



HAL
open science

Milanković's multiplicity of scales, intermittency and climate

Daniel Schertzer

► **To cite this version:**

Daniel Schertzer. Milanković's multiplicity of scales, intermittency and climate. Master. Belgrade university's Ceremonial Hall, Serbia. 2023. hal-04591547

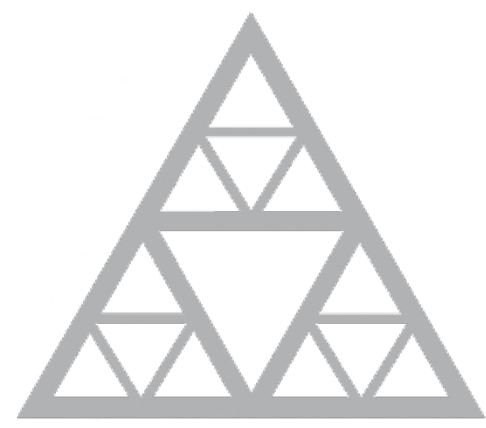
HAL Id: hal-04591547

<https://enpc.hal.science/hal-04591547v1>

Submitted on 28 May 2024

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



École des Ponts
ParisTech



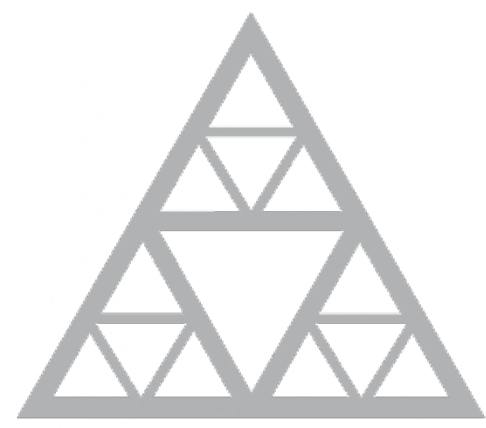
Milanković's, multiplicity of scales, intermittency and climate

Daniel Schertzer

Ecole des Ponts ParisTech (ENPC), Hydrologie Météorologie et Complexité (HM&Co)

University of Belgrade

08/05/2023



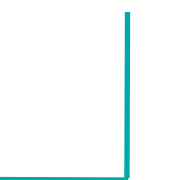
École des Ponts

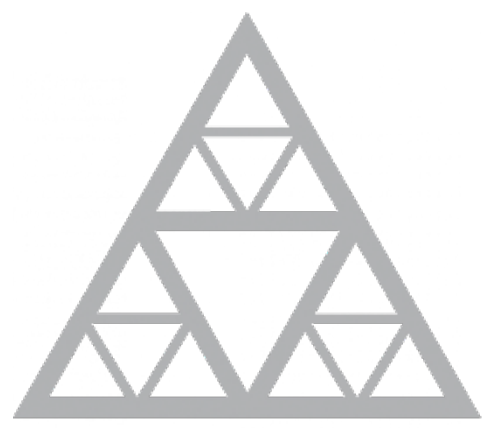
ParisTech

Main objectives



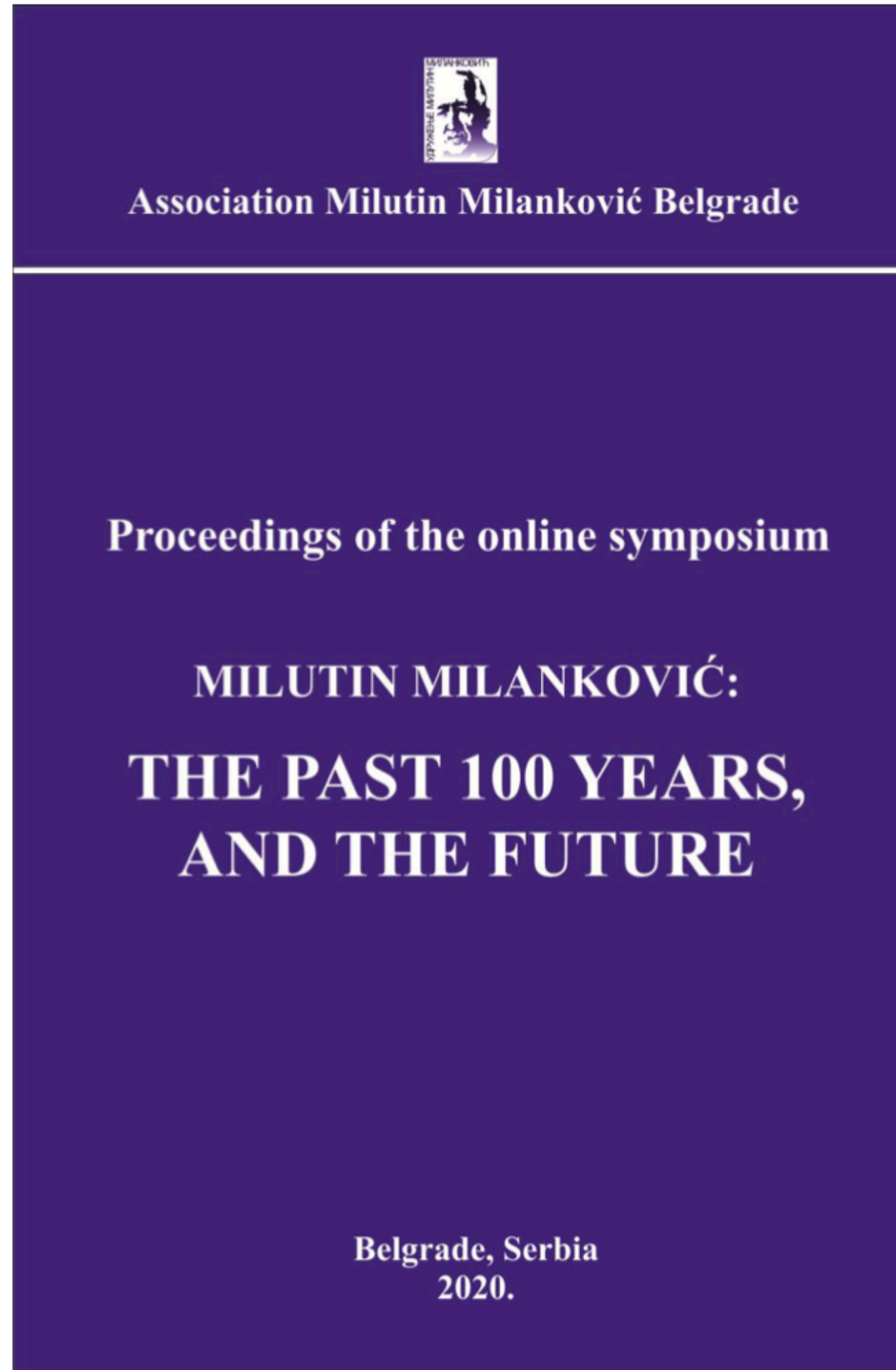
- Geosciences: an inspiring source for the development of concepts and tools
- a recurrent theme: intermittency
- recently: stronger requests for quantitative applications
 - micro-turbulence (wind energy)
 - precipitation remote sensing
 - climate
 - resilience to extreme events
- this requires clarifications and new developments on formalisms, analysis and simulation tools





École des Ponts
ParisTech

The past 100 years and the future



For the publisher
Slavko Maksimović

Editorial Committee
Zoran Stevanović
Čedo Maksimović
Slavko Maksimović
Miodrag Dinić

Consultant
Fedor Mesinger

Editors
Zoran Stevanović
Miodrag Dinić

The desing and prepress
Miodrag Dinić

Printer
Publishing Foundation
of Serbian Ortodox Church
in Belgrade

ISBN-978-86-900893-3-8

Belgrade, Serbia,
November 18, 2020

AUTHORS OF PAPERS



André Berger



Daniel Schertzer



Denis Dider
Rousseau



Michael Ghil



Jörn Thiede



Hans Peter Nachtnebel



Didier Pailard



Ratko Ristić



Dejan Radivojević



Dragoljub Antić



Zoran Stevanović



Hervé Le Treut



Attila Kovács



Jasna Plavšić



Vladimir Pešić



Fedor Mesinger



Sin Chan Chou



Gordon McBean



Slobodan Simonović



Predrag Spasojević



Čedo Maksimović



Ana Mijić



Stela Filipi Matutinović



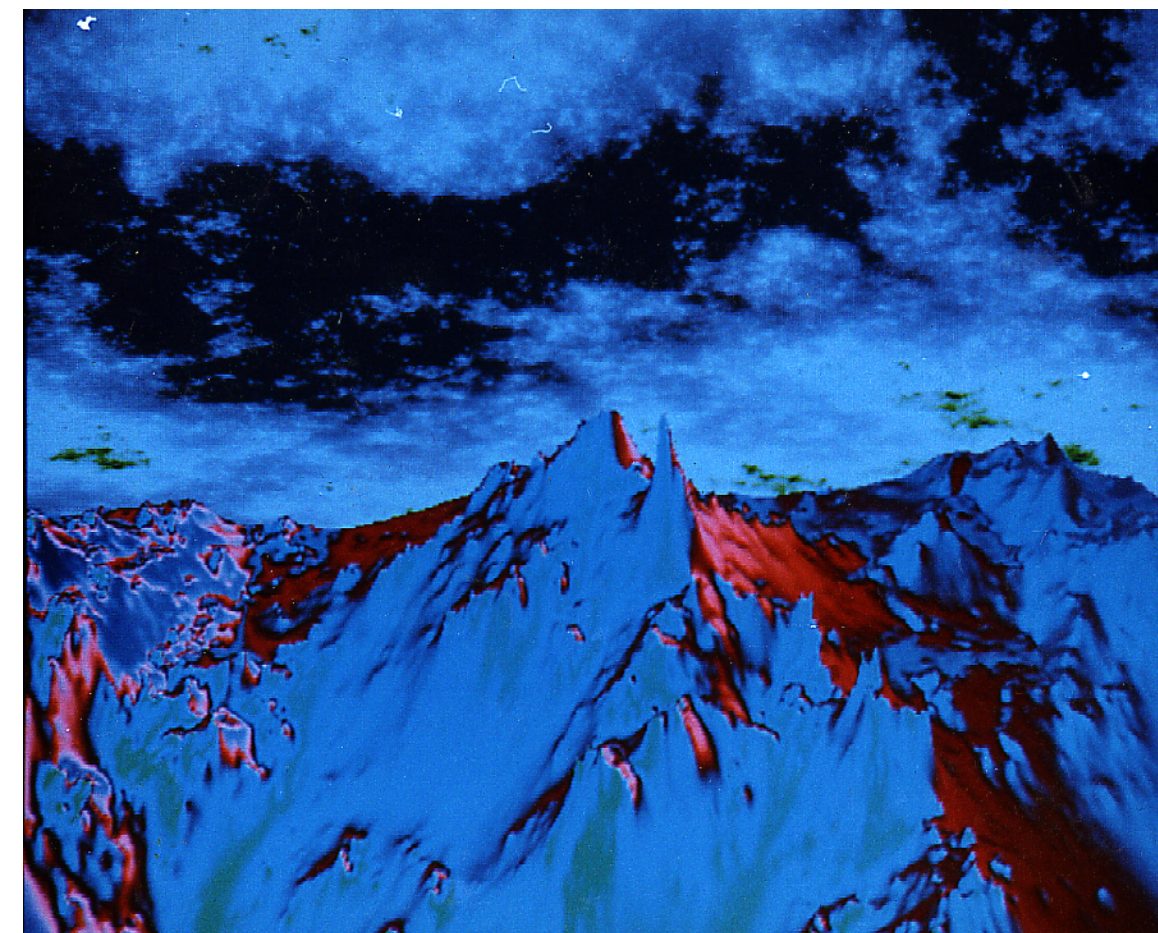
Vlado Milićević

Nobel 2021 : complexity and heterogeneity

- Nobel Prize in Physics 2021 awarded to Syukuro Manabe, Klaus Hasselmann and Giorgio Parisi "for groundbreaking contributions to our understanding of **complex physical systems**"



Congratulations to Fedor Mesinger!



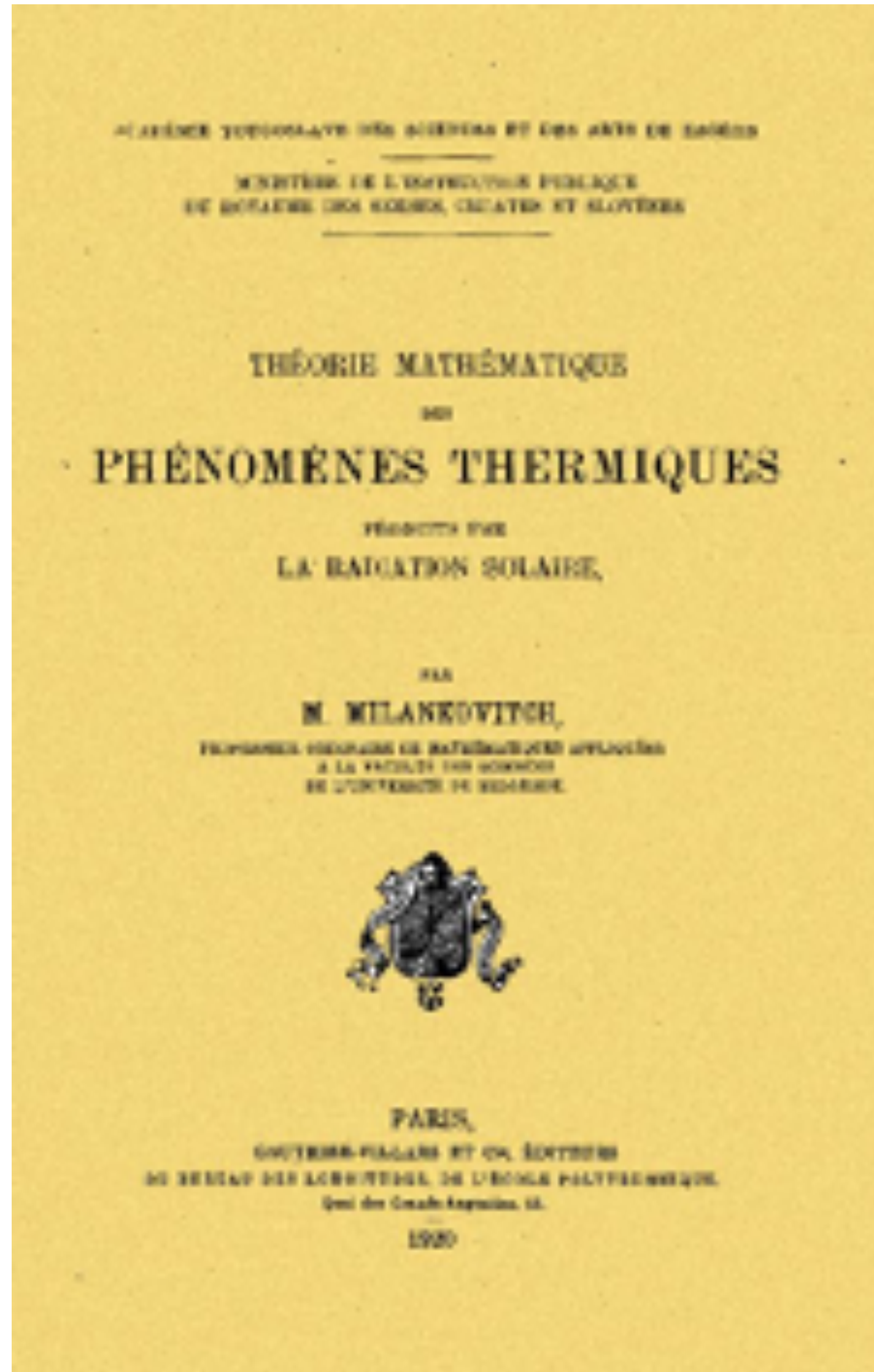
Emphasis of the “scientific background” of the prize on intermittency (*)

Why scaling and multifractality ?

СТО ГОДИНА
МИЛАНКОВИЋЕВЕ ТЕОРИЈЕ
О КЛИМАТСКИМ
ПРОМЕНАМА


1920202120

100 YEARS of
MILANKOVIC'S THEORY of
CLIMATE CHANGE

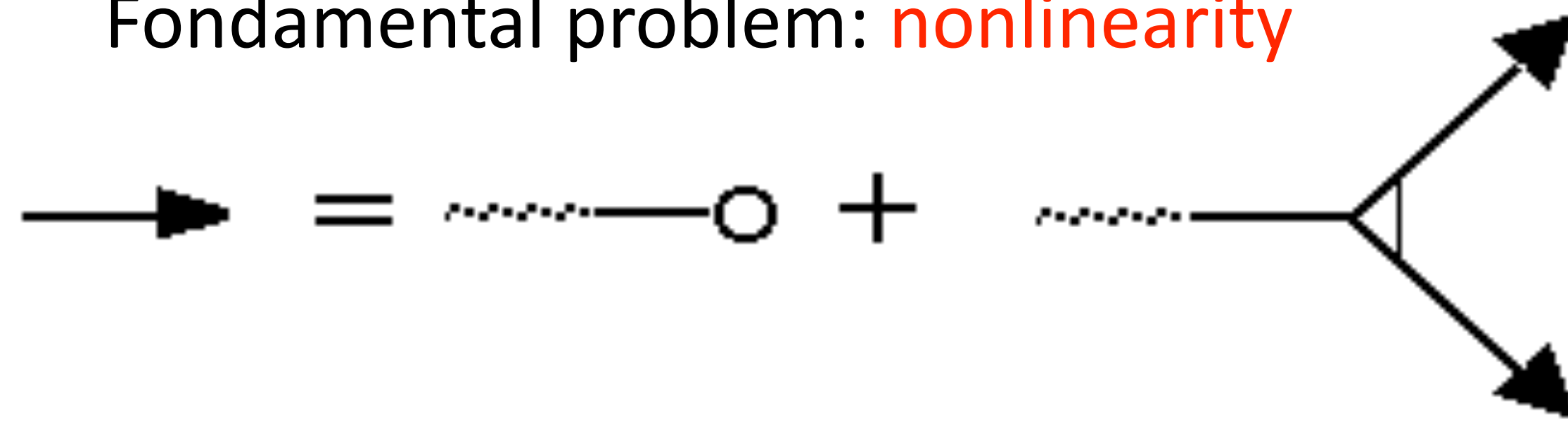


- Indeed, in order to handle the mathematical phenomena of nature, it is always necessary to admit simplifications and to neglect certain influences and irregularities. I had to do it in front of an object as complicated as our atmosphere (Milankovic, 1920)
- This variable distribution of solar radiation on the surface of the Earth is the primary cause of all meteorological phenomena, whose irregularity singularly contrasts with the regularity of their primary cause. Meteorological phenomena appear as capricious as the radiation - with which the Sun floods the Earth - follows a regular diurnal and annual course (Milankovic, 1920)
- I believe that the ultimate climate models...will be stochastic, i.e. random numbers will appear somewhere in the time derivatives (Lorenz 1975).

Why scaling and multifractality ?



Fondamental problem: **nonlinearity**



- Indeed, in order to handle the mathematical phenomena of nature, it is always necessary **to admit simplifications and to neglect certain influences and irregularities**. I had to do it in front of an object as complicated as our atmosphere (Milankovic, 1920)
- I believe that the ultimate climate models..will be stochastic, i.e. **random numbers will appear somewhere in the time derivatives** (Lorenz 1975).

 (vector-valued) **space-time field**

 (bare) propagator/Green function= **inverse of a differential operator**

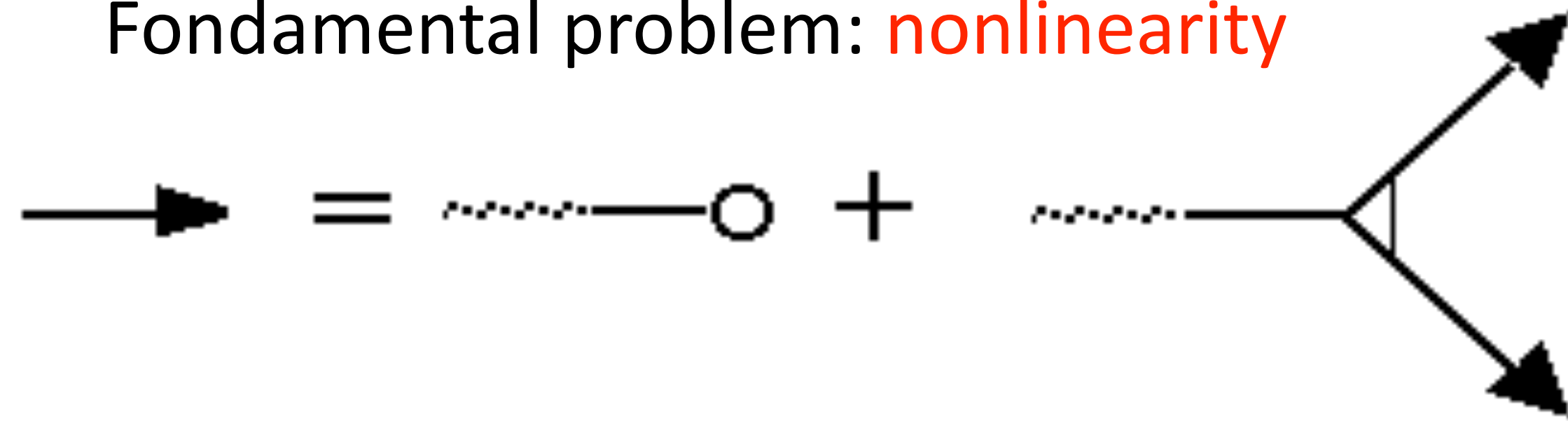
 (bare) **forcing** , including Milankovic's astronomical cycles

 (bare) **vertex**/interaction kernel

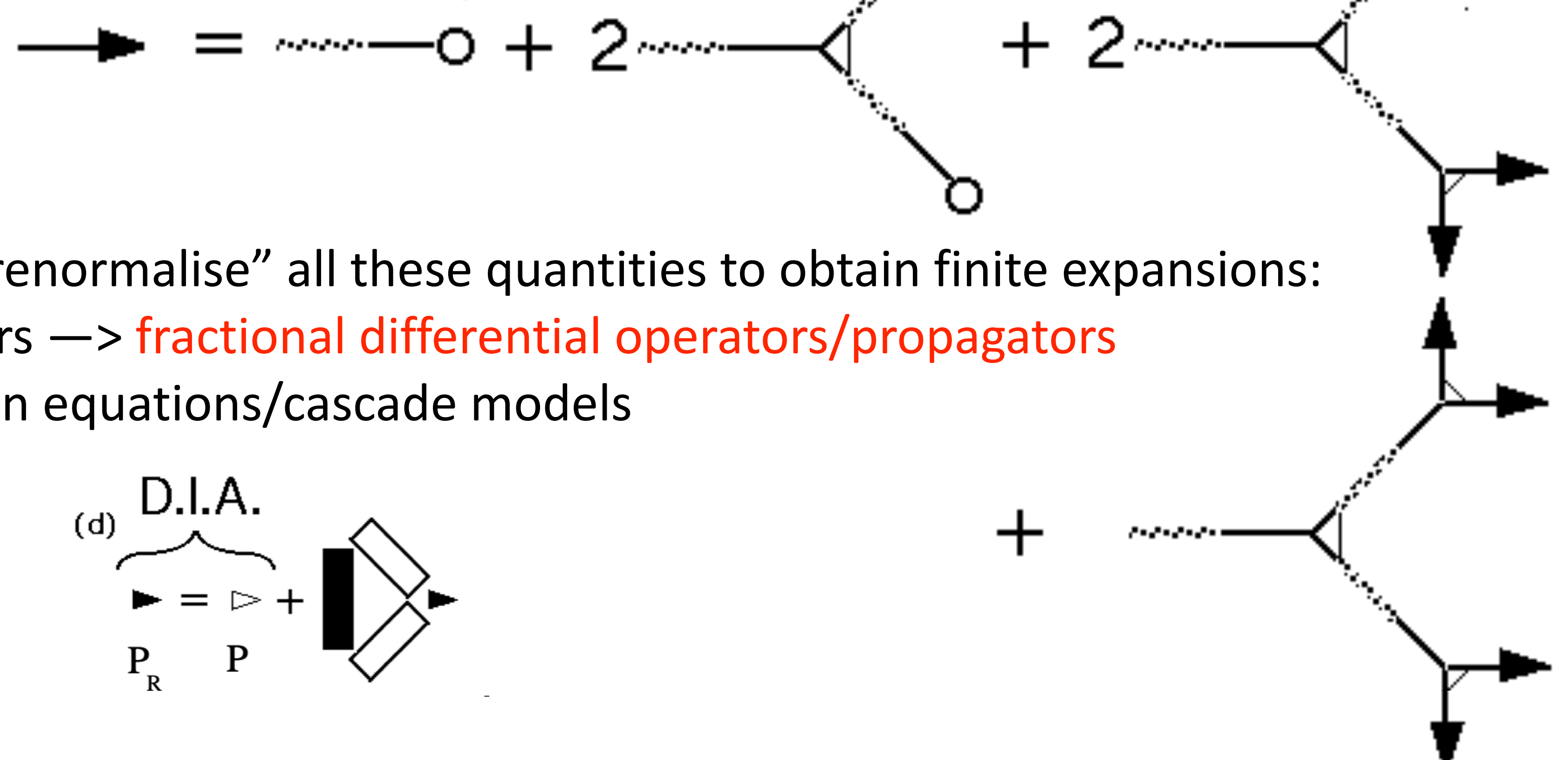
Why scaling and multifractality ?



Fondamental problem: **nonlinearity**

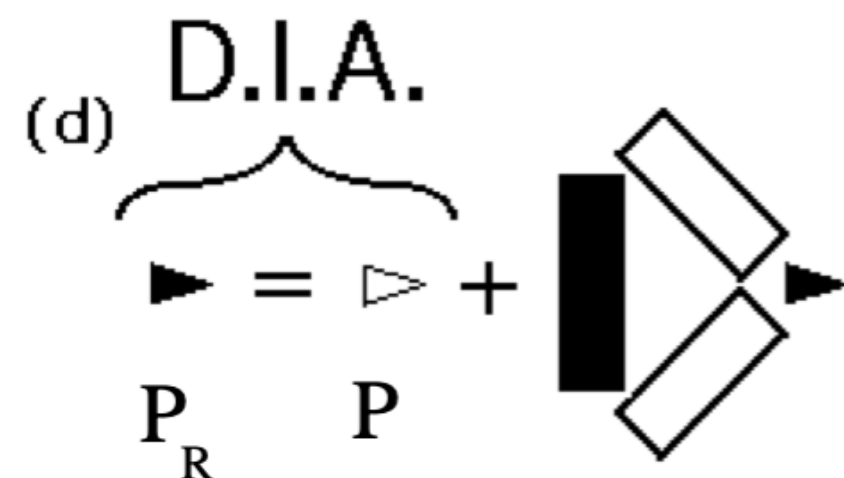


Endless proliferation of **higher and higher order diagrams**



Nevertheless, one can hope to “renormalise” all these quantities to obtain finite expansions:
differential operators/propagators \rightarrow **fractional differential operators/propagators**
e.g. non-linear fractional Langevin equations/cascade models

vertex \rightarrow **intermittent vertex**



What do the data tell us?

Scaling & multifractality of GRIP data

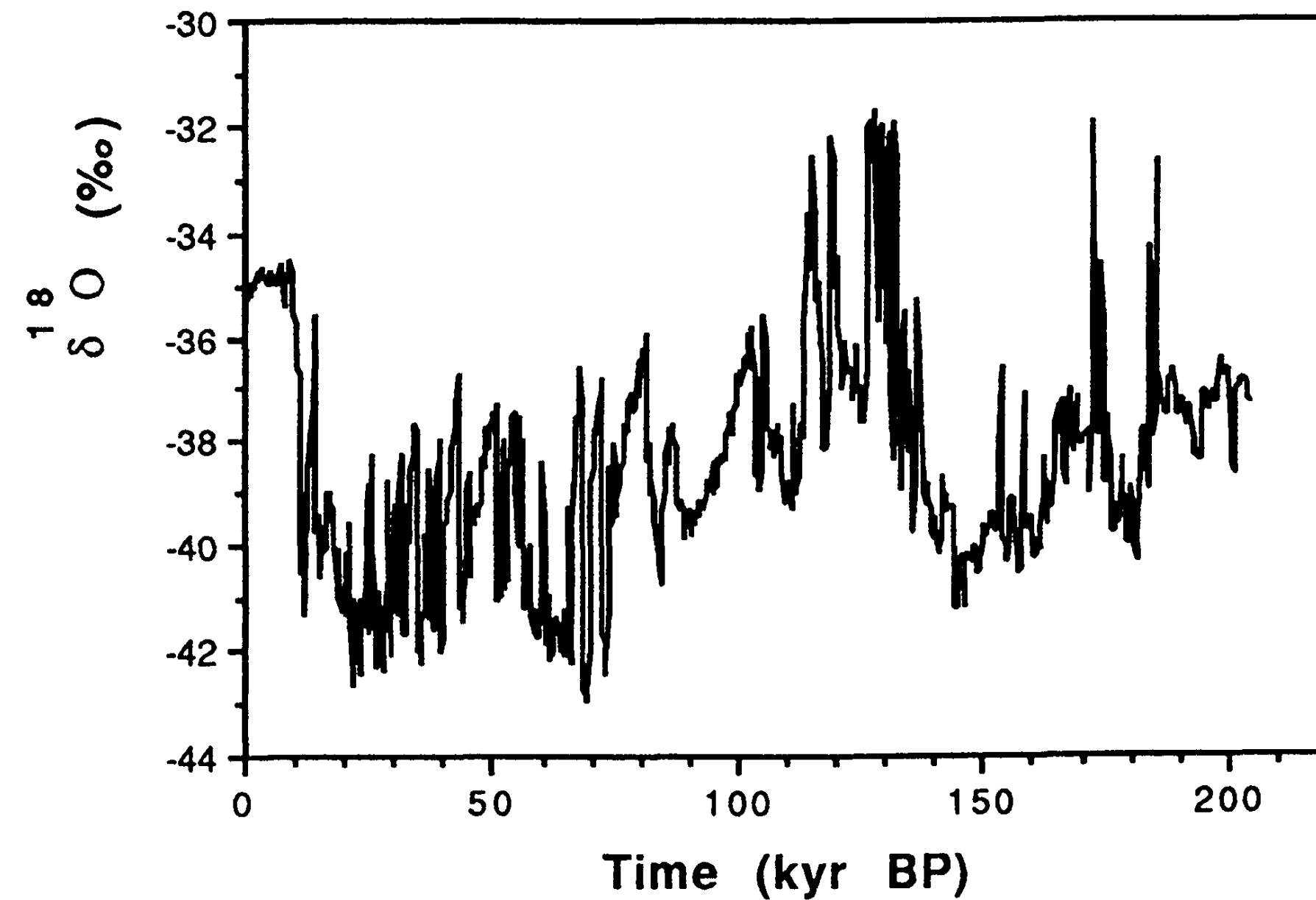


Figure 1. High resolution (200 yr average) $\delta^{18}O$ record coming from the recent GRIP Greenland ice core. Sharp fluctuations occurring on small time scales are clearly visible.

