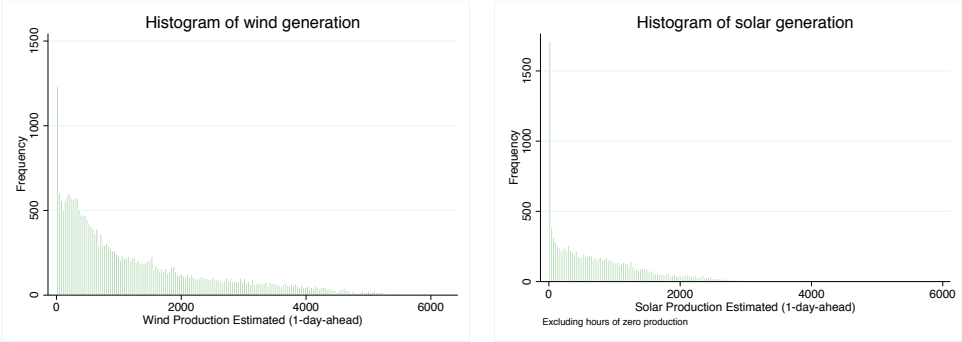



Figure 3.B.5: Histogram of predicted wind (left) and predicted solar (right) generation

For k=3 (Point of inflection)

| | (1) fxInvertQP | (2) fxInvertQP | (3) fxInvertQP | (4) fxInvertQP |
|---|---|---|---|---|
| PLUvRvarT | 0.000882 | 0.000882 | 0.00374** | 0.00508 |
| | (0.00152) | (0.00415) | (0.00155) | (0.00354) |
| PLUvRvarTsq | -0.000529 | -0.000529 | -0.00161*** | -0.00215 |
| | (0.000584) | (0.168) | (0.000603) | (0.183) |
| PLUvRvarW | 0.00790*** | 0.00790*** | 0.00647*** | 0.00574*** |
| | (0.00123) | (0.00257) | (0.00121) | (0.00207) |
| PLUvRvarWsq | -0.00235*** | -0.00235 | -0.00192*** | -0.00170 |
| | (0.000373) | (0.0644) | (0.000370) | (0.0479) |
| PLUvRvarS | -5.20e-10 | -5.20e-10 | -2.28e-09 | -2.23e-09 |
| | (2.68e-09) | (3.58e-08) | (3.16e-09) | (3.69e-08) |
| PLUvRvarSsq | 0 | 0 | 0 | 0 |
| | (0) | (0) | (0) | (0) |
| Coal | 6.90e-06*** | 6.90e-06*** | 5.18e-06*** | 6.29e-06*** |
| | (4.35e-07) | (4.64e-07) | (4.39e-07) | (6.87e-07) |
| Brent | -2.36e-05*** | -2.36e-05*** | -1.18e-05*** | -1.40e-05*** |
| | (1.51e-06) | (1.96e-06) | (1.53e-06) | (2.01e-06) |
| Gas | -2.82e-07 | -2.82e-07 | 1.37e-05*** | 1.36e-05*** |
| | (1.89e-06) | (9.41e-06) | (1.67e-06) | (2.46e-06) |
| IT2 | -2.71e-05*** | -2.71e-05 | -1.73e-05*** | -1.99e-05*** |
| | (2.17e-06) | (1.80e-05) | (1.34e-06) | (1.69e-06) |
| EUA | 7.20e-05*** | 7.20e-05*** | 2.62e-05*** | 2.71e-05*** |
| | (2.31e-06) | (4.49e-06) | (3.34e-06) | (6.84e-06) |
| Wind1DA | 1.04e-07*** | 1.04e-07*** | 1.18e-07*** | 1.25e-07*** |
| | (6.45e-09) | (1.03e-08) | (6.51e-09) | (7.63e-09) |
| Hydro | -7.55e-06*** | -7.55e-06*** | -4.08e-06*** | -5.88e-06*** |
| | (8.33e-07) | (2.24e-06) | (8.61e-07) | (1.11e-06) |
| PLUvDvarP | -0.000219*** | -0.000219 | | |
| | (4.57e-05) | (0.000203) | | |
| PLUvDvarQ | 0.000567*** | 0.000567 | | |
| | (9.44e-05) | (0.000585) | | |
| PLUvDvarKP | | | -0.000600*** | -0.000462*** |
| | | | (2.69e-05) | (4.24e-05) |
| PLUvDvarKQ | | | 0.000151*** | 0.000170** |
| | | | (3.39e-05) | (6.80e-05) |
| Constant | 0.00651*** | 0.00651*** | 0.00513*** | 0.00538*** |
| | (0.000208) | (0.000789) | (0.000195) | (0.000257) |
| Observations | 11,702 | 11,702 | 11,702 | 11,702 |
| $R^2$ | 0.200 | 0.200 | 0.233 | 0.234 |

(Standard errors in parentheses)
*** p<0.01, ** p<0.05, * p<0.1

Table 3.5: Regressions of the slope on $\text{PLU}^R$ and $\text{PLU}^D$ and $\text{PLU}^D$ at $k = 3$
*Note*: Standard errors are reported in parenthesis. Column 1 refers to the baseline specification. Column 2 reports bootstrapped results for the baseline model. Column 3 reports the results for the (weighted) regression on the kernel based $\text{PLU}^D_{\tilde{X}}$. Column 4 reports bootstrapped results of the model in column 3.

For k=1 (Left extremal point)

| | (1) fxInvertQP | (2) fxInvertQP | (3) fxInvertQP |
|---|---|---|---|
| PLUvRvarT | -4.14e-05*** | -4.14e-05 | 0.000277* |
| PLUvRvarTsq | 1.56e-05*** | 1.56e-05 | -0.0122 |
| PLUvRvarW | -6.04e-06 | -6.04e-06 | -0.000138 |
| PLUvRvarWsq | 1.71e-06 | 1.71e-06 | 0.00738 |
| PLUvRvarS | 0 | 0 | -5.70e-05 |
| PLUvRvarSsq | -0 | -0 | 0.00172 |
| Coal | -8.54e-09*** | -8.54e-09*** | (omitted) |
| Brent | 8.64e-08*** | 8.64e-08*** | (omitted) |
| Gas | -6.20e-08*** | -6.20e-08*** | (omitted) |
| IT2 | 4.95e-08*** | 4.95e-08*** | 3.50e-08 |
| EUA | -3.14e-08*** | -3.14e-08*** | 4.43e-06*** |
| Wind1DA | -3.38e-10*** | -3.38e-10*** | 2.48e-10 |
| Hydro | 4.69e-08*** | 4.69e-08*** | (omitted) |
| PLUvDvarQ | -3.87e-06*** | -3.87e-06*** | |
| PLUvDvarKQ | | | -7.21e-10*** |
| Constant | 2.11e-06*** | 2.11e-06** | -6.00e-05*** |
| Observations | 11,702 | 11,702 | 50 |
| $R^2$ | 0.152 | 0.152 | 0.681 |

Standard errors available from the authors

*** p<0.01, ** p<0.05, * p<0.1

Table 3.6: Regressions of slope on PLU$^R$ and PLU$^D$ and PLU$^D$ at $k = 1$

For k=2 (Left point of maximum curvature)

| | (1) fxInvertQP | (2) fxInvertQP | (3) fxInvertQP |
|---|---|---|---|
| PLUvRvarT | -0.00252 | -0.00252 | 0.292 |
| PLUvRvarTsq | 0.00106 | 0.00106 | -17.27 |
| PLUvRvarW | -0.00549*** | -0.00549** | 0.339 |
| PLUvRvarWsq | 0.00158*** | 0.00158 | -21.86 |
| PLUvRvarS | -6.82e-10 | -6.82e-10 | 0.0669 |
| PLUvRvarSsq | 0 | 0 | -1.968 |
| Coal | 2.36e-06*** | 2.36e-06*** | (omitted) |
| Brent | -1.86e-05*** | -1.86e-05*** | (omitted) |
| Gas | -8.94e-06*** | -8.94e-06*** | (omitted) |
| IT2 | 1.98e-05*** | 1.98e-05*** | 6.92e-05 |
| EUA | 8.69e-05*** | 8.69e-05*** | -0.000439 |
| Wind1DA | 6.13e-09 | 6.13e-09 | 6.70e-07 |
| Hydro | -5.82e-06*** | -5.82e-06*** | (omitted) |
| PLUvDvarP | -4.81e-05*** | -4.81e-05*** | |
| PLUvDvarQ | 0.000442*** | 0.000442*** | |
| PLUvDvarKP | | | -9.62e-07* |
| PLUvDvarKQ | | | -4.27e-07 |
| Constant | 0.00319*** | 0.00319*** | 0.00279 |
| Observations | 11,702 | 11,702 | 50 |
| $R^2$ | 0.158 | 0.158 | 0.414 |

Standard errors available from the authors

For k=4 (Right point of maximum curvature)

| | (1) fxInvertQP | (2) fxInvertQP | (3) fxInvertQP |
|---|---|---|---|
| PLUvRvarT | -0.00442*** | -0.00442 | 0.000559 |
| PLUvRvarTsq | 0.00149** | 0.00149 | -0.000368 |
| PLUvRvarW | -0.000137 | -0.000137 | -0.00205 |
| PLUvRvarWsq | 0.000173 | 0.000173 | 0.000739* |
| PLUvRvarS | 2.59e-09 | 2.59e-09 | 2.40e-09 |
| PLUvRvarSsq | -0 | -0 | -0 |
| Coal | 2.22e-07 | 2.22e-07 | 1.48e-06*** |
| Brent | -7.46e-06*** | -7.46e-06*** | -1.30e-05*** |
| Gas | 9.04e-06*** | 9.04e-06*** | 2.04e-05*** |
| IT2 | -1.96e-05*** | -1.96e-05*** | -2.61e-05*** |
| EUA | 4.71e-05*** | 4.71e-05*** | 3.19e-05*** |
| Wind1DA | 1.64e-08** | 1.64e-08** | 1.50e-08** |
| Hydro | -8.73e-06*** | -8.73e-06*** | -1.33e-05*** |
| PLUvDvarP | -0.000212*** | -0.000212*** | |
| PLUvDvarQ | 0.000110 | 0.000110** | |
| PLUvDvarKP | | | -0.000163*** |
| PLUvDvarKQ | | | 4.08e-05 |
| Constant | 0.00370*** | 0.00370*** | 0.00406*** |
| Observations | 11,701 | 11,701 | 11,701 |
| $R^2$ | 0.086 | 0.086 | 0.117 |

Standard errors available from the authors

*** p<0.01, ** p<0.05, * p<0.1

Table 3.8: Regressions of slope on PLU$^R$ and PLU$^D$ and PLU$^D$ at $k = 4$

For k=5 (Right extremal point)

| | (1) fxInvertQP | (2) fxInvertQP | (3) fxInvertQP |
|---|---|---|---|
| PLUvRvarT | -0.000252 | -0.000252 | 0.000734*** |
| PLUvRvarTsq | 9.10e-05 | 9.10e-05 | -0.000280*** |
| PLUvRvarW | -0.000555*** | -0.000555** | -0.000545*** |
| PLUvRvarWsq | 0.000169*** | 0.000169 | 0.000163*** |
| PLUvRvarS | -4.17e-10 | -4.17e-10 | -3.07e-10 |
| PLUvRvarSsq | 0 | 0 | 0 |
| Coal | -8.70e-07*** | -8.70e-07*** | -4.90e-07*** |
| Brent | 1.72e-06*** | 1.72e-06*** | 4.90e-07** |
| Gas | 4.53e-06*** | 4.53e-06*** | 2.96e-06*** |
| IT2 | 2.23e-06*** | 2.23e-06*** | 2.47e-06*** |
| EUA | 2.89e-06*** | 2.89e-06*** | 8.35e-06*** |
| Wind1DA | -5.41e-10 | -5.41e-10 | 3.49e-09*** |
| Hydro | 1.78e-06*** | 1.78e-06*** | 1.19e-06*** |
| PLUvDvarQ | 4.29e-05*** | 4.29e-05*** | |
| PLUvDvarKQ | | | 5.56e-05*** |
| Constant | -0.000494*** | -0.000494*** | -0.000351*** |
| Observations | 11,702 | 11,702 | 11,702 |
| $R^2$ | 0.128 | 0.128 | 0.131 |

Standard errors available from the authors

| $X_e$ | $m$ | Mean | Median | Std. dev. | Min | Max |
|---|---|---|---|---|---|---|
| Tempeff15 | 6 | 7.7 | 8 | 5 | -10 | 14 |
| Roll_Temp24 | 6 | 7.7 | 9 | 4 | -8 | 14 |
| Roll_Temp240 | 1 | 7.6 | 8 | 4 | -7 | 13 |
| suncycle | 6 | 0.3 | 0 | 0 | 0 | 1 |
| morning | 6 | 0.5 | 1 | 0 | 0 | 1 |
| deltasun | 6 | 0.1 | 0.1 | 0 | 0 | 0.4 |
| EWH | 6 | 0.3 | 0 | 0 | 0 | 1 |
| SolarRest | 6 | 5.4 | -1 | 364 | -1,337 | 2,241 |
| RteBlackBox | 6 | -0.0 | 37 | 4,755 | -16,966 | 18,209 |

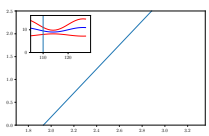
Table 3.14: Variables used in the kernel based $\text{PLU}^D$ computation
*Note*:For the PLUv51, we have excluded the variable Roll_Temp240 from the conditioning in order to increase the size of each subsample used for the calculation of the observation specific $\text{PLU}^D$. Version 52 also conditions on the variable Roll_Temp240 using $m = 6$.

# General conclusion

This thesis focused on the effect of ramping costs on the electricity market at a theoretical level, and then on the empirical analysis of market data to test the theoretical predictions.

The first chapter focused on what the introduction of ramping costs in a theoretical framework brings to the table. Ramping costs represent the fact that electricity suppliers incur costs when their production varies over time. Our main contribution has been to build and justify how these ramping costs can be tackled theoretically. First, we noted that going to a continuous time description of the problem allowed us to bring to the literature about supply function equilibria powerful mathematical tools mostly used in option pricing, that is stochastic dynamics: we want to model ramping costs, i.e. costs associated to the variation in production, while retaining the key ingredient brought by [Klemperer and Meyer, 1989], i.e. the uncertainty, through the use of brownians, and more precisely, Itō processes. In so doing we faced the issue that one cannot derive a brownian, and brought our second contribution, a physical argument about how power plants function that effectively operates as a low pass filter on our stochastic processes, and allowed us to continue to build a tractable model of ramping costs under uncertainty. Third, we found in the literature a specification of Itō processes that allowed the model to remain tractable.

From these technical contributions we obtained our economic contributions in having a rich tractable model that yields results that contrast strongly with past results from the literature. First, in the specific case of linear demand and linear costs we obtained a unique Nash equilibrium, which contrasts with the usual continuum of Nash equilibria in the supply function equilibria literature. Second, our solutions were not ex-post optimal, meaning that gathering information about the expected future evolution of demand yielded different optimal strategies for suppliers, which in turn meant that producers in our framework have a motive for submitting different supply functions from one time step to the next. Third, we have closed form solutions which yield specific predictions



about the evolution of bids under uncertainty, namely that when uncertainty increase, suppliers submit steeper supply schedules in order to transmit more of these shocks to changes in price and not quantities, which are costly due to the existence of ramping costs. Finally, and less importantly, our framework justified the existence of negative prices [57] by producers being willing to pay consumers to consume more in order to avoid facing large variations in production, in contrast to everywhere positive schedules in the case of the supply function equilibria literature. These results open the door to models being able to differentiate between day-ahead and intraday markets and therefore to offer a framework in which their interactions might be possible.

In the second chapter our main focus was on analyzing our data, on building a way to describe it, and on building proxies for the uncertainty that producers face about the residual demand they have to anticipate when bidding on the day-ahead market.

First, we noted that aggregate supply functions on the day ahead market cannot be well captured by parametric functions. Therefore, we devised a way to describe them non-parametrically: we noted that although they cannot be captured parametrically, they still have a rough S shape, and therefore four main parts, two extremal sections, and two interior ones separated by the inflection point of the curve in its middle section. We defined the transition points between these sections as the points of maximal absolute value for the derivative and second derivative of the supply schedules. This definition relied on kernel density estimates, and was therefore non-parametric. We observed that by using 5 such points, we were able to capture about 98% of the intrinsic variability of the supply schedules, and stopped there although our method can be used to define more non-parametric points. This method allowed us to define points that we considered comparable across auctions, that allowed us to perform cross-sectional analysis of our data in the third chapter.

[57] Note that such negative prices happen, a few hours a year for example in France or Germany, for example in 2017 there were 146 such hours, spread over 24 days in Germany [EPEX, 2018a]

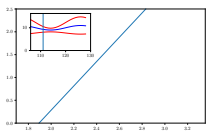
Second, we built proxies for the amount of weather uncertainty that producers face and variables that capture information that suppliers have before bidding and should therefore be controlled for. For the information available to suppliers, we noted that the effect of weather on the demand and more importantly temperature, was well understood and that we needed to control for it. To do so we built an effective temperature for France, as an average of the localised temperature weighted by the population of the spatial region considered, in order to capture the overall effect temperature has on heating.[58] The rest of our focus was on building a proxy for the uncertainty concerning renewable production. To do so we analyzed spatialized wind and sunlight data and studied it's spatial structure. We argued that spatial autocorrelation is a proxy for the uncertainty associated with weather forecasts, noting that if this data displays more spatial gradients, it was likely to be of a lesser quality due to the numerical nature of the weather simulations used to predict the weather, and therefore more uncertain.

Our contribution in the second chapter was to provide a non-parametric way to define comparable points across auctions, and a measure of the uncertainty associated with weather forecasts.

In the final chapter, we studied the impact that uncertainty about the demand plays on the shape of the aggregate supply functions suppliers bid on the French electricity market. We segmented our analysis to different parts of the supply functions in order to show how the overall shape changed with respect to our explanatory variables. We tested some of the predictions from our first chapter, mainly that the supply function should see its slope increase when uncertainty increases.

We noted that the main uncertainty is about the shape of the demand schedules itself. Therefore, we considered data available to the producers and regress the demand schedules on these variables. Next, we studied the residuals of these regressions, and

[58] France has a high level of electric heating overall, which means that demand for electricity is quite sensitive to temperature.



more specifically noted that they are heteroskedastic. We leveraged this, regressing the square of these residuals on our variables, in order to predict the expected amplitude of the residuals, that is the amplitude of the uncertainty of the demand schedule regression.

We then studied the effect of our different proxies for uncertainty on the slope of the supply schedules and noted that if our proxies about the weather uncertainty (through the channel of renewable production) have the expected effect, the results are less clear cut for our residuals on the demand schedules. As we are working with full-blown schedules in the quantity-price plane, we performed our residual analysis both on the prices and the quantities. We therefore obtained estimates for the uncertainty pertaining to the position of a given point of our demand schedule either in price or in quantity. In our theoretical framework, we made the strong assumptions that demand schedules are linear, and that demand shocks are additive, i.e. they do not impact the slope of the demand schedules. These assumptions yielded that we cannot differentiate between shocks in price or quantity, and that they should have effects in the same direction: more uncertainty implying steeper supply curves to reduce the amount of fluctuation in production. However, we observed that the effects of price and quantity uncertainty as estimated by our residuals' method yield opposite effects. Both of these assumptions, although required to obtain closed form results, are clearly not satisfied by our data, and we think that this is a clear path for improvement of the model.

The contribution of the third chapter is to provide a way to estimate the uncertainty about the demand schedules faced by suppliers, and to estimate how this uncertainty affects the shape of the supply schedules at different points along its overall length, i.e. we provide a framework to describe how the functional form of schedules is affected by estimates of the uncertainty faced by suppliers.

# Avenues of research

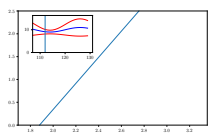
The work presented in this thesis opens new possible avenues of research, that we will outline here.

## Theoretical model

- Generalize the functional forms of the demand: we developed our model in the context of linear demand functions, and finding either general results, for example for positive and decreasing demand functions, would lend more support to our results. It would already be interesting to find whether these results hold for other specific functional forms for the demand functions. The issue is that the second order differential equations do not belong to solved for classes of equations in the cases that were tested in the course of this thesis (power demand functions for example). It is therefore unlikely that analytical results can be obtained, however numerical approaches could prove useful in this context.

- Study the impact of other stochastic processes: our results hold in the case of stochastic shocks leading to an equilibrium distribution of a quadratic form. The processes that we use to obtain our results are part of a larger class of processes, which can exhibit richer caracteristics, for example assymetric distributions. As previously, the analytical nature of our results relies partly on the specific choice of stochastic process we made, therefore analytical results are unlikely, but numerical approaches could shed light on the effect of skewed distributions.

- Study how a time discrete approach converges towards our continuous time one: the derivation is doable in the case of discrete states for demand shock. A toy model not presented in this thesis was derived in the case of a two period two valued shocks model. Although the results are consistent with those of our continuous model, the expressions derived analytically are already horribly tedious. It is therefore once again a strand of analysis that could profit from a numerical approach.



- The interaction between intraday and day-ahead markets: knowing that when one bids on the day-ahead market, it will still be possible to adjust one's position tomorrow on the intraday market is bound to impact the strategies of the suppliers. Trying to tackle this problem, if challenging, could prove very interesting. Here are key ingredients that should be taken into account:

  First, intraday bids can be submitted anytime during the day with an expiry date attached. Therefore, there is tension between, on the one hand, the will to start and correct the outcome of the day-ahead market as soon as new information enters about the demand shocks, so as to increase the likelihood for another agent to buy the intraday bid, and on the other hand, the will to wait and see as information enters to be as precise as possible on the submitted bid to correct the outcome of the day-ahead market, but therefore decreasing the likelihood to find a buyer.

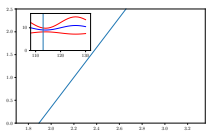
  Second, the ramping costs associated with changing production are incurred only after the net of the day-ahead market and intraday market is fixed.

- Implement the actual market clearing algorithm and study it numerically.

## Empirical analysis

- Take into account the block orders: these orders impact the bids and should be accounted for.
- Study in more detail the overall function without restriction to only 5 points, which should be doable with the increase in computational power
- Leverage the difference between weather prediction data and observations for more accurate weather uncertainty.
- Take into account the uncertainty associated with international interconnexions.

- Analyze the individual submitted points on the aggregate supply schedules to find whether it is possible to attach some of them to specific power plants with any certainty.



# References


[Anderson and Hu, 2012] Anderson, E. J. and Hu, X. (2012). Asymmetric supply function equilibria with forward contracts. *Journal of optimization theory and applications*, 152(1):198–224.

[Anderson and Xu, 2005] Anderson, E. J. and Xu, H. (2005). Supply function equilibrium in electricity spot markets with contracts and price caps. *Journal of Optimization Theory and Applications*, 124(2):257–283.

