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Camille Colas, Alessandro Perlongo, Trung Nguyen Nguyen, Jeremy Bleyer. Stress-Based Method for Concrete Solids' Reinforcement Design. TINCE 2023 – Technological Innovations in Nuclear Civil Engineering, Oct 2023, Gif-sur-Yvette, France. hal-04239714

**HAL Id: hal-04239714**

**<https://enpc.hal.science/hal-04239714v1>**

Submitted on 12 Oct 2023

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## Stress-Based Method for Concrete Solids' Reinforcement Design

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The development of computing capacities in finite element modelling has led to the possibility of modelling complex three-dimensional concrete structures, such as those in nuclear or hydraulic projects, using solid elements. These models with solid elements give only internal stresses in elements and not force results. For reinforcement design, in order to apply the traditional force-based method, the method of equivalent shells is commonly used in practice, but its applicability is limited to only uniform geometries. Therefore, a reinforcement calculation method that allows directly exploiting the stress results is reviewed and its implementation in industrial projects is studied in this paper. The study presented here was carried out for reinforcement design at the Ultimate Limit State.

With the reinforced concrete considered as a homogeneous three-dimensional medium in this method, the stresses in the structure can be decomposed as the sum of the stresses in the concrete and those in the steel. The problem is defined by using the element principal stresses, but the directions of the reinforcing bars are imposed to follow any three orthogonal directions as usually seen in practice. The Mohr-Coulomb yield criterium is applied to restrict the concrete stresses. The minimum reinforcement amount at each element is achieved by using convex optimisation.

In the present work, the algorithm was implemented with Python programming language and takes as input the internal stresses provided by a finite element software. Reinforcement calculations were then run on some simple test cases as well as on a complex industrial project, and the results were compared with those obtained by other methods such as the method of equivalent shells or the strut-and-tie one. The analyses are discussed and some recommendations when using this stress-based method are made in this paper.

**KEYWORDS:** *reinforced concrete, concrete solid, stress analysis, convex optimization*

### Introduction

The use of three-dimensional solid elements has led to a more appropriate representation of complex massive reinforced concrete structures in the nuclear or hydraulic sector, which couldn't be properly represented using one or two-dimensional elements such as beams or shells. However, the exploitation of the element internal stress fields for reinforcement design, obtained through a finite element analysis, remains under investigations in the literature. The reinforcement calculation method that is commonly used in the industry is the equivalent shells' approach [4], by integration of internal stresses components along the height of the structure to a median plane. Following this projection, the reinforcement design can be carried out by using traditional force-based method. This method has different limits: the geometry must be uniform enough; the method can't be used in discontinuity areas; and the projection can't take into account the stress components on the plans perpendicular to the equivalent shell. Another method, highlighted by the Eurocodes [3], is the strut-and-tie approach: the structure is converted into an idealized network of one-dimensional elements, working in compression (concrete strut) or in tension (tie of reinforcement). This method is made to be applied in discontinuity zones, but has a number of limitations : the scheme has to be chosen in advance, or can be automatized through a structural optimization algorithm which is costly in terms of time and computing power; each scheme is specific to a given load and has to be recalculated for each combination of loads; and the reinforcement that is obtained can be oriented in any direction, which means the solution is not realistic and would have to be adapted. A third method exists to design reinforcement in massive structures, although it has been less investigated in the literature: the stress analysis method. This approach directly

exploits the stress field by using failure criteria of each material. It can be used for every kind of geometry and load, which makes it more exploitable for industrial needs.

This paper focuses on the stress analysis method in order to design the optimized reinforcement of these massive concrete structures. The objectives are to evaluate the relevance of this method on simple test cases at the Ultimate Limit State (ULS), but also to examine whether this method can be easily applied to complex industrial projects.

### Modelling of the reinforced concrete

The stress analysis method is based on a finite elements modelling of a massive concrete structure using solid elements. Subjected to a linear elastic analysis, this modelling provides us with an internal stress field. Following the approach proposed by Foster et al. [1], the reinforced concrete can be modelled as a homogeneous three-dimensional medium that can be decomposed as the sum of the stresses in the concrete and the one from the steel, considering that steel only carries normal stresses:

$$\begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{bmatrix} = \begin{bmatrix} \sigma_{c,xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{c,yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{c,zz} \end{bmatrix} + \begin{bmatrix} \sigma_{s,xx} & 0 & 0 \\ 0 & \sigma_{s,yy} & 0 \\ 0 & 0 & \sigma_{s,zz} \end{bmatrix} \quad (1)$$

where *c* and *s* stand for *concrete* and *steel* respectively. The principal stresses of a stress tensor are classified so that  $\sigma_1 \geq \sigma_2 \geq \sigma_3$ , counting traction as positive. At the Ultimate Limit State, the structural stability of the structure is guaranteed by the compliance of failure criteria for each of the materials, following the lower bound theorem of limit stage design [5]: every stress distribution that satisfies the equilibrium conditions and the statical boundary conditions for a given load remains safe and statically admissible for this load.