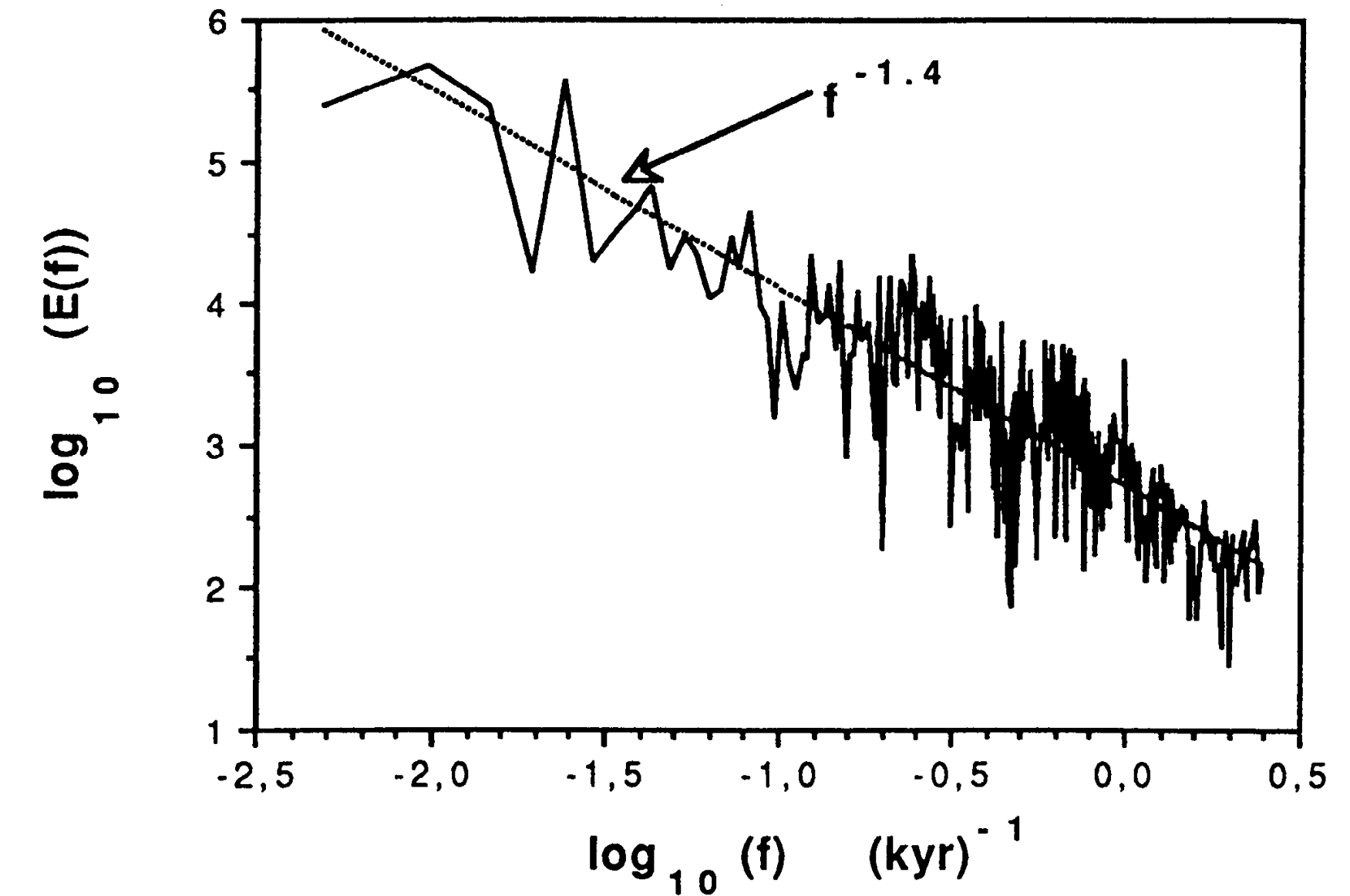


Figure 2. The power spectrum of the $\delta^{18}O$ data, shown in a log-log plot. The global straight line $f^{-1.4}$ is an indication of scaling. There is no evidence of periodic variations of frequencies of $(20 \text{ kyr})^{-1}$ or $(40 \text{ kyr})^{-1}$.

$$E(f) \approx f^{-\beta}; \quad \beta \approx 1.4$$

Scaling & multifractality of GRIP data

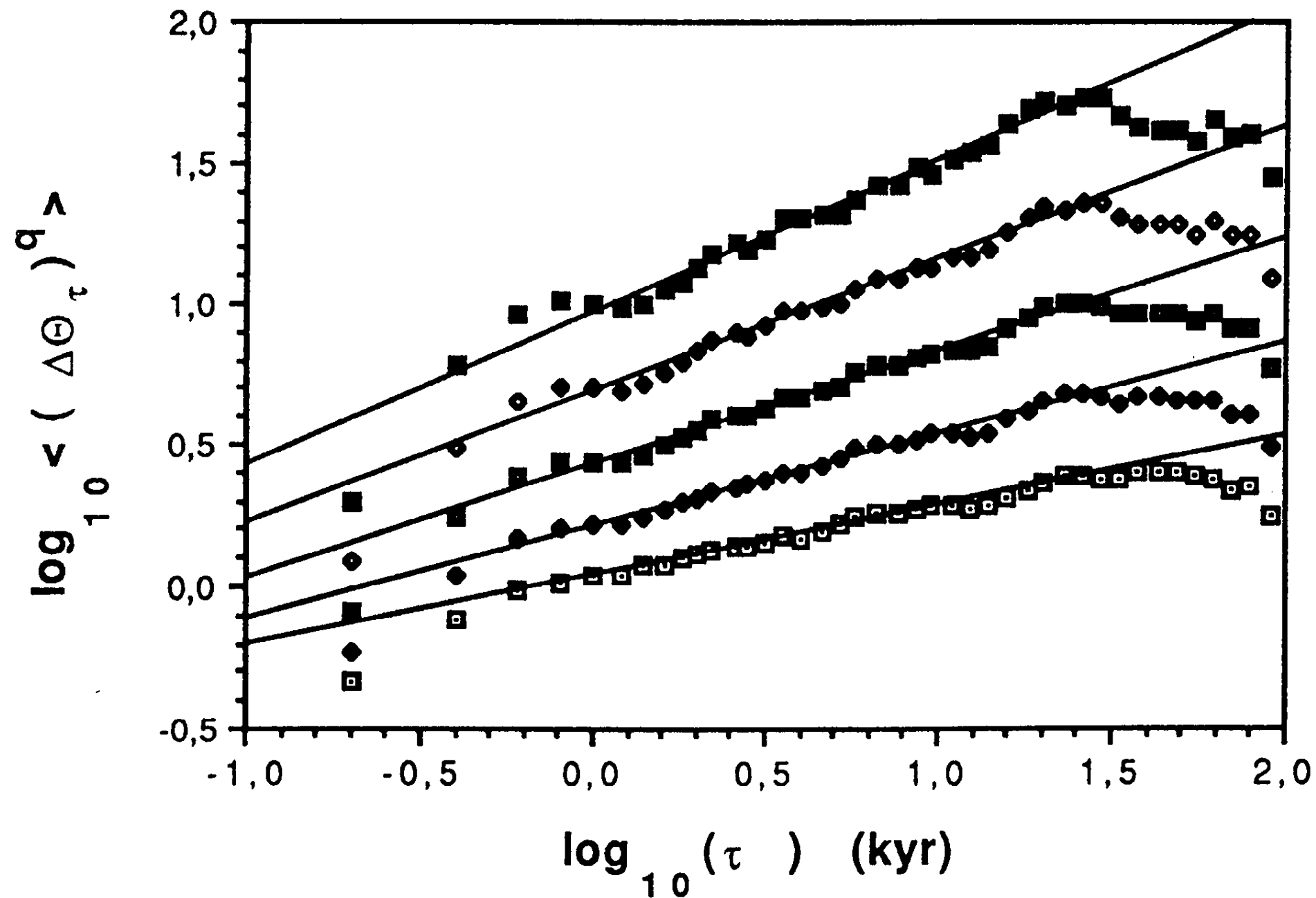


Figure 3. The structure functions $\langle (\Delta\theta_\tau)^q \rangle$ vs. τ in a log-log plot for $q=1, 1.5, 2, 2.5,$ and 3 . Linear trends are clearly visible for all order of moments, for a range of scales going from ≈ 400 yr to ≈ 40 kyr. The straight lines indicate the best linear regression over this range of scales for each value of q . This gives in particular: $H = \zeta(1) = 0.24 \pm 0.02$, and $\zeta(2) = 0.40 \pm 0.03$.

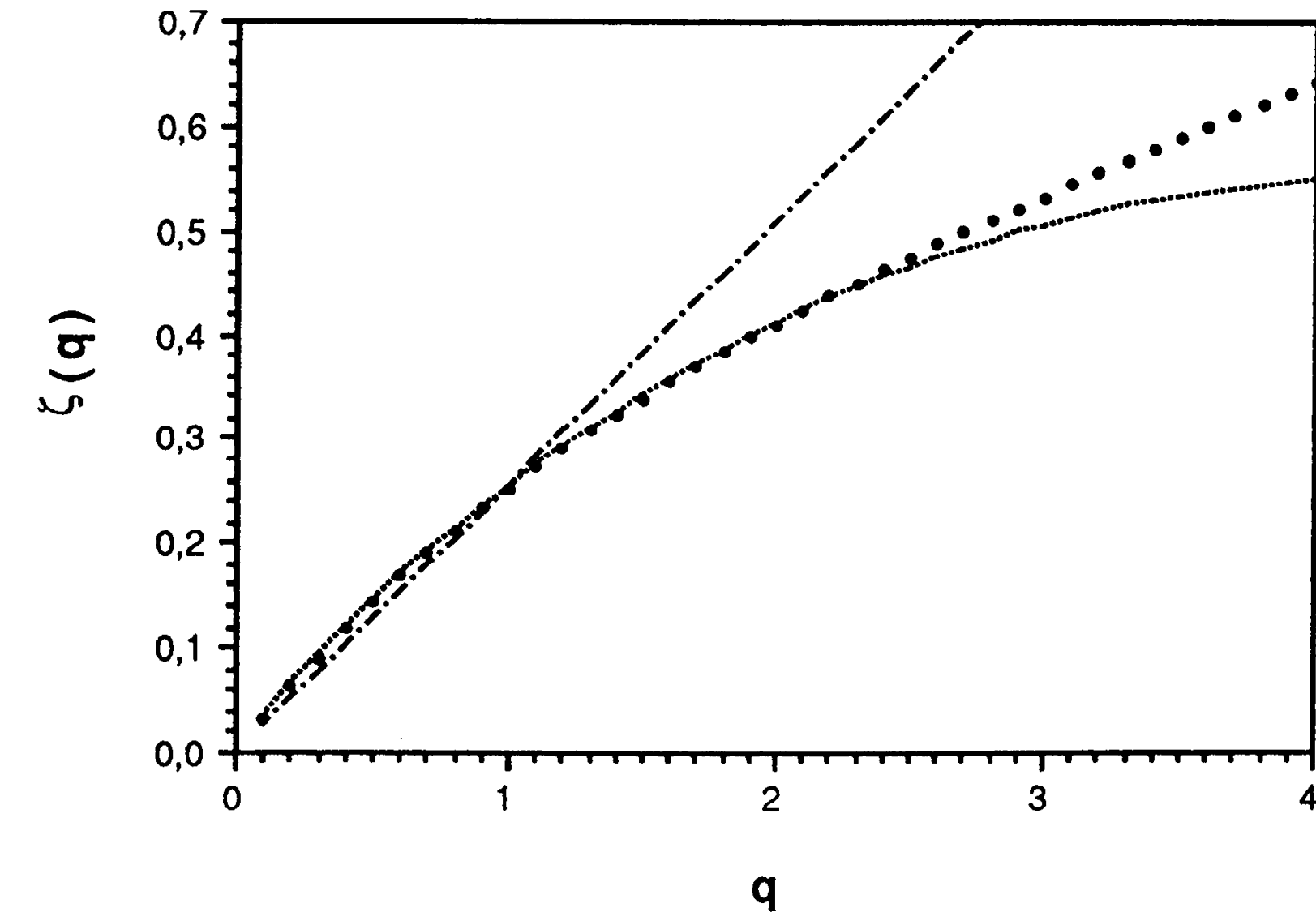


Figure 4. The scaling exponent structure function $\zeta(q)$ empirical curve (dots), compared to the monofractal curve $\zeta(q) = qH$ (dashed and dotted line), and to the universal multifractal function obtained with $\alpha = 1.6$ and $C_1 = 0.05$ in equation (2) (dotted curve). The empirical curve is nonlinear, indicating multifractality. For moments of order $q > 1$, the departure from monofractal behaviour is strong. The universal multifractal fit is excellent until moment order $q_{\max} \approx 2.5 \pm 0.3$, corresponding to a multifractal phase transition occurring because of sample limitations.

$$\left\langle \left(\Delta\theta \left(\frac{T}{\lambda} \right) \right)^q \right\rangle \approx \lambda^{-\zeta(q)} \langle (\Delta\theta(T))^q \rangle$$

(Schmitt et al., GRL, 1995)

Scaling & multifractality of GRIP data

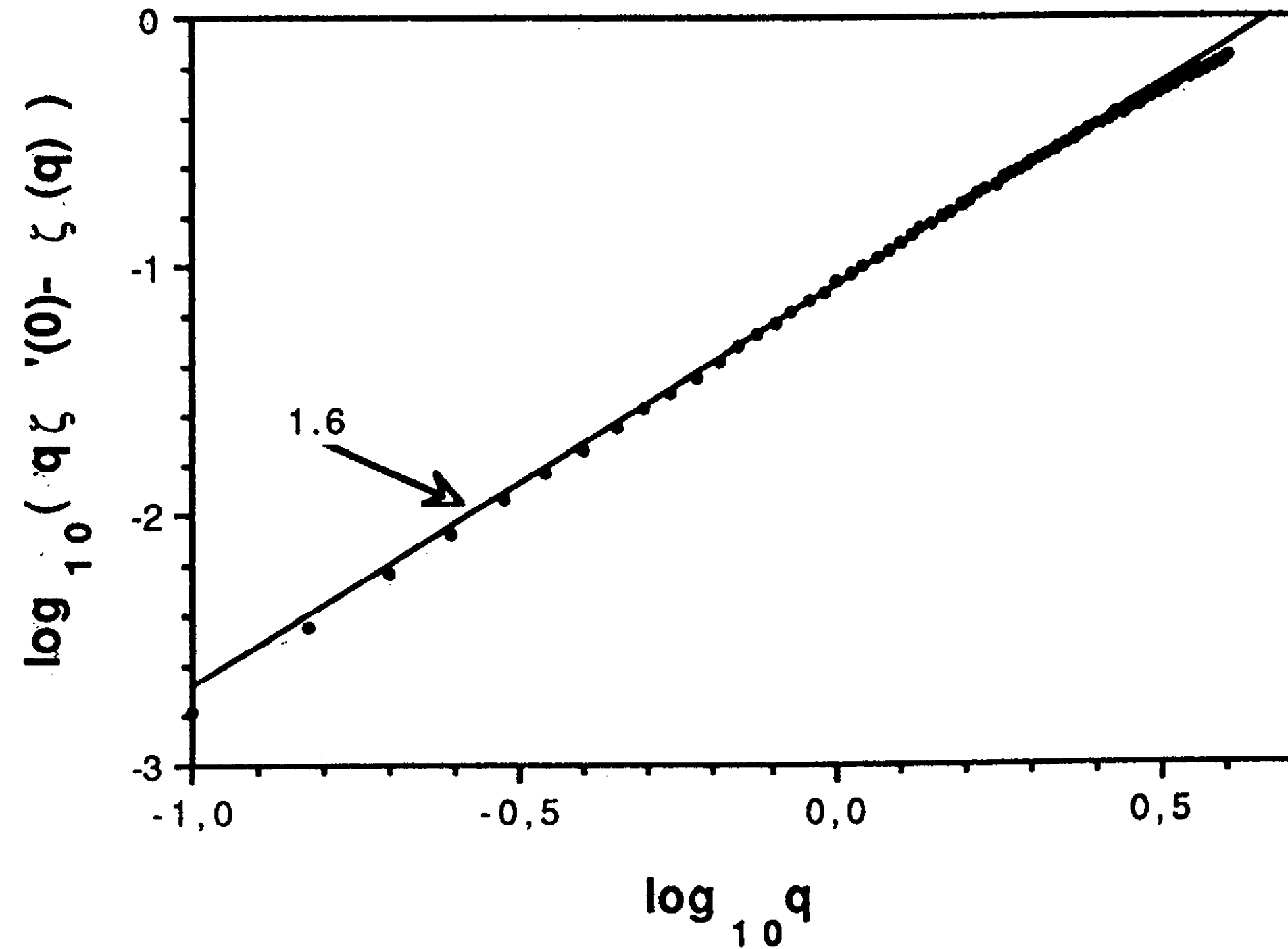


Figure 5. Empirical curve of $\log[q\zeta(0) - \zeta(q)]$ vs. $\log(q)$ giving directly the universal multifractal indices α and C_1 : α is the slope of the straight line and C_1 is estimated by the intercept. We obtain $\alpha \approx 1.6$ and $C_1 \approx 0.05$.

linear scaling,
fractional integration

nonlinear scaling
intermittency,
multifractality

$$\zeta(q) \approx Hq - C_1 \frac{q^\alpha - q}{q - 1}$$

$$\beta = 2H + 1 + K(2) \approx 2H + 1$$

H (fractional) **order of integration**

C_1 codimension of the **mean intermittency**

α **multifractality index**, concavity of $\zeta(q)$

$$H \approx 0.2, C_1 \approx 0.05, \alpha \approx 1.6$$

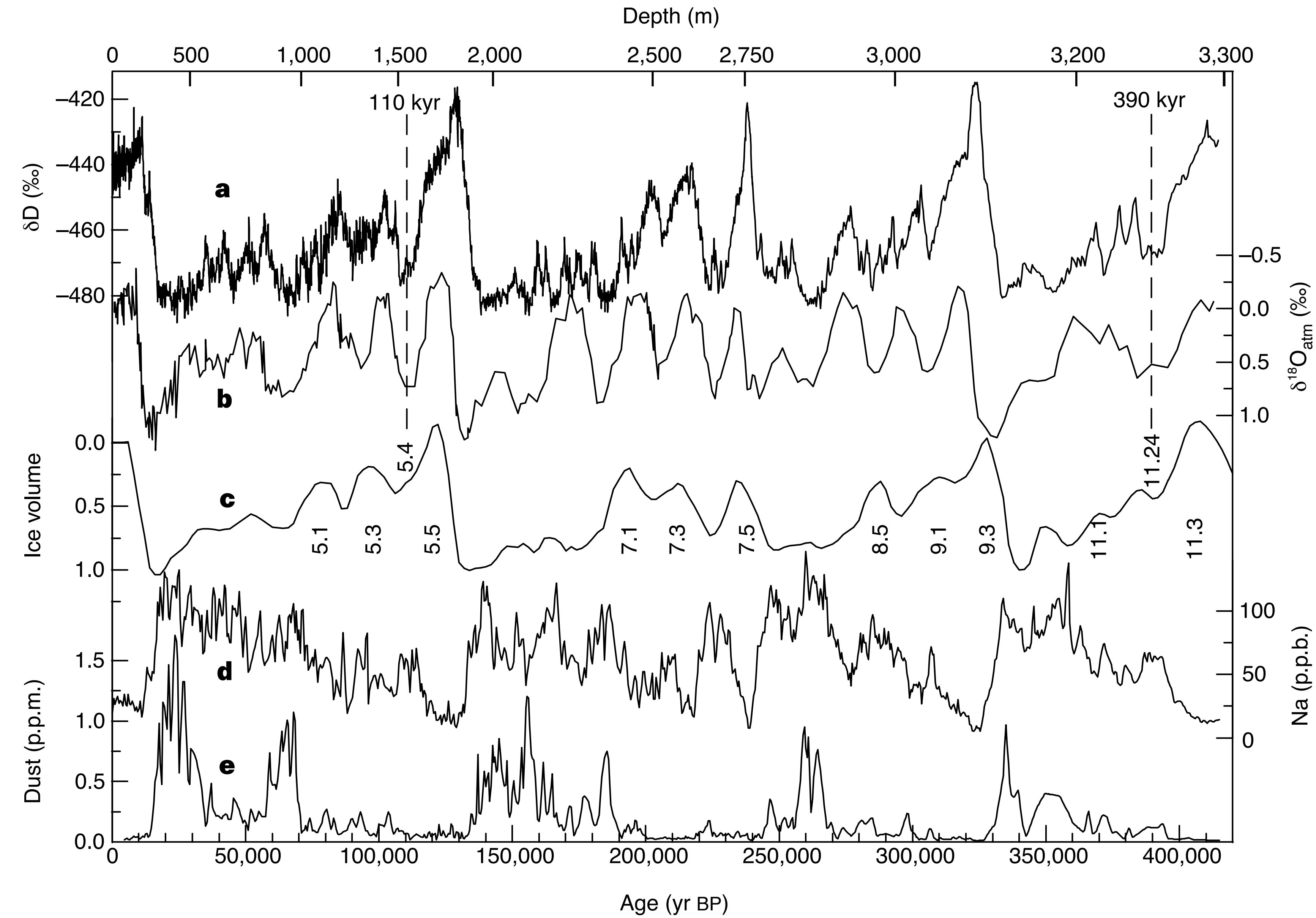
(Schmitt et al., GRL, 1995)

Vostok data

СТО ГОДИНА
МИЛАНКОВИЋЕВЕ ТЕОРИЈЕ
О КЛИМАТСКИМ
ПРОМЕНАМА

1920202120

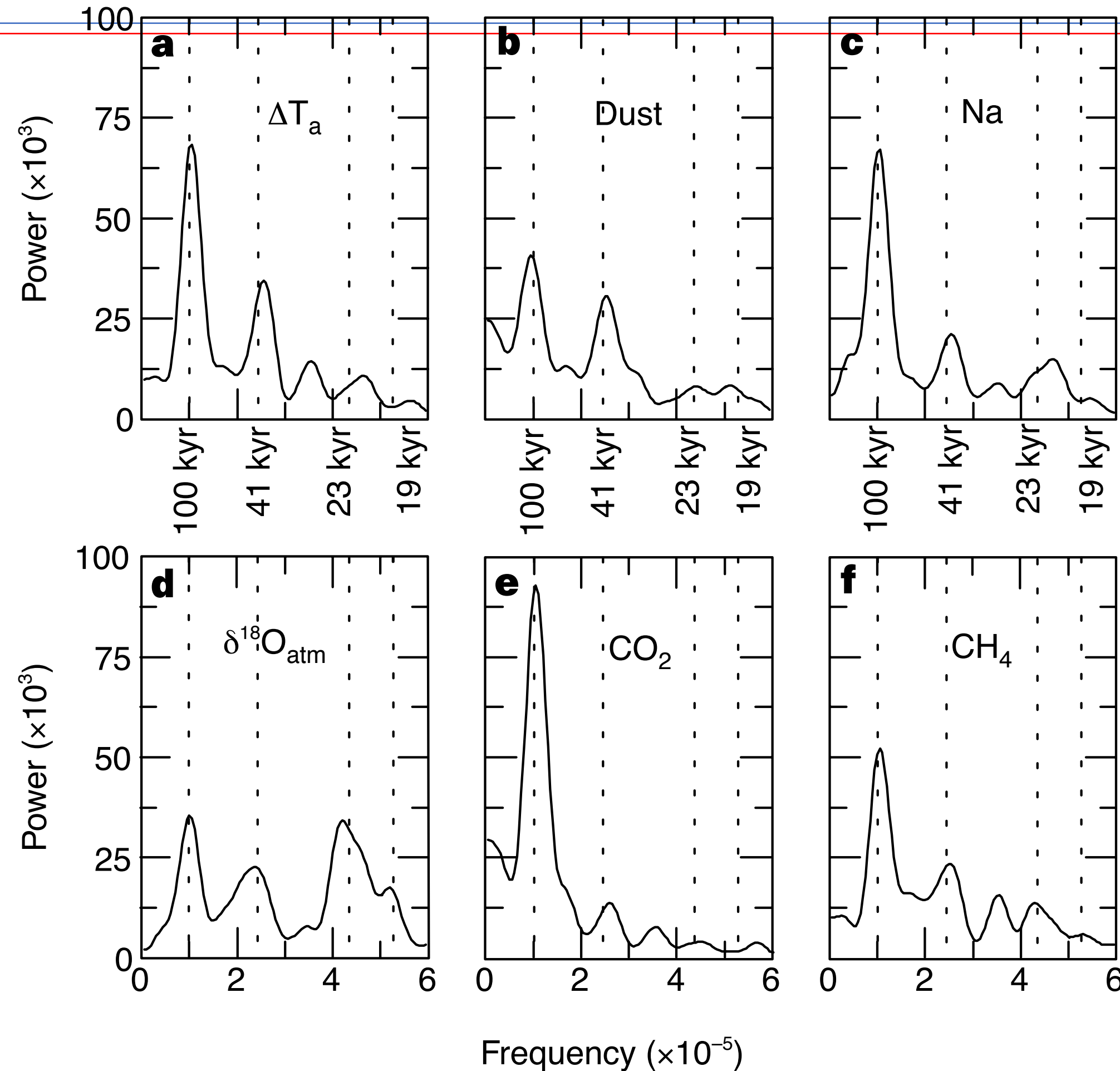
100 YEARS of
MILANKOVIC'S THEORY of
CLIMATE CHANGE



A very rich data set
over 450 000 years!

(Petit et al., Nature, 1999)

Vostok data



“integrated spectra”: $\omega E(\omega)$ vs $\text{Log}(\omega)$

well identified but broad peaks/spikes

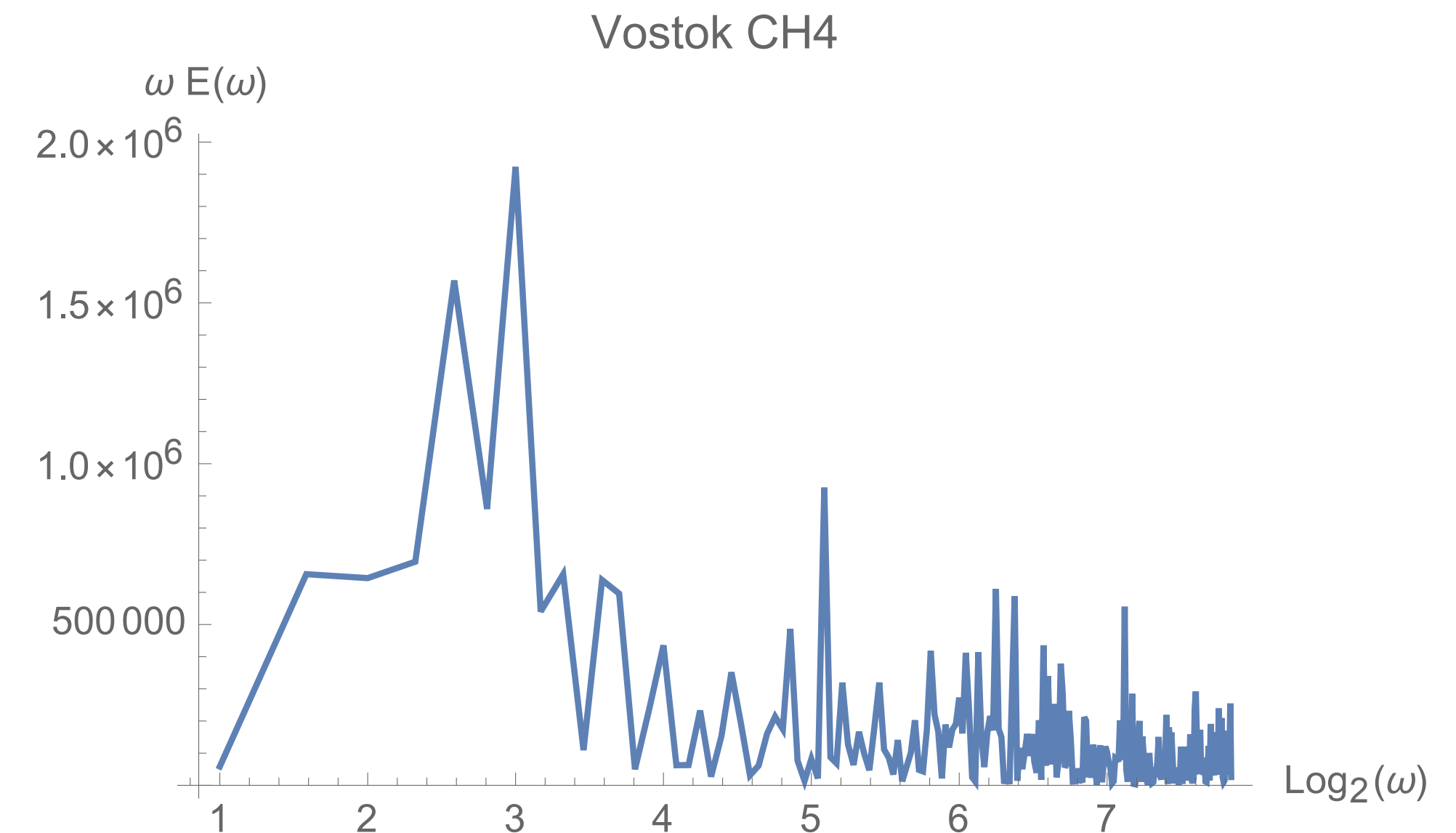


Figure 4 Spectral properties of the Vostok time series. Frequency distribution (in cycles yr^{-1}) of the normalized variance power spectrum (arbitrary units). Spectral analysis was done using the Blackman-Tukey method (calculations were performed with the Analyseries software⁴⁷): **a**, isotopic temperature; **b**, dust; **c**, sodium; **d**, $\delta^{18}\text{O}_{\text{atm}}$; **e**, CO_2 ; and **f**, CH_4 . Vertical lines correspond to periodicities of 100, 41, 23 and 19 kyr.