[APX, 2017] APX (2017). Type of orders on the epex day-ahead auction, uk. https://www.apxgroup.com/trading-clearing/uk-half-hour-day-ahead-1530-auction/.

[Arima, 2012] Arima, K. (2012). Dispatch modelling: Quantifying long term benefits via high resolution analysis. Technical report, Wärtsilä Power Plants.

[Baldick et al., 2004] Baldick, R., Grant, R., and Kahn, E. (2004). Theory and application of linear supply function equilibrium in electricity markets. *Journal of regulatory economics*, 25(2):143–167.

[Belsunce, 2015] Belsunce, H. d. (2015). *Firm behaviour under uncertainty and legal challenges*. PhD thesis, lmu.

[Berg et al., 1999] Berg, S. A., Boukai, B., and Landsberger, M. (1999). Bid functions for treasury securities; across countries comparison. Technical report, Citeseer.

[Bertoldi and Atanasiu, 2007] Bertoldi, P. and Atanasiu, B. (2007). Electricity consumption and efficiency trends in the enlarged european union, status report. Technical report, Institute for Environment and Sustainability, European Commission.

[Boiteux, 1951] Boiteux, M. (1951). La tarification au coût marginal et les demandes aléatoires. *Cahiers du Séminaire d'économétrie*, pages 56–69.

[Boiteux, 1960] Boiteux, M. (1960). Peak-load pricing. *The Journal of Business*, 33(2):157–179.

[Bolle, 1992] Bolle, F. (1992). Supply function equilibria and the danger of tacit collusion: the case of spot markets for electricity. *Energy economics*, 14(2):94–102.

[Borenstein, 2002] Borenstein, S. (2002). The trouble with electricity markets: understanding california's restructuring disaster. *The Journal of Economic Perspectives*, 16(1):191–211.

[Borenstein and Bushnell, 1999] Borenstein, S. and Bushnell, J. (1999). An empirical analysis of the potential for market power in california's electricity industry. *The Journal of Industrial Economics*, 47(3):285–323.

[Borenstein et al., 2002] Borenstein, S., Bushnell, J. B., and Wolak, F. A. (2002). Measuring market inefficiencies in california's restructured wholesale electricity market. *American Economic Review*, 92(5):1376–1405.

[Boukai and Landsberger, 1998] Boukai, B. and Landsberger, M. (1998). Market bid functions for treasury securities as logistic growth curves. Technical report, Working Paper, Indiana University–Purdue University, Indianapolis, Indiana.

[Boyle, 2007] Boyle, G. (2007). *Renewable Electricity and the Grid, the Challenge of Variability*. Routledge.

[Brown and Johnson, 1969] Brown, G. and Johnson, M. B. (1969). Public utility pricing and output under risk. *The American Economic Review*, 59(1):119–128.

[Crew et al., 1995] Crew, M. A., Fernando, C. S., and Kleindorfer, P. R. (1995). The theory of peak-load pricing: A survey. *Journal of regulatory economics*, 8(3):215–248.

[EPEX, 2014] EPEX (2014). Website of the european power exchange. https://www.epexspot.com/en/.

[EPEX, 2015] EPEX (2015). Type of orders on the epex day-ahead auction, france. https://www.epexspot.com/en/product-info/auction.

[EPEX, 2018a] EPEX (2018a). Negative prices at epex. https://www.epexspot.com/en/company-info/basics_of_the_power_market/negative_prices.

[EPEX, 2018b] EPEX (2018b). Operational rules, paragraph 9. http://www.epexspot.com/en/extras/download-center/documentation.

[EPEX, 2018c] EPEX (2018c). Price coupling of regions. https://www.epexspot.com/en/market-coupling/pcr.

[Fabrizio et al., 2007] Fabrizio, K. R., Rose, N. L., and Wolfram, C. D. (2007). Do markets reduce costs? assessing the impact of regulatory restructuring on us electric generation efficiency. *American Economic Review*, 97(4):1250–1277.

[GE, 2015] GE (2015). Heavy-duty gas turbine operating and maintenance considerations. Technical report, General Electrics Energy.

[Green, 1991] Green, R. (1991). Reshaping the cegb: electricity privatization in the uk. *Utilities Policy*, 1(3):245–254.

[Green, 1996] Green, R. (1996). Increasing competition in the british electricity spot market. *The journal of Industrial economics*, pages 205–216.

[Green, 1999] Green, R. (1999). The electricity contract market in england and wales. *The Journal of Industrial Economics*, 47(1):107–124.

[Green and Newbery, 1992] Green, R. J. and Newbery, D. M. (1992). Competition in the british electricity spot market. *Journal of political economy*, 100(5):929–953.

[Grossman, 1981] Grossman, S. J. (1981). Nash equilibrium and the industrial organization of markets with large fixed costs. *Econometrica: Journal of the Econometric Society*, pages 1149–1172.

[Hart, 1982] Hart, O. D. (1982). *Imperfect competition in general equilibrium: an overview of recent work.*

[Hertzler, 2003] Hertzler, G. (2003). A stochastic differential equation for modeling the "classical" probability distributions. In *2003 Conference (47th), February*, pages 12–14.

[Hobbs, 2001] Hobbs, B. F. (2001). *The next generation of electric power unit commitment models*, volume 36. Springer.

[Hobbs et al., 2000] Hobbs, B. F., Metzler, C. B., and Pang, J.-S. (2000). Strategic gaming analysis for electric power systems: An mpec approach. *IEEE transactions on power systems*, 15(2):638–645.

[Holmberg et al., 2008] Holmberg, P., Newbery, D. M., and Ralph, D. (2008). Supply function equilibria: Step functions and continuous representations.

[Hortacsu and Puller, 2008] Hortacsu, A. and Puller, S. L. (2008). Understanding strategic bidding in multi-unit auctions: a case study of the texas electricity spot market. *The RAND Journal of Economics*, 39(1):86–114.

[Jamasb and Pollitt, 2005] Jamasb, T. and Pollitt, M. (2005). Electricity market reform in the european union: review of progress toward liberalization & integration. *The Energy Journal*, pages 11–41.

[Kastl, 2011] Kastl, J. (2011). Discrete bids and empirical inference in divisible good auctions. *The Review of Economic Studies*, 78(3):974–1014.

[Klemperer and Meyer, 1986] Klemperer, P. and Meyer, M. (1986). Price competition vs. quantity competition: the role of uncertainty. *The RAND Journal of Economics*, pages 618–638.

[Klemperer and Meyer, 1989] Klemperer, P. and Meyer, M. (1989). Supply function equilibria in oligopoly under uncertainty. *Econometrica.*

[Kloeden and Platen, 2011] Kloeden, P. and Platen, E. (2011). *Numerical Solution of Stochastic Differential Equations.* Stochastic Modelling and Applied Probability. Springer Berlin Heidelberg.

[LaCommare and Eto, 2004] LaCommare, K. H. and Eto, J. H. (2004). Understanding the cost of power interruptions to us electricity consumers. *Lawrence Berkeley National Laboratory.*

[Meibom et al., 2009] Meibom, P., Weber, C., Barth, R., and Brand, H. (2009). Operational costs induced by fluctuating wind power production in germany and scandinavia. *Renewable Power Generation, IET*, 3(1):75–83.

[Newbery, 1997] Newbery, D. M. (1997). Privatisation and liberalisation of network utilities. *European Economic Review*, 41(3):357–383.

[Newbery, 1998] Newbery, D. M. (1998). Competition, contracts, and entry in the electricity spot market. *The RAND Journal of Economics*, pages 726–749.

[Øksendal, 2003] Øksendal, B. (2003). Stochastic differential equations. In *Stochastic differential equations*, pages 65–84. Springer.

[Özcan, 2004] Özcan, R. (2004). The logistic function approach to discriminatory and uniform price treasury auctions.

[Préget and Waelbroeck, 2005] Préget, R. and Waelbroeck, P. (2005). Treasury bill auction procedures: Empirical perspectives from french market bid functions. *Journal of International Money and Finance*, 24(7):1054–1072.

[Préget and Waelbroeck, 2005] Préget, R. and Waelbroeck, P. (2005). Treasury bill auction procedures: Empirical perspectives from french market bid functions. *Journal of International Money and Finance*, 24:1054–1072.

[Reguant, 2011] Reguant, M. (2011). The welfare effects of complementary bidding mechanisms: An application to electricity markets. Technical report, Working paper.

[Reichl et al., 2013] Reichl, J., Schmidthaler, M., and Schneider, F. (2013). Power outage cost evaluation: Reasoning, methods and an application. *Journal of Scientific Research & Reports*, 2(1):249–276.

[REN21, 2013] REN21, S. C. (2013). Renewables 2013, global status report.

[Rothwell et al., 1995] Rothwell, G., Rust, J., et al. (1995). A dynamic programming model of us nuclear power plant operations. *Manuscript, Department of Economics, University of Wisconsin*.

[RTE, 2012] RTE (2012). 2012 overview of rte, the french grid operator. https://www.rte-france.com/sites/default/files/rte_bilan_electrique_2012.pdf.

[RTE, 2014] RTE (2014). Website of rte, the french grid operator. http://www.rte-france.com/en/.

[Salgado-Ugarte et al., 1994] Salgado-Ugarte, I. H., Shimizu, M., and Taniuchi, T. (1994). Exploring the shape of univariate data using kernel density estimators. *Stata Technical Bulletin*, 3(16).

[Sannikov et al., 2016] Sannikov, Y., Skrzypacz, A., et al. (2016). Dynamic trading: Price inertia and front-running. *Preprint*.

[Schmidt, 1996] Schmidt, K. M. (1996). The costs and benefits of privatization: an incomplete contracts approach. *The Journal of Law, Economics, and Organization*, 12(1):1–24.

[Seierstad and Sydsaeter, 1987] Seierstad, A. and Sydsaeter, K. (1987). *Optimal Control Theory with Economics Applications*. North-Holland.

[Sewalt and De Jong, 2003] Sewalt, M. and De Jong, C. (2003). Negative prices in electricity markets. *Commodities Now*, 2:74–79.

[Silverman and Ramsay, 2005] Silverman, B. and Ramsay, J. (2005). *Functional Data Analysis*. Springer.

[Silverman, 1986] Silverman, B. W. (1986). *Density estimation for statistics and data analysis*, volume 26. CRC press.

[Tanaka, 2006] Tanaka, M. (2006). Real-time pricing with ramping costs: A new approach to managing a steep change in electricity demand. *Energy Policy*, 34(18):3634–3643.

[Ventosa et al., 2005] Ventosa, M., Baıllo, A., Ramos, A., and Rivier, M. (2005). Electricity market modeling trends. *Energy policy*, 33(7):897–913.

[von der Fehr and Harbord, 1993] von der Fehr, N.-H. M. and Harbord, D. (1993). Spot market competition in the uk electricity industry. *The Economic Journal*, 103(418):531–546.

[White, 1980] White, H. (1980). A heteroskedasticity-consistent covariance matrix estimator and a direct test for heteroskedasticity. *Econometrica: Journal of the Econometric Society*, pages 817–838.

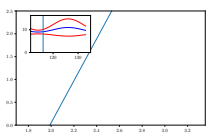
[Willems et al., 2009] Willems, B., Rumiantseva, I., and Weigt, H. (2009). Cournot versus supply functions: What does the data tell us? *Energy Economics*, 31(1):38–47.

[Wolak, 2007] Wolak, F. A. (2007). Quantifying the supply-side benefits from forward contracting in wholesale electricity markets. *Journal of Applied Econometrics*, 22(7):1179–1209.

[Wolfram, 1998] Wolfram, C. D. (1998). Strategic bidding in a multiunit auction: An empirical analysis of bids to supply electricity in england and wales. *The Rand Journal of Economics*, pages 703–725.

[Wolfram, 1999] Wolfram, C. D. (1999). Measuring duopoly power in the british electricity spot market. *American Economic Review*, pages 805–826.

[Wölfing, 2013] Wölfing, N. (2013). *Interacting markets in electricity wholesale : forward and spot, and the impact of emissions trading*. PhD thesis. Thèse de doctorat dirigée par Schubert, Katheline et Fleckinger, Pierre Sciences economiques Paris 1 2013.



# Code Annex

# Autocorrelation lengthscale

Overall, the Matlab code was written by Alexis Bergès, whereas the Stata code was written by Henri de Belsunce. All ideas, architecture choices, methods were co-developed with equal contributions from both

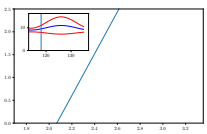
This first bit of code builds the autocorrelation lengthscales introduced in chapter 2.

It takes data from Meteofrance, that comes as files containing a row per observation, each row containing the timestamp of the observation, as well as the code of the station, and the observation itself, to which was added the latitude and longitude of the current station. For a given year, only data pertaining to this year is kept in memory, to avoid running out of RAM.

Once this first step is done, and for every timestamp, all pairs of stations are taken, and the difference of value in the observation is taken, as well as the distance between the stations.

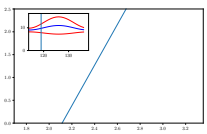
Once this treated data is generated, an exponential is fitted, and its coefficient is saved as the autocorrelation lengthscale for this given variable, and this given timestamp. The code to generate a graph of the cloud of points and the fitted function is also included.

```
clear all

filenamemaster='~/Desktop/Google Drive/Encheres Elec/For Matlab/Exogenous_treated_03.2014/Vent/
    ExoVent_numeric.csv';
M = csvread(filenamemaster);

%%Keep only year of interest
yyyy=2011;
for i=1:size(M,1)
    if mod(i,10000)==0
        disp('y of interest'), disp(i)
    end
    if M(i,5)==yyyy&&(i==1||M(i-1,5)<yyyy)
        yi=i;
    end
    if M(i,5)==yyyy&&(i==size(M,1)||M(i+1,5)>yyyy)
```



```
            yf=i;
        end
    end
    Myyyy=M(yi:yf,:);
    % clear M


    kini=1;
    kend=0;
    lautocor=[];
    for k=1:(size(Myyyy,1)-1)
        if mod(k,1000)==0
            k
        end
        if Myyyy(k,8)~=Myyyy(k+1,8 )
            kend=k;
            autocor=[];
            for i=kini:(kend-1)
                for j=(i+1):kend
                    ertemp=abs(Myyyy(i,1)-Myyyy(j,1));
                    disttemp=sqrt((Myyyy(i,9)-Myyyy(j,9))^2+(Myyyy(i,10)-Myyyy(j,10))^2);
                    autocortemp=cat(2,disttemp,ertemp);
                    autocor=cat(1,autocor,autocortemp);
                end
            end
            binranges=0:0.004:max(autocor(:,1));
            [bincounts,ind]=histc(autocor(:,1),binranges);
            autocor=cat(2,autocor,ind);
            A=sortrows(autocor,3);
            rini=1;
            rend=0;
            meanstd=[];
            for i=1:size(bincounts,1)
                if bincounts(i)>0
                    rend=rend+bincounts(i);
                    m1=mean(A(rini:rend,2));
                    m2=mean(A(rini:rend,1));
                    meantemp=cat(2,bincounts(i),m1,m2);
                    meanstd=cat(1,meanstd,meantemp);
                    rini=rend+1;
                end
            end

            y=meanstd(1:round(4/5*size(meanstd,1)),2);
            x=meanstd(1:round(4/5*size(meanstd,1)),3);
    %         y=meanstd(:,2);
    %         x=meanstd(:,3);
    %         figure, plot(meanstd(2:size(meanstd,1),3),meanstd(2:size(meanstd,1),2))
            g = fittype('a*(1-exp(-x/b))','dependent',{'y'},'independent',{'x'},...
                'coefficients',{'a','b'});
            myfit=fit(x,y,g,'Lower',[0,0],'Upper',[10,0.3],'Startpoint', [4.5     0.04]);
            coefffit=coeffvalues(myfit);
    %         figure, plot(myfit,meanstd(2:size(meanstd,1),3),meanstd(2:size(meanstd,1),2))
            kini=k+1;
            lautocortemp=cat(2,coefffit(2),Myyyy(k,5),Myyyy(k,6),Myyyy(k,7),Myyyy(k,8));
            lautocor=cat(1,lautocor,lautocortemp);
        end
    end

    %%%Convert lautocor to string to be stata friendly
```

```
dateseff=[];
for i=1:size(lautocor,1)
    if lautocor(i,4)<10
        dd=['0' num2str(lautocor(i,4))];
    else dd=num2str(lautocor(i,4));
    end
    if lautocor(i,3)<10
        mm=['0' num2str(lautocor(i,3))];
    else mm=num2str(lautocor(i,3));
    end
    if lautocor(i,5)<10
        hh=['0' num2str(lautocor(i,5))];
    else hh=num2str(lautocor(i,5));
    end
    dated=[dd '/' mm '/' num2str(lautocor(i,2))];
    datesefftemp={dated,hh,num2str(lautocor(i,1))};
    dateseff=cat(1,dateseff,datesefftemp);
end

%%Prepare the data to be written to .txt file
dateseff=dateseff;
fid = fopen(['/Users/alexisberges/Desktop/Google Drive/Encheres Elec/For Matlab/lautocor_' num2str(
    yyyy) '.txt'],'w');
fprintf(fid,'%s, %s, %s\n',dateseff{:,:});
fclose(fid);


%%%%%%%%%%%%%%% graph lautocor well
houra=[0;M(:,8)];
hourb=[M(:,8);0];
idxchange=find(houra-hourb);

lautocor=[];
hourlook=470;
kini=idxchange(hourlook);
kend=idxchange(hourlook+1)-1;
autocor=[];
k=1;
for i=kini:(kend-1)
    for j=(i+1):kend
        ertemp=abs(M(i,1)-M(j,1));
        disttemp=sqrt((M(i,9)-M(j,9))^2+(M(i,10)-M(j,10))^2)*6371;
        autocortemp=cat(2,disttemp,ertemp);
        autocor=cat(1,autocor,autocortemp);
    end
end
binranges=0:0.004*6371:max(autocor(:,1));
[bincounts,ind]=histc(autocor(:,1),binranges);
autocor=cat(2,autocor,ind);
A=sortrows(autocor,3);
rini=1;
rend=0;
meanstd=[];
for i=1:size(bincounts,1)
    if bincounts(i)>0
        rend=rend+bincounts(i);
        m1=mean(A(rini:rend,2));
        m2=mean(A(rini:rend,1));
        meantemp=cat(2,bincounts(i),m1,m2);
        meanstd=cat(1,meanstd,meantemp);
```

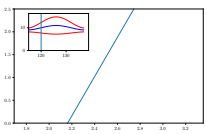


```
        rini=rend+1;
    end
end

y=meanstd(1:round(4/5*size(meanstd,1)),2);
x=meanstd(1:round(4/5*size(meanstd,1)),3);
%          y=meanstd(:,2);
%          x=meanstd(:,3);
%          figure, plot(meanstd(2:size(meanstd,1),3),meanstd(2:size(meanstd,1),2),'-','LineWidth',2)
g = fittype('a*(1-exp(-x/b))','dependent',{'y'},'independent',{'x'},...
    'coefficients',{'a','b'});
myfit=fit(x,y,g,'Lower',[0,0],'Upper',[10,0.3*6371],'Startpoint', [4.5     0.04*6371]);
coefffit=coeffvalues(myfit);
        figure
        plot1=plot(myfit,'r');
        set(plot1,'LineWidth',4)
        hold on
        plot2=plot(meanstd(2:size(meanstd,1),3),meanstd(2:size(meanstd,1),2),'b-+') ;
        set(plot2,'LineWidth',4)
        hold on
        plot3=plot(autocor(:,1),autocor(:,2),'ko','MarkerSize',2,'MarkerFaceColor',[0,0,0]);
        line1=line([0, 0.02*6371], [0,coefffit(1)/coefffit(2)*0.02*6371],'Color',[0 0 0],'LineStyle','--','LineWidth',2);
        line2=line([0, 0.2*6371], [coefffit(1),coefffit(1)],'Color',[0 0 0],'LineStyle','--','LineWidth',2);
        line3=line([coefffit(2), coefffit(2)], [0,coefffit(1)],'Color',[0 0.5 0],'LineStyle','--','LineWidth',2);
        plotleg=legend([plot3 plot2 plot1 line1 line3],
                        {'Data from the pairs' 'Kernel smoothed data' 'Fitted exponential'
                        'Derivatives of the fitted curve at 0 and \infty' 'Autocorrelation lengthscale'},
                        'Location','northwest');
        set(plotleg,'FontSize',24);
kini=k+1;
lautocortemp=cat(2,coefffit(2),M(k,5),M(k,6),M(k,7),M(k,8));
lautocor=cat(1,lautocor,lautocortemp);
```

# Regressions

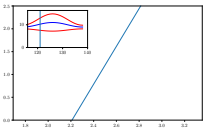
The next bit of code is the general do file for the regressions of chapter 3.

```
* COMPUTE COMPARABLE TABLES FOR SQ-RESIDUALS.

*** VERSION FEBRUARY 2015

* Final Do-file

/* Notes:
------------- MAKE SURE NORMAL VERSION RUNS LAST TO ENSURE CORRECT TABLES FOR DEMAND ESTIMATION
*/

        macro drop _all
        global YearINIT = 2011
        global YearFIN = 2013
        global HourINIT = 1
        global HourFIN = 24
        global vINPUT= "v38"
        global vOUTPUT= "v38"

        global dofiledirectoryorig= Path to directory containing do files
        global LATEXPATH = Path to directory containing latex for article
        global CLOUDPATH = Path to directory containing data

        global focusk = 5

**********
* Data Prep
**********

        use "${CLOUDPATH}v38/Temp_data/DataReady_Y${YearINIT}-Y${YearFIN}.dta", clear
        * manually merged Roll_avgT24 to dataset.
        * manually merged winddiffonly to dataset

                destring YYYY MM DD, replace
                drop Year Month
                gsort YYYY MM DD Hour SalePurchase select
                order Date Hour Price Volume SalePurchase select


        rename  COAL_IMPORTPRICE_EURpTON Coal
        rename  EST_PRICE_ELEC_EXPORT EExportPrice  /*in EUR*/
        rename BRENT_LDN_AVG Brent
        rename GAS_SPOT_GBPpTHERM Gas
        rename LcWind2011 LcWind

        label var  Roll_Temp24 "Roll\_Temp24"
        label var  Roll_Temp240 "Roll\_Temp240"
        label var   Roll_Temp720 "Roll\_Temp720"
        label var   Roll_avgT24 "Roll\_avgT24"
        label var   Roll_avgT240 "Roll\_avgT240"

* dataprep:
        gen IT2 = (PrevConsoH / 99400) * Gas  /*PrevConsoH not included in supply,  99400 is max of
              PrevConsoH */
```