For the concrete, one can use the Mohr-Coulomb criterium as failure criteria, as illustrated on Figure 1. Let  $\sigma_A$  and  $\sigma_B$  be two of the principal stresses of the considered material and  $f_c$  and  $f_t$  the strength in compression and traction, the criterium is expressed as follows:

- If  $\sigma_A \geq 0$  and  $\sigma_B \geq 0$  :

$$\sigma_A \leq f_t \text{ and } \sigma_B \leq f_t \quad (2)$$

- If  $\sigma_A \leq 0$  and  $\sigma_B \leq 0$  :

$$-\sigma_A \leq f_c \text{ and } -\sigma_B \leq f_c \quad (3)$$

- If the principal stresses have opposite signs, the compression in one direction is going to increase the traction perpendicularly and conversely, leading to a coupling of both stresses:

$$\frac{\sigma_B}{f_t} + \frac{|\sigma_A|}{f_c} \leq 1 \quad \text{if } \sigma_A \leq 0 \leq \sigma_B$$

$$\frac{\sigma_A}{f_t} + \frac{|\sigma_B|}{f_c} \leq 1 \quad \text{if } \sigma_B \leq 0 \leq \sigma_A \quad (4)$$

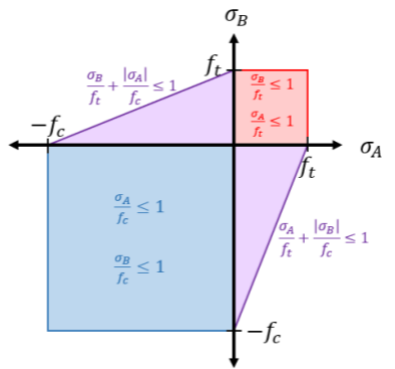


Figure 1: Mohr-Coulomb criterium

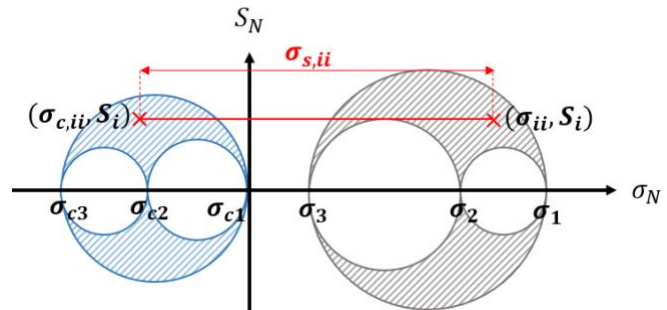


Figure 2: Representation of the behavior of the structure using a Mohr circle [1]

At the ULS, the strength of concrete is considered to be zero, which leads to the condition  $\sigma_{c,1} \leq 0$ . This condition can be illustrated using a Mohr circle representation [1] (Figure 2): the circle corresponding to the stress tensor of the concrete (in blue on Figure 2) shows an offset to the left compared to the circle formed by the stress tensor of the reinforced structure (in grey on Figure 2). The two circles are connected by horizontal red lines, corresponding to the need of reinforcement in one

direction. This reinforcement, expressed by a constraint  $\sigma_{s,ii}$ , can be related to the ratio of reinforcement  $\rho = A_s/A_{total} \geq 0$  by considering that steel is perfectly plastic:

$$\sigma_{s,ii} = \rho_i \cdot f_{yd} \quad \text{with } i = x, y, z \quad (5)$$

### Implementation of the stress analysis method

To obtain an optimized quantity of reinforcement,  $\rho_x + \rho_y + \rho_z$  must be minimized. As the problem involves principal stresses, an analytical resolution can be complex. To bypass this problem, one can propose the use of the third invariant which links principal stresses and stresses in a given reference frame [1] [2]:

$$I_3 = \sigma_{xx}\sigma_{yy}\sigma_{zz} + 2\sigma_{xy}\sigma_{xz}\sigma_{yz} - (\sigma_{xx}\sigma_{yy}^2 + \sigma_{yy}\sigma_{xx}^2 + \sigma_{zz}\sigma_{xy}^2) = \sigma_1\sigma_2\sigma_3 \quad (6)$$

For the design of reinforcement in traction, the condition  $\sigma_{c1} = 0$  corresponds to the smallest offset to be imposed to obtain a lack of traction in the concrete (as illustrated on Figure 2), which leads to the minimization of the quantity of reinforcement. This condition leads also to  $I_{c,3} = 0$ . Thus, Hoogenboom et al. [2] proposed an analytical resolution of the problem composed of 14 triplet solutions  $(\rho_x, \rho_y, \rho_z)$ , obtained by deriving the steel quantity  $\rho_x + \rho_y + \rho_z$  and using the equation  $I_{c,3} = 0$ . As  $I_{c,3} = 0$  doesn't lead necessarily to  $\sigma_{c1} = 0$ , each triplet as to be checked to verify the eligibility of the solution ( $\rho \geq 0$  in the three directions) and the best triplet can finally be chosen among the remaining solutions. Hoogenboom et al. [2] also proposed the use of an optimization problem for a more direct reinforcement sizing, working in traction but also in compression.

The problem studied in this paper is a convex optimization problem, which means it ensures the existence of a global minimum for a problem of minimisation. This convex problem corresponds to an alternative version of the problem studied by Hoogenboom et al. [2], for design at the Ultimate Limit State:

$$\begin{aligned} & \text{Minimize } \rho_x + \rho_y + \rho_z \\ & \text{s. t. } \rho_x \geq 0, \rho_y \geq 0, \rho_z \geq 0 \\ & \quad \sigma_{c1} \leq 0 \\ & \quad -\sigma_{c3} \leq f_{cd} \\ & \quad \sigma_{c,ii} = \sigma_{ii} - \rho_i f_{yd} \quad \text{with } i = x, y, z \\ & \quad \sigma_{c,ij} = \sigma_{ij} \quad \text{with } i, j = x, y, z \text{ and } i \neq j \end{aligned} \quad (7)$$

The problem was implemented in Python, by using the library CVXPY [6]. The code can be written by using mathematical tools, since the program takes care of converting it in the form required by solvers. Different solvers can also be imported through the library in addition to the ones available by default. For this paper, we used the CVXOPT solver, another solver available on Python, as it proposed the best computing time for our problem.