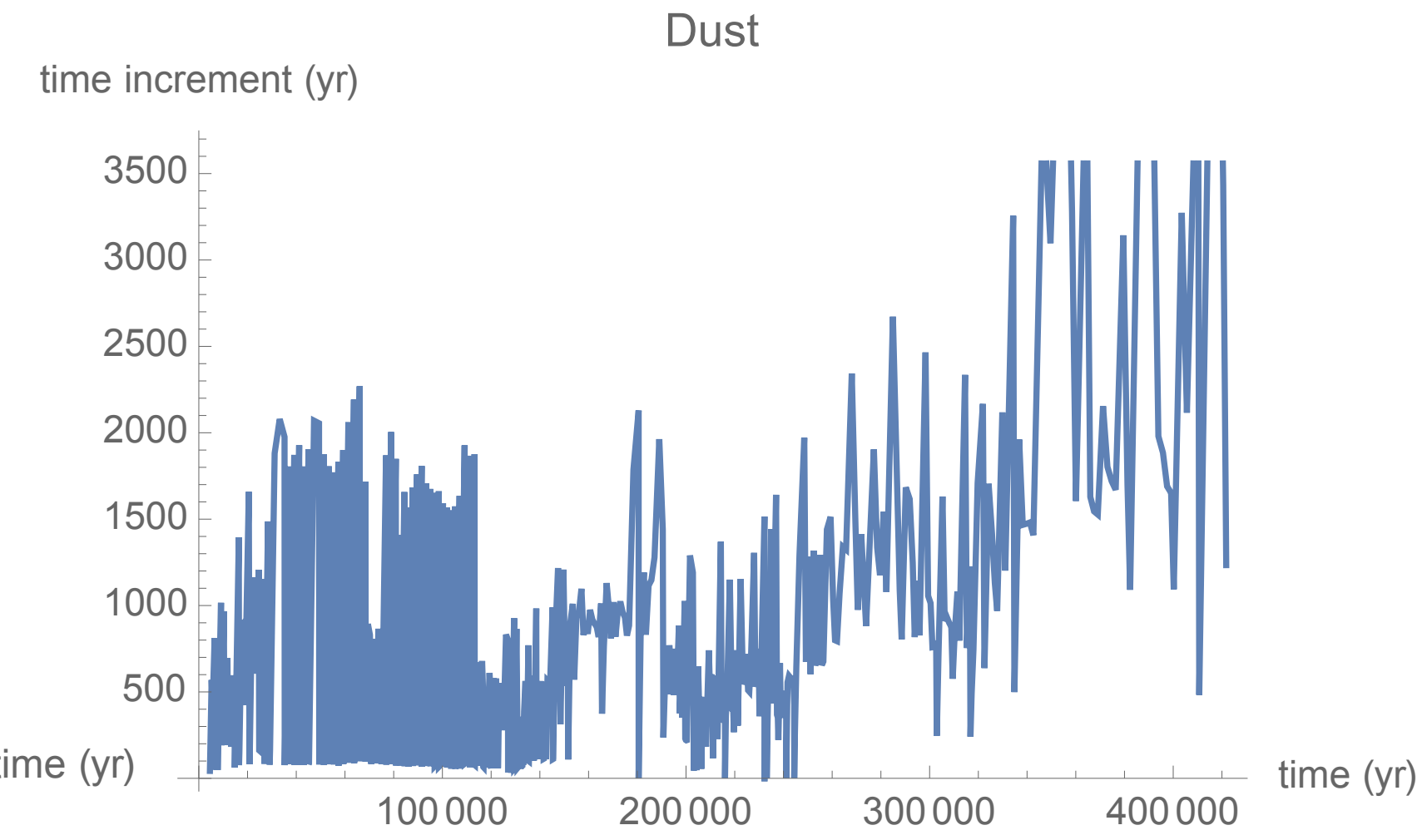
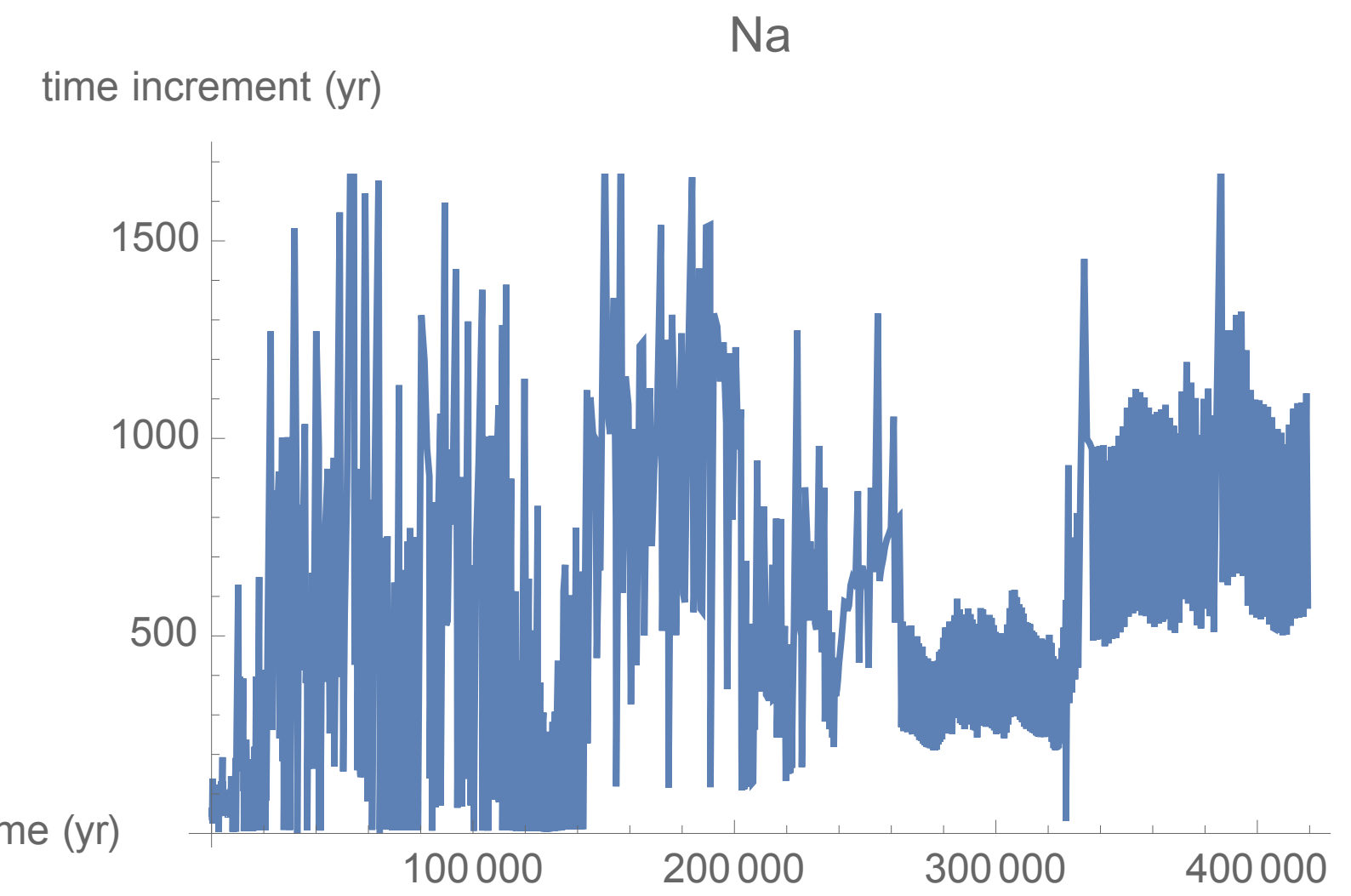
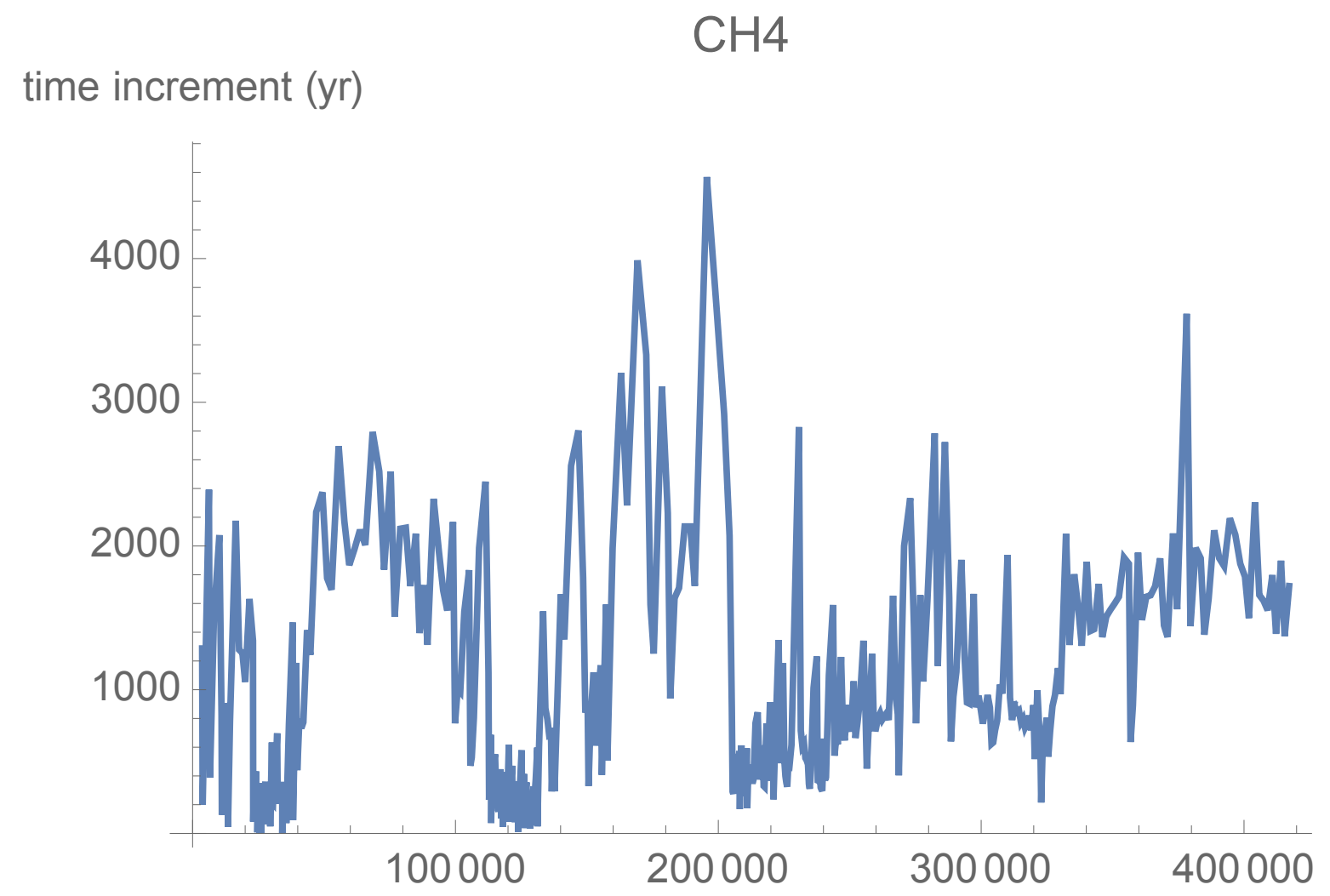
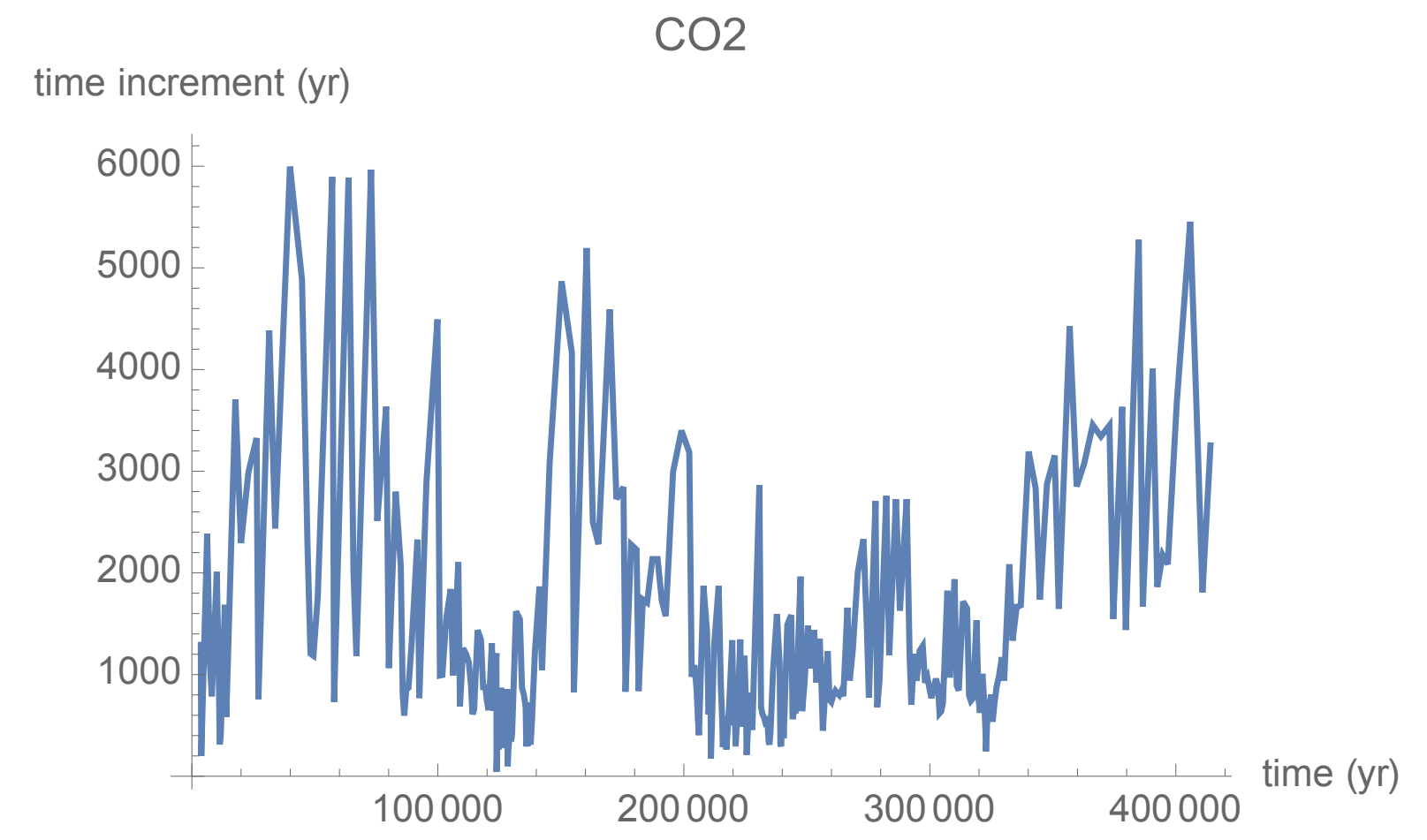
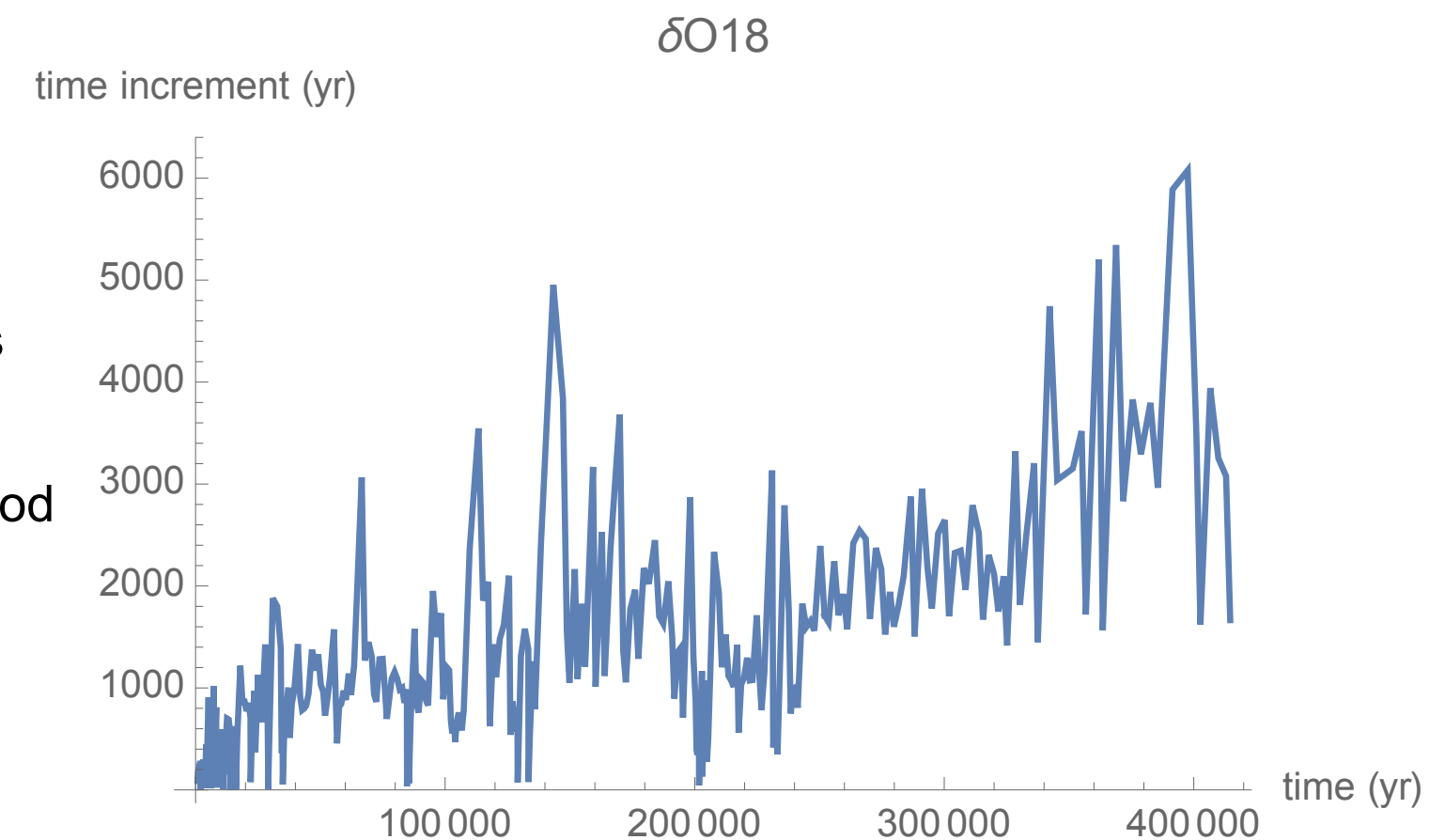
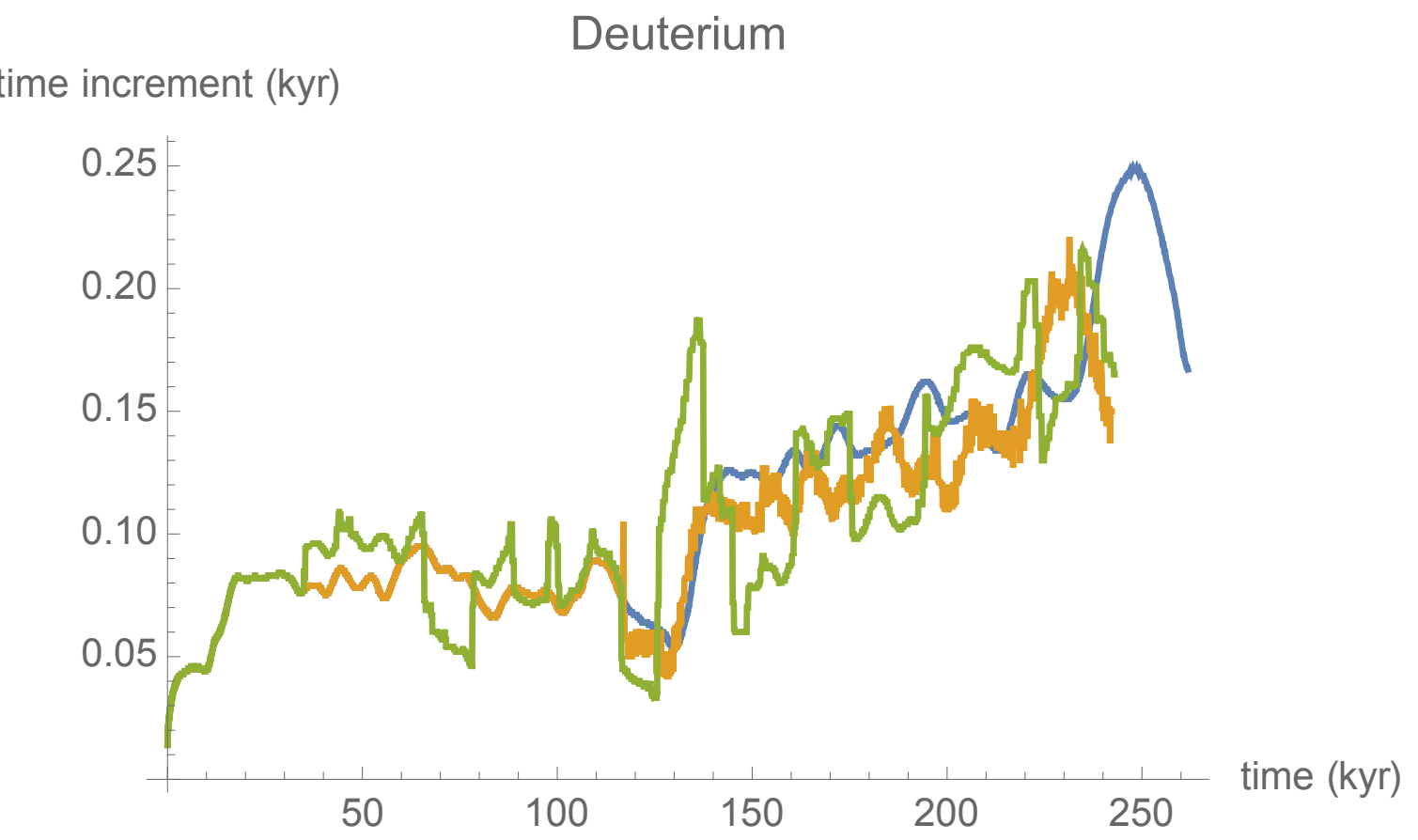
(Petit et al., Nature, 1999)