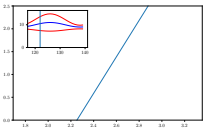
```
        gen EWH = 0
        replace EWH = 1  if Hour<=4
        replace EWH = 1  if Hour>=22
        gen dfaT15 =  Roll_Temp24 -Roll_Temp240


        * generate slope of opposite function (on Demand, add fx of supply function)
        gsort select Datestata Hour SalePurchase
        capture drop errorindic
        gen errorindic = 1 if SalePurchase[_n]==SalePurchase[_n+1]
        drop if errorindic == 1 & SalePurchase=="Purchase"
        drop errorindic
        capture drop fxInvertPQ_viaP fxInvertQP_viaP
        capture drop Price_S_viaP Volume_S_viaP
        gen fxInvertPQ_viaP = 1/( fx[_n+1]) if SalePurchase=="Purchase"
        gen fxInvertQP_viaP = fx[_n+1] if SalePurchase=="Purchase"
        gen Price_S_viaP = Price[_n+1] if SalePurchase=="Purchase"
        gen Volume_S_viaP = Volume[_n+1] if SalePurchase=="Purchase"
        gsort SalePurchase select Datestata Hour
        gen fxInvertQP = fxInvertQP_viaP
        gen fxInvertPQ = fxInvertPQ_viaP

        * make points comparable by volume (not by price!)
******************* remove to do other dimension + must rename fxinvertQP
        gen selectQ = select
        capture drop lowpk
        capture drop selectviaP
        gen lowpk = 1 if select<5  & SalePurchase=="Purchase"
        replace selectQ =9      if selectQ ==1  & SalePurchase=="Purchase" &  lowpk == 1
        replace selectQ =7      if selectQ ==3  & SalePurchase=="Purchase" &  lowpk == 1
        replace selectQ =3      if selectQ ==7  & SalePurchase=="Purchase" &  lowpk == .
        replace selectQ =1      if selectQ ==9  & SalePurchase=="Purchase" &  lowpk == .
        rename select selectviaP
        rename selectQ select
                * generate slope of opposite function (on Demand, add fx of supply function)
        gsort select Datestata Hour SalePurchase
        capture drop errorindic
        gen errorindic = 1 if SalePurchase[_n]==SalePurchase[_n+1]
        drop if errorindic == 1 & SalePurchase=="Purchase"
        drop errorindic
        capture drop fxInvertPQ_viaQ fxInvertQP_viaQ
        capture drop Price_S_viaQ Volume_S_viaQ
        gen fxInvertPQ_viaQ = 1/( fx[_n+1]) if SalePurchase=="Purchase"
        gen fxInvertQP_viaQ = fx[_n+1] if SalePurchase=="Purchase"
        gen Price_S_viaQ = Price[_n+1] if SalePurchase=="Purchase"
        gen Volume_S_viaQ = Volume[_n+1] if SalePurchase=="Purchase"
        gsort SalePurchase select Datestata Hour
        capture drop fxInvertQP fxInvertPQ
        gen fxInvertQP = fxInvertQP_viaQ
        gen fxInvertPQ = fxInvertPQ_viaQ
        drop fxInvertPQ_viaP fxInvertQP_viaP
        order Date Hour Price Volume SalePurchase select selectviaP

        *************************


**********
* Solar1DA prediction
**********
                        capture drop IT1
```

```
                        capture drop SolarRest
                        gen IT1 = suncycle * Roll_avgT240 /*Interaction term 1 = suncycle * average temp no cutoff at 15 = proxy for sunangle - decided against as no change in ry of rte black box and easier interpretation*/
                        global solarestimationvariables " suncycle"

                        reg Solar1DA $solarestimationvariables, robust
                        est store Black_3

                        capture drop SolarBlackBox
                        capture drop blackepsilon
                        capture mat drop blackalpha
                        mat blackmat = e(b)
                        mat li blackmat
                        scalar blackalpha = blackmat[1,3]
                        di  blackalpha

                        predict blackepsilon if e(sample), residuals
                        gen SolarRest = blackepsilon  /*+ blackalpha*/
                        drop blackepsilon

                btoutreg2 [Black_3] using "${LATEXPATH}SolarBlack.tex", replace tex(frag pretty ) /*stats(coef  Var se)*/ label(proper)  level(95)  sideway noparen


                *drop irrelevant OBS and variables
                        drop _est*
                        drop if SolarRest==.




**********
* Black box prediction RTE
**********
                        global blackestimationvariables1 " Tempeff15 Roll_Temp24  Roll_Temp240 SolarRest suncycle morning deltasun EWH"
                        global blackestimationvariables2 "Tempeff  Roll_avgT24  Roll_avgT240 SolarRest suncycle morning deltasun EWH "
                        global blackestimationvariables3 " Tempeff15 Roll_Temp24  Roll_Temp240 SolarRest suncycle morning deltasun   IT1 EWH CZlag EExportPlag"
                        global blackestimationvariables4 " Tempeff15 Roll_Temp24  Roll_Temp240 SolarRest suncycle morning deltasun   IT1 "

                *** INTERPRETATION : coeff on tempeff15 much larger than tempeff -> positive for us.

                        reg PrevConsoH $blackestimationvariables1, robust
                        est store Black_1

                        capture drop RteBlackBox
                        capture drop blackepsilon
                        capture mat drop blackalpha
                        mat blackmat = e(b)
                        mat li blackmat
                        scalar blackalpha = blackmat[1,9]
                        di  blackalpha

                        predict blackepsilon if e(sample), residuals
                        gen RteBlackBox = blackepsilon  /*+ blackalpha*/
                        drop blackepsilon
```



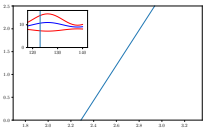
```
                        reg PrevConsoH $blackestimationvariables2 ,  robust
                        est store Black_2

                        reg PrevConsoH $blackestimationvariables3 ,  robust
                        est store Black_3

                        reg PrevConsoH $blackestimationvariables4 ,  robust
                        est store Black_4

                btoutreg2 [Black_1 Black_2 ] using "${LATEXPATH}Black1.tex", replace tex(frag pretty ) stats(coef) label(proper)  level(95)  sideway noparen
                btoutreg2 [Black_1 Black_2 Black_3 Black_4] using "${LATEXPATH}Black2.tex", replace tex(frag pretty ) stats(coef) label(proper)  level(95)  sideway noparen

                *drop irrelevant OBS and variables
                        drop _est*
                        drop if RteBlackBox==.

                *save
                        save "${CLOUDPATH}v38/Temp_data/Pre4and5.dta", replace


**********
* Generate PLU using forecast model
**********

        ******
        * DEMAND ESTIMATION + generate residuals for PLU_D
        ******

        * DEMAND ESTIMATION (no CZlag)
                local version "52"
                global runversionD `version'
                global demandestimationvariables`version' "Tempeff15 Roll_Temp24 Roll_Temp240 suncycle morning deltasun EWH SolarRest RteBlackBox "

        * run do file for demand estimation
                do "${CLOUDPATH}v38/DoFiles/107_Eqn4demand.do"




        ******
        * PLUvD :  predicted uncertainty directly using regression
        ******
                * Only predict uncertainty for demand function
                * Cannot do so for supply function since mixed with ex-post own bidding strategy.

                global DEV ${demandestimationvariables${runversionD}}

                capture drop PLUvDvarP
                capture drop PLUvDvarQ
                gen PLUvDvarP =.
                gen PLUvDvarQ =.

                forvalues k = 1(2)9{
                foreach s in /*Sell*/ Purchase{
                di "Point " `k' " Curve " "`s'"
                        reg sqresVolume ${DEV} if select==`k' & SalePurchase==""`s'""
```

```
                        predict PLUvDvartmpQ`k'a`s' if e(sample), xb
                        est store predictuncQa`k'a`s'
                        replace PLUvDvarQ = PLUvDvartmpQ`k'a`s' if e(sample)
                        drop PLUvDvartmpQ`k'a`s'
                }
                }

                forvalues k = 1(2)9{
                foreach s in /*Sell*/ Purchase{
                di "Point " `k' " Curve " "`s'"
                        reg sqresPrice ${DEV} if select==`k' & SalePurchase==" `s' "
                        predict PLUvDvartmpP`k'a`s' if e(sample), xb
                        est store predictuncPa`k'a`s'
                        replace PLUvDvarP = PLUvDvartmpP`k'a`s' if e(sample)
                        drop PLUvDvartmpP`k'a`s'
                }
                }

                capture drop PLUvDvarPabs
                capture drop PLUvDvarQabs
                gen PLUvDvarPabs =.
                gen PLUvDvarQabs =.

                forvalues k = 1(2)9{
                foreach s in /*Sell*/ Purchase{
                di "Point " `k' " Curve " "`s'"
                        reg absresVolume ${DEV} if select==`k' & SalePurchase==" `s' "
                        predict PLUvDvartmpQ`k'a`s' if e(sample), xb
                        est store predictuncQa`k'a`s'ABS
                        replace PLUvDvarQabs = PLUvDvartmpQ`k'a`s' if e(sample)
                        drop PLUvDvartmpQ`k'a`s'
                }
                }

                forvalues k = 1(2)9{
                foreach s in /*Sell*/ Purchase{
                di "Point " `k' " Curve " "`s'"
                        reg absresPrice ${DEV} if select==`k' & SalePurchase==" `s' "
                        predict PLUvDvartmpP`k'a`s' if e(sample), xb
                        est store predictuncPa`k'a`s'ABS
                        replace PLUvDvarPabs = PLUvDvartmpP`k'a`s' if e(sample)
                        drop PLUvDvartmpP`k'a`s'
                }
                }


                * (generating) tables on PLUvD
                btoutreg2 [predictuncPa5aPurchase predictuncPa5aPurchaseABS /*predictuncPa5aSell*/]
                        using "${LATEXPATH}predictunc1.tex", replace tex(frag pretty ) stats(coef)
                        label(proper) level(95)

                btoutreg2 [predictuncQa5aPurchase predictuncQa5aPurchaseABS /*predictuncQa5aSell*/]
                        using "${LATEXPATH}predictunc2.tex", replace tex(frag pretty ) stats(coef)
                        label(proper) level(95)

        * drop table data
        drop _est*
```



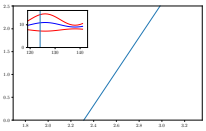
```

        * take sqrt of squared PLUs to get other order of magnitudes
                gen PLUvDrtP = sqrt(PLUvDvarP)
                gen PLUvDrtQ = sqrt(PLUvDvarQ)


        ******
        * PLUvR: LongueurCorrel Temp:
        ******

                gen PLUvRvarT = 1 / LcTemp
                gen PLUvRvarW = 1 / LcWind
                gen PLUvRvarS = 1 / LcSolar

        * add u-shaped term
                gen PLUvRvarTsq = 1/(LcTemp * LcTemp)
                gen PLUvRvarWsq = 1/(LcWind * LcWind)
                gen PLUvRvarSsq = 1/( LcSolar * LcSolar)


                /* table of extracted slopes
                        capture mat drop M
                        local variables "1 3 5 7 9"
                        local FUNC "r(mean) r(p50) r(sd) r(min) r(max)"
                        local i=0
                        foreach var of local variables{
                        local i=`i'+1
                        }
                        local j=0
                        foreach var of local FUNC{
                        local j=`j'+1
                        }
                        di `j' "  " `i'
                        mat M = J(`i',5,.)

                        local c=1
                        foreach FF of local FUNC{
                        local r=1
                        foreach VV of local variables{
                        su /* fxInvertPQ */ /* fxInvertQP  if select==`VV', detail
                        mat M[`r',`c']= `FF'
                        local r=`r'+1
                        }
                        local c=`c'+1
                        }
                        mat rownames M =   `variables'
                        mat colnames M =  Mean Median StdDev Min Max
                        mat li M

                        btouttable using "${LATEXPATH}extractedslopes", replace mat(M) asis nobox
                            caption("Estimated slopes of the supply function per point k") format
                            (%9.4fc %9.4fc %9.4fc %9.4fc %9.4fc %9.4fc) */

                *save
                        save "${CLOUDPATH}v38/Temp_data/Finaldataset.dta", replace



******
        * generate kernel based PLUs
```

```
******
                        use  "${CLOUDPATH}v38/Temp_data/Finaldataset.dta", clear

                ************* generate kernel PLUS
                 *                do "${CLOUDPATH}v38/DoFiles/107_kernelbucketreg.do"
                *************

                ************* dataset manipulations to obtain final dataset
                                 do "${CLOUDPATH}v38/DoFiles/107_PrepkernelPLUdata.do"
                *************

                *save
                        save "${CLOUDPATH}v38/Temp_data/Finalrundataset.dta", replace


*****
* Columns based on kernel PLUvD
*****
                                * do "${CLOUDPATH}v38/DoFiles/107_BootstrapKernel2702.do"
```



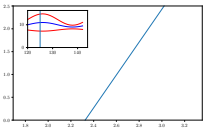
```
                                                    ************************************************
                                                    * START OF BASELINE RESULTS + BOOTSTRAP ROBUSTNESS
                                                    ************************************************



********* BASELINE


global focusk = 5


                ****************
                * Some regressions: ------------- on point k only -----------
                ****************

                        use "${CLOUDPATH}v38/Temp_data/Finalrundataset.dta", clear

*********
* FOCUS POINT
*********
        local k=${focusk}
        keep if select==`k' & SalePurchase=="Purchase"

* MACROS:
                global PLUsR "PLUvRvarT PLUvRvarTsq PLUvRvarW PLUvRvarWsq PLUvRvarS PLUvRvarSsq"
                global SupCo   "Coal Brent Gas IT2 EUA Wind1DA Hydro"

***** Program bootstrap of baseline
        capture program drop my2slsforbaselinebootstrap
        program my2slsforbaselinebootstrap
                local k=${focusk}
                capture drop  PLU_P_boot  PLU_Q_boot
                capture drop PLUvDvarP`k'resc PLUvDvarQ`k'resc
                capture noisily reg sqresPrice Tempeff15 Roll_Temp24 Roll_Temp240 suncycle morning deltasun EWH SolarRest RteBlackBox if select ==`k'  & SalePurchase=="Purchase", robust
                predict PLU_P_boot, xb
                su PLU_P_boot, meanonly
                scalar tmpP = r(mean)
                capture noisily gen PLUvDvarP`k'resc = PLU_P_boot / tmpP
                reg sqresVolume Tempeff15 Roll_Temp24 Roll_Temp240 suncycle morning deltasun EWH SolarRest RteBlackBox if select ==`k'  & SalePurchase=="Purchase", robust
                predict PLU_Q_boot, xb
                su PLU_Q_boot, meanonly
                scalar tmpQ = r(mean)
                gen PLUvDvarQ`k'resc = PLU_Q_boot / tmpQ
                if `k'==1 | `k'==9 {
                reg fxInvertQP PLUvRvarT PLUvRvarTsq PLUvRvarW PLUvRvarWsq PLUvRvarS PLUvRvarSsq Coal Brent Gas IT2 EUA Wind1DA Hydro /*PLUvDvarP`k'resc*/ PLUvDvarQ`k'resc if select ==`k'  & SalePurchase=="Purchase" , robust
                }
                else{
                reg fxInvertQP PLUvRvarT PLUvRvarTsq PLUvRvarW PLUvRvarWsq PLUvRvarS PLUvRvarSsq Coal Brent Gas IT2 EUA Wind1DA Hydro PLUvDvarP`k'resc PLUvDvarQ`k'resc     if
```

```
                    select ==`k'  & SalePurchase=="Purchase" , robust
                }
                capture drop  PLU_P_boot PLUvDvarP`k'resc PLU_Q_boot PLUvDvarQ`k'resc
        end


*FIRST TABLE:

                *col 1
                        local k=${focusk}
                        capture drop PLUvDvarP`k'resc
                        capture drop PLUvDvarQ`k'resc
                        su PLUvDvarP`k', meanonly
                        scalar tmpresc = r(mean)
                        capture noisily gen PLUvDvarP`k'resc = PLUvDvarP`k' / tmpresc
                        su PLUvDvarQ`k', meanonly
                        scalar tmpresc = r(mean)
                        gen PLUvDvarQ`k'resc = PLUvDvarQ`k' / tmpresc
                                        if `k'==1 | `k'==9 {
                        reg fxInvertQP   ${PLUsR} ${SupCo} /*PLUvDvarP`k'resc*/  PLUvDvarQ`k'resc
                        est store d1short1_`k'
                                        }
                        else{
                        reg fxInvertQP   ${PLUsR} ${SupCo} PLUvDvarP`k'resc  PLUvDvarQ`k'resc
                        est store d1short1_`k'
                        }
                *col 2
                        local k=${focusk}
                        bootstrap _b, reps(200) seed(12345): my2slsforbaselinebootstrap
                        est store bs_baseline_`k'
                        est save "${CLOUDPATH}v38/Temp_data/bs_baseline_`k'.ster", replace
                *col 3
                        local k=${focusk}
                        estimates use "${CLOUDPATH}v38/Temp_data/kernelweigthed_`k'.ster"
                        regress
                        estimates esample
                        estimates store kernel4_`k'
                *col 4
                local k=${focusk}
                if `k'==5{
                        estimates use "${CLOUDPATH}v38/Temp_data/kernelbootstrap`k'.ster"
                        regress
                        estimates esample:
                        estimates store kernel5_`k'
                        }

                * table main
                        local k=${focusk}
                        if `k'==5{
                        btoutreg2 [d1short1_`k' bs_baseline_`k' kernel4_`k' kernel5_`k'] using "${
                            LATEXPATH}main1_`k'.tex", replace tex(frag pretty ) stats(coef se)
                            label(proper)  level(95) title(For k=`k')
                        btoutreg2 [d1short1_`k' bs_baseline_`k' kernel4_`k' kernel5_`k'] using "${
                            LATEXPATH}mainNS1_`k'.tex", replace tex(frag pretty ) stats(coef) label
                            (proper)  level(95) title(For k=`k')
                        btoutreg2 [d1short1_`k' bs_baseline_`k' kernel4_`k' kernel5_`k'] using "${
                            LATEXPATH}mainoS1_`k'.tex", replace tex(frag pretty ) stats(se) label(
                            proper)  level(95) title(For k=`k')
                        }
                        else{
```



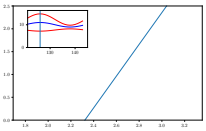
```
                        btoutreg2 [d1short1_`k' bs_baseline_`k' kernel4_`k' /*kernel5_`k'*/] using "${LATEXPATH}main1_`k'.tex", replace tex(frag pretty ) stats(coef se) label(proper)  level(95) title(For k=`k')
                        btoutreg2 [d1short1_`k' bs_baseline_`k' kernel4_`k' /*kernel5_`k'*/] using "${LATEXPATH}mainNS1_`k'.tex", replace tex(frag pretty ) stats(coef) label(proper)  level(95) title(For k=`k')
                        btoutreg2 [d1short1_`k' bs_baseline_`k' kernel4_`k' /*kernel5_`k'*/] using "${LATEXPATH}mainoS1_`k'.tex", replace tex(frag pretty ) stats(se) label(proper)  level(95) title(For k=`k')
                        }

        * table compare
        forvalues k=1(2)9{
        estimates use "${CLOUDPATH}v38/Temp_data/bs_baseline_`k'.ster"
        regress
        estimates esample:
        estimates store bs_baseline_`k'
        }
                        btoutreg2 [bs_baseline_1 bs_baseline_3 bs_baseline_5 bs_baseline_7 bs_baseline_9] using "${LATEXPATH}compare_col2.tex", replace tex(frag pretty ) stats(coef se) label(proper)  level(95) title(Comparison of col. 2)
                        btoutreg2 [bs_baseline_1 bs_baseline_3 bs_baseline_5 bs_baseline_7 bs_baseline_9] using "${LATEXPATH}compareNS_col2.tex", replace tex(frag pretty ) stats(coef) label(proper)  level(95) title(Comparison of col. 2)

        forvalues k=1(2)9{
                estimates use "${CLOUDPATH}v38/Temp_data/kernelweigthed_`k'.ster"
                regress
                estimates esample
                estimates store kernel4_`k'
        }
                        btoutreg2 [ kernel4_1 kernel4_3 kernel4_5 kernel4_7 kernel4_9] using "${LATEXPATH}compare_col3.tex", replace tex(frag pretty ) stats(coef se) label(proper)  level(95) title(Comparison of col. 3)
                        btoutreg2 [ kernel4_1 kernel4_3 kernel4_5 kernel4_7 kernel4_9] using "${LATEXPATH}compareNS_col3.tex", replace tex(frag pretty ) stats(coef) label(proper)  level(95) title(Comparison of col. 3)




                                                        ************************************************
                                                        * END OF BASELINE RESULTS + BOOTSTRAP ROBUSTNESS
                                                        ************************************************


                                                        ************************************************
                                                        * START OF DROPPING 1 PLUvDvarP or Q
                                                        ************************************************

```

```
global focusk = 5


                ****************
                * Some regressions:  -------------- on point k only -----------
                ****************

                        use "${CLOUDPATH}v38/Temp_data/Finalrundataset.dta", clear

*********
* FOCUS POINT
*********
        local k=${focusk}
        keep if select==`k' & SalePurchase=="Purchase"

* MACROS:
                global PLUsR "PLUvRvarT PLUvRvarTsq PLUvRvarW PLUvRvarWsq PLUvRvarS PLUvRvarSsq"
                global SupCo   "Coal Brent Gas IT2 EUA Wind1DA Hydro"

***** Program bootstrap of baseline
        capture program drop my2slsforbaselinebootstrap
        program my2slsforbaselinebootstrap
                local k=${focusk}
                capture drop  PLU_P_boot  PLU_Q_boot
                capture drop PLUvDvarP`k'resc PLUvDvarQ`k'resc
                capture noisily reg sqresPrice Tempeff15 Roll_Temp24 Roll_Temp240 suncycle morning
                     deltasun EWH SolarRest RteBlackBox if select ==`k'  & SalePurchase=="Purchase",
                      robust
                predict PLU_P_boot, xb
                su PLU_P_boot, meanonly
                scalar tmpP = r(mean)
                capture noisily gen PLUvDvarP`k'resc = PLU_P_boot / tmpP
                reg sqresVolume Tempeff15 Roll_Temp24 Roll_Temp240 suncycle morning deltasun EWH
                     SolarRest RteBlackBox if select ==`k'  & SalePurchase=="Purchase", robust
                predict PLU_Q_boot, xb
                su PLU_Q_boot, meanonly
                scalar tmpQ = r(mean)
                gen PLUvDvarQ`k'resc = PLU_Q_boot / tmpQ
                if `k'==1 | `k'==9 {
                reg fxInvertQP PLUvRvarT PLUvRvarTsq PLUvRvarW PLUvRvarWsq PLUvRvarS PLUvRvarSsq
                     Coal Brent Gas IT2 EUA Wind1DA Hydro /*PLUvDvarP`k'resc*/ PLUvDvarQ`k'resc
                     if select ==`k'  & SalePurchase=="Purchase"  , robust
                }
                else{
                reg fxInvertQP PLUvRvarT PLUvRvarTsq PLUvRvarW PLUvRvarWsq PLUvRvarS PLUvRvarSsq
                     Coal Brent Gas IT2 EUA Wind1DA Hydro /*PLUvDvarP`k'resc*/ PLUvDvarQ`k'resc
                     if select ==`k'  & SalePurchase=="Purchase"  , robust
                }
                capture drop  PLU_P_boot PLUvDvarP`k'resc PLU_Q_boot PLUvDvarQ`k'resc
        end


                *col 1
                        local k=${focusk}
                        capture drop PLUvDvarP`k'resc
                        capture drop PLUvDvarQ`k'resc
                        su PLUvDvarP`k', meanonly
                        scalar tmpresc = r(mean)
                        capture noisily gen PLUvDvarP`k'resc = PLUvDvarP`k' / tmpresc
                        su PLUvDvarQ`k', meanonly
```



