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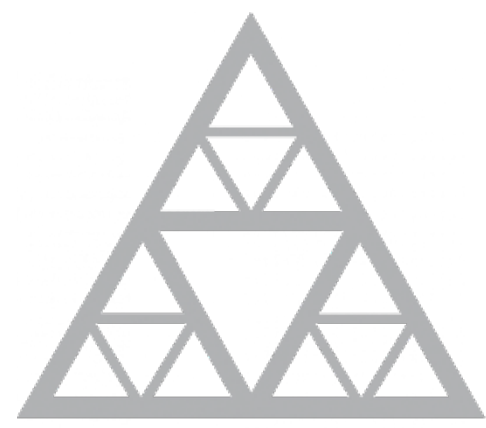
# Intermittency in water cycle and geophysics: why so important?

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**Ecole des Ponts ParisTech (ENPC), Hydrologie Météorologie et Complexité (HM&Co)**

**ERASMUS+**

**University of Tokyo, 21/06/2023**



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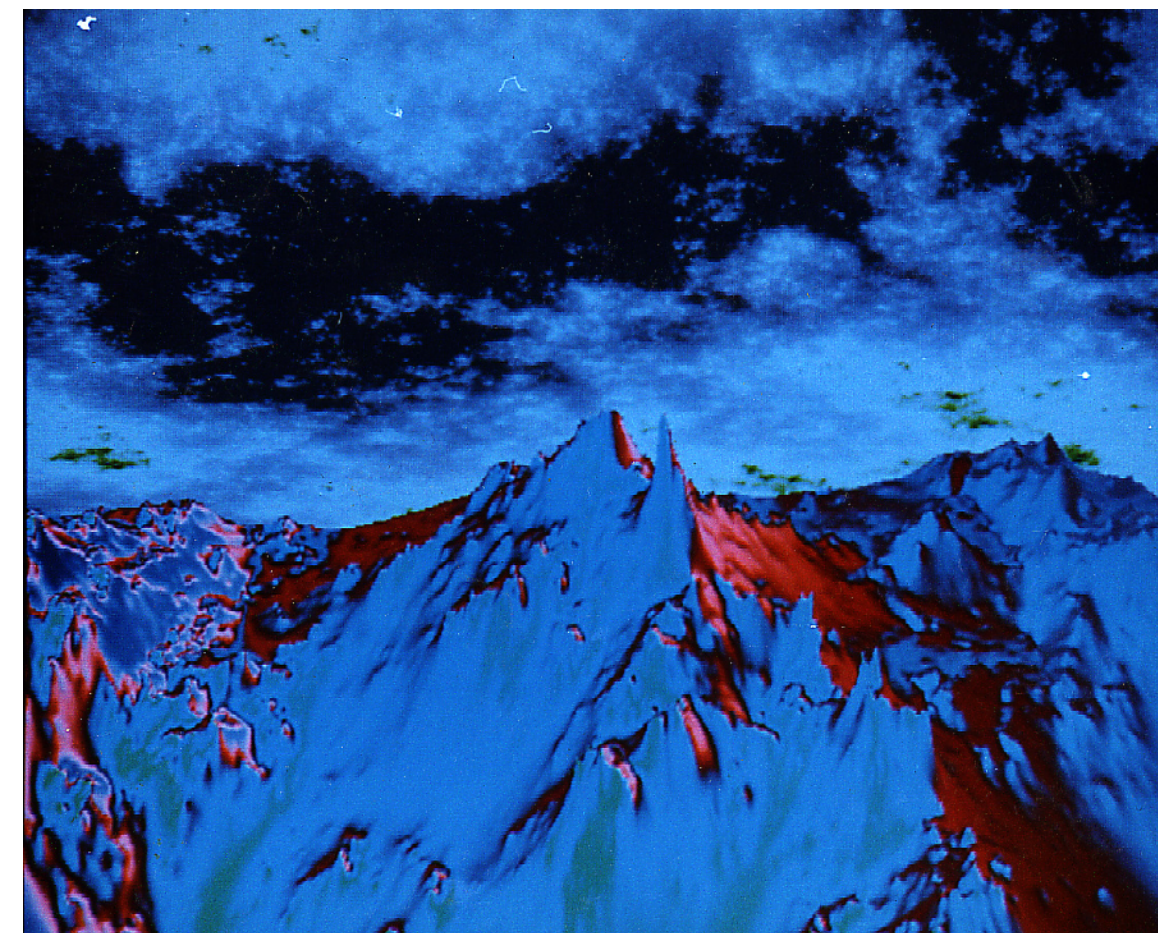
# Nobel 2021 : complexity and intermittency



- Nobel Prize in Physics 2021 awarded to Syukuro Manabe, Klaus Hasselmann and Giorgio Parisi "for groundbreaking contributions to our understanding of **complex physical systems**"



Three different approaches, but a common object: our planet!

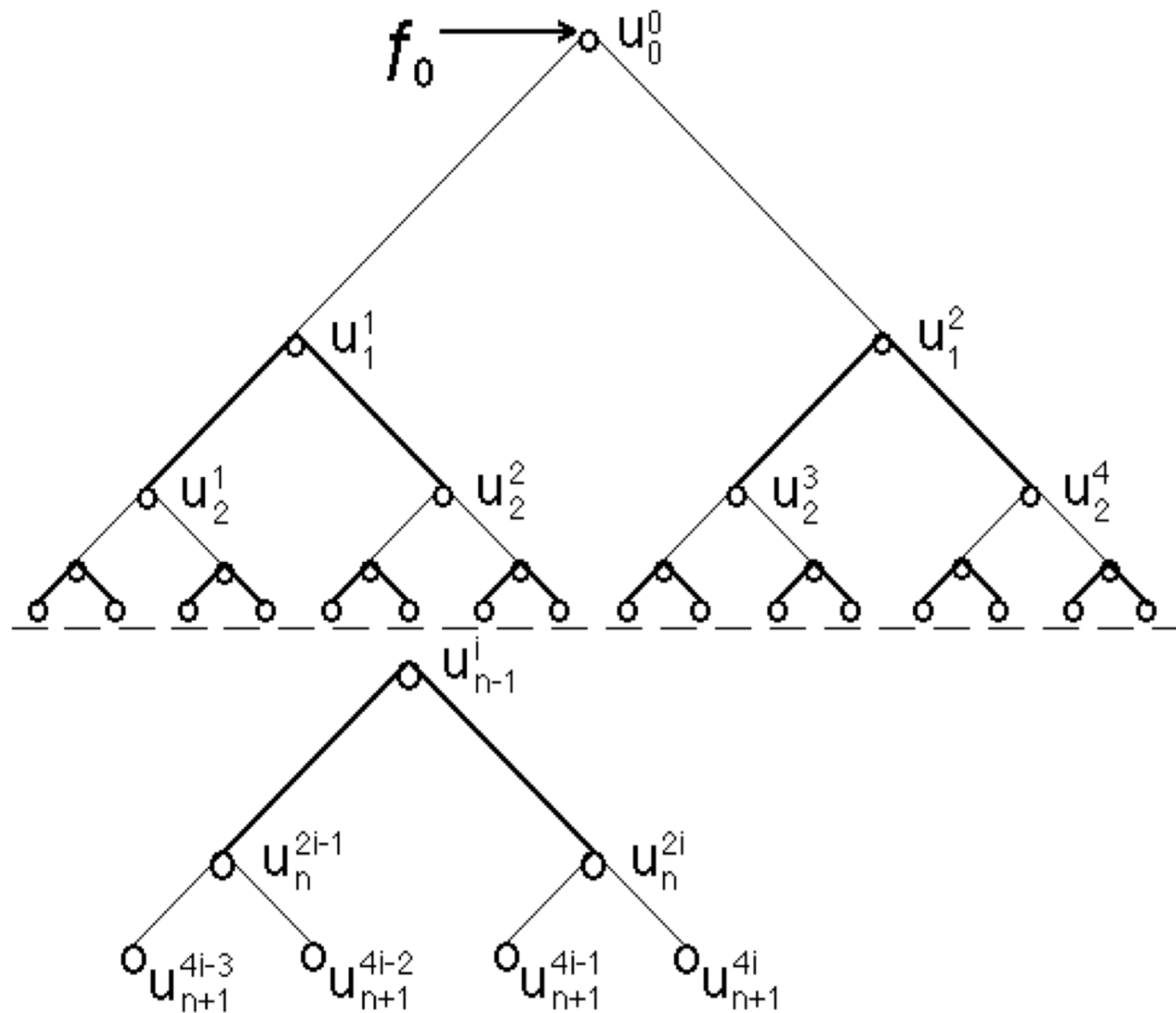


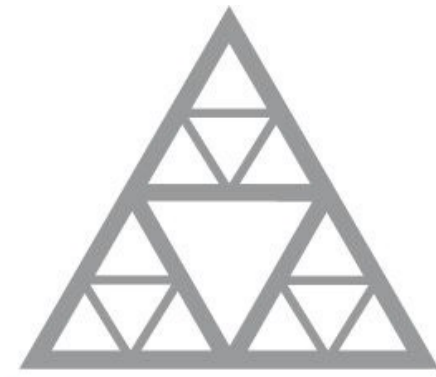
Emphasis of the prize scientific background on intermittency (\*)

(\*) Schertzer and Nicolis, Eos-2022)

# Complex systems

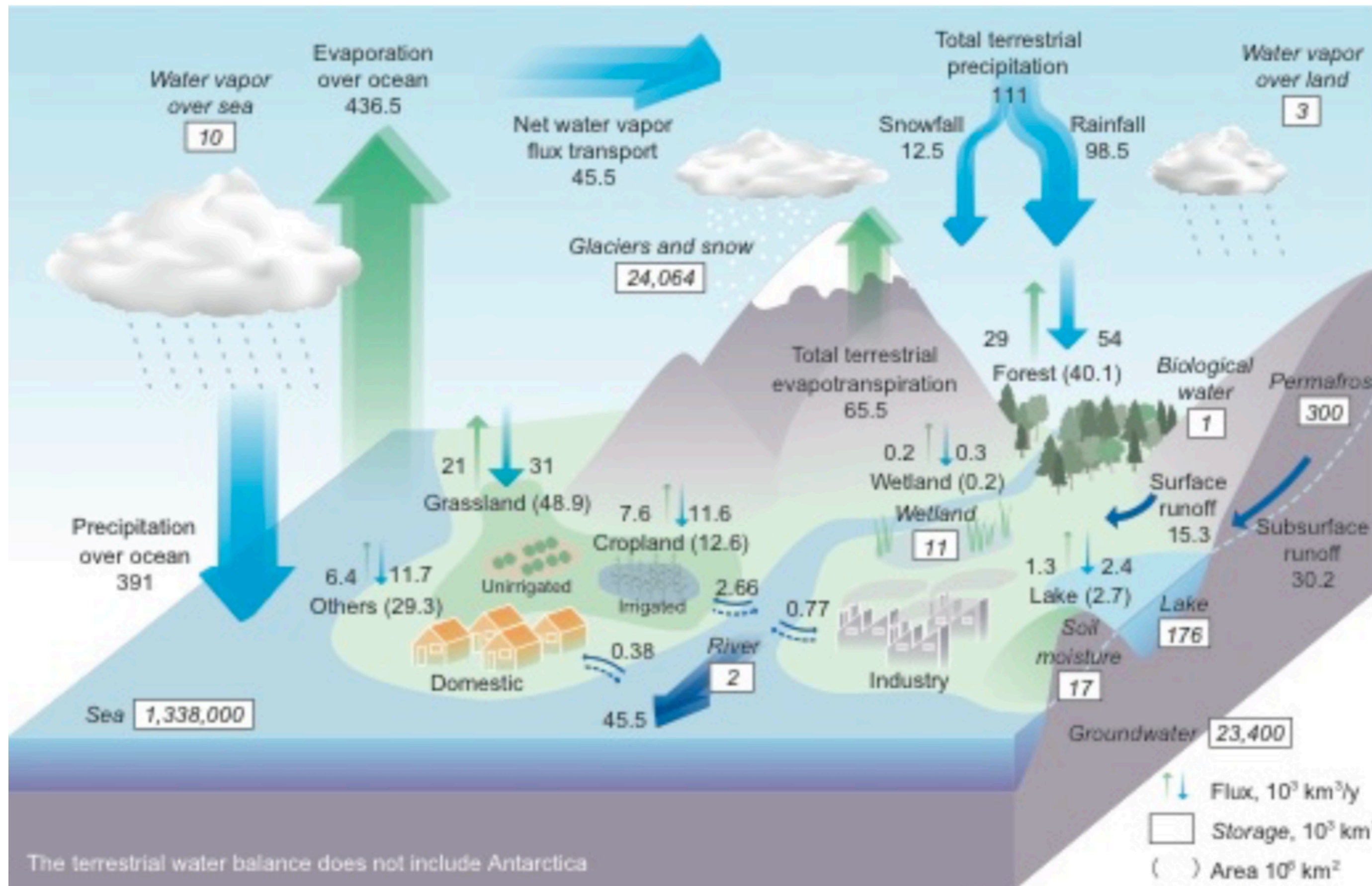
- Complex system?
  - French Roadmap for Complex Systems: “**A system comprised of a great number of heterogeneous entities, among which local interactions create multiple levels of collective structure and organization...**” .
  - explain failures to cope with their complexity at odds with reductionism (M. Gell-Man),
  - necessity to have a **multiscale** approach.



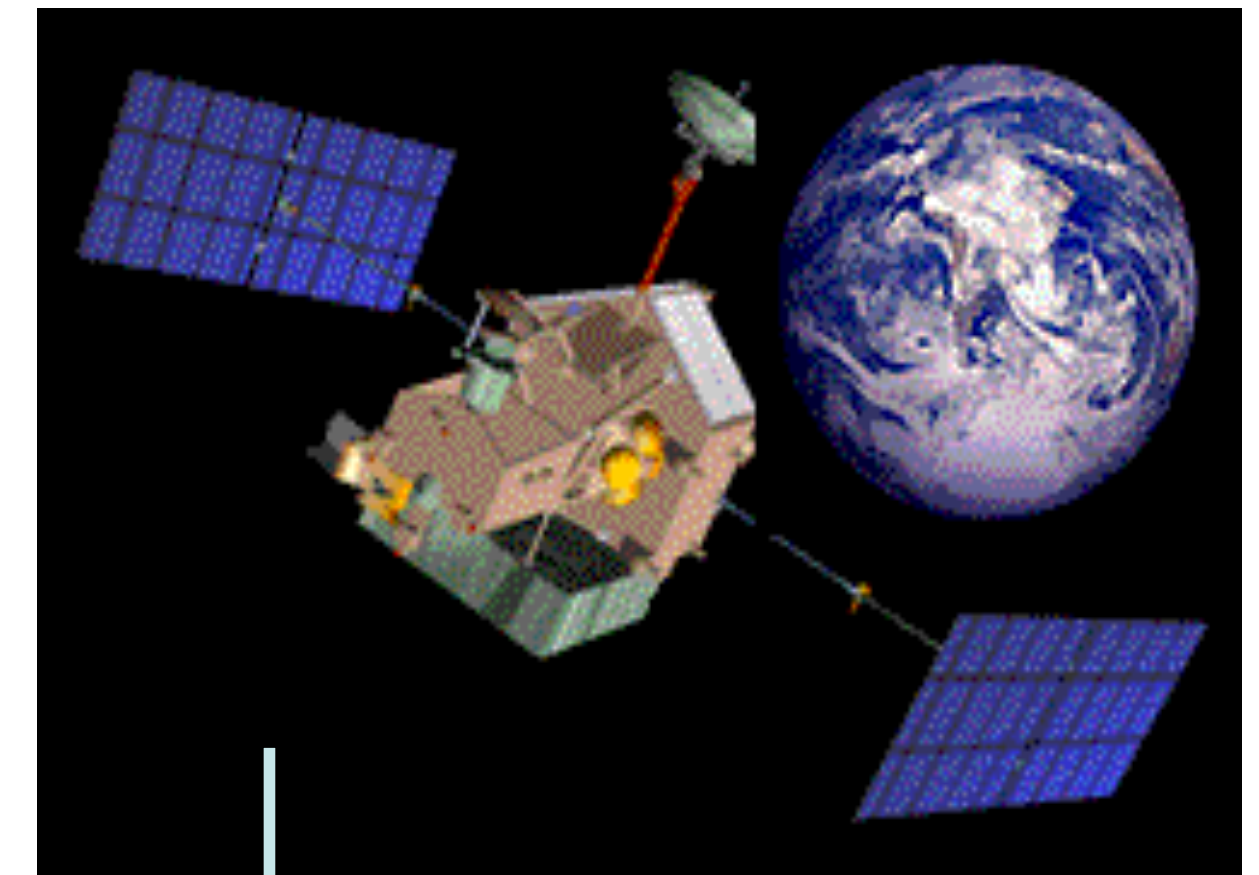


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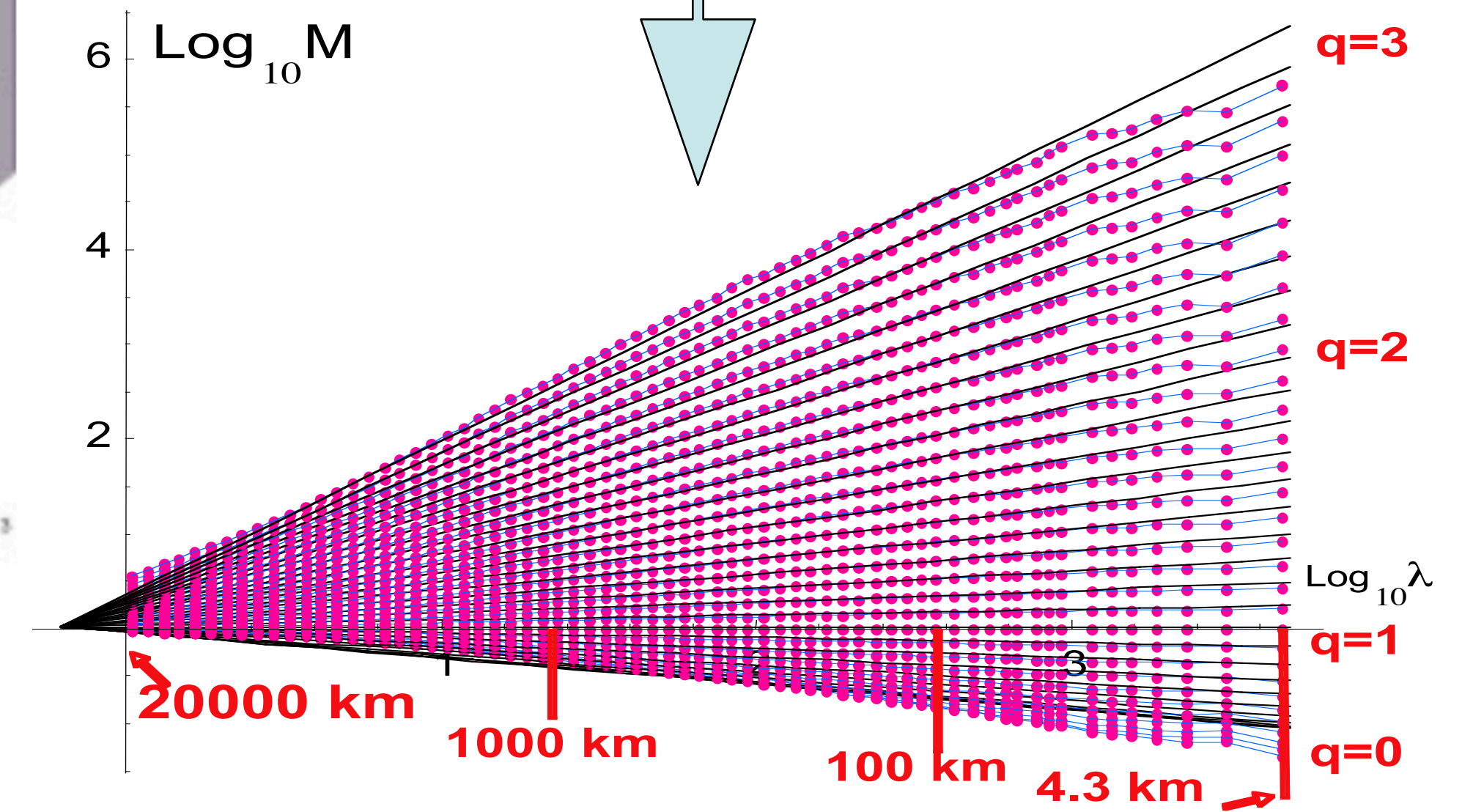
# Water cycles



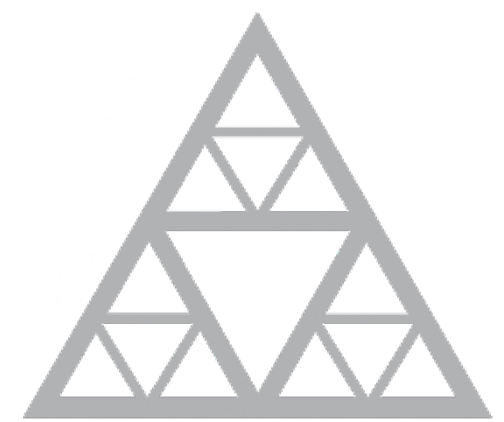
Global hydrological cycle (Oki & Kanae, 2006)



TRMM, JAXA and NASA



TRMM Multifractal analysis (L. et al., 2008)

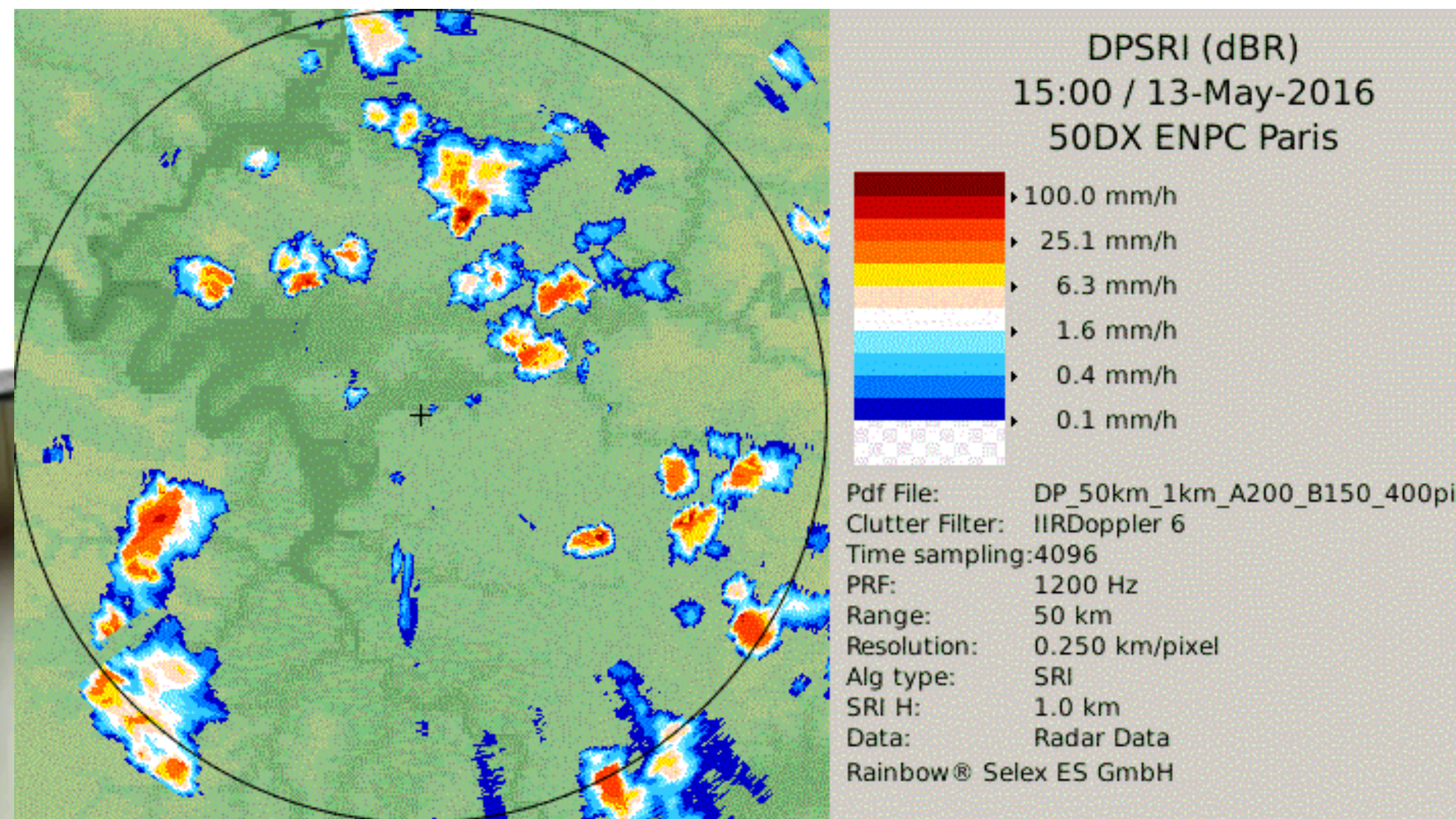


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# Intermittency



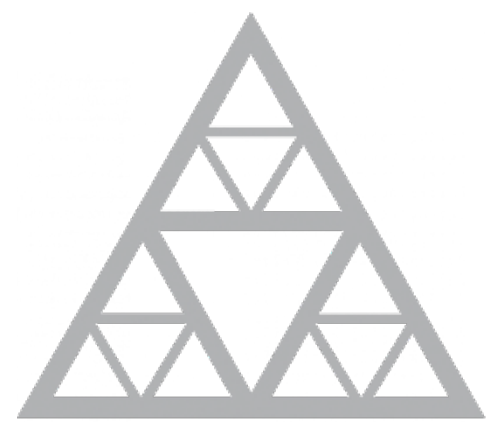
exploratorium®



**Art piece 'Windswept'** (Ch. Sowers, 2012): 612 freely rotating wind direction indicators to help a large public to understand the complexity of environment near the Earth surface

Polarimetric radar observations of heavy rainfalls over Paris region during 2016 spring (250 m resolution):

- **heaviest rain cells** are much smaller than **moderate ones**
- **complex dynamics** of their aggregation into a large front

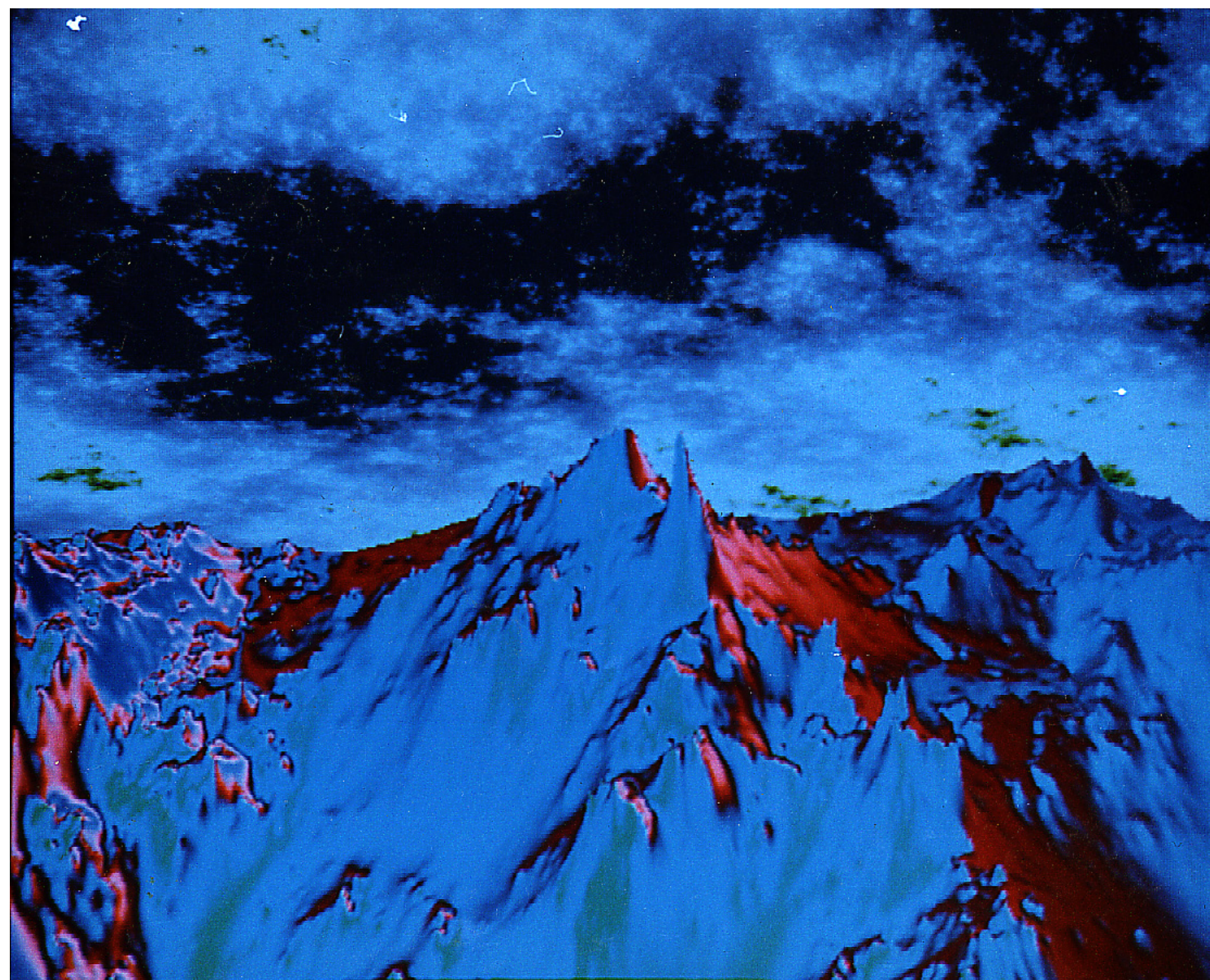


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# Intermittency

- a matter for a few turbulence specialists in the 1950s
- now a major transdisciplinary subject
- unusual features
  - extreme concentration of activity
  - structuring fluctuations
  - a multiplicity of anomalous scaling laws
  - (multifractal) phase transitions for extremes
- recent understanding of their interrelationships,
  - Legendre transform between concentration on a hierarchy of supports and the multiplicity of scaling laws
  - but also unresolved debates



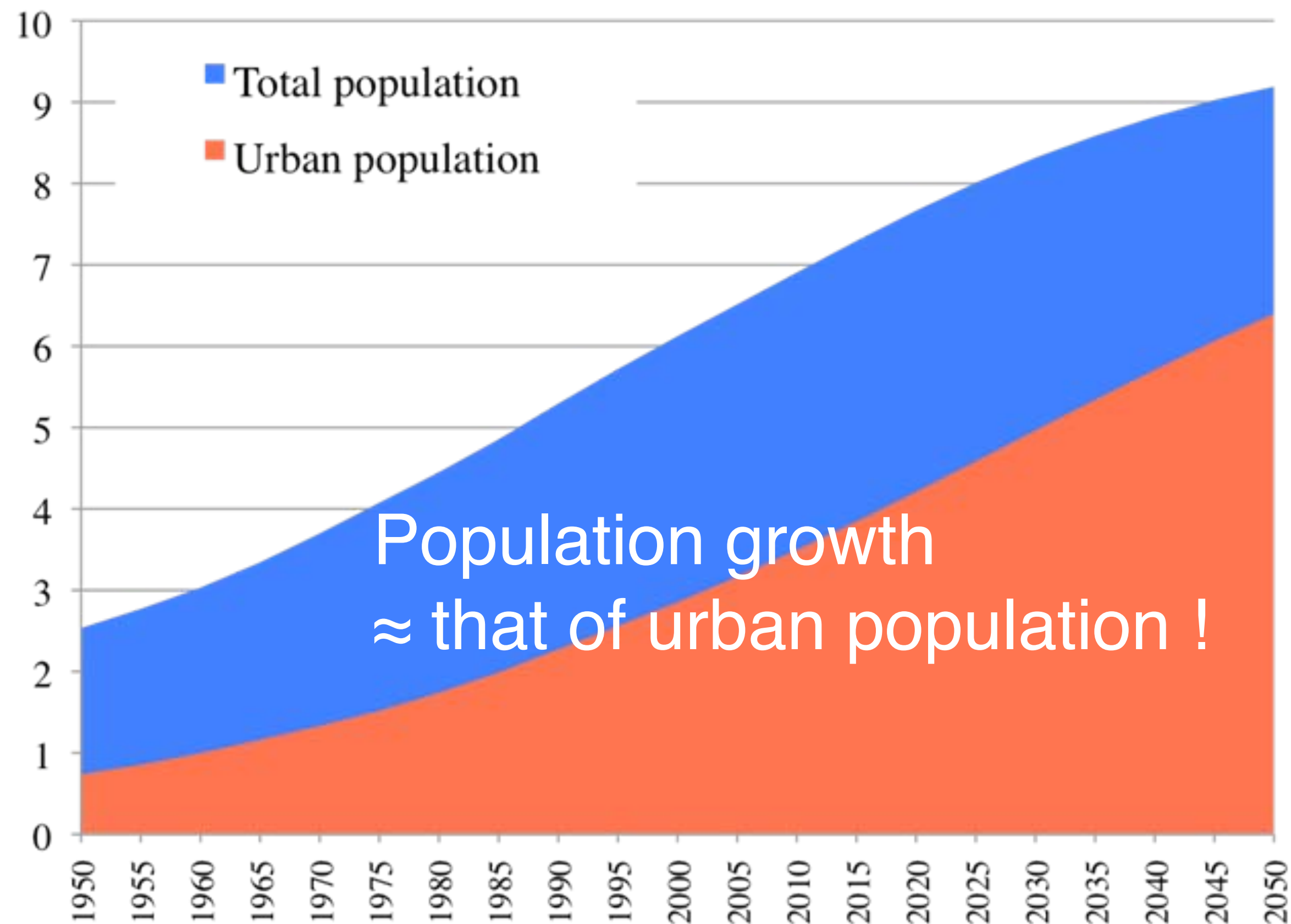
Purely random landscape, but highly structured, (S+L, 1992)

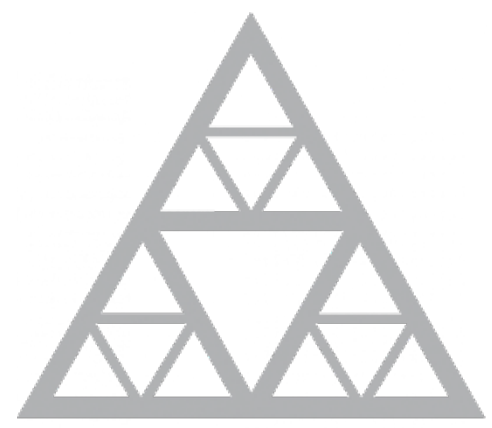




# Growing Urban Population

(Billion)



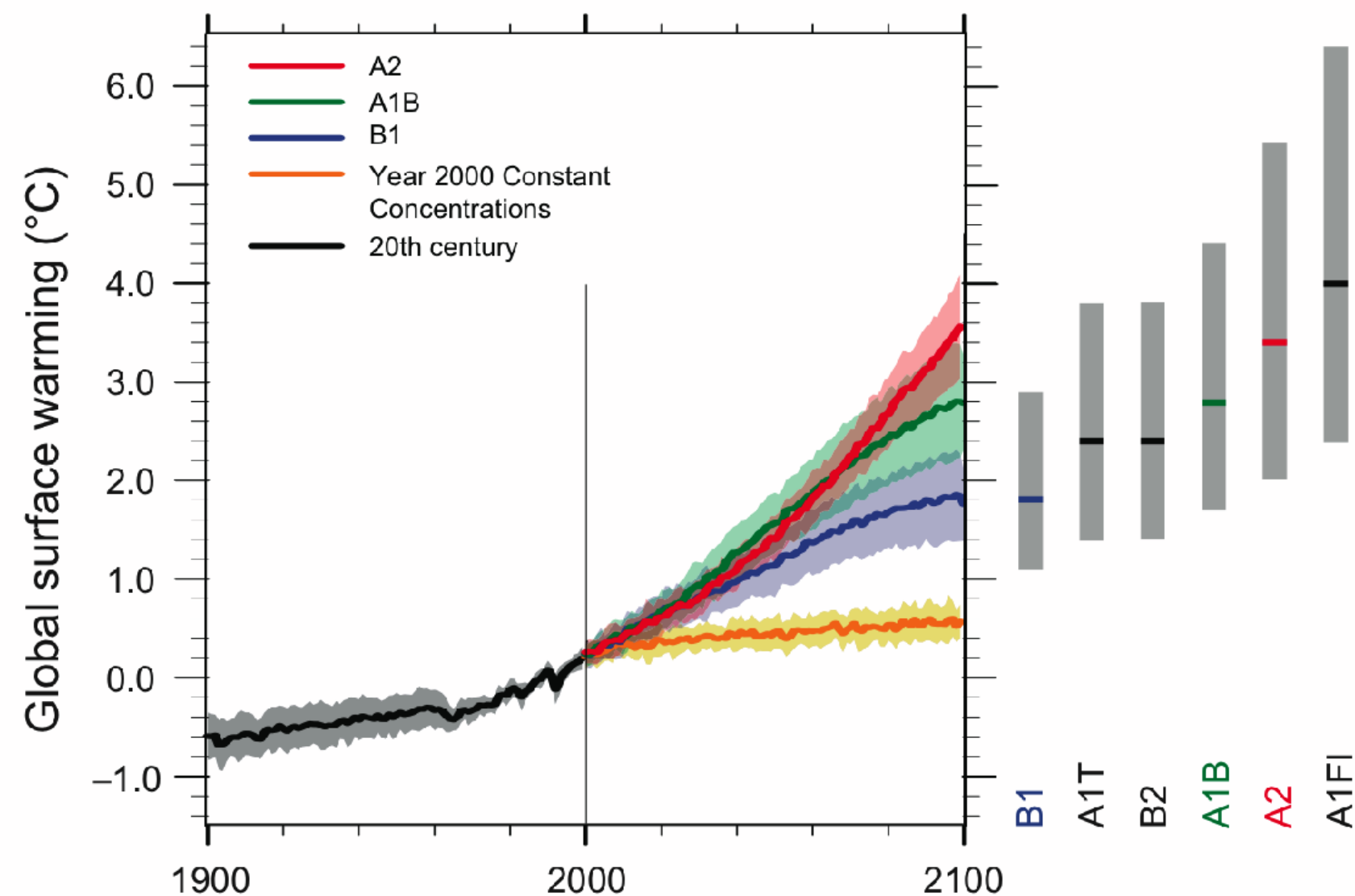


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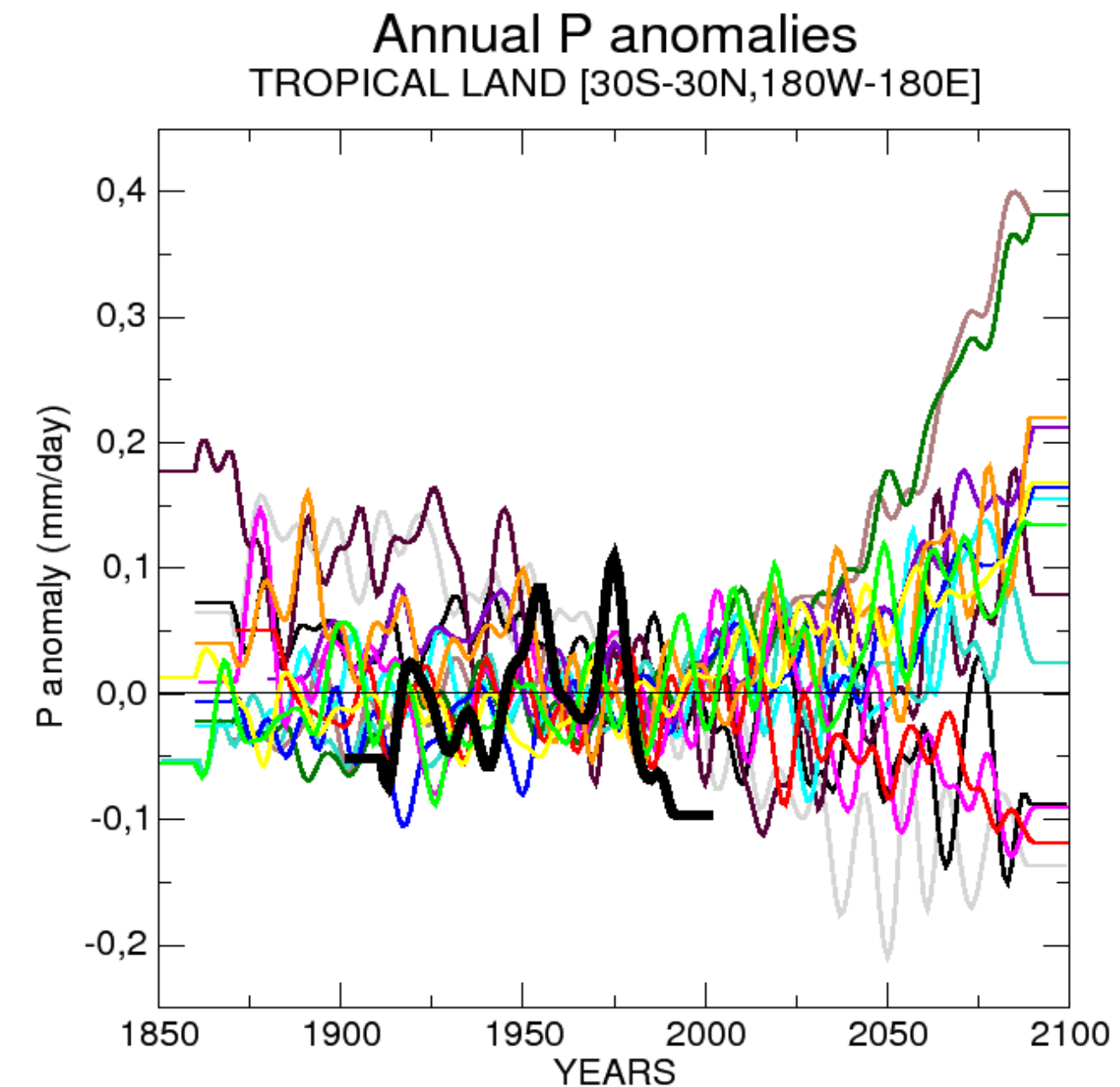
# ...in the context of climate change



- How to achieve resilient cities and mobility?
- Which strategies of reduction/adaptation to climate change?
- How to downscale climate scenarios to relevant scales?

