



HAL
open science

Prediction of fire-exposed CLT floors deflection using an advanced modeling and calibration of the Reduced Cross Section Method

Lorenzo Franzoni, Dhionis Dhima, Arthur Lebée, Florent Lyon, Gilles Forêt

► To cite this version:

Lorenzo Franzoni, Dhionis Dhima, Arthur Lebée, Florent Lyon, Gilles Forêt. Prediction of fire-exposed CLT floors deflection using an advanced modeling and calibration of the Reduced Cross Section Method. International Network on Timber Engineering Research (INTER) - Meeting 49, Graz (AUT), Aug 2016, Graz, Austria. hal-04390764

HAL Id: hal-04390764

<https://enpc.hal.science/hal-04390764>

Submitted on 16 Jan 2024

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Prediction of fire-exposed CLT floors deflection using an advanced modeling and calibration of the Reduced Cross Section Method

L. Franzoni - Laboratoire Navier ENPC/IFSTTAR/CNRS /UPE

D. Dhima - Centre Scientifique et Technique du Bâtiment (CSTB)

A. Lebéé - Laboratoire Navier ENPC/IFSTTAR/CNRS /UPE

F. Lyon - Centre Scientifique et Technique du Bâtiment (CSTB)

G. Foret - Laboratoire Navier ENPC/IFSTTAR/CNRS /UPE

Keywords: Fire design, Reduced Cross-Section Method, CLT panels, Deflection, Fire testing

1 Introduction

During last decades, the timber structures construction field has sharply increased, covering also modern residential and commercial buildings within the city agglomerates. This development takes benefits from the parallel development of the new massive timber construction method based on Cross Laminated Timber (CLT) panels (or crosslam). These prefabricated thick panels are composed by an uneven number of lumber layers stacked crosswise and glued on their upper and lower faces. The orthogonal lay-up ensures uniform mechanical and hygroscopic properties along both directions and allows the use of CLT as full-size floor, wall or roof element. The enhanced properties of crosslam compared to solid wood make this product competitive with respect to traditional mineral-based construction materials (concrete, masonry) which have a higher environmental impact than wood.

The development of timber structures has to be followed by a progressive increasing in knowledge about their fire safety. This assumes significant importance when dealing with relatively recent products such as CLT panels. Indeed, the current version of the Eurocode 5 – 1-2 (2004) includes a fire design method for timber structures derived in the '80s and originally developed for simply supported Glued Laminated Timber (GLT). Such design method, called the Reduced Cross Section Method (RCSM), was originally derived by Shaffer (1984) for GLT by means of an extensive experimen-

tal campaign. The principle is to use normal properties of wood at ambient conditions combined with a reduced cross section of the timber member due to the fire exposure. The reduction of the cross section is computed with two steps: (i) a first reduction due to a uniform charring rate and (ii) an additional reduction of a layer having no mechanical properties (also called zero strength layer - ZSTL) in order to take into account the reduced properties of wood in the region close to the char front. In the current version of the Eurocode, such additional layer is set to 7 mm according to Shaffer (1984), independently from any parameter such as the panel's geometry or time exposure. The main advantage of the RCSM compared to more advanced calculation methods lies in using normal properties of wood instead of reduced properties. This hypothesis can be successfully applied to timber structures for two main reasons: (i) wood has a very low thermal conductivity and (ii) charred wood acts as an insulating material that protects the intact wood after the char front. Therefore the temperature and reduced properties gradient ends not so distant from the char front (35 mm to 40 mm according to Shaffer, 1984).

The simple approach of the RCSM makes this method useful for practical applications by engineers. However, recent studies (Schmid 2012, 2014a, 2014b) showed that the predicted behavior of timber members exposed to fire using the RCSM is not always on the safe side, and the the "exact" value of the ZSTL is difficult to predict since it depends on a multitude of parameters like for instance the boundary conditions, the panel's geometry and the kind of acting stresses. Moreover, dealing with innovative timber products like CLT having an orthogonal lay-up, the ZSTL can be superimposed to transverse layers that do not contribute to load-carrying capacity. Additional complex phenomena can occur for the presence of glued interfaces between layers. Indeed, the presence of glue at CLT layers interfaces can influence the fire behavior because of the difference of thermal properties between glue and timber. When the char front is not yet at the glued interface, the temperature gradient can decrease glue mechanical properties and therefore can lead to premature falling-off of layers and an increasing in the charring rate. This phenomenon has been observed in CLT floors exposed to fire by Frangi et al (2009), and the proposed approach of Eurocode for initially protected surfaces seemed to well reproduce this delamination phenomenon. However, other studies found very few falling-off of layers (Craft et al 2011, Osborne et al 2012) or a falling-off phenomenon that has low influence on fire design (Klippel et al 2014). This discrepancy between test results derives from the multitude of parameters influencing the falling-off effect, like the glue thickness between layers, the type of glue, the presence of small gaps between boards of each layer, the panel's lay-up and so on.

Fire tests on loaded timber elements are useful to understand the actual thermal-mechanical behavior of the specimen, but they are expensive, time-consuming and sometimes it is not straightforward to obtain reliable information on the actual mechanical response of the specimen. This is also due to the fact that, in most of the

cases, the test is stopped before the failure of the specimen for the safety of people and equipments.

The basic principle of this paper is to use the deflection of fire-exposed CLT floors as mechanical information on the elastic properties distribution within the panel during fire exposure, in order to estimate the zero stiffness layer - ZSSL during the fire test. The deflection is predicted with both advanced modeling based on reduced properties and the simplified RCSM. The comparison with the experimental deflection of fire exposed crosslam floors shows that the RCSM of Eurocode can be improved when considering a ZSSL as a function of time exposure.