The computation chain is constructed as follow:

- Pre-treatment: The structure modelled with hexahedral elements is computed through a linear elastic finite elements analysis, using Code\_Aster (or ANSYS). The internal stress fields obtained for each load are extracted from the program and imported into Python.
- Algorithm: The program retrieves input data (stress fields, combinations of loads, material properties and parameters for the convex problem) and resolve the convex problem to obtain the envelope ratio of reinforcement at the ULS. The algorithm must be computed successively for each element of the mesh.
- Post-treatment: The 3D visualization of ratios of reinforcement in the structure can be done with GMSH, by using Code\_Aster to convert results from \*.txt to \*.pos format. Different verifications can also be done by using Python, as the stress distributions diagrams for the concrete, or the evaluation of dimensioning combinations.

The flowchart of the method is presented in Figure 3.

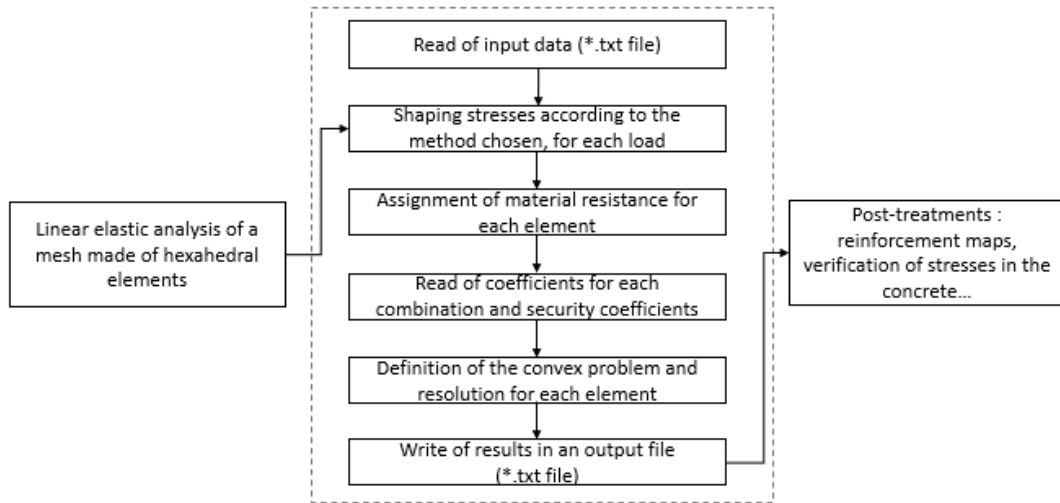


Figure 3: Flowchart of the developed program

The stress fields, as they are computed by Code\_Aster using 'MECA\_STATIQUE' solver, are obtained at Gauss points from displacement fields, and can be extrapolated at the nodes of the elements, giving in both cases 8 stress tensors for each element. Two methods have been developed to exploit these stress fields: the first method consists of calculating the average stress on the element to reduce the number of iterations of the program by computing a unique triplet of ratios for each element; the second method consists of computing the ratios at the eight nodes of each element and retrieve the average ratios then. On the second method, a shear stresses smoothing is realised within each element, to reduce local variations which tended to greatly increase the number of ratios. On an element where stresses are symmetrical on the eight points, as it can happen in bending, the first method obtains a zero average stress tensor leading to a zero quantity of reinforcement, while the second method obtains a non-zero ratio on some of the points leading to a non-zero average quantity of reinforcement for the element. In return, the second method takes eight times longer than the first one, making the two methods complementary.

Considering various combination of loads, the program must provide a single envelope triplet of ratios. One can imagine computing solutions for each combination then select the highest amount of steel obtained in each direction, but this method tends to overestimate the quantity needed, by decoupling the ratios from each other. The chosen method consists of repeating the last two constraints equations from (7) as many times as there are combinations, to let the algorithm finds by itself the envelope solution in a single computation. The new problem can be written as follows:

$$\text{Let } \bar{\rho} = \begin{pmatrix} \rho_x & 0 & 0 \\ 0 & \rho_y & 0 \\ 0 & 0 & \rho_z \end{pmatrix} \text{ be the variable of our convex problem,}$$

$$\begin{aligned} & \text{Minimize } \text{tr}(\bar{\rho}) = \rho_x + \rho_y + \rho_z \\ & \text{s. t. } \bar{\rho} \gg 0 \\ & \lambda_{\max} \left( \bar{\sigma}_i - \frac{f_{yk}}{\gamma_{s,i}} * \bar{\rho} \right) \leq 0 \quad \text{for } i \in \llbracket 1, \text{nb\_of\_combinations} \rrbracket \\ & -\lambda_{\min} \left( \bar{\sigma}_i - \frac{f_{yk}}{\gamma_{s,i}} * \bar{\rho} \right) \leq \frac{f_{ck}}{\gamma_{c,i}} \quad \text{for } i \in \llbracket 1, \text{nb\_of\_combinations} \rrbracket \end{aligned} \quad (8)$$

In the above formulation,  $\lambda_{\max}$  and  $\lambda_{\min}$  are the functions that compute respectively the higher and the lower eigenvalues of a tensor. The strengths of concrete and steel are written using their characteristic value to show that safety coefficient depends on the combination according to Eurocodes [3].

**Test cases of shells**

Similar tests were then conducted on shells, in order to compare the method using stress analysis (SAM) with the equivalent shells method (ESM) on complex structures. Let consider a 5.0m long shell (in the x direction) with a width of 2.0m (in the y direction) and a thickness of 0.2m (in the z direction). The same properties of materials are considered as for beam testing.