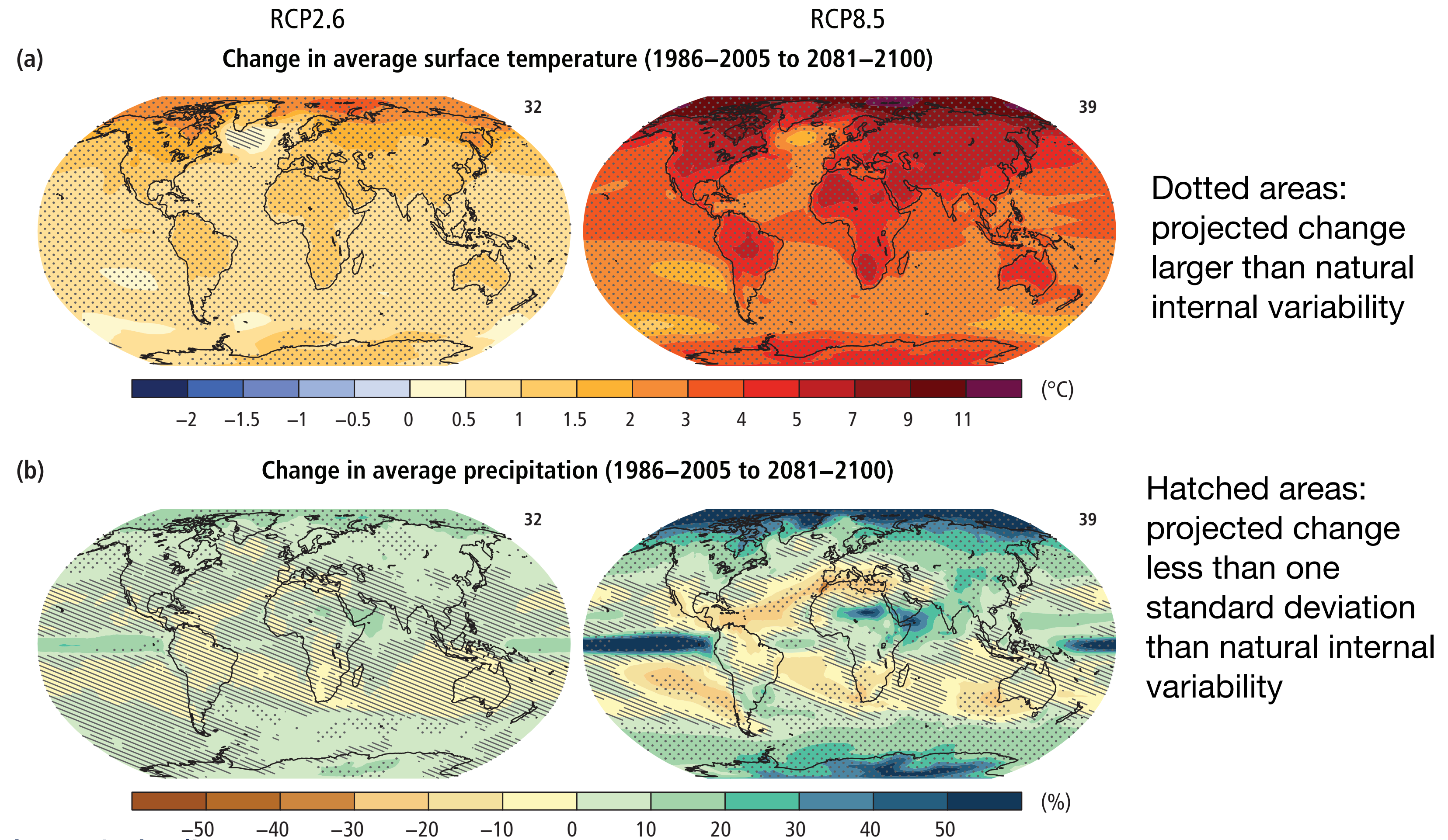


Strong agreement of models  
on temperature increases...



but strong disagreement on the  
evolution  
of precipitation extremes





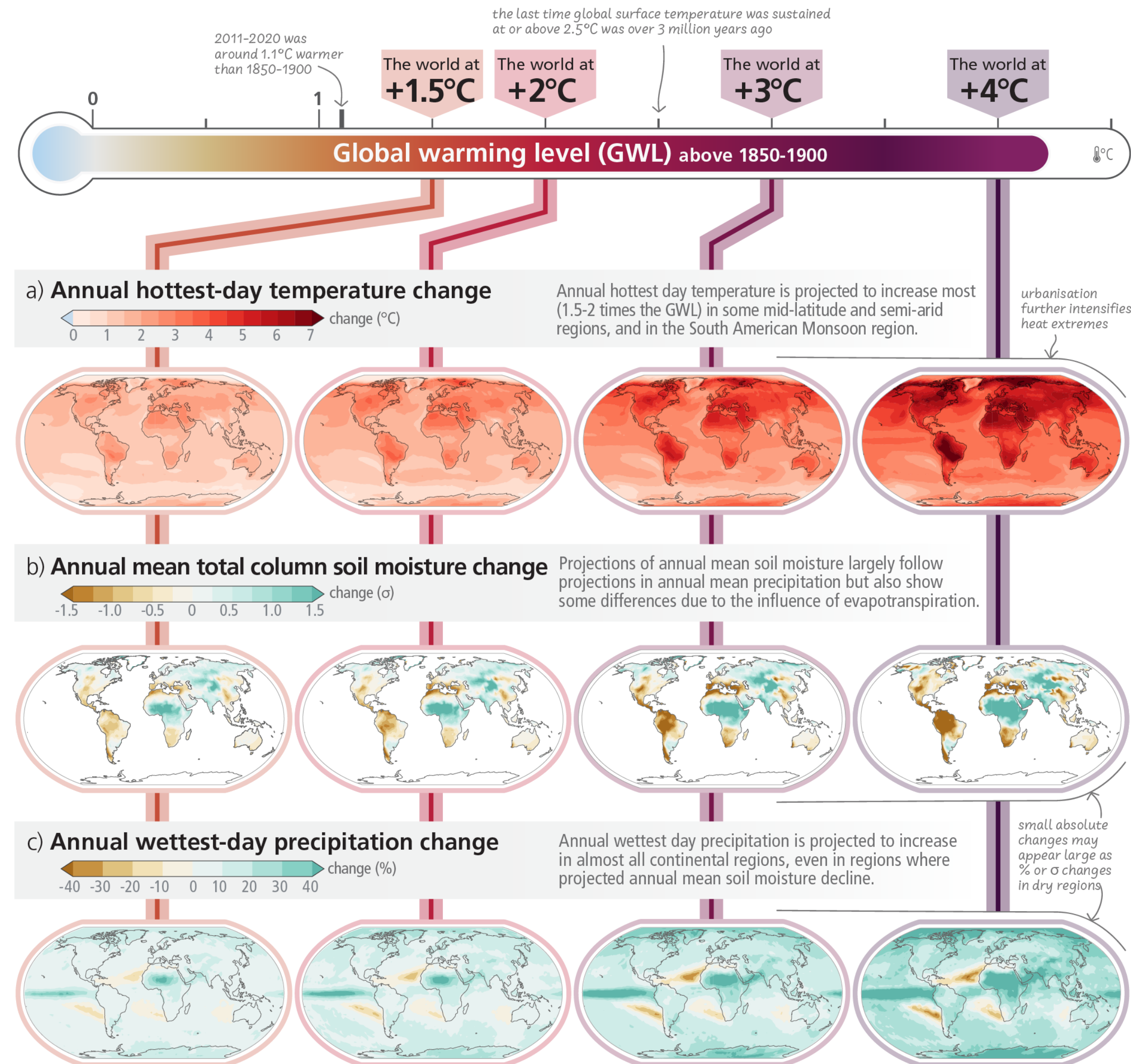
## SPM 2.2 Projected changes in the climate system

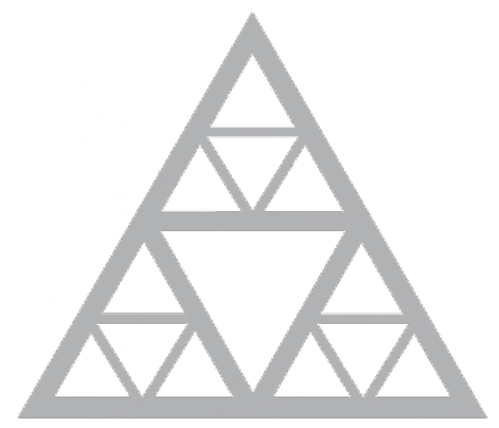
Surface temperature is projected to rise over the 21st century under all assessed emission scenarios. It is *very likely* that heat waves will occur more often and last longer, and that extreme precipitation events will become more intense and frequent in many regions. The ocean will continue to warm and acidify, and global mean sea level to rise. {2.2}

**Figure SPM.7 |** Change in average surface temperature (a) and change in average precipitation (b) based on multi-model mean projections for 2081–2100 relative to 1986–2005 under the RCP2.6 (left) and RCP8.5 (right) scenarios.

# IPCC AR6

With every increment of global warming, regional changes in mean climate and extremes become more widespread and pronounced





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# Two centuries ... since Navier (1822)



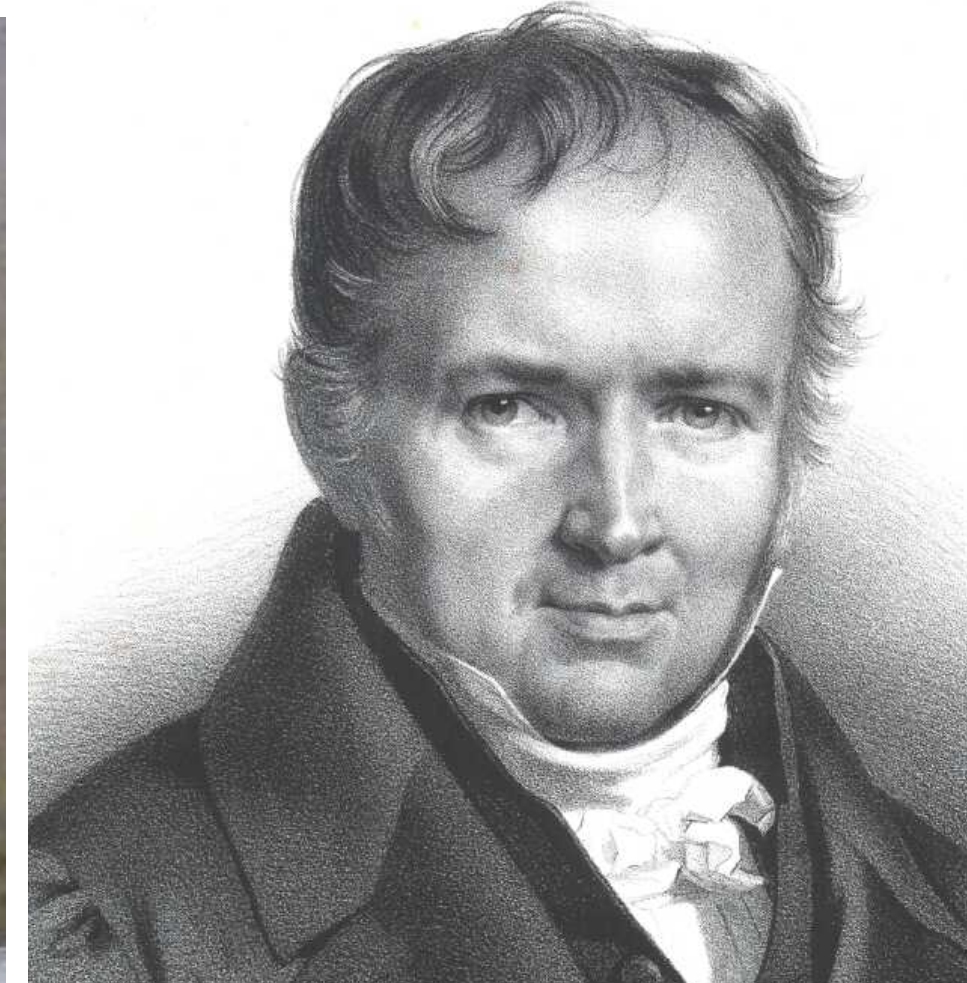
Louis Navier  
(1822)



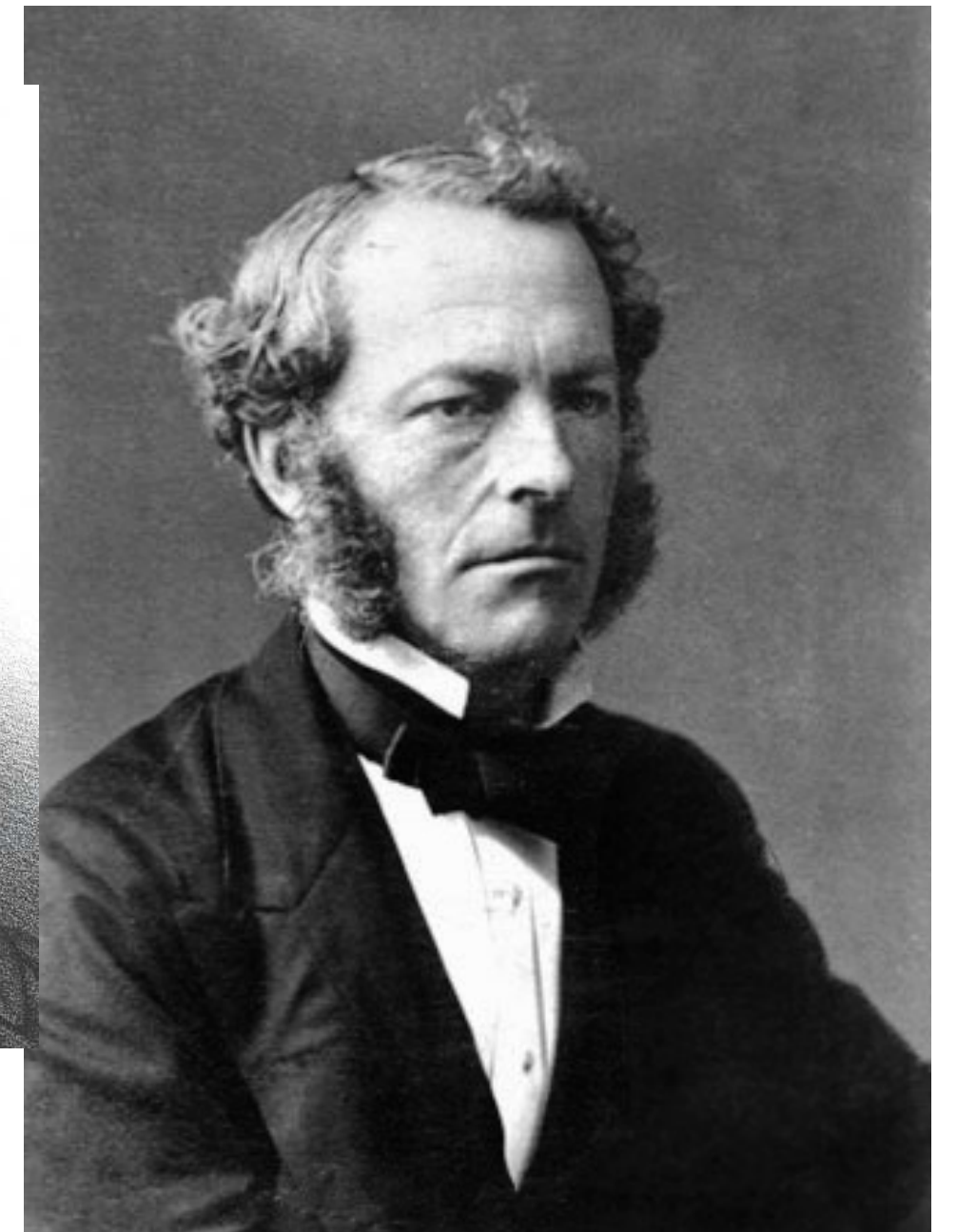
Augustin-Louis  
Cauchy



Adhémar Jean Claude  
Barré de Saint-Venant



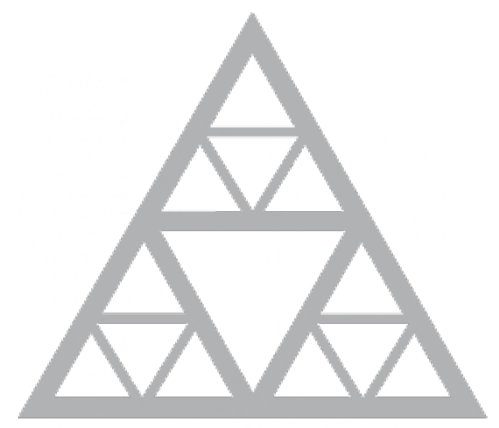
Simeon Denis  
Poisson



Sir George Stokes  
(1843)

The fundamental equation of fluid dynamics derived at Ecole Nationale des Ponts et Chaussées



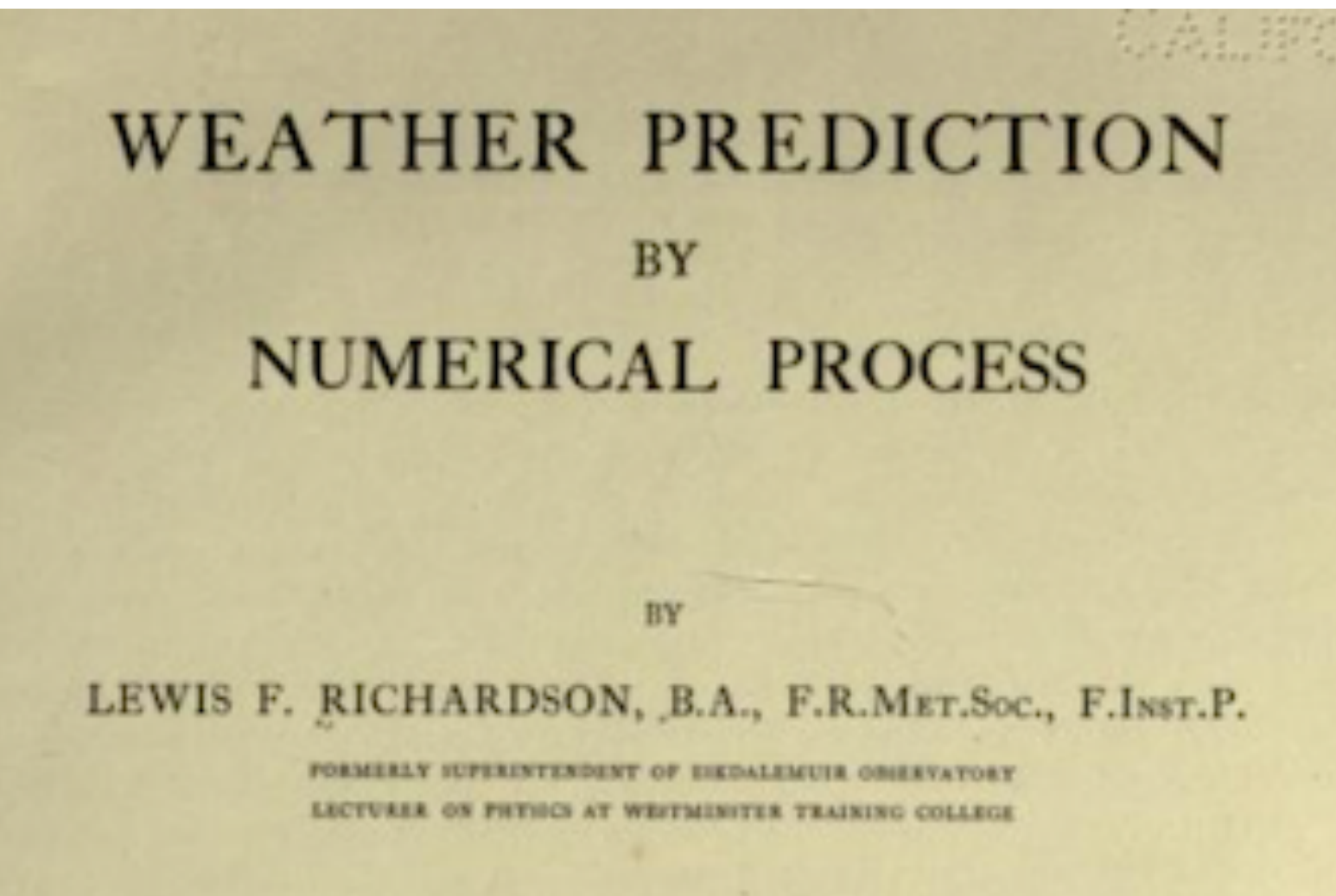


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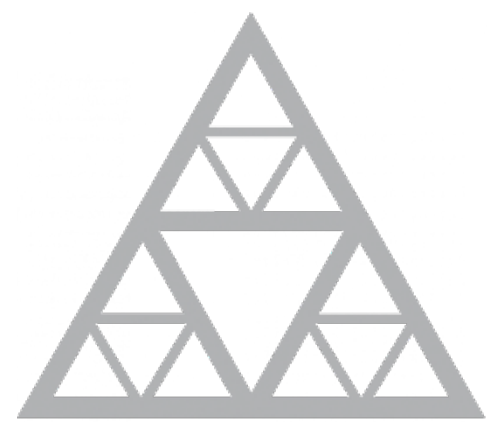
# Lewis Fry Richardson's "dream" (1922)



64.000 « human computers» synchronised by a « coordinator »



(François Schuiten, 2000)



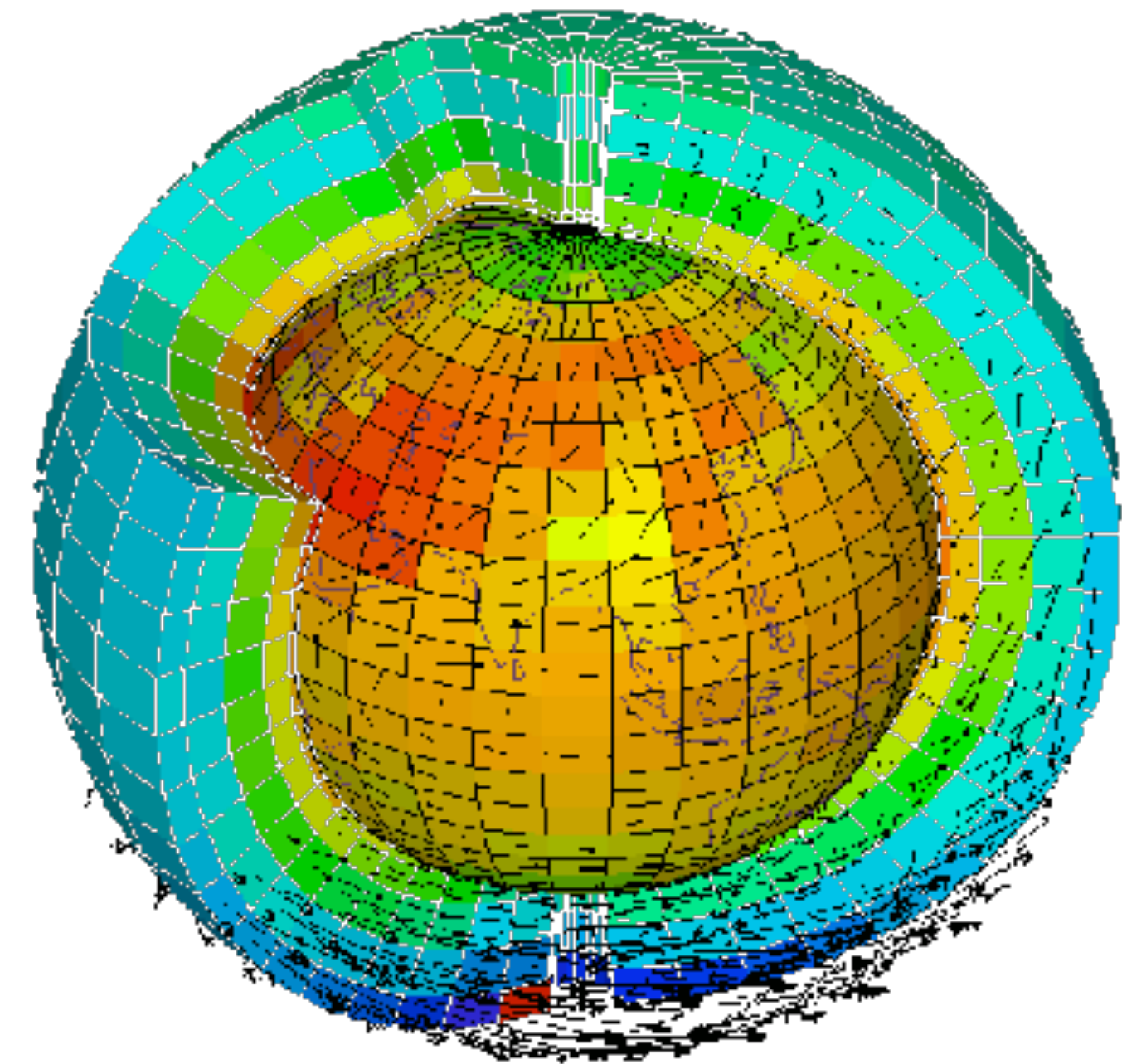
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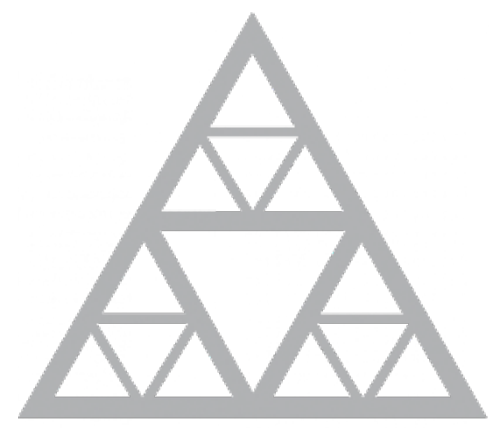
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# Modelling with brute force?



- Multiscale complexity vs. computing brute force
  - (nonlinear) equations discretised on  $N$  voxels ( $\approx$  cubes)
  - # voxels ( $1\text{mm}^3$ ) to reach the **viscous scale** ( $\approx 1\text{mm}$ ):
    - atmosphere  $\approx 10\text{ km}$  high x  $(10,000\text{ km})^2$  horizontal
    - $\Rightarrow N \approx 10^7(10^{10})^2 = 10^{27}$
    - much larger than  $N_A = 10^{23}$  !!
- Why to be deterministic... over only a small range of scales?





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# Two centuries ... since Navier (1822)



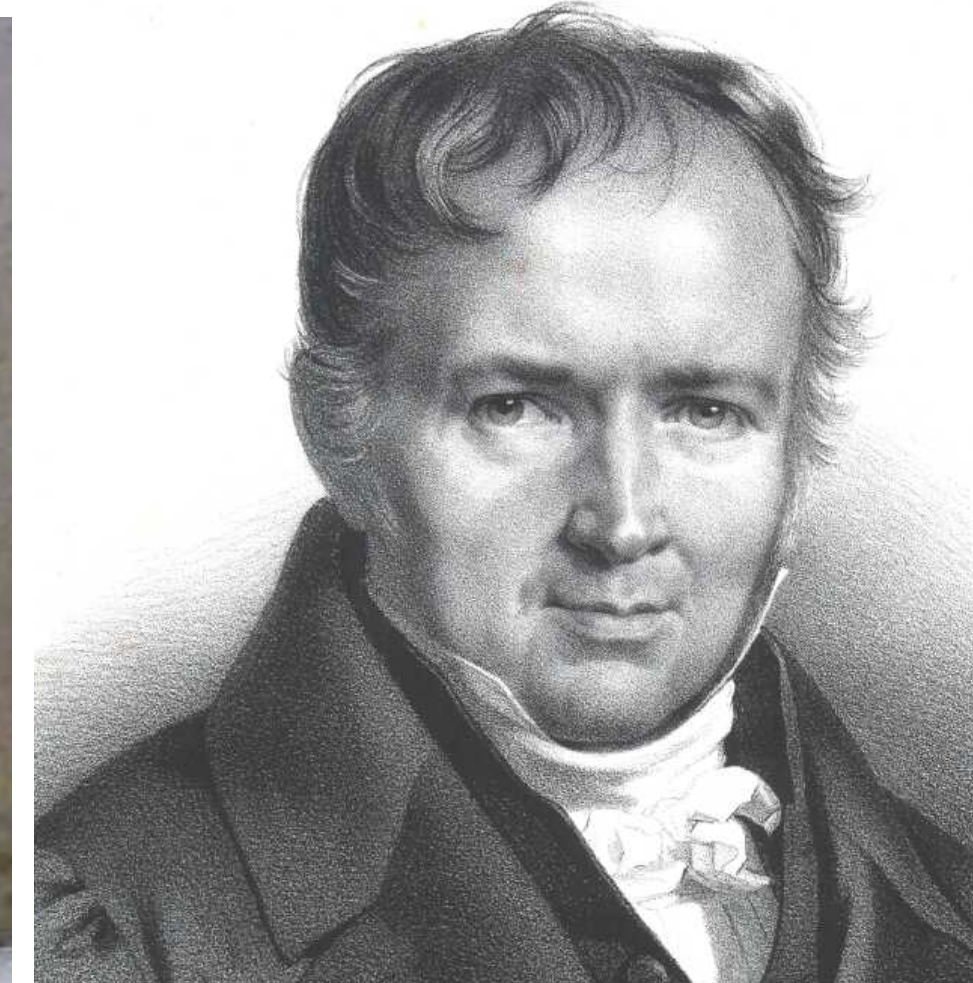
Louis Navier  
(1822)



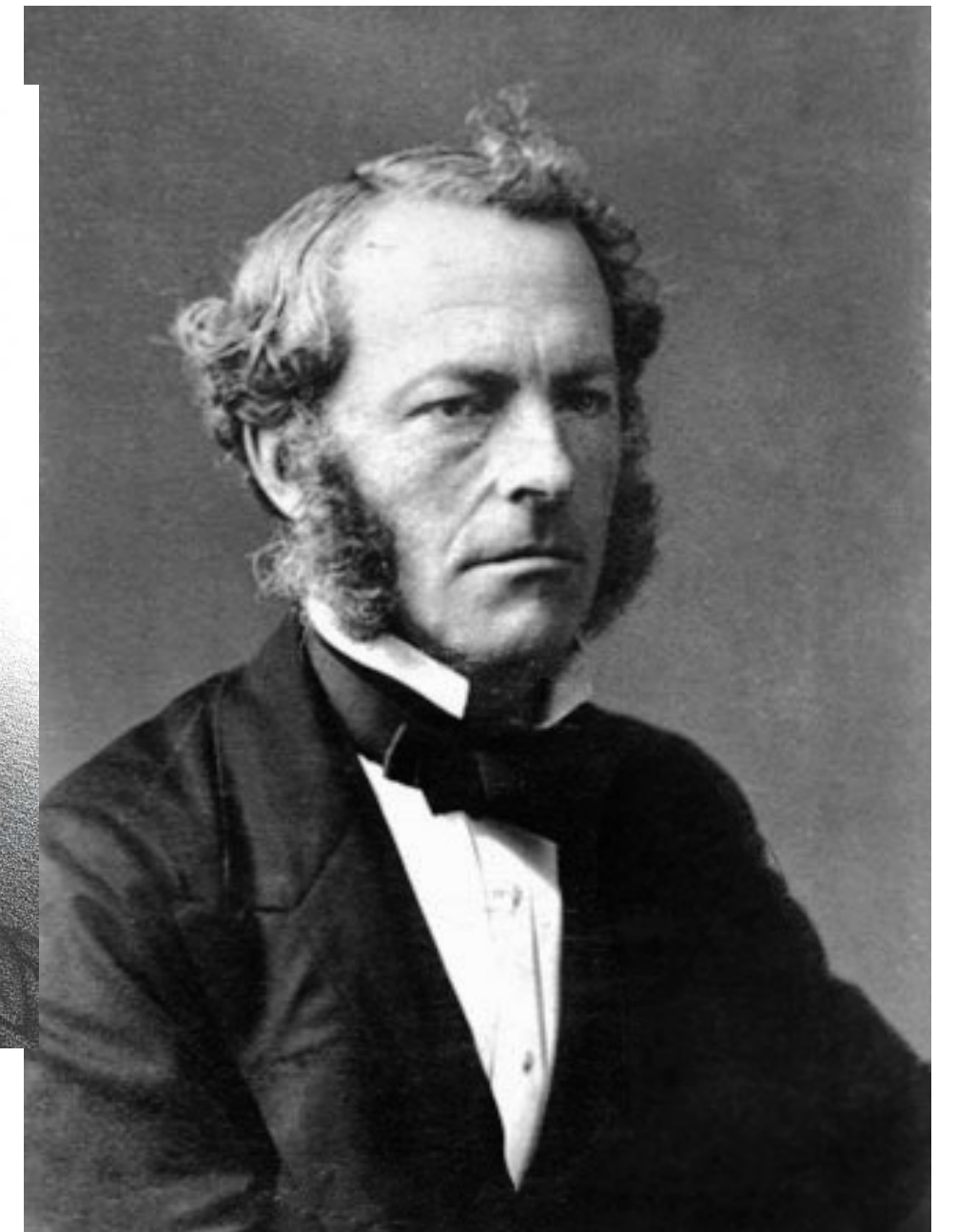
Augustin-Louis  
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Barré de Saint-Venant



Simeon Denis  
Poisson

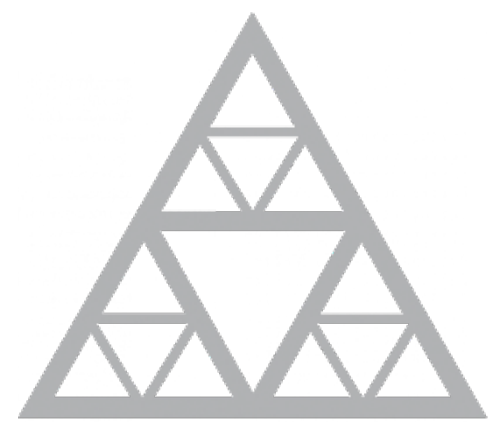


Sir George Stokes  
(1843)



A **millenium problem** raised at Ecole Nationale des Ponts et Chaussées, recent episodes:  
Otelbaev (2013) and Tao (2015)





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# A century of cascades !



WEATHER PREDICTION  
BY  
NUMERICAL PROCESS

BY  
LEWIS F. RICHARDSON, B.A., F.R.MET.SOC., F.INST.P.  
FORMERLY SUPERINTENDENT OF SHEDDALEMUR OBSERVATORY  
LECTURER ON PHYSICS AT WESTMINSTER TRAINING COLLEGE

the sketch can be completed. We realize thus that: big whirls have little whirls that feed on their velocity, and little whirls have lesser whirls and so on to viscosity—in the molecular sense.

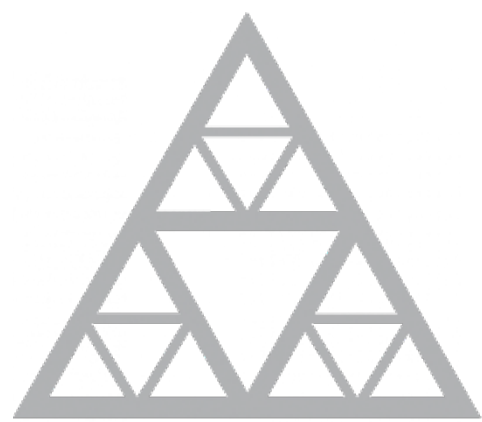
*The Supply of Energy from and to Atmospheric Eddies.*

By LEWIS F. RICHARDSON.

(Communicated by Sir Napier Shaw, F.R.S. Received March 9, 1920.)

*Introduction.*

This paper extends Osborne Reynold's theory of the Criterion of Turbulence, to make it apply to the case in which work is done by the eddies, acting as thermodynamic engines in a gravitating atmosphere. For



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# Scale symmetry and equations



Whereas elementary mathematical properties of Navier-Stokes solutions are still unknown (existence, uniqueness):

$$\frac{\partial \underline{u}}{\partial t} + \underline{u} \cdot \underline{grad}(\underline{u}) = \underline{f} - \frac{1}{\rho} \underline{grad}(p) + \nu \Delta \underline{u}$$

one can point out a **scale symmetry** (\*):  $T_\lambda: \underline{x} \mapsto \underline{x}/\lambda$

$$\tilde{T}_\lambda: \begin{aligned} t &\mapsto t/\lambda^{1-\gamma}; \underline{u} \mapsto \underline{u}/\lambda^\gamma; \\ \underline{f} &\mapsto \underline{f}/\lambda^{2\gamma-1}; \nu \mapsto \nu/\lambda^{1+\gamma}; \rho \mapsto \rho/\lambda^{\gamma'} \end{aligned}$$

Kolmogorov's scaling (K41)  
obtained with:

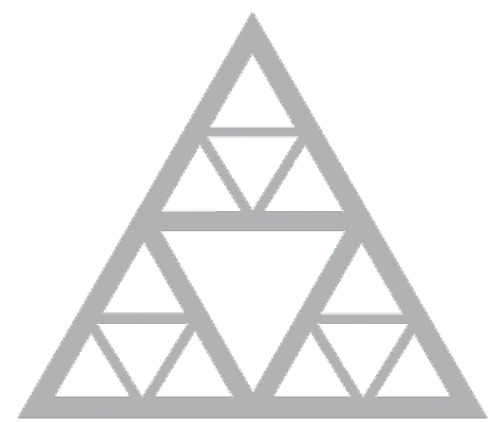
$$\varepsilon(\ell) \approx \frac{\delta u^3}{\ell} \approx \bar{\varepsilon} \Rightarrow \gamma_K = 1/3$$

General case: multiple singularities  $\gamma$ 's:  $\Pr(\gamma' > \gamma) \approx \lambda^{-c(\gamma)}$

(\*) from to self-similarity (Sedov, 1961), symmetry (Parisi +Frisch, 1985),  
to generalised Galilean invariance (S+al, 2010)

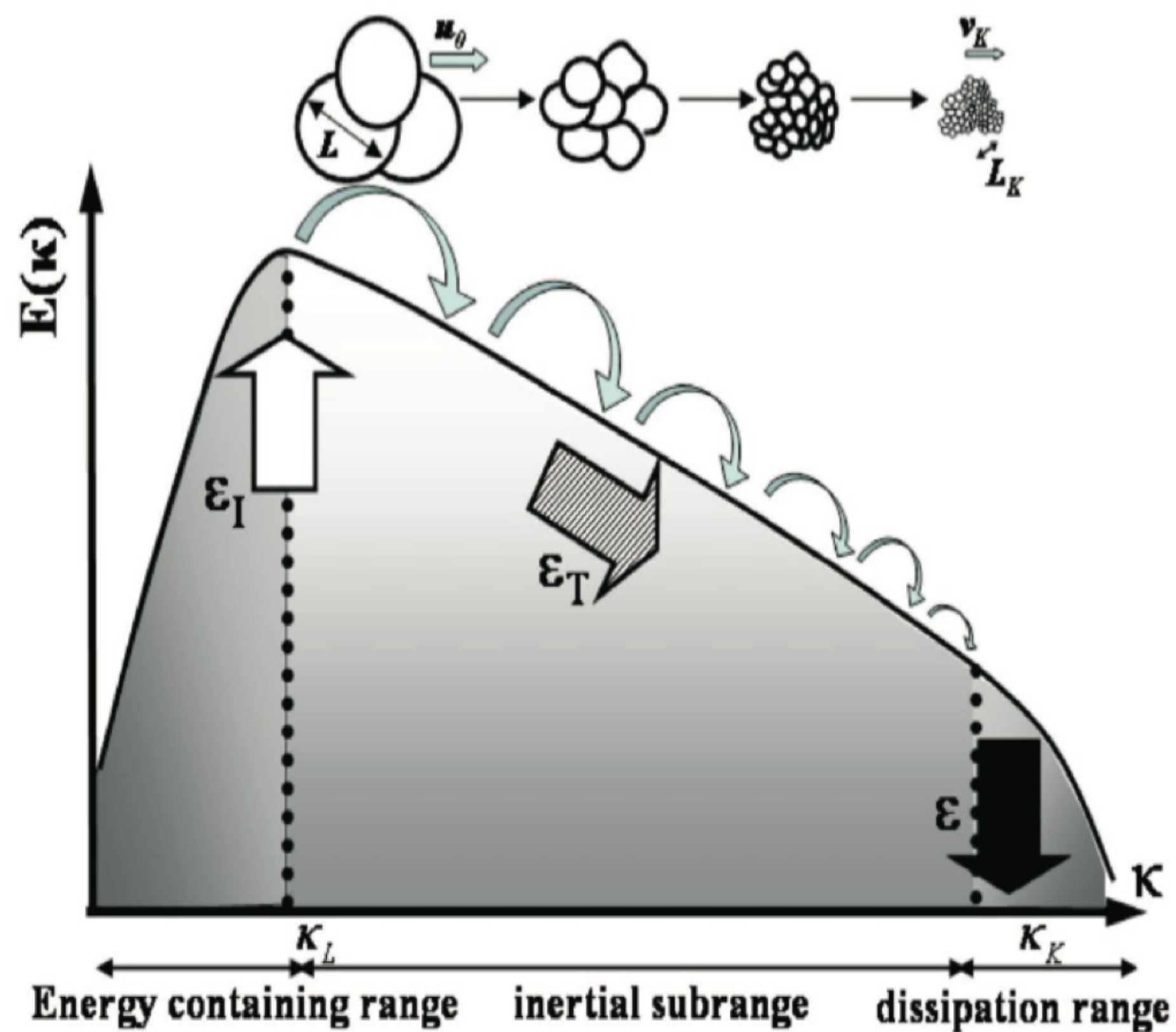






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# Symmetry in the Fourier space



$$\underline{x} \mapsto \underline{x}/\lambda; \quad \underline{u} \mapsto \underline{u}/\lambda^\gamma$$

$$\hat{u}(\underline{k}) = \int_{-\infty}^{\infty} d\underline{x} \exp(i\underline{k} \cdot \underline{x}) \underline{u}(\underline{x})$$

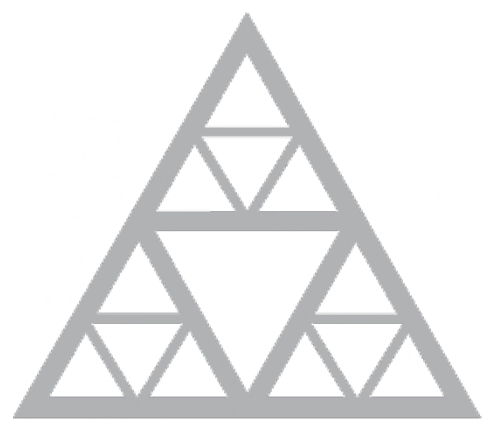
$$\underline{k} \mapsto \underline{k}\lambda; \quad \hat{u} \mapsto \hat{u}/\lambda^{\hat{\gamma}}$$

$$\Rightarrow \hat{\gamma} = d + \gamma$$

Spectral scaling:  $\langle u^2 \rangle = \int_0^\infty dk E(k)$

$$E(k) \propto k^{-2\gamma} k^{-1} = k^{-5/3}$$

Kolmogorov-Obukhov spectrum (K41)