First, in Section 2, the available experimental data of bending tests on fire exposed crosslam floors is presented. The following Section 3 presents both advanced and simplified modeling in order to reproduce the experimental deflection. The comparison between the predicted and experimental behavior is showed in Section 4, while Section 5 includes a discussion on obtained results.

2 Fire tests on CLT floors

2.1 Description of fire tests on CLT floors

Three CLT floors from three different producers were submitted to bending and exposed to ISO fire on their lower face in tension. The out-of-plane load remained constant during fire exposure and the panels were simply supported on two sides. Thermocouples were placed in several sections and over the thickness in order to measure the temperature profiles. The deflection was measured with LVDTs at panel mid-span and the displacement rate was constantly monitored. The end of tests was determined when reaching the safety criterion of maximal displacement rate, when rupture occurred or when reaching an established time exposure without satisfying any safety criterion. Table 2.1 shows the main properties of the fire tests on CLT floors.

Table 2.1 Properties of the tested specimens of CLT floors

Test number:	1	2	3
Span L [m]	4.2	4.5	4.6
Thickness H [mm]	195	160	182
Load [kN]	40	40	60
Kind of load	4-points	4-points	Uniform
N° of sections	9	1	4
N° of thermocouples per section	9	4	10
Time exposure [min]	150	86	90
Safety criterion reached	Displacement rate	Failure	None

All panels were made of Norway spruce lamellas of strength class C24 and glued with one-component polyurethane glue. Suddenly after the end of fire test, the specimen was removed to fire exposure and the fire on its exposed side extinguished with water. Figure 2.1 presents a typical residual cross section after removing the exceeding charcoal.



Figure 2.1 Residual cross section of the Test-1 floor

Figure 2.1 shows that there were no alteration of panels' supports geometry due to fire exposure, allowing therefore the hypothesis of respected simple supported boundary condition for the subsequent modeling step.

2.2 Temperature profiles

The temperature evolution within the panels during fire exposure has been measured by means of thermocouples along the cross section. Figure 2.2 presents the position of thermocouples over the thickness for the three tested floors.

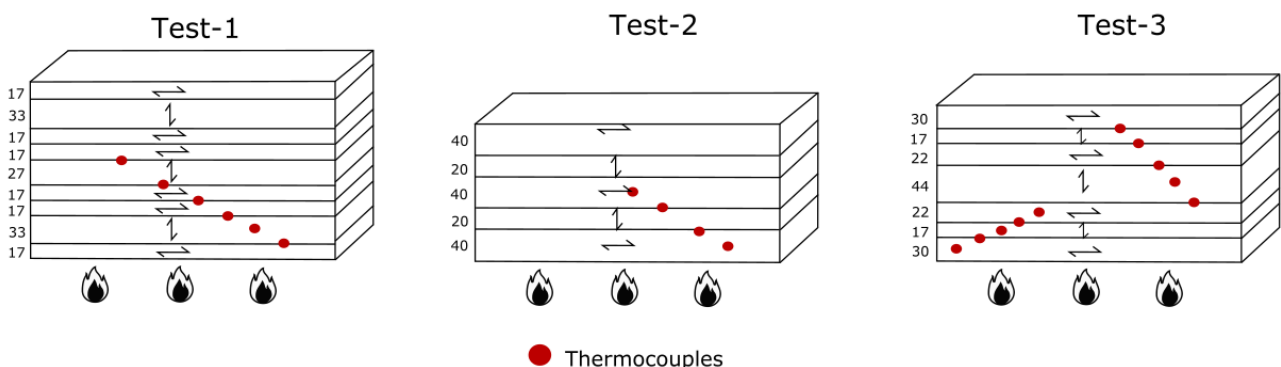


Figure 2.2 Position of thermocouples across the thickness of tested crosslam floors. Dimensions in mm

Due to its high number of thermocouples (9 sections along the span and 9 thermocouples per section), Test-1 has the most certain documentation on temperature

profiles over the CLT thickness. In Figure 2.3 are plotted the registrations of Test-1 thermocouples placed at 84 mm from the bottom exposed side during the test.

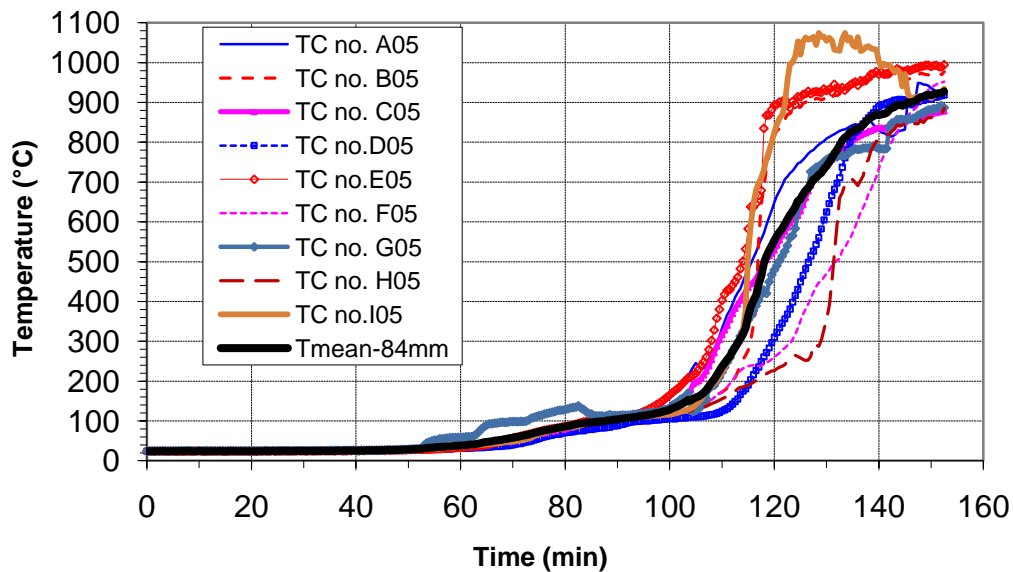


Figure 2.3 Evolution of the measured temperature in Test-1 at 84mm from the bottom side for each of the nine sections and considered mean temperature (solid black line)