Vostok data: glaciological timescale increments

СТО ГОДИНА
МИЛАНКОВИЋЕВЕ ТЕОРИЈЕ
О КЛИМАТСКИМ
ПРОМЕНАМА

1920202120

100 YEARS of
MILANKOVIC'S THEORY of
CLIMATE CHANGE



Vostok data: spectral scaling

СТО ГОДИНА
МИЛАНКОВИЋЕВЕ ТЕОРИЈЕ
О КЛИМАТСКИМ
ПРОМЕНАМА

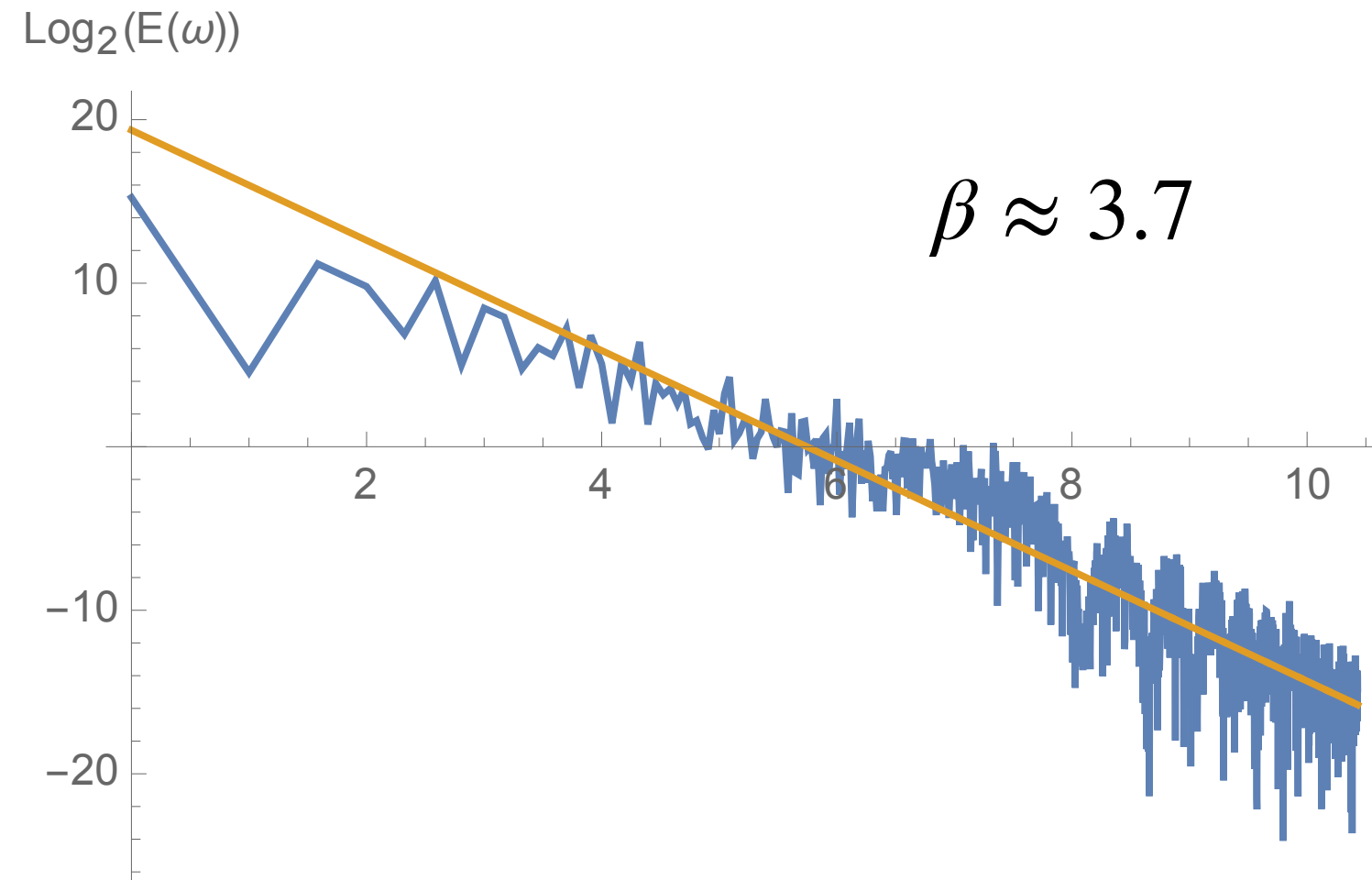
1920202120

100 YEARS of
MILANKOVIC'S THEORY of
CLIMATE CHANGE

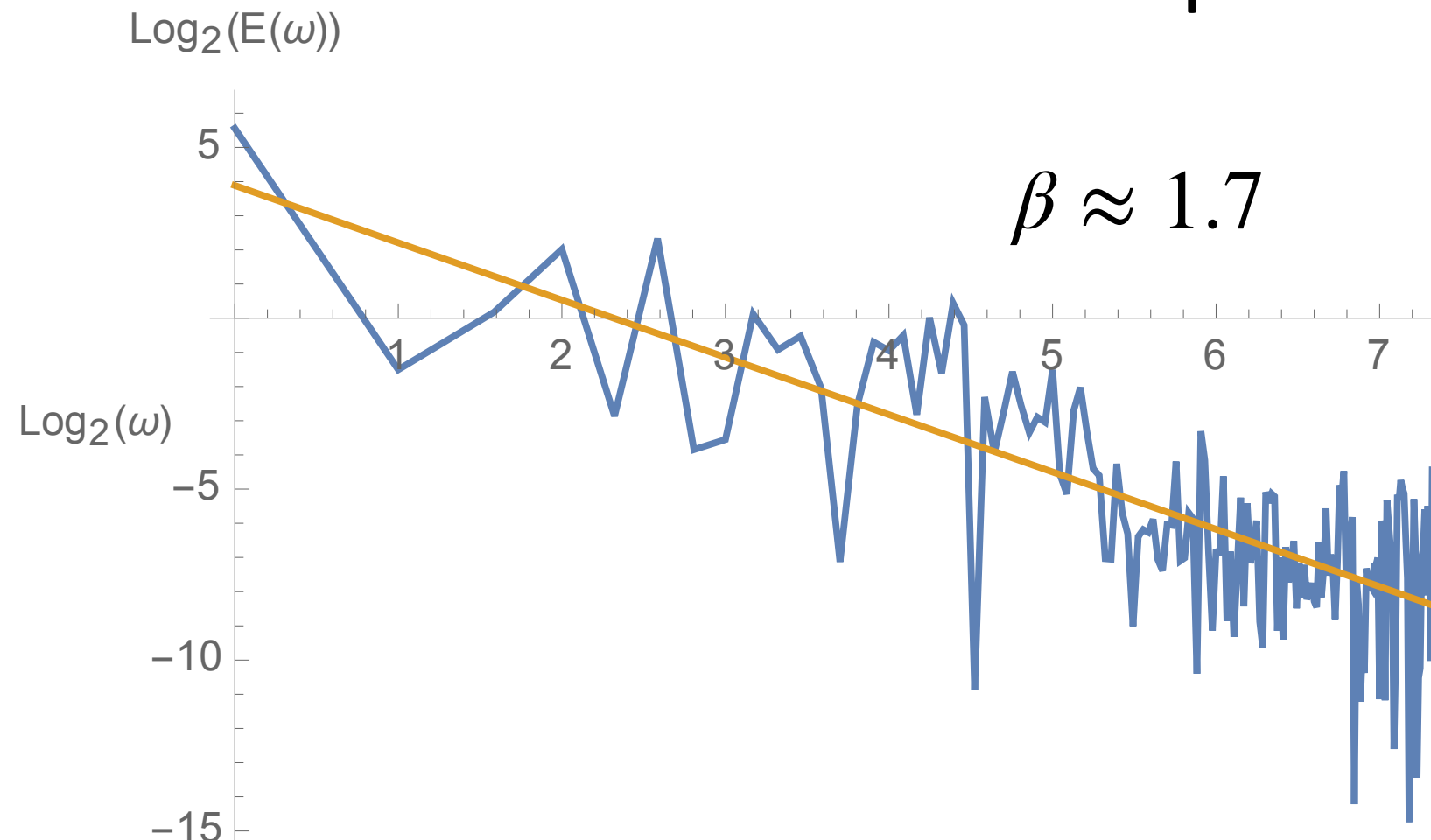


“spectra”: $\text{Log}(E(\omega))$ vs. $\text{Log}(\omega)$

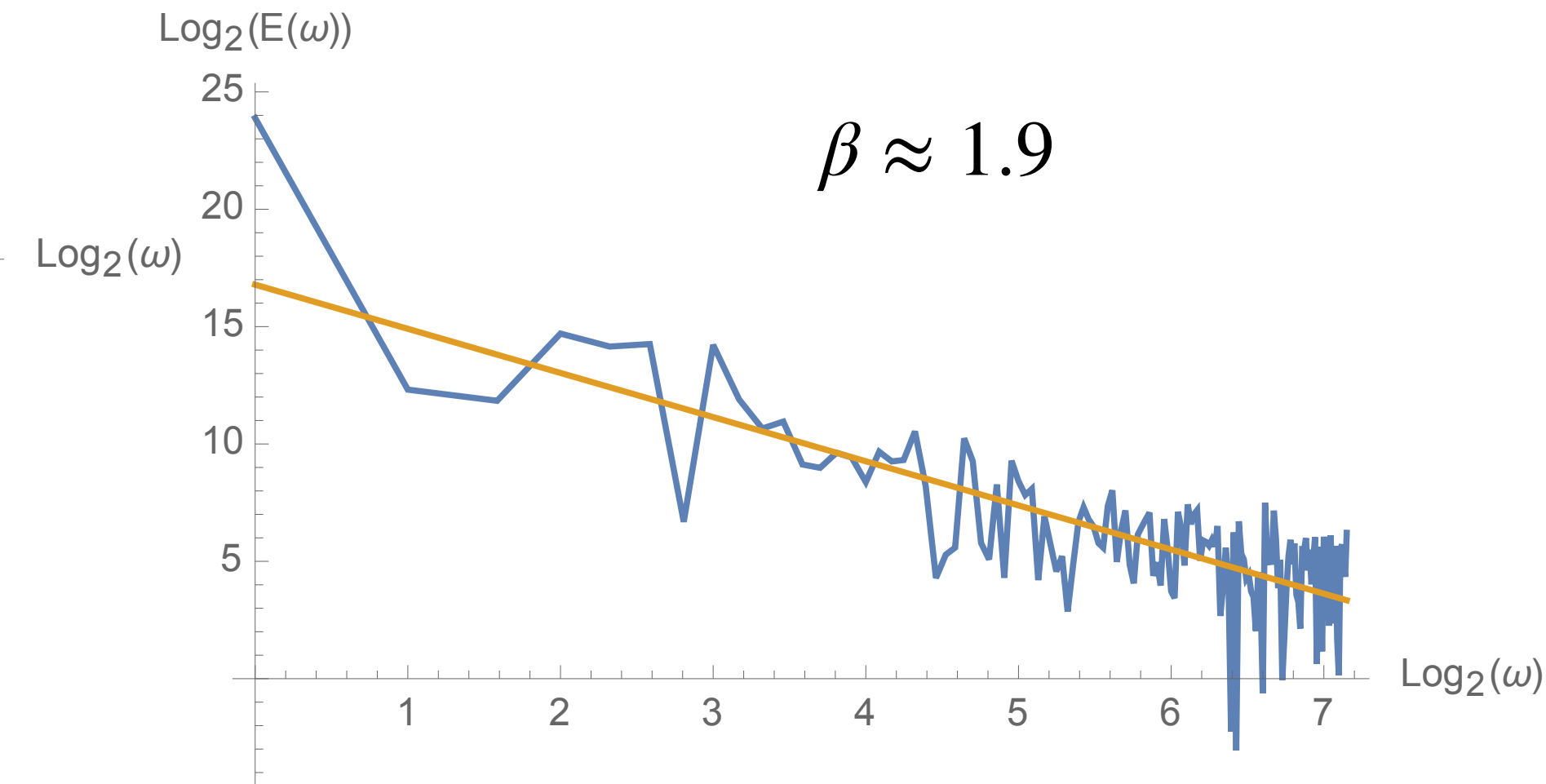
Vostok deuterium



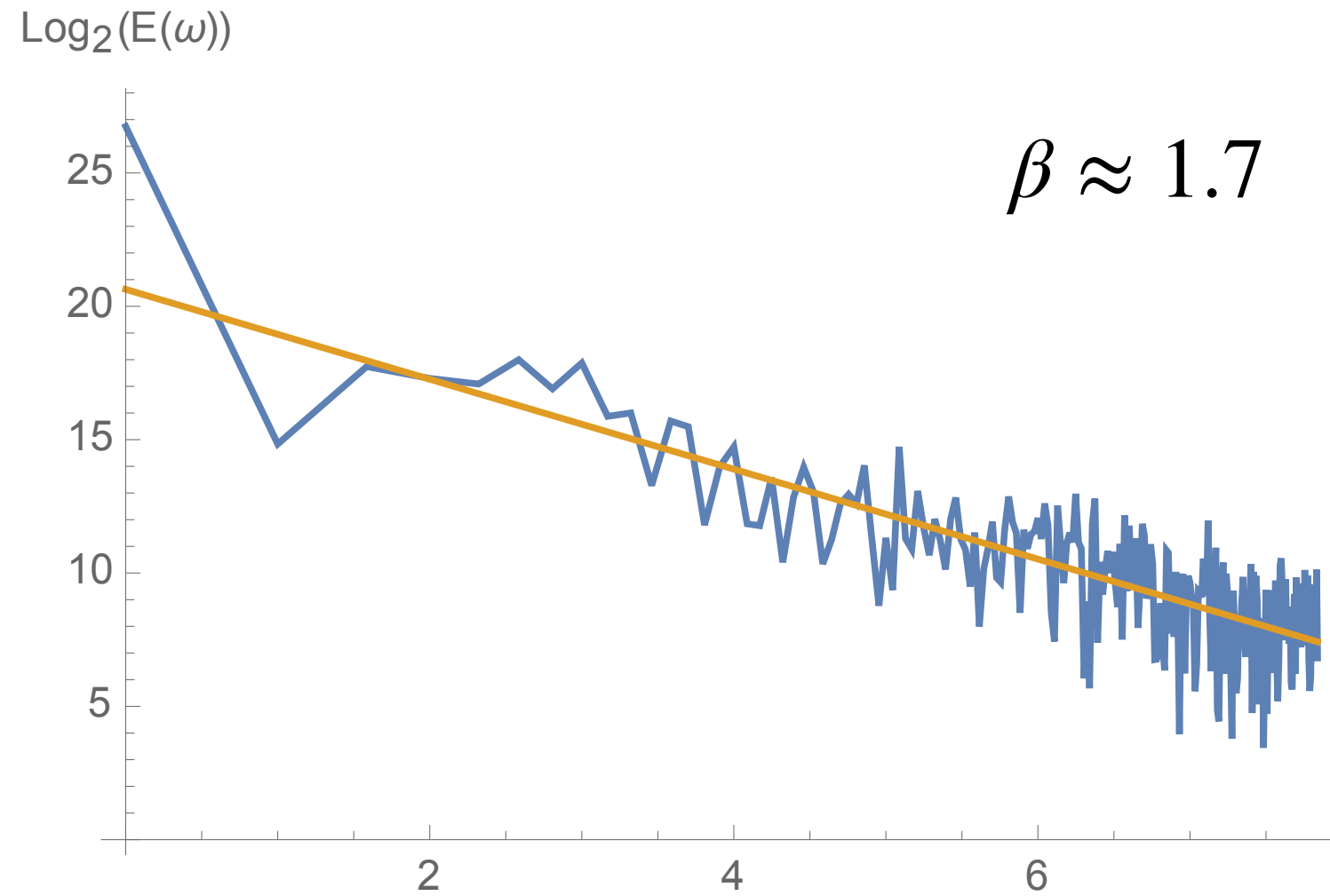
Vostok δO_18



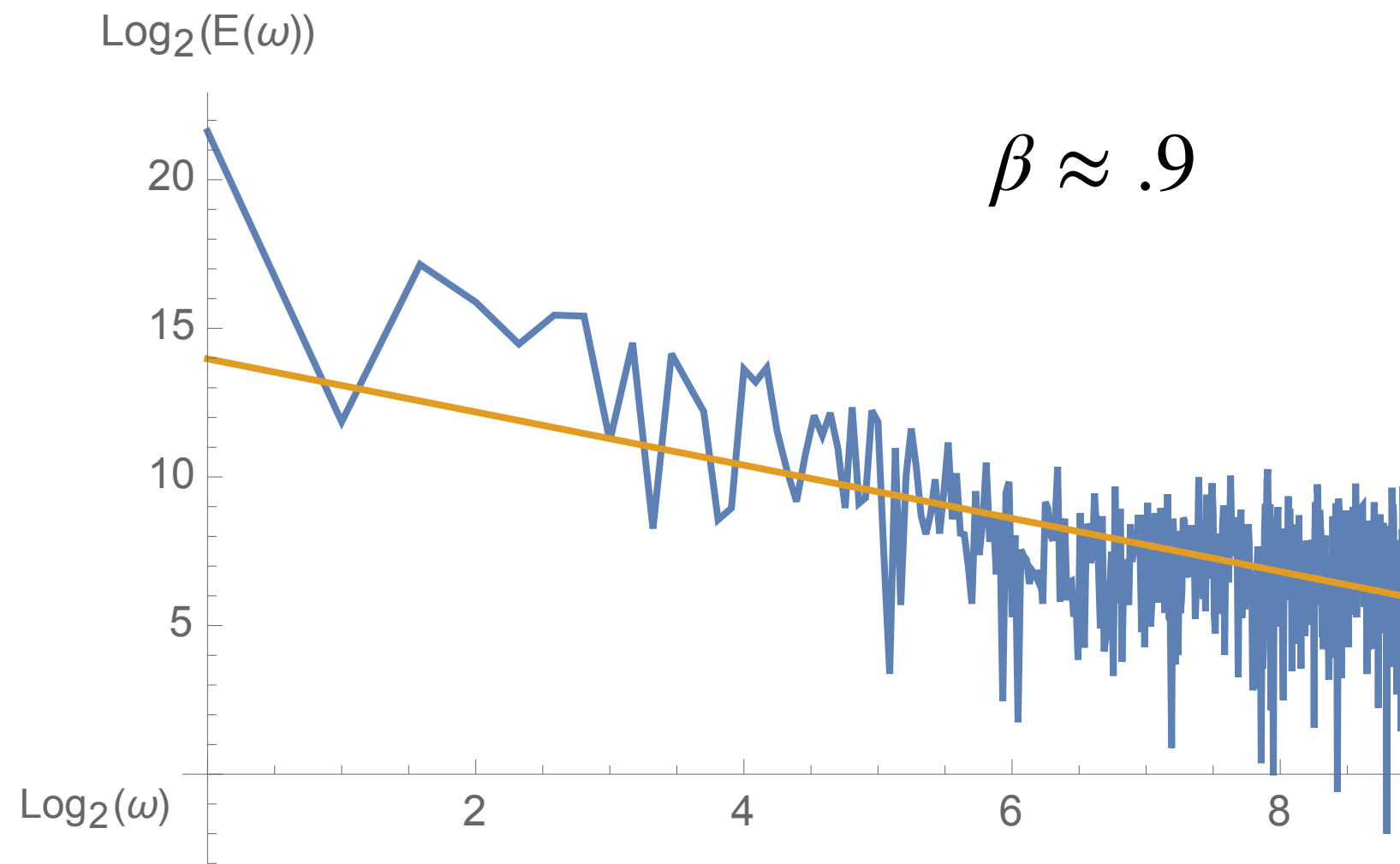
Vostok CO2



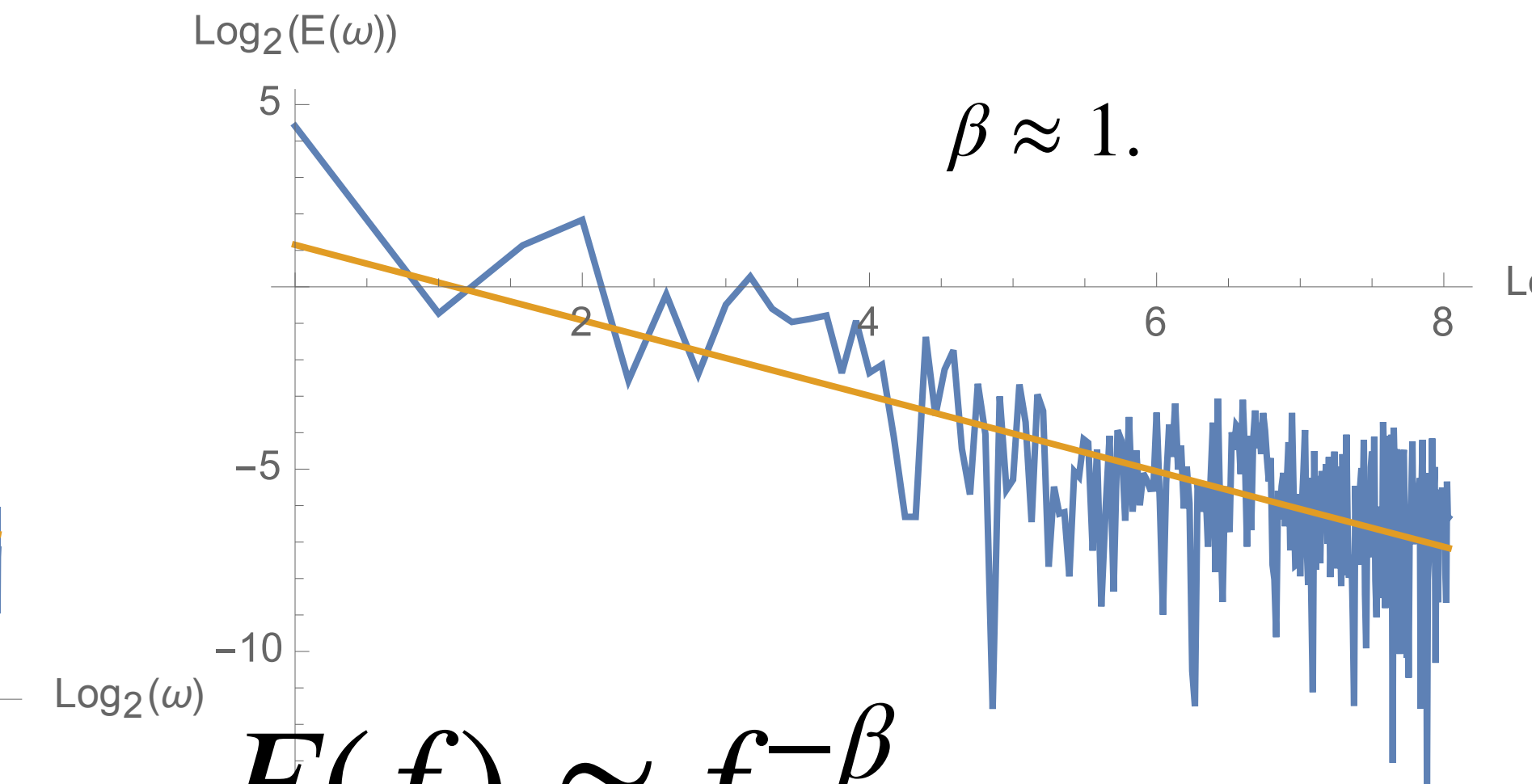
Vostok CH4



Vostok Na

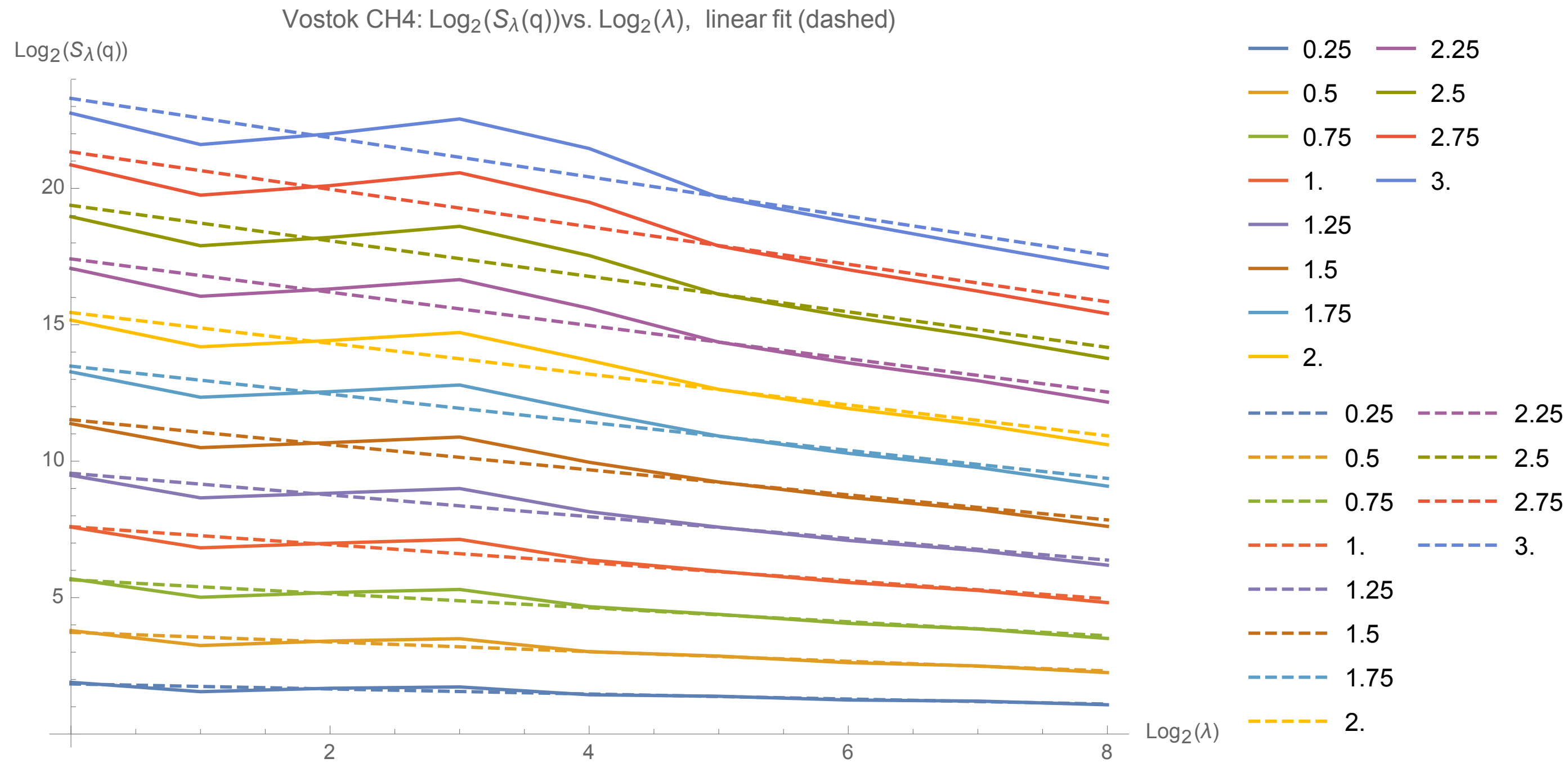


Vostok Dust

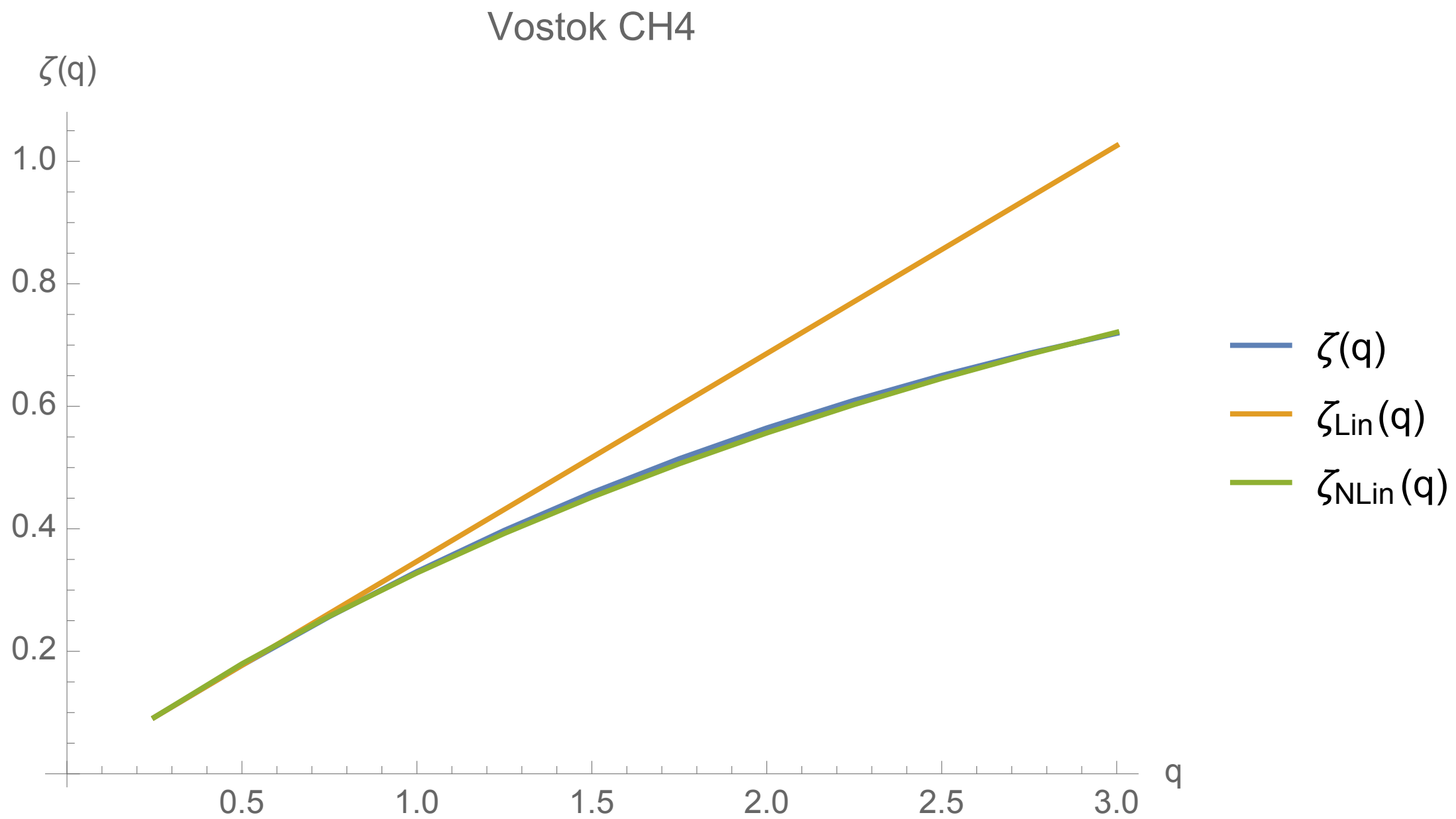


$$E(f) \approx f^{-\beta}$$

Vostok data: multi-scaling



(a)

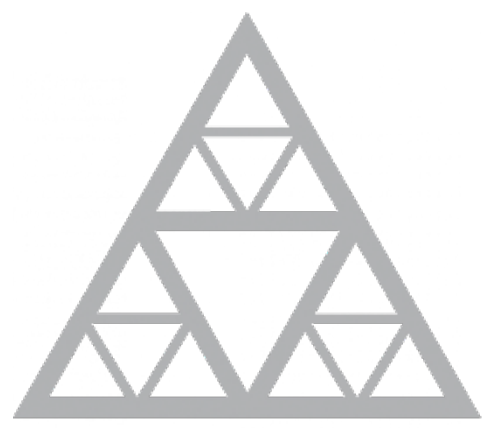


(a)

curvature => multifractality
 $H \approx 0.34, C_1 \approx 0.07, \alpha \approx 1.4$

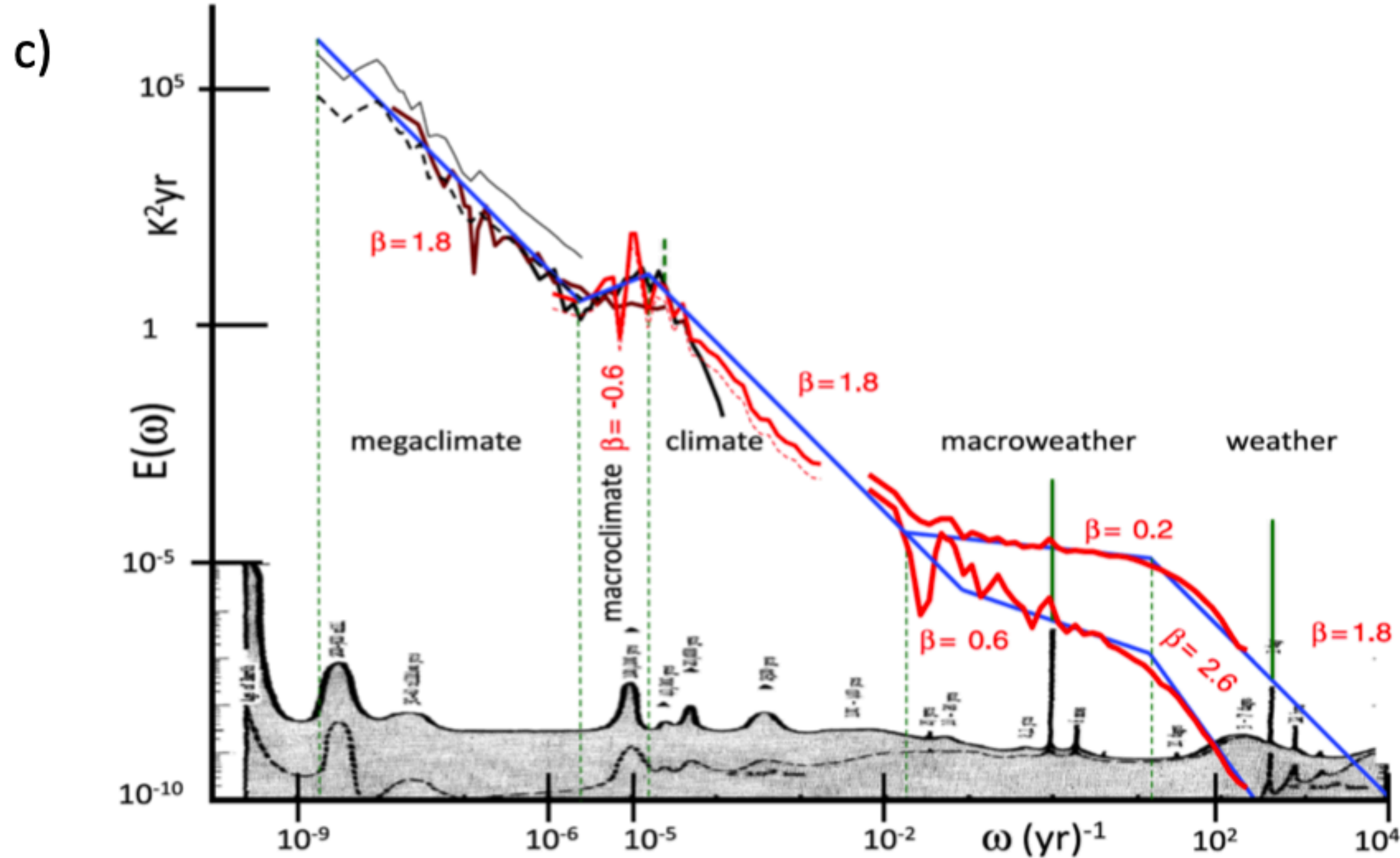
Structure functions \approx generalisation of Fourier spectra ($q \neq 2$):

$$S_\lambda(q) = \left\langle \left(\Delta\theta \left(\frac{T}{\lambda} \right)^q \right) \right\rangle \approx \lambda^{-\zeta(q)}$$



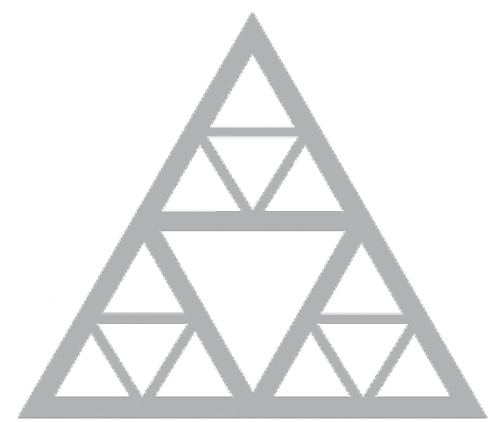
École des Ponts
ParisTech

Multiscale variability



Scaling regimes in atmospheric spectrum, Lovejoy and Schertzer (2013)

A long quest...
since antiquity,
Renaissance...



École des Ponts
ParisTech

A century of cascades !



WEATHER PREDICTION

BY

NUMERICAL PROCESS

BY

LEWIS F. RICHARDSON, B.A., F.R.MET.SOC., F.INST.P.

FORMERLY SUPERINTENDENT OF SHEDDALEMUR OBSERVATORY
LECTURER ON PHYSICS AT WESTMINSTER TRAINING COLLEGE

the sketch can be completed. We realize thus that: big whirls have little whirls that feed on their velocity, and little whirls have lesser whirls and so on to viscosity—in the molecular sense.

The Supply of Energy from and to Atmospheric Eddies.

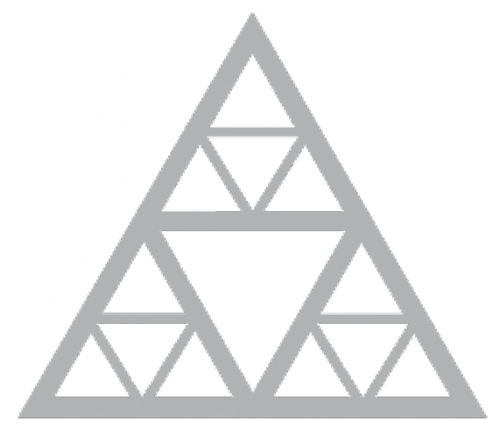
By LEWIS F. RICHARDSON.

(Communicated by Sir Napier Shaw, F.R.S. Received March 9, 1920.)

Introduction.

This paper extends Osborne Reynold's theory of the Criterion of Turbulence, to make it apply to the case in which work is done by the eddies, acting as thermodynamic engines in a gravitating atmosphere. For

“Weather Forecast as a Mathematical and Physical Problem”
“Numerical Weather Forecast: A Problem, Where Are We And What Can We Expect? Eta Model Lessons”
(Mesinger, May 24, 2001)



École des Ponts
ParisTech

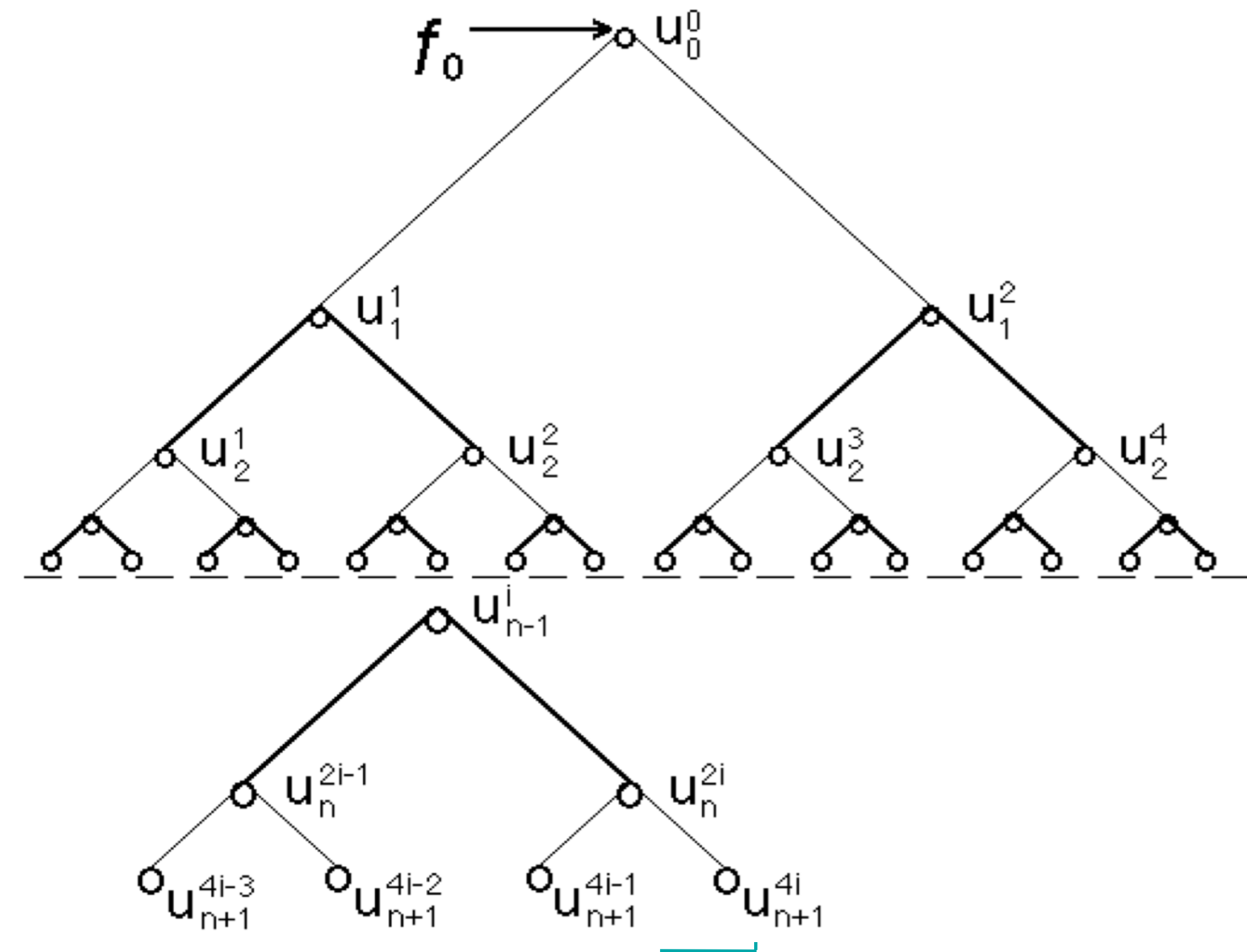
A century of cascades !



the sketch can be completed. We realize thus that: big whirls have little whirls that feed on their velocity, and little whirls have lesser whirls and so on to viscosity—in the molecular sense.

Cascade paradigm not limited to atmospheric turbulence, but rather generic of the geosphere and complex systems that "**comprise a great number of heterogenous entities**"

(French roadmap of complex systems, Bourguine et al., 2009)



From scaling analysis to cascade processes

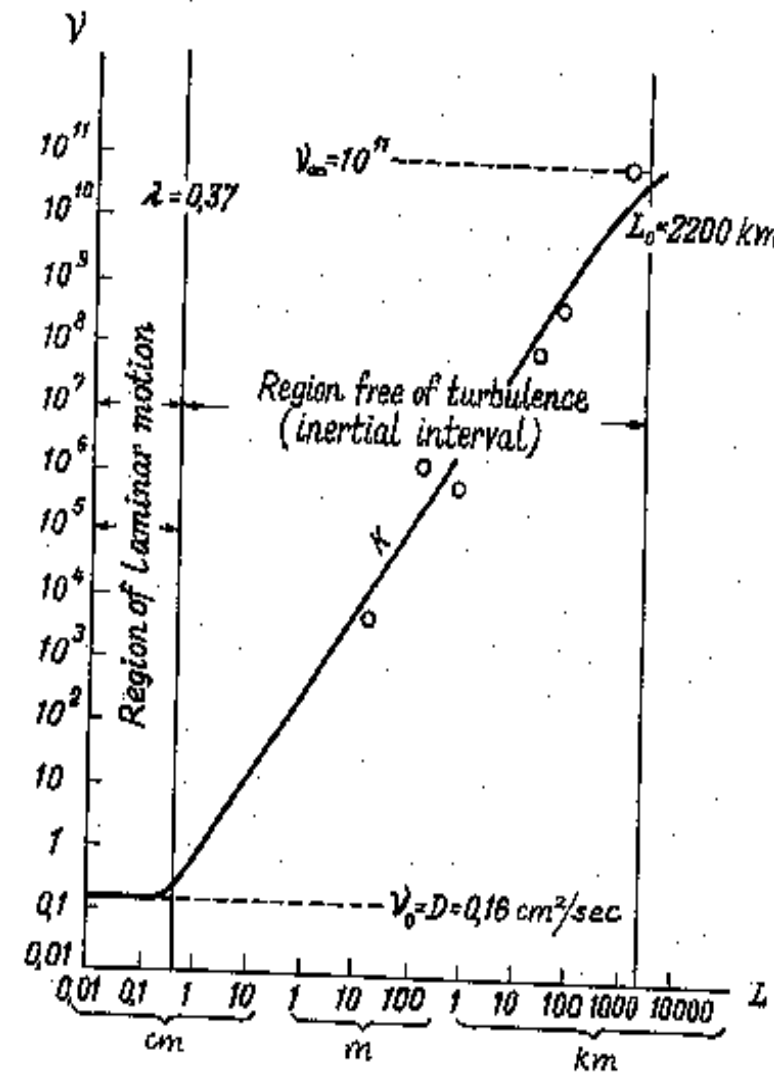
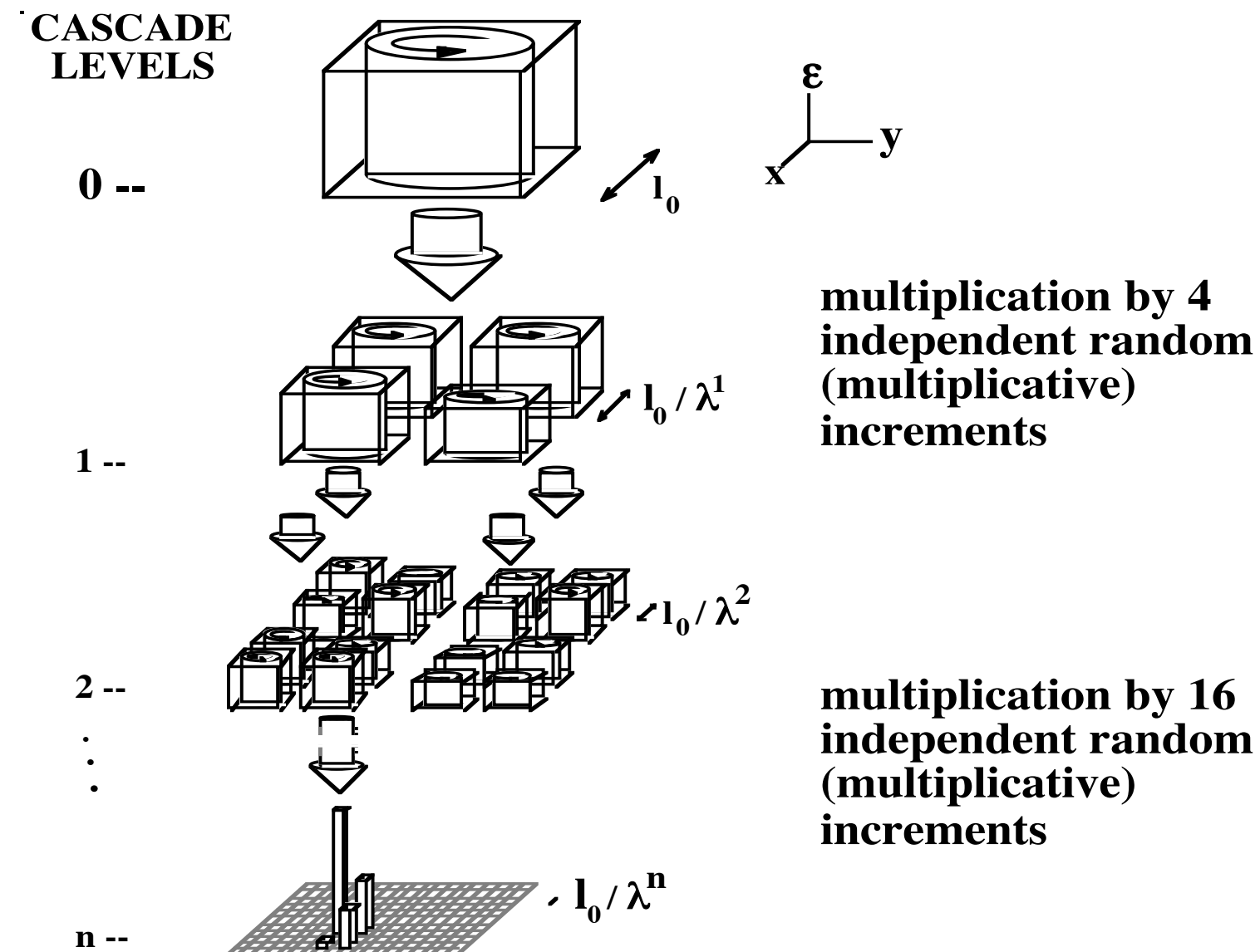


Fig. 1 The vertical diffusion coefficient $v(L)$ as a function of the turbulence scale L . Empirical points after Richardson.¹⁹

Richardson, 1926



Schertzer & Lovejoy, 1989b

- Scaling analysis
 - Passive scalar dispersion: Richardson (1926)
 - Structure function: Kolmogorov (1941)
 - Energy spectrum: Obukhov (1941)
 - Higher order structure functions: Kolmogorov (1962), Obukhov (1962)
 - Renyi dimensions: Grassberger and Procaccia (1983), Hentschel, and Procaccia (1983)
 - Legendre transform to dimensions: Parisi and Frisch (1985)
 - Fractal measures: Halsey et al. (1987)
 -

- Cascade processes
 - β -model: Novikov and Stewart (1964), Frisch et al. (1978)
 - Log normal model: Yaglom (1966)
 - Limit log-normal: Mandelbrot (1974)
 - a-model: S+L (1984, 1985)
 - Multiplicative chaos: Kahane (1985)
 - Universal multifractals/Levy multiplicative chaos: S+L (1987a&b, 1997), Fan (1987)
 - Log-Poisson model: Dubrulle (1994)
 -

Varenna summer school (1983)

- “Turbulence and Predictability in Geophysical Fluid Dynamics” organised by M. Ghil, R. Benzi et G. Parisi
 - a primary version of the multifractal formalism of Parisi and Frisch (1985) was presented there after many informal discussions. This paper concludes by: **“Still the multifractal model appears to be somewhat more restrictive than Mandelbrot’s weighted-curdling model which does include the lognormal case”**.