To compare the two methods, the ratios obtained by SAM are converted into quantities of steel distributed between two reinforcement fibers (a superior one and an inferior one). Considering longitudinal reinforcement in X direction, conversion between ratios and quantities is made by summing the quantities of reinforcement for each element ( $A_{s,element} = \rho_{x,element} * A_{element}$ ) on each half. These quantities are located on the center of inertia of steel. Considering transversal reinforcement in Z direction, the maximal ratio for each column of elements pointing in Z direction is selected as the reinforcement to be put in place. In the following example, ASX and AIX are the reinforcement in X direction respectively on upper and lower fiber, ASY and AIY are the reinforcement in Y direction respectively on upper and lower fiber, and AZ is the shear reinforcement.

For tension and compression tests, the shell was recessed on one extremity (X=0m), and a resultant force in the X direction was applied under the form of a volumetric charge in the totality of the structure, to avoid punctual loads that represent discontinuities. As results are symmetrical between upper fiber and lower fiber, results focused on only one fiber. Results obtained in traction are presented Figure 7. Despite similar distribution, stress analysis method (SAM) leads to an increase of 14% and 3% of the global volume of reinforcement respectively in X and Y directions, compared to ESM. In compression, no reinforcement was computed in X direction. In Y direction, the differences observed on Figure 8 are amplified by the fact that the quantity of reinforcement is small, leading to a reduction of the volume of reinforcement of 81% by SAM compared to ESM. This last example illustrates the difficulty to estimate reinforcement needed for low-reinforcement structures.

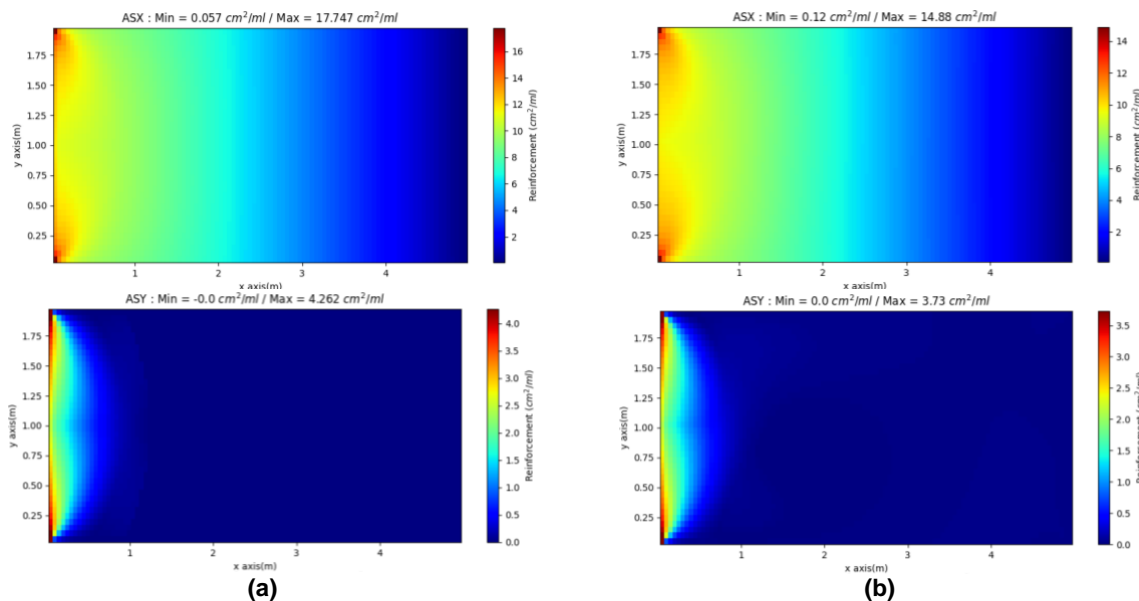


Figure 7: Longitudinal reinforcement for the shell in traction using: (a) SAM / (b) ESM.

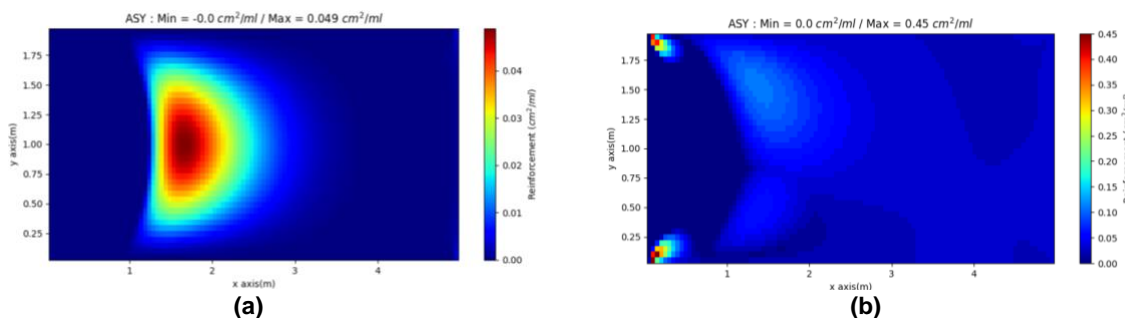


Figure 8: Longitudinal reinforcement for the shell in compression using: (a) SAM / (b) ESM.

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The bending example consists of a shell recessed in  $x = 0m$  and  $x = 5.0m$ , submitted to ten times its own weight. Reinforcement was computed using Averaged stresses approach and Shear smoothing +Averaged ratios approach and leads to close results. Thus, only results using the first method are presented in this paper, as it is the one with the smaller computation time. Results presented on Figure 9 show that SAM always leads to a higher maximal quantity of reinforcement for every direction. Distributions present small variations, especially for reinforcement in Y direction, that appears to be more widespread with ESM.