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# Scale symmetry and equations



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# From scaling analysis to scalar cascade processes

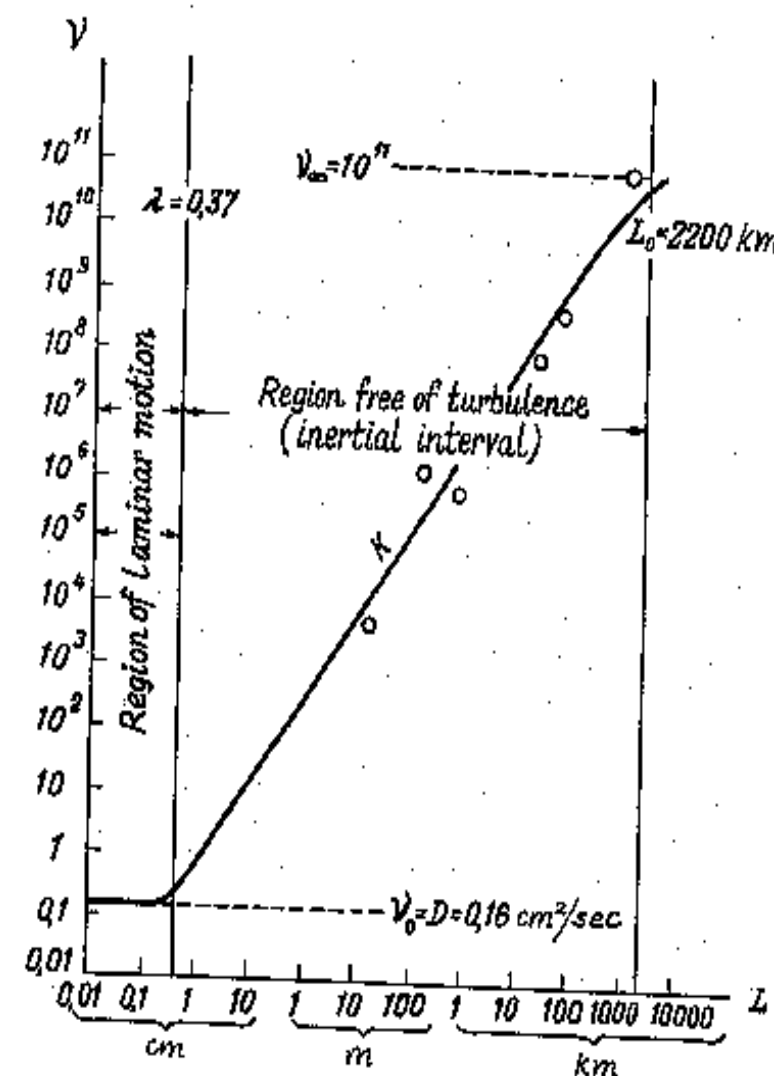
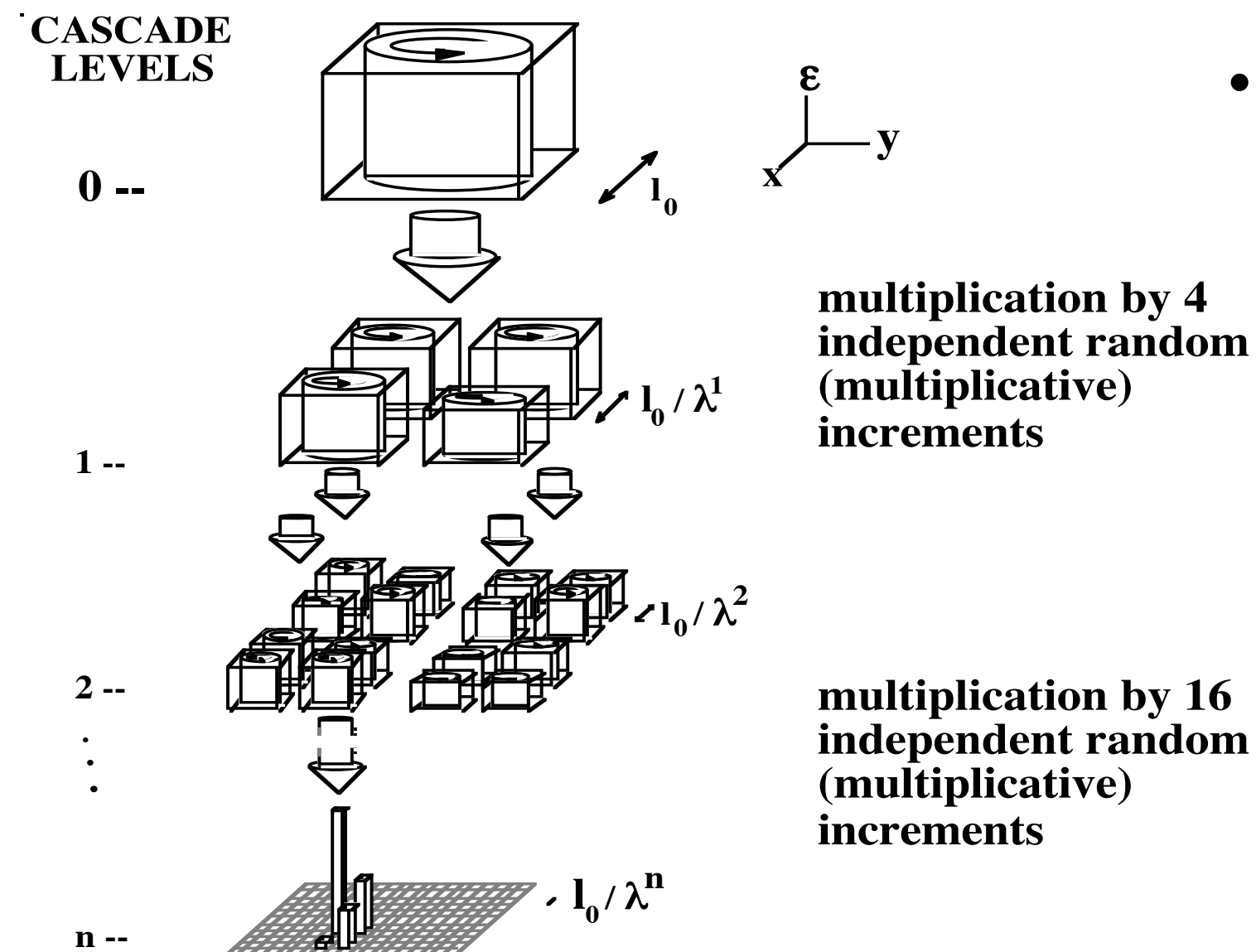


Fig. 1 The vertical diffusion coefficient  $v(L)$  as a function of the turbulence scale  $L$ . Empirical points after Richardson.<sup>19</sup>

Richardson, 1926



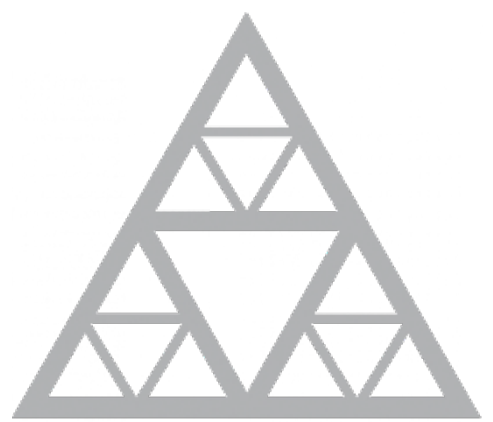
Schertzer & Lovejoy, 1989b

## Scaling analysis

- Passive scalar dispersion: Richardson (1926)
- Structure function: Kolmogorov (1941)
- Energy spectrum: Obukhov (1941)
- Higher order structure functions: Kolmogorov (1962), Obukhov (1962)
- Renyi dimensions: Grassberger and Procaccia (1983), Hentschel, and Procaccia (1983)
- Legendre transform to dimensions: Parisi and Frisch (1985)
- Fractal measures: Halsey et al. (1987)
- ....

## Cascade processes

- $\beta$ -model: Novikov and Stewart (1964), Frisch et al. (1978)
- Log normal model: Yaglom (1966)
- Limit log-normal: Mandelbrot (1974)
- $\alpha$ -model: S+L (1984, 1985)
- Multiplicative chaos: Kahane (1985)
- Universal multifractals/Levy multiplicative chaos: S+L (1987a&,b, 1997)), Fan (1987)
- Log-Poisson model: Dubrulle (1994)
- ...

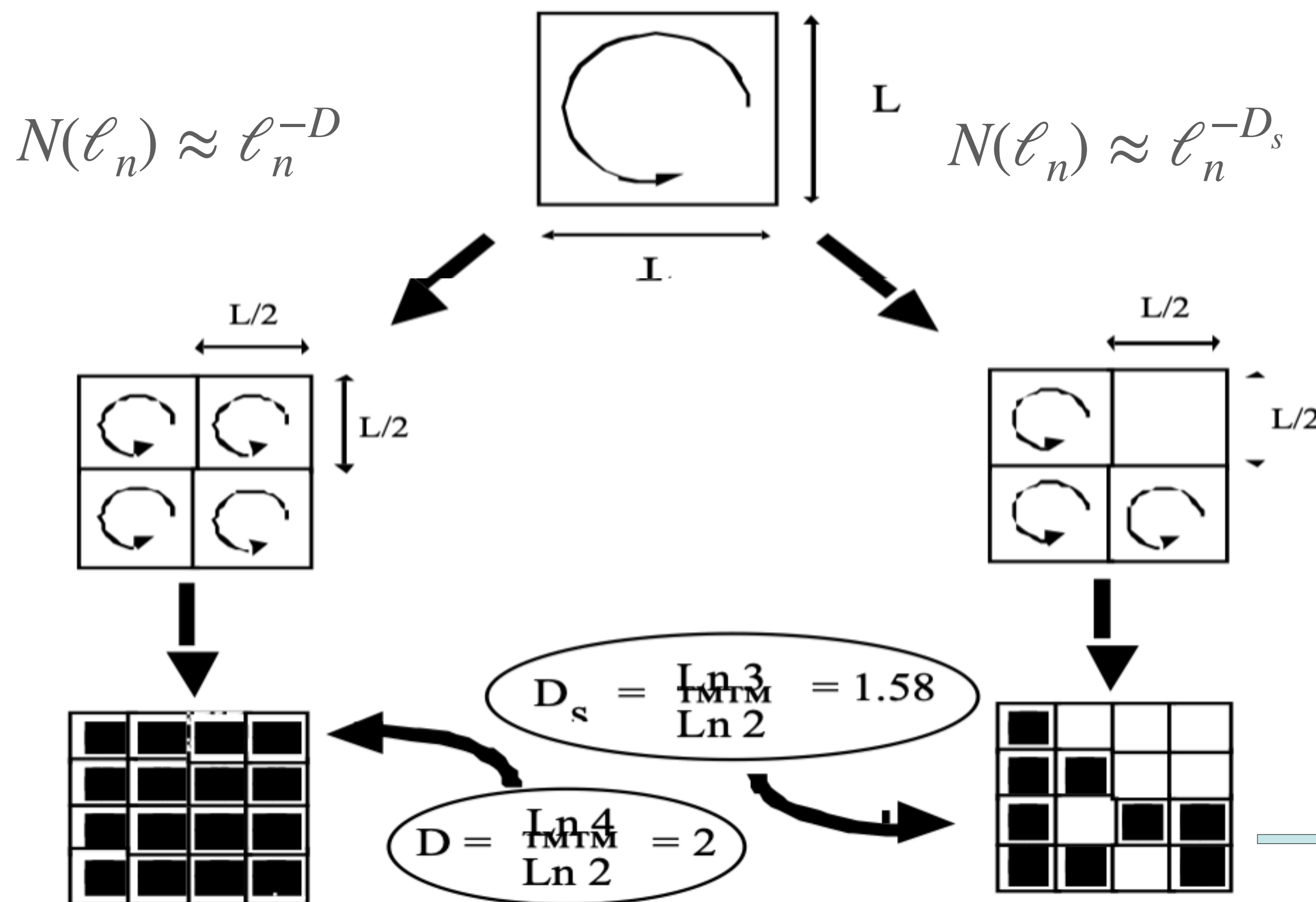


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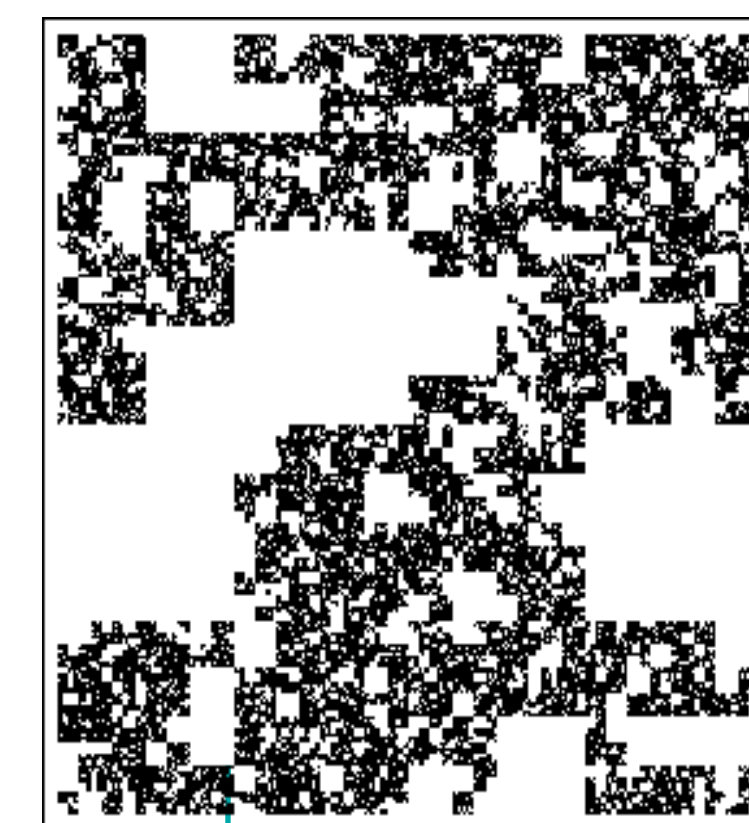
# $\beta$ -model

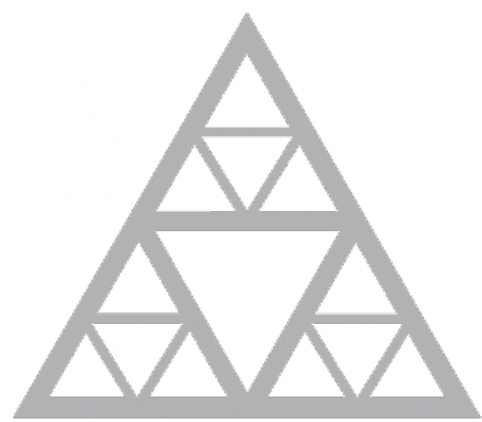


Non-intermittent  
("homogeneous")  
cascade



$\beta$ -model of  
intermittency :  
sub-eddies are  
either "alive" or  
"dead"





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# Varenna summer school (1983)

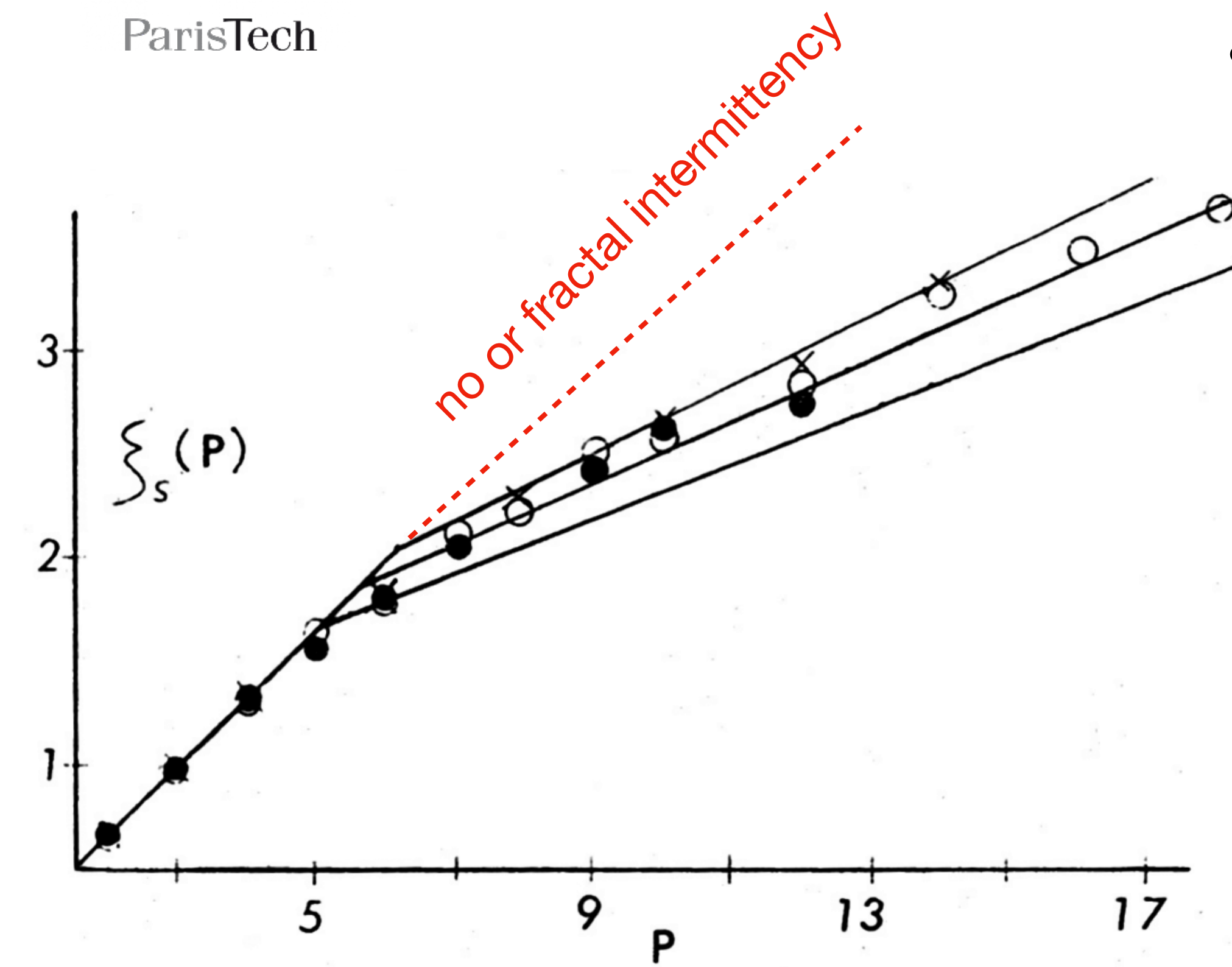
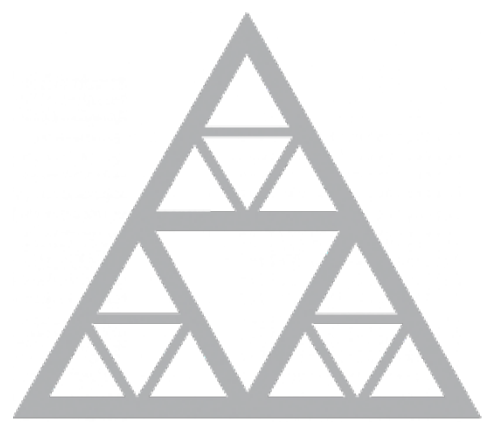


Fig.1 S+L (1984) fitted the  $\alpha$ -model to the experimental data of Anselmet et al (1983)

- “Turbulence and Predictability in Geophysical Fluid Dynamics” organised by M. Ghil, R. Benzi et G. Parisi
  - a primary version of the multifractal formalism of Parisi and Frisch (1985) that concluded: **“Still the multifractal model appears to be somewhat more restrictive than Mandelbrot’s weighted-curdling model which does include the lognormal case”**.
  - S+L’s short presentation (Varenna proceedings (1985)) refers to S+L (1984):
    - a small perturbation of the  $\beta$ -model is **no longer limited to a unique dimension** ( $\alpha$ -model)
    - the **divergence of higher order moments is rather generic**
    - it introduces spurious scaling, an analytical approximation depending on a unique scaling exponent  $H$  and the critical order  $\alpha$  was proposed:
$$\xi(p) = pH + \theta(p - \alpha)(1 - p/\alpha)$$
  - it was shown to fit the experimental points from Anselmet et al. (1983), see fig. 1 with  $H = 1/3, \alpha = 5, 5.5, 6$



# Multiplicity of anomalous scaling laws

- Example of Kolmogorov's refined scaling laws (K62,  $0 \leq q \leq q_D$ ):

$$S_q(r) := \langle \delta_r u^q \rangle \propto \bar{\epsilon}^{q/3} r^{q/3} (r/L)^{C_1(q/3-1)q/3}$$

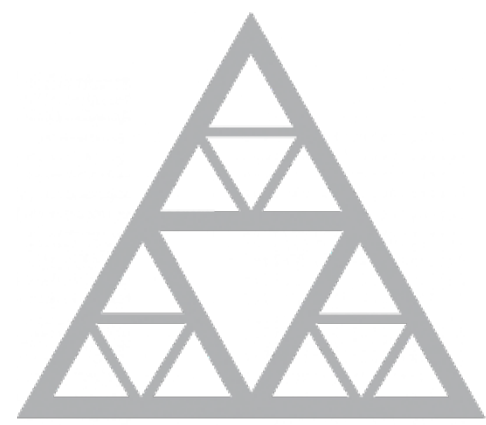
Structure function of order  $q$

$$\delta_r u := u(x+r) - u(x)$$

Normal/dimensional scaling (K41)  
unique law  
(linear exponent wrt  $q$ )

Anomalous/adimensional scaling  
multiple laws  
(nonlinear exponent wrt  $q$ )

refined scaling (K62)



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# Multifractal Fields



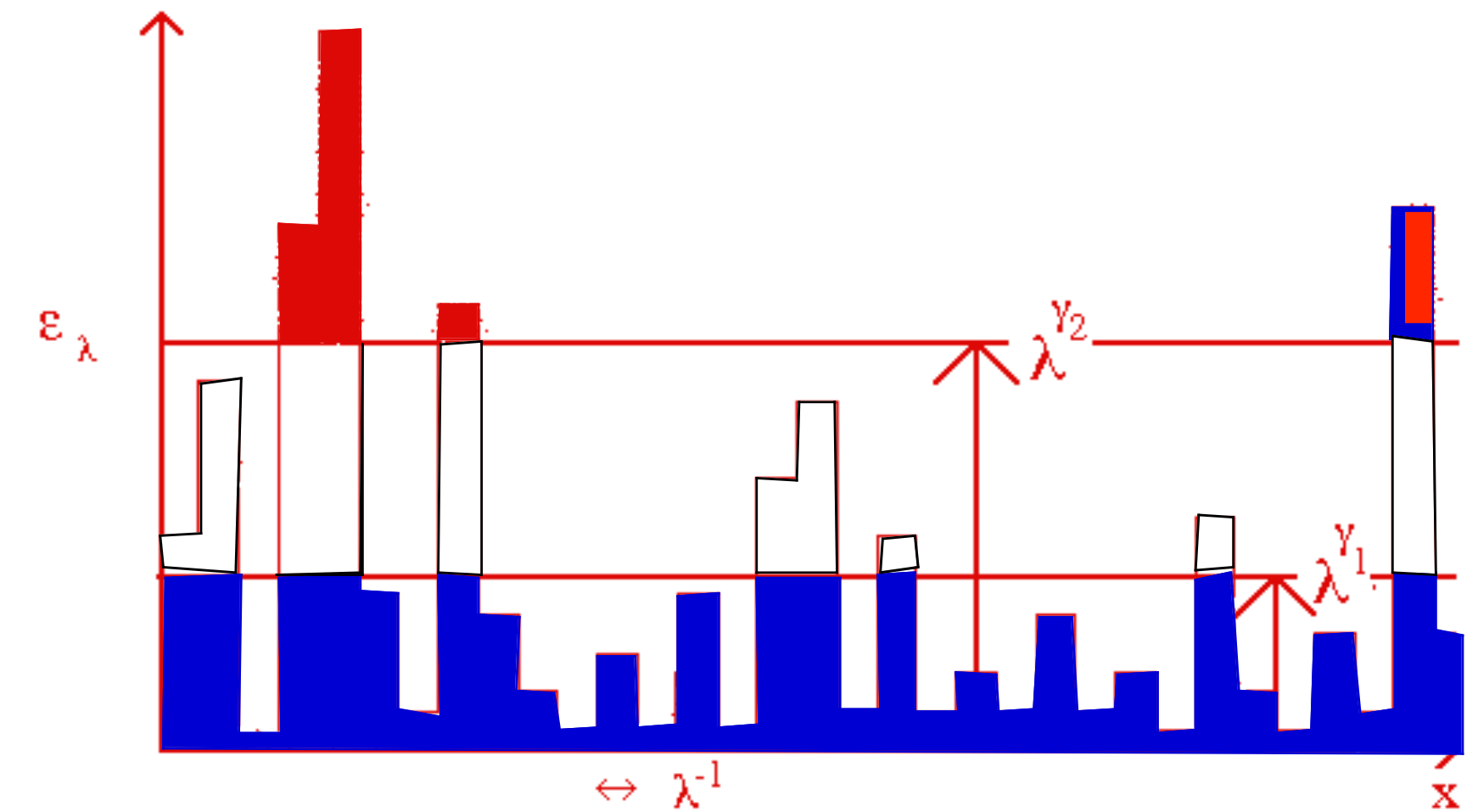
**Measure of the Fraction** at resolution  $\lambda$   
( $\ell = L/\lambda$ ) of the **Probability Space**  
corresponding to the event  $\{\epsilon_\lambda \geq \lambda^\gamma \bar{\epsilon}\}$ :

$$\Pr(\epsilon_\lambda \geq \lambda^\gamma \bar{\epsilon}) \approx \lambda^{-c(\gamma)}$$

$c(\gamma)$  is a codimension,  $\gamma$  a singularity

(Schertzer and Lovejoy, 1987)

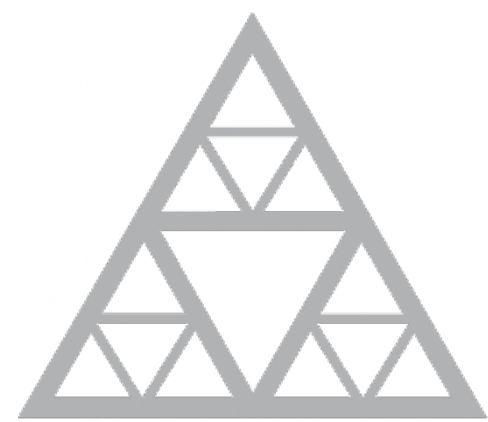
or a 'Cramer's function' (Mandelbrot, 1991)



high singularities

medium singularities

low singularities



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# Cascades and statistical physics



Transformation of a measure  $\sigma$  with the help of a “density”  $\varepsilon$  into another measure  $\Pi$  :

$$d\Pi = \varepsilon d\sigma$$

Generalisation with a non trivial limit  $\varepsilon$  of densities  $\varepsilon_\lambda$  of increasing resolution:  $\lambda = L/\ell \rightarrow \infty$

$$\varepsilon_\lambda = e^{\Gamma_\lambda}; \mathbb{E}e^{q\Gamma_\lambda} = Z_\lambda(q) = e^{K_\lambda(q)} \approx \lambda^{K(q)} \xleftrightarrow{\text{Mellin}} \mathbb{P}(\varepsilon_\lambda > \lambda^\gamma) \approx \lambda^{-c(\gamma)}$$



Legendre

$$K(q) \longleftrightarrow c(\gamma)$$

main “trick”:

log-divergence of the generator

## Cascades Statistical physics analogues

$\Gamma$ : generator  $\approx$  hamiltonian

$q$  : statistical order  $\approx$  inverse of temperature

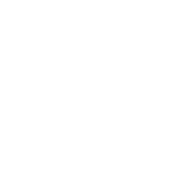
$Z$ : 1st characteristic or moment generating function  $\approx$  partition function

$K$ : 2nd characteristic or cumulant generating function  $\approx$  Gibbs free energy

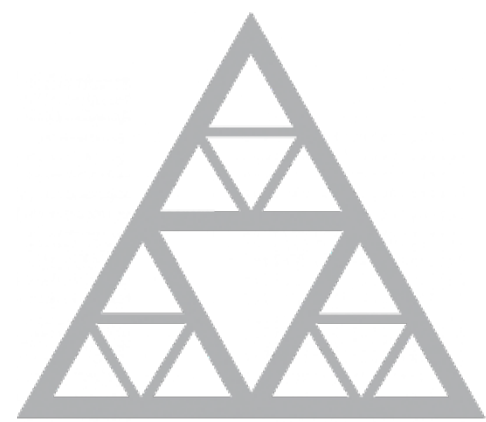
$c$ : codimension or Kramer function  $\approx$  entropy

$\gamma$ : singularity or Hölder exponent  $\approx$  energy

$$K(q) = \max_{\gamma} [q\gamma - c(\gamma)]; c(\gamma) = \max_q [q\gamma - K(q)]$$







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# To downscale climate scenarios?



Consensus on the need to downscale simulations:

- to manage water
- below a kilometer to model cities ?

but a fundamental obstacle :

- relationships large/small scales
- *stationnary?* (« stationnarity is dead ! »)
- *Implicit hypothesis* in:

- correlations (linear or not) large/small scales or neural networks: training on « *non perturbed* » data sets;
- weather types: *stationnary basis*, only frequencies change;
- GCM: *stationnary parametrisations* of small scales...

**Alternative:**

- Analyze and take into account the *evolution of fluxes through scales!*
- Example: **rainrate** for the water cycle.



# Scaling of precipitations

## 1. Parsimonious description: only 3 exponents:

- intermittency:
  - **average intermittency**  $C_1$  : how sparse is the average rainfall?  
 $C_1 \neq 0$ : it does not rain everyday, everywhere!
  - **intermittency variability**  $\alpha$  : diversity of rainfall regimes  
 $\alpha \neq 0$  : not only the alternative rain/no rain!
- scale dependence  $H$  of the average rainfall  $\langle R_1 \rangle$  ?  
For rainfall  $H \approx 0$

### a Trivial conséquences for the extremes:

$C_1$  and  $\alpha \uparrow \Rightarrow$  extremes  $\uparrow$     or     $C_1$  and  $\alpha \downarrow \Rightarrow$  extremes  $\downarrow$

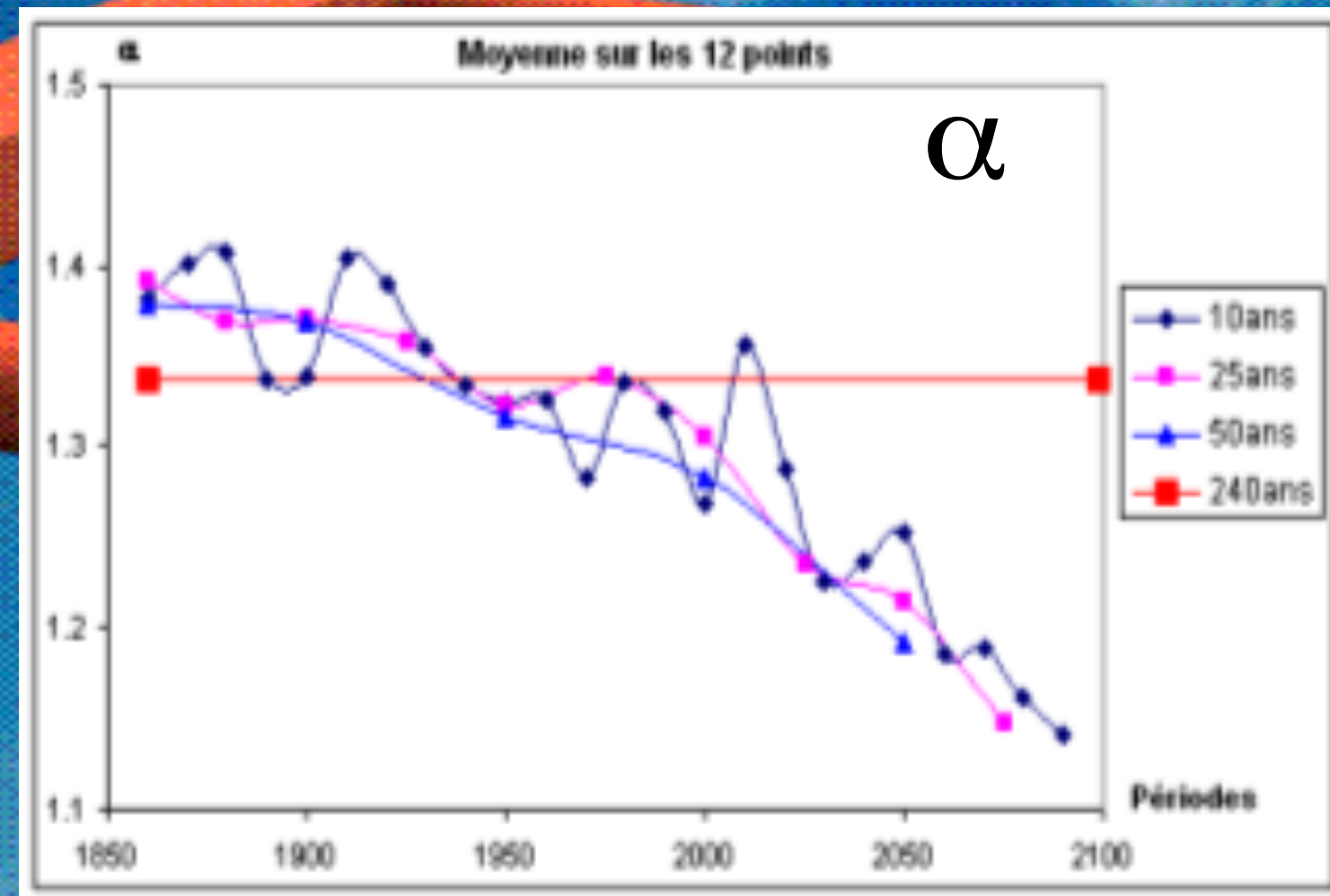
### a More generally:

- these exponents define the Intensity-Duration-Frequency (IDF) curves over a wide range of scale and intensities (Benjdoudi et al., 1987, Tchiguirinskaia et al 2010) ;
- allow to compare quantiles over various durations

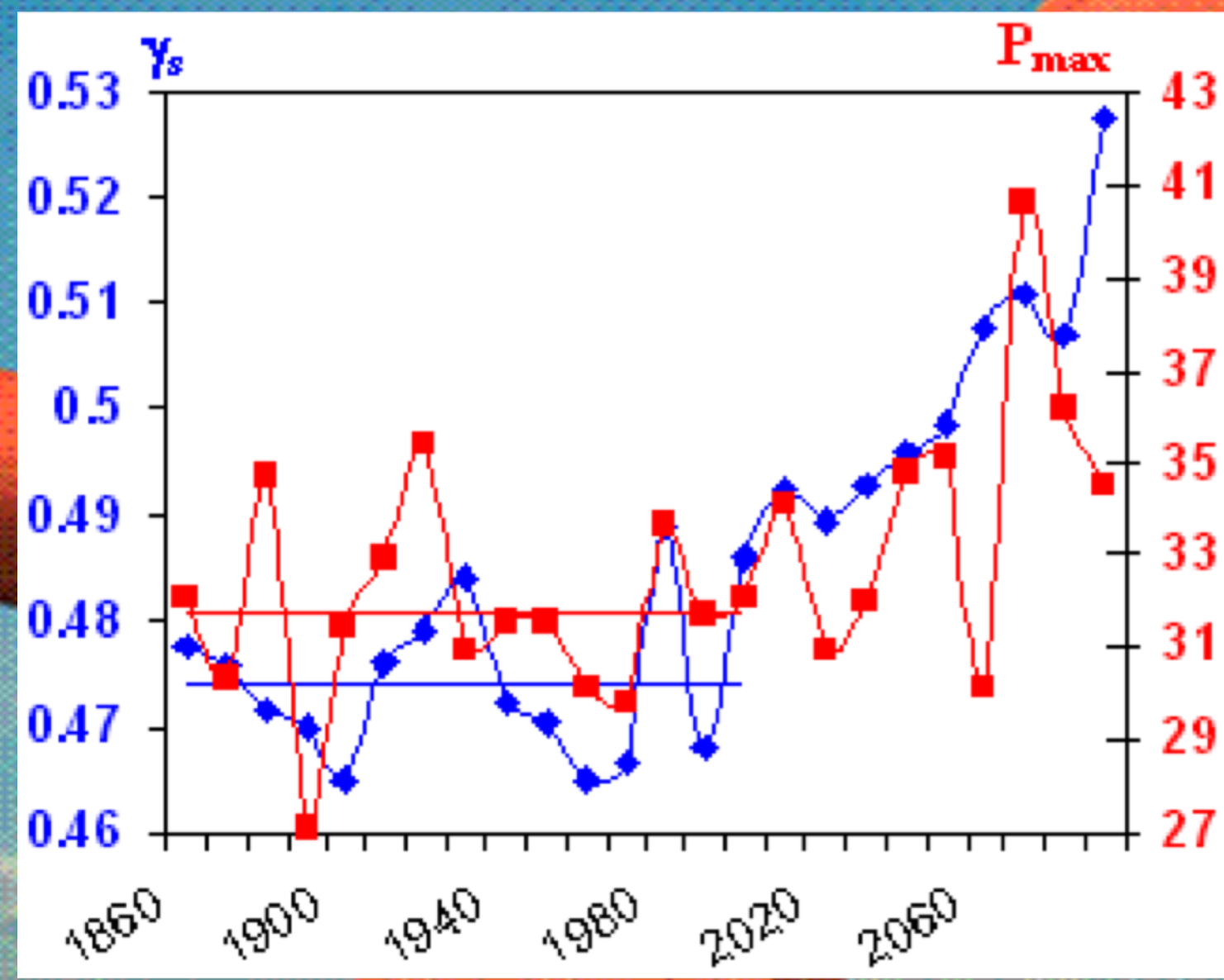
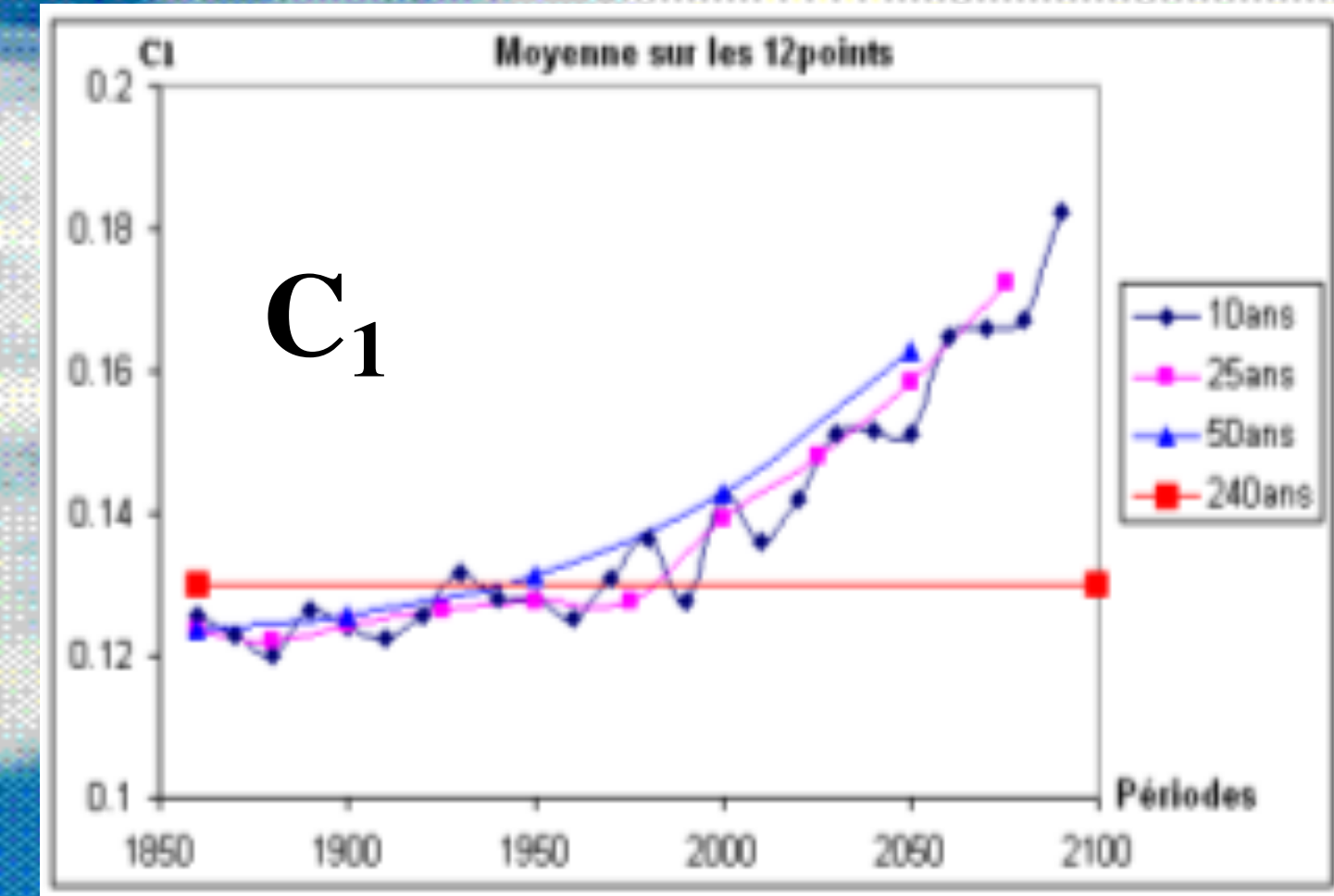


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# Analysis of the scenario A2

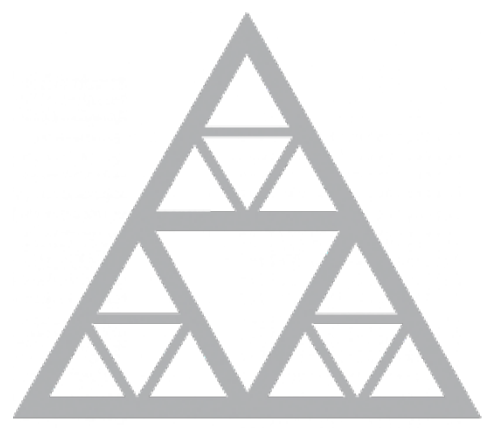


- average intermittency  $C_1 \uparrow$
- intermittency variability  $\alpha \downarrow$ ,  
=> difficulty to evaluate extremes of precipitations