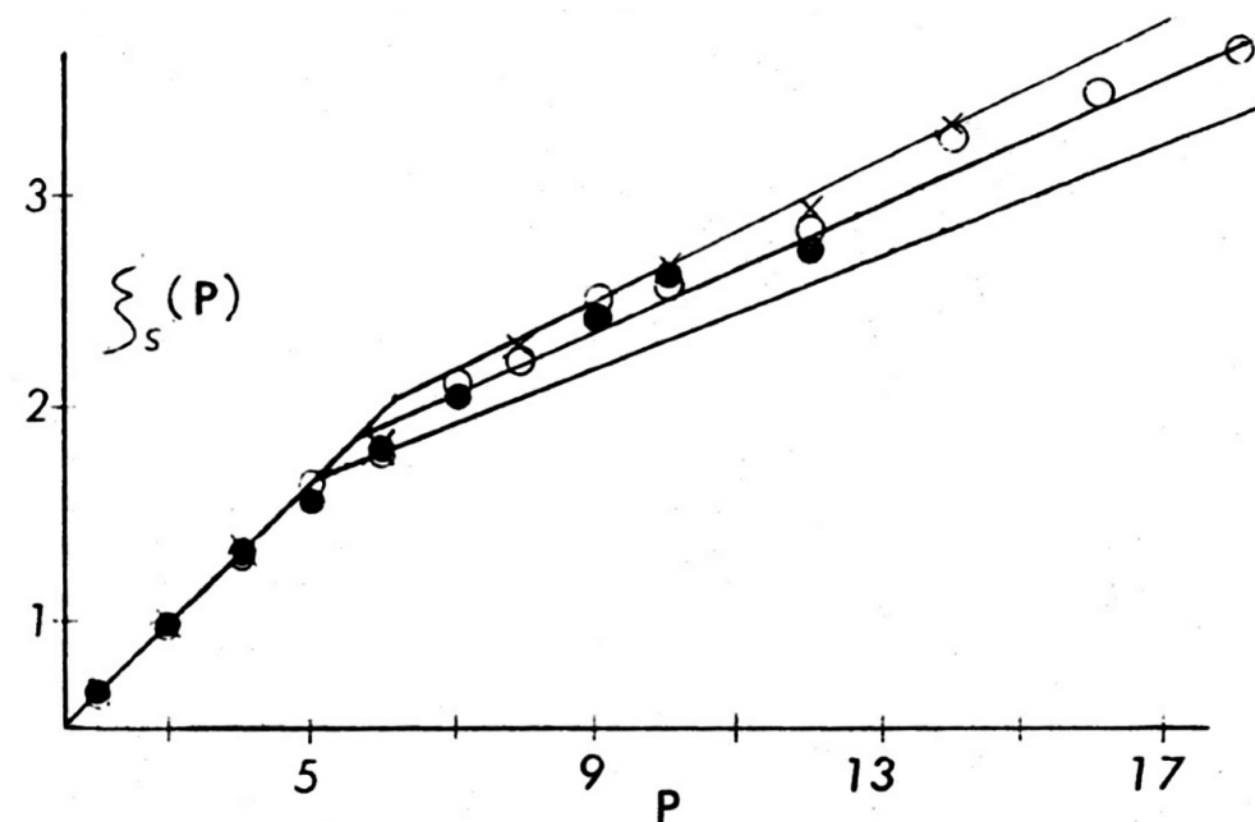
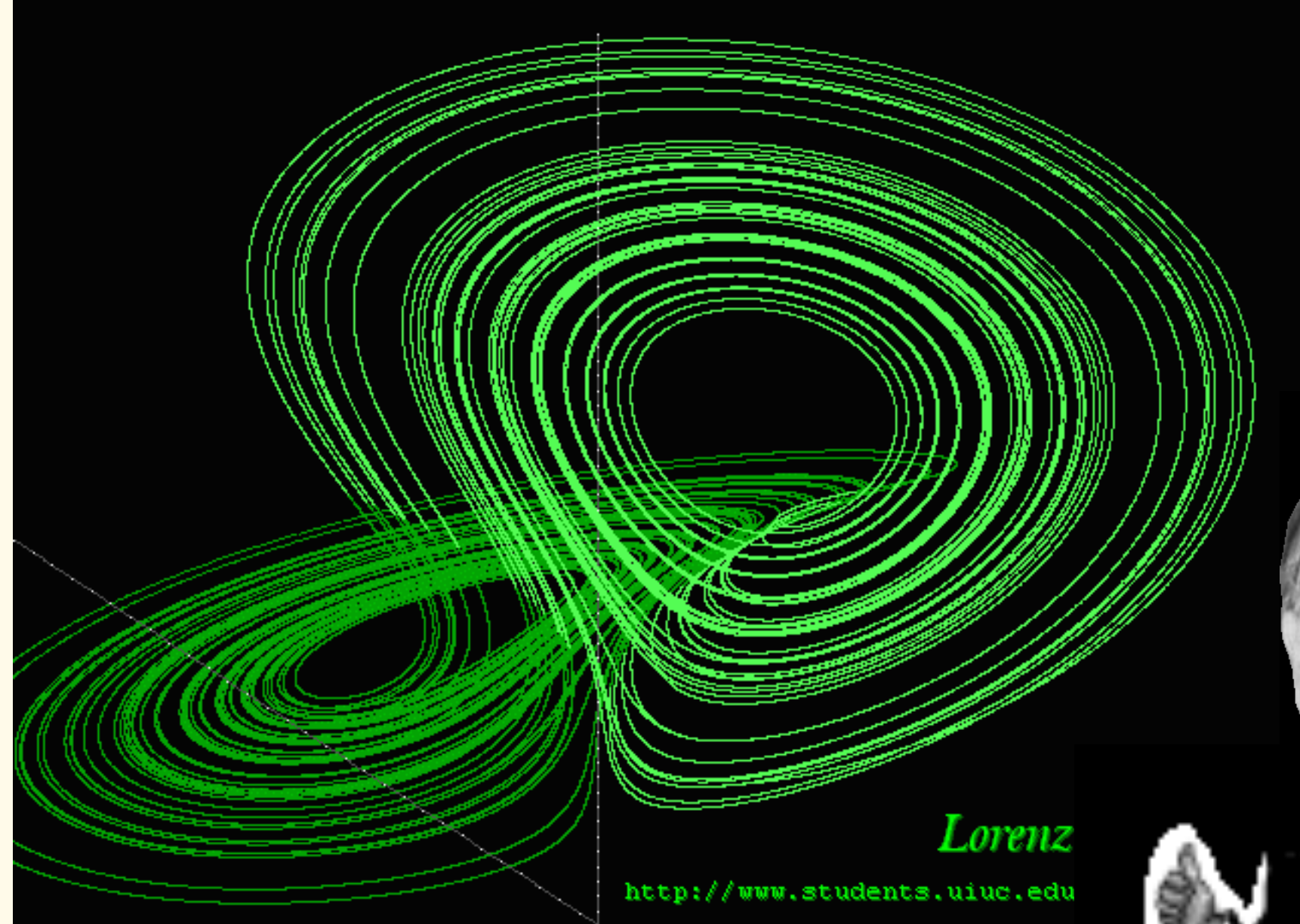


Fig.1 from S+L (1984)

- we had the opportunity to deliver a short formal presentation (the conference proceedings (1985) refers to S+L (1984)):
 - a small perturbation of the β -model is no longer limited to a unique dimension (α -model)
 - the divergence of higher order moments is rather generic in cascade models
 - the later can introduce spurious scaling, an analytical approximation depending on a unique scaling exponent H and the critical order α was proposed:

$$\xi(p) = pH + \theta(p - \alpha)(1 - p/\alpha)$$

- it was shown to fit the experimental points from Anselmet et al. (1983), see fig. 1 with $H = 1/3, \alpha = 5, 5.5, 6$



Lorenz
<http://www.students.uiuc.edu>

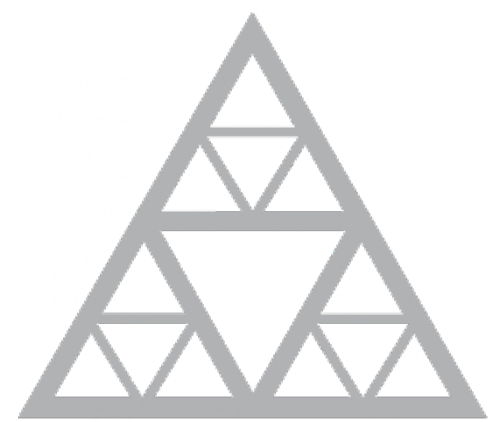
More you average
wind spectra, more
you obtain Kolmogorov's
spectrum...



Ed's statement to cool
down a hot debate
(Varena, 1983)



2008 AGU
Lorenz lecture



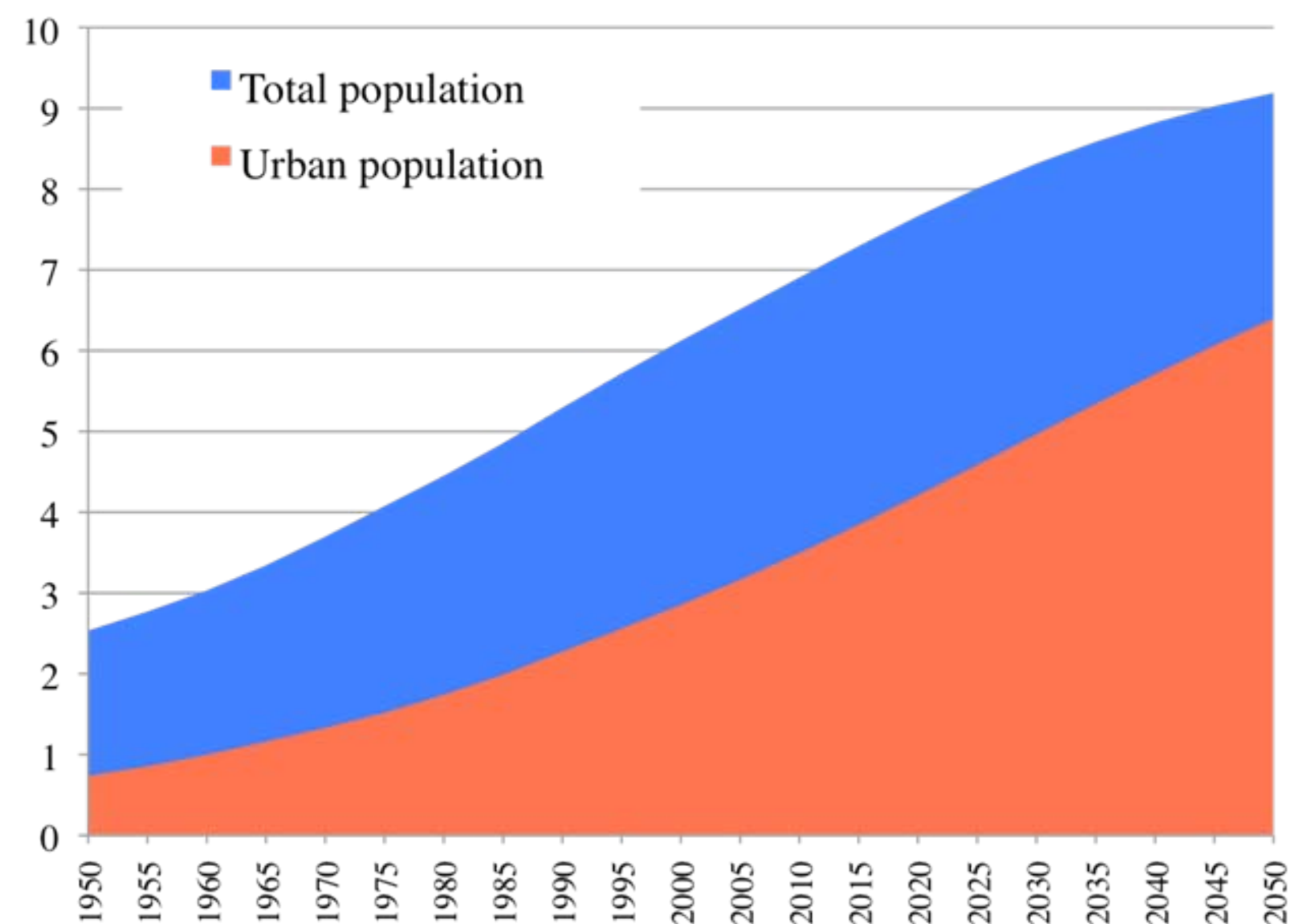
École des Ponts
ParisTech

Urban and rural populations 1950-2050 (UN projections)

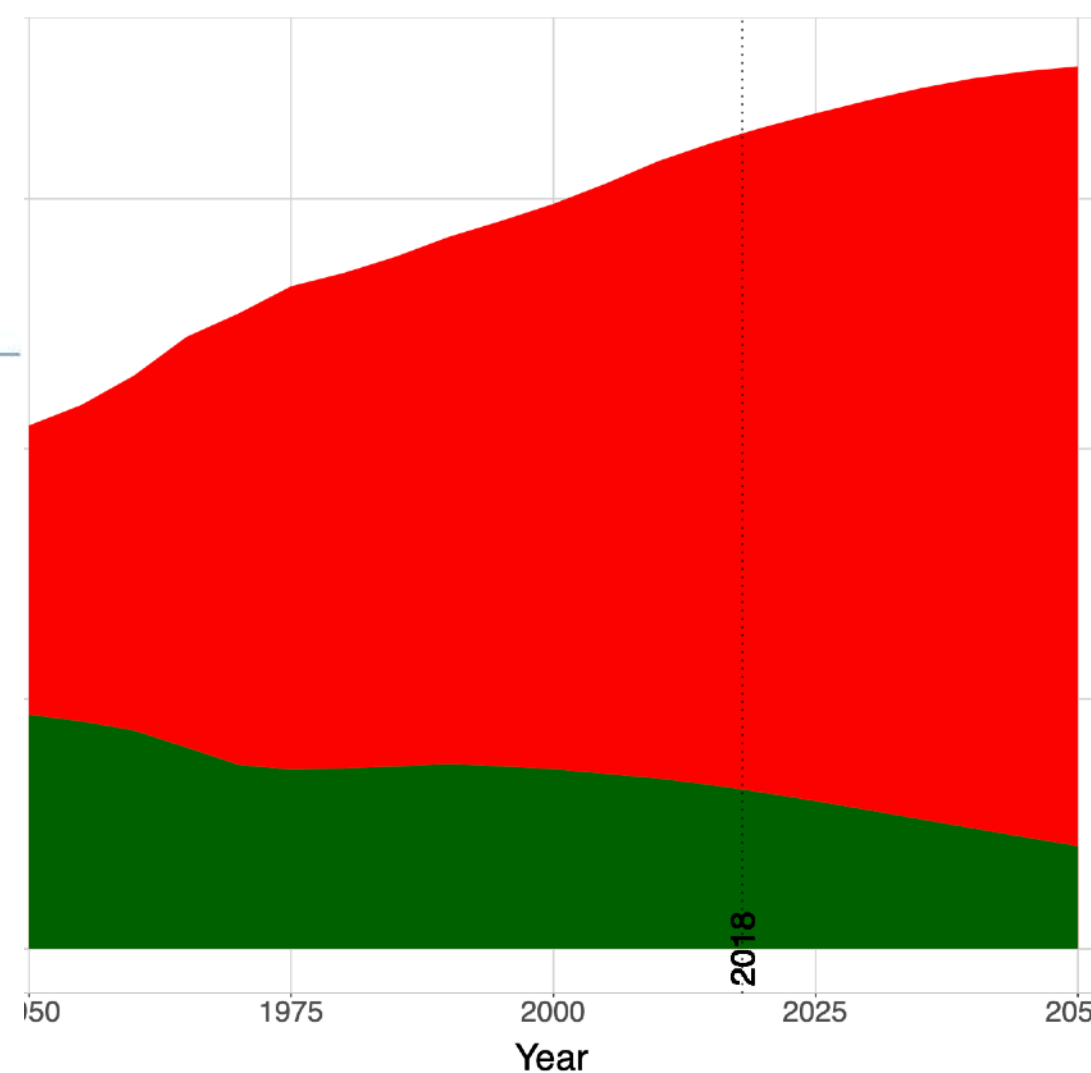


Growing Urban Population

(Billion)

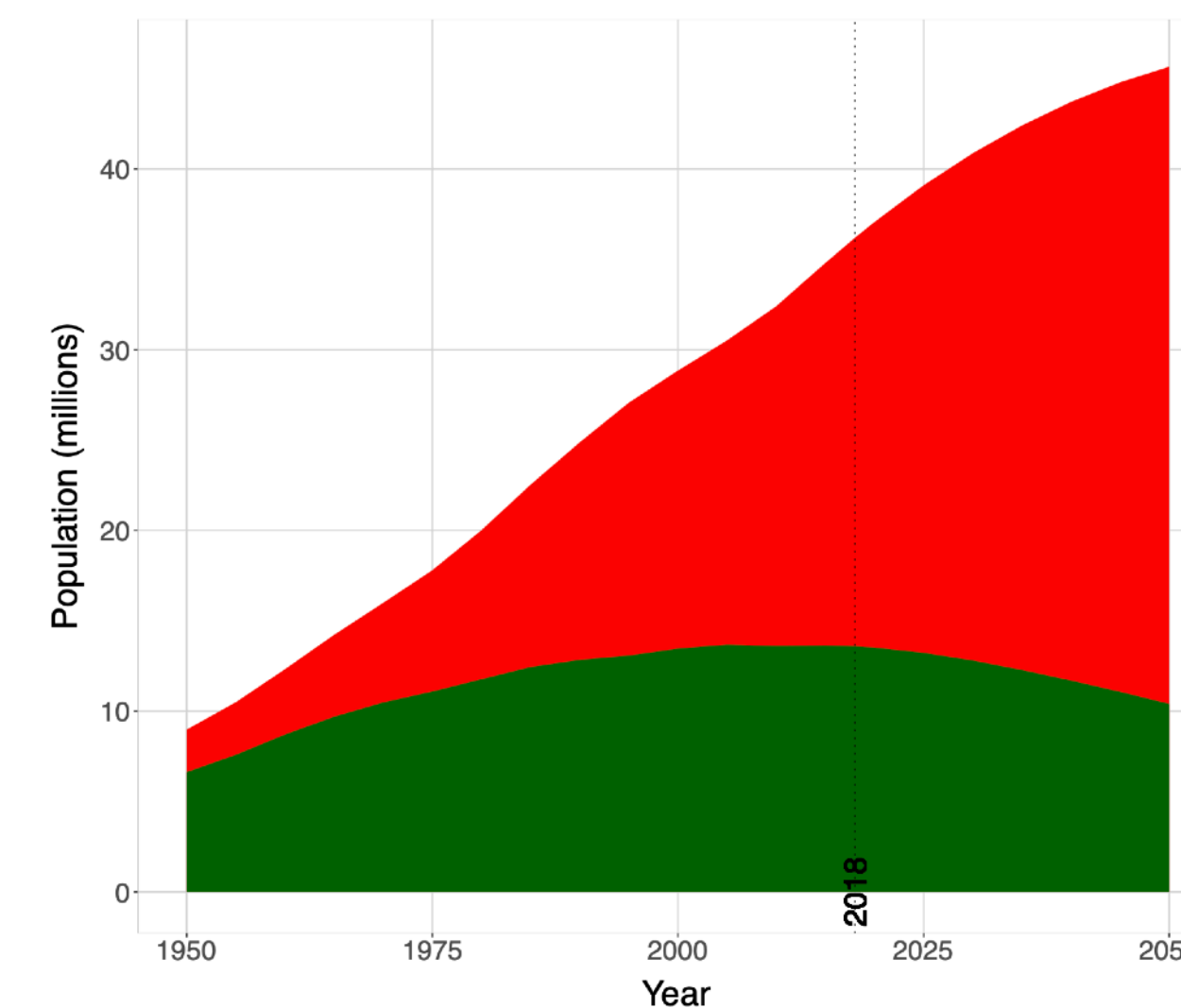


Urban and rural population
France



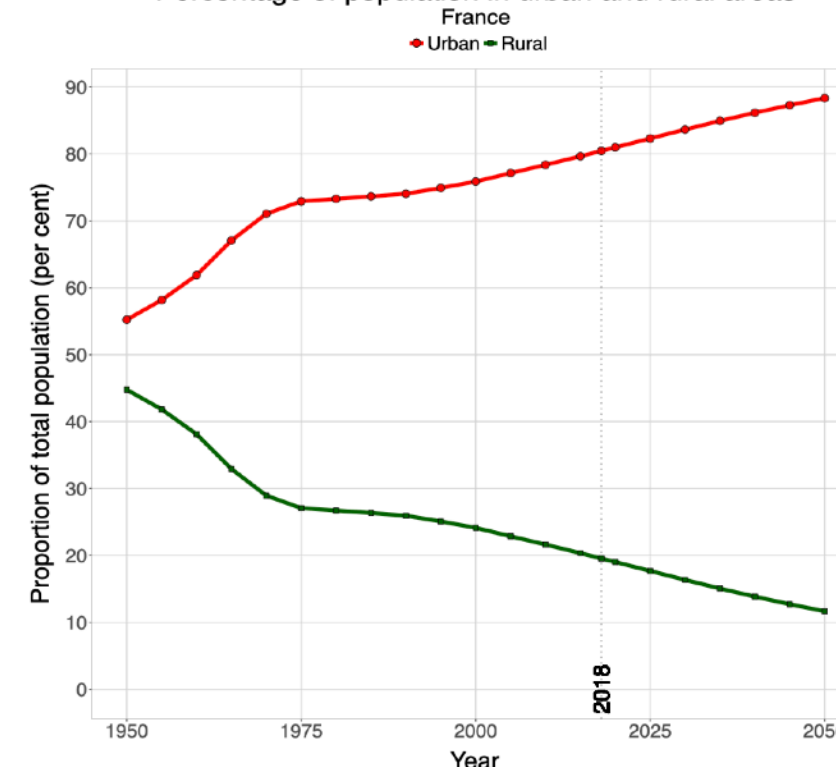
18 United Nations, DESA, Population Division. Licensed under Creative Commons license CC BY 3.0 IGO.

Urban and rural population
Morocco



© 2018 United Nations, DESA, Population Division. Licensed under Creative Commons license CC BY 3.0 IGO.

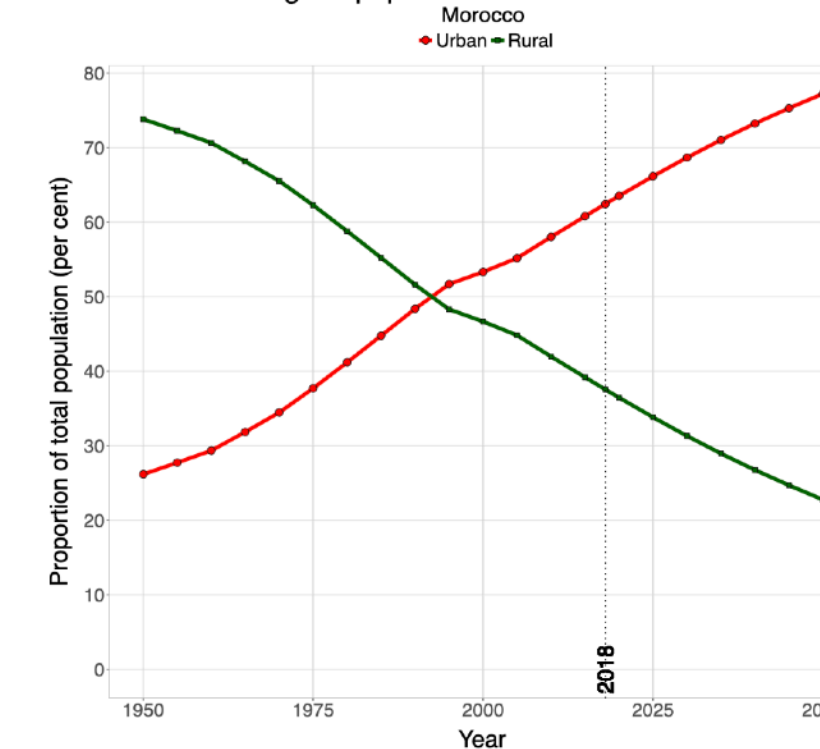
Percentage of population in urban and rural areas
France



© 2018 United Nations, DESA, Population Division. Licensed under Creative Commons license CC BY 3.0 IGO.

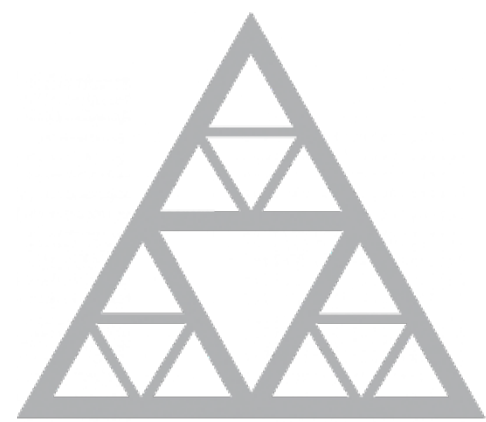
Note: Urban and rural population in the current country or area as a percentage of the total population, 1950 to 2050.

Percentage of population in urban and rural areas
Morocco



© 2018 United Nations, DESA, Population Division. Licensed under Creative Commons license CC BY 3.0 IGO.

Note: Urban and rural population in the current country or area as a percentage of the total population, 1950 to 2050.



École des Ponts
ParisTech

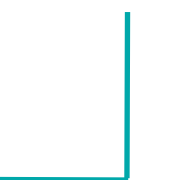
Complicated ...but also complex

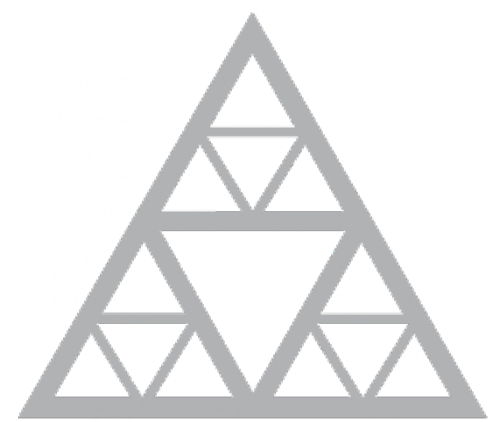


Léonard de Vinci (1452–1519)

**description of a
turbulent stream
flow (1513)**

structures and nonlinear
interactions at all scales,
nonlinear response..





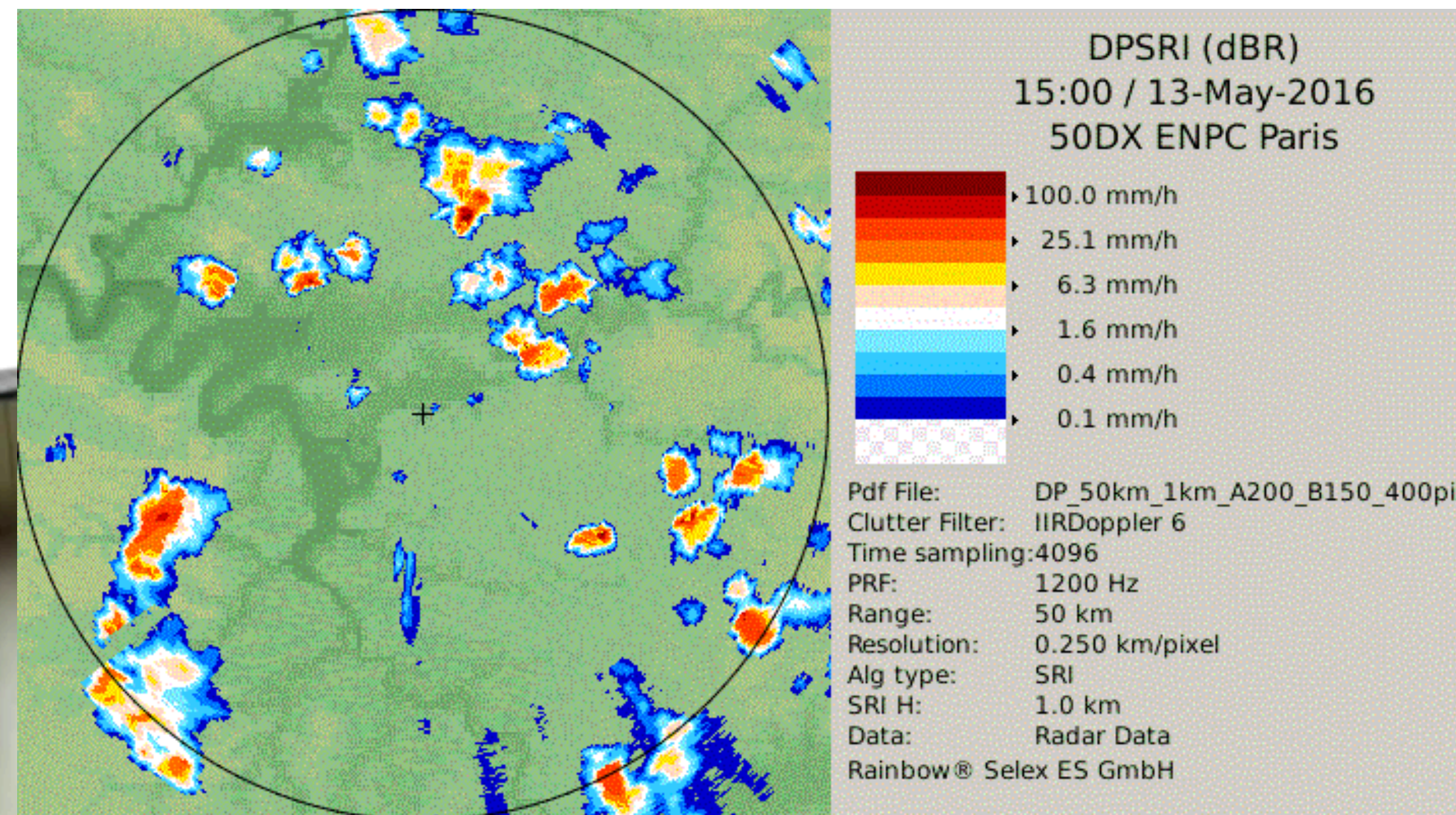
École des Ponts
ParisTech

exploratorium®



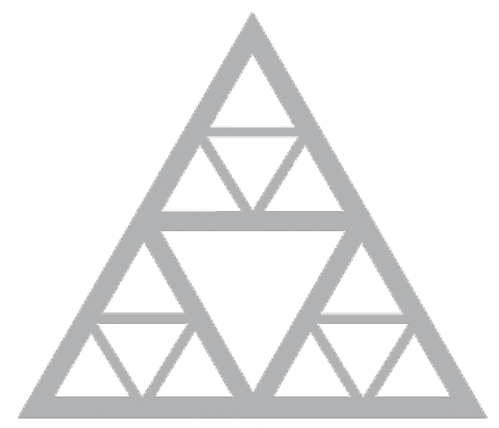
Art piece 'Windswept' (Ch. Sowers, 2012): 612 freely rotating wind direction indicators to help a large public to understand the complexity of environment near the Earth surface

Millenium problem of turbulence !