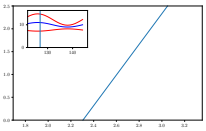
```
                        scalar tmpresc = r(mean)
                        gen PLUvDvarQ`k'resc = PLUvDvarQ`k' / tmpresc
                                        if `k'==1 | `k'==9 {
                        reg fxInvertQP   ${PLUsR} ${SupCo} /*PLUvDvarP`k'resc*/  PLUvDvarQ`k'resc
                        est store d1short1_`k'
                                        }
                        else{
                        reg fxInvertQP   ${PLUsR} ${SupCo} /*PLUvDvarP`k'resc*/  PLUvDvarQ`k'resc
                        est store d1short1_`k'
                        }
                *col 2
                        local k=${focusk}
                        bootstrap _b, reps(200) seed(12345): my2slsforbaselinebootstrap
                        est store bs_baseline_`k'
                        est save "${CLOUDPATH}v38/Temp_data/bs_baseDROP_P_`k'.ster", replace
                *col 3
                        local k=${focusk}
                        estimates use "${CLOUDPATH}v38/Temp_data/kernelweigDROP_P_`k'.ster"
                        regress
                        estimates esample
                        estimates store kernel4_`k'
                *col 4
/*              local k=${focusk}
                if `k'==5{
                        estimates use "${CLOUDPATH}v38/Temp_data/kernelbootDROP_P_`k'.ster"
                        regress
                        estimates esample:
                        estimates store kernel5_`k'
                        }*/

                * table main
                        local k=${focusk}
                        if `k'==5{
                        btoutreg2 [d1short1_`k' bs_baseline_`k' kernel4_`k' /*kernel5_`k'*/] using "${LATEXPATH}main1DROP_P_`k'.tex", replace tex(frag pretty ) stats(coef se) label(proper)  level(95) title(For k=`k')
                        btoutreg2 [d1short1_`k' bs_baseline_`k' kernel4_`k' /*kernel5_`k'*/] using "${LATEXPATH}mainNS1DROP_P_`k'.tex", replace tex(frag pretty ) stats(coef) label(proper)  level(95) title(For k=`k')
                        btoutreg2 [d1short1_`k' bs_baseline_`k' kernel4_`k' /*kernel5_`k'*/] using "${LATEXPATH}mainoS1DROP_P_`k'.tex", replace tex(frag pretty ) stats(se) label(proper)  level(95) title(For k=`k')
                        }
                        else{
                        btoutreg2 [d1short1_`k' bs_baseline_`k' kernel4_`k' /*kernel5_`k'*/] using "${LATEXPATH}main1DROP_P_`k'.tex", replace tex(frag pretty ) stats(coef se) label(proper)  level(95) title(For k=`k')
                        btoutreg2 [d1short1_`k' bs_baseline_`k' kernel4_`k' /*kernel5_`k'*/] using "${LATEXPATH}mainNS1DROP_P_`k'.tex", replace tex(frag pretty ) stats(coef) label(proper)  level(95) title(For k=`k')
                        btoutreg2 [d1short1_`k' bs_baseline_`k' kernel4_`k' /*kernel5_`k'*/] using "${LATEXPATH}mainoS1DROP_P_`k'.tex", replace tex(frag pretty ) stats(se) label(proper)  level(95) title(For k=`k')
                        }


                        use "${CLOUDPATH}v38/Temp_data/Finalrundataset.dta", clear

*********
* FOCUS POINT
```

```
*********
        local k=${focusk}
        keep if select==`k' & SalePurchase=="Purchase"

* MACROS:
                global PLUsR "PLUvRvarT PLUvRvarTsq PLUvRvarW PLUvRvarWsq PLUvRvarS PLUvRvarSsq"
                global SupCo   "Coal Brent Gas IT2 EUA Wind1DA Hydro"

***** Program bootstrap of baseline
        capture program drop my2slsforbaselinebootstrap
        program my2slsforbaselinebootstrap
                local k=${focusk}
                capture drop  PLU_P_boot  PLU_Q_boot
                capture drop PLUvDvarP`k'resc PLUvDvarQ`k'resc
                capture noisily reg sqresPrice Tempeff15 Roll_Temp24 Roll_Temp240 suncycle morning
                     deltasun EWH SolarRest RteBlackBox if select ==`k'  & SalePurchase=="Purchase",
                      robust
                predict PLU_P_boot, xb
                su PLU_P_boot, meanonly
                scalar tmpP = r(mean)
                capture noisily gen PLUvDvarP`k'resc = PLU_P_boot / tmpP
                reg sqresVolume Tempeff15 Roll_Temp24 Roll_Temp240 suncycle morning deltasun EWH
                     SolarRest RteBlackBox if select ==`k'  & SalePurchase=="Purchase", robust
                predict PLU_Q_boot, xb
                su PLU_Q_boot, meanonly
                scalar tmpQ = r(mean)
                gen PLUvDvarQ`k'resc = PLU_Q_boot / tmpQ
                if `k'==1 | `k'==9 {
                reg fxInvertQP PLUvRvarT PLUvRvarTsq PLUvRvarW PLUvRvarWsq PLUvRvarS PLUvRvarSsq
                     Coal Brent Gas IT2 EUA Wind1DA Hydro PLUvDvarP`k'resc /*PLUvDvarQ`k'resc*/
                     if select ==`k'  & SalePurchase=="Purchase"  , robust
                }
                else{
                reg fxInvertQP PLUvRvarT PLUvRvarTsq PLUvRvarW PLUvRvarWsq PLUvRvarS PLUvRvarSsq
                     Coal Brent Gas IT2 EUA Wind1DA Hydro PLUvDvarP`k'resc /*PLUvDvarQ`k'resc*/
                     if select ==`k'  & SalePurchase=="Purchase"  , robust
                }
                capture drop  PLU_P_boot PLUvDvarP`k'resc PLU_Q_boot PLUvDvarQ`k'resc
        end


        *col 1
                        local k=${focusk}
                        capture drop PLUvDvarP`k'resc
                        capture drop PLUvDvarQ`k'resc
                        su PLUvDvarP`k', meanonly
                        scalar tmpresc = r(mean)
                        capture noisily gen PLUvDvarP`k'resc = PLUvDvarP`k' / tmpresc
                        su PLUvDvarQ`k', meanonly
                        scalar tmpresc = r(mean)
                        gen PLUvDvarQ`k'resc = PLUvDvarQ`k' / tmpresc
                                        if `k'==1 | `k'==9 {
                        reg fxInvertQP   ${PLUsR} ${SupCo} PLUvDvarP`k'resc  /*PLUvDvarQ`k'resc */
                        est store d1short1_`k'
                                        }
                        else{
                        reg fxInvertQP   ${PLUsR} ${SupCo} PLUvDvarP`k'resc  /*PLUvDvarQ`k'resc */
                        est store d1short1_`k'
                        }
                *col 2
```



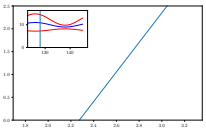
```
                        local k=${focusk}
                        bootstrap _b, reps(200) seed(12345): my2slsforbaselinebootstrap
                        est store bs_baseline_`k'
                        est save "${CLOUDPATH}v38/Temp_data/bs_baseDROP_Q_`k'.ster", replace
                *col 3
                        local k=${focusk}
                        estimates use "${CLOUDPATH}v38/Temp_data/kernelweigDROP_Q_`k'.ster"
                        regress
                        estimates esample
                        estimates store kernel4_`k'
                *col 4
/*              local k=${focusk}
                if `k'==5{
                        estimates use "${CLOUDPATH}v38/Temp_data/kernelbootDROP_Q_`k'.ster"
                        regress
                        estimates esample:
                        estimates store kernel5_`k'
                        }*/

                * table main
                        local k=${focusk}
                        if `k'==5{
                        btoutreg2 [d1short1_`k' bs_baseline_`k' kernel4_`k' /*kernel5_`k'*/] using "${LATEXPATH}main1DROP_Q_`k'.tex", replace tex(frag pretty ) stats(coef se) label(proper)  level(95) title(For k=`k')
                        btoutreg2 [d1short1_`k' bs_baseline_`k' kernel4_`k' /*kernel5_`k'*/] using "${LATEXPATH}mainNS1DROP_Q_`k'.tex", replace tex(frag pretty ) stats(coef) label(proper)  level(95) title(For k=`k')
                        btoutreg2 [d1short1_`k' bs_baseline_`k' kernel4_`k' /*kernel5_`k'*/] using "${LATEXPATH}mainoS1DROP_Q_`k'.tex", replace tex(frag pretty ) stats(se) label(proper)  level(95) title(For k=`k')
                        }
                        else{
                        btoutreg2 [d1short1_`k' bs_baseline_`k' kernel4_`k' /*kernel5_`k'*/] using "${LATEXPATH}main1DROP_Q_`k'.tex", replace tex(frag pretty ) stats(coef se) label(proper)  level(95) title(For k=`k')
                        btoutreg2 [d1short1_`k' bs_baseline_`k' kernel4_`k' /*kernel5_`k'*/] using "${LATEXPATH}mainNS1DROP_Q_`k'.tex", replace tex(frag pretty ) stats(coef) label(proper)  level(95) title(For k=`k')
                        btoutreg2 [d1short1_`k' bs_baseline_`k' kernel4_`k' /*kernel5_`k'*/] using "${LATEXPATH}mainoS1DROP_Q_`k'.tex", replace tex(frag pretty ) stats(se) label(proper)  level(95) title(For k=`k')
                        }




                                                        ************************************************
                                                        * END OF DROPPING 1 PLUvDvarP or Q
                                                        ************************************************

                                                        ************************************************
                                                        * REST NOT RELEVANT
```

```
                                                        ************************************************



********* testing on combined PLU using new ones.


global focusk = 5


                ****************
                * Some regressions:  ------------- on point k only -----------
                ****************

                        use "${CLOUDPATH}v38/Temp_data/Finalrundataset.dta", clear



        ***
        * scalefactor to adjust fx to slope
        ***
        capture drop group
                egen group= group(Datestatafrac)
        gsort group Datestata  Hour  SalePurchase select
        capture drop slopeDpost slopeDpre slopeDatk fxscalefactor
                gen slopeDpost =.
                gen slopeDpre =.
                gen slopeDatk =.
                gen fxscalefactor =.

forvalues k= 1(2)9{
        *note in p-q dimension!
by group: replace slopeDpost  = (Volume[_n+1] - Volume[_n])/(Price[_n+1]- Price[_n]) if select==`k'

by group:        replace slopeDpre = (Volume[_n] - Volume[_n-1])/(Price[_n]- Price[_n-1]) if select
    ==`k'
by group:        replace slopeDatk = abs(slopeDpost[_n]+ slopeDpre[_n])/2 if select==`k'

        capture drop tmp1 tmp2
         egen tmp1 = mean(slopeDatk ) if select==`k'
         egen tmp2 = mean(fx) if select==`k'
        replace fxscalefactor = tmp1 / tmp2 if select==`k'
        drop tmp1 tmp2
        }
        capture drop fxscaled
        gen fxscaled = fx * fxscalefactor


        * scalefactor to adjust fx to slope
        capture drop group
                egen group= group(Datestatafrac)
                        gsort group Datestata  Hour  SalePurchase select
        capture drop slopeDpostQP slopeDpreQP slopeDatkQP fxscalefactorQP
                gen slopeDpostQP =.
                gen slopeDpreQP =.
                gen slopeDatkQP =.
                gen fxscalefactorQP =.
                        gen fxQP = (1/fx)

```



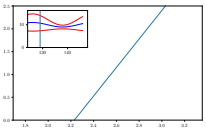
```
forvalues k= 1(2)9{
        *note in q-p dimension!
        by group: replace slopeDpostQP  = (Price[_n+1]- Price[_n])/(Volume[_n+1] - Volume[_n]) if select==`k'
         by group: replace slopeDpreQP = (Price[_n]- Price[_n-1])/(Volume[_n] - Volume[_n-1]) if select==`k'
         by group: replace slopeDatkQP = abs(slopeDpostQP[_n]+ slopeDpreQP[_n])/2 if select==`k'
        capture drop tmp1 tmp2
         egen tmp1 = mean(slopeDatkQP ) if select==`k'
         egen tmp2 = mean(fxQP) if select==`k'
        replace   fxscalefactorQP = tmp1 / tmp2 if select==`k'
        drop tmp1 tmp2
        }
        capture drop fxscaledQP
        gen fxscaledQP = fxQP * fxscalefactorQP
        gen comparisonfx = 1/ fxscaled
* SCALING ONLY APPROPRIATE FOR K=5, otherwise too much mixing flat and vertical section.





        local k=${focusk}
        keep if select==`k' & SalePurchase==”Purchase”

        *gen check1 = sqrt(PLUvDvarP) / (1 / fxscaled )
        *gen check2= sqrt(PLUvDvarQ)
        capture drop PLUvDcomb
        capture noisily gen PLUvDcomb   = (( sqrt(PLUvDvarP) / (1 / fxscaled ) ) + sqrt(PLUvDvarQ) )^2
        capture drop PLUvDcombK
        capture noisily gen PLUvDcombK   = (( sqrt(PLUv51avarPsq) / (1 / fxscaled ) ) + sqrt( PLUv51avarQsq) )^2

        *rescale combined
        capture drop PLUvDvarC
        su PLUvDcomb, meanonly
        scalar tmpresc = r(mean)
        capture noisily gen PLUvDvarC = PLUvDcomb / tmpresc

        capture drop PLUvDvarCK
        su PLUvDcombK, meanonly
        scalar tmpresc = r(mean)
        capture noisily gen PLUvDvarCK = PLUvDcomb / tmpresc


*** REGS
        reg fxInvertQP PLUvRvarT PLUvRvarTsq PLUvRvarW PLUvRvarWsq PLUvRvarS PLUvRvarSsq Coal Brent Gas IT2 EUA Wind1DA Hydro PLUvDvarC    if select ==`k' & SalePurchase==”Purchase” , robust
        reg fxInvertQP PLUvRvarT PLUvRvarTsq PLUvRvarW PLUvRvarWsq PLUvRvarS PLUvRvarSsq Coal Brent Gas IT2 EUA Wind1DA Hydro PLUvDvarC    if select ==`k' & SalePurchase==”Purchase” , vce(bootstrap, reps(300) seed(12345))


        reg fxInvertQP PLUvRvarT PLUvRvarTsq PLUvRvarW PLUvRvarWsq PLUvRvarS PLUvRvarSsq Coal Brent Gas IT2 EUA Wind1DA Hydro PLUvDvarCK    if select ==`k' & SalePurchase==”Purchase” [aweight = PointsInBinv51a]