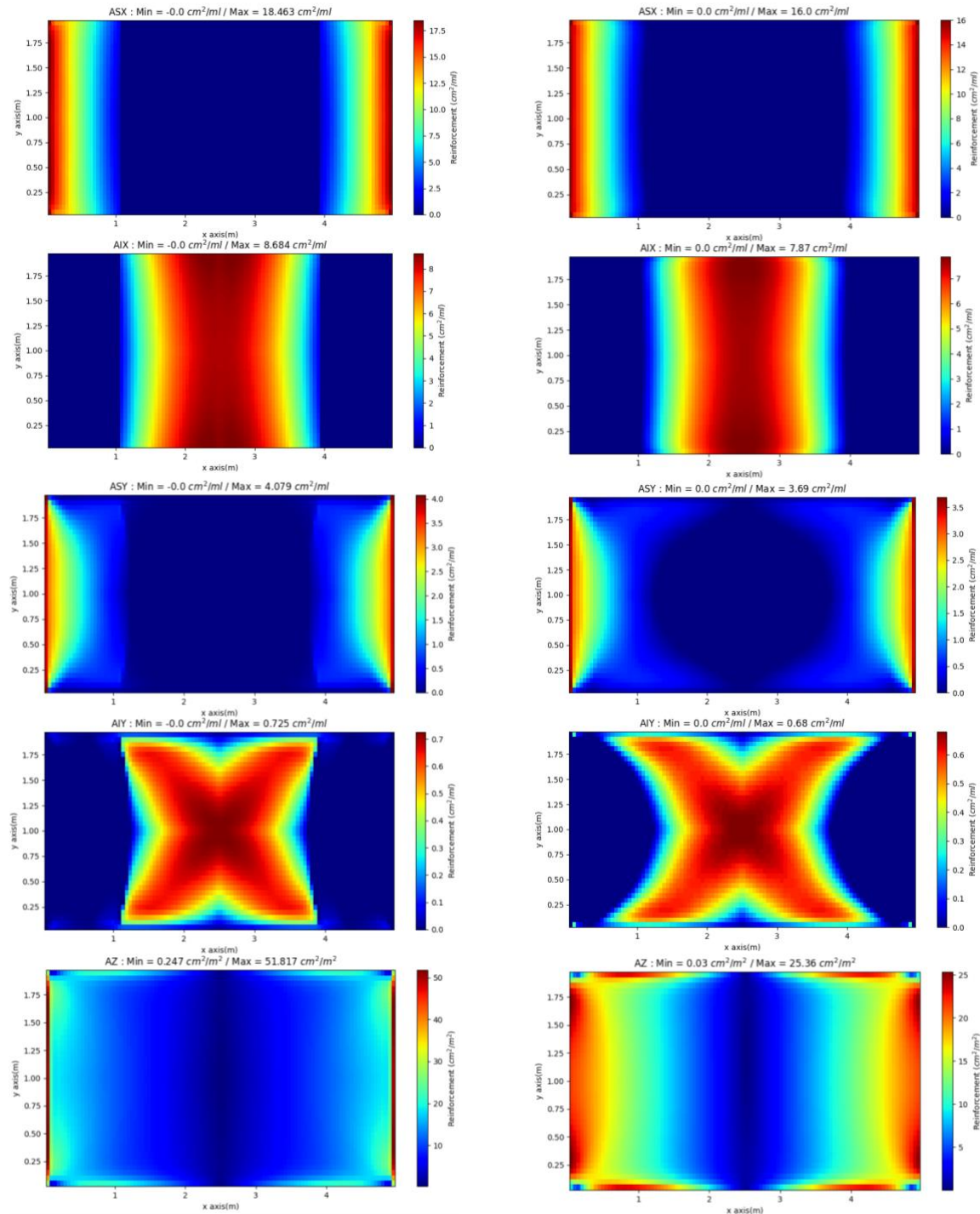


Figure 9: Reinforcement for the shell in bending using: (a) SAM / (b) ESM.

The SAM leads to an increase of 18 % of the global volume of longitudinal reinforcement compared to ESM: +24% for reinforcement in x direction on upper fiber, +17% for reinforcement in x direction on

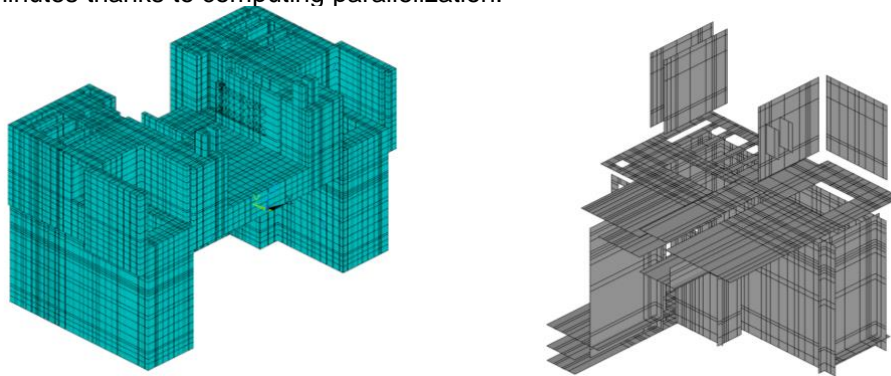
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lower fiber, +2% for reinforcement in y direction on upper fiber and -4% for reinforcement in y direction on lower fiber.

The shear reinforcement computed by ESM corresponds to the maximal quantity of reinforcement needed, without considering the design resistive shear force under which no reinforcement is required according to Eurocode Standards (§6.2.2. of [3]). Under this hypothesis, both methods present the same results with a global reinforcement under  $30 \text{ cm}^2/\text{m}$ , except along recesses where SAM leads to a peak of reinforcement. The global increase of the volume of shear reinforcement with SAM is up to 3% (taking into account the peak of reinforcement). If we consider the design resistive shear force to reduce reinforcement for ESM, the gap rises from 3% to 33%.