Polarimetric radar observations of heavy rainfalls over Paris region during 2016 spring (250 m resolution):

- **heaviest rain cells** are much smaller than **moderate ones**
- **complex dynamics** of their aggregation into a large front



École des Ponts

Two centuries ... since Navier (1822)



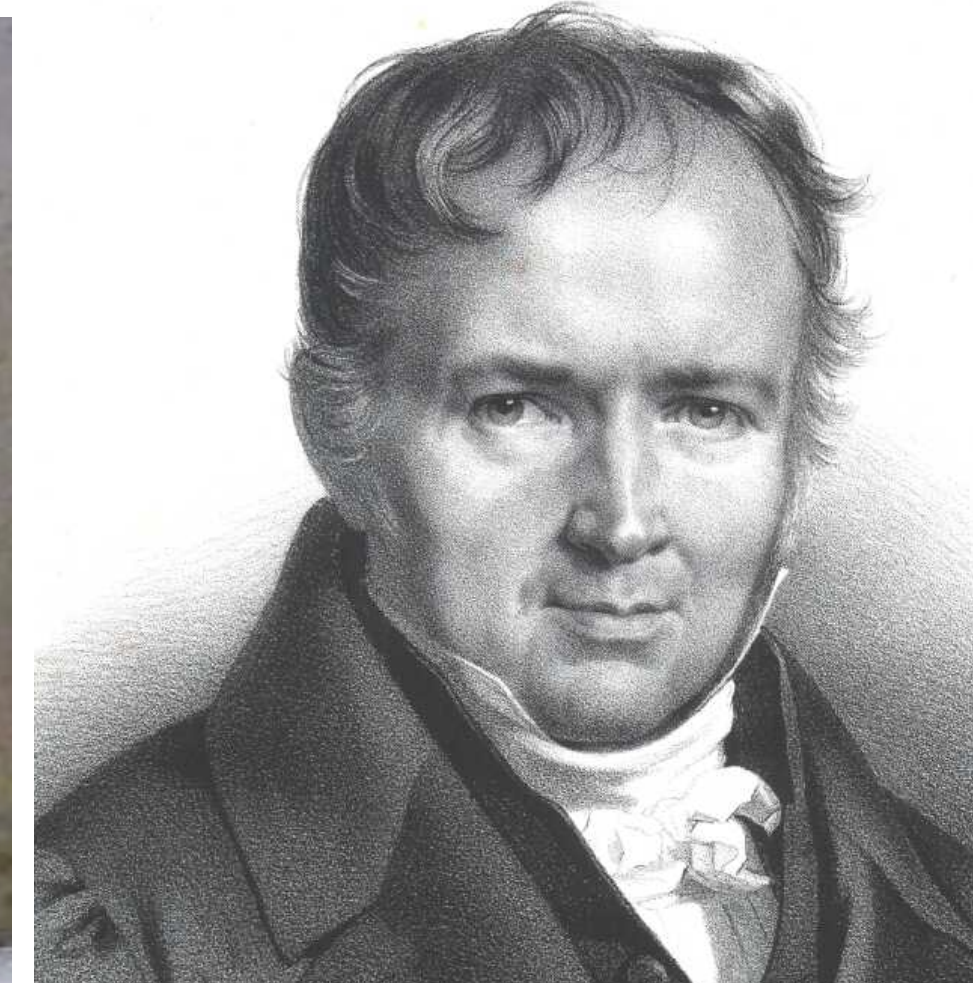
Louis Navier
(1822)



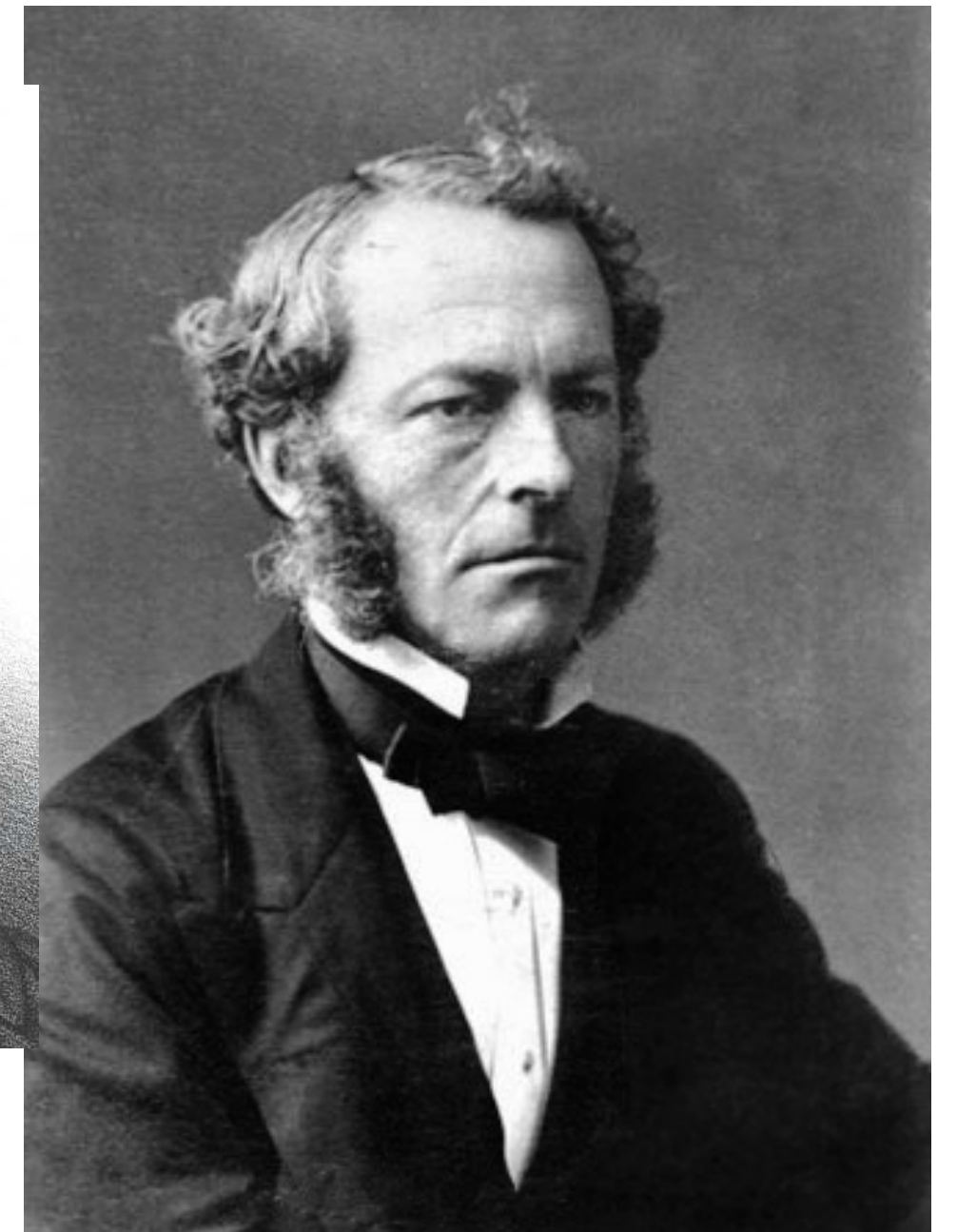
Augustin-Louis
Cauchy



Adhémar Jean Claude
Barré de Saint-Venant



Simeon Denis
Poisson

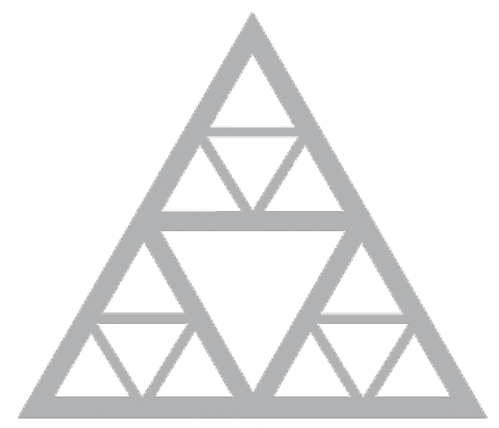


Sir George Stokes
(1843)



A **millenium problem** raised at Ecole Nationale des Ponts et Chaussées, recent episodes:
Otelbaev (2013) and Tao (2015)





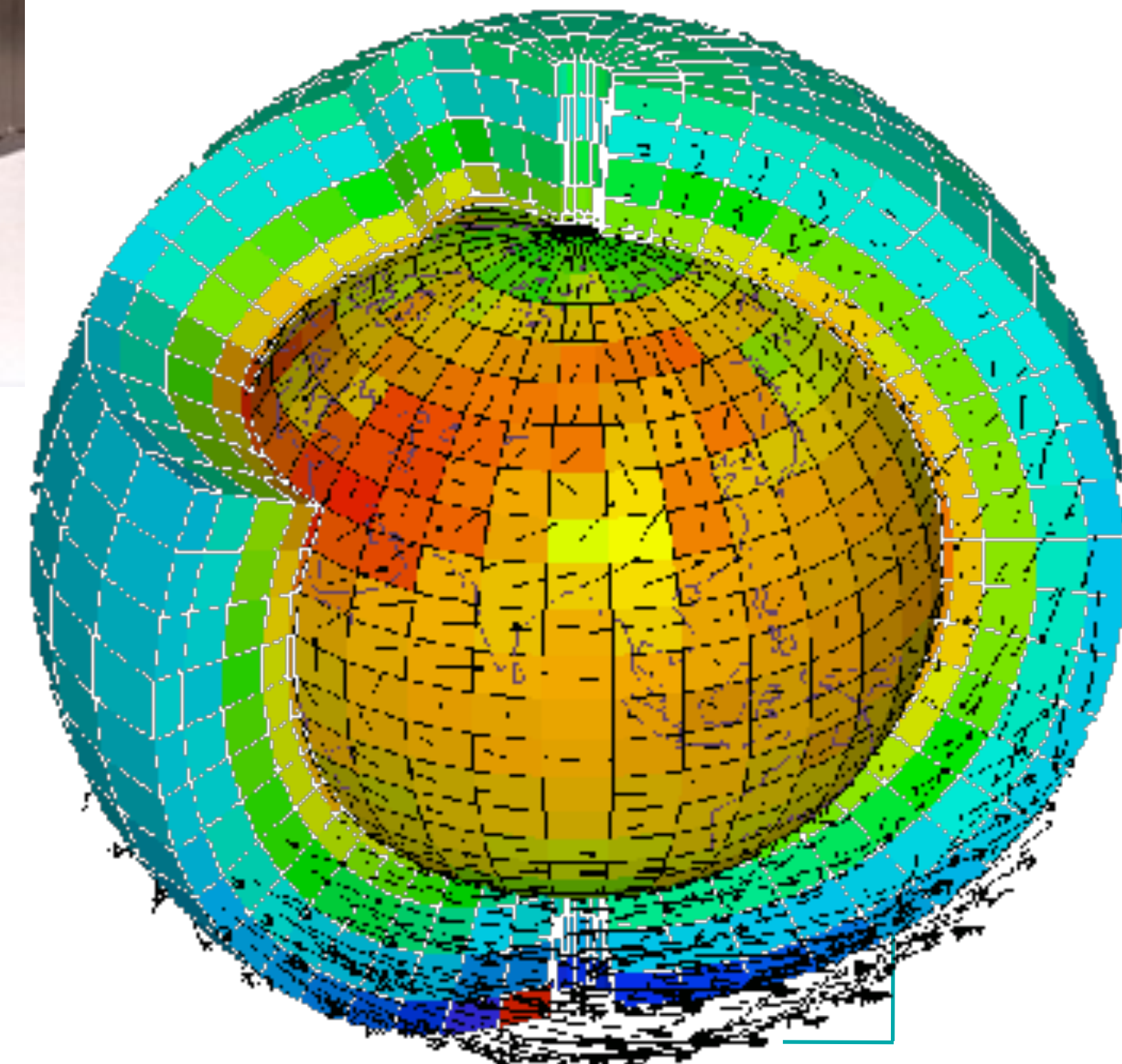
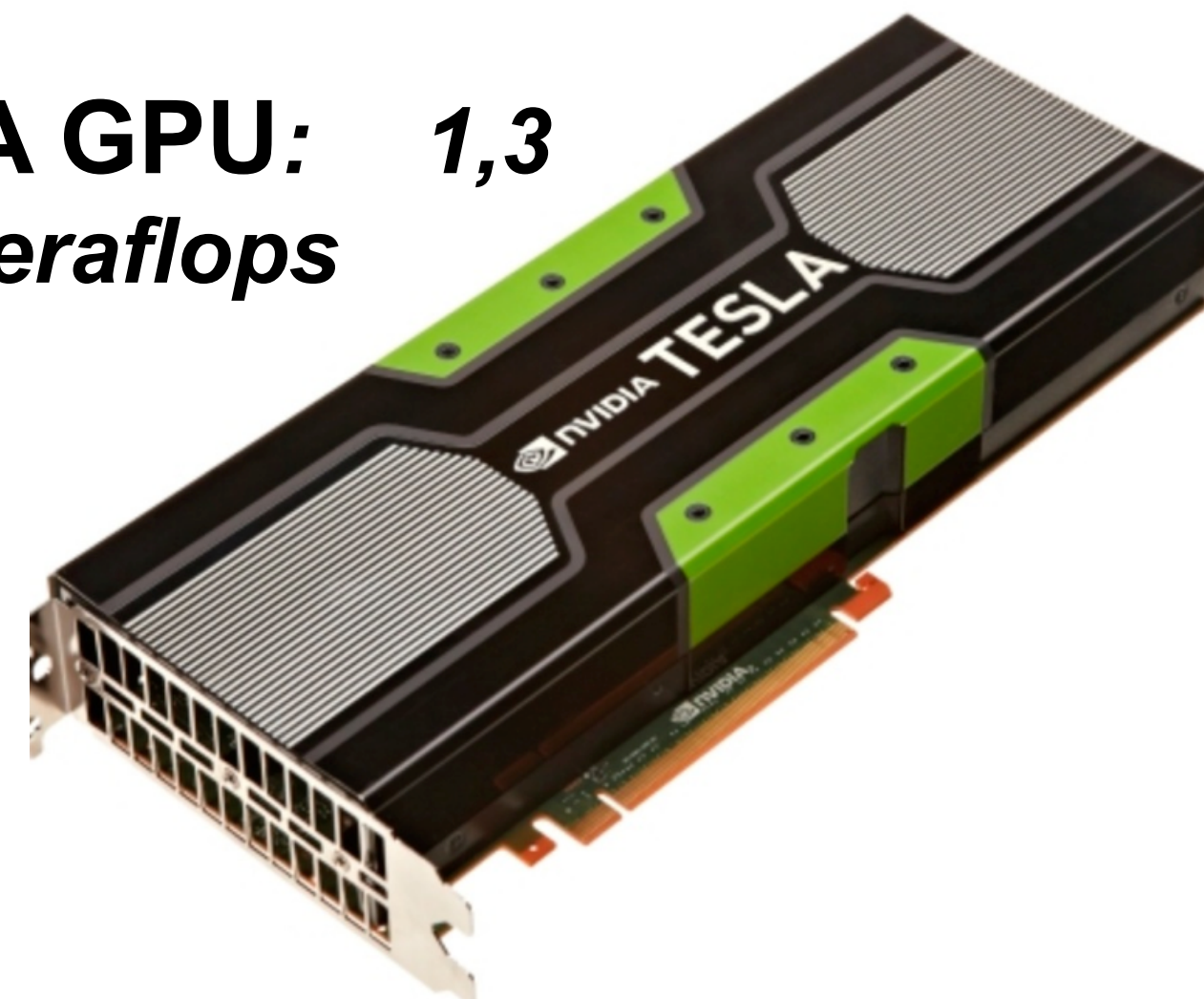
École des Ponts
ParisTech

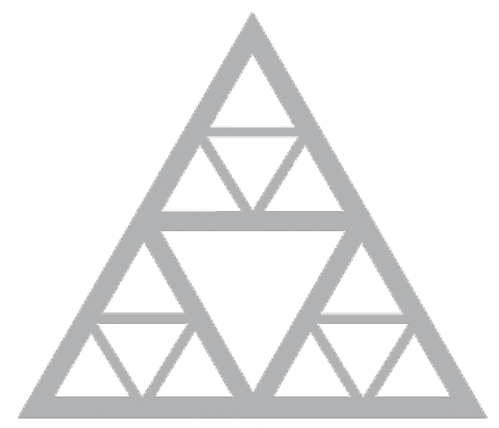
Modeling with brute computing force?



Fujitsu VPP 5000 :
0,3 Teraflops

NVIDIA GPU: 1,3
Teraflops





École des Ponts
ParisTech

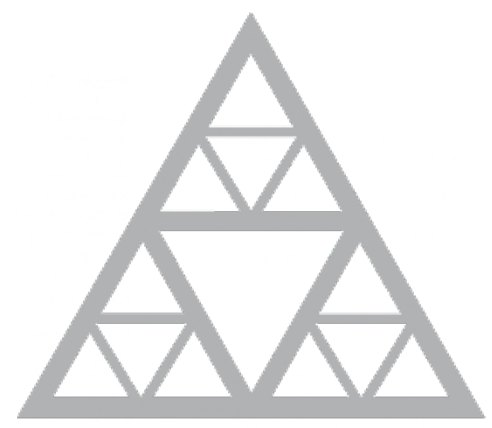
Lewis Fry Richardson's "dream" (1922)



64.000 « human computers» synchronised by a « coordinator »



(François Schuiten, 2000)

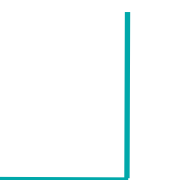
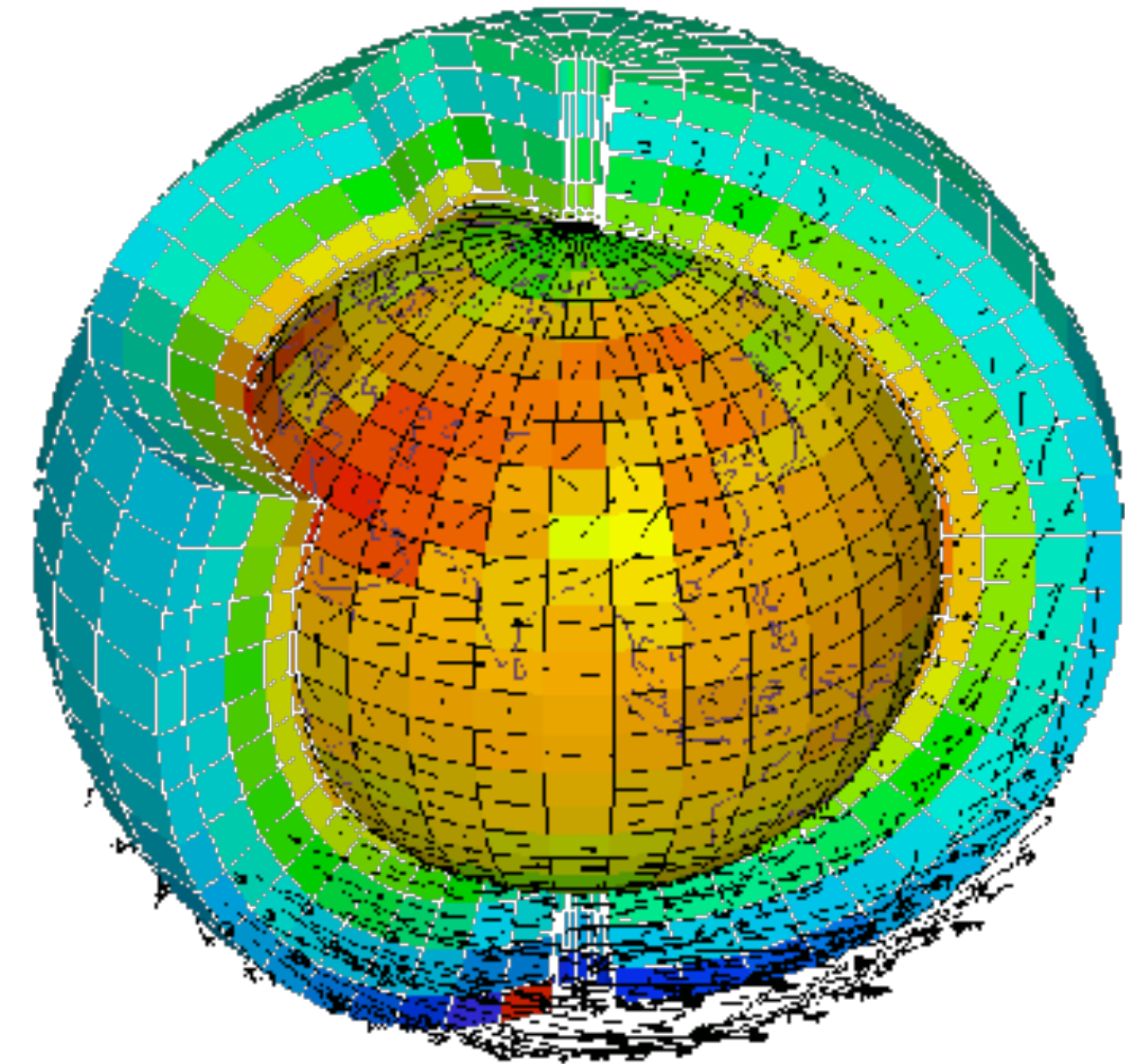


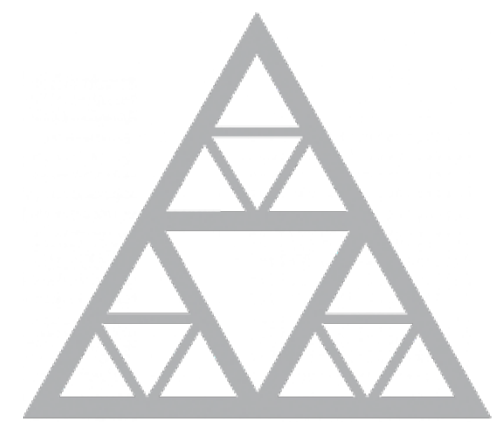
École des Ponts
ParisTech

Modelling with brute force?



- Multiscale complexity vs. computing brute force
 - (nonlinear) equations discretised on N voxels (\approx cubes)
 - # voxels (1mm^3) to reach the **viscous scale** ($\approx 1\text{mm}$):
 - atmosphere $\approx 10\text{ km}$ high x $(10,000\text{ km})^2$ horizontal
 - $\Rightarrow N \approx 10^7(10^{10})^2 = 10^{27}$
 - much larger than $N_A = 10^{23}$!!
- Why to be deterministic... over only a small range of scales?





École des Ponts

ParisTech

Scale symmetry and equations



Whereas elementary mathematical properties of Navier-Stokes solutions are still unknown (existence, uniqueness):

$$\frac{\partial \underline{u}}{\partial t} + \underline{u} \cdot \underline{grad}(\underline{u}) = \underline{f} - \frac{1}{\rho} \underline{grad}(p) + \nu \Delta \underline{u}$$

one can point out a **scale symmetry** (*): $T_\lambda: \underline{x} \mapsto \underline{x}/\lambda$

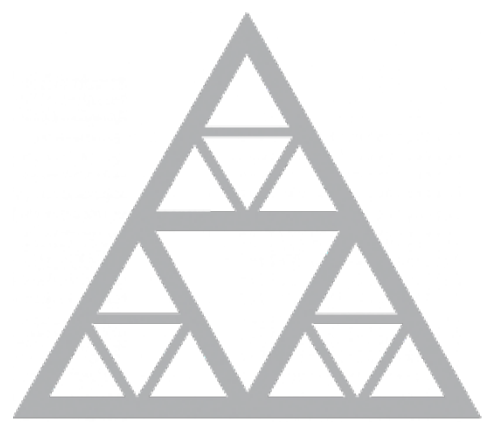
$$\tilde{T}_\lambda: \begin{aligned} t &\mapsto t/\lambda^{1-\gamma}; \underline{u} \mapsto \underline{u}/\lambda^\gamma; \\ \underline{f} &\mapsto \underline{f}/\lambda^{2\gamma-1}; \nu \mapsto \nu/\lambda^{1+\gamma}; \rho \mapsto \rho/\lambda^{\gamma'} \end{aligned}$$

Kolmogorov's scaling (K41) obtained with: $\varepsilon(\ell) \approx \frac{\delta u^3}{\ell} \approx \bar{\varepsilon} \Rightarrow \gamma_K = -1/3$

General case: multiple singularities γ 's: $\Pr(\gamma' > \gamma) \approx \lambda^{-c(\gamma)}$

(*) from self-similarity (Sedov, 1961), symmetry (Parisi + Frisch, 1985), to generalised Galilean invariance (S+al, 2010)





École des Ponts

ParisTech

Scale symmetry and equations



Whereas elementary mathematical properties of Navier-Stokes solutions are still unknown (existence, uniqueness):

$$\frac{\partial \underline{u}}{\partial t} + \underline{u} \cdot \underline{grad}(\underline{u}) = \underline{f} - \frac{1}{\rho} \underline{grad}(p) + \nu \Delta \underline{u}$$

one can point to **scale symmetry (*)**: $T_\lambda: \underline{x} \mapsto \underline{x}/\lambda$

$$\tilde{T}_\lambda: \underline{t} \mapsto \underline{t}/\lambda^{1-\gamma}; \underline{u} \mapsto \underline{u}/\lambda^\gamma; \underline{f} \mapsto \underline{f}/\lambda^{2\gamma-1}; \nu \mapsto \nu/\lambda^{1+\gamma}; \rho \mapsto \rho/\lambda^{\gamma'}$$

Kolmogorov's scaling (K41) obtained with: $\varepsilon(\ell) \approx \frac{\delta u^3}{\ell} \approx \bar{\varepsilon} \Rightarrow \dots 1/3$

General case: multiple singularities γ 's: $\Pr(\gamma' > \gamma) \approx \lambda^{-c(\gamma)}$

(*) from self-similarity (Sedov, 1961), symmetry (Parisi + Frisch, 1985), to generalised Galilean invariance (S+al, 2010)

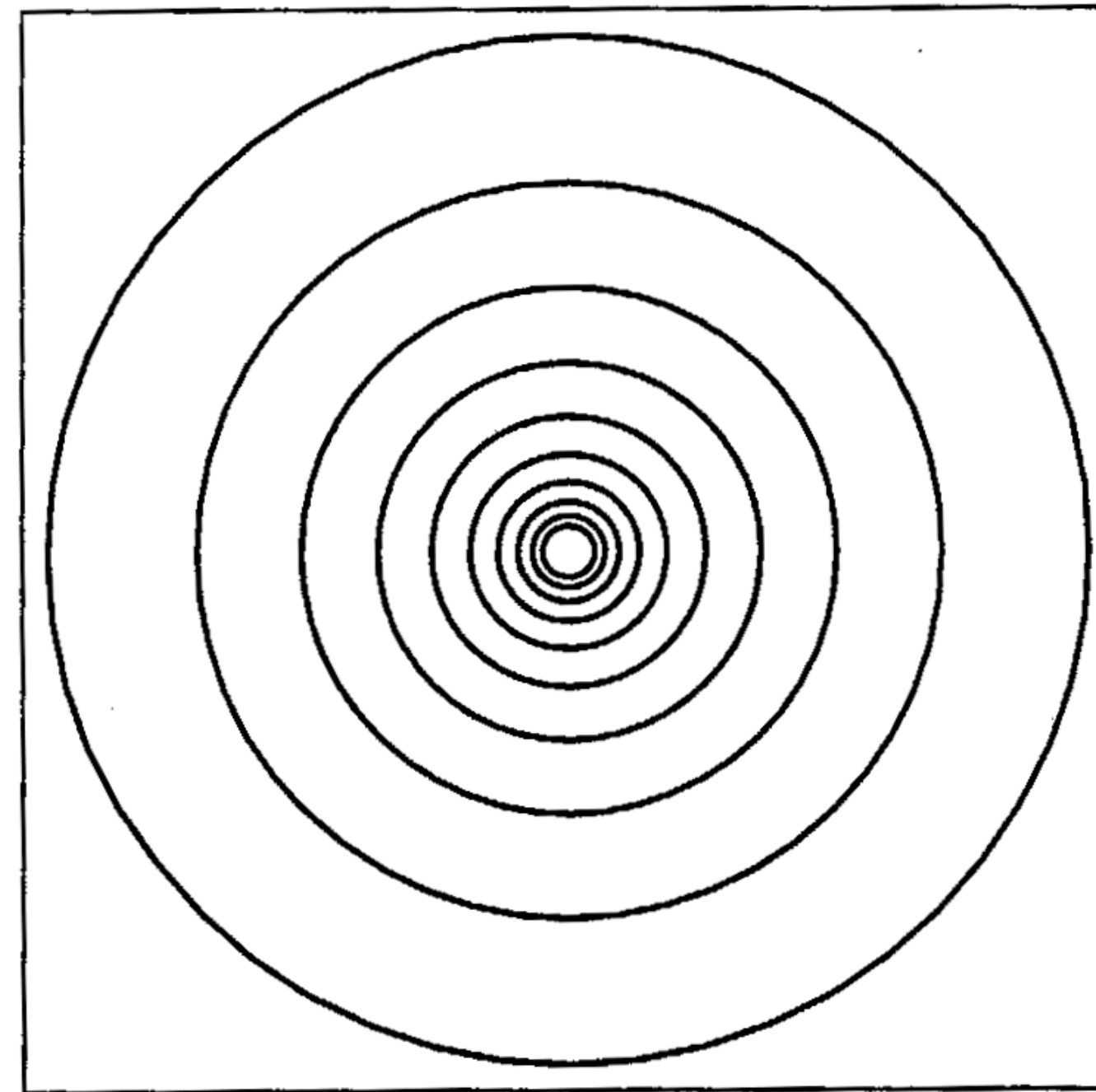


Euclidean scales vs. physical scales (23/9-D atmospheric turbulence)

- stratification
- flattening of structures at larger scales

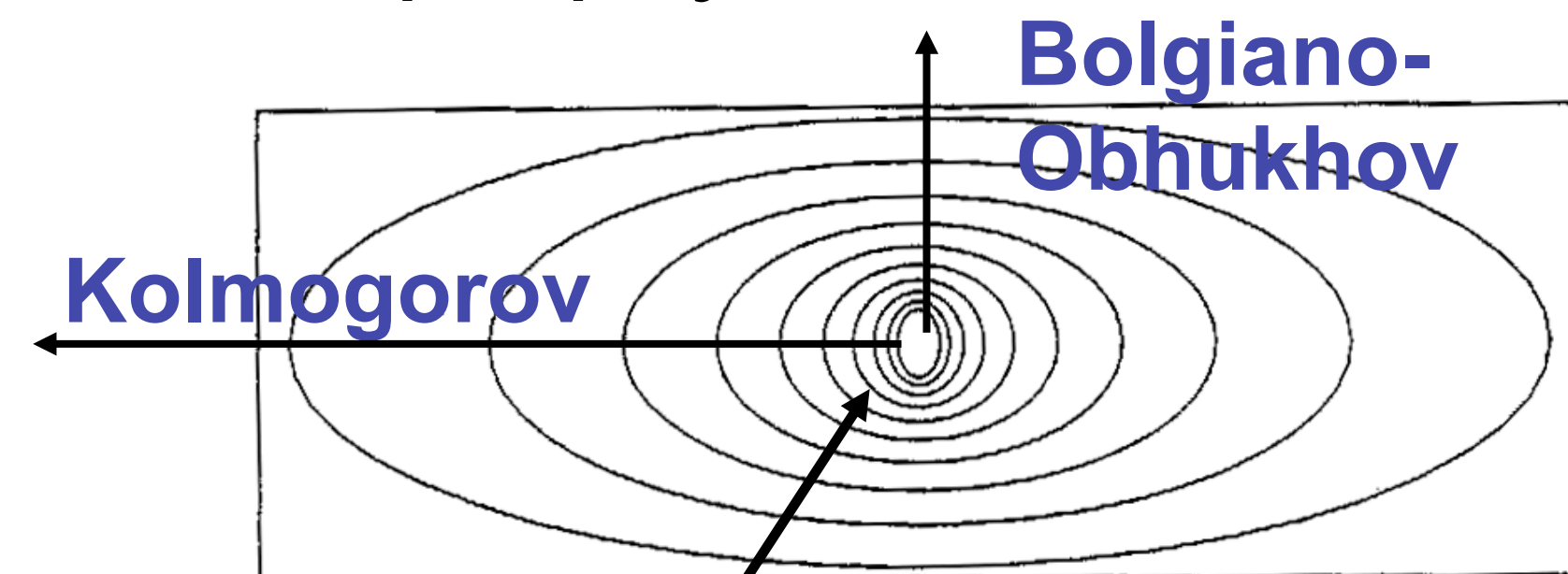
$$\|\Delta r\| = l_s \left(\left(\frac{\Delta y}{l_s} \right)^2 + \left(\frac{\Delta x}{l_s} \right)^2 + \left(\frac{\Delta z}{l_s} \right)^{2/H_z} \right)^{1/2}$$

Isotropic scales



$H_z=1$

Anisotropic physical scales

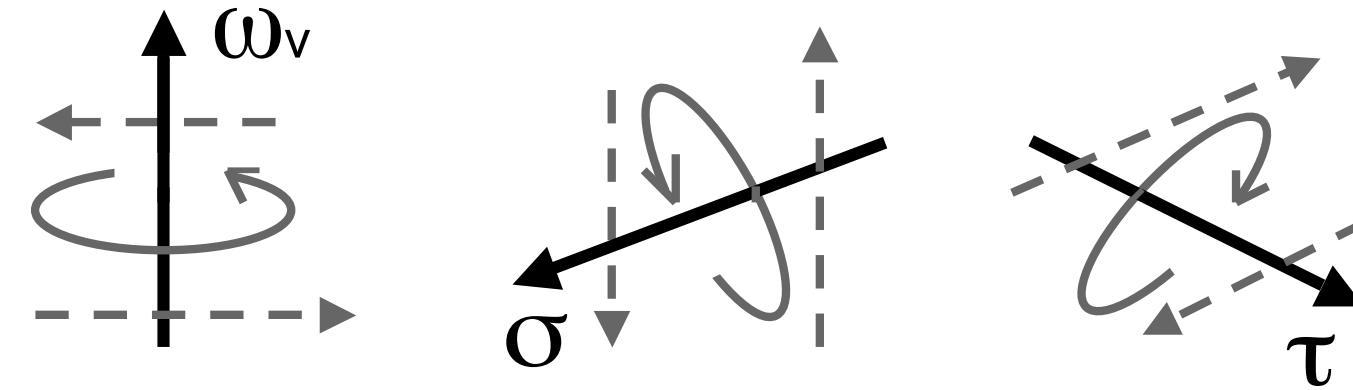
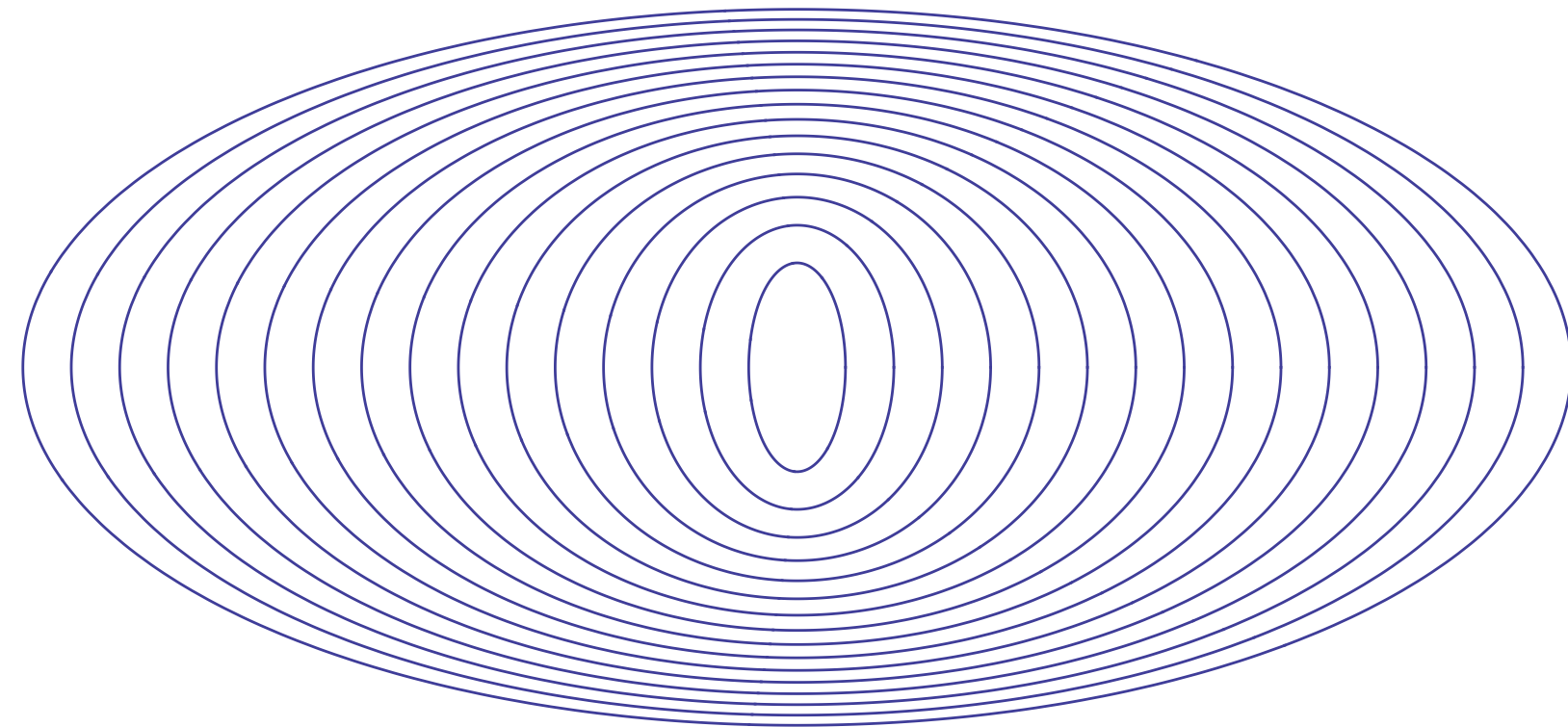


$H_z=5/9$

D-dimensional vorticity equation

$$D=2+Hz \quad (0 < Hz < 1)$$

Stratified atmosphere:



$$D\vec{\sigma}/Dt = (\vec{\sigma} \cdot \vec{\nabla}_h)\vec{u}_h$$

$$D\vec{\tau}/Dt = (\vec{\tau} \cdot \vec{\nabla}_h + \underbrace{\vec{\omega}_v \cdot \vec{\nabla}_v})\vec{u}_h$$

$$D\vec{\omega}_v/Dt = (\vec{\tau} \cdot \vec{\nabla}_h + \vec{\omega}_v \cdot \vec{\nabla}_v)\vec{u}_v$$

Strong interactions between *local generalized* scales,

= *strongly non local* (Euclidean) scales !