The solid black line is the considered mean temperature at 84 mm thickness between the nine sections along the panel’s span. The same mean temperatures between sections are considered in Test-3, while the specimen of Test-2 had only one section of thermocouples over the thickness. The significant difference between measured temperatures from different sections derives from the local falling-off of timber pieces that yields a local sharp increase of temperatures.

2.3 Charring rates

Starting from the temperature profiles registered by the thermocouples during the test and assuming 300 °C to be the wood charring temperature, one can simply derive the charring rates β over the CLT thickness. Table 2.2 presents the computed charring rates for several layers of Test-1 during the fire exposure, considering mean temperatures from the 9 sections (Fig. 2.3).

Layer	Thickness [mm]	$t^+(\theta=300)$ [min]	$t^-(\theta=300)$ [min]	β_{mean} [mm/min]
1	17	25,5	0,00	0,67
2	16,5	37,5	25,50	1,38
2	16,5	66	37,50	0,58
3	17	88	66,00	0,77
4	17	113	88,00	0,68

Table 2.2 Charring rates of Test-1 during the fire test for the first four layers

The great increase in the charring rate when passing from the first to the second layer clearly shows that a falling-off of layers occurred. Moreover, the charring rate within layer 2, directly exposed to fire after the falling-off of layer 1, is the double of the previous charring rate, exactly as established by the design approach of initially protected surfaces of Eurocode 5. Concerning the interfaces layers 2/3 and layers 3/4, the charring rate of solid wood (0.65 mm/min in Eurocode 5) is in average respected and therefore no significant falling-off phenomena should be occurred.

Dealing with Test-2 and Test-3, charring rate variations similar to the rates presented in Table 2.2 have been found, showing that also during the two other considered tests, the falling-off phenomenon occurred.

2.4 Evolution of deflection during fire tests

During the fire test on loaded crosslam floors, the mid-span, deflection was constantly measured by means of Linear Variable Displacement Transducers (LVDT) placed on the upper side in compression. Figure 2.4 plots the evolution of mid-span deflection during fire exposure of the three tested CLT.

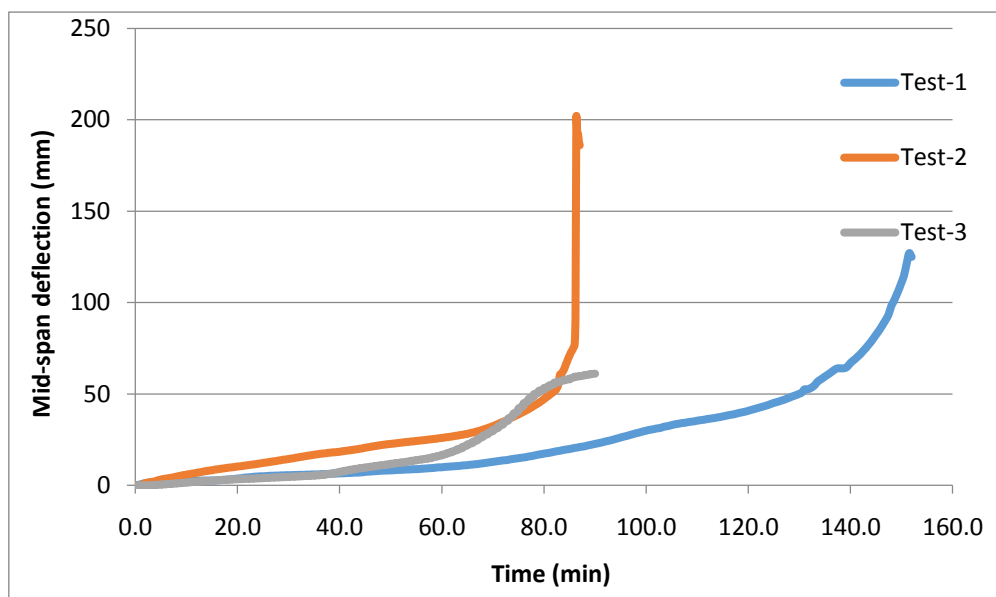


Figure 2.3 Evolution of the mid-span deflection during the three fire tests on CLT floors

Test-1 reached the safety criterion of maximum displacement rate at 150 minutes, highlighted by the sharp increase of its mid-span displacement curve versus time. This means also that the specimen was close to the failure point at that time of exposure. The specimen of Test-2 failed brittly on the tension side at 86 min of fire test, as the respective displacement curve shows. Finally, Test-3 was stopped after 90 minutes without reaching any safety criterion.

3 Modeling

In this section the modeling procedure to reproduce the deflection of tested CLT floors is presented, as well as the compared design approaches based on the RCSM.

3.1 Advanced modeling based on reduced properties

The advanced modeling is based on temperature prediction with Safir software and mid-span deflection prediction using a plate theory combined with reduced properties by means of the Eurocode laws.