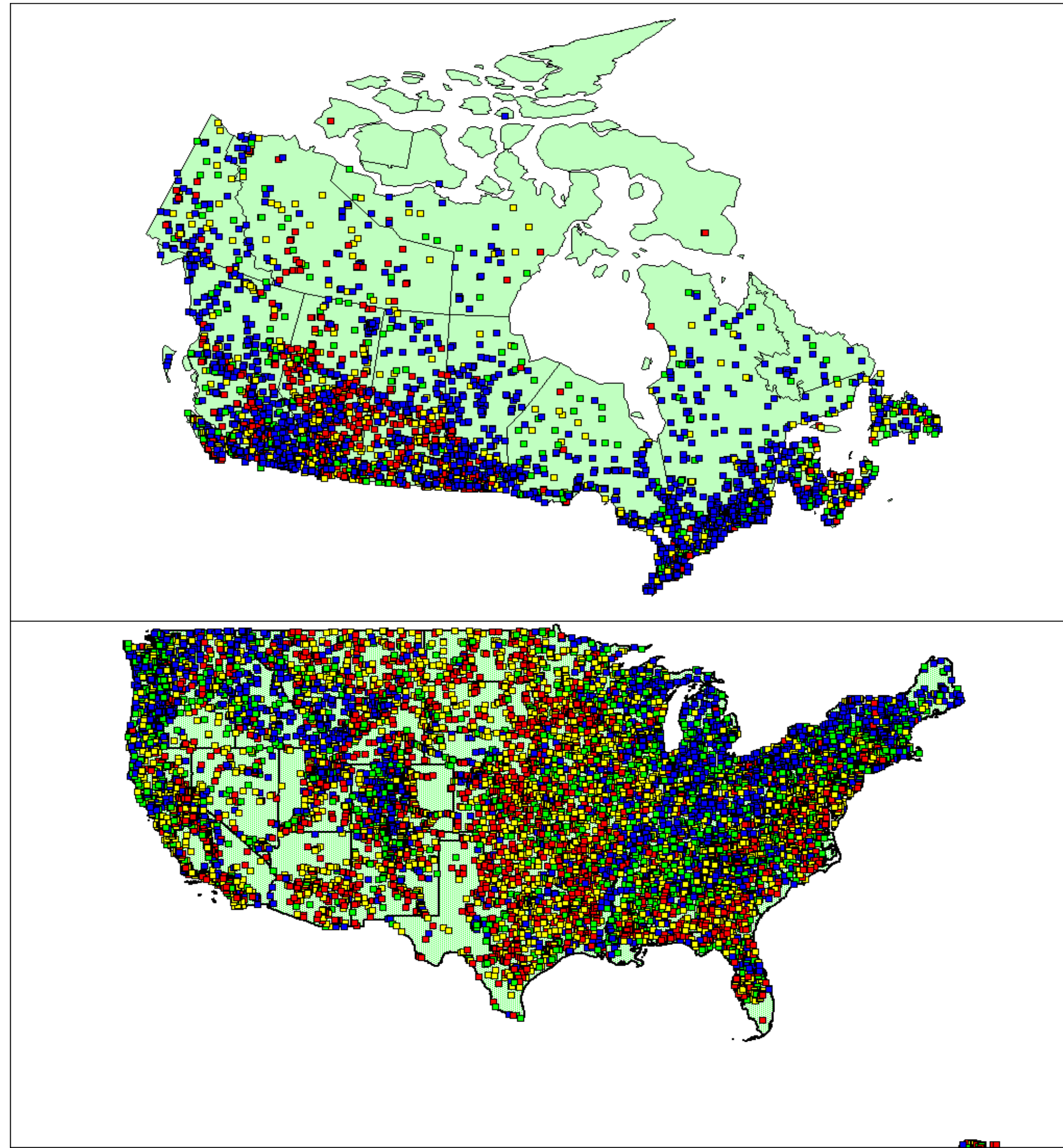
=> refined analysis :

- time volution of the **Most Probable Singularity**  $\gamma_s$  (Hubert et al, 1993; Douglas & Barros, 2003):
- a **scale invariant statistic**, more stable than the maximal simulated precipitation  $P_{max}$ .
- Unable us to conclude: **extremes**  $\uparrow$  (Royer et al., 2008),
- **seasonality can be taken into account** (Royer et al., 2010)



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# Mapping H



Hurst parameter  $H$ :  
scaling exponent of the  
mean field ( $H = 0$  for  
conservative multiplicative  
cascade):

for Canada ( ~equally  
25%):

0.6-0.25 (red);

0.25-0.2 (yellow);

0.2-0.17 (green);

0.01-0.17 (blue)

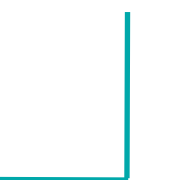
and for US:

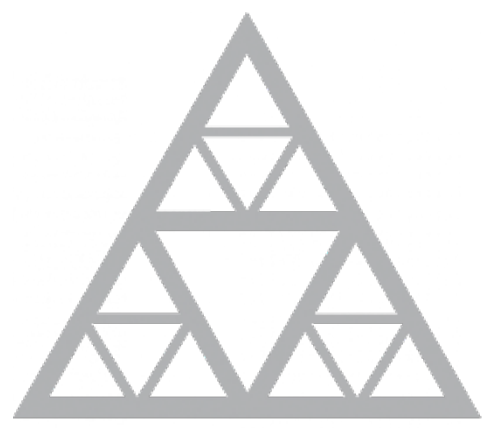
0.6-0.25 (~ 19%, red);

0.25-0.2 (~ 19%, yellow);

0.2-0.17 (~ 14%, green);

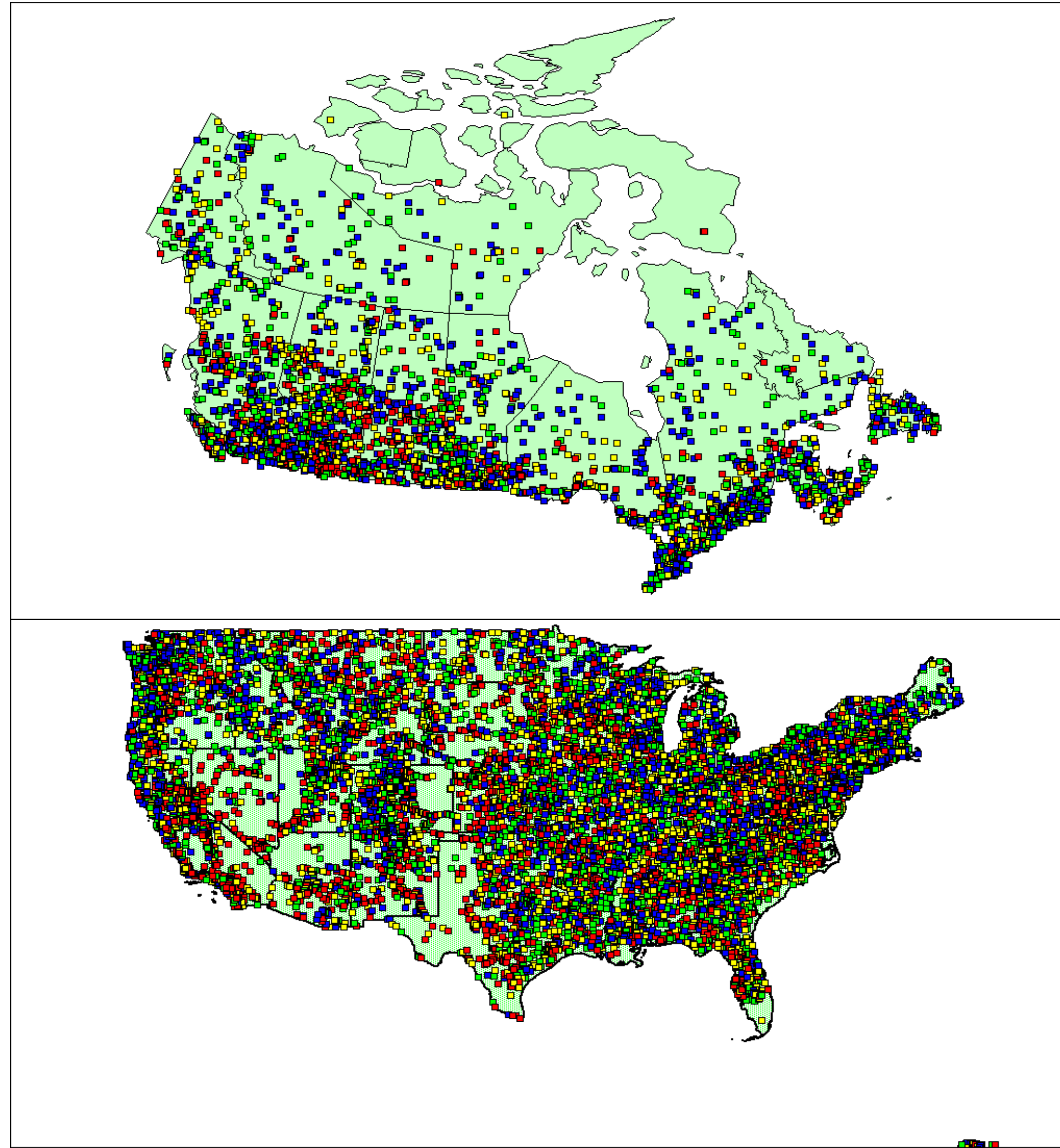
0.01-0.17 (~ 48%, blue)





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# Mapping C1



Codimension C1:  
how intermittent is the  
mean field

( ~equally 25%):

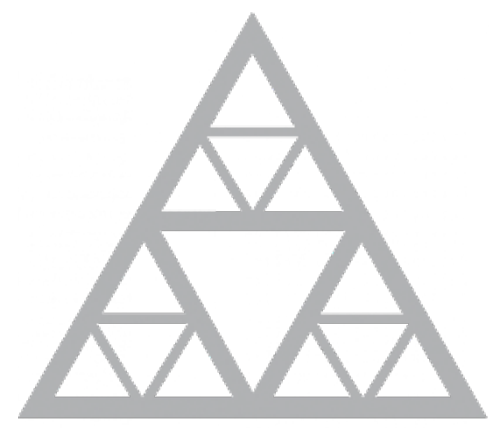
0.9-0.2 (red);

0.15-0.2 (yellow);

0.1-0.15 (green);

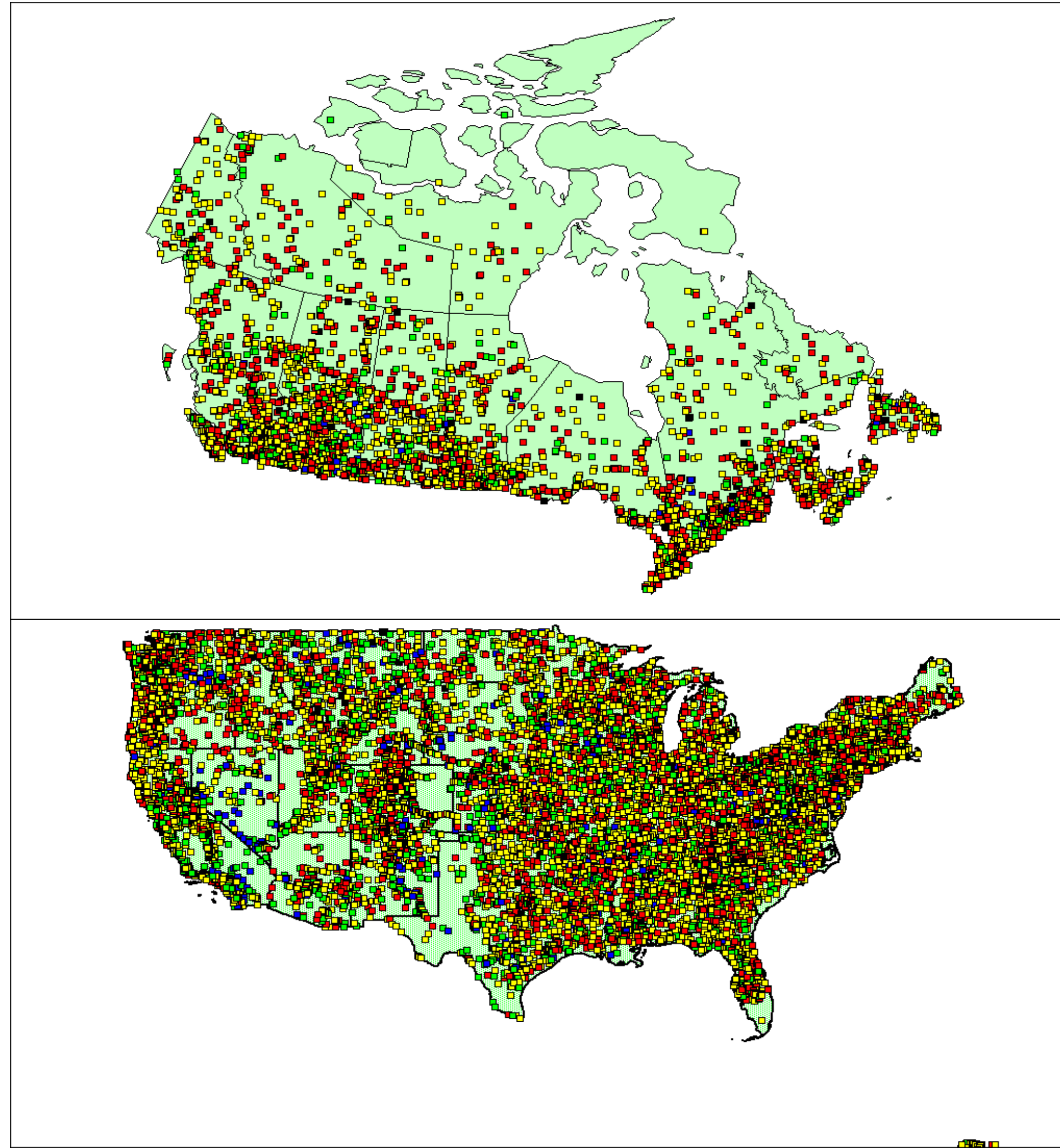
0.01-0.1 (blue)





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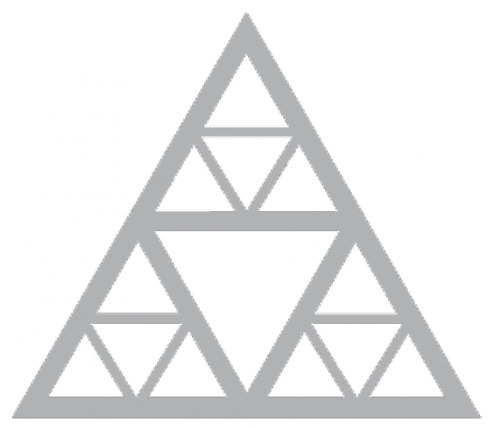
# Mapping $\alpha$



Multifractality index  $\alpha$

how multifractal  
is the process

- $>2$  (~ 1%; black);
- 2.0-1.5 (~ 30%, red);
- 1.5-1.0 (~ 52%, yellow);
- 1.0-0.6 (~ 15%, green);
- $<0.6$  (~ 2%, blue)



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# Histograms of multifractal parameters

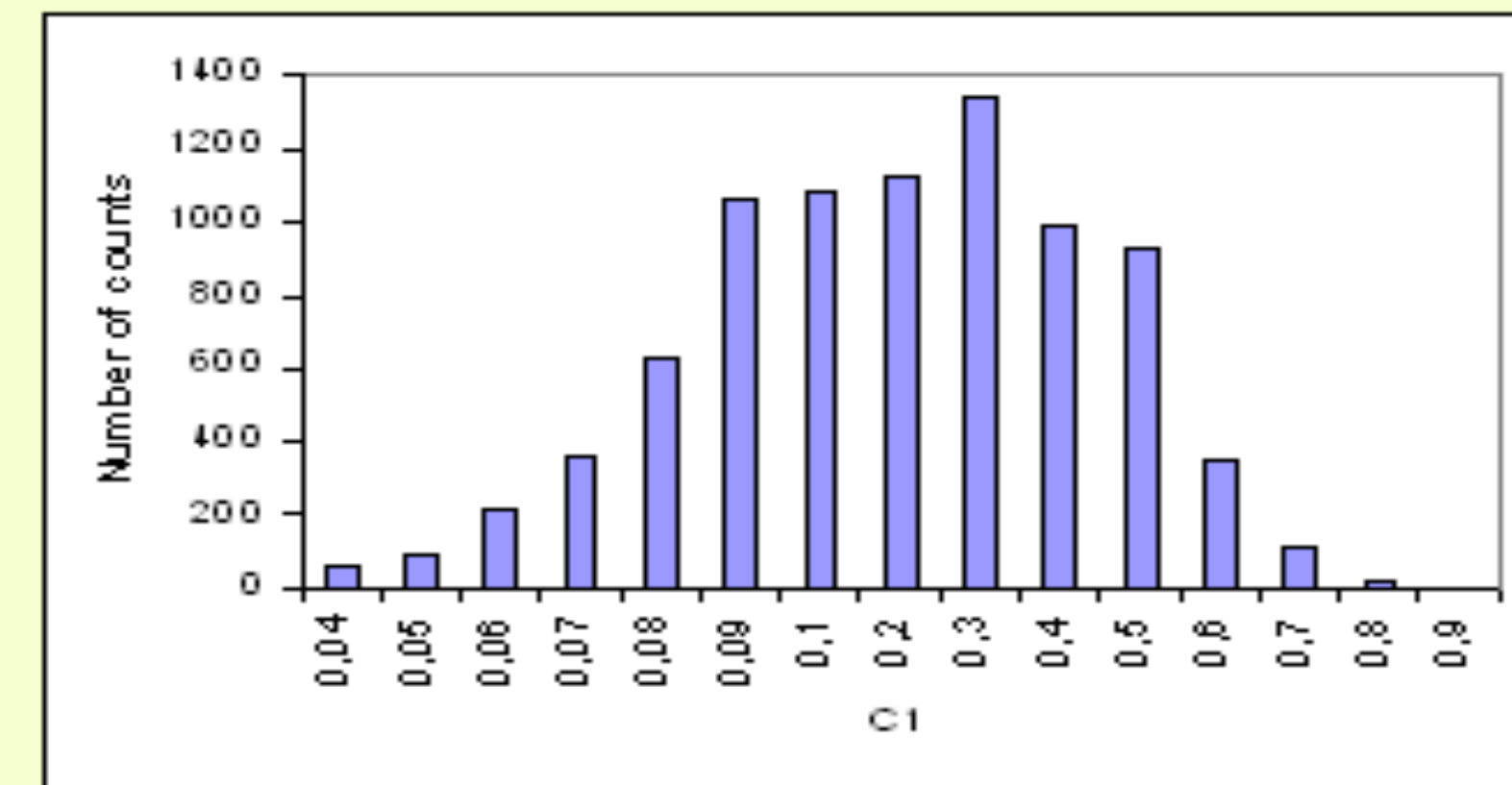
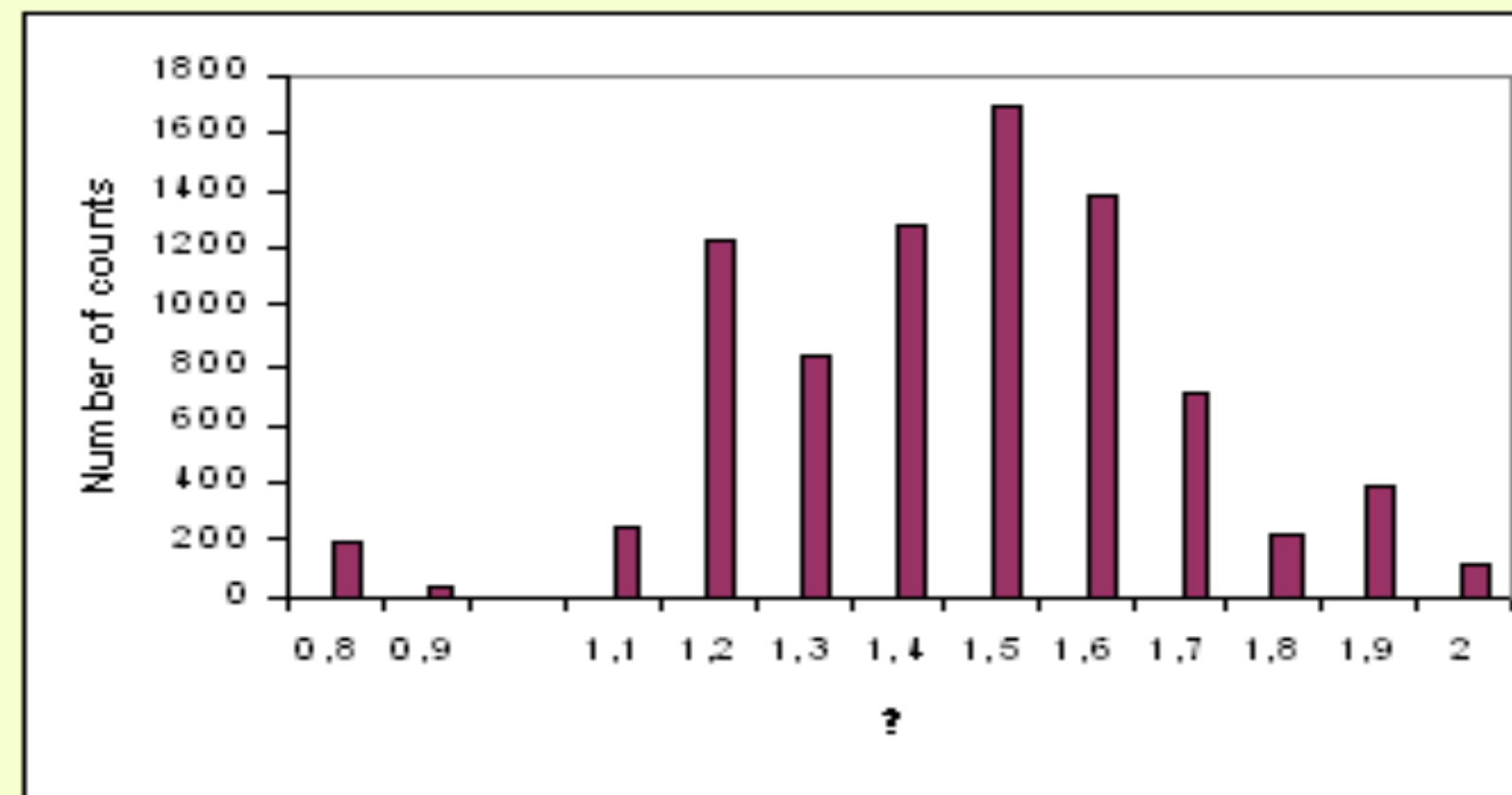


## 3. Main results

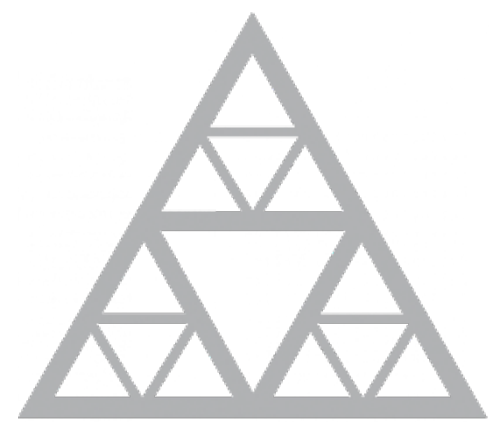
### 3.1 Database

### 3.2 Analysis

## Multifractal parameter distribution

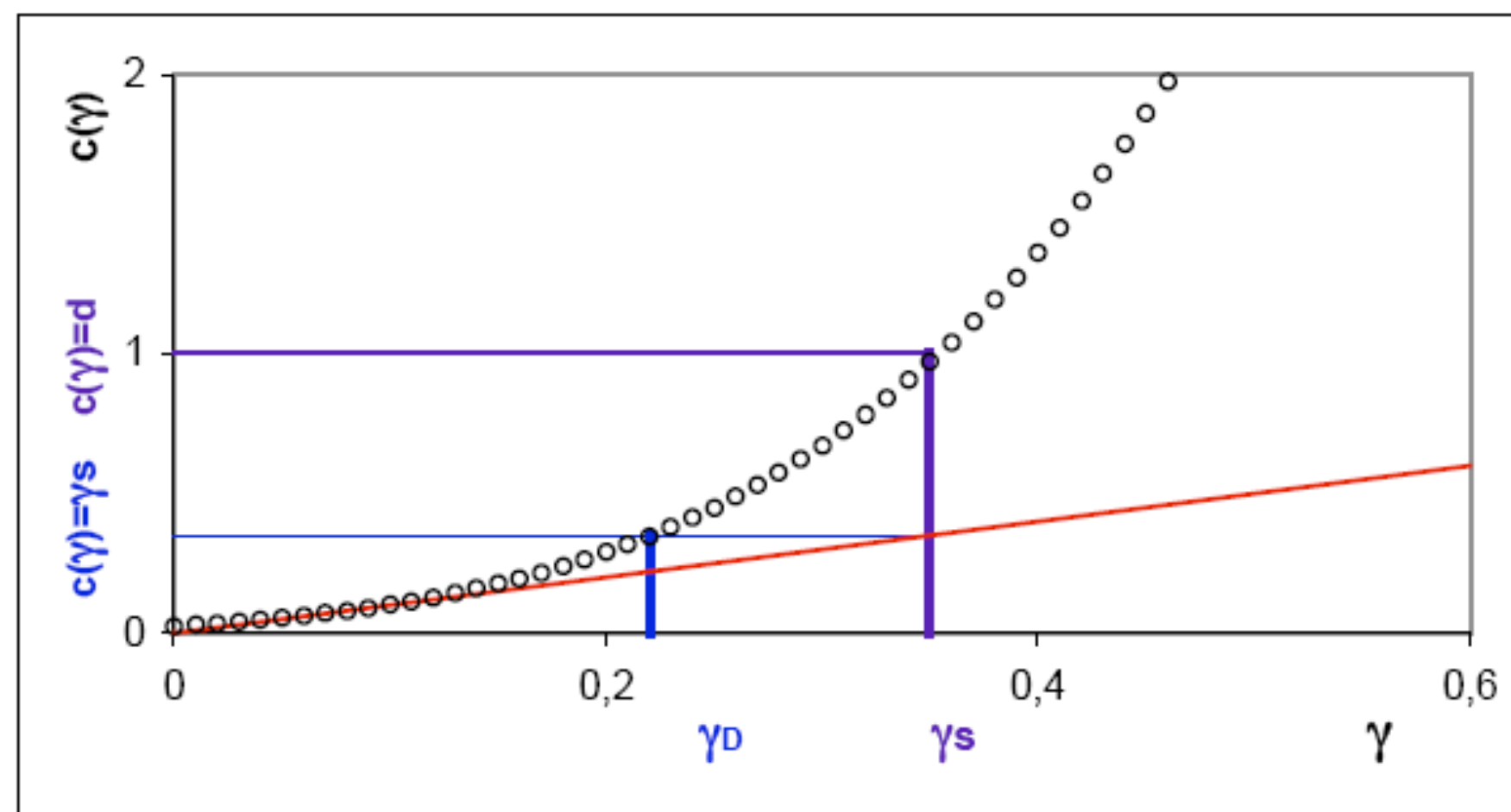


Parameter distributions for empirical data →  
'prior' probabilities  $\Pr(\alpha, C1)$

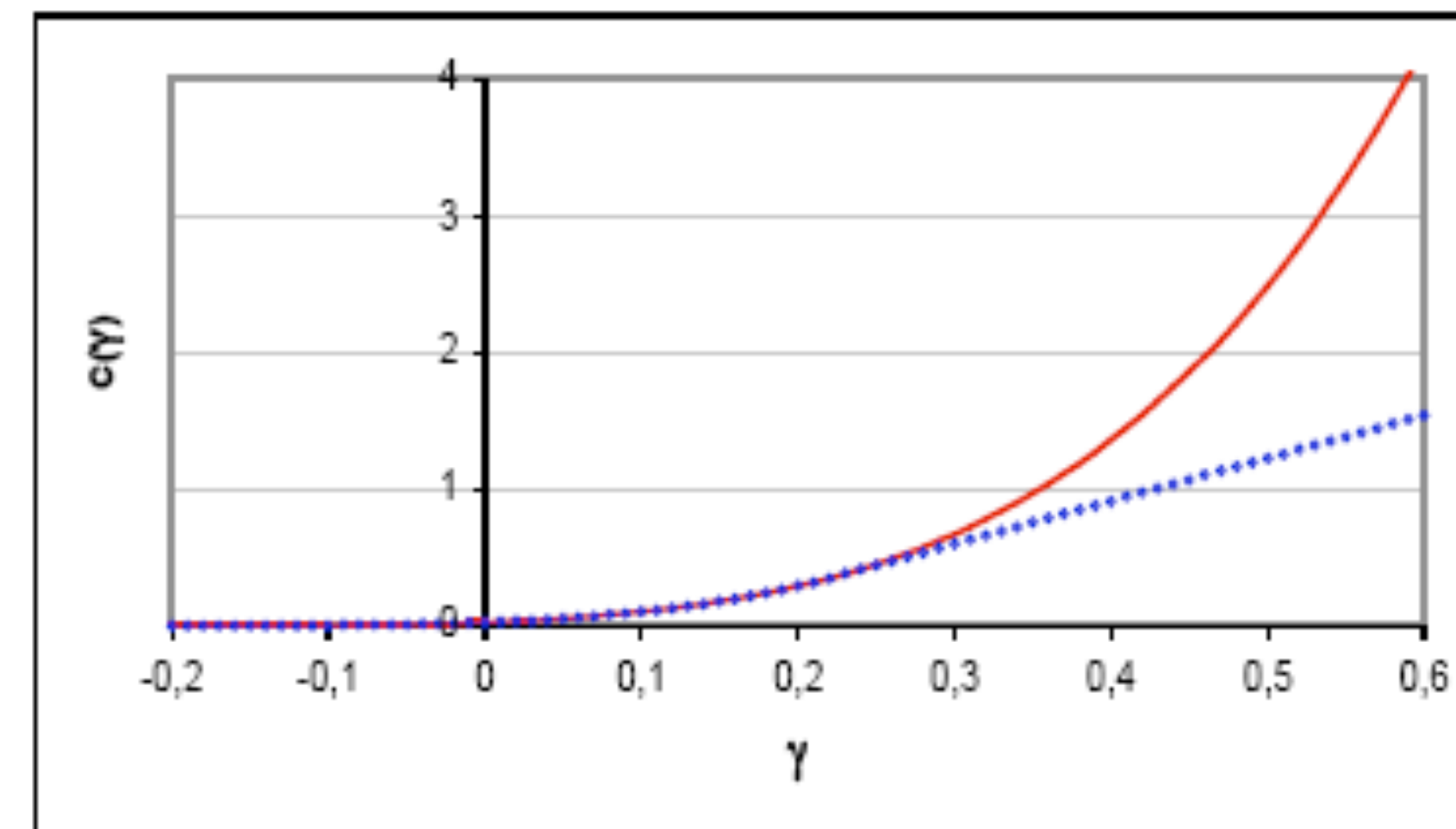


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# Multifractal Flood Frequency analysis: quantile plots and codimension



$\alpha, C_1, H \rightarrow$  critical singularity  $\gamma_D$



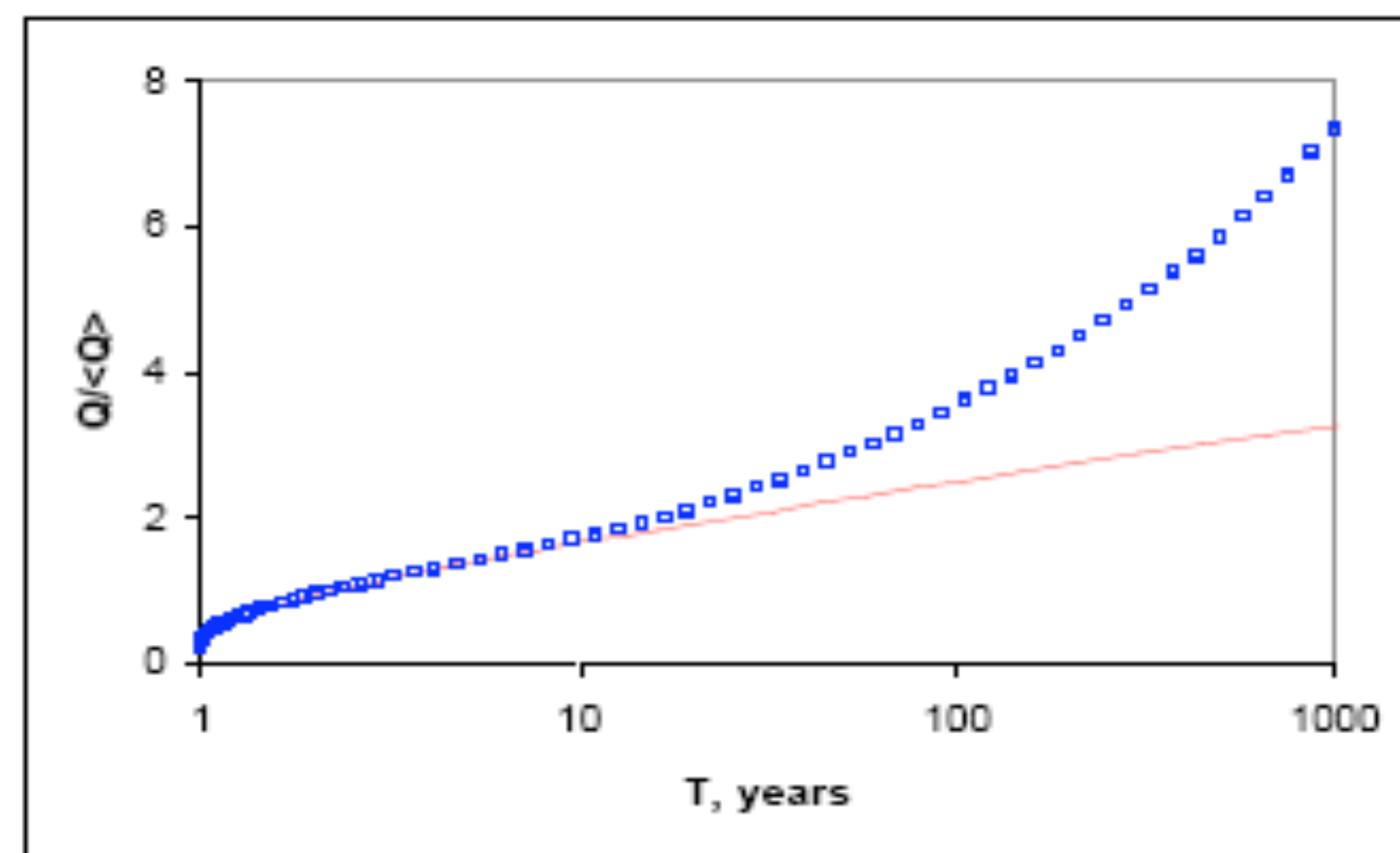
Predict shape of multifractal  
codimension function:

$$c(\gamma) = C_1 \left[ \frac{1}{\alpha} \left( \frac{(\alpha - 1)}{C_1} \gamma + 1 \right)^{\frac{\alpha}{\alpha - 1}} \right]^{\alpha - 1}$$

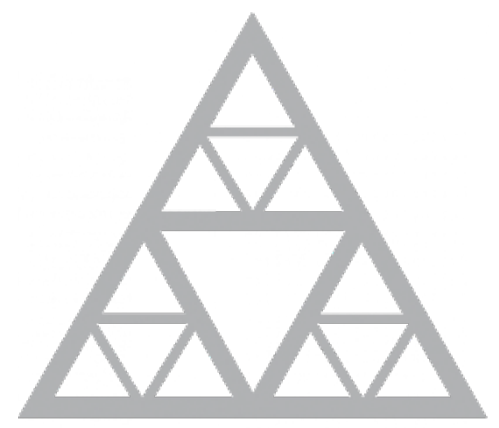
Transform into probability space:

$$T = \lambda^{c(\gamma)}$$

$$Q / \langle Q \rangle = \lambda^{\gamma - H}$$

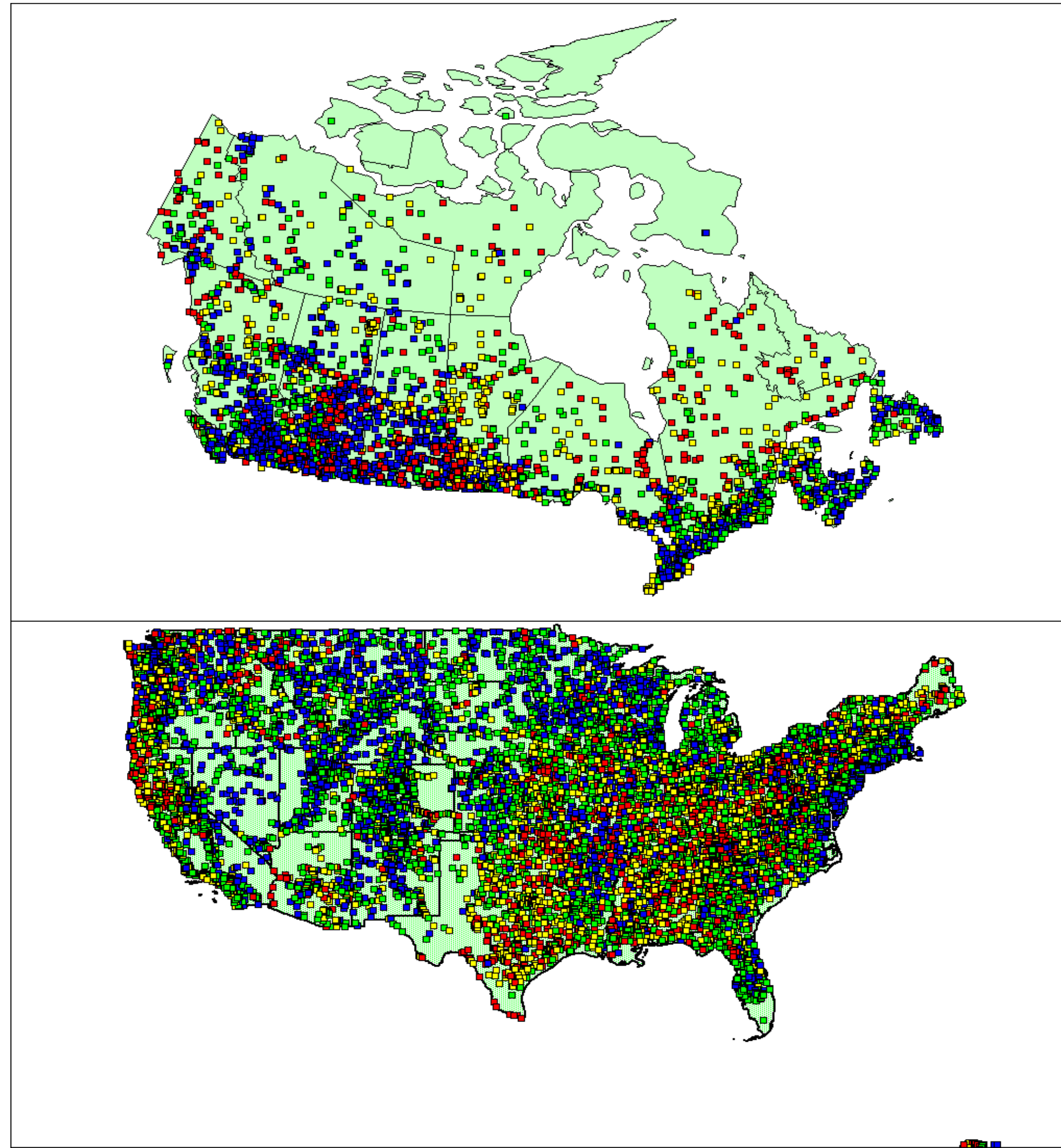






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# Mapping the reference discharges: $Q(\gamma_D)$



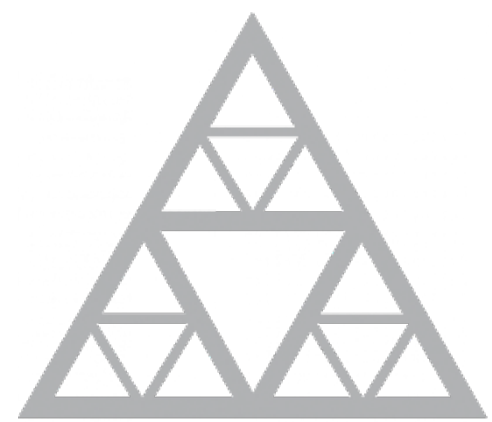
## Reference discharge

for Canada:

- >1000 (~14%, red);
- 300-1000 (~17%, yellow);
- 50-300 (~24%, green);
- <50 (~45%, blue)

for US:

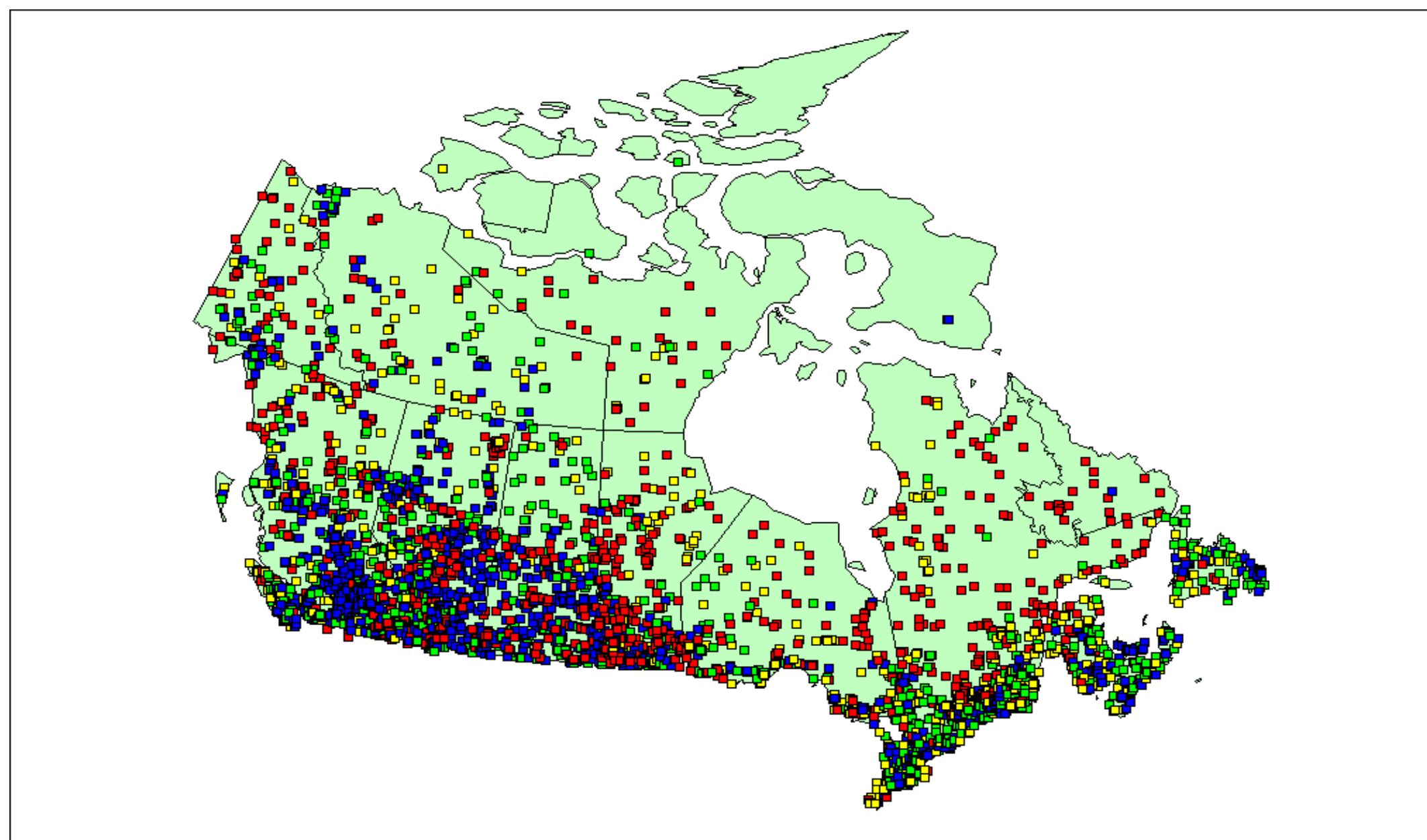
- >30000 (~14%, red);
- 8000-30000 (~22%, yellow);
- 1000-8000 (~38%, green);
- <1000 (~26%, blue)



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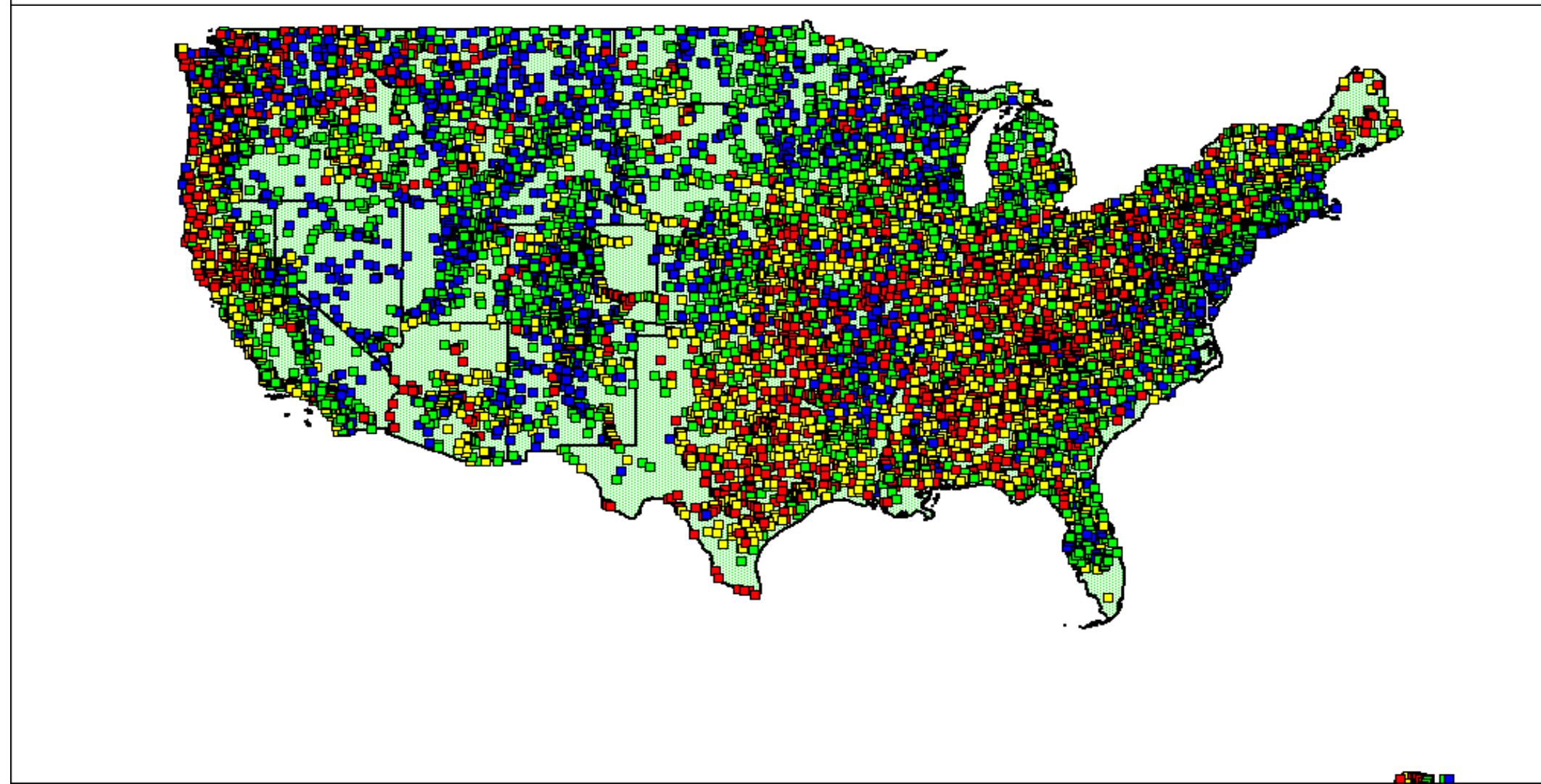
# Mapping the reference discharges: T=100



Discharges corresponding to 100 year return period

for Canada:

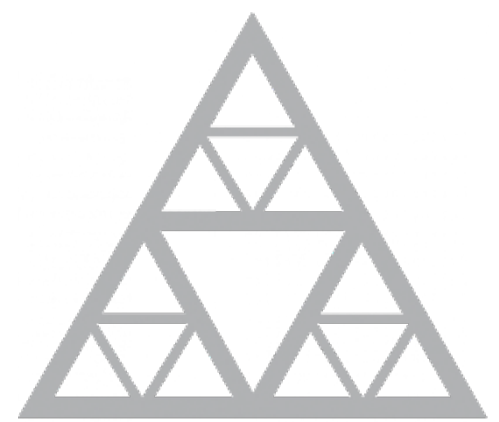
>1000 (~23%, red);  
300-1000 (~16%, yellow);  
50-300 (~26%, green);  
<50 (~35%, blue)



For US:

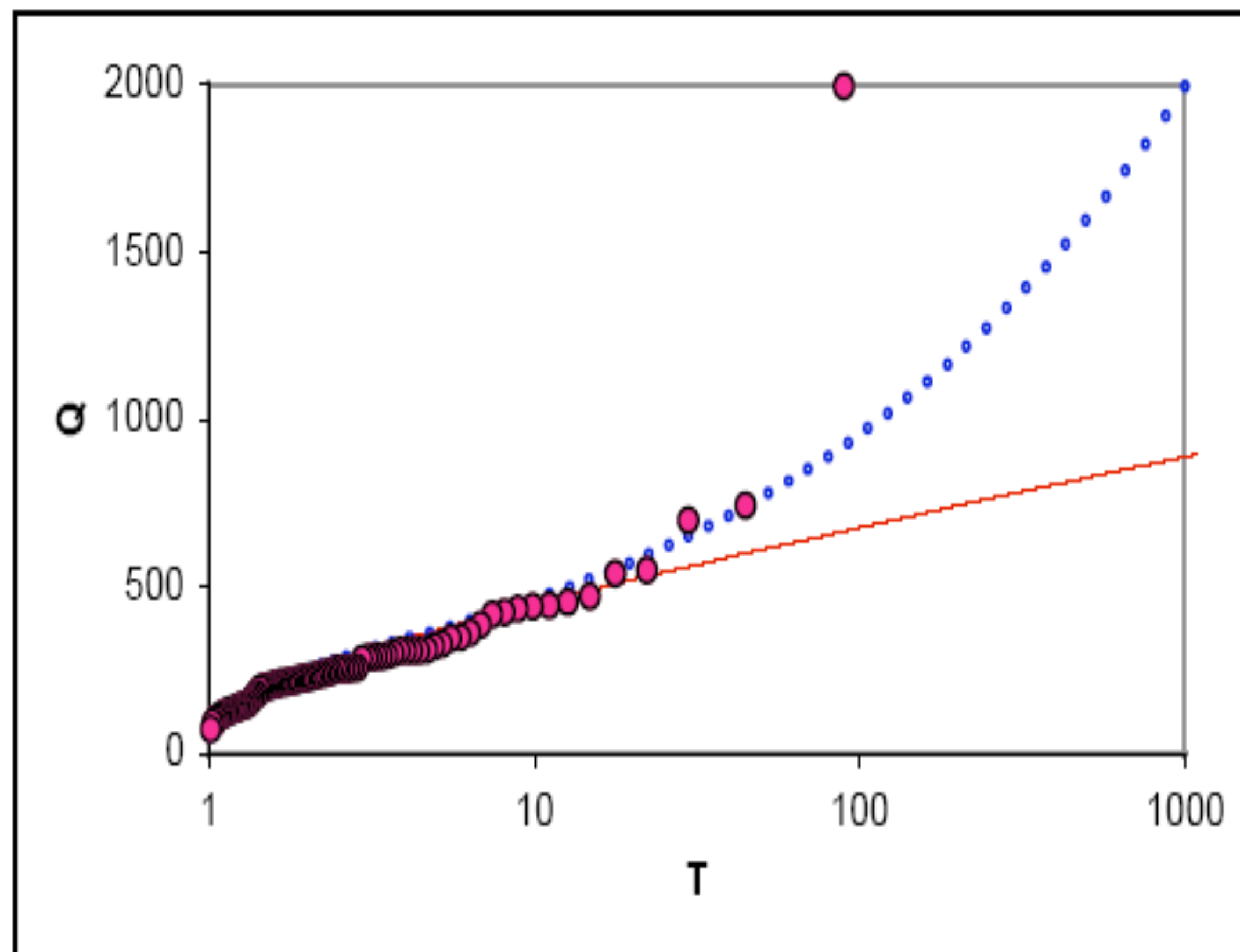
>30000 (~18%, red);  
8000-30000 (~27%, yellow);  
1000-8000 (~36%, green);  
<1000 (~19%, blue)





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# Multifractal Flood Frequency analysis: Kenogami series



Red dots are empirical frequencies for data sample;  
Red line is "classical" approach;  
Blue dots are MMFA results;

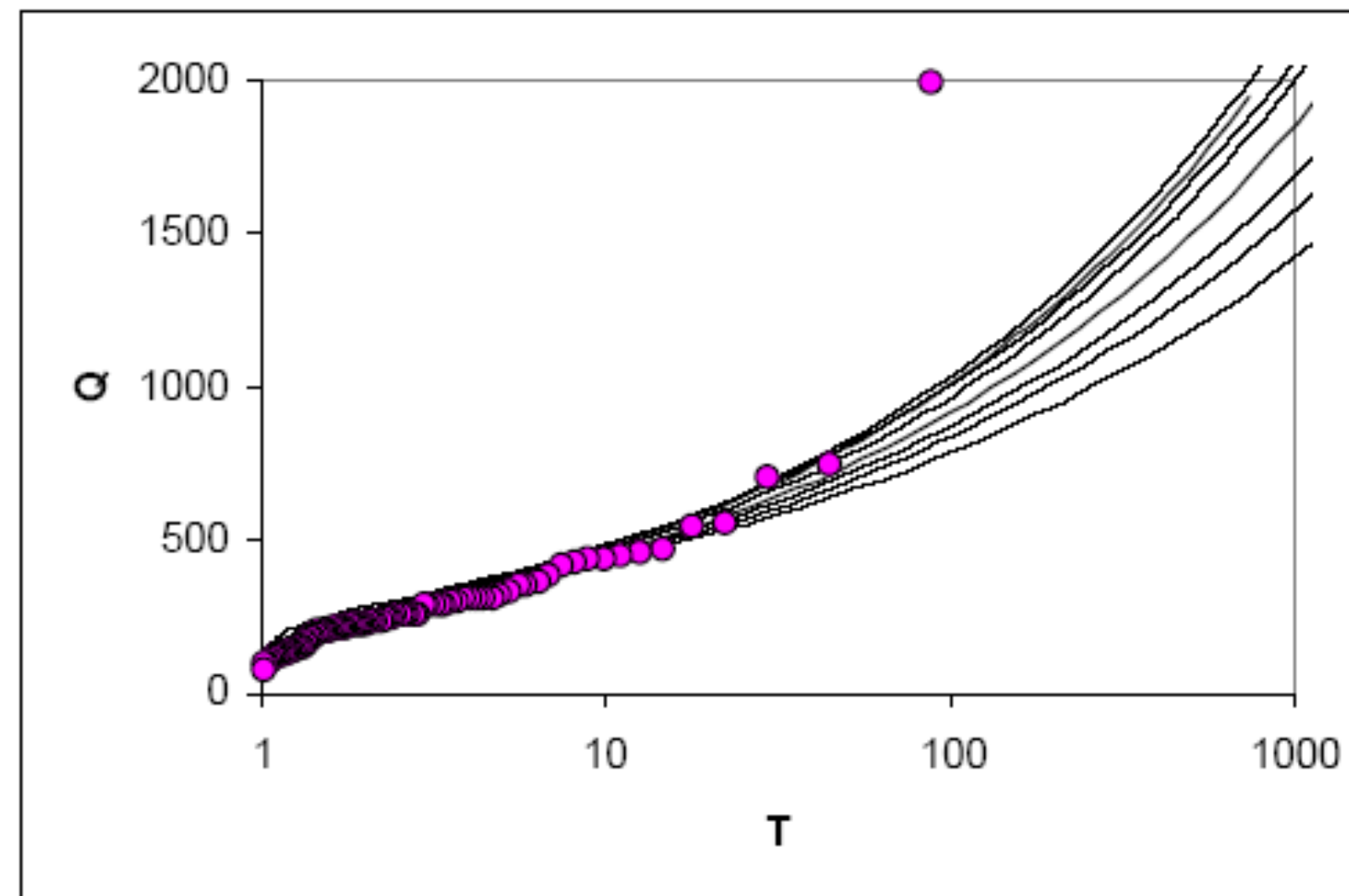
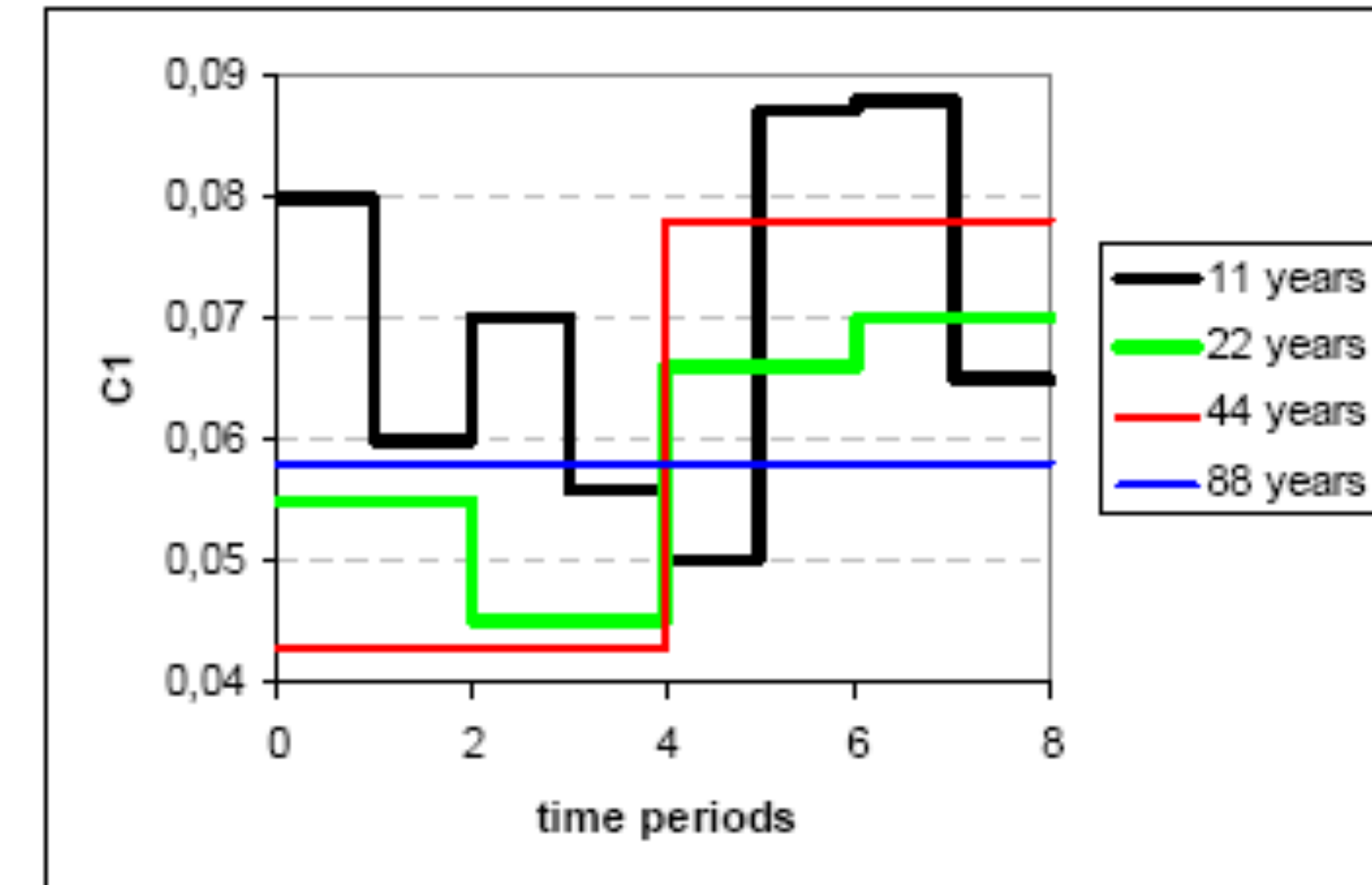
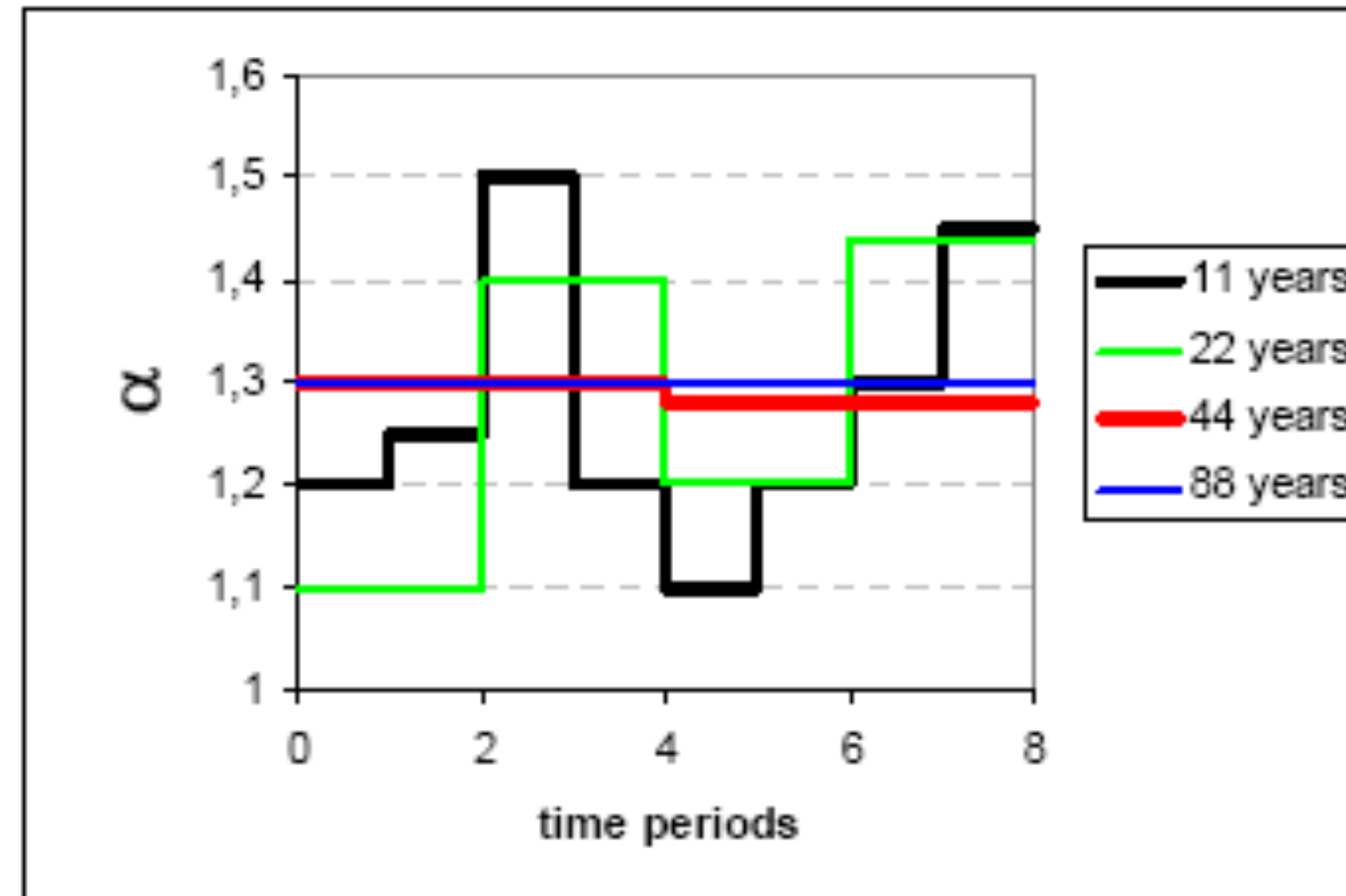
1996 Saguenay flood of 2000 m<sup>3</sup>/s has return period of:  
-1000 years for UM,  
-billions of yeas for classical analysis





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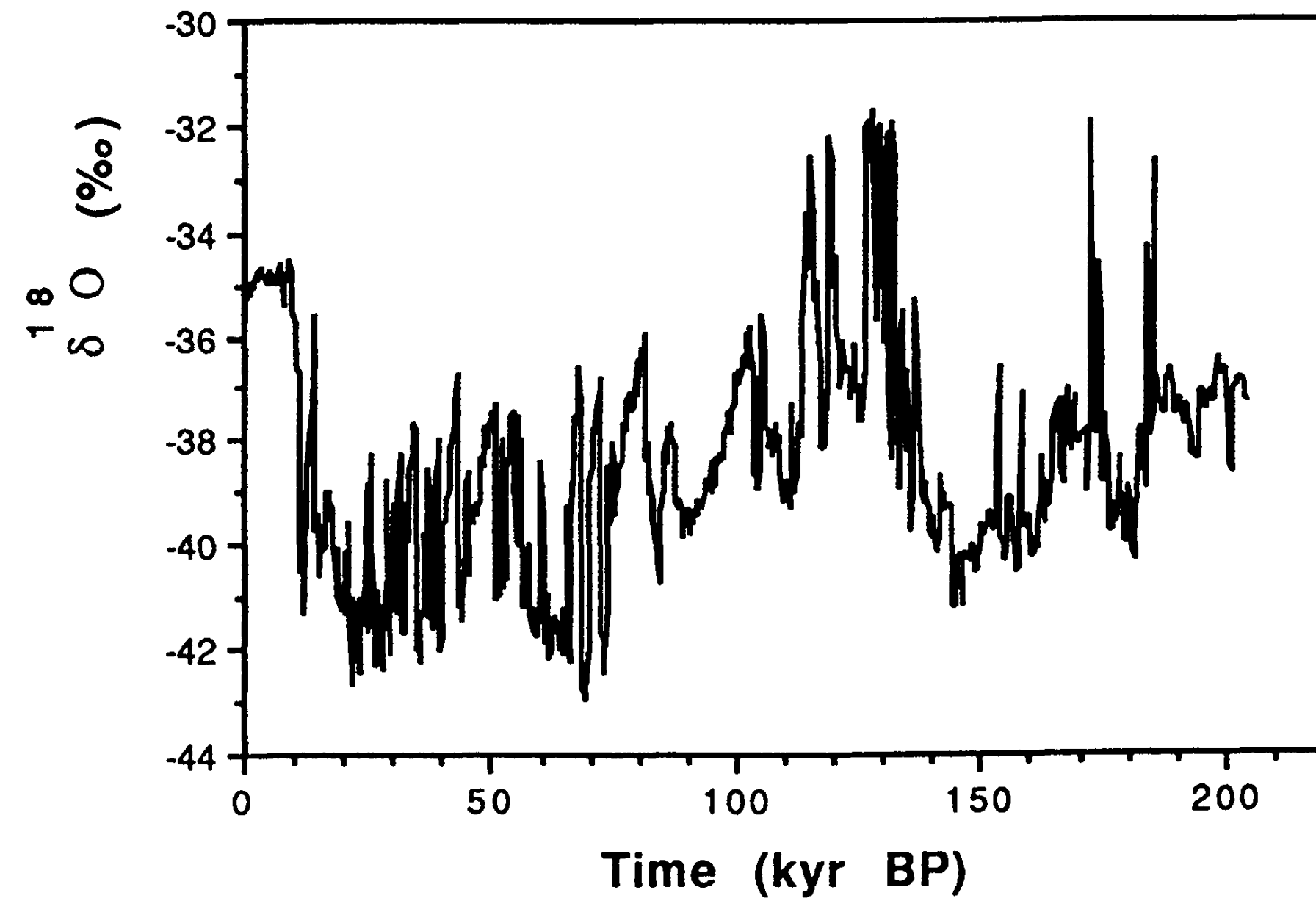
# Testing acceptability criteria on annual maxima: sample size



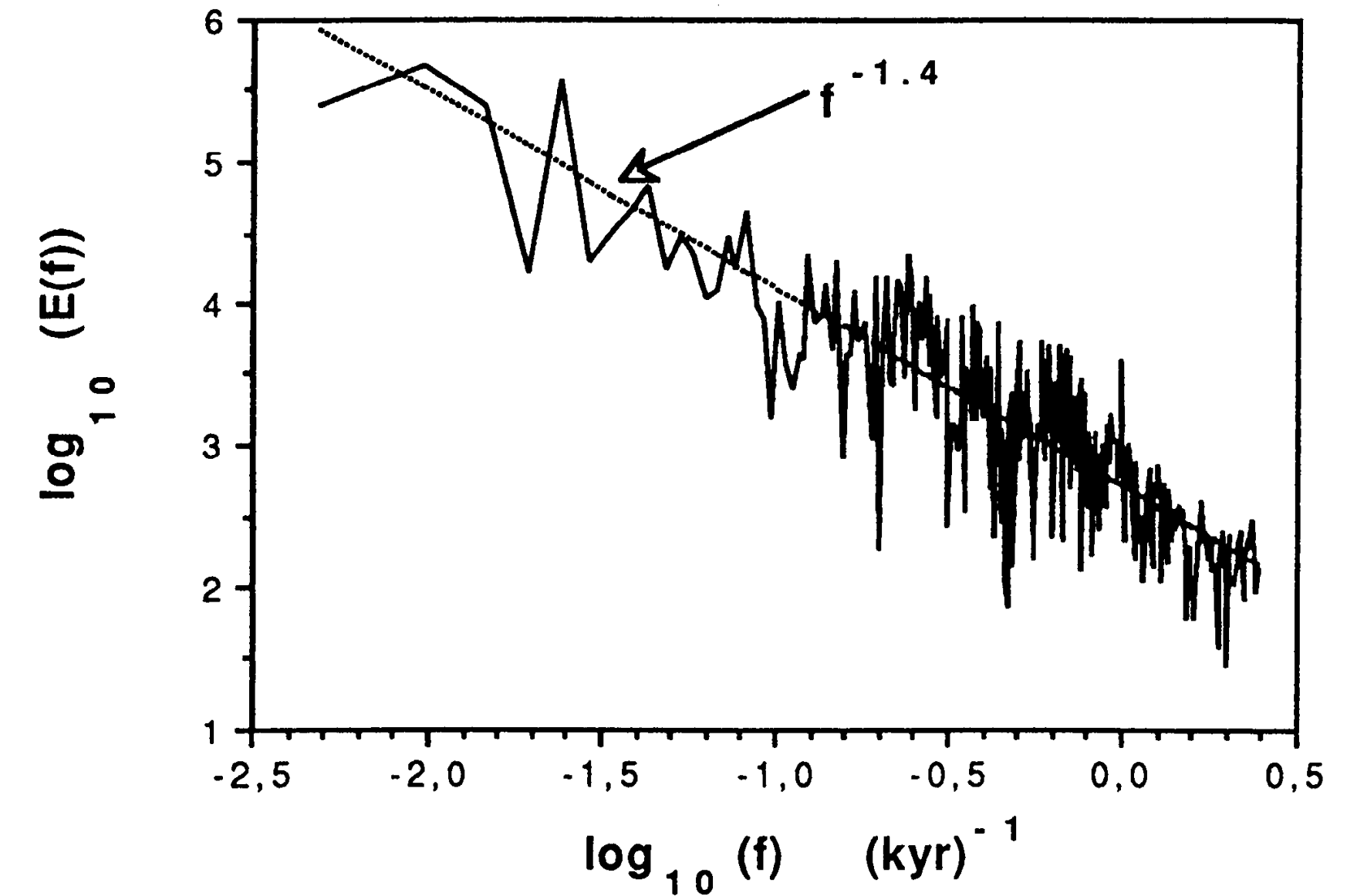
These parameter fluctuations for Kenogami series give quite a bit of dispersion for high return periods

→ uncertainty/reliability ?

# Scaling & multifractality of GRIP data



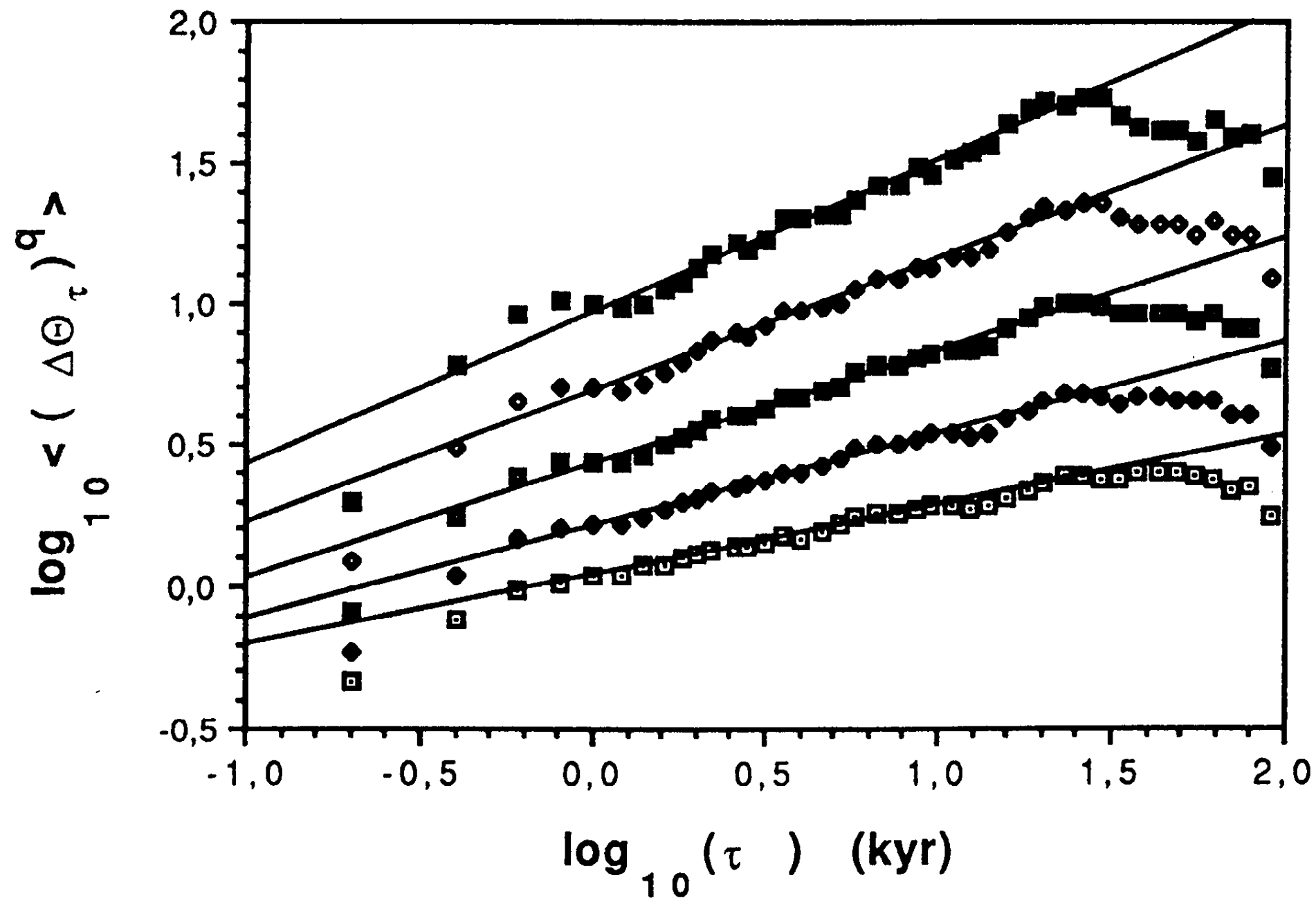
**Figure 1.** High resolution (200 yr average)  $\delta^{18}O$  record coming from the recent GRIP Greenland ice core. Sharp fluctuations occurring on small time scales are clearly visible.



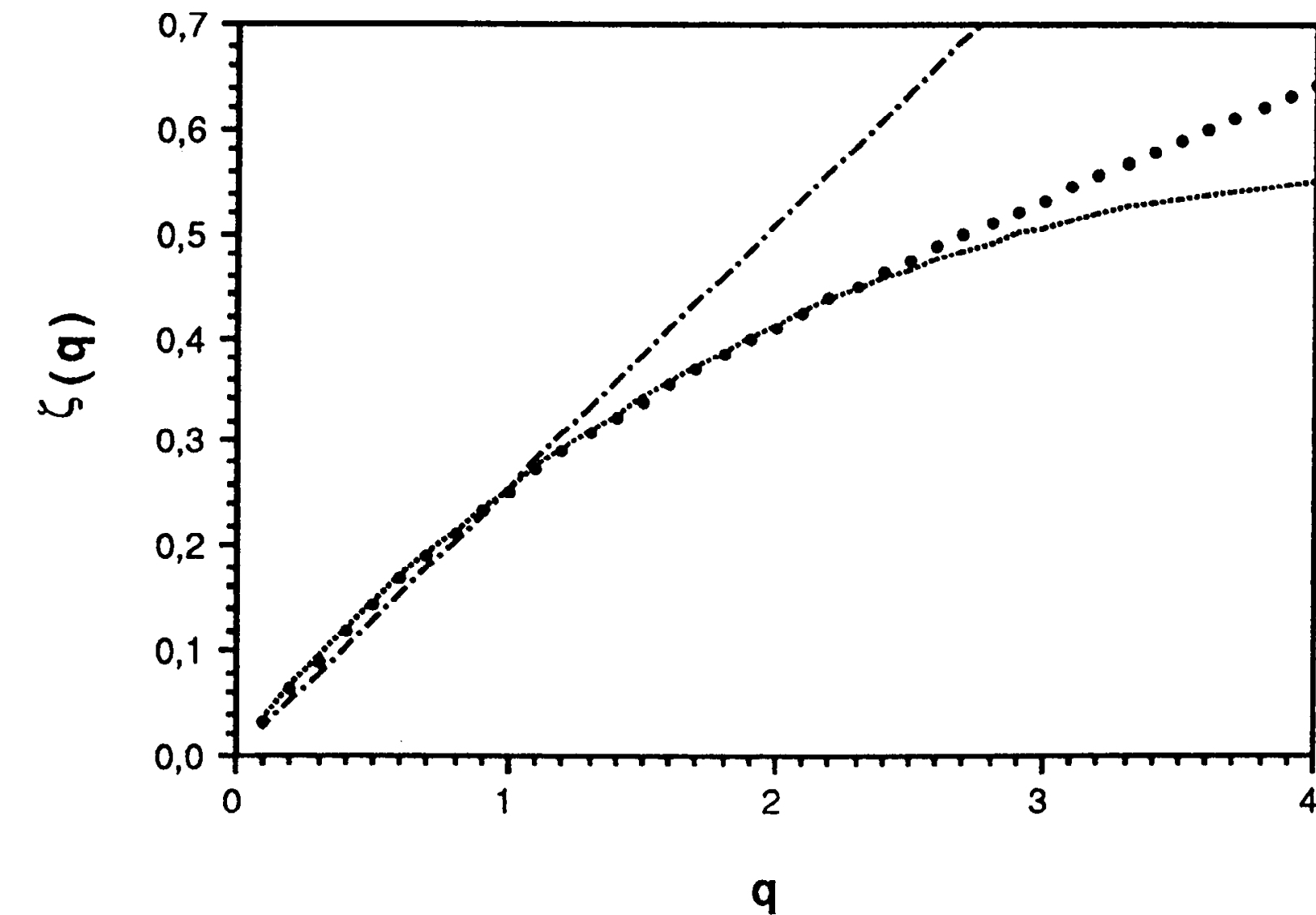
**Figure 2.** The power spectrum of the  $\delta^{18}O$  data, shown in a log-log plot. The global straight line  $f^{-1.4}$  is an indication of scaling. There is no evidence of periodic variations of frequencies of  $(20 \text{ kyr})^{-1}$  or  $(40 \text{ kyr})^{-1}$ .

$$E(f) \approx f^{-\beta}; \quad \beta \approx 1.4$$

# Scaling & multifractality of GRIP data



**Figure 3.** The structure functions  $\langle (\Delta\theta_\tau)^q \rangle$  vs.  $\tau$  in a log-log plot for  $q=1, 1.5, 2, 2.5,$  and  $3$ . Linear trends are clearly visible for all order of moments, for a range of scales going from  $\approx 400$  yr to  $\approx 40$  kyr. The straight lines indicate the best linear regression over this range of scales for each value of  $q$ . This gives in particular:  $H = \zeta(1) = 0.24 \pm 0.02$ , and  $\zeta(2) = 0.40 \pm 0.03$ .