```

```
*******
* generate 1-dim PLUs:
*******
        capture drop PLU_COMB*
        capture noisily gen PLU_COMBa_Dresc   = sqrt( (PLUvDvarPresc)^2 + (PLUvDvarQresc)^2 )
        capture noisily gen PLU_COMBa_Dabsresc   = sqrt( (PLUvDvarPabsresc)^2 + (PLUvDvarQabsresc)
            ^2 )
        capture noisily gen PLU_COMBa_Drtresc   = sqrt( (PLUvDrtPresc)^2 + (PLUvDrtQresc)^2 )

        /* version b: translation approach - not allowed for rescaled variables!!
        capture noisily gen PLU_COMBb_D   =((PLUvDvarP)/ (1 / fxscaled ) ) + (PLUvDvarQ)
        capture noisily gen PLU_COMBb_Dabs   = ((PLUvDvarPabs)/ (1 / fxscaled) ) + (PLUvDvarQabs)
        capture noisily gen PLU_COMBb_Drt   = ((PLUvDrtP)/ (1 / fxscaled ) ) + (PLUvDrtQ) */

/*      capture noisily gen PLU_COMBb_D   =((PLUvDvarP)/ (fxscaledQP ) ) + (PLUvDvarQ)
```



```
        capture noisily gen PLU_COMBb_Dabs   = ((PLUvDvarPabs)/ (fxscaledQP) ) + (PLUvDvarQabs)
        capture noisily gen PLU_COMBb_Drt   = ((PLUvDrtP)/ (fxscaledQP ) ) + (PLUvDrtQ) */


****
* defining variables
****
                local versionS "61"
                global runversionS `versionS'
                        global demandestimationvariables`version' "Tempeff15 Roll_Temp24 Roll_Temp240 suncycle morning deltasun EWH SolarRest RteBlackBox "
                global supplyestimationvariables`versionS' "Coal Brent Gas IT2 EUA suncycle morning deltasun EWH Wind1DA SolarRest Hydro RteBlackBox"
                global uncertaintyproxies`versionS' "PLUvDvarP PLUvDvarQ PLUvDvarPabs PLUvDvarQabs PLUvDrtP PLUvDrtQ PLUvRvarT PLUvRvarW PLUvRvarS PLUvRvarTsq PLUvRvarWsq PLUvRvarSsq PLUvDvarQresc PLUvDvarQabsresc PLUvDvarPresc PLUvDvarPabsresc PLUvDrtPresc PLUvDrtQresc PLU_COMBa_Dresc PLU_COMBa_Dabsresc PLU_COMBa_Drtresc PLU_COMBb_D PLU_COMBb_Dabs PLU_COMBb_Drt "
                global PLUsD "PLUvDvarP PLUvDvarQ PLUvDvarPabs PLUvDvarQabs PLUvDrtP PLUvDrtQ PLUvDvarPresc PLUvDvarQresc PLUvDvarPabsresc PLUvDvarQabsresc   PLUvDrtPresc PLUvDrtQresc PLU_COMBa_Dresc PLU_COMBa_Dabsresc PLU_COMBa_Drtresc PLU_COMBb_D PLU_COMBb_Dabs PLU_COMBb_Drt "
                                global PLUsDP "PLUvDvarP`k' PLUvDvarPabs`k' PLUvDrtP`k'   "
                                global PLUsDQ " PLUvDvarQ`k' PLUvDvarQabs`k' PLUvDrtQ`k' "
                global PLUsR "PLUvRvarT PLUvRvarTsq PLUvRvarW PLUvRvarWsq PLUvRvarS PLUvRvarSsq"
                global SEV ${supplyestimationvariables${runversionS}}
                global UCP ${uncertaintyproxies${runversionS}}
                di $SEV
                di $UCP


***reg on demand slope as cross check to interpretation from level functional regressions:
        capture drop negfx
        gen negfx = -fx
        reg negfx   $demandestimationvariables`version', robust
        reg fx   $demandestimationvariables`version', robust
        est store demandslopepred${focusk}
                btoutreg2 [demandslopepred${focusk} ] using "${LATEXPATH}demandslopepred${focusk}.tex", replace tex(frag pretty ) stats(coef) label(proper)  level(95) title(Demand-slope-regression at k=${focusk})
                drop _est*


******
* REGRESSION 1 :

reg fxInvertQP  ${PLUsR}, robust
est store onlyplus
* all plu_renouvelable are significant, only plu wind of correct sign
reg fxInvertQP  ${PLUsR} $SEV, robust
est store onlyplur1
reg fxInvertQP  ${PLUsR} Coal Brent Gas IT2 EUA Wind1DA Hydro, robust
est store onlyplur2
reg fxInvertQP  ${PLUsR} Coal Brent Gas IT2 EUA Wind1DA Hydro $SEV, robust
est store onlyplur3
* when adding supply controls, only wind stay significant with correct sign, others non-sig. thats good. :)

                        foreach UCP of global PLUsD{
                        reg fxInvertQP   ${PLUsR} Coal Brent Gas IT2 EUA Wind1DA Hydro   `UCP'
```

```
                        est store linreg1`UCP'
                        }

                        foreach UCP1 of global PLUsDP{
                        foreach UCP2 of global PLUsDQ{
                        reg fxInvertQP   ${PLUsR} Coal Brent Gas IT2 EUA Wind1DA Hydro `UCP1' `UCP2'
                        est store `UCP1'`UCP2'
                        }
                        }




                        reg fxInvertQP   ${PLUsR} Coal Brent Gas IT2 EUA Wind1DA Hydro PLUvDvarP`k' resc  PLUvDvarQ`k'resc
                        est store d1short1
                        *reg fxInvertQP   ${PLUsR} Coal Brent Gas IT2 EUA Wind1DA Hydro PLUvDvarP`k' resc  PLUvDvarQ`k'resc , vce(bootstrap, rep(500))
                        *est store b1short1
                        reg fxInvertQP   ${PLUsR} Coal Brent Gas IT2 EUA Wind1DA Hydro PLUvDvarPabs `k'resc PLUvDvarQabs`k'resc
                        est store d1short2
                        reg fxInvertQP   ${PLUsR} Coal Brent Gas IT2 EUA Wind1DA Hydro  PLUvDrtP`k' resc PLUvDrtQ`k'resc
                        est store d1short3

                * (generating) regression output
/* Shows: PLU_temperature never significant
        - wind1da : pos + sig (more wind, more uncertainty)
        - plu wind: sig + positive effect only for PLUs_on_P (longer autocorrelation wind-> more uncertainty)
        - plu wind squared : very neg + sig only for PLUs_on_P (very short or long autocorrel = low uncertainty, errors cancel out)
        -PLU-solar never sig
        - solar1da included in plusD
        - HAVE EXCLUDED DAYTIME CONTROLS (but they are strongly included in PLUsD
        - all input prices has sig effect: coal positive and all other negative (interpretation?)
        - plu_D_on_P have negative, sigificant effects, plu_D_on_Q have positive effects, when very significant
        */
                btoutreg2 [onlyplus  onlyplur2 onlyplur1] using "${LATEXPATH}onlypluRs.tex", replace tex(frag pretty ) stats(coef) label(proper)  level(95) title(regression for k=`k')

                btoutreg2 [/*linreg1PLUvDvarP linreg1PLUvDvarPabs linreg1PLUvDrtP*/ linreg1PLUvDvarPresc    linreg1PLUvDvarPabsresc linreg1PLUvDrtPresc    ] using "${LATEXPATH}linregsummary1P_`k'.tex", replace tex(frag pretty ) stats(coef) label(proper)  level(95) title(regression for k=`k')

                btoutreg2 [/*linreg1PLUvDvarQ linreg1PLUvDvarQabs    linreg1PLUvDrtQ */ linreg1PLUvDvarQresc    linreg1PLUvDvarQabsresc linreg1PLUvDrtQresc ] using "${LATEXPATH}linregsummary1Q_`k'.tex", replace tex(frag pretty ) stats(coef) label(proper)  level(95) title(regression for k=`k')


                btoutreg2 [/*PLUvDvarP`k'PLUvDvarQ`k' PLUvDvarPabs`k'PLUvDvarQabs`k' PLUvDrtP`k' PLUvDrtQ`k' */  d1short1 d1short2 d1short3] using "${LATEXPATH}doublereg1_`k'.tex", replace tex(frag pretty ) stats(coef) label(proper)  level(95) title(regression for k=`k')
```



```
994
995
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1003
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1007
1008
1009
1010                ****************
1011                * some robustness regressions to plu_d specifications
1012                ****************
1013
1014 *****
1015 *****
1016
1017        /*
1018                ********************************KERNEL BASED PLU = v52
1019 */
1020
1021        use "${CLOUDPATH}v38/Temp_data/Finalrundataset.dta", clear
1022
1023
1024 * Generate 1 dimensional proxies for kernel based PLUs
1025 **********
1026
1027        ********
1028        * COMBINE P and Q uncertainty into single value on kernel based proxies.
1029        ********
1030                        capture drop PLU_COMB*
1031                * version a: Hypothenuse approach
1032
1033        foreach versionD in "52a" "52b"{
1034                        foreach switch in /*""*/ "abs"{
1035                        forvalues k = 1(2)9{
1036 capture noisily gen PLU_COMBa_v`versionD'_`switch'`k'resc   = sqrt( (PLUv`versionD'varP`switch'`k'
     resc)^2 + (PLUv`versionD'varQ`switch'`k'resc)^2 )
1037        }
1038        }
1039        }
1040
1041                * version b: translation approach - not correct conversion anymore after rescaling
                        !!!!
1042                                        forvalues k = 1(2)9{
1043        capture noisily gen PLU_COMBb_v52a`k'   = ((PLUv52avarPabs`k')/ (1 / fxscaled ) ) + (
            PLUv52avarQabs`k')
1044        capture noisily gen PLU_COMBb_v52b`k'   = ((PLUv52bvarPabs`k')/ (1 / fxscaled ) ) + (
            PLUv52bvarQabs`k')
1045        }
1046
1047 /*
1048                                                forvalues k = 1(2)9{
1049        capture noisily gen PLU_COMBb_v52a`k'   = ((PLUv52avarPabs`k')/ ( fxscaledQP ) ) + (
```
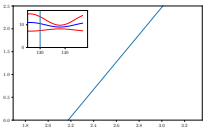
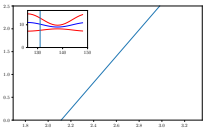

```
            PLUv52avarQabs`k')
        capture noisily gen PLU_COMBb_v52b`k'   = ((PLUv52bvarPabs`k')/ ( fxscaledQP ) ) + (
            PLUv52bvarQabs`k')
        }
*/

*********
* FOCUS POINT
*********
        local k=${focusk}
        keep if select==`k' & SalePurchase=="Purchase"

/*      * general placeholders
         capture drop PLUvDvarQresc PLUvDvarQabsresc PLUvDvarPresc PLUvDvarPabsresc PLUvDrtPresc
             PLUvDrtQresc
        gen PLUvDvarQresc = PLUvDvarQ`k'resc
        gen PLUvDvarQabsresc = PLUvDvarQabs`k'resc
        gen PLUvDvarPresc = PLUvDvarP`k'resc
        gen PLUvDvarPabsresc = PLUvDvarPabs`k'resc
        gen PLUvDrtPresc = PLUvDrtP`k'resc
        gen PLUvDrtQresc = PLUvDrtQ`k'resc*/


****
* defining variables
****
                local versionS "61"
                global runversionS `versionS'
                        global demandestimationvariables`version' "Tempeff15 Roll_Temp24
                            Roll_Temp240 suncycle morning deltasun EWH SolarRest RteBlackBox "
                global supplyestimationvariables`versionS' "Coal Brent Gas IT2 EUA suncycle morning
                     deltasun EWH Wind1DA SolarRest Hydro RteBlackBox"
                *global uncertaintyproxies`versionS' "PLUvDvarP PLUvDvarQ PLUvDvarPabs PLUvDvarQabs
                     PLUvDrtP PLUvDrtQ PLUvRvarT PLUvRvarW PLUvRvarS PLUvRvarTsq PLUvRvarWsq
                    PLUvRvarSsq PLUvDvarQresc PLUvDvarQabsresc PLUvDvarPresc PLUvDvarPabsresc
                    PLUvDrtPresc PLUvDrtQresc PLU_COMBa_Dresc PLU_COMBa_Dabsresc PLU_COMBa_Drtresc
                    PLU_COMBb_D PLU_COMBb_Dabs PLU_COMBb_Drt "
                *global PLUsD "PLUvDvarP PLUvDvarQ PLUvDvarPabs PLUvDvarQabs PLUvDrtP PLUvDrtQ
                    PLUvDvarPresc PLUvDvarQresc PLUvDvarPabsresc PLUvDvarQabsresc   PLUvDrtPresc
                    PLUvDrtQresc PLU_COMBa_Dresc PLU_COMBa_Dabsresc PLU_COMBa_Drtresc PLU_COMBb_D
                    PLU_COMBb_Dabs PLU_COMBb_Drt "
                global PLUsR "PLUvRvarT PLUvRvarTsq PLUvRvarW PLUvRvarWsq PLUvRvarS PLUvRvarSsq"
                global PLUsROB "PLUv52avarPabs`k' PLUv52avarQabs`k' PLUv52bvarPabs`k'
                    PLUv52bvarQabs`k' PLUv52avarPabs`k'resc PLUv52bvarPabs`k'resc PLUv52avarQabs`k'
                    resc PLUv52bvarQabs`k'resc PLU_COMBa_v52a_abs`k'resc PLU_COMBa_v52b_abs`k'resc
                    PLU_COMBb_v52a`k' PLU_COMBb_v52b`k'"
                global PLUsROBa "PLUv52avarPabs`k' PLUv52avarQabs`k' PLUv52avarPabs`k'resc
                    PLUv52avarQabs`k'resc  PLU_COMBa_v52a_abs`k'resc  PLU_COMBb_v52a`k' "
                global PLUsROBb " PLUv52bvarPabs`k' PLUv52bvarQabs`k' PLUv52bvarPabs`k'resc
                    PLUv52bvarQabs`k'resc  PLU_COMBa_v52b_abs`k'resc  PLU_COMBb_v52b`k'"
                global SEV ${supplyestimationvariables${runversionS}}
                global UCP ${uncertaintyproxies${runversionS}}
                di $SEV
                di ${PLUsROB}



*** first regression

reg fxInvertQP  ${PLUsR}, robust
```



```
est store onlyplus
reg fxInvertQP  Coal Brent Gas IT2 EUA Wind1DA Hydro, robust
est store onlycontrols
* all plu_renouvelable are sigigifcant, only plu wind of correct sign
reg fxInvertQP  ${PLUsR} Coal Brent Gas IT2 EUA Wind1DA Hydro, robust
* when adding supply controls, only wind stay significant with correct sign, others non-sig. thats good. :)

                        foreach UCP of global PLUsROB{
                        reg fxInvertQP   ${PLUsR} Coal Brent Gas IT2 EUA Wind1DA Hydro /*${SEV}*/ `UCP'
                        est store r1`UCP'
                        }

                * including weighting using Pointsperbin
                        foreach UCP of global PLUsROBa{
                        reg fxInvertQP   ${PLUsR} Coal Brent Gas IT2 EUA Wind1DA Hydro /*${SEV}*/ `UCP' [aweight=PointsInBinv52a]
                        est store w1`UCP'
                        }
                        foreach UCP of global PLUsROBb{
                        reg fxInvertQP   ${PLUsR} Coal Brent Gas IT2 EUA Wind1DA Hydro /*${SEV}*/ `UCP' [aweight=PointsInBinv52b]
                        est store w1`UCP'
                        }


*simultaneous reg on PLu_P and PLU-Q
                        reg fxInvertQP   ${PLUsR} Coal Brent Gas IT2 EUA Wind1DA Hydro PLUv52avarPabs`k'resc  PLUv52avarQabs`k'resc [aweight=PointsInBinv52a]
                                est store w2_52a
                        reg fxInvertQP   ${PLUsR} Coal Brent Gas IT2 EUA Wind1DA Hydro PLUv52bvarPabs`k'resc  PLUv52bvarQabs`k'resc [aweight=PointsInBinv52b]
                                est store w2_52b


                * (generating) regression output
/* Shows:
        - plu_r as before
        - PLU_temperature never significant
        - wind1da : pos + sig (more wind, more uncertainty)
        - plu wind: sig + positive effect only for PLUs_on_P (longer autocorrelation wind-> more uncertainty)
        - plu wind squared : very neg + sig only for PLUs_on_P (very short or long autocorrel = low uncertainty, errors cancel out)
        -PLU-solar never sig
        - solar1da included in plusD
        - HAVE EXCLUDED DAYTIME CONTROLS (but they are strongly included in PLUsD
        - all input prices has sig effect: coal positive and all other negative (interpretation?)
        - plu_D_on_P have negative, sigificant effects, plu_D_on_Q have positive effects, when very significant
        ***  PROMISING RESUTLS HERE ON ROBUSTNESS!
        - resc variables have nonsignificnat effect when combined, significant and pos for quantities plus when individual effect.
        */
                btoutreg2 [/*onlyplus*/ onlycontrols r1PLUv52avarPabs`k' r1PLUv52avarQabs`k' r1PLUv52bvarPabs`k' r1PLUv52bvarQabs`k' /* r1PLUv52avarPabs`k'resc r1PLUv52bvarPabs`k'resc r1PLUv52avarQabs`k'resc r1PLUv52bvarQabs`k'resc */ r1PLU_COMBa_v52a_abs`k'resc r1PLU_COMBa_v52b_abs`k'resc r1PLU_COMBb_v52a`k' r1PLU_COMBb_v52b`k'] using "${LATEXPATH}r1_`k'.tex", replace tex(frag pretty )
```

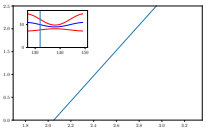
```
                    stats(coef) label(proper)  level(95) title(For k=`k')
                btoutreg2 [/*onlyplus*/ onlycontrols w1PLUv52avarPabs`k' w1PLUv52avarQabs`k'
                    w1PLUv52bvarPabs`k' w1PLUv52bvarQabs`k' /* w1PLUv52avarPabs`k'resc
                    w1PLUv52bvarPabs`k'resc w1PLUv52avarQabs`k'resc w1PLUv52bvarQabs`k'resc */
                    w1PLU_COMBa_v52a_abs`k'resc w1PLU_COMBa_v52b_abs`k'resc w1PLU_COMBb_v52a`k'
                    w1PLU_COMBb_v52b`k'] using "${LATEXPATH}w1_`k'.tex", replace tex(frag pretty )
                    stats(coef) label(proper)  level(95) title(For k=`k')

        *separate tables for individual P or Q

        btoutreg2 [w1PLUv52avarPabs`k' w1PLUv52avarQabs`k' w1PLUv52bvarPabs`k' w1PLUv52bvarQabs`k']
             using "${LATEXPATH}w1a_`k'.tex", replace tex(frag pretty ) stats(coef) label(proper)
            level(95) title(For k=`k')

        btoutreg2 [w2_52a  w2_52b  w1PLU_COMBa_v52a_abs`k'resc w1PLU_COMBa_v52b_abs`k'resc /*
            w1PLU_COMBb_v52a`k' w1PLU_COMBb_v52b`k' */] using "${LATEXPATH}w1b_`k'.tex", replace
            tex(frag pretty ) stats(coef) label(proper)  level(95) title(For k=`k')


        /* run individually if only running first part
                drop _est*
*/
```

```
                ****************
                * some robustness regressions to plu_d specifications
                ****************

*****
*****

        /*
                *************************   kernel based plus. = v51
```



```
*/

*       use "${CLOUDPATH}v38/Temp_data/FinalrundatasetK1.dta", clear
*       use "${CLOUDPATH}v38/Temp_data/Finalrundataset.dta", clear


* Generate 1 dimensional proxies for kernel based PLUs
**********

        ********
        * COMBINE P and Q uncertainty into single value on kernel based proxies.
        ********
                        capture drop PLU_COMB*
                * version a: Hypothenuse approach

        foreach versionD in "51a" "51b"{
                        foreach switch in /*""*/ "sq"{
                        forvalues k = 1(2)9{
capture noisily gen PLU_COMBa_v`versionD'_`switch'`k'resc   = sqrt( (PLUv`versionD'varP`switch'`k'resc)^2 + (PLUv`versionD'varQ`switch'`k'resc)^2 )
        }
        }
        }

                * version b: translation approach - not correct conversion anymore after rescaling !!!!
                                        forvalues k = 1(2)9{
        capture noisily gen PLU_COMBb_v51a`k'   = ((PLUv51avarPsq`k')/ (1 / fxscaled ) ) + (PLUv51avarQsq`k')
        capture noisily gen PLU_COMBb_v51b`k'   = ((PLUv51bvarPsq`k')/ (1 / fxscaled ) ) + (PLUv51bvarQsq`k')
        }

/*
                                                forvalues k = 1(2)9{
        capture noisily gen PLU_COMBb_v51a`k'   = ((PLUv51avarPsq`k')/ ( fxscaledQP ) ) + (PLUv51avarQsq`k')
        capture noisily gen PLU_COMBb_v51b`k'   = ((PLUv51bvarPsq`k')/ ( fxscaledQP ) ) + (PLUv51bvarQsq`k')
        }
*/

*********
* FOCUS POINT
*********
        local k=${focusk}
        keep if select==`k' & SalePurchase=="Purchase"

/*      * general placeholders
         capture drop PLUvDvarQresc PLUvDvarQsqresc PLUvDvarPresc PLUvDvarPsqresc PLUvDrtPresc PLUvDrtQresc
        gen PLUvDvarQresc = PLUvDvarQ`k'resc
        gen PLUvDvarQsqresc = PLUvDvarQsq`k'resc
        gen PLUvDvarPresc = PLUvDvarP`k'resc
        gen PLUvDvarPsqresc = PLUvDvarPsq`k'resc
        gen PLUvDrtPresc = PLUvDrtP`k'resc
        gen PLUvDrtQresc = PLUvDrtQ`k'resc*/


****
```

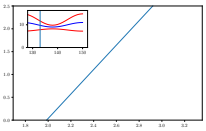
```
* defining variables
****
                local versionS "61"
                global runversionS `versionS'
                        global demandestimationvariables`version' "Tempeff15 Roll_Temp24 Roll_Temp240 suncycle morning deltasun EWH SolarRest RteBlackBox "
                global supplyestimationvariables`versionS' "Coal Brent Gas IT2 EUA suncycle morning deltasun EWH Wind1DA SolarRest Hydro RteBlackBox"
                *global uncertaintyproxies`versionS' "PLUvDvarP PLUvDvarQ PLUvDvarPsq PLUvDvarQsq PLUvDrtP PLUvDrtQ PLUvRvarT PLUvRvarW PLUvRvarS PLUvRvarTsq PLUvRvarWsq PLUvRvarSsq PLUvDvarQresc PLUvDvarQsqresc PLUvDvarPresc PLUvDvarPsqresc PLUvDrtPresc PLUvDrtQresc PLU_COMBa_Dresc PLU_COMBa_Dsqresc PLU_COMBa_Drtresc PLU_COMBb_D PLU_COMBb_Dsq PLU_COMBb_Drt "
                *global PLUsD "PLUvDvarP PLUvDvarQ PLUvDvarPsq PLUvDvarQsq PLUvDrtP PLUvDrtQ PLUvDvarPresc PLUvDvarQresc PLUvDvarPsqresc PLUvDvarQsqresc  PLUvDrtPresc PLUvDrtQresc PLU_COMBa_Dresc PLU_COMBa_Dsqresc PLU_COMBa_Drtresc PLU_COMBb_D PLU_COMBb_Dsq PLU_COMBb_Drt "
                global PLUsR "PLUvRvarT PLUvRvarTsq PLUvRvarW PLUvRvarWsq PLUvRvarS PLUvRvarSsq"
                global PLUsROB "PLUv51avarPsq`k' PLUv51avarQsq`k' PLUv51bvarPsq`k' PLUv51bvarQsq`k' PLUv51avarPsq`k'resc PLUv51bvarPsq`k'resc PLUv51avarQsq`k'resc PLUv51bvarQsq`k'resc PLU_COMBa_v51a_sq`k'resc PLU_COMBa_v51b_sq`k'resc PLU_COMBb_v51a`k' PLU_COMBb_v51b`k'"
                global PLUsROBa "PLUv51avarPsq`k' PLUv51avarQsq`k' PLUv51avarPsq`k'resc PLUv51avarQsq`k'resc  PLU_COMBa_v51a_sq`k'resc  PLU_COMBb_v51a`k' "
                global PLUsROBb " PLUv51bvarPsq`k' PLUv51bvarQsq`k' PLUv51bvarPsq`k'resc PLUv51bvarQsq`k'resc  PLU_COMBa_v51b_sq`k'resc  PLU_COMBb_v51b`k'"
                global SEV ${supplyestimationvariables${runversionS}}
                global UCP ${uncertaintyproxies${runversionS}}
                di $SEV
                di ${PLUsROB}



*** first regression

reg fxInvertQP  ${PLUsR}, robust
est store onlyplus
reg fxInvertQP  Coal Brent Gas IT2 EUA Wind1DA Hydro, robust
est store onlycontrols
* all plu_renouvelable are sigigifcant, only plu wind of correct sign
reg fxInvertQP  ${PLUsR} Coal Brent Gas IT2 EUA Wind1DA Hydro, robust
* when adding supply controls, only wind stay significant with correct sign, others non-sig. thats good. :)

                        foreach UCP of global PLUsROB{
                        reg fxInvertQP   ${PLUsR} Coal Brent Gas IT2 EUA Wind1DA Hydro /*${SEV}*/ `UCP'
                        est store r1`UCP'
                        }

                * including weighting using Pointsperbin
                        foreach UCP of global PLUsROBa{
                        reg fxInvertQP   ${PLUsR} Coal Brent Gas IT2 EUA Wind1DA Hydro /*${SEV}*/ `UCP' [aweight=PointsInBinv51a]
                        est store w1`UCP'
                        }
                        foreach UCP of global PLUsROBb{
                        reg fxInvertQP   ${PLUsR} Coal Brent Gas IT2 EUA Wind1DA Hydro /*${SEV}*/ `UCP' [aweight=PointsInBinv51b]
                        est store w1`UCP'
```



```
                        }


*simultaneous reg on PLu_P and PLU-Q
                        reg fxInvertQP   ${PLUsR} Coal Brent Gas IT2 EUA Wind1DA Hydro PLUv51avarPsq`k'resc  PLUv51avarQsq`k'resc [aweight=PointsInBinv51a]
                            est store w2_51a
                        reg fxInvertQP   ${PLUsR} Coal Brent Gas IT2 EUA Wind1DA Hydro PLUv51bvarPsq`k'resc  PLUv51bvarQsq`k'resc [aweight=PointsInBinv51b]
                            est store w2_51b


                * (generating) regression output
/* Shows:
        - plu_r as before
        - PLU_temperature never significant
        - wind1da : pos + sig (more wind, more uncertainty)
        - plu wind: sig + positive effect only for PLUs_on_P (longer autocorrelation wind-> more uncertainty)
        - plu wind squared : very neg + sig only for PLUs_on_P (very short or long autocorrel = low uncertainty, errors cancel out)
        -PLU-solar never sig
        - solar1da included in plusD
        - HAVE EXCLUDED DAYTIME CONTROLS (but they are strongly included in PLUsD
        - all input prices has sig effect: coal positive and all other negative (interpretation?)
        - plu_D_on_P have negative, sigificant effects, plu_D_on_Q have positive effects, when very significant
        ***  PROMISING RESUTLS HERE ON ROBUSTNESS!
        - resc variables have nonsignificnat effect when combined, significant and pos for quantities plus when individual effect.
        */
                btoutreg2 [/*onlyplus*/ onlycontrols r1PLUv51avarPsq`k' r1PLUv51avarQsq`k' r1PLUv51bvarPsq`k' r1PLUv51bvarQsq`k' /* r1PLUv51avarPsq`k'resc r1PLUv51bvarPsq`k'resc r1PLUv51avarQsq`k'resc r1PLUv51bvarQsq`k'resc */ r1PLU_COMBa_v51a_sq`k' resc r1PLU_COMBa_v51b_sq`k'resc r1PLU_COMBb_v51a`k' r1PLU_COMBb_v51b`k'] using "${LATEXPATH}k1_`k'.tex", replace tex(frag pretty ) stats(coef) label(proper) level(95) title(For k=`k')
                btoutreg2 [/*onlyplus*/ onlycontrols w1PLUv51avarPsq`k' w1PLUv51avarQsq`k' w1PLUv51bvarPsq`k' w1PLUv51bvarQsq`k' /* w1PLUv51avarPsq`k'resc w1PLUv51bvarPsq`k'resc w1PLUv51avarQsq`k'resc w1PLUv51bvarQsq`k'resc */ w1PLU_COMBa_v51a_sq`k' resc w1PLU_COMBa_v51b_sq`k'resc w1PLU_COMBb_v51a`k' w1PLU_COMBb_v51b`k'] using "${LATEXPATH}k2_`k'.tex", replace tex(frag pretty ) stats(coef) label(proper) level(95) title(For k=`k')

        *separate tables for individual P or Q

        btoutreg2 [w1PLUv51avarPsq`k' w1PLUv51avarQsq`k' w1PLUv51bvarPsq`k' w1PLUv51bvarQsq`k'] using "${LATEXPATH}k2a_`k'.tex", replace tex(frag pretty ) stats(coef) label(proper) level(95) title(For k=`k')

        btoutreg2 [w2_51a  w2_51b  w1PLU_COMBa_v51a_sq`k'resc w1PLU_COMBa_v51b_sq`k'resc /* w1PLU_COMBb_v51a`k' w1PLU_COMBb_v51b`k' */] using "${LATEXPATH}k2b_`k'.tex", replace tex(frag pretty ) stats(coef) label(proper)  level(95) title(For k=`k')

        btoutreg2 [w2_51a  w2_51b w2_52a  w2_52b  ] using "${LATEXPATH}k5152_`k'.tex", replace tex(frag pretty ) stats(coef) label(proper)  level(95) title(For k=`k')



        /* run individually if only running first part
```