### Application to the design of a lock head

The Stress Analysis Method was then applied for the design of a lock head, submitted to various loads like its weight, land and water pressures, temperatures, exploitation loads or boats impact. Around fifty combinations of loads at the Ultimate Limit State were selected for this study, to evaluate the computing time needed, using problem as it is defined on equation (8). As the structure is symmetrical, only a half of the structure had to be study, composed of 40 000 hexahedral elements. The immersed part of the structure is composed of C25/30 while the upper part of the structure is made of C30/37. The computation was realized on an Intel(R) Xeon(R) W-2135 CPU @ 3.70GHz processor and took around 45 minutes thanks to computing parallelization.



**Figure 10: Massive Finite Element Modelling of the lock head (left) and equivalent shells (right)**

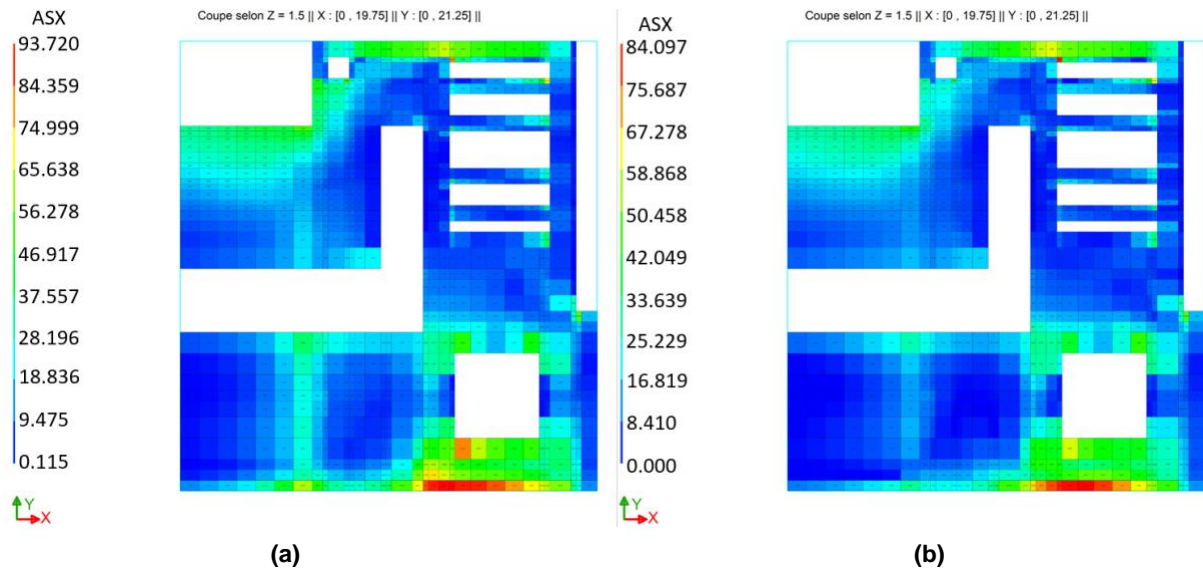
The results of SAM were compared to the ones obtained by the ESM, in areas that were regular enough to be cut out. Figure 11 illustrates ASX reinforcement obtained by each method on a horizontal cut of the structure, showing similar distributions. Overall, up to 17 equivalent shells were computed, distributed in the three directions (Figure 10, right), all showing similar distributions for longitudinal reinforcement. Table 2 shows variations in volume of longitudinal reinforcement of SAM compared to ESM: results are classified according to the number of elements in the thickness of the equivalent shell and the dimension of this thickness. The results of reinforcement are given according to the local frame of each equivalent shell, considering X and Y as directions in the plane.

Results show variations of global longitudinal reinforcement from -38% (for component 10) to 27% (for component 17). A pattern seems to appear in the results: SAM seems to reduce the quantity of longitudinal reinforcement for thin components (components 1, 7, 10 and 11) and increases it for thicker components (components 16 and 17). High increases of reinforcement in one direction are visible in few components (components 4, 7 and 16), without this being visible on the total variation of reinforcement: this are caused by low-reinforcement areas, as it was experienced for shell in compression test cases.

Shear reinforcement was not presented for this study as results show important variations depending on whether design resistive shear force is considered or not, as experimented during bending shell test case. Differences can also be explained by the fact that, with SAM, the direction for shear reinforcement is considered as an equivalent direction as for longitudinal reinforcement, whereas calculations are conducted differently with force-based methods as the one used by ESM.



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**Figure 11: ASX reinforcement obtained for component 15 using: (a) SAM / (b) ESM.**

	Number of elements In the thickness	Thickness (m)	Local X direction		Local Y direction		Total
			Sup. Fiber	Inf. Fiber	Sup. Fiber	Inf. Fiber	
1	2	0.75	-49%	-50%	-15%	-16%	-30%
2	2	0.75	3%	-13%	14%	5%	1%
3	2	1	-15%	18%	-11%	51%	5%
4	2	1	-13%	108%	-22%	102%	13%
5	2	1.25	6%	5%	8%	10%	7%
6	2	1.5	9%	12%	24%	17%	15%
7	2	1.5	-19%	12%	-41%	126%	-7%
8	2	1.75	21%	13%	-6%	-35%	9%
9	3	1	-30%	-50%	-10%	-15%	-24%
10	3	1	-25%	19%	-35%	34%	-12%
11	4	2.25	6%	31%	15%	41%	24%
12	4	2.25	26%	38%	12%	15%	20%
13	4	2.25	11%	1%	39%	5%	9%
14	4	2.5	-12%	37%	-11%	82%	21%
15	4	3	27%	27%	17%	15%	22%
16	6	2	7%	24%	127%	52%	29%
17	6	2.5	28%	25%	36%	33%	29%

**Table 2: Variation of volume of longitudinal reinforcement of SAM compared to ESM.**

### Comparison with strut-and-tie approach

It was then decided to compare the developed methodology with the strut-and-tie approach, as this second approach represents the principal alternative for massive reinforcement design, despite limits explained in the introduction part. A simple way to proceed is to consider a 2D problem, for which a realistic scheme can be calculated by hand. The solution proposed would remain admissible thanks to the lower bound theorem [5] but would not necessarily be the most optimized solution: the scheme is only chosen to correspond to the reinforcement pattern obtained numerically.