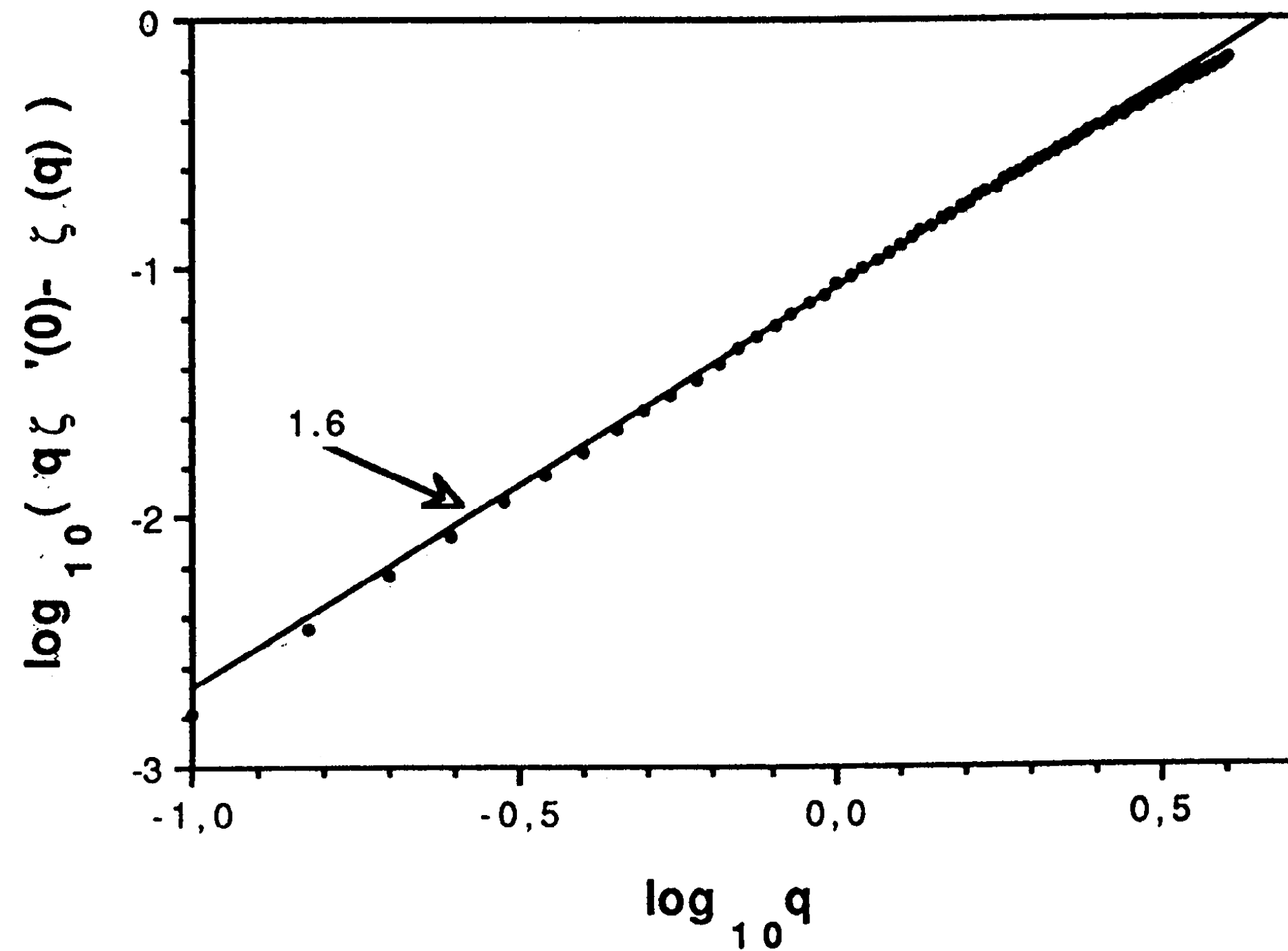


**Figure 4.** The scaling exponent structure function  $\zeta(q)$  empirical curve (dots), compared to the monofractal curve  $\zeta(q) = qH$  (dashed and dotted line), and to the universal multifractal function obtained with  $\alpha = 1.6$  and  $C_1 = 0.05$  in equation (2) (dotted curve). The empirical curve is nonlinear, indicating multifractality. For moments of order  $q > 1$ , the departure from monofractal behaviour is strong. The universal multifractal fit is excellent until moment order  $q_{\max} \approx 2.5 \pm 0.3$ , corresponding to a multifractal phase transition occurring because of sample limitations.

$$\left\langle \left( \Delta\theta \left( \frac{T}{\lambda} \right) \right)^q \right\rangle \approx \lambda^{-\zeta(q)} \langle (\Delta\theta(T))^q \rangle$$

(Schmitt et al., GRL, 1995)

# Scaling & multifractality of GRIP data



**Figure 5.** Empirical curve of  $\log[q\zeta(0) - \zeta(q)]$  vs.  $\log(q)$  giving directly the universal multifractal indices  $\alpha$  and  $C_1$ :  $\alpha$  is the slope of the straight line and  $C_1$  is estimated by the intercept. We obtain  $\alpha \approx 1.6$  and  $C_1 \approx 0.05$ .

linear scaling,  
fractional integration

nonlinear scaling  
intermittency,  
multifractality

$$\zeta(q) \approx Hq - C_1 \frac{q^\alpha - q}{q - 1}$$

$$\beta = 2H + 1 + K(2) \approx 2H + 1$$

$H$  (fractional) **order of integration**

$C_1$  codimension of the **mean intermittency**

$\alpha$  **multifractality index**, concavity of  $\zeta(q)$

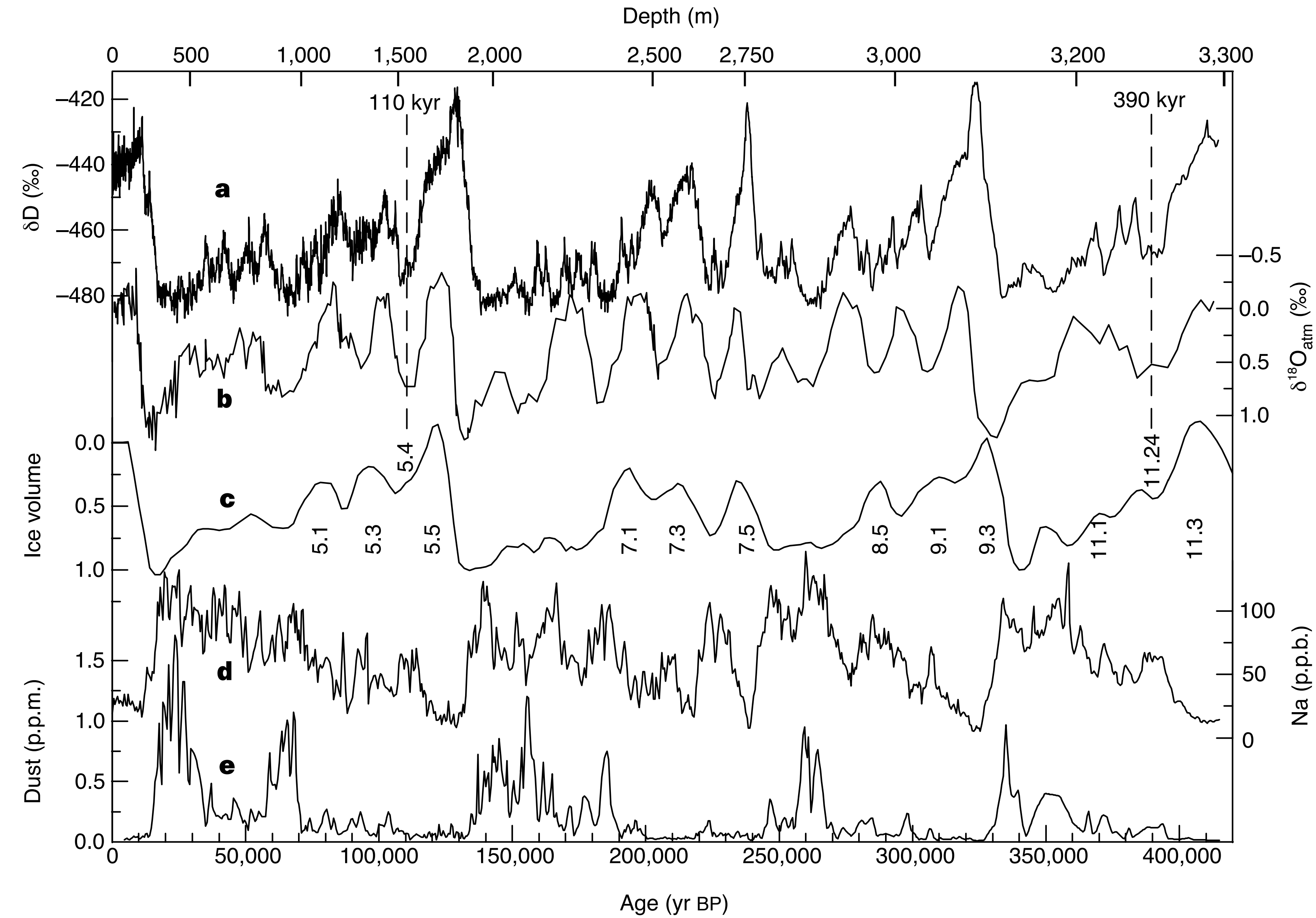
$$H \approx 0.2, C_1 \approx 0.05, \alpha \approx 1.6$$

# Vostok data

СТО ГОДИНА  
МИЛАНКОВИЋЕВЕ ТЕОРИЈЕ  
О КЛИМАТСКИМ  
ПРОМЕНАМА

1920202120

100 YEARS of  
MILANKOVIC'S THEORY of  
CLIMATE CHANGE

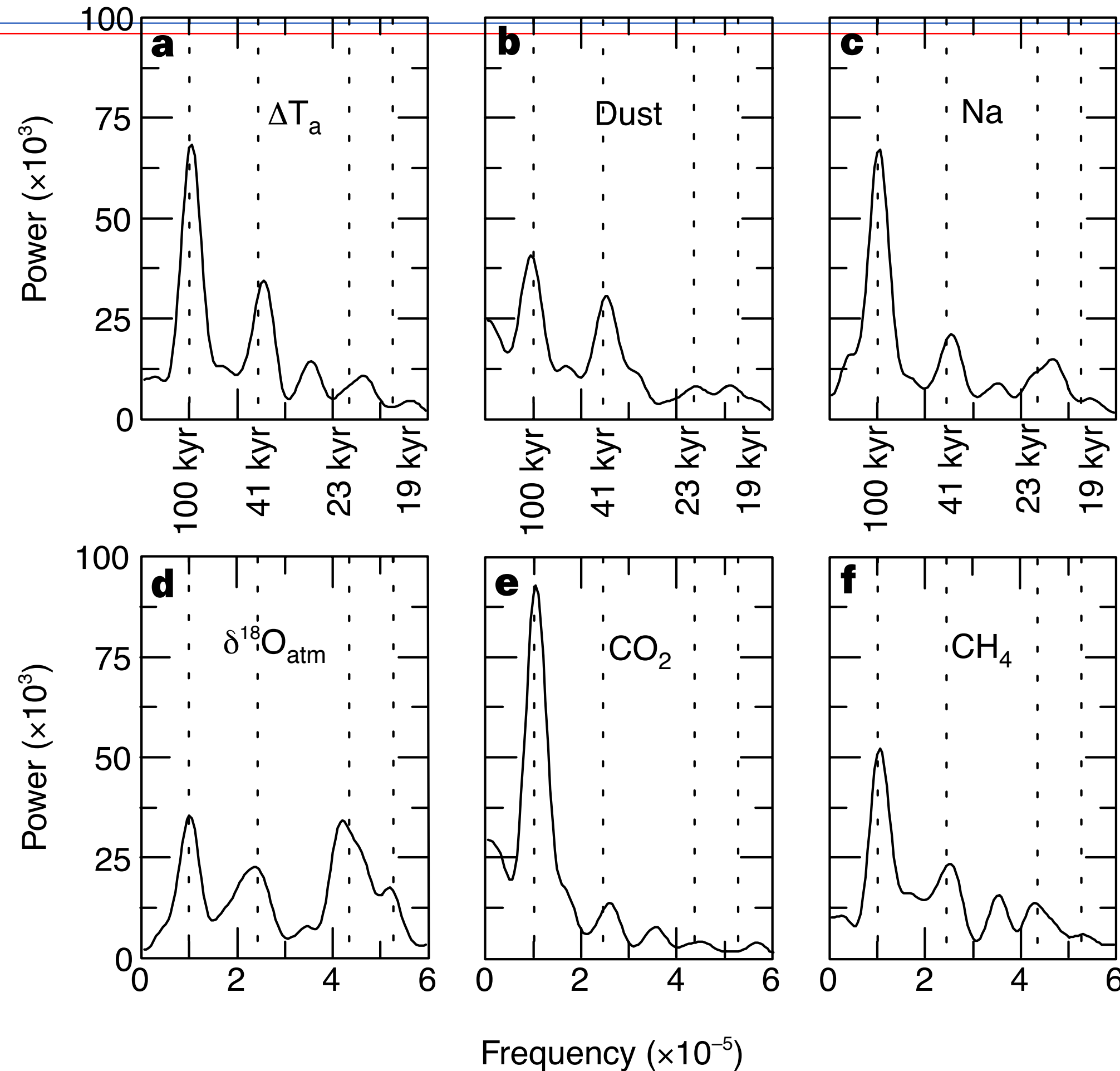


A very rich data set  
over 450 000 years!

(Petit et al., Nature, 1999)

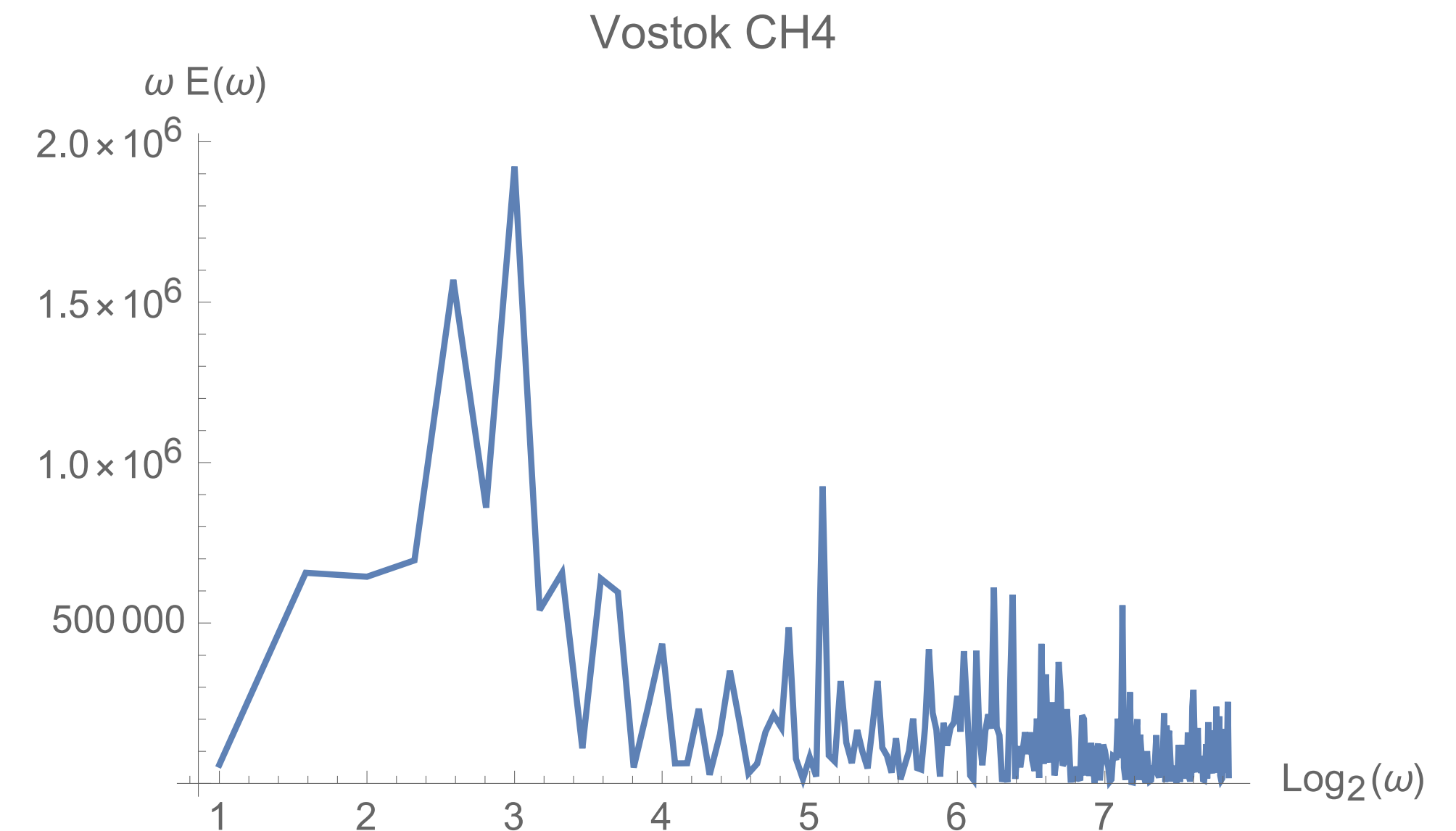


# Vostok data



“integrated spectra”:  $\omega E(\omega)$  vs  $\text{Log}(\omega)$

well identified but broad peaks/spikes



**Figure 4** Spectral properties of the Vostok time series. Frequency distribution (in cycles  $\text{yr}^{-1}$ ) of the normalized variance power spectrum (arbitrary units). Spectral analysis was done using the Blackman-Tukey method (calculations were performed with the Analyseries software<sup>47</sup>): **a**, isotopic temperature; **b**, dust; **c**, sodium; **d**,  $\delta^{18}\text{O}_{\text{atm}}$ ; **e**,  $\text{CO}_2$ ; and **f**,  $\text{CH}_4$ . Vertical lines correspond to periodicities of 100, 41, 23 and 19 kyr.

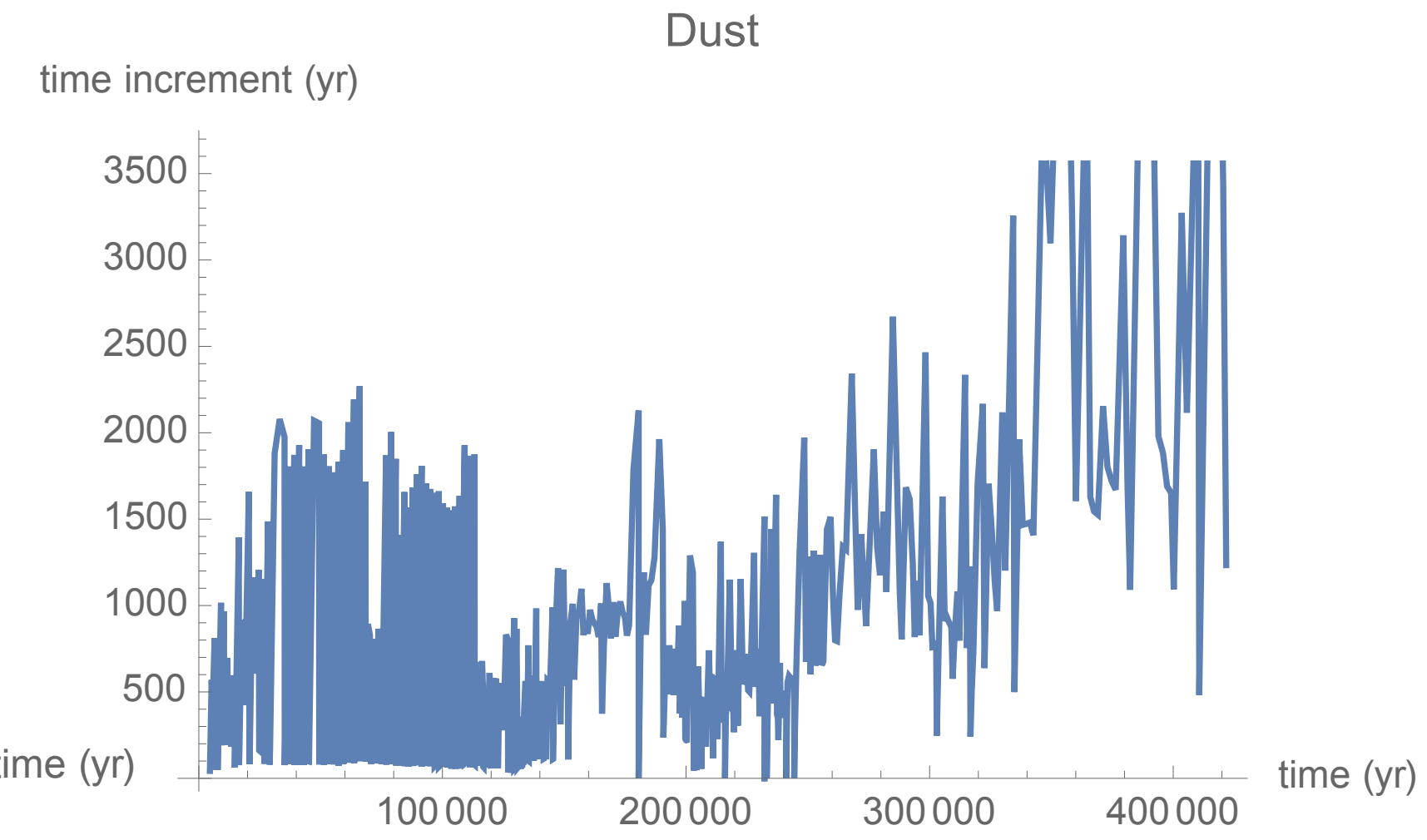
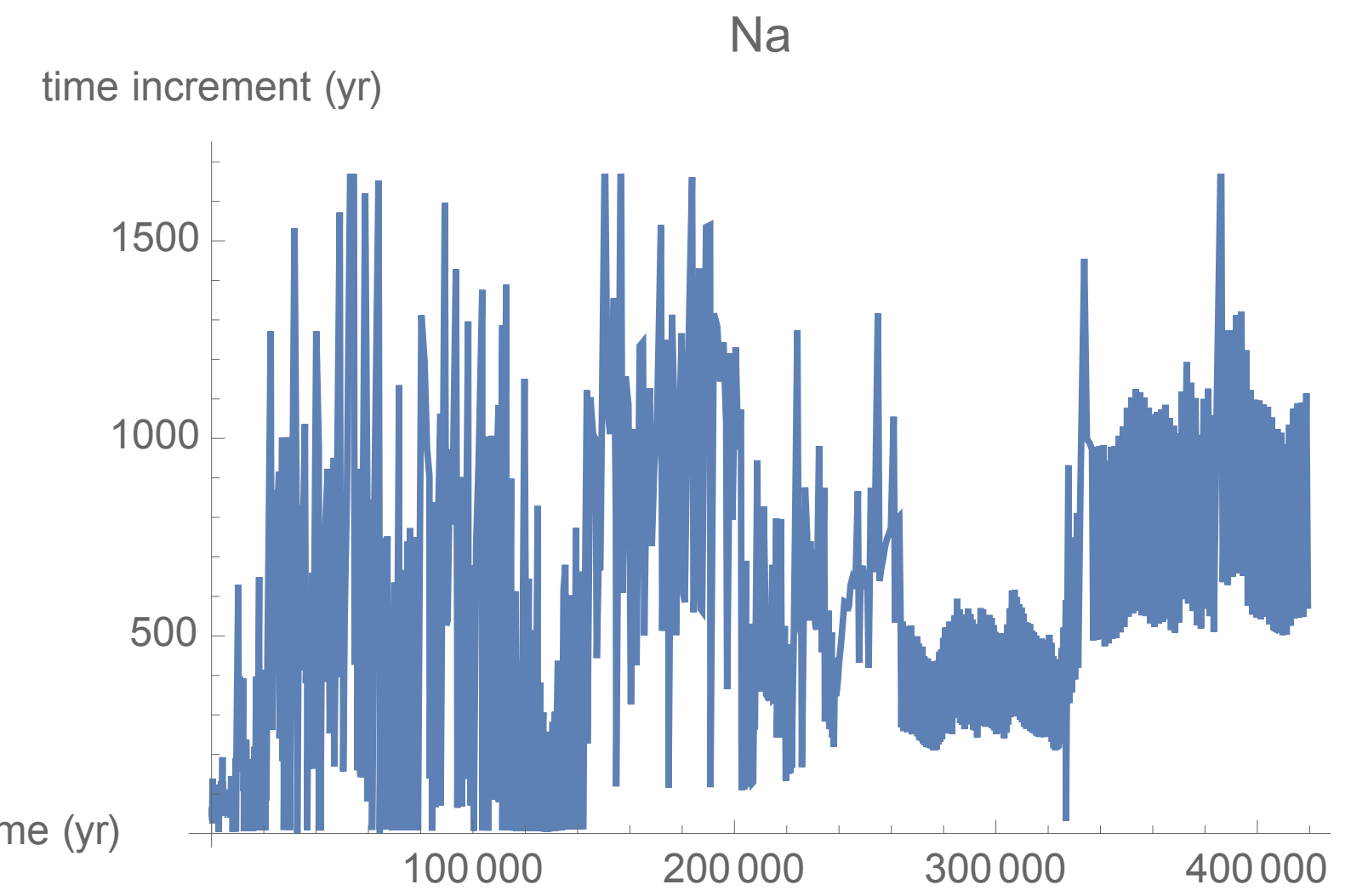
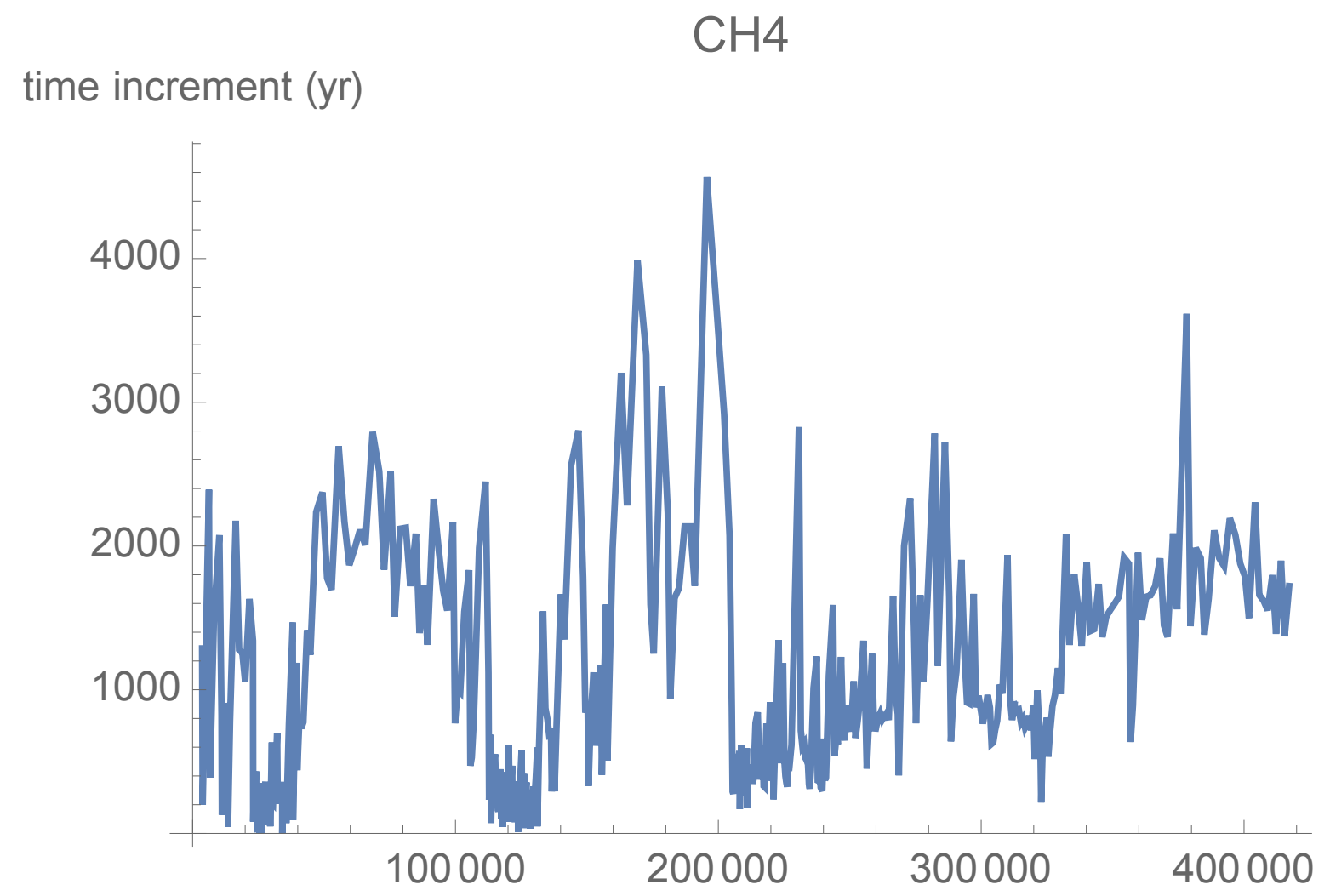
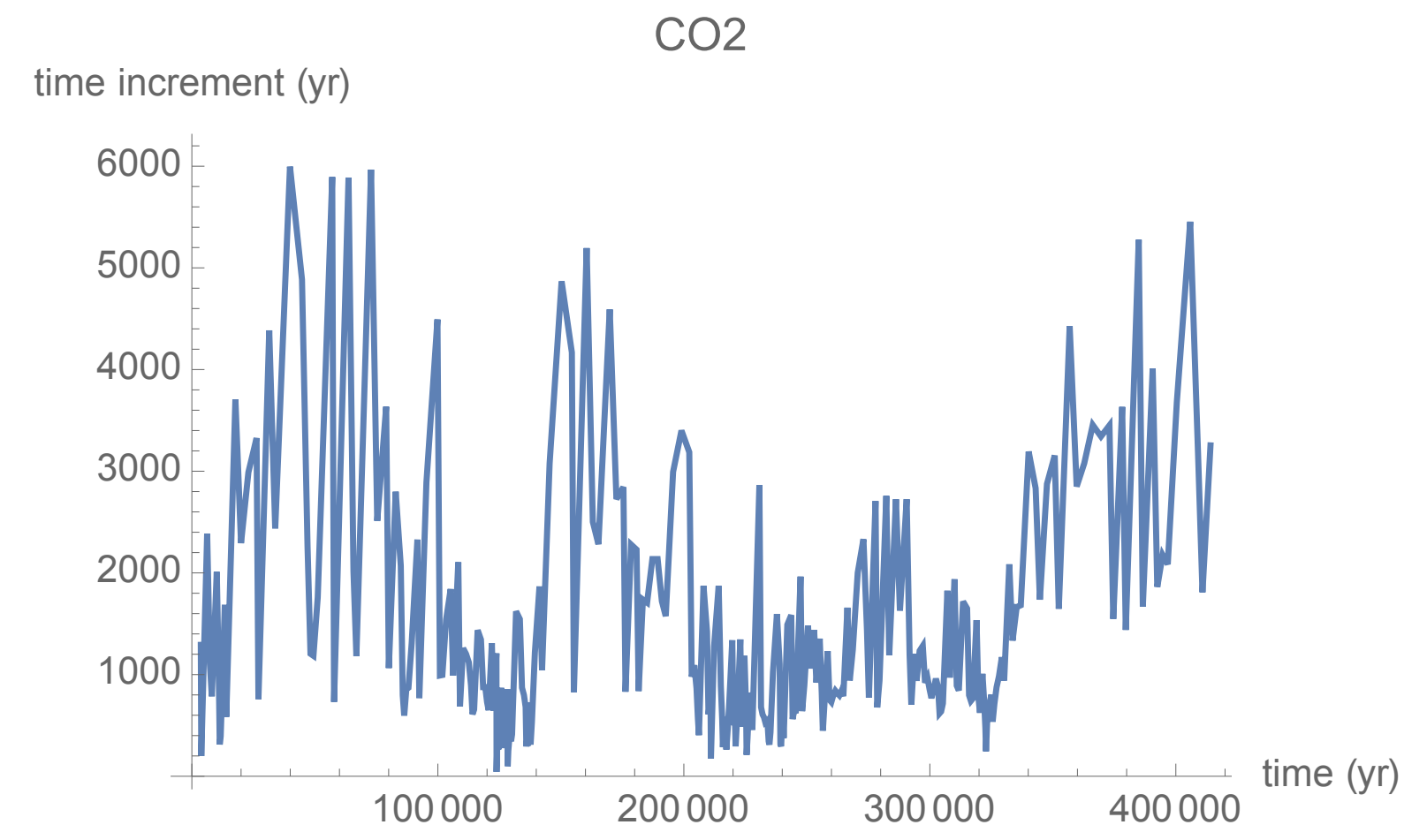
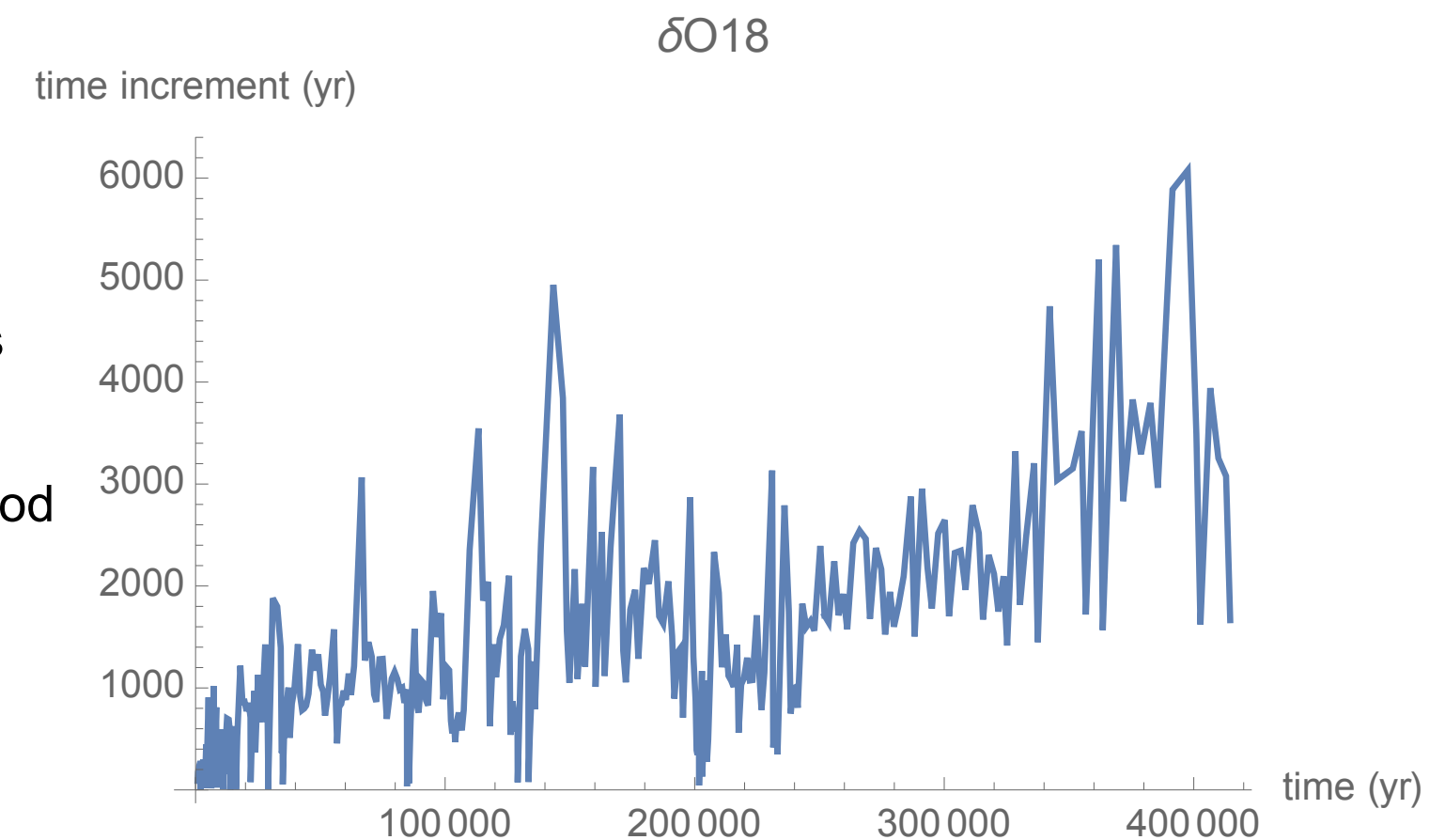
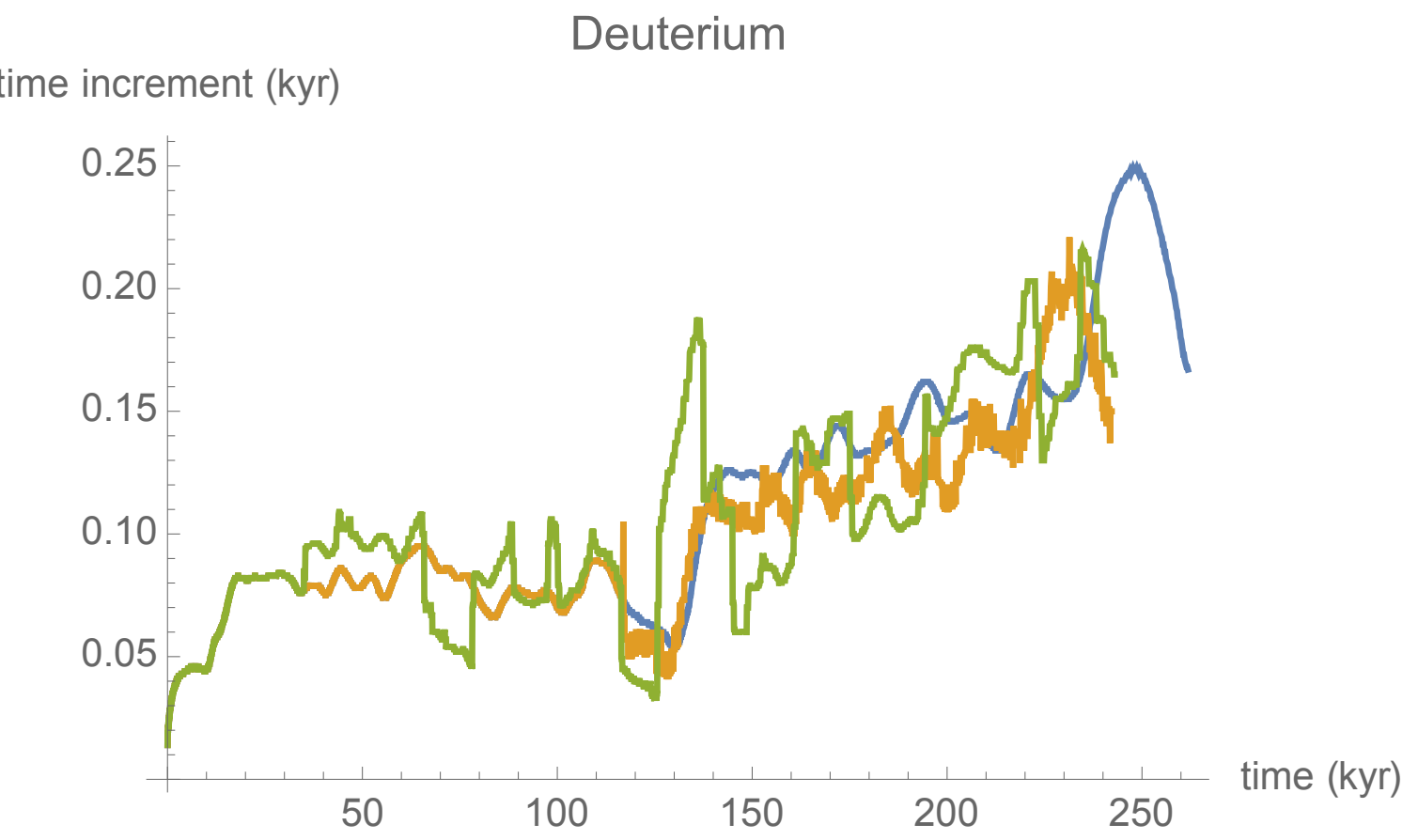
(Petit et al., Nature, 1999)

# Vostok data: glaciological timescale increments

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# Vostok data: spectral scaling