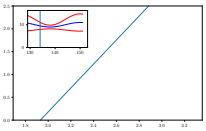
```
                drop _est*
*/
```

```
******* alternative pairing  - not relevant for k=${focusk}
```



```
*** BASELINE RESULTS:

        local k=${focusk}
                btoutreg2 [d1short1 bs_baseline`k' w2_51a  w2_51b] using "${LATEXPATH}
                    comparableregs_`k'.tex", replace tex(frag pretty ) stats(coef) label(proper)
                    level(95) title(regression for k=`k')

* OLD CODE (to delete)

** test in single step: ----- NOT CORRECT CODE; SINCE INCLUDE ALL EXOGENOUS IN PLU PREDICTION.
/*ivregress 2sls  fxInvertQP PLUvRvarT PLUvRvarTsq PLUvRvarW PLUvRvarWsq PLUvRvarS PLUvRvarSsq Coal
     Brent Gas IT2 EUA Wind1DA Hydro (sqresPrice sqresVolume = Tempeff15 Roll_Temp24 Roll_Temp240
    suncycle morning deltasun EWH SolarRest RteBlackBox)  if select ==5  & SalePurchase=="Purchase"
     , robust first

ivregress 2sls  fxInvertQP PLUvRvarT PLUvRvarTsq PLUvRvarW PLUvRvarWsq PLUvRvarS PLUvRvarSsq Coal
    Brent Gas IT2 EUA Wind1DA Hydro (sqresPrice sqresVolume = Tempeff15 Roll_Temp24 Roll_Temp240
    suncycle morning deltasun EWH SolarRest RteBlackBox)  if select ==5  & SalePurchase=="Purchase"
     , vce(bootstrap, rep(200))


***** bootstrap example 1
/*capture drop yhat PLU_boot
```

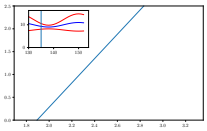
```
capture program drop my2slsforboot
program my2slsforboot
        reg sqresVolume Tempeff15 Roll_Temp24 Roll_Temp240 suncycle morning deltasun EWH SolarRest
            RteBlackBox if select ==5 & SalePurchase=="Purchase", robust
        predict PLU_boot, xb
        reg fxInvertQP PLUvRvarT PLUvRvarTsq PLUvRvarW PLUvRvarWsq PLUvRvarS PLUvRvarSsq Coal Brent
             Gas IT2 EUA Wind1DA Hydro PLU_boot   if select ==5 & SalePurchase=="Purchase" ,
            robust
        drop yhat PLU_boot
end
  bootstrap _b[PLU_boot] _se[PLU_boot], reps(50) seed(10): my2slsforboot
  bootstrap , bca reps(50) seed(10): my2slsforboot
di _se[PLU_boot]



***** bootstrap example 2
capture drop  PLU_boot
capture drop  volhat
capture program drop my2slsforboot
program my2slsforboot
        reg Volume Tempeff15 Roll_Temp24 Roll_Temp240 suncycle morning deltasun EWH SolarRest
            RteBlackBox if select ==5 & SalePurchase=="Purchase", robust
        predict yhat, xb
        gen PLU_boot = (Volume - yhat)^2
        reg fxInvertQP PLUvRvarT PLUvRvarTsq PLUvRvarW PLUvRvarWsq PLUvRvarS PLUvRvarSsq Coal Brent
             Gas IT2 EUA Wind1DA Hydro PLU_boot   if select ==5 & SalePurchase=="Purchase" ,
            robust
        drop yhat PLU_boot
end
  bootstrap _b[PLU_boot] _se[PLU_boot], reps(50) seed(10): my2slsforboot
  bootstrap , bca reps(50) seed(10): my2slsforboot
di _se[PLU_boot]

*/



        ***
        * generate table of kernel variables
        ***
/*              capture mat drop M
                mat M = J(9,6,.)
                local variables "  ${demandestimationvariables`version'} "
                local FUNC " r(mean) r(p50) r(sd) r(min) r(max)"
                local c=2
                foreach FF of local FUNC{
                local r=1
                foreach VV of local variables{
                su `VV', detail
                mat M[`r',`c']= `FF'
                local r=`r'+1
                }
                local c=`c'+1
                }
                mat rownames M =   `variables'
                mat colnames M = NumberBin Mean Median StdDev Min Max
                mat li M
                forvalues mm = 1/9{
                mat M[`mm', 1] =  `mm'
```



```
                }
                                mat li M
*/
*               Table for Variables used in the kernel based PLU$^D$ computation:
*               btouttable using "${LATEXPATH}multikernel", replace mat(M) asis nobox format(%9.0fc
      %9.1fc %9.0fc %9.0fc %9.0fc %9.0fc) longtable
```

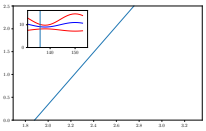
Next is the script named `107_Eqn4demand.do` and called in the general file.

```
* pull in do-file: needs to draw in following global macros:

                global DEV ${demandestimationvariables${runversionD}}


* open base file:
                use "${CLOUDPATH}v38/Temp_data/Pre4and5.dta", clear


**********
* 1st step: demand  estimation
**********


* initiate variables

                capture drop absres*
                capture drop sqres*
                capture drop normalres*
                capture drop stdres*
                capture drop tmp

        foreach m in Price Volume{
                gen absres`m'=.
                gen sqres`m'=.
                gen normalres`m'=.
                gen stdres`m'=.
        }


        **********
        * eqn 4 : DEMAND
        **********

* open loop for measure of uncertainty
        foreach m in Price Volume{

                * open loop for points and marketside
                forvalues i=9(-2)1{
                foreach k in Purchase{
                forvalues XXX=1/1{              /*Irrelevant in this setting, left for copy
                    convenience*/
                di "Next: " "`k'" "`i'" " " `XXX'

                        * reg 1a: retrieve absolute prediction errors

                        reg `m' ${DEV} if select ==`i' & SalePurchase==""`k'"" , robust

                        est store DE_`m'_`k'_`i'

                        * White test for heteroskedasticity

                        estat imtest, white
```



```
                        capture mat drop whitestat
                                mat whitestat = r(chi2_h)
                                di whitestat[1,1]
                                global  a`m'a`k'a`i' = whitestat[1,1]
                        di "next global"
                        di   ${a`m'a`k'a`i'}
                                if whitestat[1,1] == . {
                                global  a`m'a`k'a`i' = 99999999
                                }
                        di "next global"
                        di   ${a`m'a`k'a`i'}

                        * predict errors
                        predict tmp if e(sample), residuals

                        * gen deviations of residuals
                        replace absres`m' = abs(tmp) if e(sample)
                        replace sqres`m' = tmp*tmp if e(sample) /*consistent with white*/
                        replace normalres`m' = tmp if e(sample)

                        * gen Stdev of residuals /*over all residuals of that regression, thus single value for all -> add only to */
                        tabstat tmp if e(sample), stat(sd) save
                        mat tmpstdev = r(StatTotal)
                        di tmpstdev[1,1]
                        replace stdres`m' = tmpstdev[1,1] if e(sample)

                        drop tmp
                                }
                        }
                }
        }









        **********
        * generate tables for demand estimation, incl white test
        **********

                *******
                * Tables for k=1...5
                *******


                foreach m in Price{
                forvalues i=9/9{
                foreach k in Purchase {
                        btoutreg2 [DE_`m'_`k'_`i'] using "${LATEXPATH}PriceDEPur${runversionD}.tex", replace tex(frag pretty landscape) label(proper)  addstat(White, ${a`m'a`k'a`i'} )
                        }
                        }
                        }
                        foreach m in Price{
                forvalues i=7(-2)1{
                foreach k in Purchase {
```

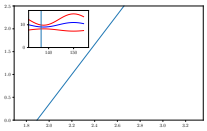
```
                        btoutreg2 [DE_`m'_`k'_`i'] using "${LATEXPATH}PriceDEPur${runversionD}.tex"
                            , append tex(frag pretty landscape) label(proper)  addstat(White, ${a`m
                            'a`k'a`i'} )
                        }
                        }
                        }


                foreach m in Volume{
                forvalues i=9/9{
                foreach k in Purchase {
                        btoutreg2 [DE_`m'_`k'_`i'] using "${LATEXPATH}VolDEPur${runversionD}.tex",
                            replace tex(frag pretty landscape) label(proper)  addstat(White, ${a`m'
                            a`k'a`i'} )
                        }
                        }
                        }
                foreach m in Volume{
                forvalues i=7(-2)1{
                foreach k in Purchase {
                        btoutreg2 [DE_`m'_`k'_`i'] using "${LATEXPATH}VolDEPur${runversionD}.tex",
                            append tex(frag pretty landscape) label(proper)  addstat(White, ${a`m'a
                            `k'a`i'} )
                        }
                        }
                        }




        **********
        * generate tables for heteroskedasticity test
        **********

                * can do later, already incl above
                * if so, then create matrix with inputs, then export.


                drop _est*


* save
                save "${CLOUDPATH}v38/Temp_data/PreKernel_D_${runversionD}.dta", replace
```



Next is the script named `107_kernelbucketreg.do` and called in the general file.

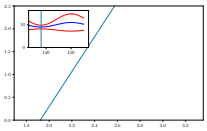
```
*DO file for robustness - 23.02

*using bucket specific linear regression.

* checking results with kernel based plus.
        global dofiledirectoryorig= Path to directory containing do files
        global LATEXPATH = Path to directory containing latex for article
        global CLOUDPATH = Path to directory containing data

****************************************
* version
                local binsetting "a"
                global runversionD `binsetting'
                global demandestimationvariables`binsetting' "Tempeff15 Roll_Temp24 Roll_Temp240
                     suncycle morning deltasun EWH SolarRest RteBlackBox "

****************************************

* Define macros
                local sensitivity 0
                local endofdata = _N
                local REP1 SolarRest
                local Numbin`REP1' 6*(1-`sensitivity')
                local REP2 deltasun
                local Numbin`REP2' 6*(1-`sensitivity')
                local REP3 Tempeff15
                local Numbin`REP3' 6*(1-`sensitivity')
                local REP4 Roll_Temp24
                local Numbin`REP4' 6*(1-`sensitivity')
                local REP5 Roll_Temp240
                local Numbin`REP5' 1*(1-`sensitivity')
                local REP6 suncycle
                local Numbin`REP6' 6*(1-`sensitivity')
                local REP7 morning
                local Numbin`REP7' 6*(1-`sensitivity')
                local REP8 EWH
                local Numbin`REP8' 6*(1-`sensitivity')
                local REP9 RteBlackBox
                local Numbin`REP9' 6*(1-`sensitivity')
                global variablesusedkernel "${demandestimationvariables`binsetting'}"

                        **********
                        * Execution PLU
                        **********

                                        * open saving loop for speeding up computation
                                        forvalues k = 1(2)9{
                                        foreach s in Purchase{
                                        use "${CLOUDPATH}v38/Temp_data/Pre4and5.dta", clear
                                                keep if select==`k' & SalePurchase == "`s'"
                                                local endofdata = _N

                        * run
                                quietly{
                                        gsort SalePurchase select Datestata Hour

```

```
56                          * gen variables to fill
57                          foreach m in Price Volume{
58                                          capture drop  Kabsres`m'
59                                          capture drop  Ksqres`m'
60                                          capture drop  PLUvKDvarabsres`m'
61                                          capture drop  PLUvKDvarsqres`m'
62                                          capture drop Ksamplesizeabsres`m'
63                                          capture drop Ksamplesizesqres`m'
64                                          gen Kabsres`m'=0
65                                          gen Ksqres`m'=0
66                                          gen PLUvKDvarabsres`m'=0
67                                          gen PLUvKDvarsqres`m'=0
68                                          gen Ksamplesizeabsres`m'=0
69                                          gen Ksamplesizesqres`m'=0
70
71
72
73                          forvalues obs= 1/`endofdata'{
74                          * su Tempeff if _n==`obs'                    /*crosscheck*/
75                          * su select if _n==`obs'                     /*crosscheck*/
76                          local CompPoint = select[`obs']
77                          local CompFunc = "SalePurchase[`obs']"
78                          * di `CompFunc'                               /*crosscheck*/
79
80                  foreach controlfactor in $variablesusedkernel {
81                          * find bincentre per controlfactor for given observation
82                          local Bincentre_`controlfactor' = `controlfactor'[`obs']
83                          * find binwidth per control factor
84                          su `controlfactor', meanonly
85                          local topendrange`controlfactor' = r(max)
86                          local lowendrange`controlfactor' = r(min)
87                          local binwidth`controlfactor' = (r(max) - r(min))/ `=`Numbin`
                                controlfactor'''
88
89                          di "----------------     `controlfactor'"
90                          di `=`Numbin`controlfactor'''
91                          di "Max: " `topendrange`controlfactor''
92                          di "Min: " `lowendrange`controlfactor''
93                          di "Binwidth: "`binwidth`controlfactor''
94                          di "Bincentre from current obs: " `Bincentre_`controlfactor''
95                          }
96
97                  capture reg `m' ${demandestimationvariables`version'}  ///
98                  if select == `CompPoint' & SalePurchase== `CompFunc' ///
99                  & `REP1'<= `=`Bincentre_`REP1''+ `binwidth`REP1''' & `REP1'>=`=`Bincentre_`
                        REP1''- `binwidth`REP1''' ///
100                 & `REP2'<= `=`Bincentre_`REP2''+ `binwidth`REP2''' & `REP2'>=`=`Bincentre_`
                        REP2''- `binwidth`REP2''' ///
101                 & `REP3'<= `=`Bincentre_`REP3''+ `binwidth`REP3''' & `REP3'>=`=`Bincentre_`
                        REP3''- `binwidth`REP3''' ///
102                 & `REP4'<= `=`Bincentre_`REP4''+ `binwidth`REP4''' & `REP4'>=`=`Bincentre_`
                        REP4''- `binwidth`REP4''' ///
103                 & `REP5'<= `=`Bincentre_`REP5''+ `binwidth`REP5''' & `REP5'>=`=`Bincentre_`
                        REP5''- `binwidth`REP5''' ///
104                 & `REP6'<= `=`Bincentre_`REP6''+ `binwidth`REP6''' & `REP6'>=`=`Bincentre_`
                        REP6''- `binwidth`REP6''' ///
105                 & `REP7'<= `=`Bincentre_`REP7''+ `binwidth`REP7''' & `REP7'>=`=`Bincentre_`
                        REP7''- `binwidth`REP7''' ///
106                 & `REP8'<= `=`Bincentre_`REP8''+ `binwidth`REP8''' & `REP8'>=`=`Bincentre_`
                        REP8''- `binwidth`REP8''' ///
```



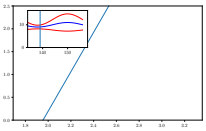
```
                    & `REP9'<= `='Bincentre_`REP9''+ `binwidth`REP9''' & `REP9'>=`='Bincentre_`
                        REP9''- `binwidth`REP9''' ///
                    , robust

                    *predict residuals locally
                    capture  predict tmp`m' if e(sample), residuals

                    * gen deviations of residuals
                    capture replace Kabsres`m' = abs(tmp) if _n==`obs'
                    capture replace Ksqres`m' = tmp*tmp if _n==`obs' /*consistent with white*/
                    capture drop tmp`m'

                    foreach g in "absres" "sqres"{
                    capture reg K`g'`m' ${demandestimationvariables`version'} ///
                    if select == `CompPoint' & SalePurchase== `CompFunc' ///
                    & `REP1'<= `='Bincentre_`REP1''+ `binwidth`REP1''' & `REP1'>=`='Bincentre_`
                        REP1''- `binwidth`REP1''' ///
                    & `REP2'<= `='Bincentre_`REP2''+ `binwidth`REP2''' & `REP2'>=`='Bincentre_`
                        REP2''- `binwidth`REP2''' ///
                    & `REP3'<= `='Bincentre_`REP3''+ `binwidth`REP3''' & `REP3'>=`='Bincentre_`
                        REP3''- `binwidth`REP3''' ///
                    & `REP4'<= `='Bincentre_`REP4''+ `binwidth`REP4''' & `REP4'>=`='Bincentre_`
                        REP4''- `binwidth`REP4''' ///
                    & `REP5'<= `='Bincentre_`REP5''+ `binwidth`REP5''' & `REP5'>=`='Bincentre_`
                        REP5''- `binwidth`REP5''' ///
                    & `REP6'<= `='Bincentre_`REP6''+ `binwidth`REP6''' & `REP6'>=`='Bincentre_`
                        REP6''- `binwidth`REP6''' ///
                    & `REP7'<= `='Bincentre_`REP7''+ `binwidth`REP7''' & `REP7'>=`='Bincentre_`
                        REP7''- `binwidth`REP7''' ///
                    & `REP8'<= `='Bincentre_`REP8''+ `binwidth`REP8''' & `REP8'>=`='Bincentre_`
                        REP8''- `binwidth`REP8''' ///
                    & `REP9'<= `='Bincentre_`REP9''+ `binwidth`REP9''' & `REP9'>=`='Bincentre_`
                        REP9''- `binwidth`REP9''' ///
                    , robust
                     capture predict PLUvKDtmp`g'`m' if _n==`obs', xb
                    capture scalar sizetmp = e(N)
                    capture replace Ksamplesize`g'`m' = sizetmp if _n==`obs'
                    capture replace PLUvKDvar`g'`m' = PLUvKDtmp`g'`m' if _n==`obs'
                    capture drop PLUvKDtmp`g'`m'
                    }

                    di `obs'
                                        } /*close observations loop*/
                            } /* closes quietly*/
                    } /*close `m'*/

                    * gen variables to fill
                            capture drop PLUv51`binsetting'var1
                            capture drop PLUv51`binsetting'var2
                            capture drop PLUv52`binsetting'var1
                            capture drop PLUv52`binsetting'var2
                            capture drop PointsInBinv51`binsetting'
                            capture drop PointsInBinv52`binsetting'
                            gen PLUv51`binsetting'var1 =.
                            gen PLUv51`binsetting'var2 =.
                            gen PLUv52`binsetting'var1 =.
                            gen PLUv52`binsetting'var2 =.
                            gen PointsInBinv51`binsetting' =.
                            gen PointsInBinv52`binsetting' =.
                    * generate PLUs
```

```
                                replace PLUv51`binsetting'var1    = PLUvKDvarsqresPrice
                                replace PLUv51`binsetting'var2    = PLUvKDvarsqresVolume
                                replace PLUv52`binsetting'var1    = PLUvKDvarabsresPrice
                                replace PLUv52`binsetting'var2    = PLUvKDvarabsresVolume
                                replace PointsInBinv51`binsetting'  = KsamplesizesqresVolume
                                replace PointsInBinv52`binsetting'  = KsamplesizeabsresVolume
                                drop Ksamplesize*



                                * close saving loop for speeding up computation
                                                keep Date Hour SalePurchase select PLUv51`
                                                     binsetting'var1 PLUv51`binsetting'var2 PLUv52`
                                                     binsetting'var1 PLUv52`binsetting'var2
                                                     PointsInBinv51`binsetting'  PointsInBinv52`
                                                     binsetting' KabsresPrice KsqresPrice
                                                     KabsresVolume KsqresVolume PLUvKDvarabsresPrice
                                                      PLUvKDvarsqresPrice PLUvKDvarabsresVolume
                                                     PLUvKDvarsqresVolume
                                                save "${CLOUDPATH}v38/Temp_data/KERNEL`binsetting'
                                                     buck`k'and`s'.dta", replace
                                        }
                                        }


****************************************
* version
                local binsetting "b"
                global runversionD `binsetting'
                global demandestimationvariables`binsetting' "Tempeff15 Roll_Temp24 Roll_Temp240
                     suncycle morning deltasun EWH SolarRest RteBlackBox "

****************************************

* Define macros
                local sensitivity 0
                local endofdata = _N
                local REP1 SolarRest
                local Numbin`REP1' 6*(1-`sensitivity')
                local REP2 deltasun
                local Numbin`REP2' 6*(1-`sensitivity')
                local REP3 Tempeff15
                local Numbin`REP3' 6*(1-`sensitivity')
                local REP4 Roll_Temp24
                local Numbin`REP4' 6*(1-`sensitivity')
                local REP5 Roll_Temp240
                local Numbin`REP5' 6*(1-`sensitivity')
                local REP6 suncycle
                local Numbin`REP6' 6*(1-`sensitivity')
                local REP7 morning
                local Numbin`REP7' 6*(1-`sensitivity')
                local REP8 EWH
                local Numbin`REP8' 6*(1-`sensitivity')
                local REP9 RteBlackBox
                local Numbin`REP9' 6*(1-`sensitivity')
                global variablesusedkernel "${demandestimationvariables`binsetting'}"

                        **********
                        * Execution PLU
                        **********
```



```
                                        * open saving loop for speeding up computation
                                        forvalues k = 1(2)9{
                                        foreach s in Purchase{
                                        use "${CLOUDPATH}v38/Temp_data/Pre4and5.dta", clear
                                                keep if select==`k' & SalePurchase == "`s'"
                                                local endofdata = _N

                        * run
                                quietly{
                                        gsort SalePurchase select Datestata Hour

                                * gen variables to fill
                                foreach m in Price Volume{
                                                capture drop  Kabsres`m'
                                                capture drop  Ksqres`m'
                                                capture drop  PLUvKDvarabsres`m'
                                                capture drop  PLUvKDvarsqres`m'
                                                capture drop Ksamplesizeabsres`m'
                                                capture drop Ksamplesizesqres`m'
                                                gen Kabsres`m'=0
                                                gen Ksqres`m'=0
                                                gen PLUvKDvarabsres`m'=0
                                                gen PLUvKDvarsqres`m'=0
                                                gen Ksamplesizeabsres`m'=0
                                                gen Ksamplesizesqres`m'=0



                                forvalues obs= 1/`endofdata'{
                                * su Tempeff if _n==`obs'                        /*crosscheck*/
                                * su select if _n==`obs'                         /*crosscheck*/
                                local CompPoint = select[`obs']
                                local CompFunc = "SalePurchase[`obs']"
                                * di `CompFunc'                                   /*crosscheck*/

                        foreach controlfactor in $variablesusedkernel {
                                * find bincentre per controlfactor for given observation
                                local Bincentre_`controlfactor' = `controlfactor'[`obs']
                                * find binwidth per control factor
                                su `controlfactor', meanonly
                                local topendrange`controlfactor' = r(max)
                                local lowendrange`controlfactor' = r(min)
                                local binwidth`controlfactor' = (r(max) - r(min))/ `=`Numbin`
                                    controlfactor'''

                                di "----------------     `controlfactor'"
                                di `=`Numbin`controlfactor'''
                                di "Max: " `topendrange`controlfactor''
                                di "Min: " `lowendrange`controlfactor''
                                di "Binwidth: "`binwidth`controlfactor''
                                di "Bincentre from current obs: " `Bincentre_`controlfactor''
                                }

                        capture reg `m' ${demandestimationvariables`version'}  ///
                        if select == `CompPoint' & SalePurchase== `CompFunc' ///
                        & `REP1'<= `=`Bincentre_`REP1''+ `binwidth`REP1''' & `REP1'>=`=`Bincentre_`
                            REP1''- `binwidth`REP1''' ///
                        & `REP2'<= `=`Bincentre_`REP2''+ `binwidth`REP2''' & `REP2'>=`=`Bincentre_`
                            REP2''- `binwidth`REP2''' ///
```