3.1.1 Prediction of temperature profiles

The temperature evolution over the panels' thickness during fire exposure was predicted with Safir software. Crosslam panels were modeled as solid wood, with perfect connections between layers. According to EN 338, C24 solid wood was modeled with a mean density of 420 Kg/m^3 , while the initial relative humidity of the panel was set to 12%. The emissivity (ϵ) of the modeled wood was set to 0.8, while the coefficients of convection of heated (h_h) and unheated (h_c) surfaces were assumed to be respectively $25 \text{ W/m}^2\text{K}$ and $4 \text{ W/m}^2\text{K}$. Parameter studies not reported in this paper showed that the density and relative humidity are the parameters that most influence the predicted temperatures, while the variation of the remaining coefficients has a negligible influence.

One dimensional constant mesh of 1 mm was applied as a discretization over the panel thickness for the temperature prediction. Figure 3.1 plots the comparison between predicted and experimental temperature profiles during Test-1 across the specimen thickness.

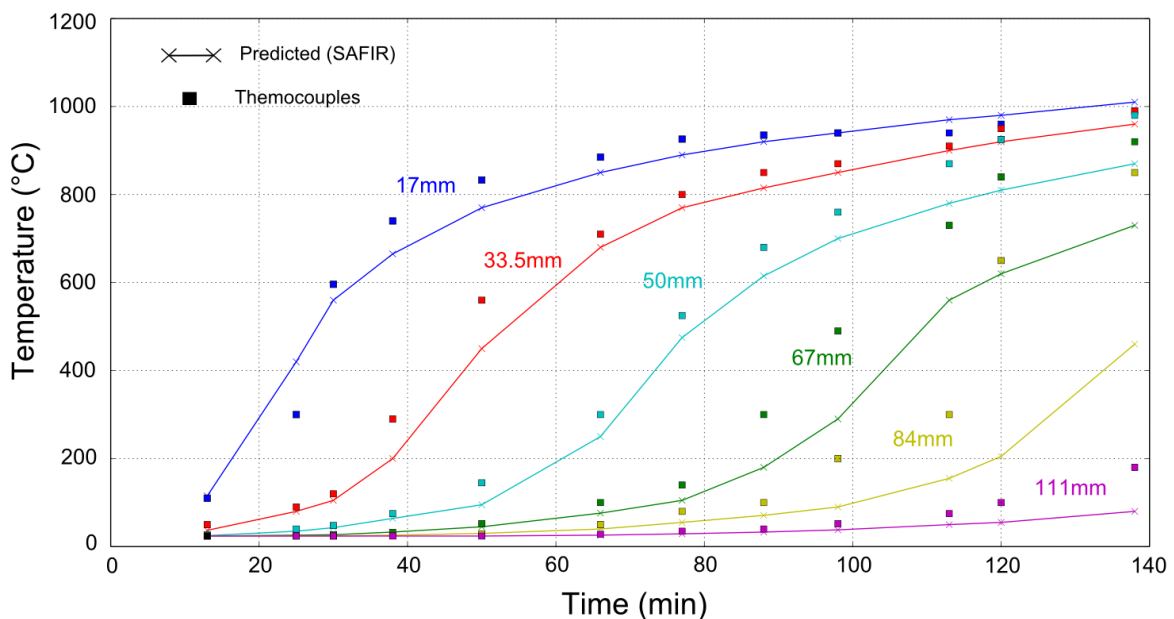


Figure 3.1 Evolution of the predicted and measured temperatures within the floor of Test-1

3.1.2 Thermo-mechanical coupling

Dealing with the mechanical properties of panels, no characterization of the raw material has been done. Hence, elastic moduli of wood according to strength class C24 of EN 338 and a *rolling* shear modulus of 50 Mpa were considered. Once the temperature profile is established for each considered time exposure, the CLT floor is considered as a “multilayer” panel having all thicknesses of 1mm with different values of elastic moduli. Such elastic moduli are defined as a function of temperature using the reduction coefficient k_{θ} given by the Eurocode 5 laws (Figure 3.2). Different reductions of Young’s modulus for the upper or lower part of the crosslam floor have been taken into account. Since no reduction diagrams are present in Eurocode, the same law of compressive Young’s modulus has been adopted for shear moduli.

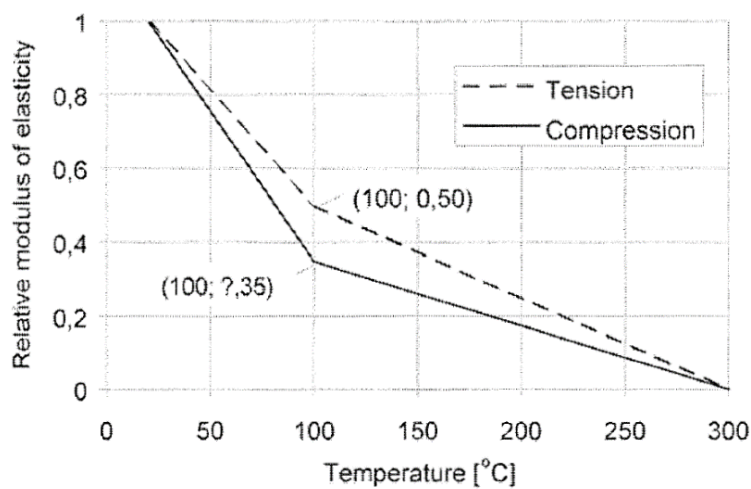


Figure 3.2 Eurocode 5 laws for the determination of reduction coefficient k_{θ} for softwood stiffness

Finally, with the reduced properties for each 1mm layer, the mid-span deflection is computed with a plate theory for thick layered plates (Lebée and Sab, 2011). This theory is an extension to laminated plates of the well-known Reissner-Mindlin plate theory. Therefore transverse shear effects, which sometimes play a crucial role in CLT panels in bending, can be well predicted with this approach.