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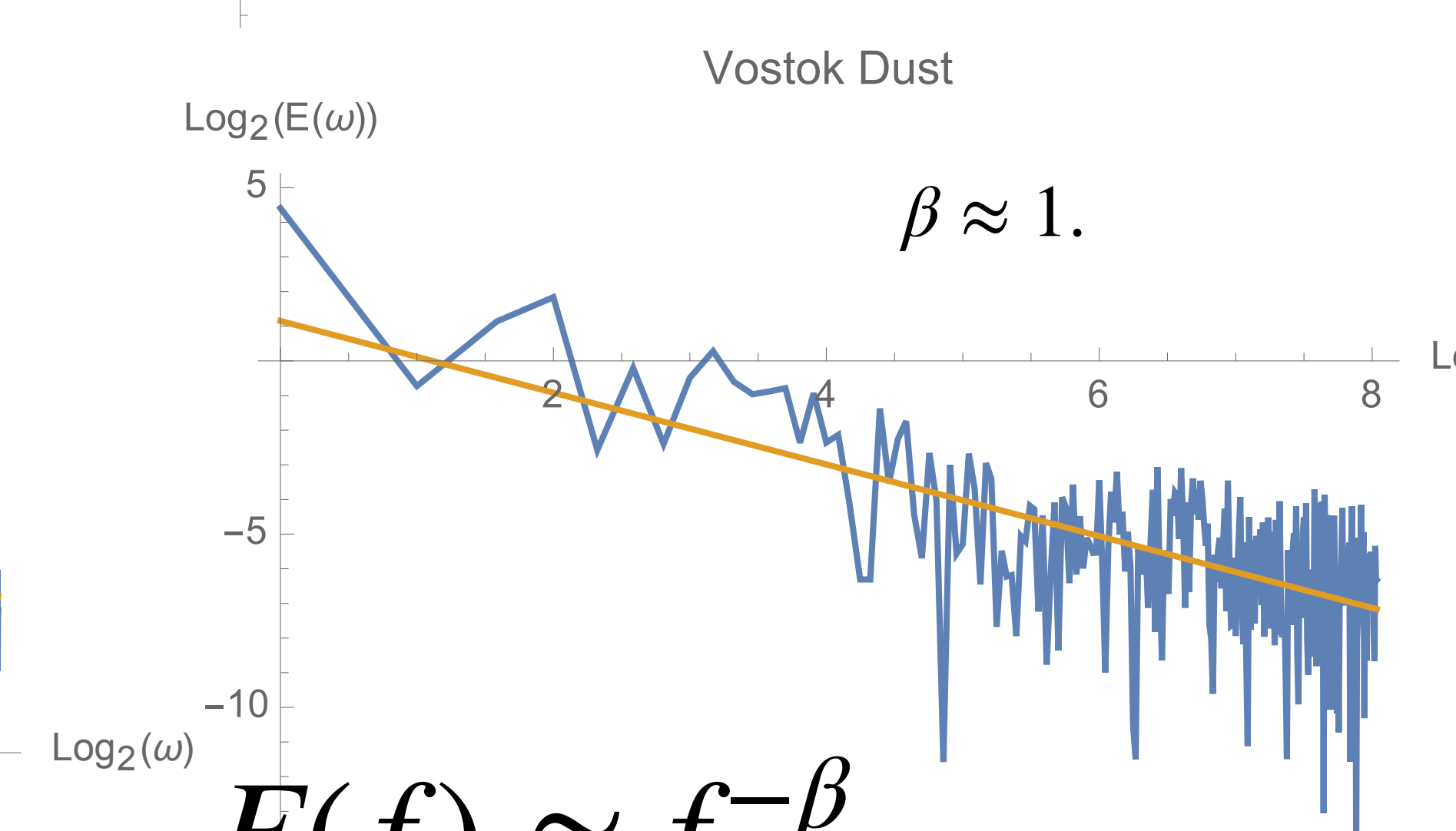
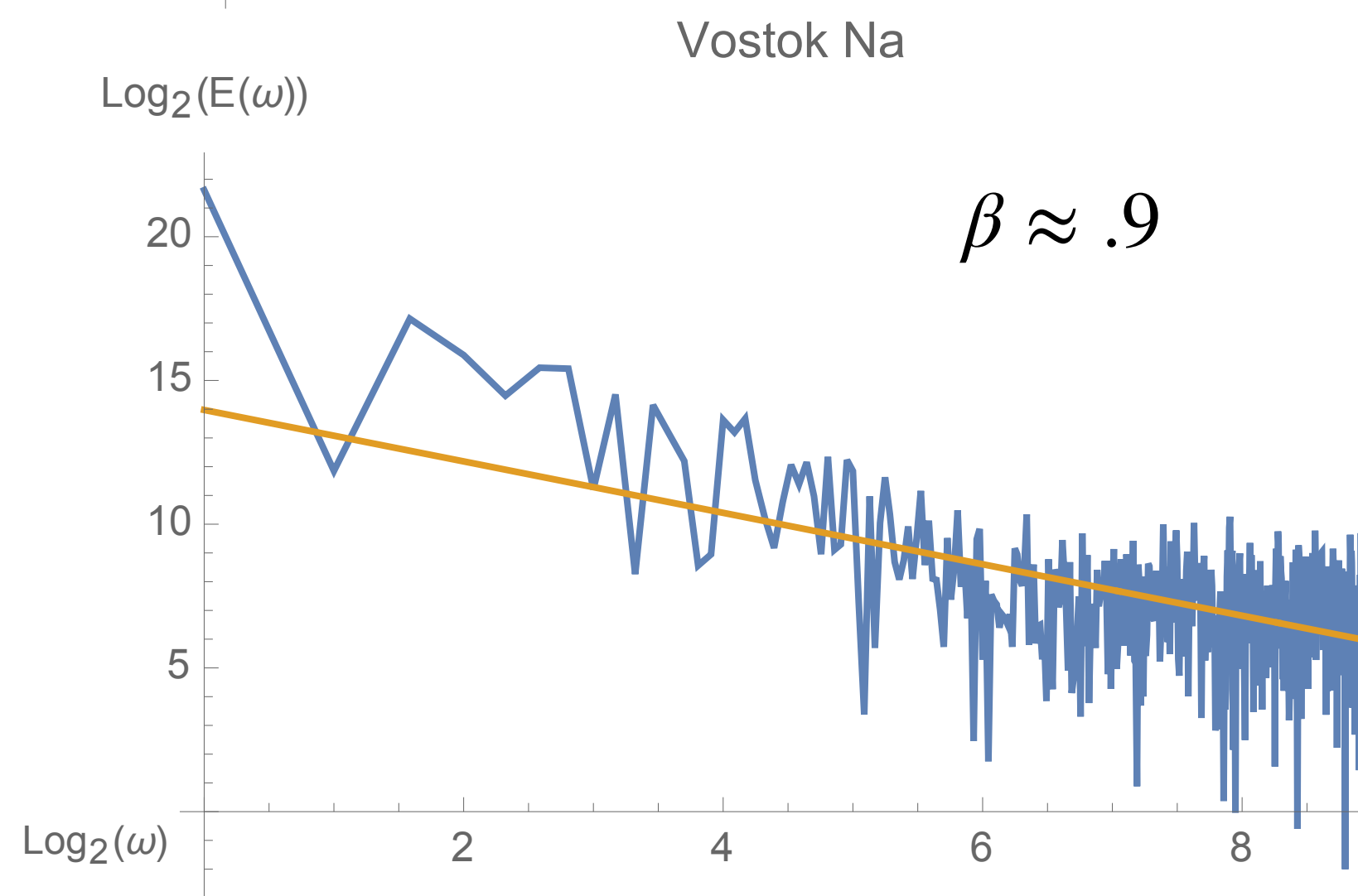
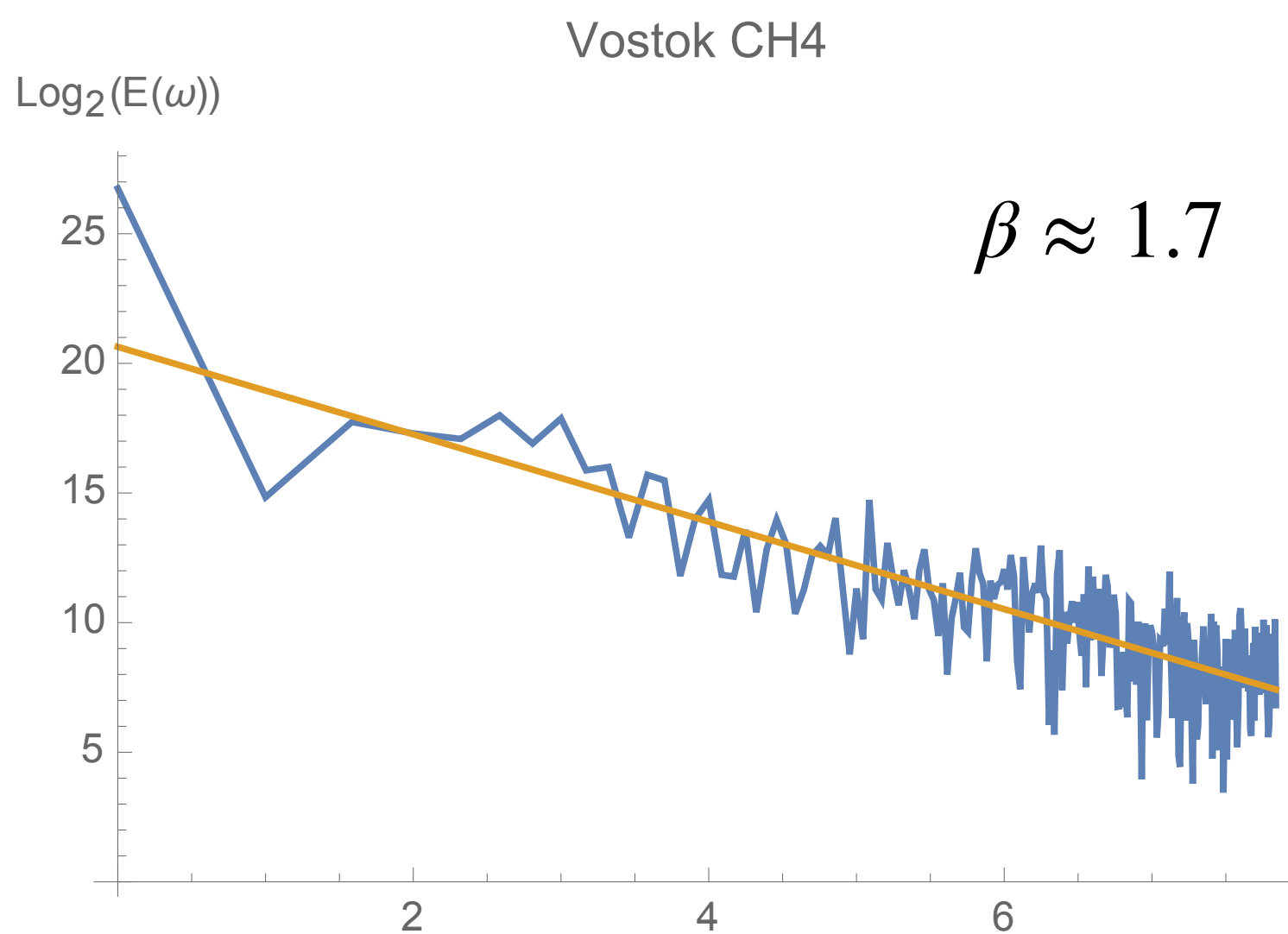
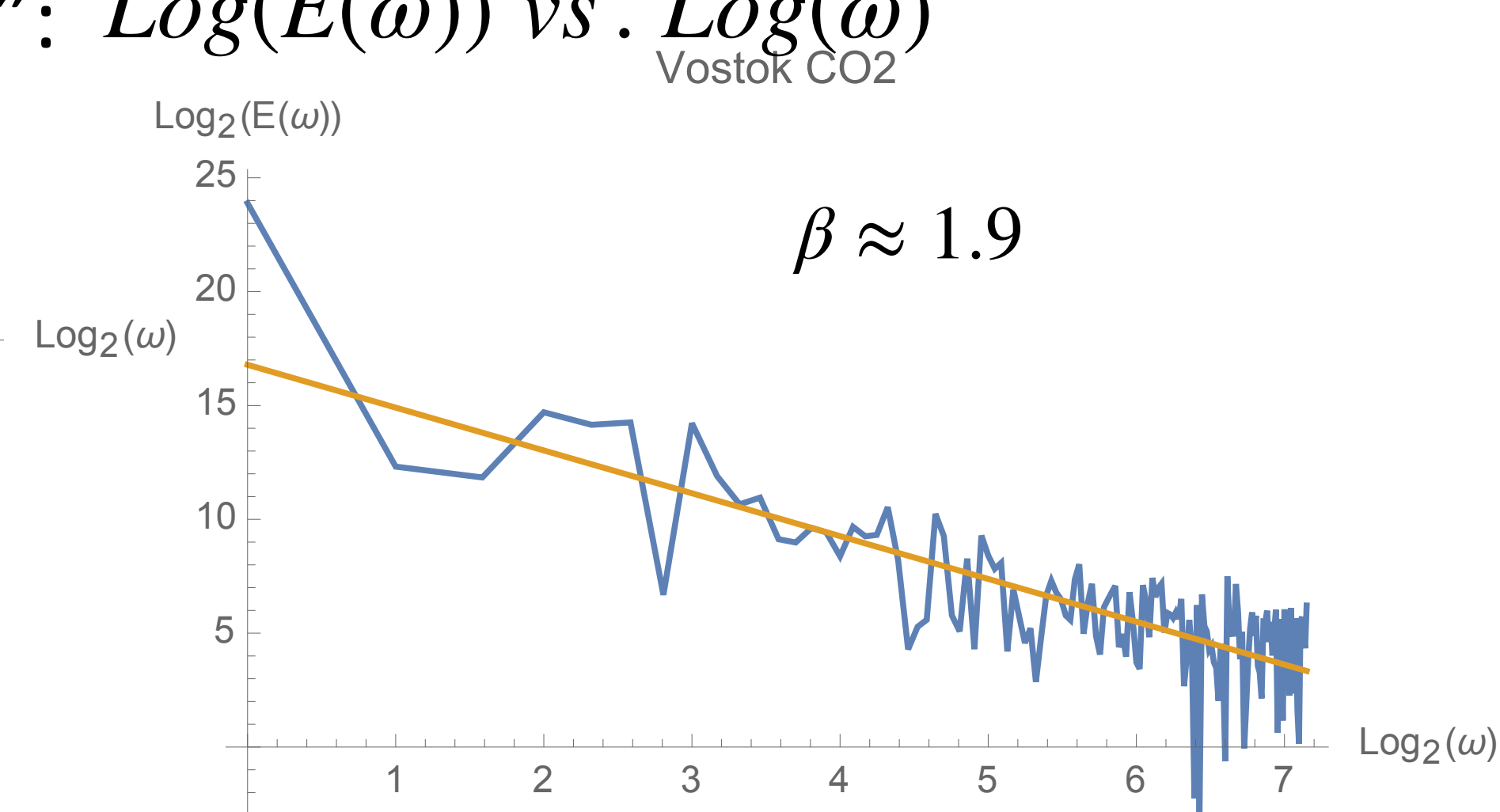
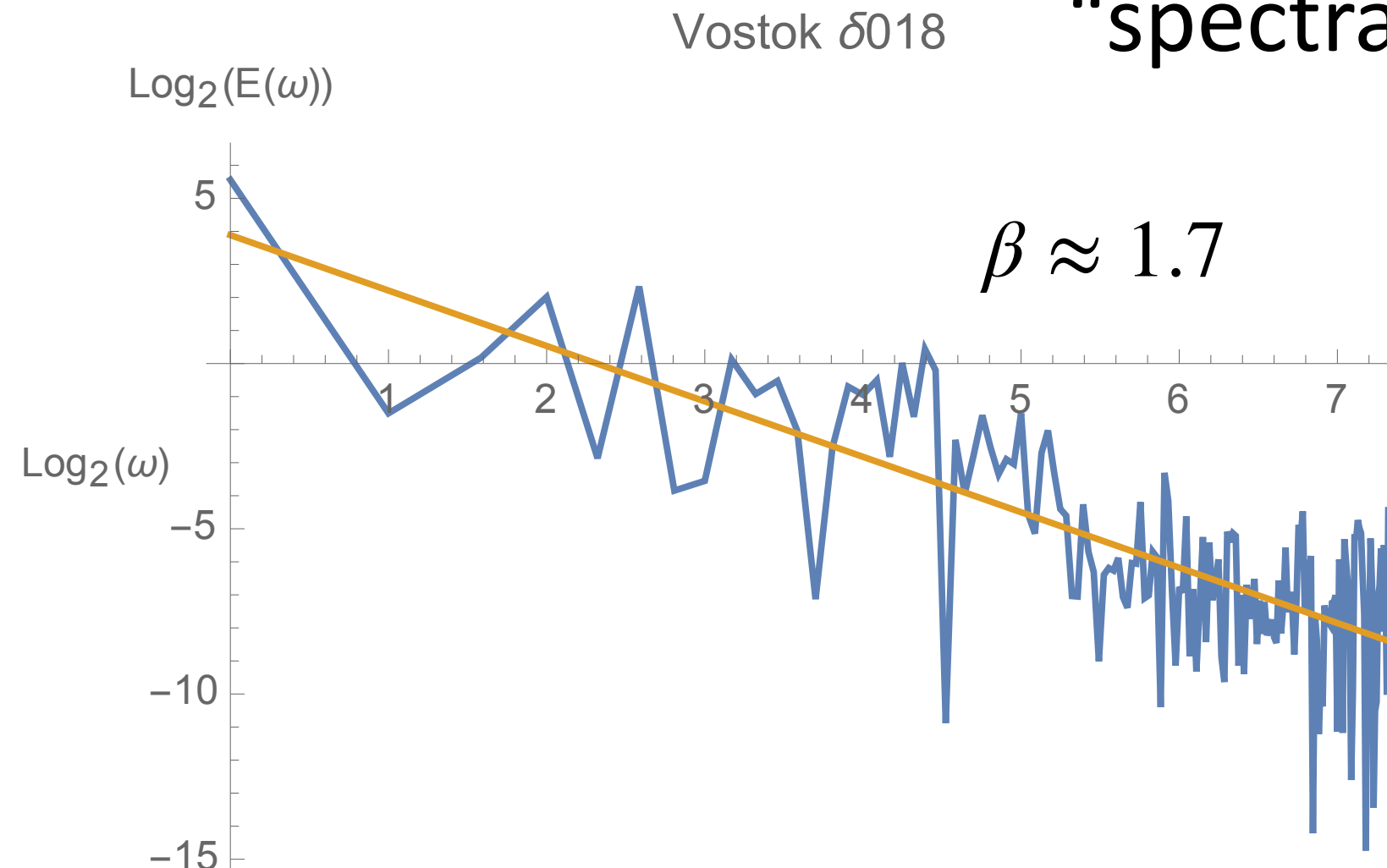
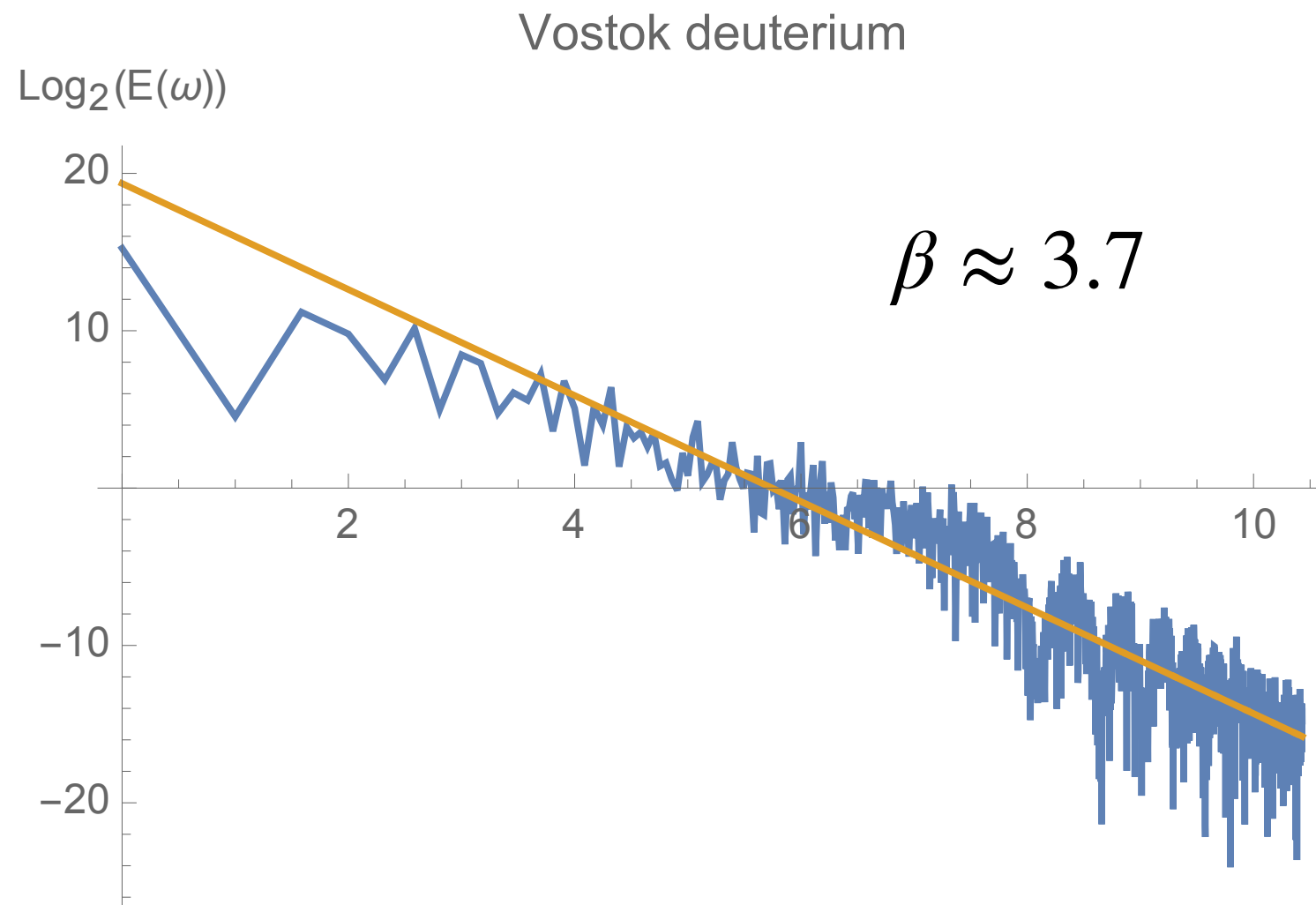
1920202120

100 YEARS of  
MILANKOVIC'S THEORY of  
CLIMATE CHANGE



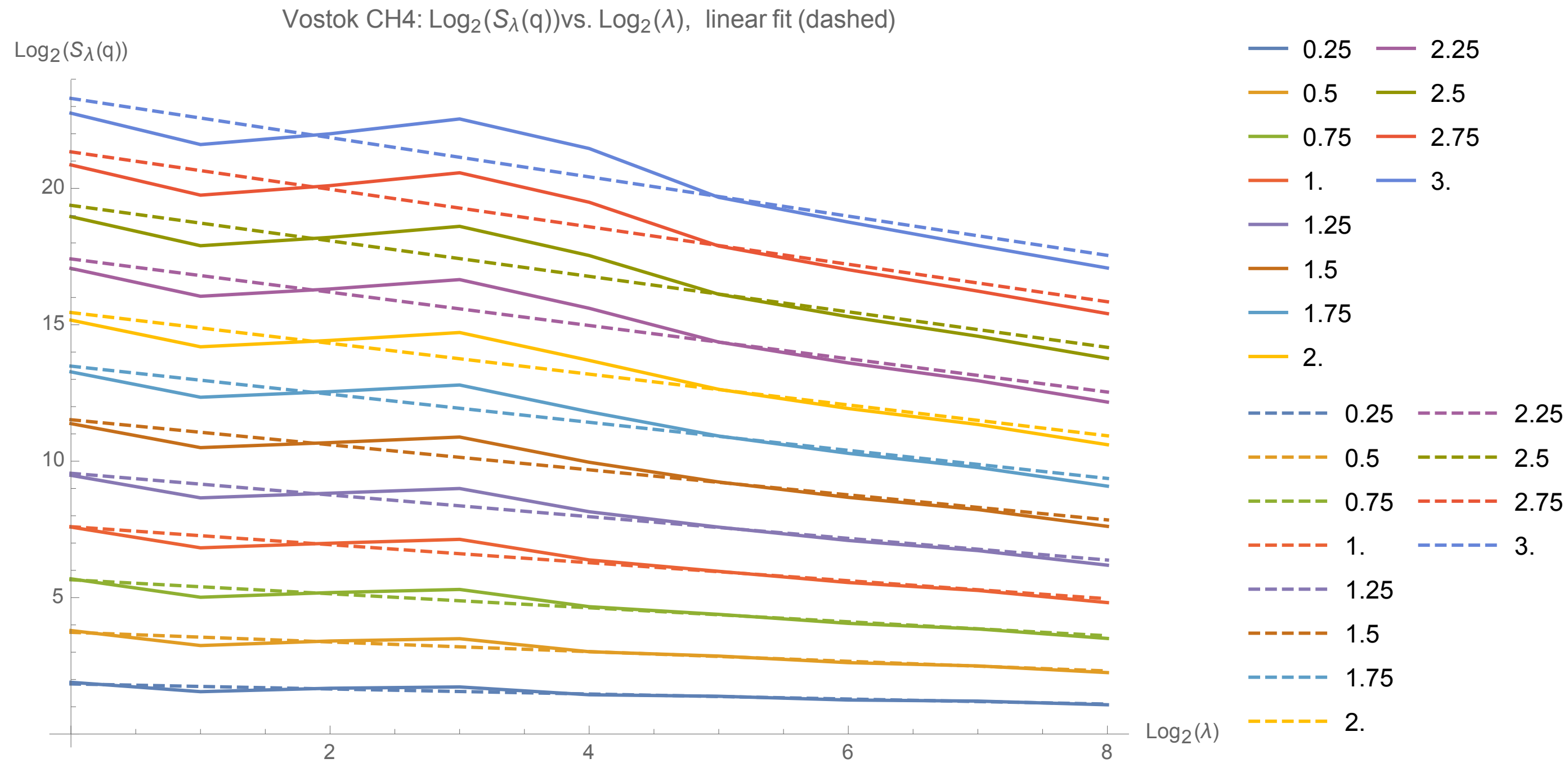
“spectra”:  $\text{Log}(E(\omega))$  vs.  $\text{Log}(\omega)$

Vostok CO2

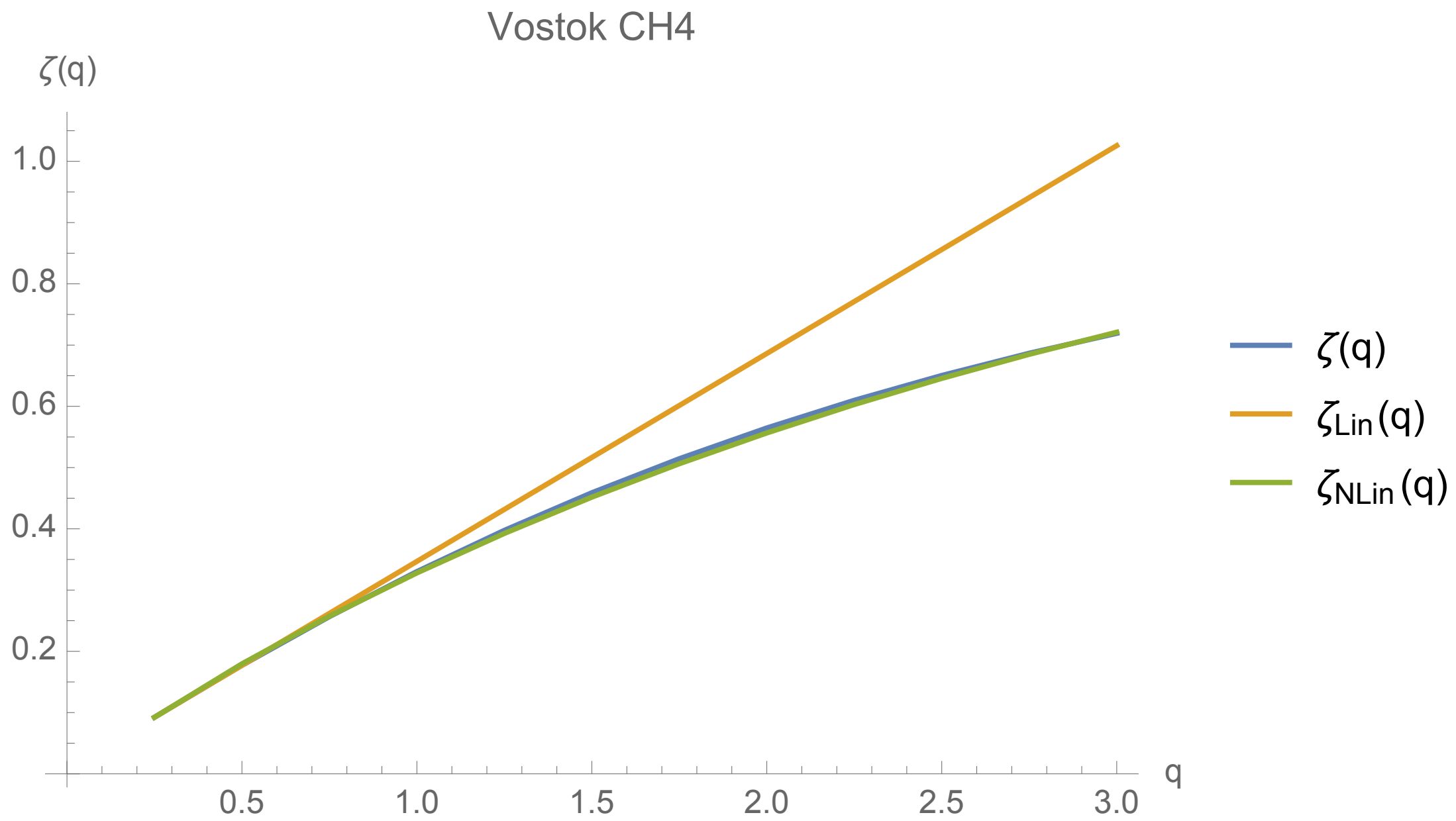


$$E(f) \approx f^{-\beta}$$

# Vostok data: multi-scaling



(a)

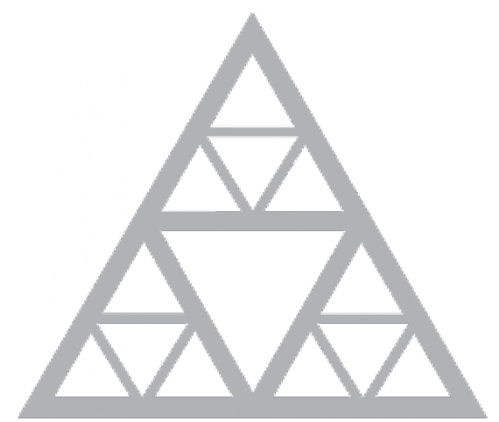


(a)

curvature => multifractality  
 $H \approx 0.34, C_1 \approx 0.07, \alpha \approx 1.4$

Structure functions  $\approx$  generalisation of Fourier spectra ( $q \neq 2$ ):

$$S_\lambda(q) = \left\langle \left( \Delta\theta \left( \frac{T}{\lambda} \right)^q \right) \right\rangle \approx \lambda^{-\zeta(q)}$$

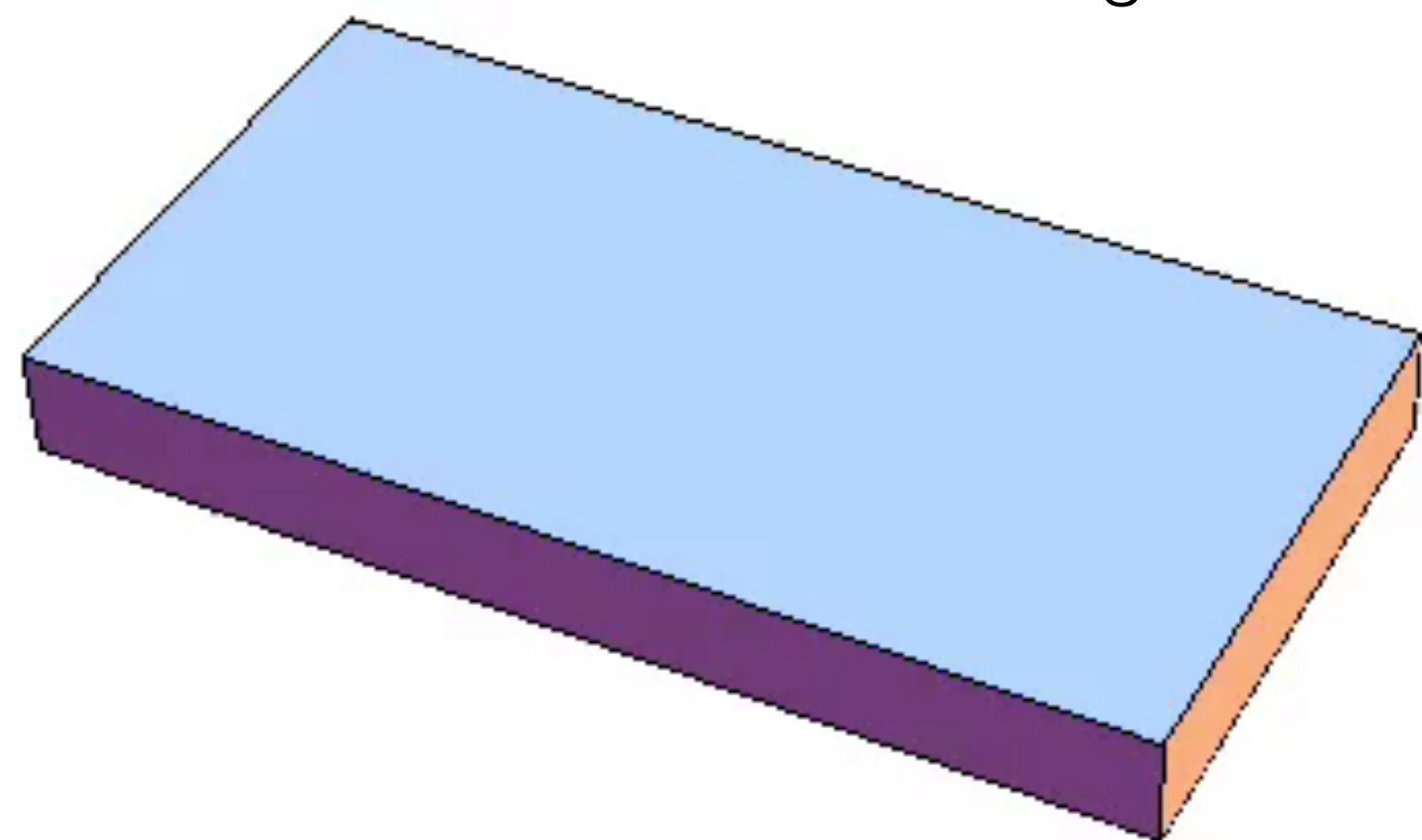


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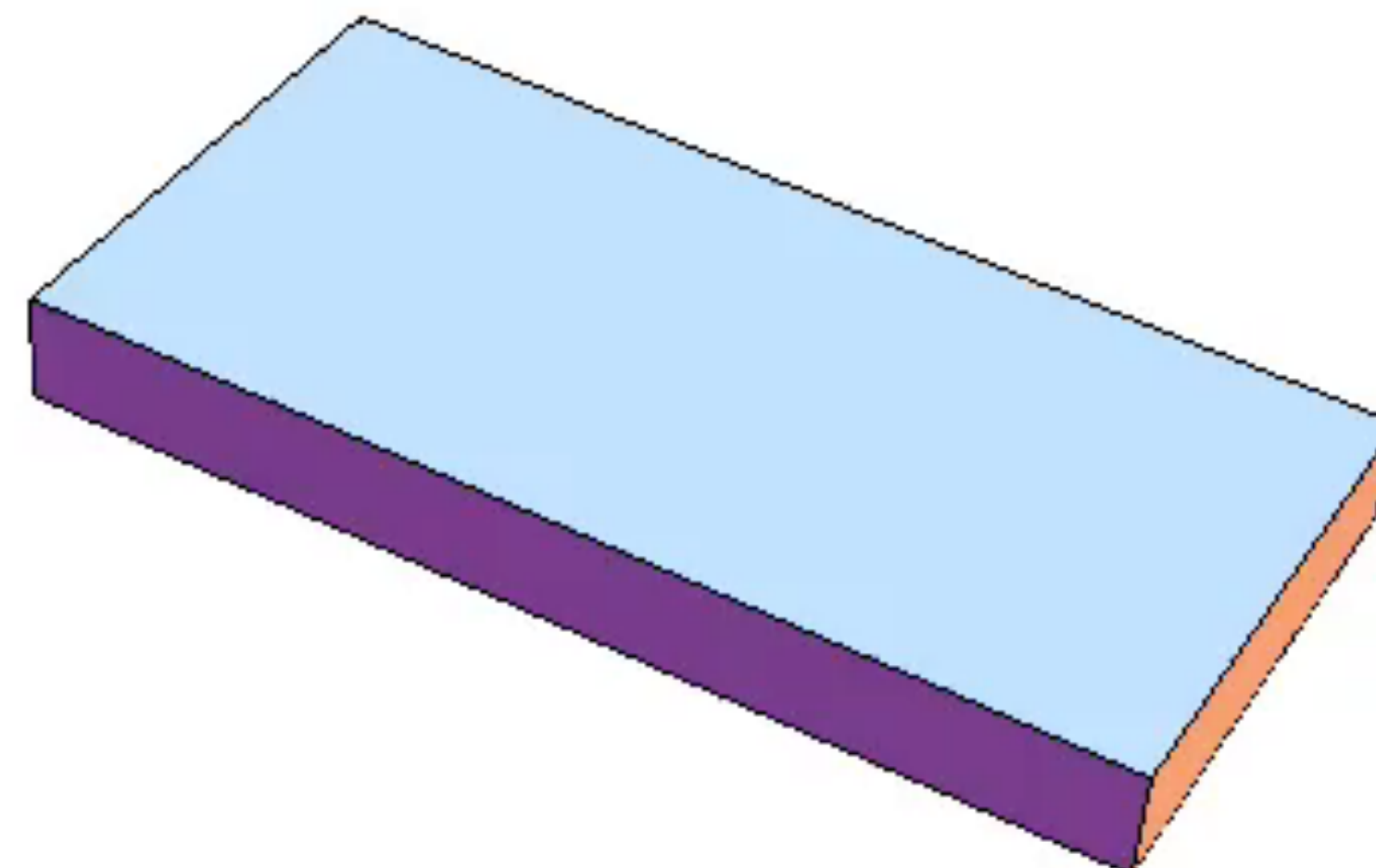
# Gyroscopic instability



initial rotation along the main axis

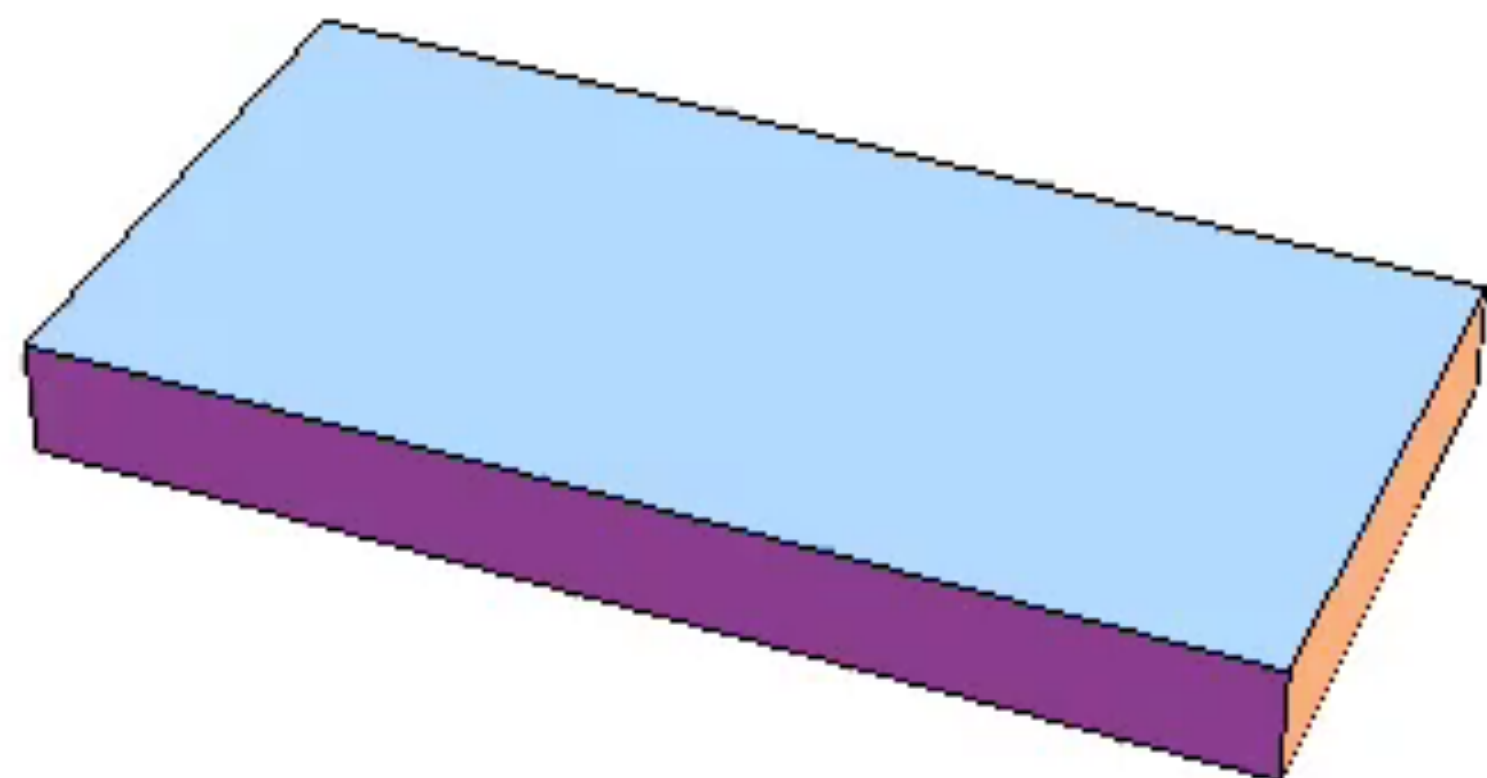


initial rotation along the medium axis



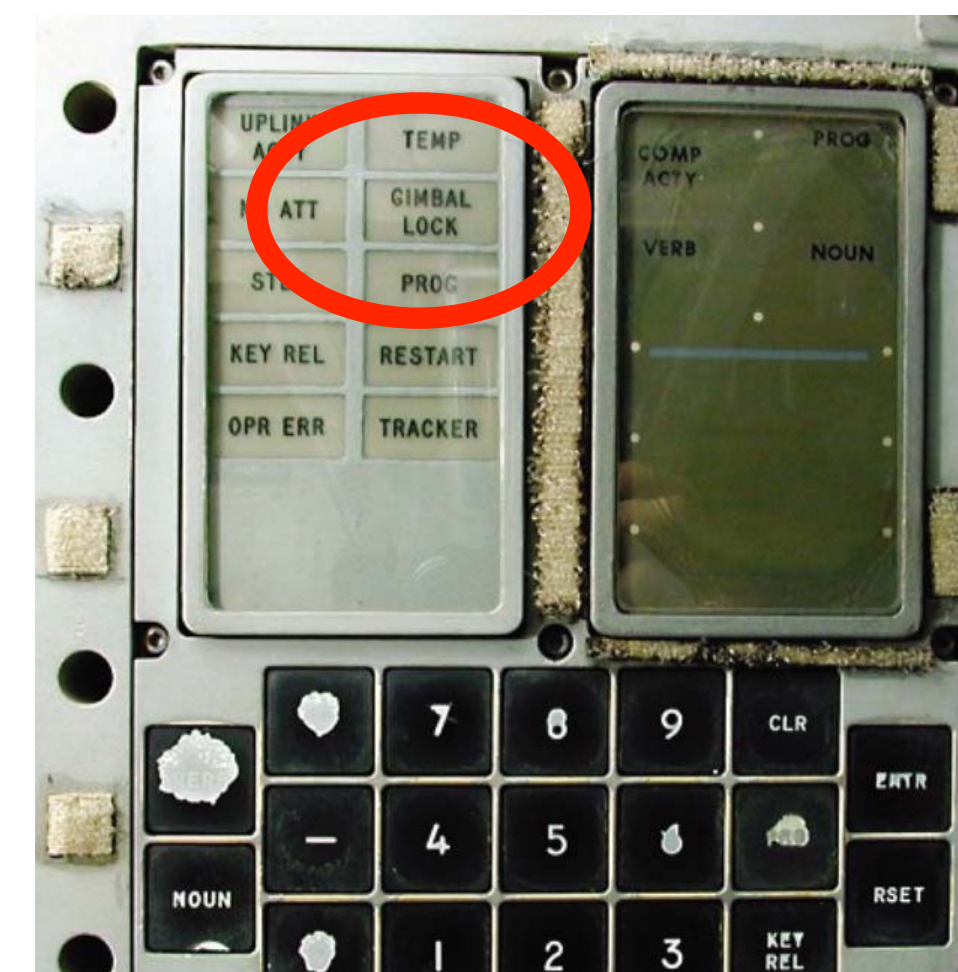
wobble & gimbal lock

initial rotation along the smallest axis



Apollo 13 guidance  
computer console

Hyperion





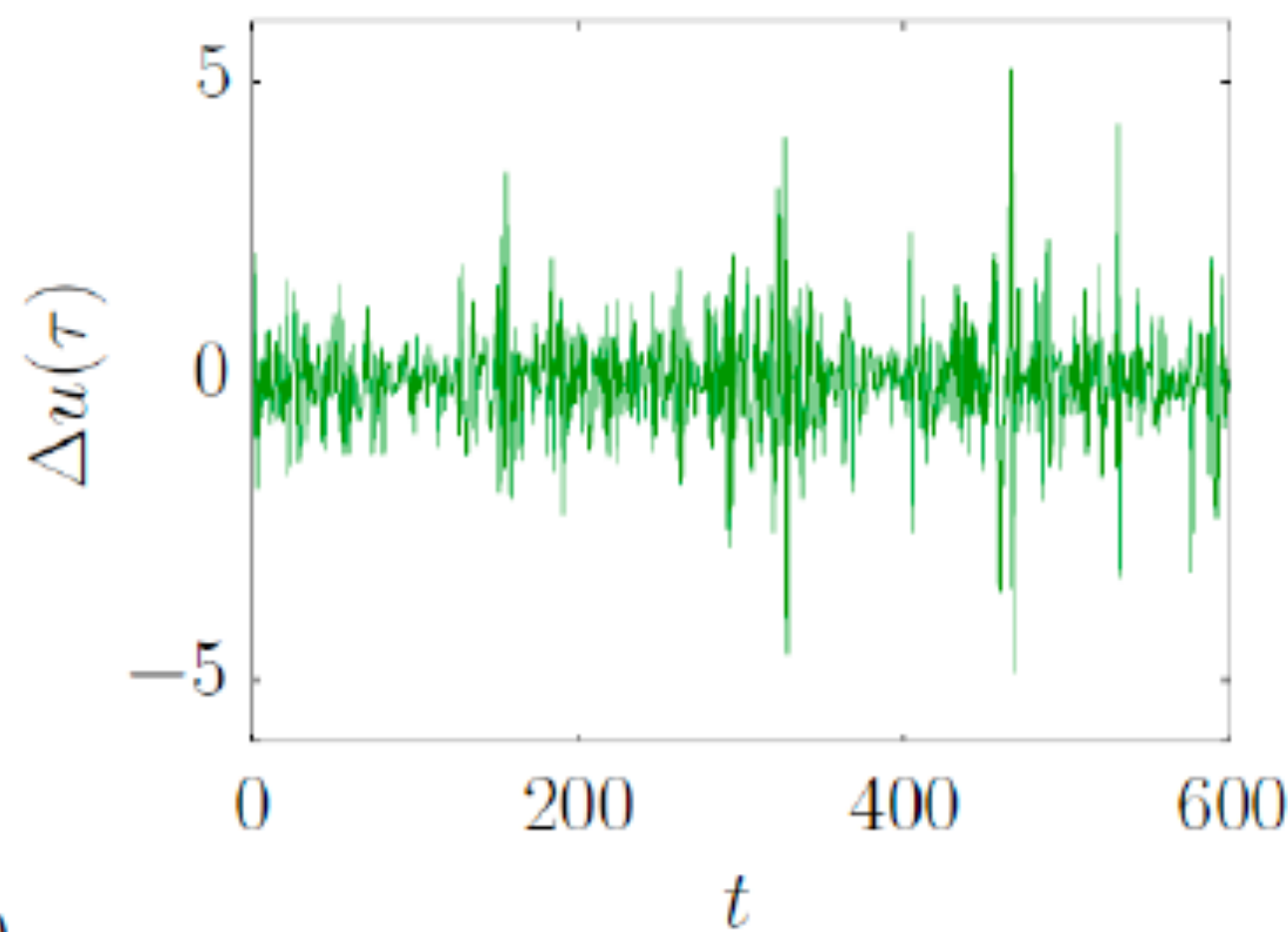
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# 3D Scaling Gyroscope Cascade

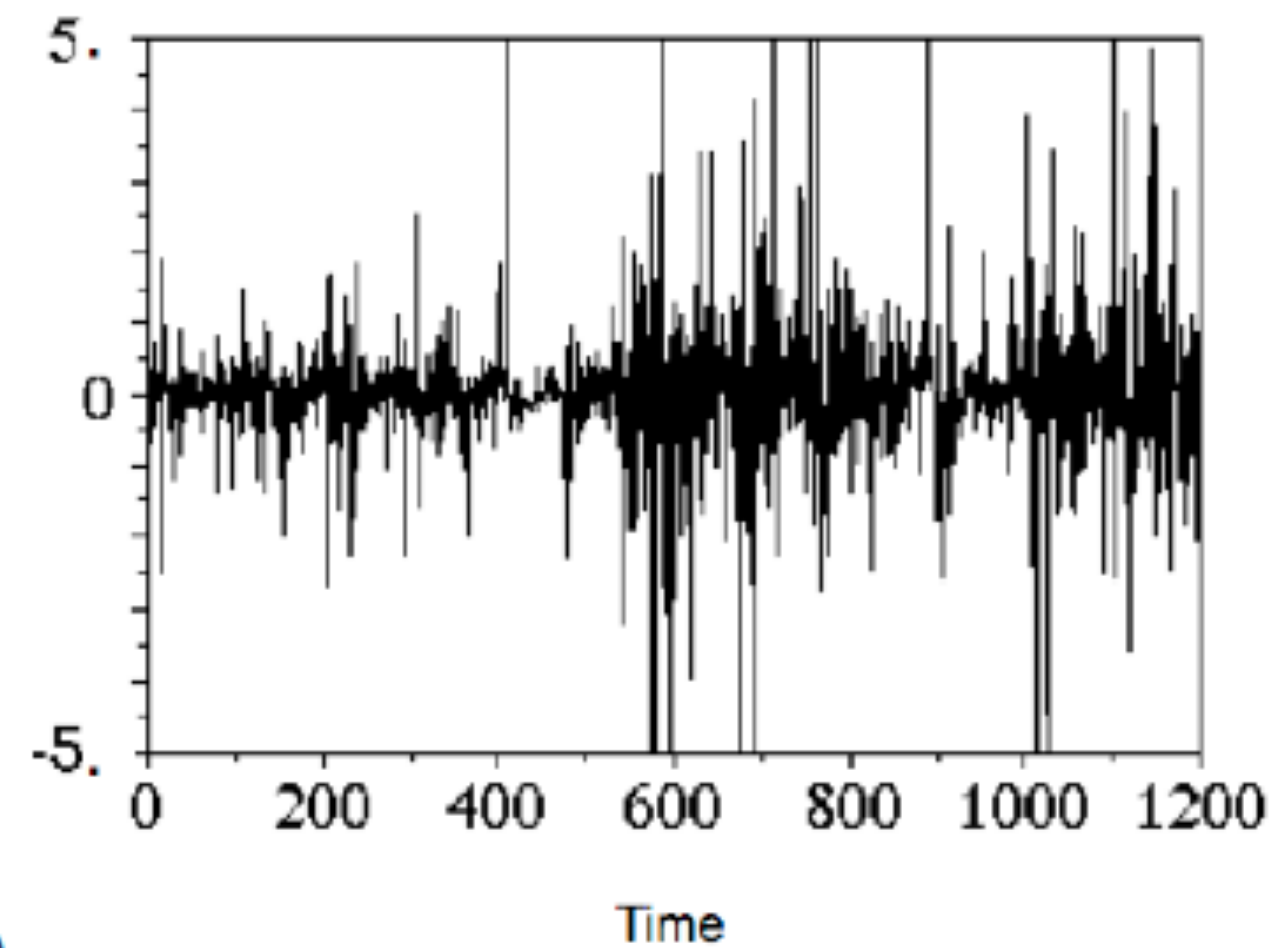


$$\left(\frac{d}{dt} + \nu k_n^2\right) \hat{u}_n^i = i \{ k_{n+1} [ |\hat{u}_{n+1}^{2i-1}|^2 - |\hat{u}_{n+1}^{2i}|^2 ] + (-1)^i k_n \hat{u}_n^i * \hat{u}_{n-1}^{a(i)} \}$$

$a(i)$  is an ancestor.