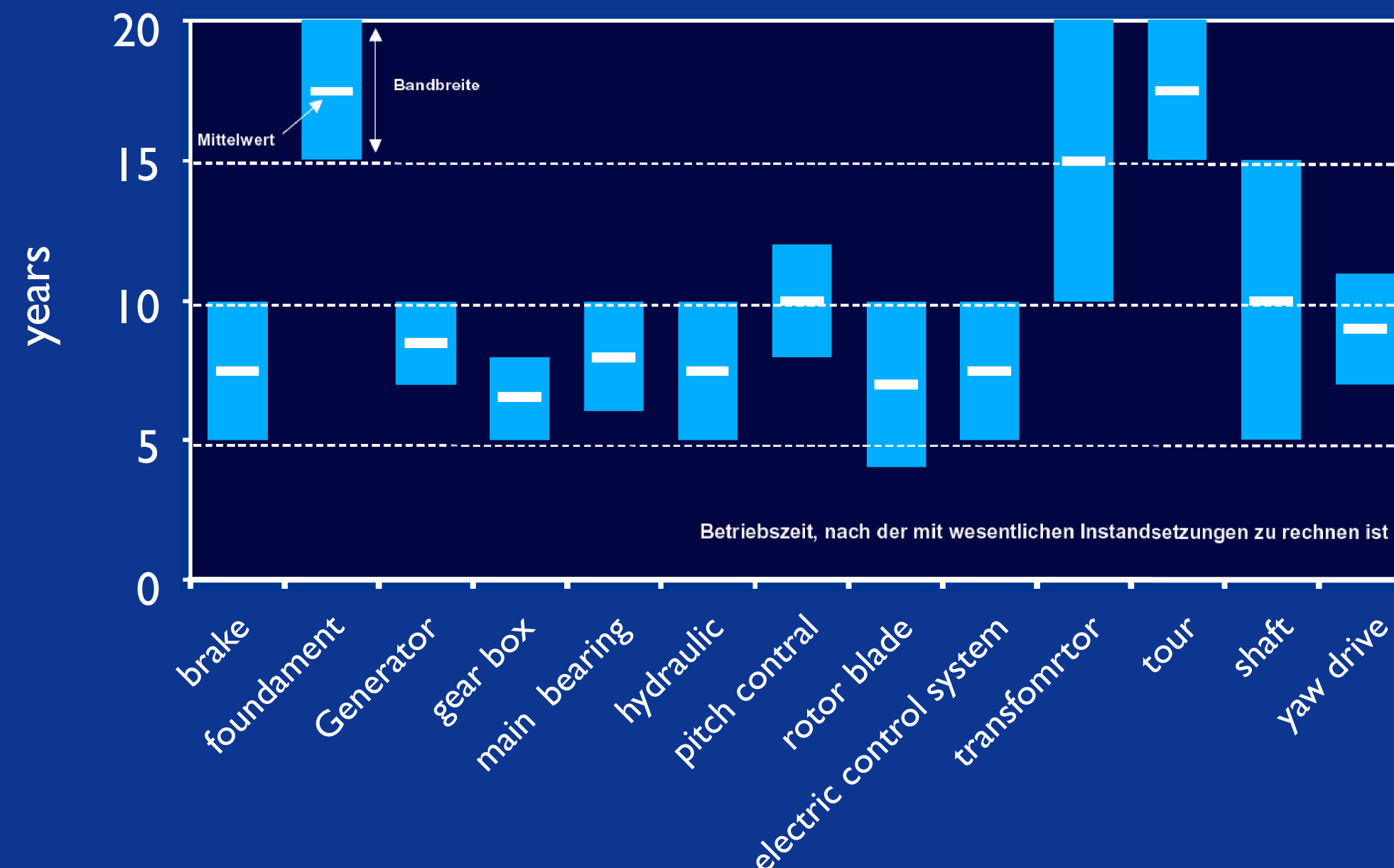
- a difficulty for direct numerical simulations ?
- easy for stochastic simulations

Wind Energy - on- and offshore - 2030 - > 300GW

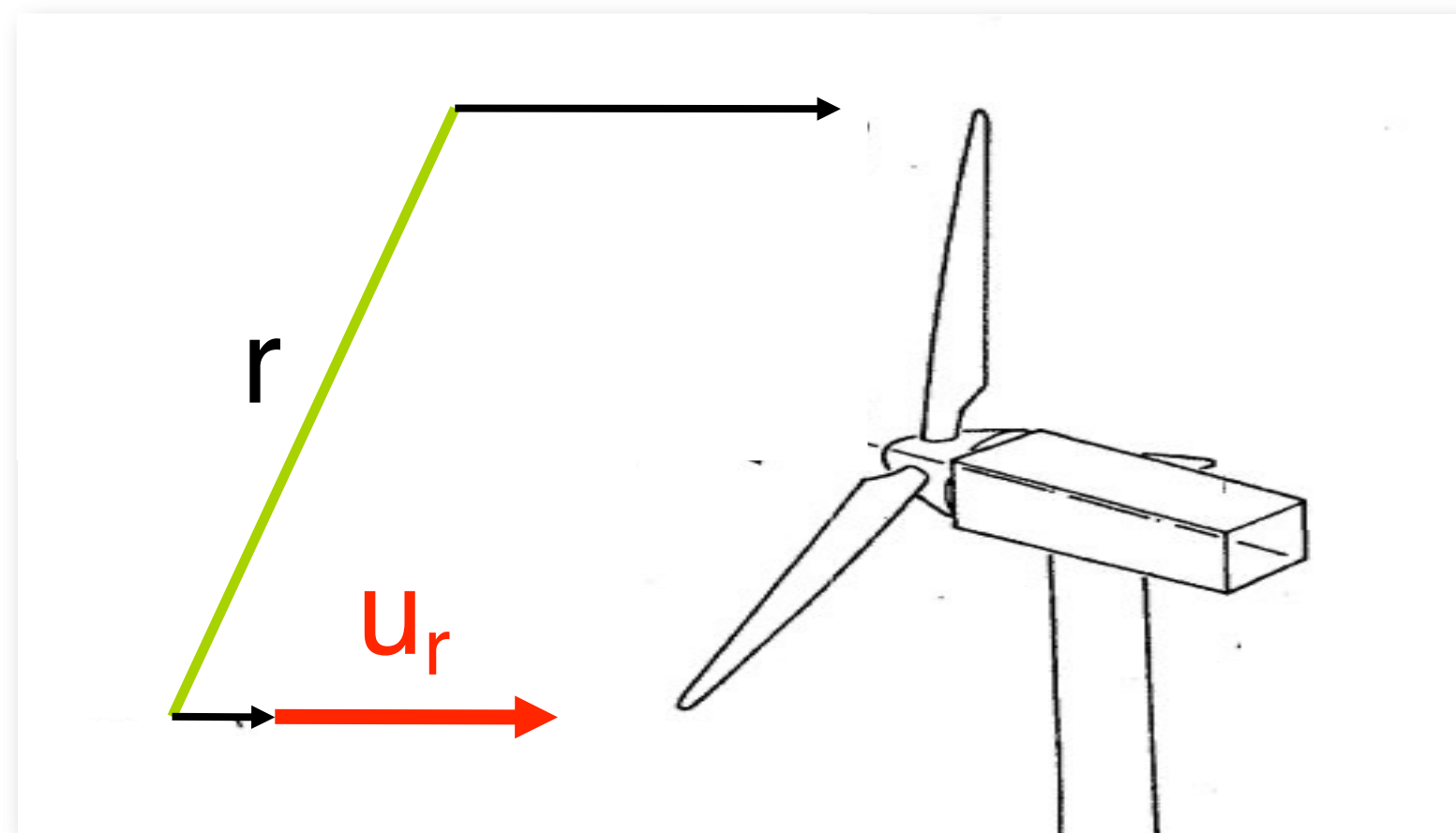
- chances: 50% and more of our electr. energy demand
- reliability problem due to extrem events?



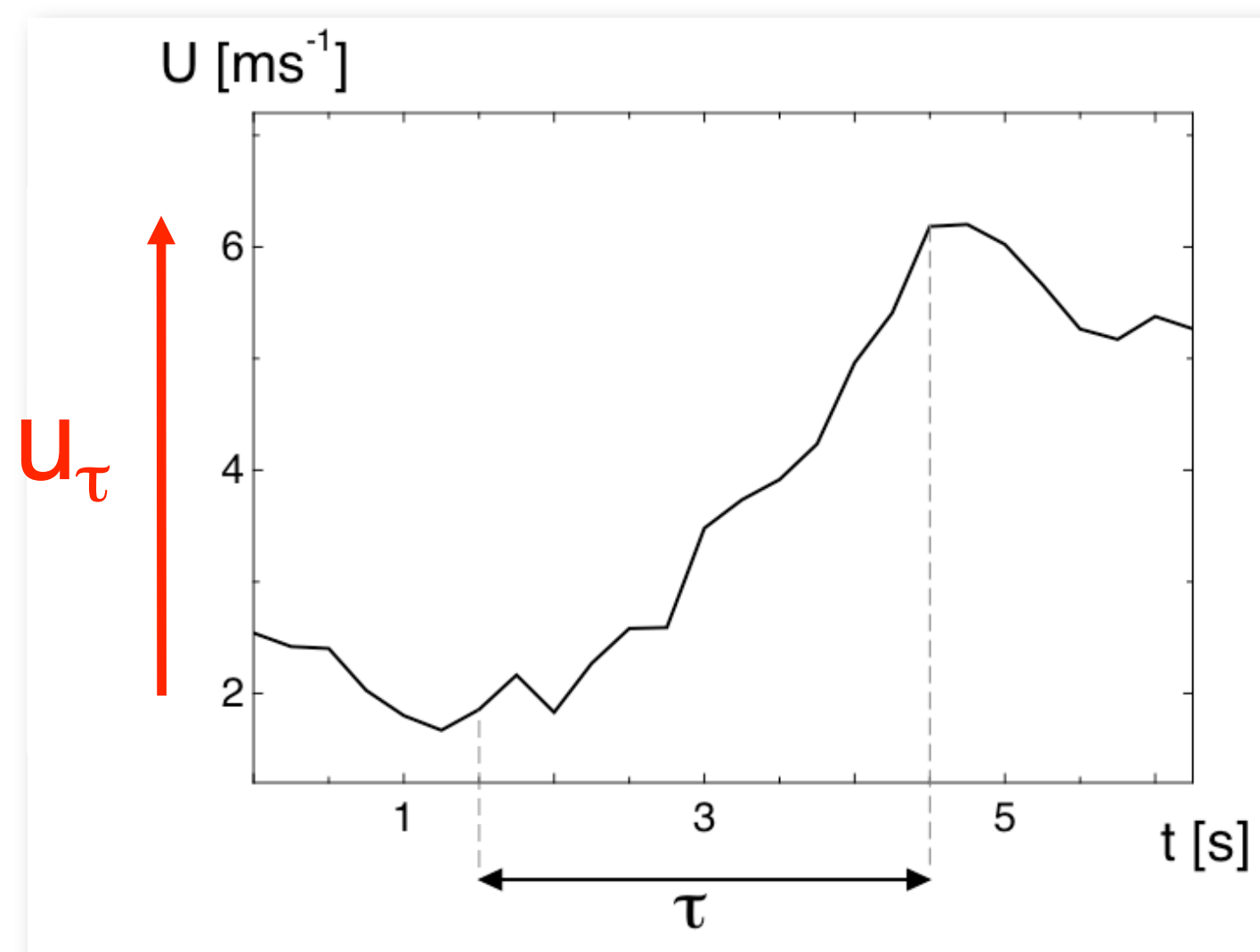
failure statistics EAWE Ispra
good part not pre-visible failures



Wind Turbulence: Scales, Intermittency & Extreme events



$$u_r = U(x + r) - U(x)$$



$$u_\tau = U(t + \tau) - U(t)$$

Wind Turbulence: Scales, Intermittency & Extreme events

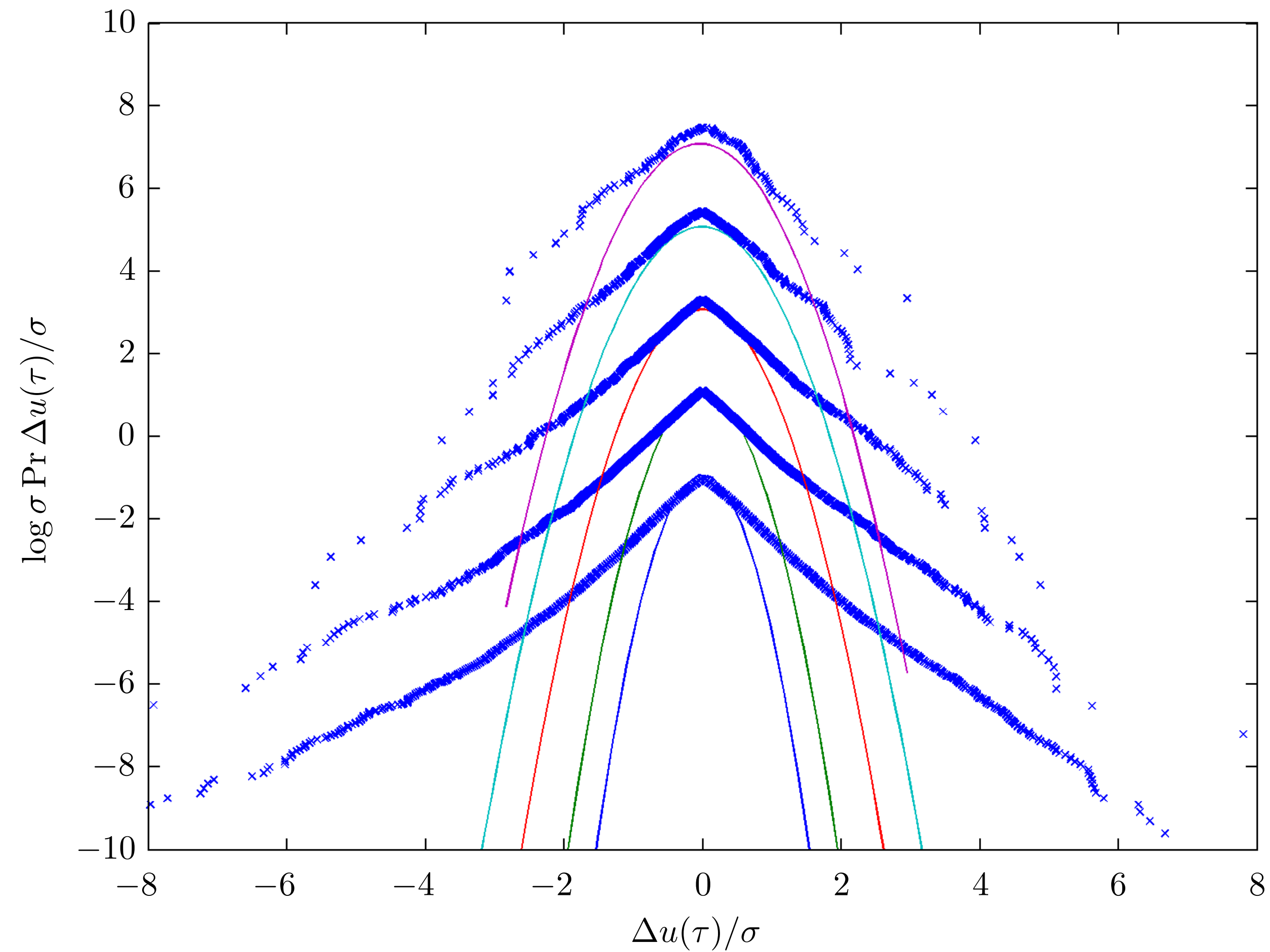


Figure 1.2: Same distribution as in figure 1.1 but with a logarithmic vertical axis and for increasing time-scales: $\tau = 0.1, 0.4, 1.6, 6.4$ seconds.

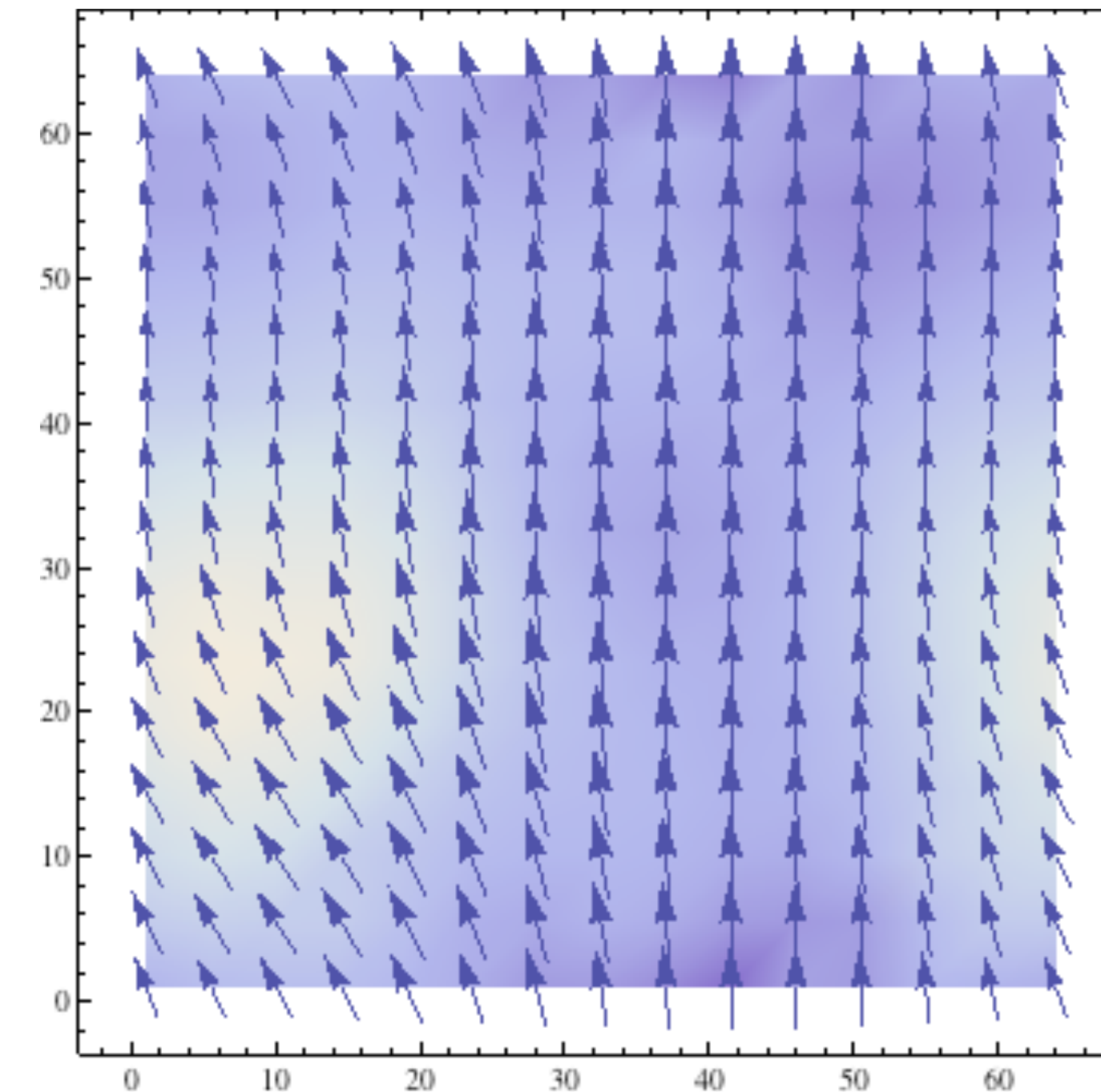
Surface layer complexity!

explOatorium®



Art piece 'Windswept' (Ch. Sowers, 2012): 612 freely rotating wind direction indicators to help a large public to understand the complexity of environment near the Earth surface

WAUDIT Wind resource assessment
audit and standardization



Multifractal FIF simulation (S et al., 2013) of a 2D+1 cut of wind and its vorticity (color). This stochastic model has only a few parameters that are physically meaningful.

Both movies illustrate the challenge of the near surface wind that plays a key role in the heterogeneity of the precipitations... and wind energy!

Fractionnaly Integrated Flux model (FIF, vector version)

FIF assumes that both the renomalized propagator G_R and force f_R are known:

$$G_R^{-1} * u = f_R$$

where:

$$f_R = \varepsilon^a$$

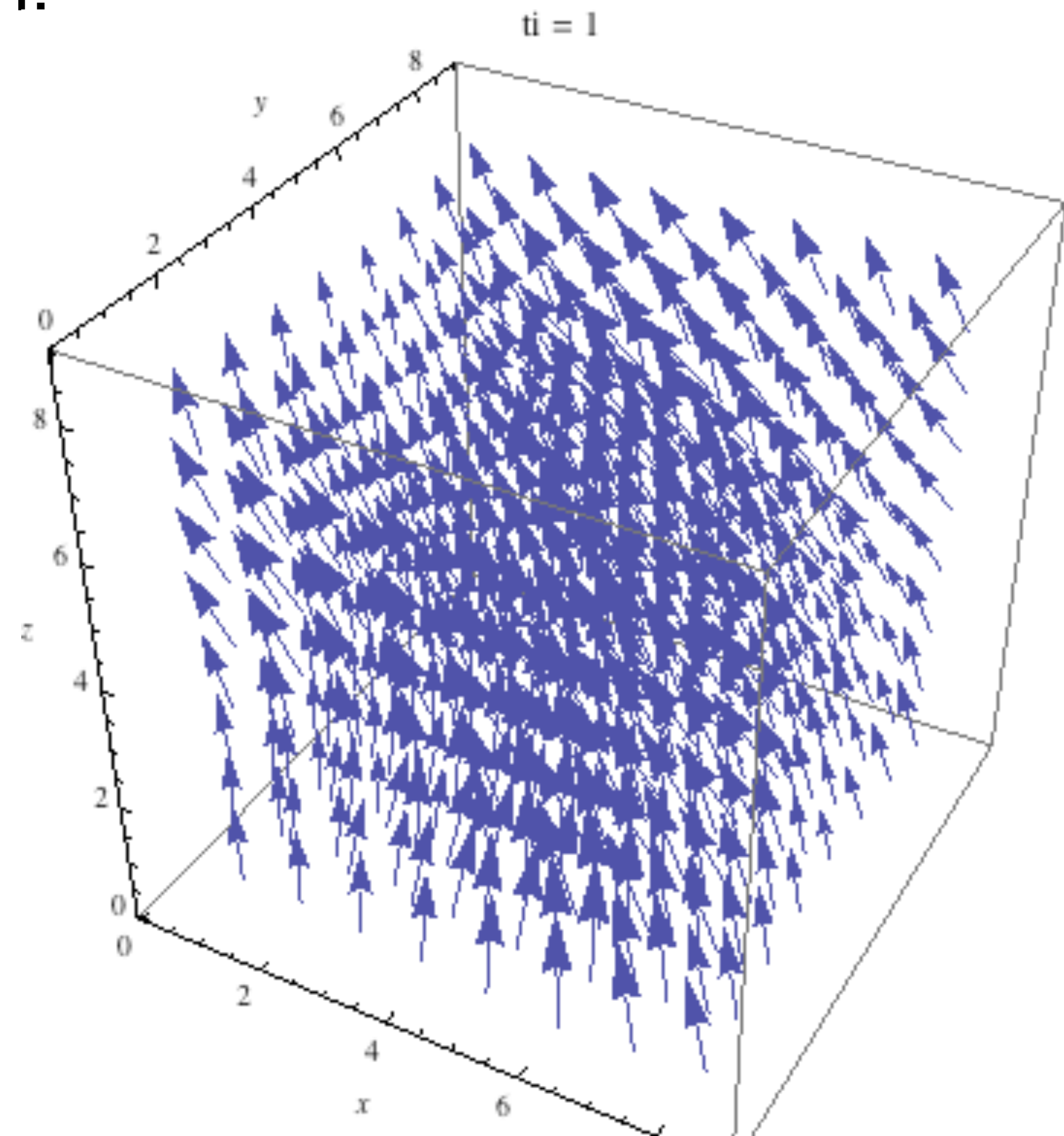
$$G_R^{-1}$$

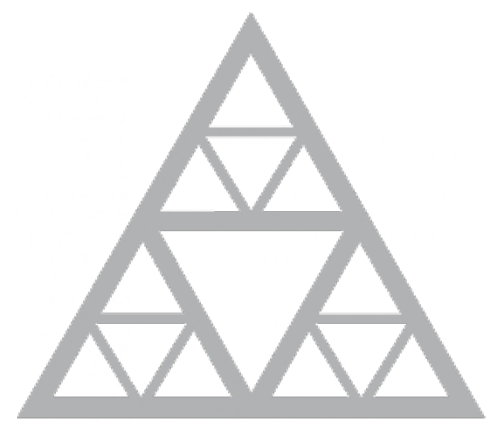
is a fractionnal differential operator

$$\varepsilon$$

results from a continuous, vector, multiplicative cascade (Lie cascade)

3D FIF wind simulation based on a Clifford algebra





École des Ponts

ParisTech

To downscale climate scenarios?



Consensus on the need to downscale simulations:

- to manage water
- below a kilometer to model cities ?

but a fundamental obstacle :

- relationships large/small scales
- *stationnary?* (« stationnarity is dead ! »)
- *Implicit hypothesis* in:

- correlations (linear or not) large/small scales or neural networks: training on « *non perturbed* » data sets;
- weather types: *stationnary basis*, only frequencies change;
- GCM: *stationnary parametrisations* of small scales...

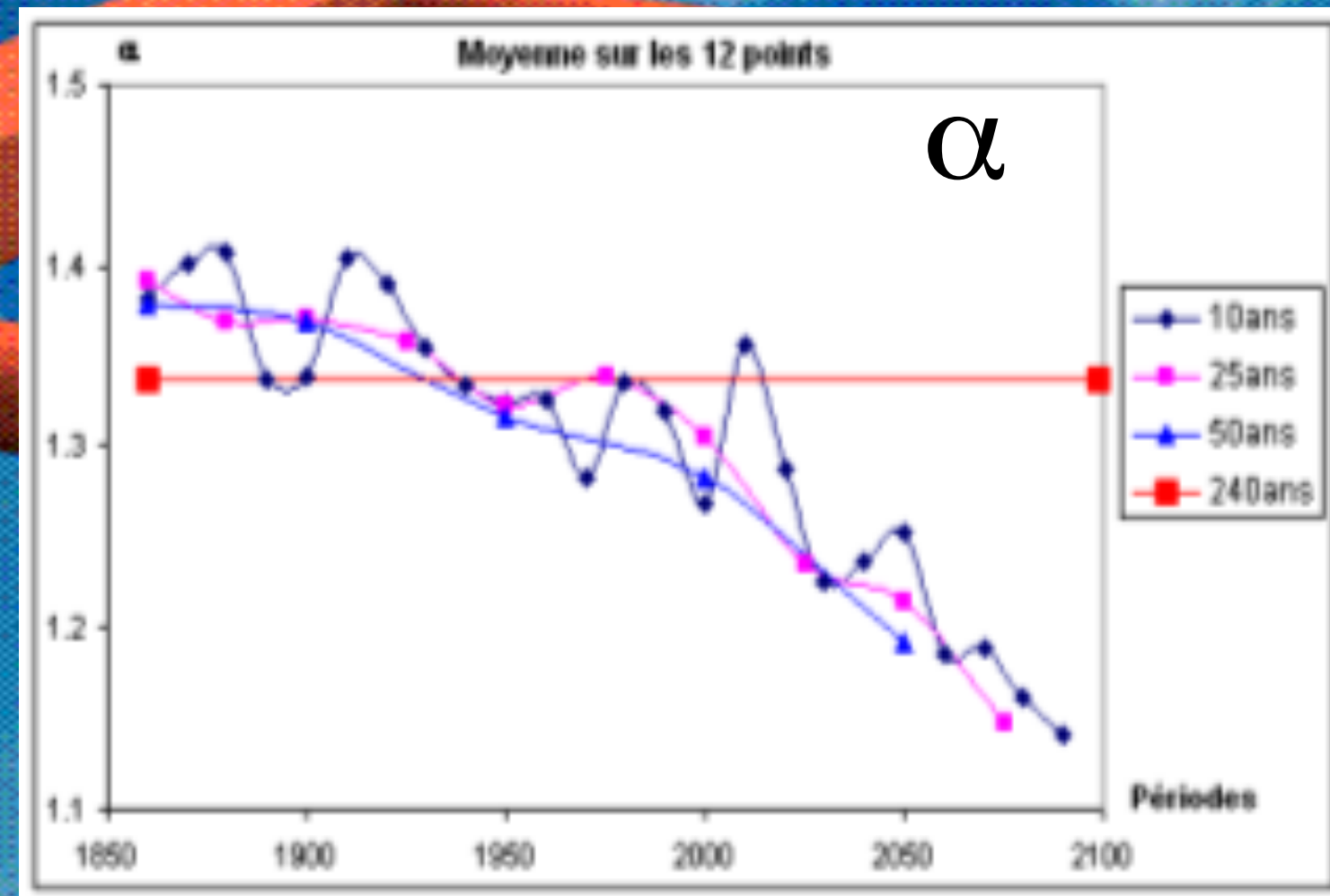
Alternative:

- Analyze and take into account the *evolution of fluxes through scales!*
- Example: **rainrate** for the water cycl.