3.2 RCSM modeling

The simplified design approach of the RCSM implemented in the Eurocode considers reduction of panel’s geometry instead of properties during fire exposure. The cross section of the timber member is reduced by the charring rate during fire exposure and by an additional zero strength layer – ZSTL in order to take into account the reduced properties of wood close to the char front. In the actual version of the Eurocode 5, the ZSTL is set to 7 mm and recent studies pointed out that such approach is not always conservative (Schmid, 2014). However, several attempts to improve the RCSM without changing its simple principle have been done. In the next paragraphs, three methods to determine the geometry of the effective cross section are presented. Once the effective geometry is established, normal properties of C24 timber

from EN 338 are combined with the plate theory in order to predict the mid-span deflection according to the RCSM.

3.2.1 Existing approaches

- **RCSM-1.** The first existing RCSM approach has been derived by Schmid et al, (2012) fitting results of advanced numerical simulations on timber members in bending and exposed to fire on their tension side. According to this method, the charring rate is uniform and as prescribed in the Eurocode 5 (0.65 mm/min), while the ZSTL is derived as a function of panel's total thickness by means of a logarithmic relationship. Applying this approach to the three tested panels leads to the following ZSTLs: Panel-1 = 11 mm; Panel-2 = 10.7 mm; Panel-3 = 10.8 mm.
- **RCSM-2.** The second simplified design model (Frangi et al., 2009) is based on the initially protected surfaces approach of the Eurocode, in order to take into account the falling-off phenomenon discussed in the previous section. Hence, a double charring rate (1.3 mm/min) is considered suddenly after a glued interface between layers and until 25 mm of charring depth. After 25 mm of charring depth after a glued interface, the charring rate returns at 0.65 mm/min. Within this method, the ZSTL is taken as in the Eurocode (constant at 7 mm).

3.2.2 Calibration of the RCSM

More than the two presented existing models, a new simplified approach based on the RCSM is suggested by the authors (**RCSM-3**). The principle is to use the reduction coefficient for material properties k_{ϑ} found within the advanced modeling to reduce the cross section geometry. The basic hypothesis is that, for each discretized mesh within the advanced modeling, the value $(1 - k_{\vartheta})$ multiplied by the thickness of each mesh (1 mm) returns the thickness of wood that has no mechanical properties (Figure 3.3). The sum of each reduced thickness finally leads to the ZSSL for each considered time of fire exposure.

$h = 1\text{mm}$	$\theta = 20^{\circ}\text{C}$	$k_{\theta} = 1.0$	$k_{\theta}d_0 = (1-1)*1 = 0\text{ mm}$	0	+
$h = 1\text{mm}$	$\theta = 30^{\circ}\text{C}$	$k_{\theta} = 0.9$	$k_{\theta}d_0 = (1-0.9)*1 = 0.1\text{mm}$	0.1	+
				...	
				...	
$h = 1\text{mm}$	$\theta = 180^{\circ}\text{C}$	$k_{\theta} = 0.3$	$k_{\theta}d_0 = (1-0.3)*1 = 0.7\text{mm}$	0.7	+
$h = 1\text{mm}$	$\theta = 240^{\circ}\text{C}$	$k_{\theta} = 0.15$	$k_{\theta}d_0 = (1-0.15)*1 = 0.85\text{mm}$	0.85	=
$h = 0.65*\text{time}$ $\theta > 300^{\circ}\text{C}$ charred wood				ZSSL [mm]	




Figure 3.3 Principle of the suggested derivation of the zero stiffness layer. ϑ = temperature

The suggested derivation of the ZSSL is therefore based on the hypothesis that at established reduction of stiffnesses corresponds the same reduction of geometry (or “loss of material”). The studies of Franzoni et al. (2015, 2016) showed that this hypothesis is valid when dealing with the bending deflection, without contribution of transverse shear. On the contrary, such hypothesis is not anymore valid concerning the transverse shear deflection. However, since the geometry and the lay-up of the considered panels yield a low contribution of transverse shear to global deflection (in the range of 10%-12%), the principle showed on Figure 3.3 can be applied to the tested panels with a low margin of error. When plotting the estimated ZSSL versus the time of fire exposure, a plot like the one showed in Figure 3.4 can be found. From Figure 3.4 it is clear that the ZSSL increases during time exposure, reaching values much higher than the constant 7 mm prescribed in the Eurocode 5.

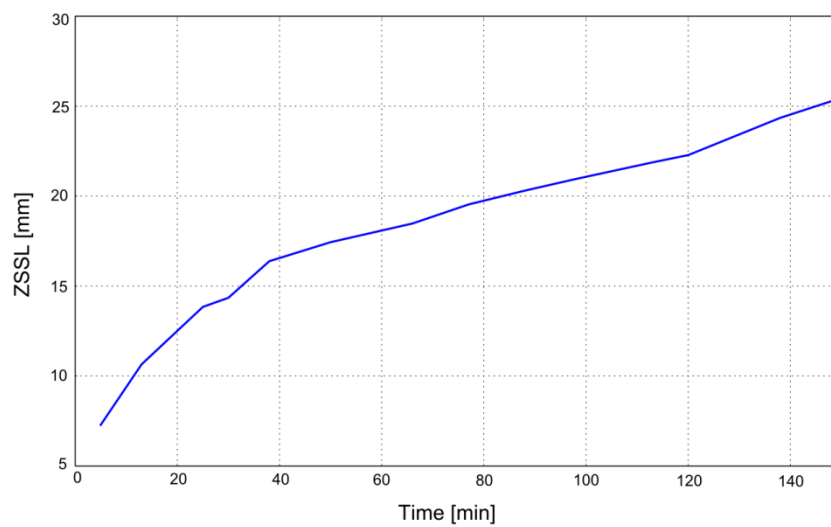


Figure 3.4 Variation of the ZSSL during time exposure of Panel-1

On the basis of Figure 3.3, values of ZSSL as a function of time are therefore suggested, in order to take into account the increasing heat flux received by the CLT panel during fire exposure. Finally, a simplified design approach can be obtained setting four values of zero stiffness layer for four ranges of time as in Table 3.1.