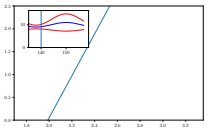
```
                    & `REP3'<= `='Bincentre_`REP3''+ `binwidth`REP3''' & `REP3'>=`='Bincentre_`
                        REP3''- `binwidth`REP3''' ///
                    & `REP4'<= `='Bincentre_`REP4''+ `binwidth`REP4''' & `REP4'>=`='Bincentre_`
                        REP4''- `binwidth`REP4''' ///
                    & `REP5'<= `='Bincentre_`REP5''+ `binwidth`REP5''' & `REP5'>=`='Bincentre_`
                        REP5''- `binwidth`REP5''' ///
                    & `REP6'<= `='Bincentre_`REP6''+ `binwidth`REP6''' & `REP6'>=`='Bincentre_`
                        REP6''- `binwidth`REP6''' ///
                    & `REP7'<= `='Bincentre_`REP7''+ `binwidth`REP7''' & `REP7'>=`='Bincentre_`
                        REP7''- `binwidth`REP7''' ///
                    & `REP8'<= `='Bincentre_`REP8''+ `binwidth`REP8''' & `REP8'>=`='Bincentre_`
                        REP8''- `binwidth`REP8''' ///
                    & `REP9'<= `='Bincentre_`REP9''+ `binwidth`REP9''' & `REP9'>=`='Bincentre_`
                        REP9''- `binwidth`REP9''' ///
                    , robust

                    *predict residuals locally
                    capture  predict tmp`m' if e(sample), residuals

                    * gen deviations of residuals
                    capture replace Kabsres`m' = abs(tmp) if _n==`obs'
                    capture replace Ksqres`m' = tmp*tmp if _n==`obs' /*consistent with white*/
                    capture drop tmp`m'

                    foreach g in "absres" "sqres"{
                    capture reg K`g'`m' ${demandestimationvariables`version'} ///
                    if select == `CompPoint' & SalePurchase== `CompFunc' ///
                    & `REP1'<= `='Bincentre_`REP1''+ `binwidth`REP1''' & `REP1'>=`='Bincentre_`
                        REP1''- `binwidth`REP1''' ///
                    & `REP2'<= `='Bincentre_`REP2''+ `binwidth`REP2''' & `REP2'>=`='Bincentre_`
                        REP2''- `binwidth`REP2''' ///
                    & `REP3'<= `='Bincentre_`REP3''+ `binwidth`REP3''' & `REP3'>=`='Bincentre_`
                        REP3''- `binwidth`REP3''' ///
                    & `REP4'<= `='Bincentre_`REP4''+ `binwidth`REP4''' & `REP4'>=`='Bincentre_`
                        REP4''- `binwidth`REP4''' ///
                    & `REP5'<= `='Bincentre_`REP5''+ `binwidth`REP5''' & `REP5'>=`='Bincentre_`
                        REP5''- `binwidth`REP5''' ///
                    & `REP6'<= `='Bincentre_`REP6''+ `binwidth`REP6''' & `REP6'>=`='Bincentre_`
                        REP6''- `binwidth`REP6''' ///
                    & `REP7'<= `='Bincentre_`REP7''+ `binwidth`REP7''' & `REP7'>=`='Bincentre_`
                        REP7''- `binwidth`REP7''' ///
                    & `REP8'<= `='Bincentre_`REP8''+ `binwidth`REP8''' & `REP8'>=`='Bincentre_`
                        REP8''- `binwidth`REP8''' ///
                    & `REP9'<= `='Bincentre_`REP9''+ `binwidth`REP9''' & `REP9'>=`='Bincentre_`
                        REP9''- `binwidth`REP9''' ///
                    , robust
                     capture predict PLUvKDtmp`g'`m' if _n==`obs', xb
                    capture scalar sizetmp = e(N)
                    capture replace Ksamplesize`g'`m' = sizetmp if _n==`obs'
                    capture replace PLUvKDvar`g'`m' = PLUvKDtmp`g'`m' if _n==`obs'
                    capture drop PLUvKDtmp`g'`m'
                    }

                    di `obs'
                                        } /*close observations loop*/
                            } /* closes quietly*/
                    } /*close `m'*/

                    * gen variables to fill
                            capture drop PLUv51`binsetting'var1
```



```
309                                 capture drop PLUv51`binsetting'var2
310                                 capture drop PLUv52`binsetting'var1
311                                 capture drop PLUv52`binsetting'var2
312                                 capture drop PointsInBinv51`binsetting'
313                                 capture drop PointsInBinv52`binsetting'
314                                 gen PLUv51`binsetting'var1 =.
315                                 gen PLUv51`binsetting'var2 =.
316                                 gen PLUv52`binsetting'var1 =.
317                                 gen PLUv52`binsetting'var2 =.
318                                 gen PointsInBinv51`binsetting' =.
319                                 gen PointsInBinv52`binsetting' =.
320                         * generate PLUs
321                                 replace PLUv51`binsetting'var1   = PLUvKDvarsqresPrice
322                                 replace PLUv51`binsetting'var2   = PLUvKDvarsqresVolume
323                                 replace PLUv52`binsetting'var1   = PLUvKDvarabsresPrice
324                                 replace PLUv52`binsetting'var2   = PLUvKDvarabsresVolume
325                                 replace PointsInBinv51`binsetting'  = KsamplesizesqresVolume
326                                 replace PointsInBinv52`binsetting'  = KsamplesizeabsresVolume
327                                 drop Ksamplesize*
328
329
330
331                                 * close saving loop for speeding up computation
332                                             keep Date Hour SalePurchase select PLUv51`
                                                    binsetting'var1 PLUv51`binsetting'var2 PLUv52`
                                                    binsetting'var1 PLUv52`binsetting'var2
                                                    PointsInBinv51`binsetting'  PointsInBinv52`
                                                    binsetting' KabsresPrice KsqresPrice
                                                    KabsresVolume KsqresVolume PLUvKDvarabsresPrice
                                                     PLUvKDvarsqresPrice PLUvKDvarabsresVolume
                                                    PLUvKDvarsqresVolume
333                                             save "${CLOUDPATH}v38/Temp_data/KERNEL`binsetting'
                                                    buck`k'and`s'.dta", replace
334                                         }
335                                         }
```

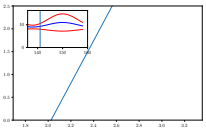
Next is the script named `107_PrepkernelPLUdata.do` and called in the general file.

```
*PrepkernelPLUdata


        global dofiledirectoryorig= Path to directory containing do files
        global LATEXPATH = Path to directory containing latex for article
        global CLOUDPATH = Path to directory containing data


        ****
        * Append kernel PLUs
        ****

                        * on sq residuals
        foreach binsetting in "a" "b"{
                        use "${CLOUDPATH}v38/Temp_data/KERNEL`binsetting'buck1andPurchase.dta",
                            clear
                        *append
                        forvalues k = 3(2)9{
                        foreach s in Purchase{
                        append using "${CLOUDPATH}v38/Temp_data/Kernel`binsetting'buck`k'
                            andPurchase.dta"
                                }
                                }

                duplicates list  SalePurchase select Date Hour
                duplicates drop  SalePurchase select Date Hour, force

* -  51a. = sqres + 8 bins.
* -  51b. = sqres + 9 bins.
* -  52a. = absres + 8 bins.
* -  52b. = absres + 9 bins.

                                capture rename PLUv51avar1 PLUv51avarPsq
                                capture rename PLUv51avar2 PLUv51avarQsq
                                capture rename PLUv52avar1 PLUv52avarPabs
                                capture rename PLUv52avar2 PLUv52avarQabs

                                capture rename PLUv51bvar1 PLUv51bvarPsq
                                capture rename PLUv51bvar2 PLUv51bvarQsq
                                capture rename PLUv52bvar1 PLUv52bvarPabs
                                capture rename PLUv52bvar2 PLUv52bvarQabs

                                drop PLUvKD*
                                save "${CLOUDPATH}v38/Temp_data/PLUKernel`binsetting'.dta", replace
                                }

        ****
        * Merge PLUs with dataset
        ****

                use "${CLOUDPATH}v38/Temp_data/Finaldataset.dta", clear
                duplicates list  SalePurchase select Date Hour
                duplicates drop  SalePurchase select Date Hour, force
        foreach binsetting in "a" "b"{
                /* here add other versions */
        merge 1:1 SalePurchase select Date Hour using "${CLOUDPATH}v38/Temp_data/PLUKernel`
```



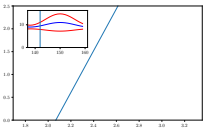
```
            binsetting'.dta", nogenerate
                }

        *save
                save "${CLOUDPATH}v38/Temp_data/Int1.dta", replace


        ****
        * transpose uncertainty of k points.
        ****

                use "${CLOUDPATH}v38/Temp_data/Int1.dta", clear
                        keep Date SalePurchase select Hour Datestata PLU*
                        reshape wide PLU* , i(SalePurchase Datestata Hour) j(select)
                        order _all, sequential
                         gsort Datestata Hour SalePurchase
                        duplicates list Date Hour SalePurchase
                        duplicates drop Date Hour SalePurchase, force
                save "${CLOUDPATH}v38/Temp_data/Int2.dta", replace

                use "${CLOUDPATH}v38/Temp_data/Int1.dta", clear
                merge m:1 Date Hour SalePurchase using "${CLOUDPATH}v38/Temp_data/Int2.dta",
                    nogenerate

                * drop constant plu_r
                drop PLUvRvarS1 PLUvRvarS3 PLUvRvarS5 PLUvRvarS7 PLUvRvarS9 PLUvRvarSsq1
                    PLUvRvarSsq3 PLUvRvarSsq5 PLUvRvarSsq7 PLUvRvarSsq9 PLUvRvarT1 PLUvRvarT3
                    PLUvRvarT5 PLUvRvarT7 PLUvRvarT9 PLUvRvarTsq1 PLUvRvarTsq3 PLUvRvarTsq5
                    PLUvRvarTsq7 PLUvRvarTsq9 PLUvRvarW1 PLUvRvarW3 PLUvRvarW5 PLUvRvarW7
                    PLUvRvarW9 PLUvRvarWsq1 PLUvRvarWsq3 PLUvRvarWsq5 PLUvRvarWsq7 PLUvRvarWsq9

                        gsort Datestata  Hour  SalePurchase select
                        order Date Datestata Hour SalePurchase select , first
                        order PLU* Poin*, last
                save "${CLOUDPATH}v38/Temp_data/Int3.dta", replace



                        ***
                        * table of Points in bin and PLU and PDU
                        ***
                                        forvalues k = 5/5{
                                        foreach versionD in "51a" "51b"{
                                        foreach versionS in ""{
                                        foreach switch in /*"abs"*/ "sq"{


                                capture mat drop M`versionD'
                                local variables "PLUv`versionD'varP`switch'`k' PLUv`versionD'varQ`
                                    switch'`k'  PointsInBinv`versionD' "
                                local FUNC "r(mean) r(p50) r(sd) r(min) r(max)"
                                local i=0
                                foreach var of local variables{
                                local i=`i'+1
                                }
                                local j=0
                                foreach var of local FUNC{
                                local j=`j'+1
                                }
                                di `j' "  " `i'
```

```
                                mat M`versionD' = J(`i',5,.)

                                local c=1
                                foreach FF of local FUNC{
                                local r=1
                                foreach VV of local variables{
                                su `VV', detail
                                mat M`versionD'[`r',`c']= `FF'
                                local r=`r'+1
                                }
                                local c=`c'+1
                                }
                                mat rownames M`versionD' =   `variables'
                                mat colnames M`versionD' =  Mean Median StdDev Min Max
                                mat li M`versionD'
                }
                }
                }
                                capture mat drop Mtogether
                                mat Mtogether = J(6,5,.)
                                mat Mtogether = M51a \ M51b
                                mat li Mtogether

* Summary Statistics of PLUs and PDUs:
                btouttable using "${LATEXPATH}suPDUPLU", replace mat(Mtogether) label asis nobox
                    format(%9.1fc %9.0fc %9.0fc %9.0fc %9.0fc) footnote(Proxies based on multi-
                    variate kernels) longtable caption(Summary statistics of kernel based PLU$^D$
                    for k=`k')
                }


        **********
        * all variables of interest in purchase obs.
        **********

                drop if SalePurchase=="Sell"
```



```
* OLD CODE
/*

        ***
        * scalefactor to adjust fx to slope
        ***
/*              capture drop group
                egen group= group(Datestatafrac)
        gsort group Datestata  Hour  SalePurchase select
        capture drop slopeDpost slopeDpre slopeDatk fxscalefactor
                gen slopeDpost =.
                gen slopeDpre =.
                gen slopeDatk =.
                gen fxscalefactor =.

forvalues k= 1(2)9{
        *note in p-q dimension!
by group: replace slopeDpost  = (Volume[_n+1] - Volume[_n])/(Price[_n+1]- Price[_n]) if select==`k'

by group:        replace slopeDpre = (Volume[_n] - Volume[_n-1])/(Price[_n]- Price[_n-1]) if select
    ==`k'
by group:        replace slopeDatk = abs(slopeDpost[_n]+ slopeDpre[_n])/2 if select==`k'

        capture drop tmp1 tmp2
         egen tmp1 = mean(slopeDatk ) if select==`k'
         egen tmp2 = mean(fx) if select==`k'
        replace fxscalefactor = tmp1 / tmp2 if select==`k'
        drop tmp1 tmp2
        }
        capture drop fxscaled
        gen fxscaled = fx * fxscalefactor


        * scalefactor to adjust fx to slope
        capture drop group
                egen group= group(Datestatafrac)
                        gsort group Datestata  Hour  SalePurchase select
        capture drop slopeDpostQP slopeDpreQP slopeDatkQP fxscalefactorQP
                gen slopeDpostQP =.
                gen slopeDpreQP =.
                gen slopeDatkQP =.
                gen fxscalefactorQP =.
```

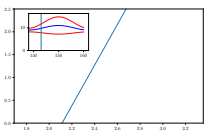
```
                        gen fxQP = (1/fx)

forvalues k= 1(2)9{
        *note in q-p dimension!
        by group: replace slopeDpostQP  = (Price[_n+1]- Price[_n])/(Volume[_n+1] - Volume[_n]) if select==`k'
         by group: replace slopeDpreQP = (Price[_n]- Price[_n-1])/(Volume[_n] - Volume[_n-1]) if select==`k'
         by group: replace slopeDatkQP = abs(slopeDpostQP[_n]+ slopeDpreQP[_n])/2 if select==`k'
        capture drop tmp1 tmp2
         egen tmp1 = mean(slopeDatkQP ) if select==`k'
         egen tmp2 = mean(fxQP) if select==`k'
        replace   fxscalefactorQP = tmp1 / tmp2 if select==`k'
        drop tmp1 tmp2
        }
        capture drop fxscaledQP
        gen fxscaledQP = fxQP * fxscalefactorQP
        gen comparisonfx = 1/ fxscaled
* SCALING ONLY APPROPRIATE FOR K=5, otherwise too much mixing flat and vertical section.
*/



/*
        *******
        * rescale variables
        *******
                        foreach versionD in "D" "51a" "51b"{
                        foreach switch in "" "abs"{
                        foreach switch2 in "var" "rt"{
                        foreach dim in "P" {
                        forvalues k = 3(2)7{
                        capture noisily su PLUv`versionD'`switch2'`dim'`switch'`k', detail
                        capture noisily scalar meanPLUv`versionD'`switch2'`dim'`switch'`k' = r(mean)
                        capture noisily di  meanPLUv`versionD'`switch2'`dim'`switch'`k'
                        capture noisily gen PLUv`versionD'`switch2'`dim'`switch'`k'resc = PLUv`versionD'`switch2'`dim'`switch'`k' / meanPLUv`versionD'`switch2'`dim'`switch'`k'
        }
        }
        }
        }
        }

                        foreach versionD in "D" "52a" "52b"{
                        foreach switch in "" "abs"{
                        foreach switch2 in "var" "rt"{
                        foreach dim in "P" {
                        forvalues k = 1(8)9{
                        capture noisily su PLUv`versionD'`switch2'`dim'`switch'`k', detail
                        capture noisily scalar meanPLUv`versionD'`switch2'`dim'`switch'`k' = r(mean)
                        capture noisily di  meanPLUv`versionD'`switch2'`dim'`switch'`k'
                        capture noisily gen PLUv`versionD'`switch2'`dim'`switch'`k'resc = PLUv`versionD'`switch2'`dim'`switch'`k'
        }
        }
        }
        }
```



```
        }

                        foreach versionD in "D" "52a" "52b"{
                        foreach switch in "" "abs"{
                        foreach switch2 in "var" "rt"{
                        foreach dim in "Q" {
                        forvalues k = 1(2)9{
                        capture noisily su PLUv`versionD''switch2''dim''switch''k', detail
                        capture noisily scalar meanPLUv`versionD''switch2''dim''switch''k' = r(mean
                            )
                        capture noisily di  meanPLUv`versionD''switch2''dim''switch''k'
                        capture noisily gen PLUv`versionD''switch2''dim''switch''k'resc = PLUv`
                            versionD''switch2''dim''switch''k' / meanPLUv`versionD''switch2''dim''
                            switch''k'
        }
        }
        }
        }
        }

                        order Date Datestata Hour SalePurchase select , first
                        order PLU* Poin*, last
*/
```

Next is the script named `107_BootstrapKernel2702.do` and called in the general file.

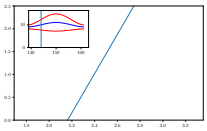
```
** kernel based PLU



* Notes:
* -  51a. = sqres + 8 bins.
* -  51b. = sqres + 9 bins.
* -  52a. = absres + 8 bins.
* -  52b. = absres + 9 bins.


        global dofiledirectoryorig= Path to directory containing do files
        global LATEXPATH = Path to directory containing latex for article
        global CLOUDPATH = Path to directory containing data

        global focusk = 5
        keep if select== ${focusk}
        global VVV = 51
        global APP = "sq"
                *global VVV = 52
                *global APP = "abs"

        * drop observations that will not be used for final reg anyway. (dropped 2989 obs) = last 6 months approx.
        drop if PLUvRvarW==.
        drop if Wind1DA==.




************** (Col. 3)
* for comparison, without bootstrap but weighted:
**************

                                use "${CLOUDPATH}v38/Temp_data/Finalrundataset.dta", clear
                                local binsetting "a"
                                local k = ${focusk}
                                local VVV = ${VVV}
                                local APP = "${APP}"
                                keep if select== `k'

                *gen rescaled PLU
                capture drop  PLU_P_boot PLU_P_resc PLU_Q_boot PLU_Q_resc
                gen  PLU_P_boot = PLUv`VVV'`binsetting'varP`APP'
                        su PLU_P_boot, meanonly
                        scalar tmpP = r(mean)
                        gen PLU_P_resc = PLU_P_boot / tmpP
                gen  PLU_Q_boot = PLUv`VVV'`binsetting'varQ`APP'
                        su PLU_Q_boot, meanonly
                        scalar tmpQ = r(mean)
                        gen PLU_Q_resc = PLU_Q_boot / tmpQ

                if `k'==1 | `k'==9 {
                                local k = ${focusk}
                                reg fxInvertQP   PLUvRvarT PLUvRvarTsq PLUvRvarW PLUvRvarWsq
```



```
                                PLUvRvarS PLUvRvarSsq Coal Brent Gas IT2 EUA Wind1DA Hydro /*
                                PLU_P_resc*/ PLU_Q_resc     if select ==`k' & SalePurchase==”
                                Purchase”, robust
                            est store kernel3_`k'
                            reg fxInvertQP   PLUvRvarT PLUvRvarTsq PLUvRvarW PLUvRvarWsq
                                PLUvRvarS PLUvRvarSsq Coal Brent Gas IT2 EUA Wind1DA Hydro /*
                                PLU_P_resc*/ PLU_Q_resc     if select ==`k' & SalePurchase==”
                                Purchase” [aweight=PointsInBinv51`binsetting']
                            est store kernel4_`k'
                            est save ”${CLOUDPATH}v38/Temp_data/kernelweigthed_`k'.ster”,
                                replace
                }
                else{
                            local k = ${focusk}
                            reg fxInvertQP   PLUvRvarT PLUvRvarTsq PLUvRvarW PLUvRvarWsq
                                PLUvRvarS PLUvRvarSsq Coal Brent Gas IT2 EUA Wind1DA Hydro
                                PLU_P_resc PLU_Q_resc     if select ==`k' & SalePurchase==”
                                Purchase”, robust
                            est store kernel3_`k'
                            reg fxInvertQP   PLUvRvarT PLUvRvarTsq PLUvRvarW PLUvRvarWsq
                                PLUvRvarS PLUvRvarSsq Coal Brent Gas IT2 EUA Wind1DA Hydro
                                PLU_P_resc PLU_Q_resc     if select ==`k' & SalePurchase==”
                                Purchase” [aweight=PointsInBinv51`binsetting']
                            est store kernel4_`k'
                            est save ”${CLOUDPATH}v38/Temp_data/kernelweigthed_`k'.ster”,
                                replace
}


******************************** NOW SAME BUT DROPPING 1 PLUvD (COL.3)

                            use ”${CLOUDPATH}v38/Temp_data/Finalrundataset.dta”, clear
                            local binsetting ”a”
                            local k = ${focusk}
                            local VVV = ${VVV}
                            local APP = ”${APP}”
                            keep if select== `k'

                *gen rescaled PLU
                capture drop  PLU_P_boot PLU_P_resc PLU_Q_boot PLU_Q_resc
                gen  PLU_P_boot = PLUv`VVV'`binsetting'varP`APP'
                        su PLU_P_boot, meanonly
                        scalar tmpP = r(mean)
                        gen PLU_P_resc = PLU_P_boot / tmpP
                gen  PLU_Q_boot = PLUv`VVV'`binsetting'varQ`APP'
                        su PLU_Q_boot, meanonly
                        scalar tmpQ = r(mean)
                        gen PLU_Q_resc = PLU_Q_boot / tmpQ

                if `k'==1 | `k'==9 {
                            local k = ${focusk}
                            reg fxInvertQP   PLUvRvarT PLUvRvarTsq PLUvRvarW PLUvRvarWsq
                                PLUvRvarS PLUvRvarSsq Coal Brent Gas IT2 EUA Wind1DA Hydro /*
                                PLU_P_resc*/ PLU_Q_resc     if select ==`k' & SalePurchase==”
                                Purchase”, robust
                            est store kernel3_`k'
                            reg fxInvertQP   PLUvRvarT PLUvRvarTsq PLUvRvarW PLUvRvarWsq
                                PLUvRvarS PLUvRvarSsq Coal Brent Gas IT2 EUA Wind1DA Hydro /*
                                PLU_P_resc*/ PLU_Q_resc     if select ==`k' & SalePurchase==”
                                Purchase” [aweight=PointsInBinv51`binsetting']
```