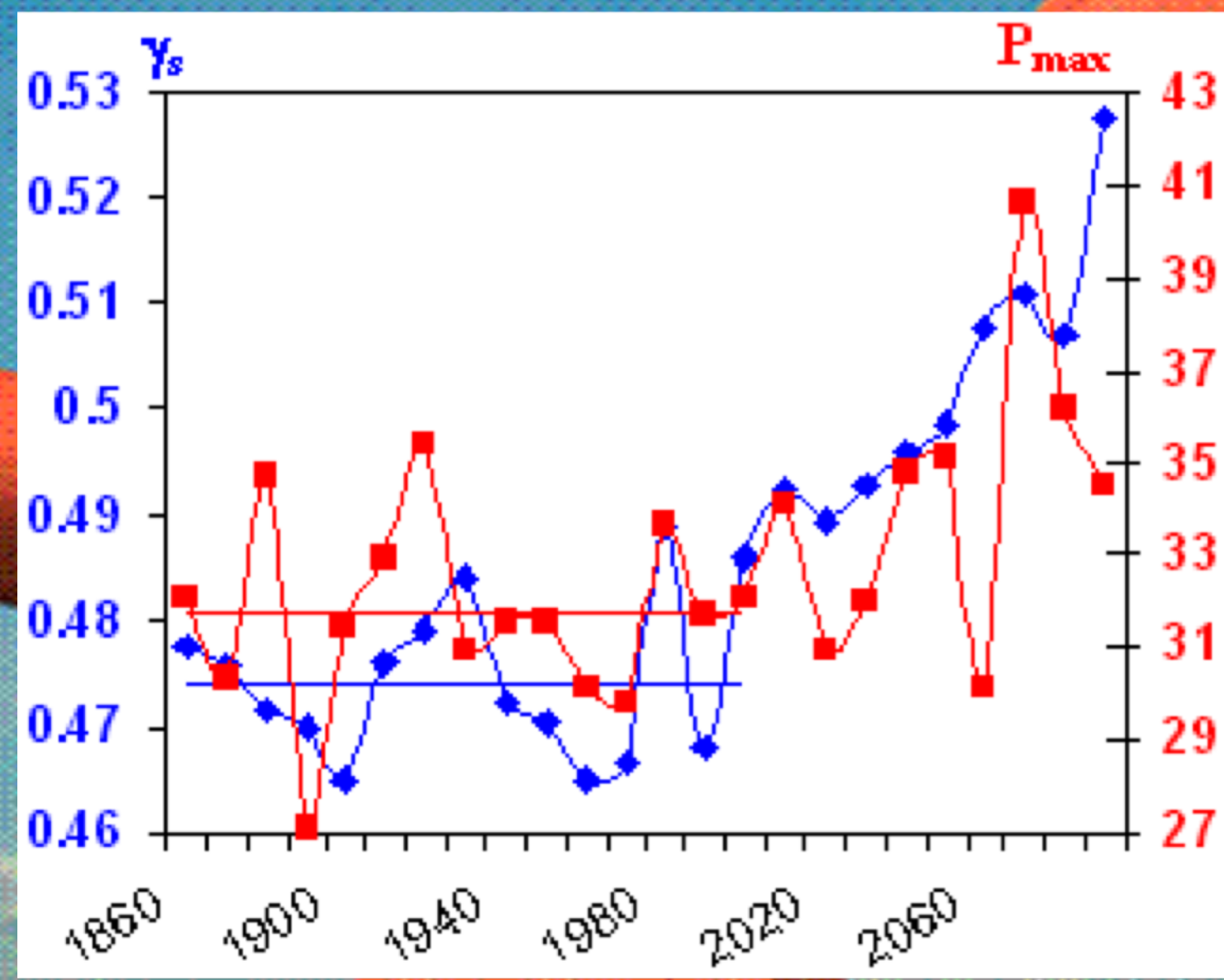
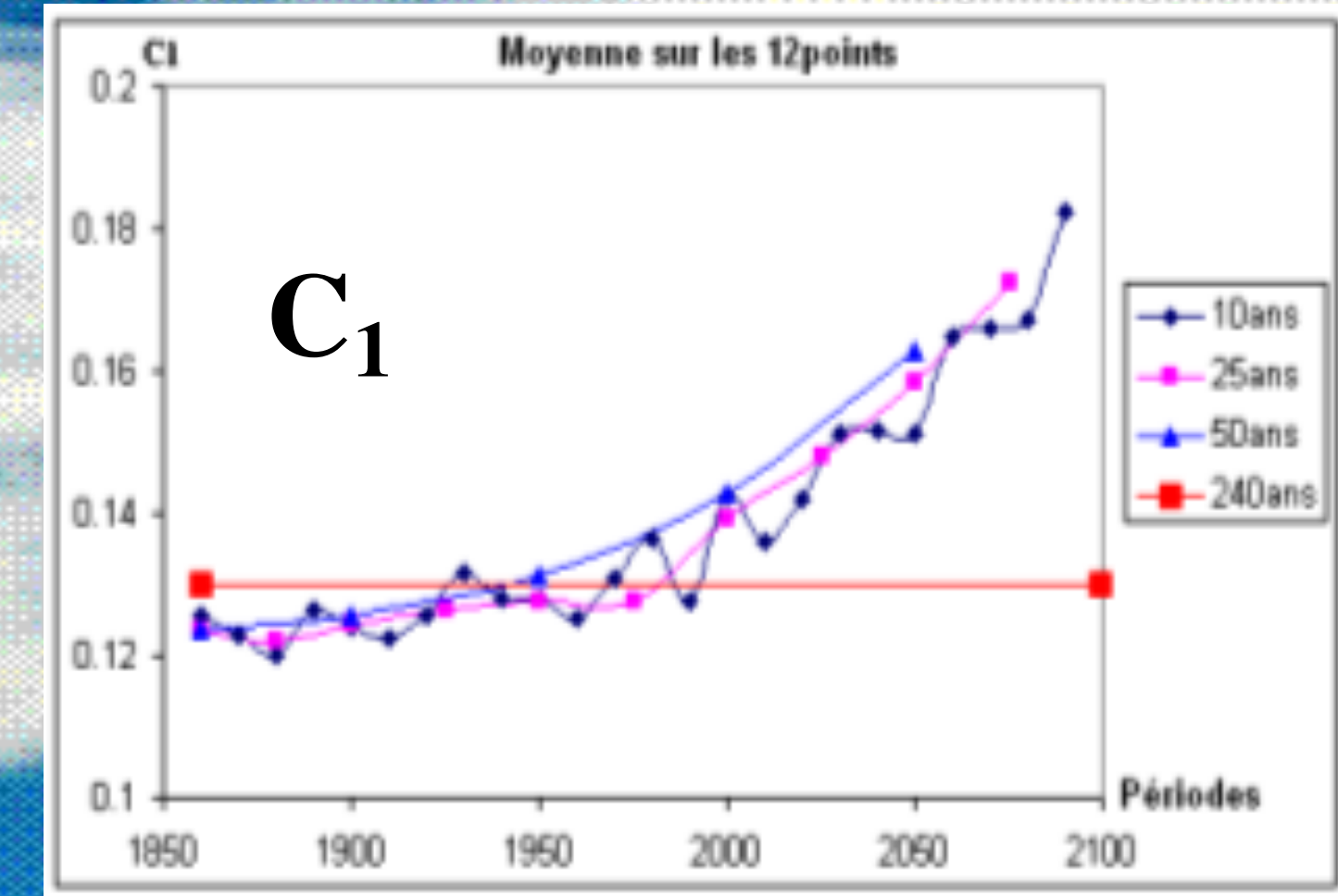


École des Ponts
ParisTech

Analysis of the scenario A2

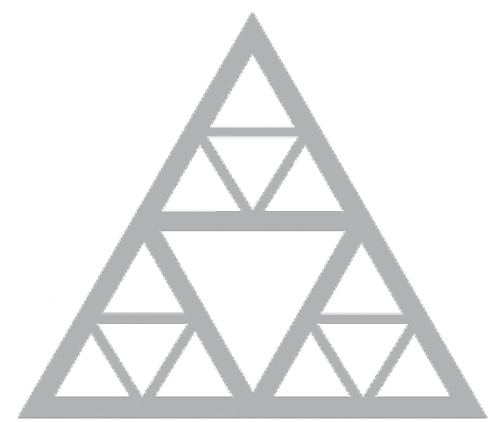


- average intermittency $C_1 \uparrow$
- intermittency variability $\alpha \downarrow$,
=> difficulty to evaluate extremes of precipitations



=> refined analysis :

- time volution of the **Most Probable Singularity** γ_s (Hubert et al, 1993; Douglas & Barros, 2003):
- a **scale invariant statistic**, more stable than the maximal simulated precipitation P_{max} .
- Unable us to conclude: **extremes** \uparrow (Royer et al., 2008),
- **seasonality can be taken into account** (Royer et al., 2010)



Cascades and statistical physics

Transformation of a measure σ with the help of a “density” ε into another measure Π :

$$d\Pi = \varepsilon d\sigma$$

Generalisation with a non trivial limit ε of densities ε_λ of increasing resolution: $\lambda = L/\ell \rightarrow \infty$

$$\varepsilon_\lambda = e^{\Gamma_\lambda}; \mathbb{E}e^{q\Gamma_\lambda} = Z_\lambda(q) = e^{K_\lambda(q)} \approx \lambda^{K(q)} \xleftrightarrow{\text{Mellin}} \mathbb{P}(\varepsilon_\lambda > \lambda^\gamma) \approx \lambda^{-c(\gamma)}$$



Legendre

$$K(q) \longleftrightarrow c(\gamma)$$

main “trick”:
log-divergence of the generator

Γ : generator \approx hamiltonian

q : statistical order \approx inverse of temperature

Z : 1st characteristic or moment generating function \approx partition function

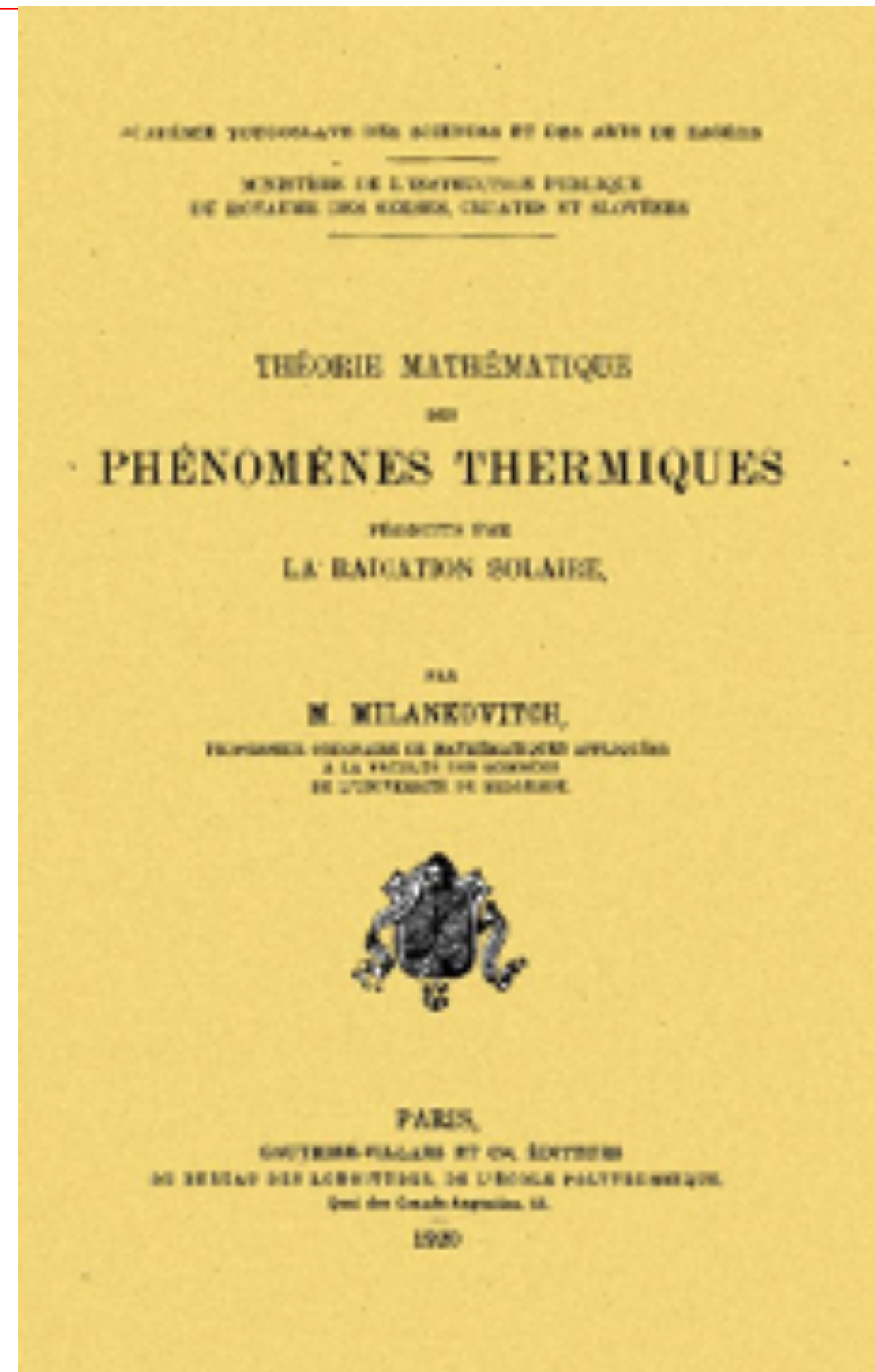
K : 2nd characteristic or cumulant generating function \approx Gibbs free entropy

c : codimension or Kramer function \approx entropy

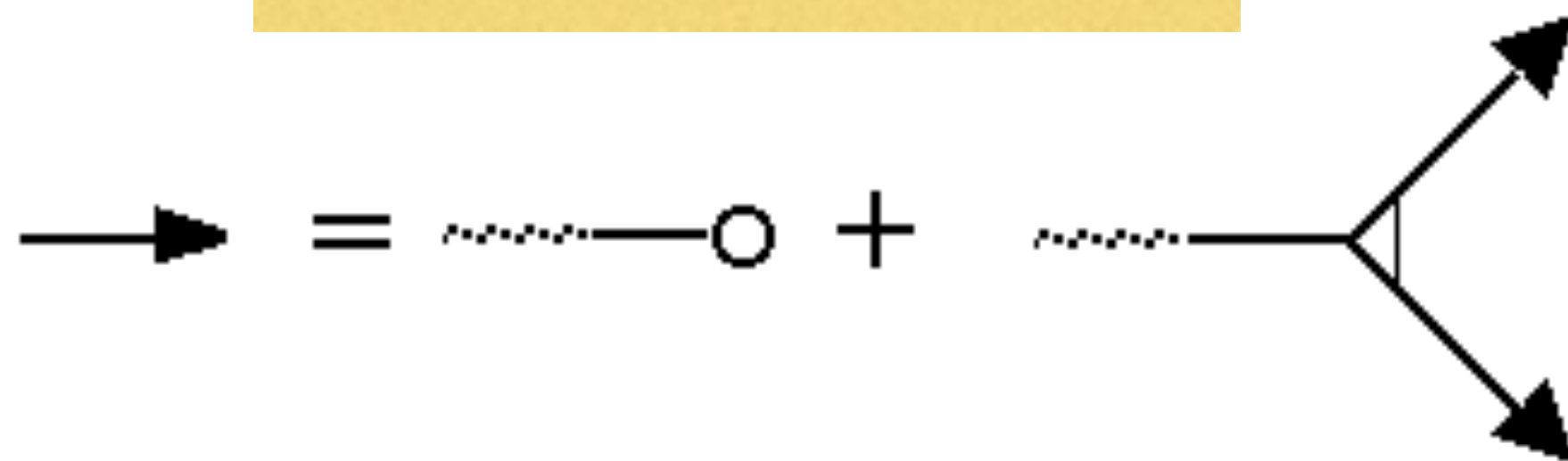
γ : singularity or Hölder exponent \approx energy

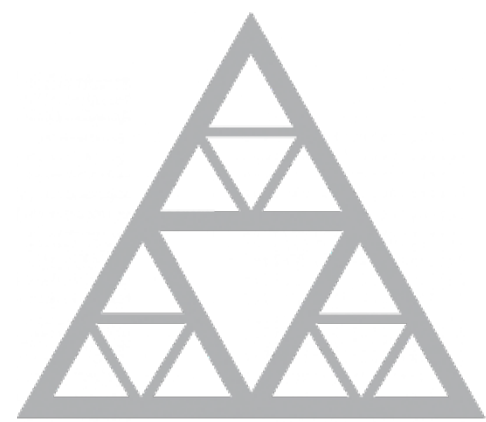


Conclusions



- This variable distribution of solar radiation on the surface of the Earth is the primary cause of **all meteorological phenomena, whose irregularity singularly contrasts with the regularity of their primary cause**. Meteorological phenomena appear as capricious as the radiation - with which the Sun floods the Earth - follows a regular diurnal and annual course (Milankovic, 1920)
- What seems to be more recently established:
 - the **climate response** to forcing peaks is broad, **not narrow**
 - scaling is present: **fractional operators** are therefore expected
 - furthermore there is **intermittency**: **linear fractional operators are not sufficient**
 - not a property of a single component (temperature proxy), but of **multiple interacting components**: scalar valued multifractals are insufficient—> **multifractal operators / cascade of operators**
- Cascades/multifractals based on **Clifford algebra** are at least an excellent training case...
- Available data need to be more investigated

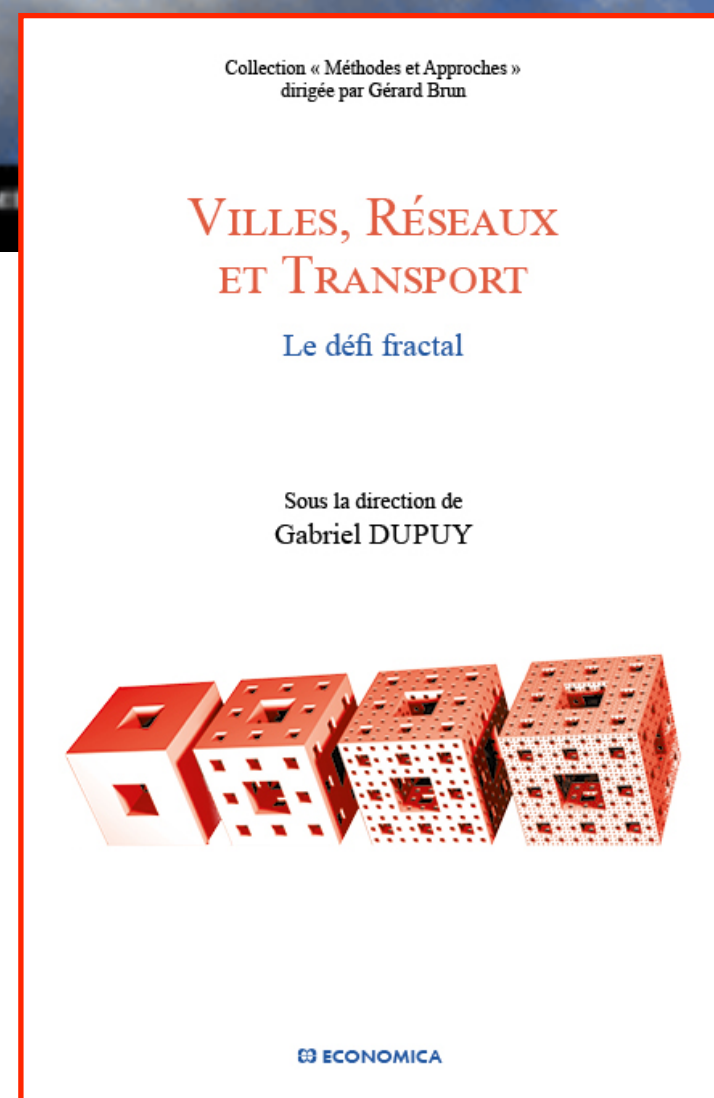
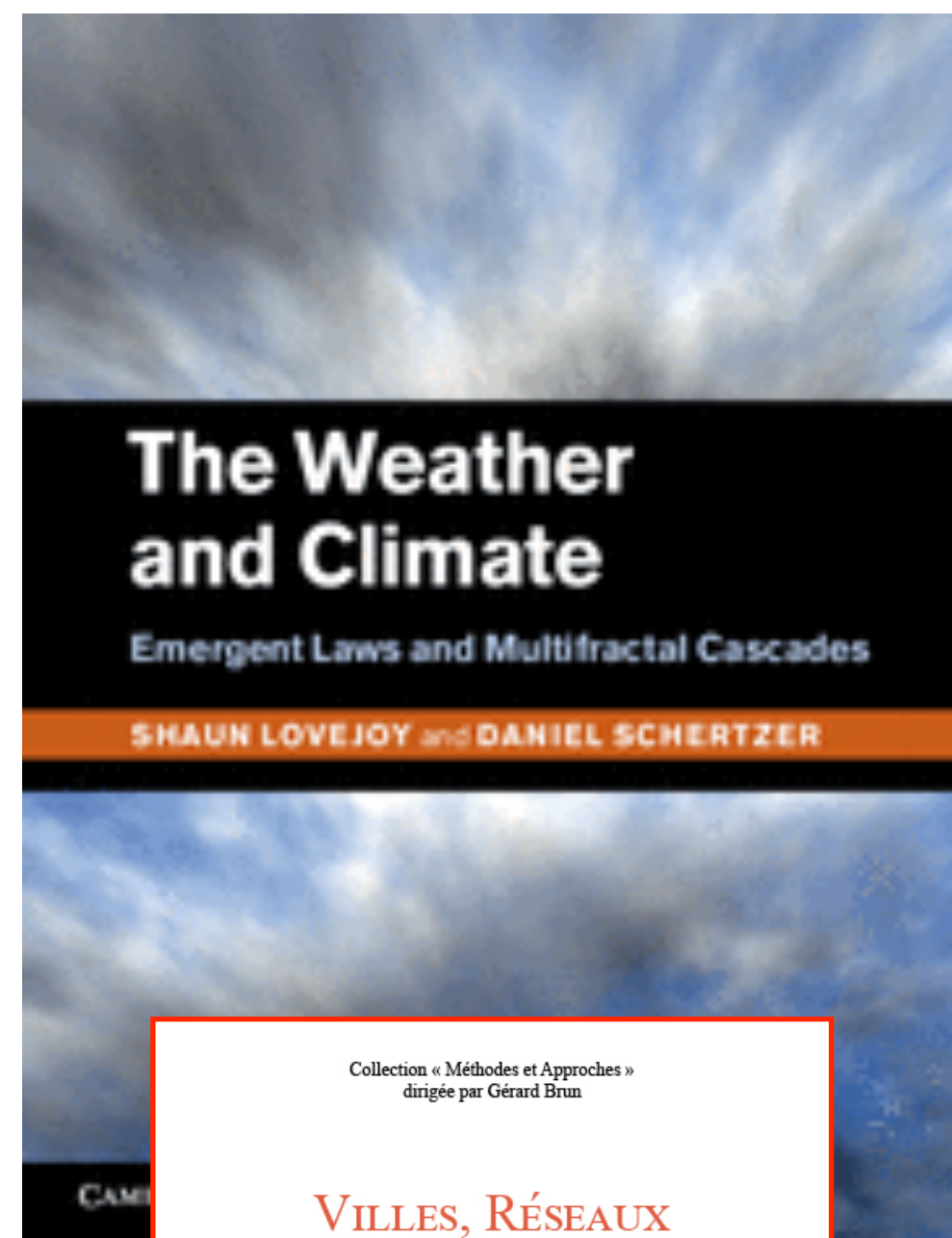




École des Ponts

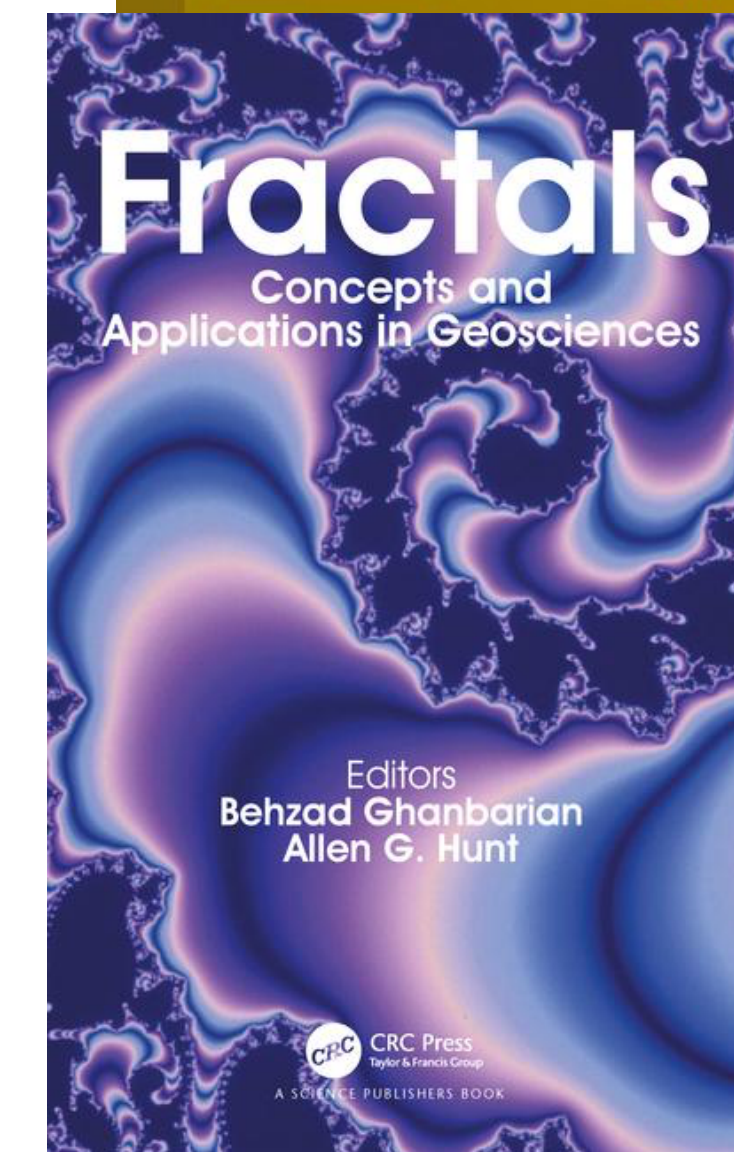
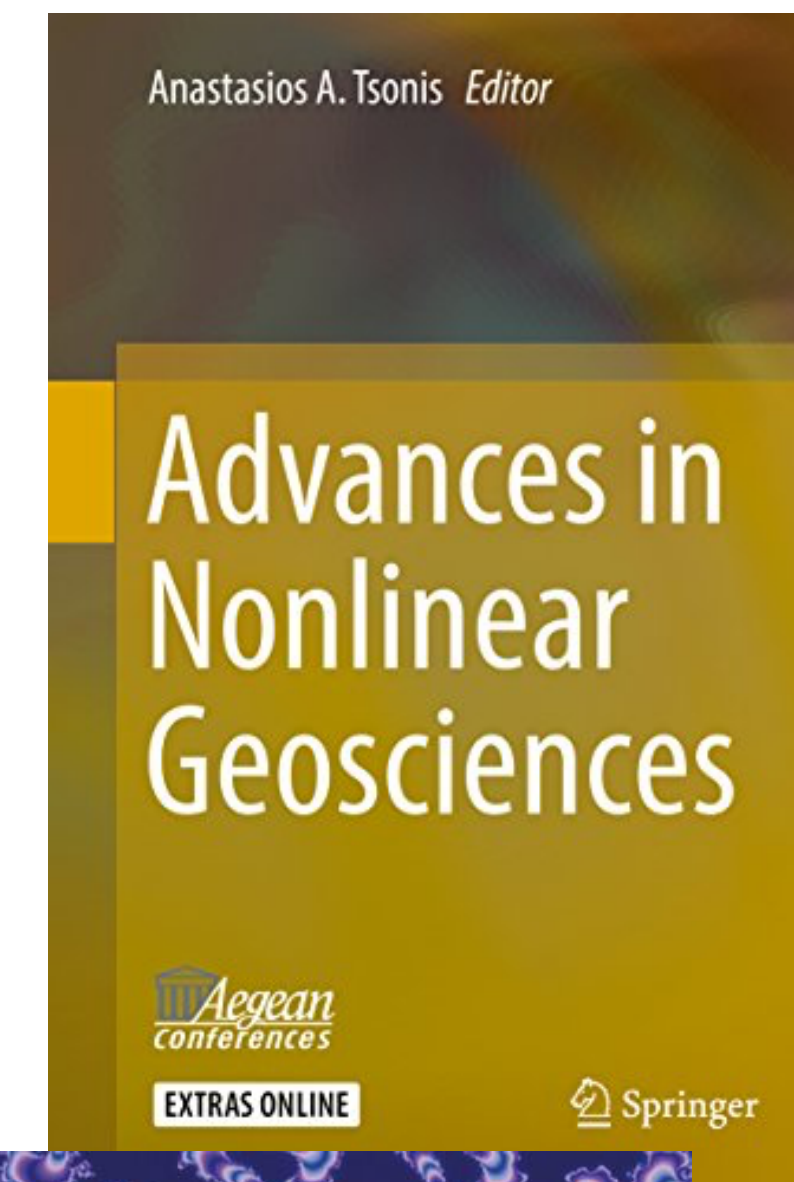
ParisTech

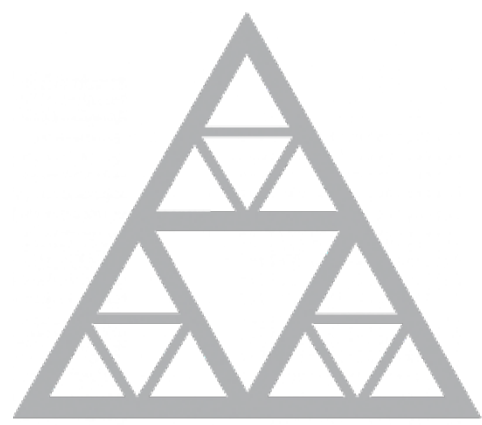
A paradigm shift on complexity and decision making



- a more and more direct dialogue between policy and research
- based on unifying scientific concepts and methodologies
- for both geophysical and urban systems:
 - nonlinearity
 - non-stationarity
 - heterogeneity, intermittency
 - multiplicity of scales, scaling
 - resilience to extreme weather, water and climate change

CNRS GDR “Multifractal Analysis”
I-SITE project “Multifrac”
CNRS Complex System Institute





École des Ponts

ParisTech

Conclusions

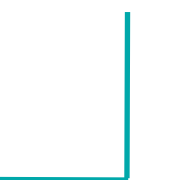


SUSTAINABLE
DEVELOPMENT
GOALS

17 GOALS TO TRANSFORM OUR WORLD

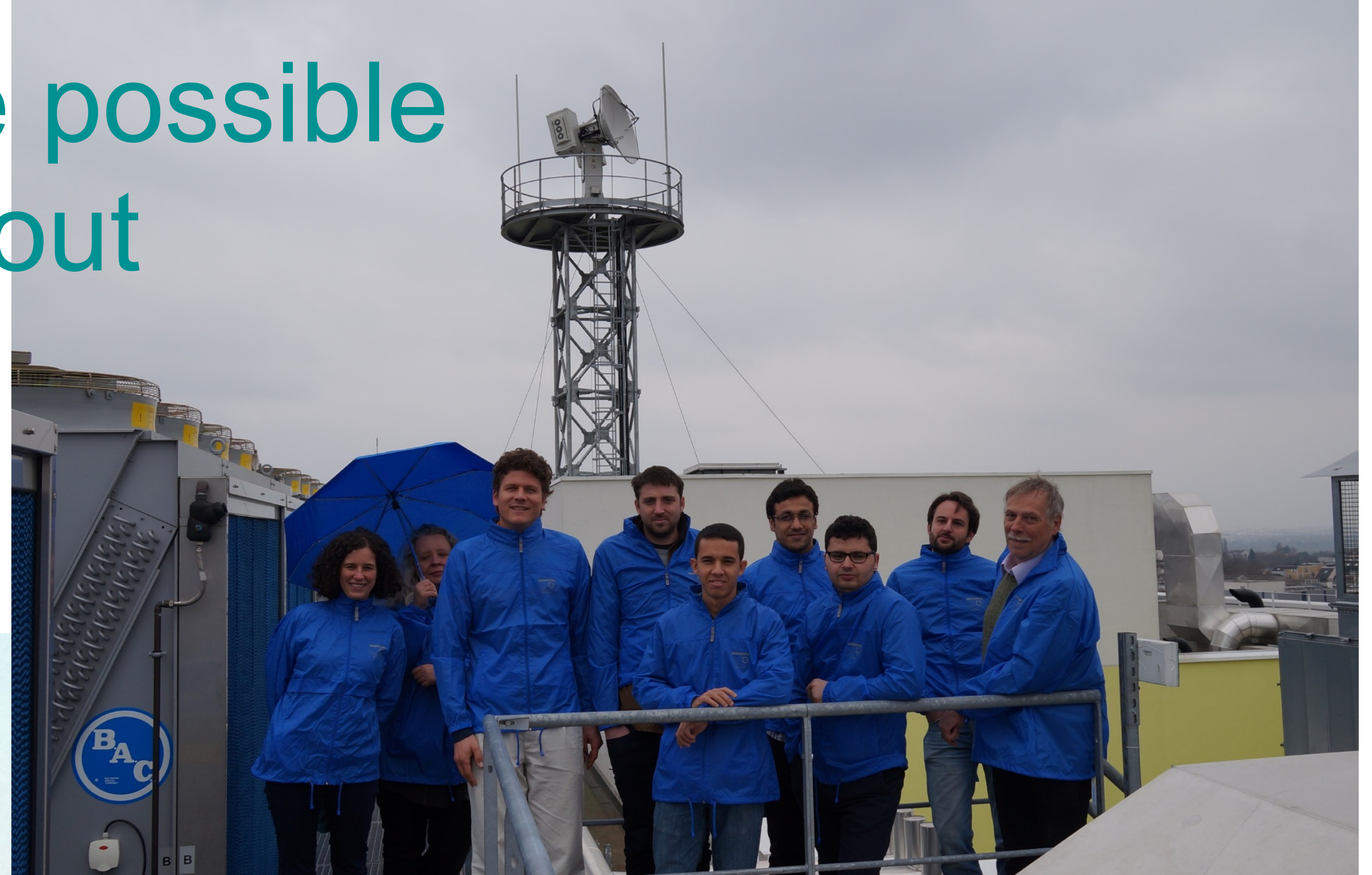


- UN 2030 Agenda and follow-ups: a **vibrant call to engineering intelligence**
- not achievable without
 - **advanced observation technologies** (e.g. small radars and lidars)
 - **innovations** (e.g., Blue Green/Nature Based solutions)
 - **disruptive methodologies** (multiscale analysis, simulation and modelling)
 - **synergic, integrative approaches** overcoming the traditional silo thinking
 - similar to PUB decade, but at much smaller scales
 - requires common concepts (e.g. resilience) and tools (integrative platforms, e.g. Hydro)
- Paris olympics (2024) and GPE as **show cases of Urban Geo**



Would not be possible without

Hydrology Meteorology and
Complexity team (HM&Co),
Ecole des Ponts ParisTech
Avril 2015



Group of Nonlinear Analysis (GANG)
EGS 1992, Edinburgh UK

... and EGU/NP and AGU/NG!