Table 3.1 Suggested values of ZSSL as a function of time exposure

Time t (min)	0<t<20	20<t<40	40<t<60	> 60
ZSSL (mm)	10	14	17	20

The same principle can be applied to strength using the Eurocode 5 laws for tensile, compressive and shear strengths reduction factors k_{ϑ} . With this approach, ZSTL for fire exposed members in tension, compression or submitted to shear as a function of time like the values presented in Table 3.2 can be derived.

Table 3.2 Variation of ZSTL as a function of time exposure for timber members in compression, tension and shear

ZSTL Time	25 min	50 min	75 min	100 min
Tension [mm]	9.6	12	13.5	14.5
Compression [mm]	16	19.5	22	23
Shear [mm]	13.5	17	19.5	20.5

4 Comparison of predicted deflection

The comparison between the predicted and experimental mid-span deflection for the three fire tests on CLT are showed from Figure 4.1 to Figure 4.3. As already introduced in 3.2, the RCSM-1 method is according to Schmid, 2012, the RCSM-2 is according to Frangi, 2009 and RCSM-3 considers the ZSSL as a function of time (Table 3.1). For the three approaches based on reduced cross section, the ZSSL is computed as in the EC 5: $ZSSL = k_0 \cdot ZSSL$, where $k_0 = \text{time}/20$ for time < 20 minutes and $k_0 = 1$ for time > 20 minutes.