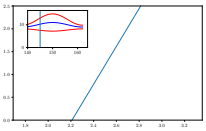
```
                                    est store kernel4_`k'
                                    est save "${CLOUDPATH}v38/Temp_data/kernelweigDROP_P_`k'.ster", replace
                    }
                    else{
                                    local k = ${focusk}
                                    reg fxInvertQP   PLUvRvarT PLUvRvarTsq PLUvRvarW PLUvRvarWsq PLUvRvarS PLUvRvarSsq Coal Brent Gas IT2 EUA Wind1DA Hydro /*PLU_P_resc*/ PLU_Q_resc     if select ==`k'  & SalePurchase==”Purchase”, robust
                                    est store kernel3_`k'
                                    reg fxInvertQP   PLUvRvarT PLUvRvarTsq PLUvRvarW PLUvRvarWsq PLUvRvarS PLUvRvarSsq Coal Brent Gas IT2 EUA Wind1DA Hydro /*PLU_P_resc*/ PLU_Q_resc     if select ==`k'  & SalePurchase==”Purchase” [aweight=PointsInBinv51`binsetting’]
                                    est store kernel4_`k'
                                    est save "${CLOUDPATH}v38/Temp_data/kernelweigDROP_P_`k'.ster", replace
                                    reg fxInvertQP   PLUvRvarT PLUvRvarTsq PLUvRvarW PLUvRvarWsq PLUvRvarS PLUvRvarSsq Coal Brent Gas IT2 EUA Wind1DA Hydro PLU_P_resc /*PLU_Q_resc*/     if select ==`k'  & SalePurchase==”Purchase”, robust
                                    est store kernel3_`k'
                                    reg fxInvertQP   PLUvRvarT PLUvRvarTsq PLUvRvarW PLUvRvarWsq PLUvRvarS PLUvRvarSsq Coal Brent Gas IT2 EUA Wind1DA Hydro PLU_P_resc /*PLU_Q_resc*/     if select ==`k'  & SalePurchase==”Purchase” [aweight=PointsInBinv51`binsetting’]
                                    est store kernel4_`k'
                                    est save "${CLOUDPATH}v38/Temp_data/kernelweigDROP_Q_`k'.ster", replace
}
```

```
************
* for bootstrapping of kernel based PLUvD

        ***** bootstrap of kernel based equation 4 forecasts
************

        use "${CLOUDPATH}v38/Temp_data/Finalrundataset.dta", clear
        global focusk = 5
        keep if select== ${focusk}
        * drop observations with missing values (dropped 2989 obs) = last 6 months approx.
        drop if PLUvRvarW==.
        drop if Wind1DA==.
```



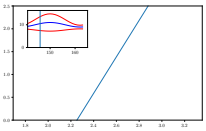
```
        capture drop  PLU_P_boot
        capture drop  PLU_P_resc
        capture drop  PLU_Q_boot
        capture drop  PLU_Q_resc



********************************************************** START PROG
        capture program drop my2slsforbootkernel
        program my2slsforbootkernel

* version
                local binsetting "a"
                local k = ${focusk}
                global runversionD `binsetting'
                global demandestimationvariables`binsetting' "Tempeff15 Roll_Temp24 Roll_Temp240
                    suncycle morning deltasun EWH SolarRest RteBlackBox "

***************************************

* Define macros
                local sensitivity 0
                local endofdata = _N
                local REP1 SolarRest
                local Numbin`REP1' 6*(1-`sensitivity')
                local REP2 deltasun
                local Numbin`REP2' 6*(1-`sensitivity')
                local REP3 Tempeff15
                local Numbin`REP3' 6*(1-`sensitivity')
                local REP4 Roll_Temp24
                local Numbin`REP4' 6*(1-`sensitivity')
                local REP5 Roll_Temp240
                local Numbin`REP5' 1*(1-`sensitivity')
                local REP6 suncycle
                local Numbin`REP6' 6*(1-`sensitivity')
                local REP7 morning
                local Numbin`REP7' 6*(1-`sensitivity')
                local REP8 EWH
                local Numbin`REP8' 6*(1-`sensitivity')
                local REP9 RteBlackBox
                local Numbin`REP9' 6*(1-`sensitivity')
                global variablesusedkernel "${demandestimationvariables`binsetting'}"


************************************************** stage 1 -> bucket specfic reg. 1
local endofdata = _N
                        quietly{
                        gsort SalePurchase select Datestata Hour

                                * gen variables to fill
                                foreach m in Price Volume{
                                                capture drop  Kabsres`m'
                                                capture drop  Ksqres`m'
                                                capture drop  PLUvKDvarabsres`m'
                                                capture drop  PLUvKDvarsqres`m'
                                                capture drop Ksamplesizeabsres`m'
                                                capture drop Ksamplesizesqres`m'
                                                gen Kabsres`m'=0
                                                gen Ksqres`m'=0
```

```
                                        gen PLUvKDvarabsres`m'=0
                                        gen PLUvKDvarsqres`m'=0
                                        gen Ksamplesizeabsres`m'=0
                                        gen Ksamplesizesqres`m'=0


                    forvalues obs= 1/`endofdata'{
                    * su Tempeff if _n==`obs'                          /*crosscheck*/
                    * su select if _n==`obs'                           /*crosscheck*/
                    local CompPoint = select[`obs']
                    local CompFunc = "SalePurchase[`obs']"
                    * di `CompFunc'                                     /*crosscheck*/

            foreach controlfactor in $variablesusedkernel {
                    * find bincentre per controlfactor for given observation
                    local Bincentre_`controlfactor' = `controlfactor'[`obs']
                    * find binwidth per control factor
                    su `controlfactor', meanonly
                    local topendrange`controlfactor' = r(max)
                    local lowendrange`controlfactor' = r(min)
                    local binwidth`controlfactor' = (r(max) - r(min))/ `=`Numbin`
                        controlfactor'''

                    di "----------------     `controlfactor'"
                    di `=`Numbin`controlfactor'''
                    di "Max: " `topendrange`controlfactor''
                    di "Min: " `lowendrange`controlfactor''
                    di "Binwidth: "`binwidth`controlfactor''
                    di "Bincentre from current obs: " `Bincentre_`controlfactor''
                    }

            capture reg `m' ${demandestimationvariables`version'}  ///
            if select == `CompPoint' & SalePurchase== `CompFunc' ///
            & `REP1'<= `=`Bincentre_`REP1''+ `binwidth`REP1''' & `REP1'>=`=`Bincentre_`
                REP1''- `binwidth`REP1''' ///
            & `REP2'<= `=`Bincentre_`REP2''+ `binwidth`REP2''' & `REP2'>=`=`Bincentre_`
                REP2''- `binwidth`REP2''' ///
            & `REP3'<= `=`Bincentre_`REP3''+ `binwidth`REP3''' & `REP3'>=`=`Bincentre_`
                REP3''- `binwidth`REP3''' ///
            & `REP4'<= `=`Bincentre_`REP4''+ `binwidth`REP4''' & `REP4'>=`=`Bincentre_`
                REP4''- `binwidth`REP4''' ///
            & `REP5'<= `=`Bincentre_`REP5''+ `binwidth`REP5''' & `REP5'>=`=`Bincentre_`
                REP5''- `binwidth`REP5''' ///
            & `REP6'<= `=`Bincentre_`REP6''+ `binwidth`REP6''' & `REP6'>=`=`Bincentre_`
                REP6''- `binwidth`REP6''' ///
            & `REP7'<= `=`Bincentre_`REP7''+ `binwidth`REP7''' & `REP7'>=`=`Bincentre_`
                REP7''- `binwidth`REP7''' ///
            & `REP8'<= `=`Bincentre_`REP8''+ `binwidth`REP8''' & `REP8'>=`=`Bincentre_`
                REP8''- `binwidth`REP8''' ///
            & `REP9'<= `=`Bincentre_`REP9''+ `binwidth`REP9''' & `REP9'>=`=`Bincentre_`
                REP9''- `binwidth`REP9''' ///
            , robust

            *predict residuals locally
            predict tmp`m' if e(sample), residuals

            * gen deviations of residuals
            replace Kabsres`m' = abs(tmp) if _n==`obs'
            replace Ksqres`m' = tmp*tmp if _n==`obs' /*consistent with white*/
            capture drop tmp`m'
```



```


**************************************************** stage 2 -> bucket specfic reg. 2 (heteroskedasticity)
                        foreach g in /*"absres"*/ "sqres"{
                        capture reg K`g'`m' ${demandestimationvariables`version'} ///
                        if select == `CompPoint' & SalePurchase== `CompFunc' ///
                        & `REP1'<= `='Bincentre_`REP1''+ `binwidth`REP1''' & `REP1'>=`='Bincentre_`REP1''- `binwidth`REP1''' ///
                        & `REP2'<= `='Bincentre_`REP2''+ `binwidth`REP2''' & `REP2'>=`='Bincentre_`REP2''- `binwidth`REP2''' ///
                        & `REP3'<= `='Bincentre_`REP3''+ `binwidth`REP3''' & `REP3'>=`='Bincentre_`REP3''- `binwidth`REP3''' ///
                        & `REP4'<= `='Bincentre_`REP4''+ `binwidth`REP4''' & `REP4'>=`='Bincentre_`REP4''- `binwidth`REP4''' ///
                        & `REP5'<= `='Bincentre_`REP5''+ `binwidth`REP5''' & `REP5'>=`='Bincentre_`REP5''- `binwidth`REP5''' ///
                        & `REP6'<= `='Bincentre_`REP6''+ `binwidth`REP6''' & `REP6'>=`='Bincentre_`REP6''- `binwidth`REP6''' ///
                        & `REP7'<= `='Bincentre_`REP7''+ `binwidth`REP7''' & `REP7'>=`='Bincentre_`REP7''- `binwidth`REP7''' ///
                        & `REP8'<= `='Bincentre_`REP8''+ `binwidth`REP8''' & `REP8'>=`='Bincentre_`REP8''- `binwidth`REP8''' ///
                        & `REP9'<= `='Bincentre_`REP9''+ `binwidth`REP9''' & `REP9'>=`='Bincentre_`REP9''- `binwidth`REP9''' ///
                        , robust
                        predict PLUvKDtmp`g'`m' if _n==`obs', xb
                        scalar sizetmp = e(N)
                        replace Ksamplesize`g'`m' = sizetmp if _n==`obs'
                        replace PLUvKDvar`g'`m' = PLUvKDtmp`g'`m' if _n==`obs'
                        capture drop PLUvKDtmp`g'`m'
                        }

                        di `obs'
                                                } /*close observations loop*/
                                } /* closes quietly*/
                        } /*close `m'*/

        * gen variables to fill
                                capture drop PLUv51`binsetting'var1
                                capture drop PLUv51`binsetting'var2
                                capture drop PLUv52`binsetting'var1
                                capture drop PLUv52`binsetting'var2
                                capture drop PointsInBinv51`binsetting'
                                capture drop PointsInBinv52`binsetting'
                                gen PLUv51`binsetting'var1 =.
                                gen PLUv51`binsetting'var2 =.
                                gen PLUv52`binsetting'var1 =.
                                gen PLUv52`binsetting'var2 =.
                                gen PointsInBinv51`binsetting' =.
                                gen PointsInBinv52`binsetting' =.
                        * generate PLUs
                                replace PLUv51`binsetting'var1   = PLUvKDvarsqresPrice
                                replace PLUv51`binsetting'var2   = PLUvKDvarsqresVolume
                                *replace PLUv52`binsetting'var1   = PLUvKDvarabsresPrice
                                *replace PLUv52`binsetting'var2   = PLUvKDvarabsresVolume
                                replace PointsInBinv51`binsetting'  = KsamplesizesqresVolume
                                *replace PointsInBinv52`binsetting'  = KsamplesizeabsresVolume
                                drop Ksamplesize*

```

```
gen  PLU_P_boot = PLUv51`binsetting'var1
        su PLU_P_boot, meanonly
        scalar tmpP = r(mean)
        gen PLU_P_resc = PLU_P_boot / tmpP
gen  PLU_Q_boot = PLUv51`binsetting'var2
        su PLU_Q_boot, meanonly
        scalar tmpQ = r(mean)
        gen PLU_Q_resc = PLU_Q_boot / tmpQ


************************************************** stage 3 -> final reg


                reg fxInvertQP   PLUvRvarT PLUvRvarTsq PLUvRvarW PLUvRvarWsq PLUvRvarS PLUvRvarSsq
                     Coal Brent Gas IT2 EUA Wind1DA Hydro PLU_P_resc PLU_Q_resc     if select ==`k'
                      & SalePurchase=="Purchase" [aweight=PointsInBinv51`binsetting']

                        drop PLU_P_boot PLU_P_resc  PLU_Q_boot  PLU_Q_resc
                        drop PointsInBinv51`binsetting'

********************************************************** END PROG
end


************************************************** Run once and save dataset (not inside program)
                *run prog
                my2slsforbootkernel

                local k= ${focusk}
                save  "${CLOUDPATH}v38/Temp_data/Finaldataset_withKERNELpluDa`k'.dta", replace

************************************************** Bootstrap (approx. 15h for 50reps)

                        bootstrap _b, reps(50) seed(12345): my2slsforbootkernel

                        est save "${CLOUDPATH}v38/Temp_data/kernelbootstrap`k'.ster", replace
```



```
358
359
360
361 *OLD CODE:
362 /*              * gen variables to fill
363                                 capture drop PLUv51`binsetting'var1
364                                 capture drop PLUv51`binsetting'var2
365                                 capture drop PLUv52`binsetting'var1
366                                 capture drop PLUv52`binsetting'var2
367                                 capture drop PointsInBinv51`binsetting'
368                                 capture drop PointsInBinv52`binsetting'
369                                 gen PLUv51`binsetting'var1 =.
370                                 gen PLUv51`binsetting'var2 =.
371                                 gen PLUv52`binsetting'var1 =.
372                                 gen PLUv52`binsetting'var2 =.
373                                 gen PointsInBinv51`binsetting' =.
374                                 gen PointsInBinv52`binsetting' =.
375                         * generate PLUs
376                                 capture {
377                                 replace PLUv51`binsetting'var1   = PLUvKDvarsqresPrice
378                                 replace PLUv51`binsetting'var2   = PLUvKDvarsqresVolume
379                                 replace PLUv52`binsetting'var1   = PLUvKDvarabsresPrice
380                                 replace PLUv52`binsetting'var2   = PLUvKDvarabsresVolume
381                                 replace PointsInBinv51`binsetting'  = KsamplesizesqresVolume
382                                 replace PointsInBinv52`binsetting'  = KsamplesizeabsresVolume
383                                 drop Ksamplesize*
384                                 }
```

# List of Figures


1 This graph is adapted from the original paper by Klemperer and Meyer and illustrates the admissible region of solutions to the differential equation so as to verify the constraints on the slope of the supply function. . . . . 10

2 Ce graphique est adapté du papier original de Klemperer et Meyer et illustre la région de solutions de l'équation différentielle admissibles dans le cadre de notre problème, c'est-à-dire vérifiant les contraintes sur leur pente. . . . . 24

1.1 Static symmetric oligopoly . . . . . 57

1.2 This graph plots $S^*(p)$ for different values of the ramping cost parameter, and compares them to the set of equilibria obtained in KM's framework. Four optimal supply schedules are plotted. The black curve (full line) corresponds to the case where $\gamma \to 0$. As before, as $\gamma$ increases the optimal schedules get steeper and steeper until in the limit of $\gamma \to \infty$, the optimal schedule attains a vertical slope. In addition, we show the set of available equilibria in KM's model in light green, and the extremal demand schedules in dashed black. . 60

1.3 On the left, this graph plots an envelope of constant width $\omega(t)$ but varying average. The insert represents this envelope, while the graph itself represents the supply curves associated with the points in time represented by the green and orange dotted lines on the insert. On the right, this graph plots an envelope of constant average but varying width $\omega(t)$. . . . . 73

2.2.1 Example of an asymetric aggregate supply function. The x axis is the quantity in MWh, the y axis if the price in €. In red is the actual aggregate function, in green is an estimated logistic function showcasing the large discrepancies that can arise with this parametric approach. The blue point is the market outcome. . . . . 91
2.2.2 Comparison of two aggregate demand functions for the same hour . . . 94
2.2.3 Steps of the point selection process . . . . . 95
2.2.4 Heat map on selected, comparable demand and supply points . . . . . 97
2.2.5 Distribution of selected demand (left) and supply (right) points . . . . . 98
2.2.6 Histogram of slopes per point type . . . . . 99
2.2.7 Error bars as a function of the number of extracted points . . . . . 101
2.2.8 Proxy for degrees of freedom on master curve . . . . . 102
2.2.9 Overall (left) and zoomed (right) Mastercurve with confidence interval . 103
2.3.1 Stations for which we have hourly data. Left: temperature, center: wind speed, right: light intensity. . . . . 105
2.3.2 Left: Voronoi's algorithm is applied once on the reference points highlighted in green to obtain the white surfaces, and a second time on the same points to which is added the query point in the center to obtain the new blue cell. The green circles, which represent the interpolating weights, are generated using the ratio of the shaded area to that of the cell area of the surrounding points. Center left: example of a reference surface (color mapped) to be reconstructed through a natural neighbor interpolation. Center right: interpolated surface with a reference set of 16 evenly organized points, represented in black. Right: interpolated surface with a reference set of 16 unevenly organized points, represented in black. From 16 points one is able to reconstruct the color mapped surfaces which are approximations of the reference one, represented in the center left image. . . . . 106
2.3.3 Temperature forecast from a simulation run by the GFS at 6 a.m. on the 3rd of November 2011, for a forecast at 22 p.m. . . . . 107

2.3.4 Left: reference image. Center left: borders and numbers are removed. Center right: edge recognition. Right: final fitted surface. . . . . 109
2.3.5 Auto-correlation lengthscale computation. In black are the points obtained from all the pairs from our original data, that is absolute wind speed differences as a function of the distance between the two points. In blue is the kernel smoothed function from those points. In red is the exponential fit. In black are the derivatives of the fit at 0 and $\infty$. In green is the recovered auto-correlation lengthscale. The unit for the lengthscale is in km. . . . . 112
2.3.6 Typical response curves of different wind turbines . . . . . 114
2.3.7 All curves are hourly production data. The origin of the hours is the first of January 2011, and the production is in MWh. In blue: the observed wind production. In dark red: the day-ahead predictions from the grid operator. In light red: the day-ahead predictions from weather data. . . . . 114
2.3.8 Daily electricity consumption in France as a function of the temperature, [RTE, 2012] . . . . . 115
2.3.9 Hourly solar production in MWh. The time origin is the first of January 2011. In blue: observed production by RTE. In dark red: reconstructed production from observed weather data. . . . . 116
2.A.1 Comparison of bandwidths: Large bandwidth in first stage . . . . . 125
2.A.2 Comparison of bandwidths: Small bandwidth in first stage . . . . . 126
2.A.3 Histogram of volume variation between points . . . . . 127

3.1.1 Illustrating the effect of increased uncertainty. . . . . 138
3.2.1 Traded volume plotted against total annual consumption . . . . . 141
3.3.1 Example aggregate demand and supply bid functions . . . . . 145
3.3.2 Aggregate bid functions for 20 consecutive days . . . . . 145
3.4.1 Selected points on original bid functions . . . . . 149
3.4.2 Temperature based seasonality controls . . . . . 161
3.4.3 Continuous controls for daily patterns . . . . . 161

3.B.1 Plotted average realised Volume (left) and Price (right) per Hour with 95% confidence intervals. . . . . 185
3.B.2 Distribution of observed market equilibria . . . . . 186
3.B.3 Distribution of minimum and maximum production volumes (and corresponding range) bid in an hourly auction. . . . . 186
3.B.4 Distribution of number of bid function steps . . . . . 186
3.B.5 Histogram of predicted wind (left) and predicted solar (right) generation 187

# Table of Contents - Long

**Remerciements—Acknowledgements** **i**

**Summary** **v**

**General Introduction** **1**

**Chapter 1: Dynamics of the Electricity Day-Ahead Market : Supply Function Equilibria and Ramping Costs** **33**

1.1 Introduction 35

1.1.1 Litterature review . . . . . 37

1.1.2 The day-ahead markets . . . . . 40

1.2 Heuristic Description of the Model 43

1.3 Stochastic Dynamics 47

1.3.1 The stochastic process . . . . . 47

1.3.2 The ramping costs . . . . . 49

1.3.3 Discussion of the approximations . . . . . 52

1.3.4 The maximization program . . . . . 52

1.4 The Monopoly 53

1.4.1 Results . . . . . 54

1.5 The Symmetric Oligopoly 57

1.5.1 Discussion . . . . . 62

1.6 Dynamic Behavior of the Bids 63

1.6.1 The setup . . . . . 65

1.6.2 Results . . . . . 68

1.6.3 Discussion . . . . . 71

1.7 Limits 73

1.8 Concluding Remarks 74

**Appendix 77**

Appendix 1.A Proof of Equation 1.11 77

Appendix 1.B Proof of Proposition 1.4.1 78

Appendix 1.C Proof of Proposition 1.4.2 80

Appendix 1.D Proof of Proposition 1.5.1 80

Appendix 1.E Proof of proposition 1.5.2 83

Appendix 1.F Proof of proposition 1.6.3 84

Appendix 1.G Proof of proposition 1.6.1 85

Appendix 1.H Proof of proposition 1.6.2 86

**Chapter 2: Methodological Tools for Non-Parametric Functional data Evaluation and Weather Data Usage 87**

2.1 Introduction 89

2.2 Point Selection on Functional Data: a Non-Parametric Approach 89

2.2.1 Information about our data . . . . . 92

2.2.2 Point selection algorithm . . . . . 93

2.2.3 Results of the point selection methodology . . . . . 96

2.2.4 Observations of bidding frictions . . . . . 98

2.2.5 Discussion . . . . . 103

2.3 Methodology to Aggregate Geographically Dispersed Information on a National Level 104

2.3.1 Dealing with meteorological data . . . . . 105

2.3.2 Aggregation of local information . . . . . 111

2.4 Conclusion 120

**Appendix 121**

Appendix 2.A Technical Details 121

2.A.1 Using the kernel density estimation (KDE) in our setting . . . . . 121

2.A.2 Outlier detection and removal . . . . . 126

**Chapter 3: Investigating the Impact of Uncertainty on Firms with Dynamic Costs: A Case Study of the French Electricity Market 129**

3.1 Introduction 131

3.1.1 Literature review and contribution . . . . . 134

3.1.2 Theoretical prediction . . . . . 137

3.2 The EPEX Spot Market 140

3.2.1 General background . . . . . 140

3.2.2 Auction rules and mechanism . . . . . 142

3.3 Our Data Explained 144

3.4 Methodology 147

3.4.1 Regression methodology . . . . . 150

3.5 Results 165

3.5.1 Demand estimation . . . . . 165

3.5.2 Final regression . . . . . 171

3.6 Discussion 175

3.6.1 Findings . . . . . 176

3.6.2 Internal and external validity . . . . . 178

3.6.3 Endogeneity . . . . . 180

3.7 Conclusion 182

**Appendix 183**

Appendix 3.A Summary Statistics of Selected Points 183

Appendix 3.B Computational Details and Descriptives 184

3.B.1 Hard choices in the PLU computation . . . . . 184

3.B.2 Descriptive Statistics . . . . . 185

**General conclusion 193**

**References 203**

**Code Annex** **209**

**List of Figures** **269**

**Table of Contents - Long** **273**