The studied structure is a corbel whose geometry is illustrated Figure 12. The thickness of the structure is set up at 1.0m and a load  $F$  of  $-0.5$  MN directed in the Y direction is linearly distributed along the thickness. For the analytical resolution, the loading area is assumed to be 0.05m-width. The corresponding three-dimensional model for FEM analysis is presented Figure 13, using elements of 0.05m.

The strut-and-tie scheme is formed of hydrostatic nodes from A to D. The resolution of the problem led to one vertical tie of  $A_V = 21.6\text{cm}^2$  and one horizontal tie of  $A_H = 9.2\text{cm}^2$ , corresponding to respective volumes of steel of  $V_V = 3538\text{cm}^3$  and  $V_H = 816\text{cm}^3$ , to which we must add reinforcement caused by secondary ties perpendicular to the struts : the left one requires a steel cross-section of  $A_L = 4.9\text{cm}^2$  ( $V_L = 223\text{cm}^3$ ) oriented at an angle  $\theta_L = 67^\circ$  to the Y axis ; the right one requires a steel cross-section of  $A_R = 3.3\text{cm}^2$  ( $V_R = 161\text{cm}^3$ ) oriented at an angle  $\theta_R = 51.5^\circ$  to the Y axis. By combining principal and secondary-tie reinforcement projected on X and Y axis, we obtain a global reinforcement of the structure of  $V_X = 1103\text{cm}^3$  and  $V_Y = 3635\text{cm}^3$ , for a total of  $V_{X+Y} = 4738\text{cm}^3$ .

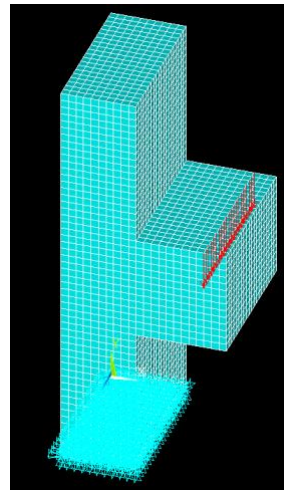
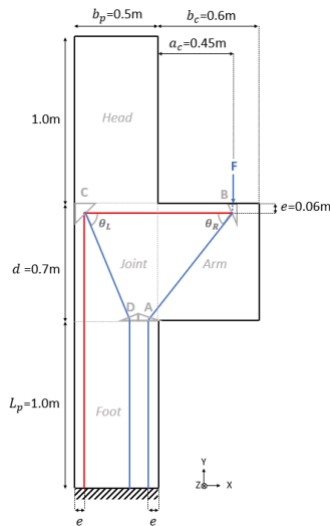


Figure 12: Strut-and-Tie Model for the corbel      Figure 13: Finite element modelling of the corbel

Results obtained by SAM are presented orientated on general (X, Y, Z) axis (Figure 14), but also on (X', Y', Z) axis which represent a rotation of  $\theta_R = 51.5^\circ$  of the general orientation around Z axis (Figure 15). Cuts are made at medium thickness to avoid edge effects.

Volumes of reinforcement in the different parts of the structure, obtained by SAM are presented in Table 3. A small quantity of reinforcement appears in Z direction, especially in discontinuity zones and around boundary conditions, corresponding to 5% of the total quantity of reinforcement. The total quantity of reinforcement obtained by SAM is  $V_{X+Y+Z} = 4608\text{cm}^3$  and  $V_{X+Y} = 4346\text{cm}^3$  if we only consider X and Y reinforcement, consistent with the quantity found by strut-and-tie modelling. The appearance of the results obtained by SAM also matched with proposed strut-and-tie scheme: Figure 14 shows maximum of reinforcement at the position of principal ties, and diffuse areas of reinforcement around secondary ties, in X and in Y direction. The rotation of the axis proposed Figure 14 in the direction of right strut makes it easier to distinguish secondary ties, as reinforcement is only oriented on X' axis for the left strut, and on Y' axis for the right strut. Z reinforcement for both computation remains identical. Quantitatively, we obtain a cross-section of  $1929/(1 * 100) = 19.3\text{cm}^2$  for the vertical tie. Horizontal tie can't be measured as the reinforcement is mixed with the one of the secondary ties on Y direction. One can estimate the volume of the right secondary tie in Y' direction by only selecting the zone for which  $\rho_{X'} = 0$ , which gives us  $320\text{cm}^3$ , around the double of the analytical quantity.