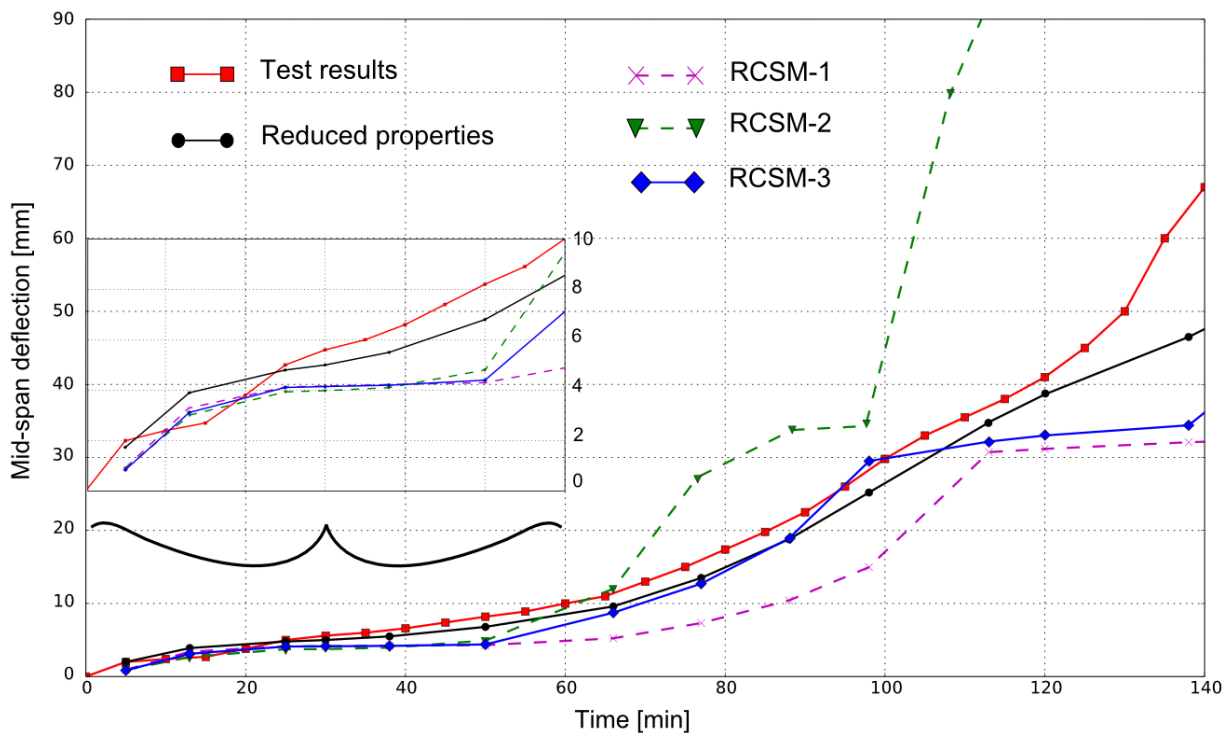


Figure 4.1 Comparison between predicted and experimental deflection of Test-1

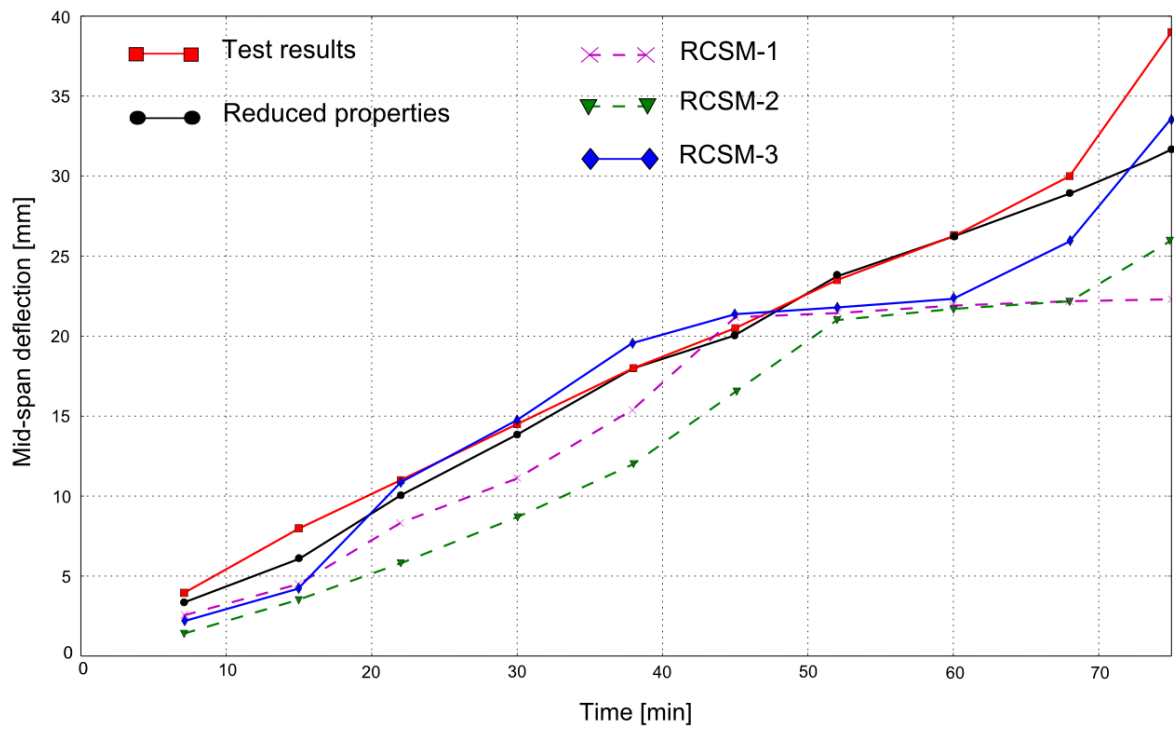


Figure 4.2 Comparison between predicted and experimental deflection of Test-2

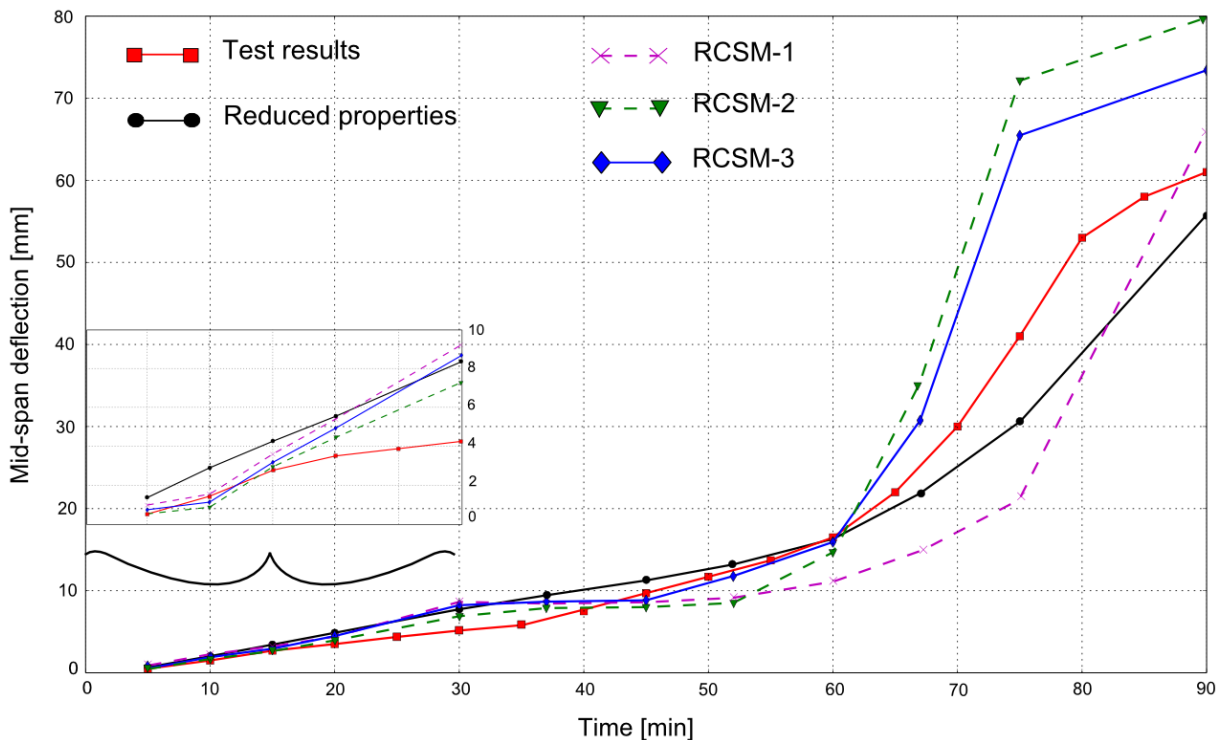


Figure 4.2 Comparison between predicted and experimental deflection of Test-3

The advanced modeling based on reduced properties returns the mid-span deflection that less deviates from experimental results for the three tests. Moreover, contrary to RCSM approaches, it does not show plateau corresponding to the non-contribution of cross layers to global stiffness. This confirms that the actual phenomenon is a progressive reduction of properties and not a reduction of geometry. The RCSM-1 approach underestimates the deflection of the three fire tests. Figures 4.1 to

4.3 show that the RCSM of the Eurocode can be improved when considering a ZSSL as a function of time exposure.