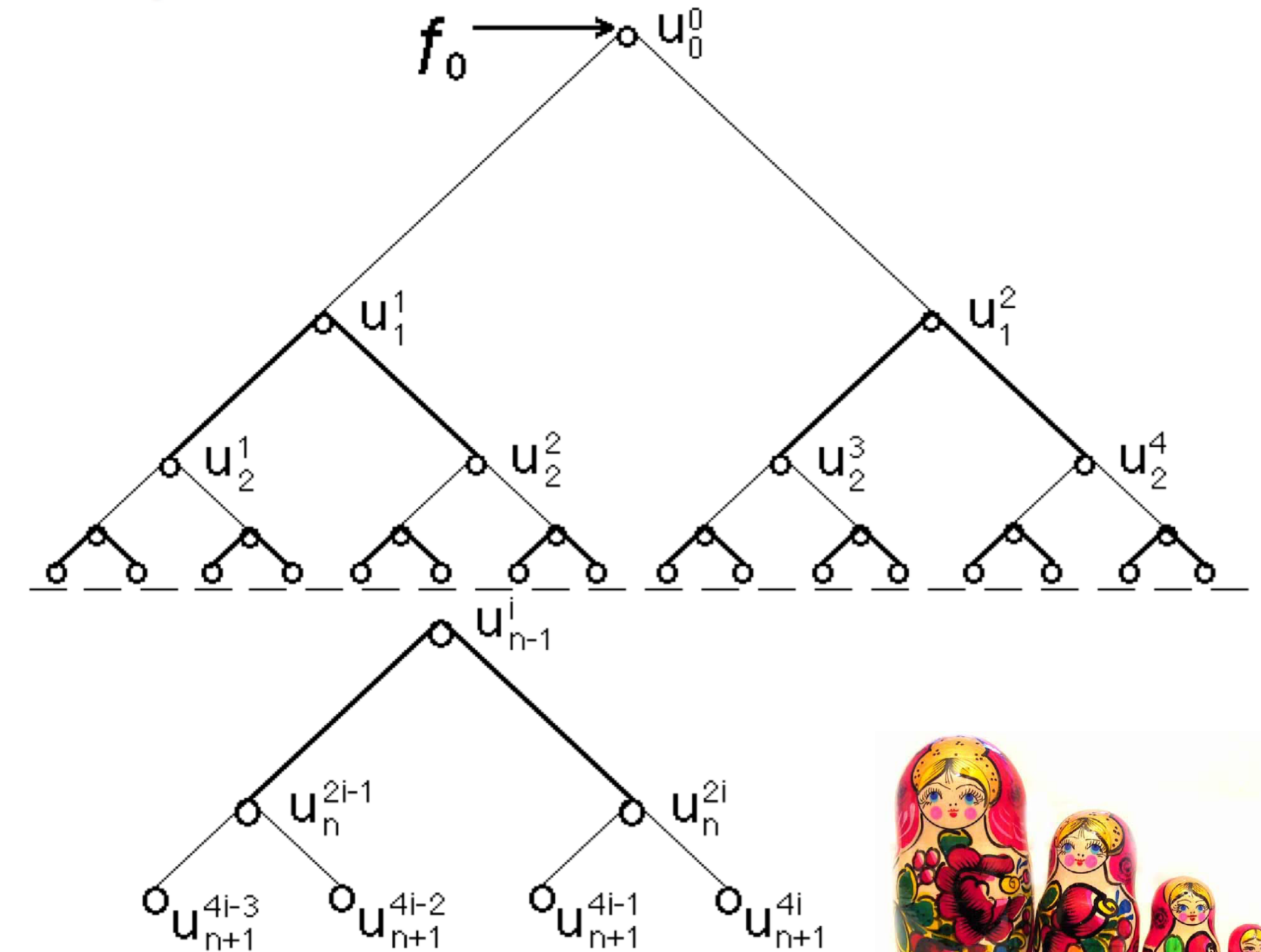


(a)



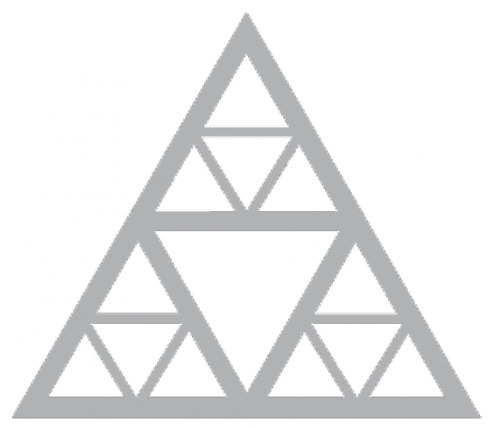
(b)

Figure 2: Comparison of fluctuations: (a) atmospheric turbulence at 100m (Fitton, 2013) and (b) SGC simulation for  $n=6$  (Chigirinskaya and Schertzer, 1996), both display somehow similar strong intermittency.



Local flux of energy:

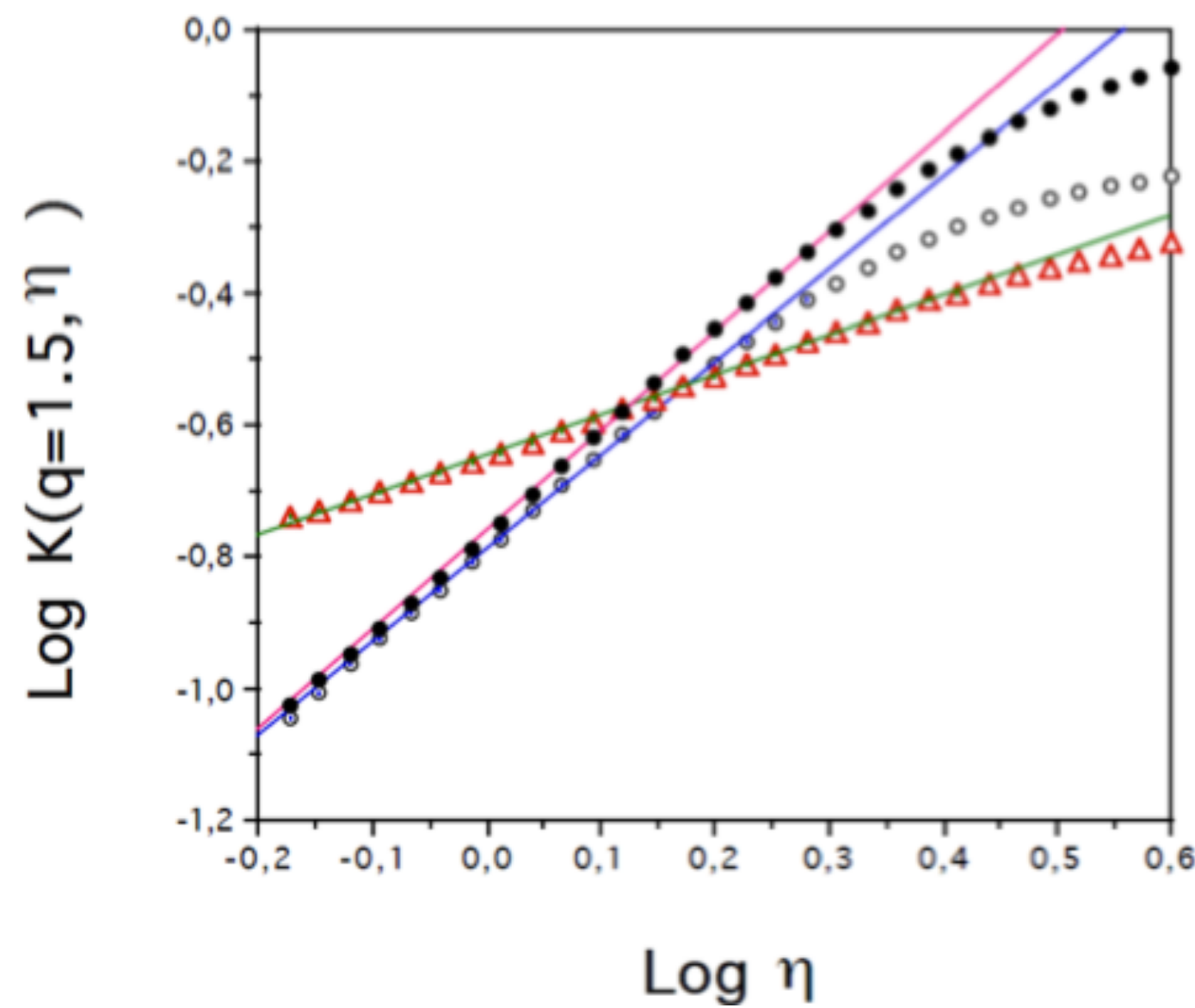
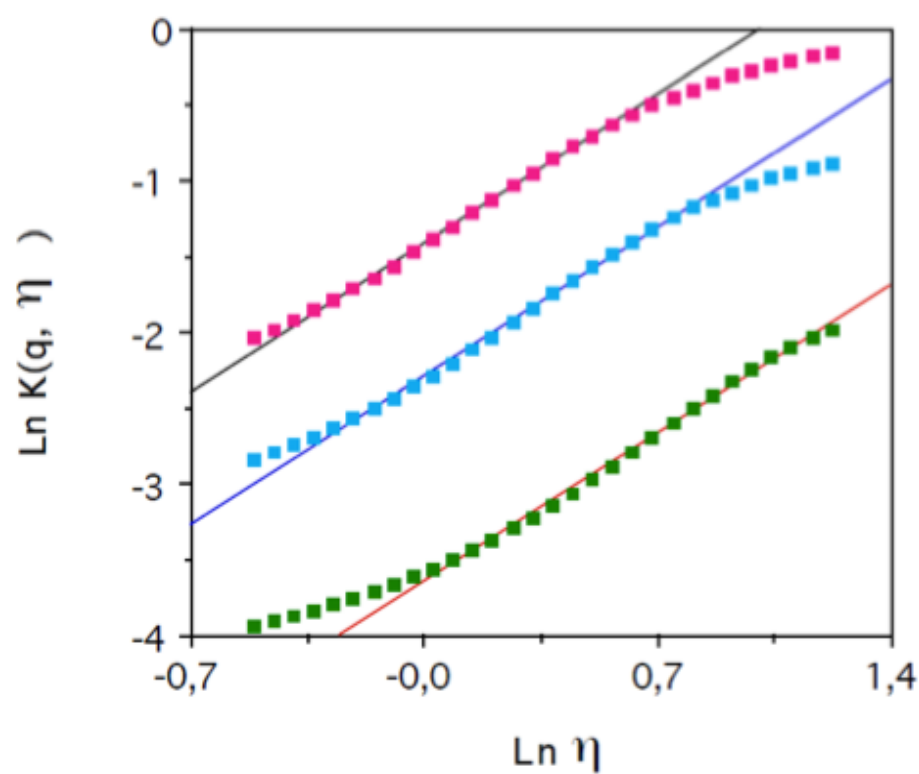
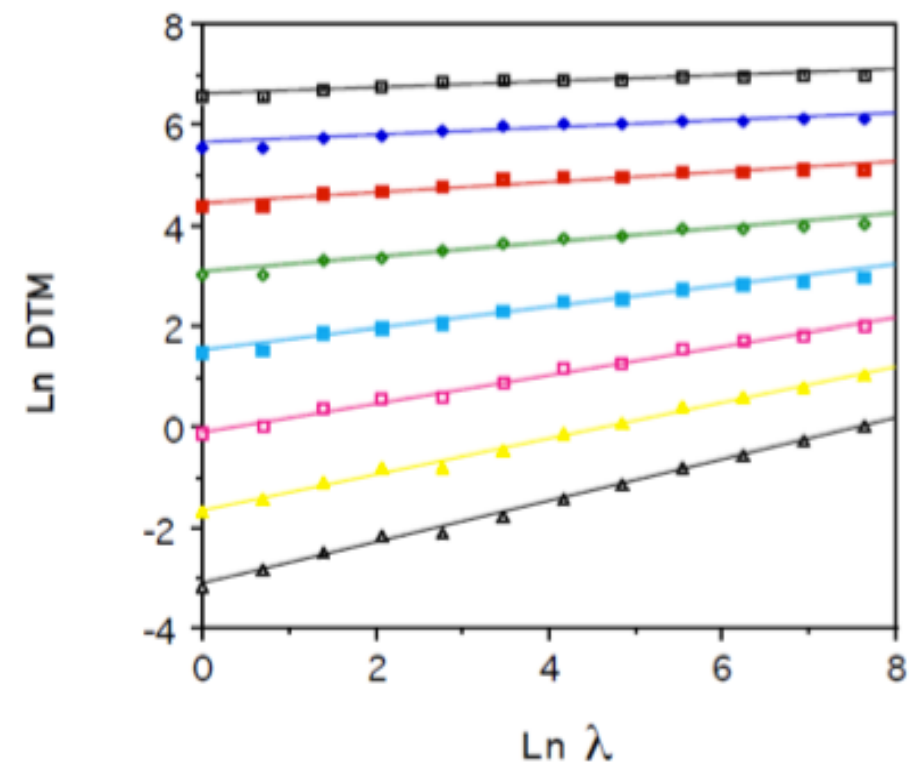
$$\epsilon_n^i = - \sum_{r=0}^n k_{n-r+1} \left[ |\hat{u}_{n-r+1}^{2a^r(i)-1}|^2 - |\hat{u}_{n-r+1}^{2a^r(i)}|^2 \right] \text{Im}(\hat{u}_n^{a^r(i)}) + (-1)^{a^r(i)+1} k_{n-r} \left| \hat{u}_n^{a^{r+1}(i)} \right|^2 \text{Im}(\hat{u}_{n-1}^{a^{r+1}(i)})$$



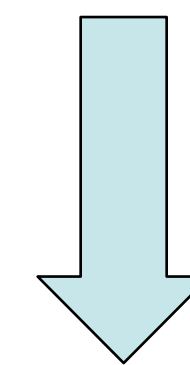
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# SGC vs. shell-models

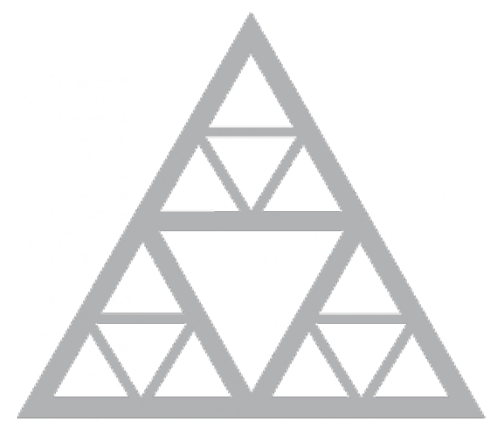
Scaling of DTM curves with  $q = 1.5$ ,  $\eta = \overline{0.5, 3.5}$  (for simulated energy flux).



Comparison between:	Lévy index of the generator	codimension of the mean fractality
SGC-3D space	$\alpha = 1.4 \pm 0.05$	$c_1 = 0.25 \pm 0.05$
SGC-3D time	$\alpha = 1.5 \pm 0.05$	$c_1 = 0.25 \pm 0.05$
Shell-model of the SGC (time)	$\alpha = 0.6 \pm 0.05$	$c_1 = 0.4 \pm 0.05$



SGC-3D has a 1st order phase transition: FAT TAILED distribution  
Shell model (SGC-1D) has not !



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# Mr Jourdain and Lie cascades



What is general and theoretically straightforward:

- $exp : Lie\ algebra \mapsto Lie\ group$

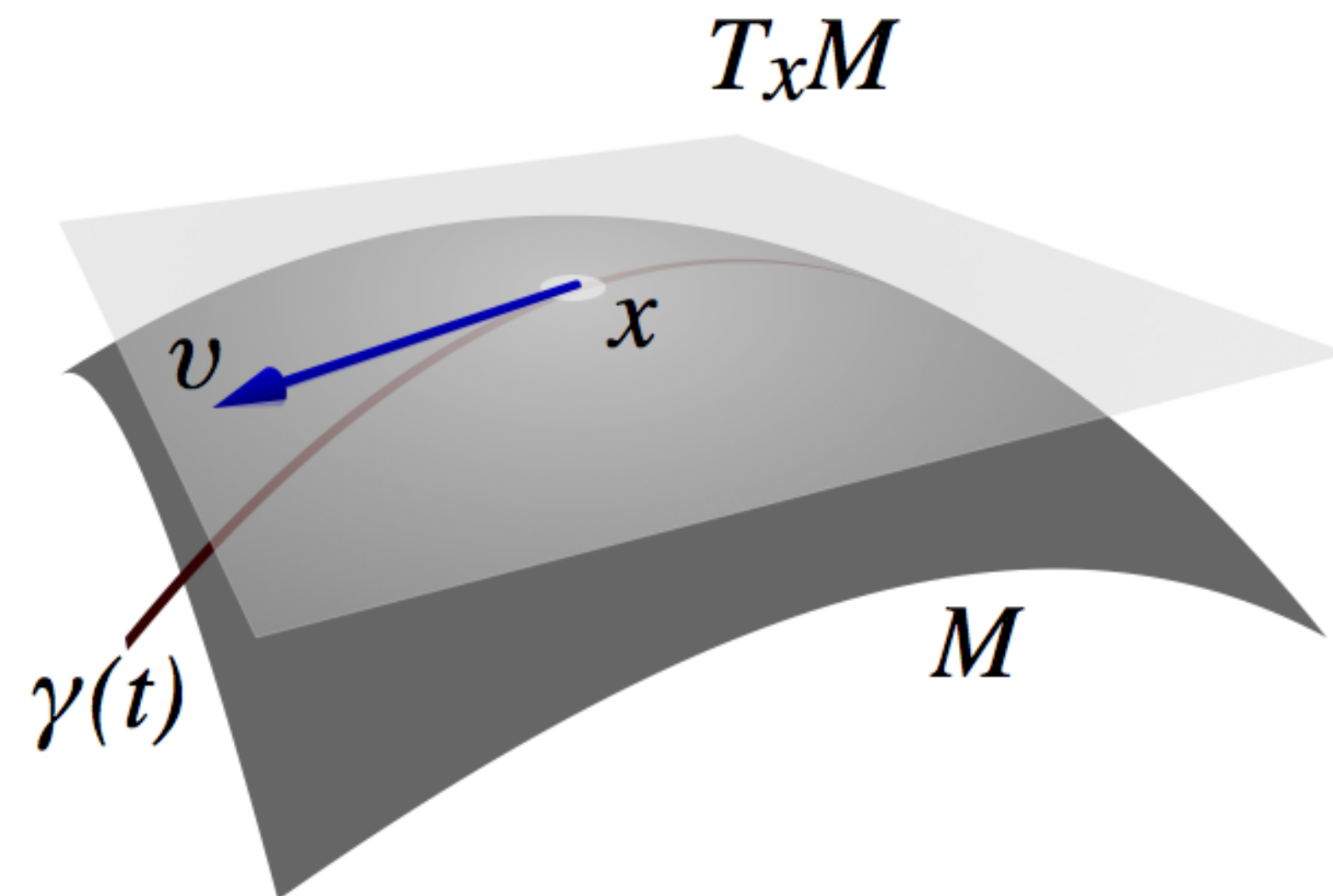
scalar valued cascade:  $R^d \dashrightarrow R^+$

- Lie group: smooth manifold
- Lie algebra: tangent space to the group at the identity
- therefore a vector space with a skew product that satisfy the Jacobi identity:

$$[X, [Y, Z]] + [Y, [Z, X]] + [Z, [X, Y]] = 0$$

exemple: commutator of matrices

$$[X, Y] = XY - YX \quad [X, Y] = 0 \Rightarrow \exp(X + Y) = \exp(X) \exp(Y)$$





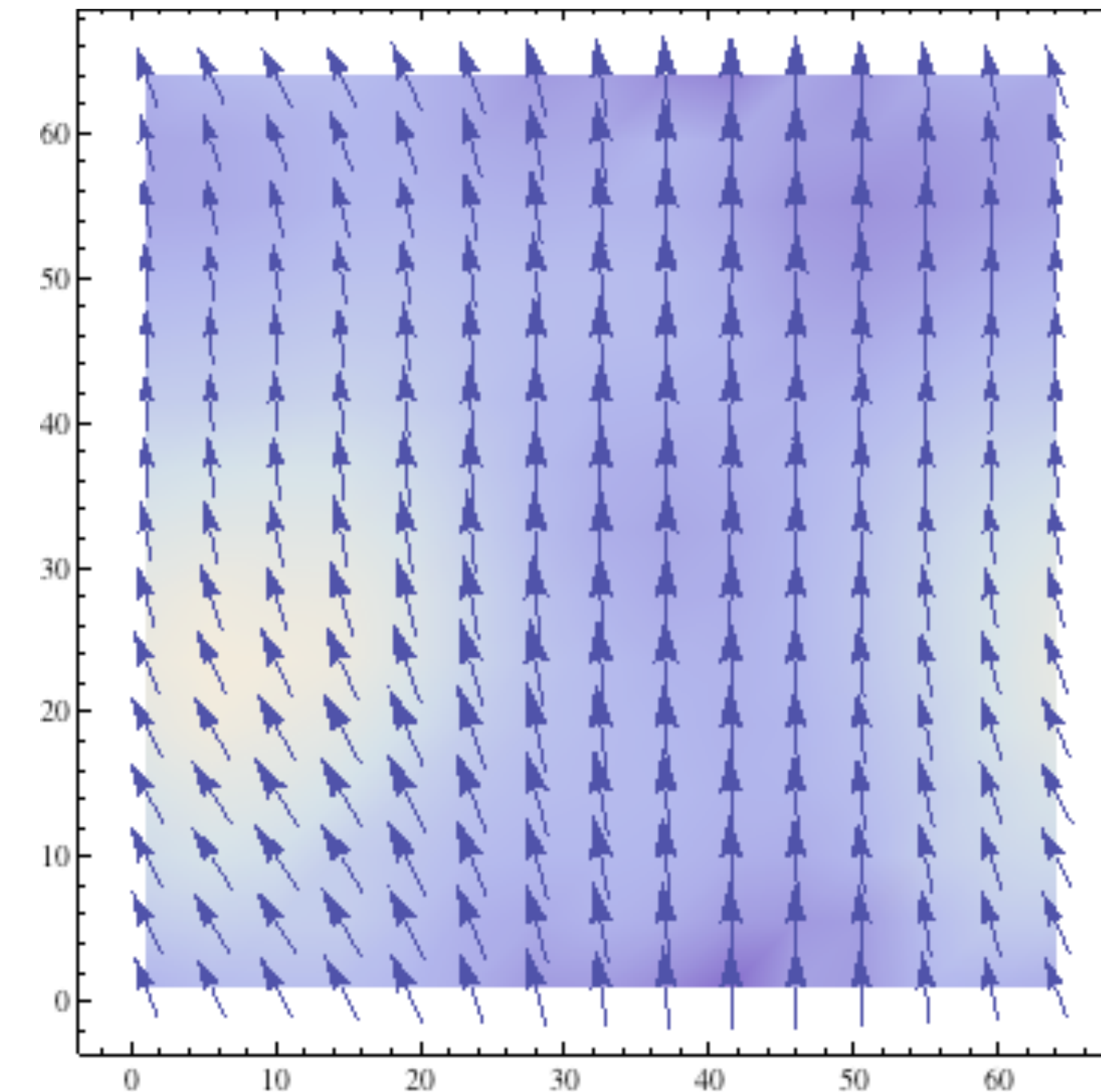
# Surface layer complexity!

explOatorium®



Art piece 'Windswept' (Ch. Sowers, 2012): 612 freely rotating wind direction indicators to help a large public to understand the complexity of environment near the Earth surface

WAUDIT Wind resource assessment  
audit and standardization



Multifractal FIF simulation (S et al., 2013) of a 2D+1 cut of wind and its vorticity (color). This stochastic model has only a few parameters that are physically meaningful.

Both movies illustrate the challenge of the near surface wind that plays a key role in the heterogeneity of the precipitations... and wind energy!

# Fractionnaly Integrated Flux model (FIF, vector version)

FIF assumes that both the renomalized propagator  $G_R$  and force  $f_R$  are known:

$$G_R^{-1} * u = f_R$$

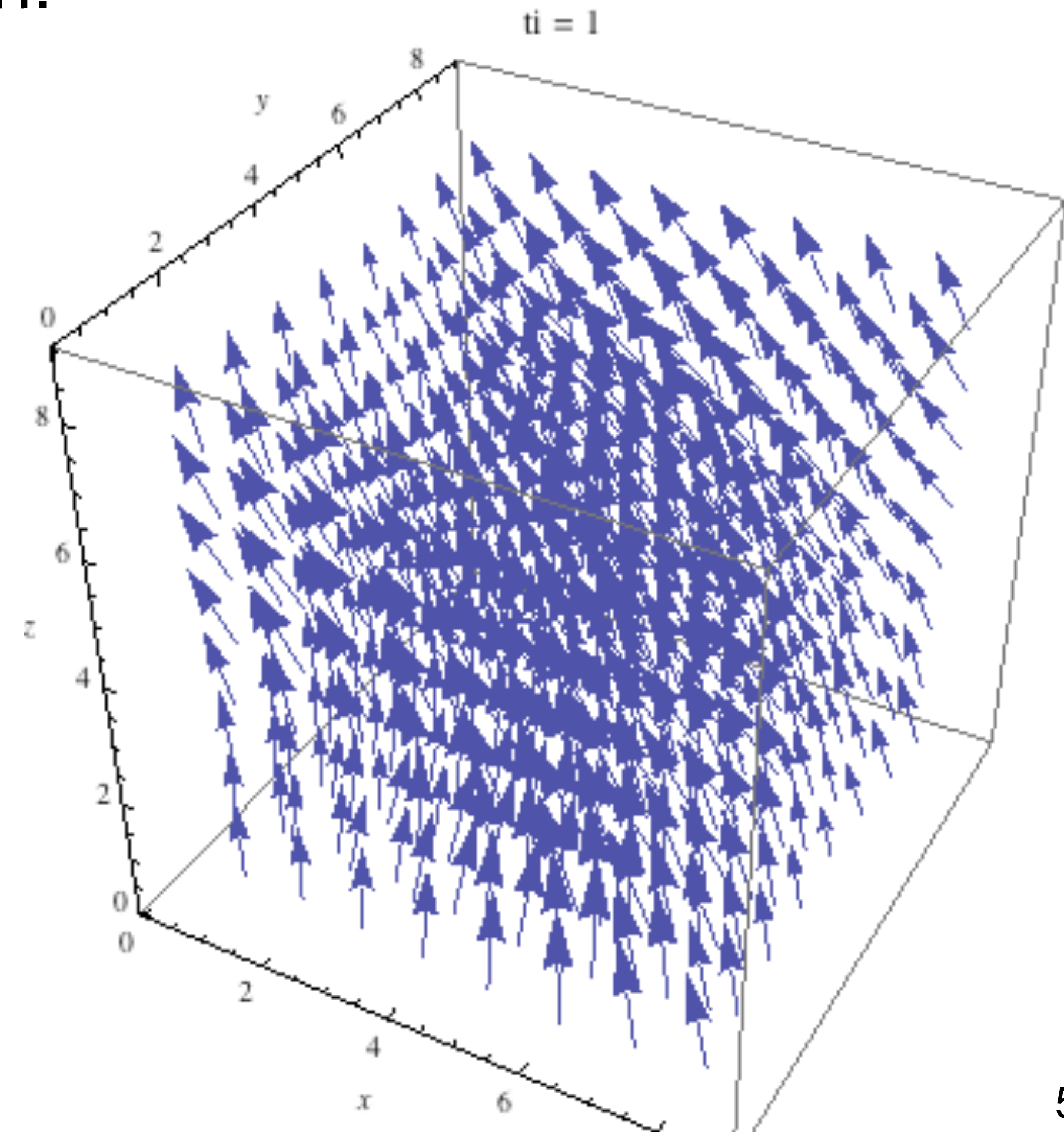
where:

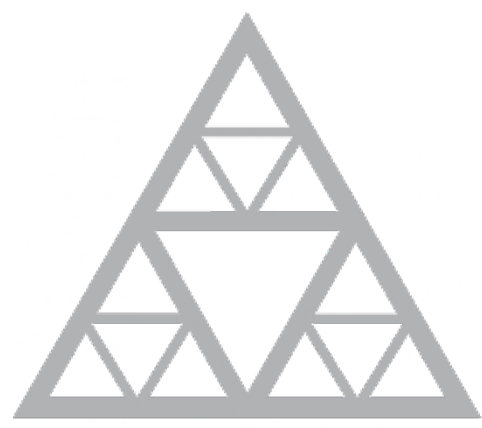
$$f_R = \varepsilon^a$$

$G_R^{-1}$  is a fractionnal differential operator

$\varepsilon$  results from a continuous, vector, multiplicative cascade (Lie cascade)

3D FIF wind simulation based on quaternions





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# UN SDGs

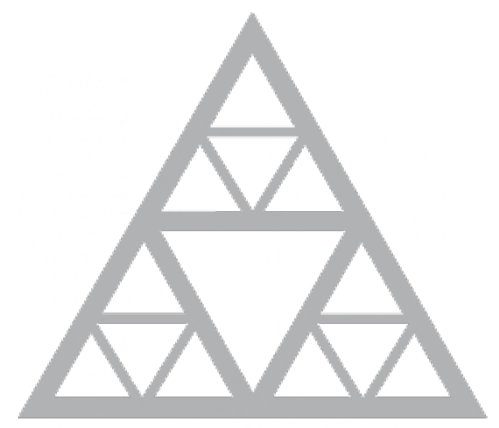


HYDROLOGY METEOROLOGY and COMPLEXITY

- UN 2030 Agenda and follow-ups: a **vibrant call to engineering intelligence**
- not achievable without
  - **advanced observation technologies** (e.g. small radars and lidars)
  - **innovations** (e.g., Blue Green/Nature Based solutions)
  - **synergic, integrative approaches** overcoming the traditional silo thinking
  - **disruptive methodologies** (multiscale analysis, simulation and modelling) for full deployment of the above

SUSTAINABLE DEVELOPMENT GOALS  
17 GOALS TO TRANSFORM OUR WORLD





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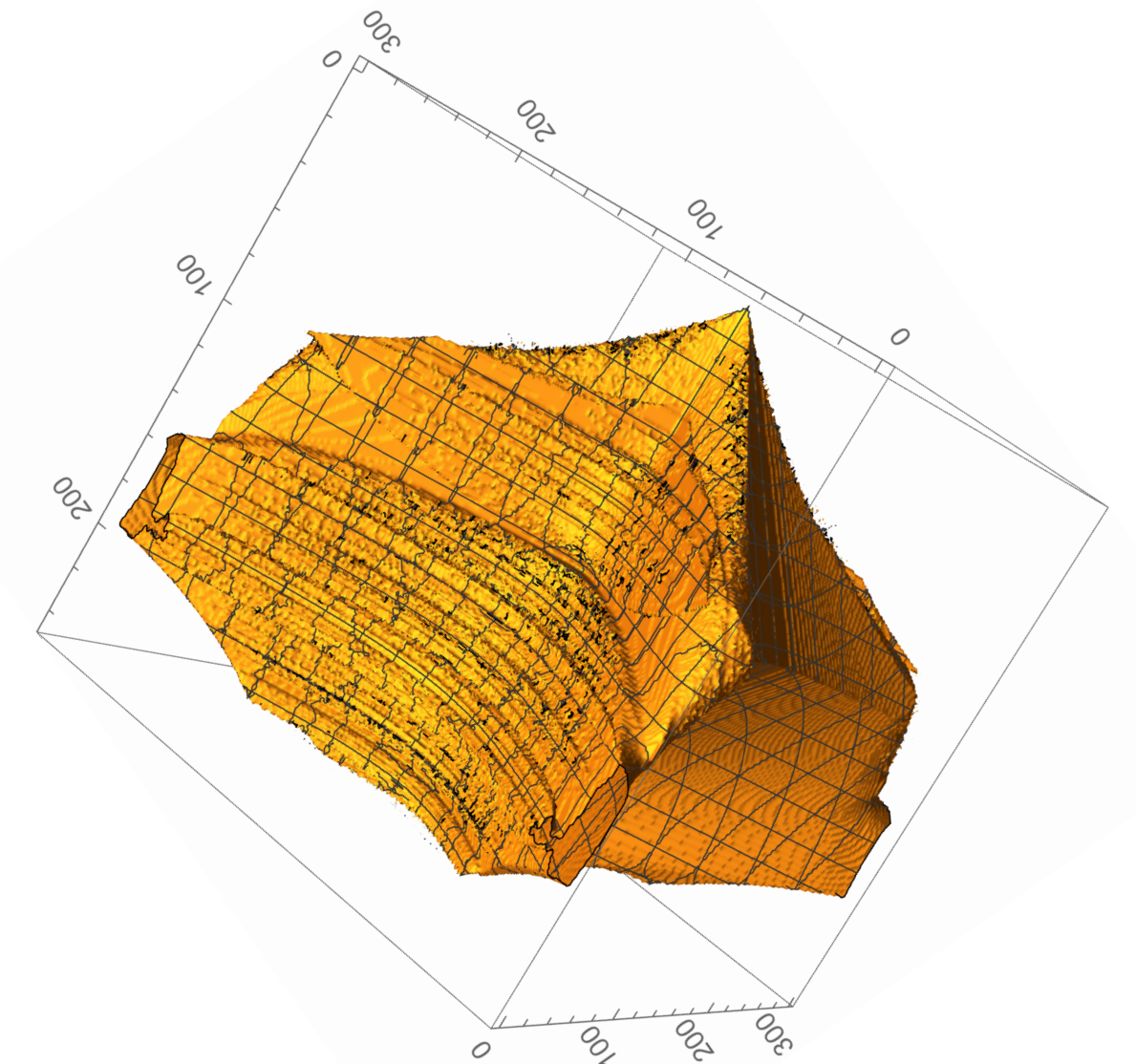
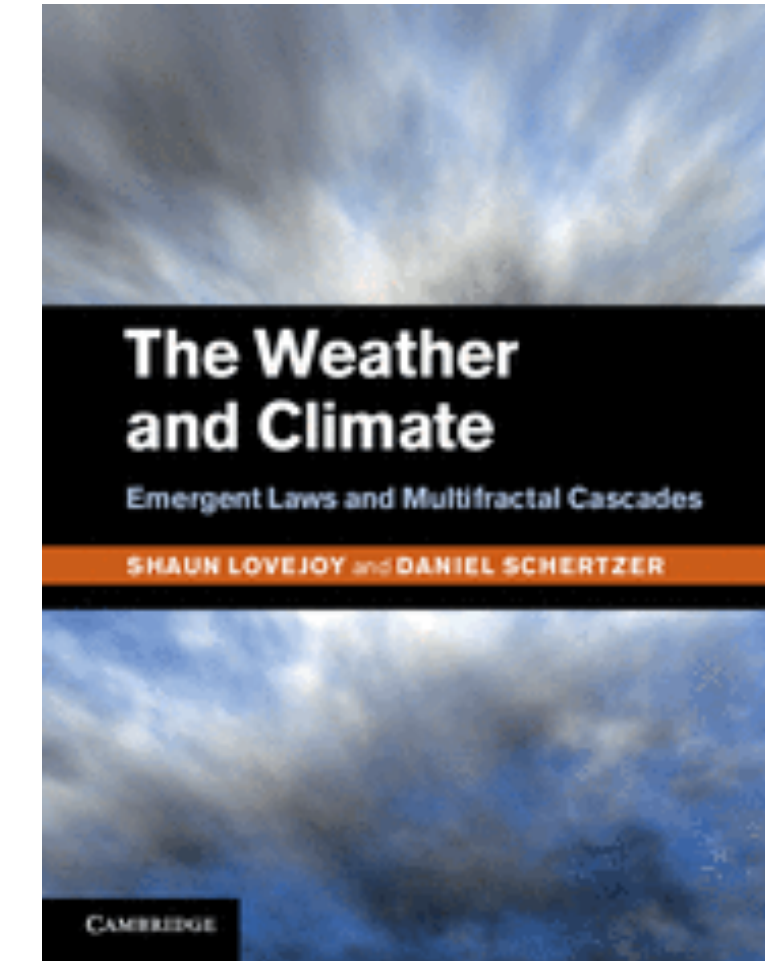
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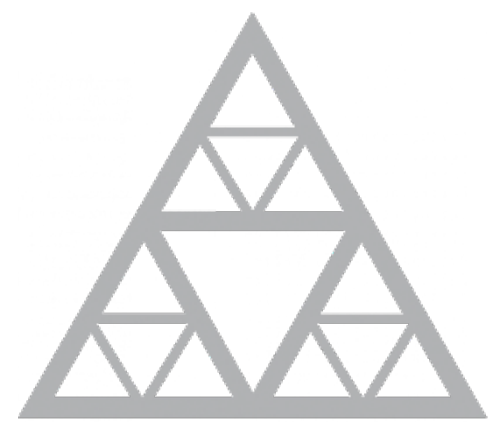
# Conclusions

S&T, Earth& Space, 2020  
Chaos 2015, S&aI. ACP, 2012,  
S&L, IJBC, 2011,  
Fitton&aI., JMI 2013



- Intermittency: a key issue in physics, mechanics, geophysics and elsewhere
- Breakthroughs with cascades and multifractals
  - intermittency across scales
  - New approaches of extremes, predictability and predictions
  - Beyond commonalities significant differences of formalisms
- Beyond scalar valued fields
  - Scaling Gyroscopes Cascades
  - Multifractal operators: exponentiation of a stochastic Lie algebra of generators
  - Multifractal multivariate analysis





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# Conclusions



Final conclusion: Philip Anderson (Phys. Today 41 526 1988):

**“A real scientific mystery is worth pursuing to the ends of the Earth for its own sake, independently of any obvious practical importance or intellectual glamour.”**

Intermittency is without doubt such a mystery, but not without multifaceted practical importance and numerous corresponding contributions.

This is more than being illustrated by thousands of communications in EGS/EGU NP since 1988, in particular those of our colleagues of HM&Co.