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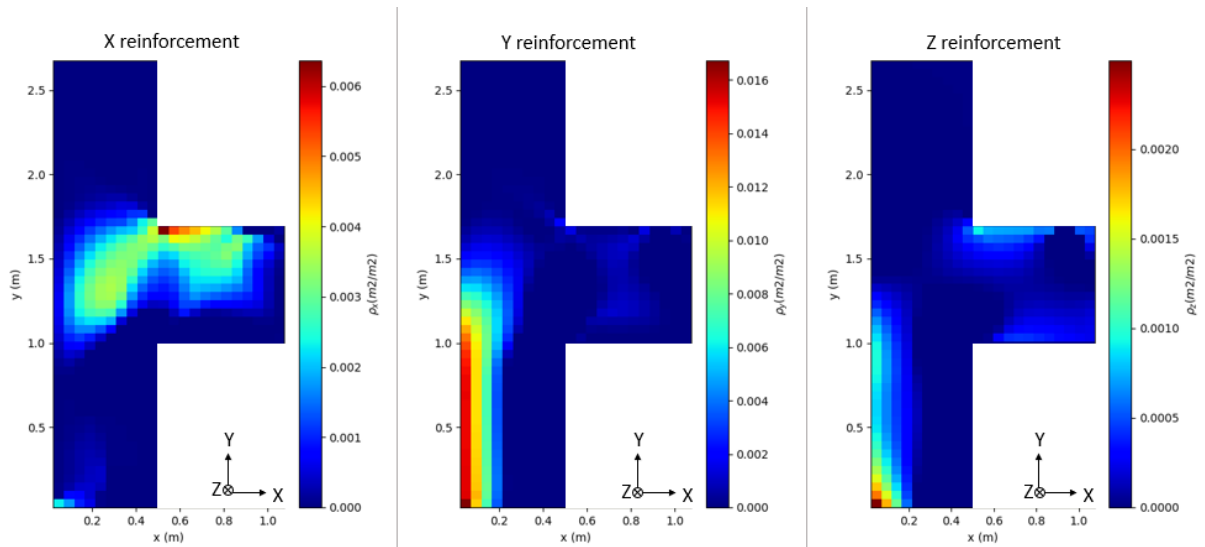


Figure 14: Density of reinforcement in the corbel obtained by SAM, in  $Z=0.45m$

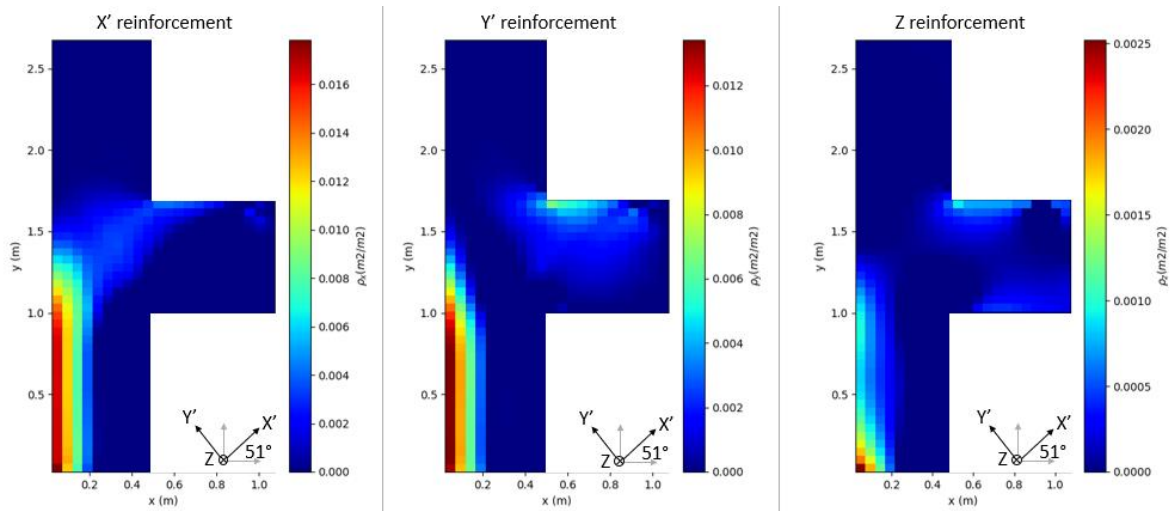


Figure 14: Density of reinforcement in the corbel obtained by SAM on rotated axis, in  $Z=0.45m$

	$V_x$ (cm <sup>3</sup> )	$V_y$ (cm <sup>3</sup> )	$V_z$ (cm <sup>3</sup> )	Total (cm <sup>3</sup> )
Arm	586	122	70	778
Head	56	27	4	87
Joint	697	854	45	1596
Foot	75	1929	143	2147
Full structure	1414	2932	262	4608

Table 3: Volume of reinforcement obtained in the corbel by SAM.

It remains important to note that the total quantity obtained for the structure computed on  $(X', Y', Z)$  axis is up to 6042cm<sup>3</sup>, representing an increase of 24%: this is caused by the fact that the principal ties are projected on two directions instead of one, which is less optimized. While working with imposed directions of reinforcement, the chosen orientation remains an important parameter for reinforcement optimisation. This example illustrates the capability of the Stress Analysis Method to compute reinforcement for complex geometry, with the same relevance as a Strut-and-Tie scheme. The savings

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in reinforcement that can be achieved by using strut-and-tie approach must be balanced against the need to achieve a realist distribution along imposed axis.

### Conclusion

In this paper, stress analysis method was investigated, as an alternative approach to design optimized reinforcement for massive concrete structures. A Python program has been set up aimed at computing reinforcement design through a convex optimization problem. The methodology presented was first applied on simple structures like beams and shells to estimate the relevance of this approach, by comparing results with traditional force-based reinforcement design. The program was then applied on a lock head model to test the capacities of the method on bigger meshes submitted to various loads and combinations. This method was compared to the equivalent shell method. The ultimate test was carried out on a corbel for which a strut-and-tie scheme had been calculated by hand. The stress analysis method appears to be a relevant method for reinforcement design at the Ultimate Limit States, as results appear consistent with the other methods introduced in tests. The developed program may be further extended to design of reinforced concrete structure at the Service Limit States, in order to obtain a complete method.

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