Despite of the falling-off effect in Test-1 highlighted in section 2.3, the RCSM-2 (derived to take into account such phenomenon) overestimates the Test-1 deflection. The deflection of Test-2 is underestimated by RCSM-2 due to the charring of almost only the first layer. Dealing with Test-3, the final part of the deflection evolution in Figure 4.3 clearly shows an increase of deflection when the char front passes from first to second layer and then the slope returns smooth, highlighting the falling-off phenomenon. Hence in this case the delamination phenomenon has an influence on global elastic response, contrary to Test-1 results. Therefore the reduced properties method, based on temperature prediction of solid wood without delamination, cannot correctly predict the deflection. In this case, the RCSM-2 and RCSM-3 seem to well reproduce the sharp increase of deflection, even if in a quite conservative way.

In the final parts of Figure 4.1 and Figure 4.2, all methods underestimate the deflection. This is because both of specimens were close to failure at those times exposure and hence the increasing non-linear contributions to deflection cannot be taken into account by the plate theory.

5 Discussion and Conclusion

In the present paper, the deflection of fire exposed CLT floors has been used to obtain information on the mechanical response during fire exposure. The experimental results have been compared to advanced modeling based on reduced properties and to the simplified RCSM of Eurocode 5. The advanced modeling globally returned a better agreement with test results than two existing RCSM approaches. However, the predicted results can be affected by the variation of material properties such as wood density, relative humidity and density. Unfortunately, the raw material of crosslam has not been tested and the input parameters for the modeling are based on mean properties given by C24 strength class in EN 338. Being the predicted deflection based on a linear elastic plate model, for a given variation of wood stiffness, the same variation of global deflection occurs. Dealing with the reduced properties modeling, further parameter studies not reported in this paper showed that the predicted temperature profiles by Safir software, and consequently the reduced stiffnesses, are strongly influenced by input wood density and RH. For instance, for a variation of 10% of wood density, a variation between 20% and 50% of the predicted deflection occurs, depending on the exposure time.

However, it is clear from section 4 that the RCSM based on reduced geometry deviates from experimental results. This is also due to the reduced properties after the char front taken into account by a constant value of zero stiffness layer. Such limitation can be improved when considering a ZSSL that varies during fire exposure, as showed in section 4.

The phenomenon of layers falling-off is a complex mechanism influenced by a multitude of parameters and therefore very difficult to predict. It can occur locally, with delamination of small pieces of wood, or with a complete falling-off of layer. The charring rate estimation with thermocouples registration presented in section 2.3 pointed out falling-off phenomena for the three considered test. However, only Test-3 showed an influence of such effect on the mid-span deflection evolution, while the other two tests showed no increase on deflection evolution. Hence, more research on influencing parameters on delamination phenomenon is needed.

The suggested improvement of RCSM is based on zero stiffness layer estimation as a function of time exposure. The fire design of timber members is often governed by ULS and therefore the determination of zero strength layer instead of zero stiffness layer has a great importance. However, dealing with panels' strength and load-carrying capacity, a direct comparison with experimental results is not as simple as in the case of mid-span deflection. A starting point for future studies may be assuming a zero strength layer as a function of time derived as the zero stiffness layer in this paper. Therefore, values similar to those presented in Table 3.2 can be obtained. Of course, this approach needs further validation with advanced modeling and available experimental data, but it may be a research path to follow, considering also the ongoing revision process of the Eurocode 5.

References

- Craft, S., Desjardins, R., Mehaffey, J. (2011): Investigation of the behaviour of CLT panels exposed to fire. *Fire and Materials*
- Eurocode 5 (2004): Design of timber structures - Part 1-2: Structural fire design. CEN. (EN 1995-1-2)
- Frangi, A., Fontana, M., Hugi, E., Jobstl, R. (2009) Experimental analysis of cross laminated timber panels in fire. *Fire safety Journal*
- Franzoni, L., Lebée, A., Lyon, F., Foret, G. (2015) Advanced modeling for design helping of heterogeneous CLT panels in bending. 2nd INTER meeting, Sibenik (CRO)
- Franzoni, L., Lebée, A., Lyon, F., Foret, G. (2016) Bending behavior of regularly spaced CLT panels. *World Conference on Timber Engineering*, Vienna (AUT)
- Klippel, M., Leyder, C., Frangi, A., Fontana, M., Lam, F., Ceccotti, A. (2014) Fire tests on loaded Cross Laminated Timber wall and floor elements. 11th international symposium on Fire Safety Science
- Osborne, L., Dagenais, C., Bénichou, N. (2012): Preliminary CLT Fire Resistance Testing Report. Tech. rep., Advanced Building Systems – Serviceability and Fire Group, Point-Claire, Canada

- Schmid, J., Konig, J., Just, A. (2012) The Reduced Cross Section Method for the design of timber structures exposed to fire – Background, Limitations and new Developments. Structural Engineering International
- Schmid, J., Klippel, M., Just, A., Frangi, A. (2014) Review and analysis of fire resistance tests of timber members in bending, tension and compression with respect to the Reduced Cross Section Method. Fire Safety Journal
- Schmid, J., Just, A., Klippel, M., Fragiacomio, M. (2015) The Reduced Cross Section Method for evaluation of the fire resistance of timber members: discussion and determination of the Zero-Strength layer. Fire Technology
- Shaffer, EL (1984) Structural fire design: wood. US Dept of Agriculture, Madison WI