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1 Soil-water retention behaviour of fine/coarse soil mixture with varying coarse
2 grain contents and fine soil dry densities

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22 **Abstract**

23 An interlayer soil identified in the French conventional rail track corresponded to a mixture of
24 fine soil and coarse grains. To investigate the role of fines in the soil-water retention property
25 of such mixture, different coarse grain contents f_v and dry densities of fine soil ρ_{d-f} were
26 considered. The filter paper method was applied to measure the matric suction. Mercury
27 intrusion porosimetry tests were performed for the microstructure observation of fine soil. In
28 terms of gravimetric water content of fine soil w_f with matric suction ψ , the soil-water
29 retention curve (SWRC) was significantly affected by ρ_{d-f} for $\psi < 715$ kPa, while independent
30 of ρ_{d-f} for $\psi > 715$ kPa. Interestingly, this threshold ψ corresponded to a delimiting pore
31 diameter of bi-modal microstructure of fine soil, which separated micro-pores from macro-
32 pores. In terms of degree of saturation S_r with ψ , the SWRC was significantly affected by ρ_{d-f}
33 in the full suction range, while independent of f_v . These findings help better understand the
34 results on samples with the dry density of mixture ρ_d kept constant: an increase of f_v resulted
35 in a decrease of ρ_{d-f} and the suction changed accordingly. In that case, both f_v and ψ affected
36 the mechanical behavior.

37 **Keywords:** soil-water retention; matric suction; pore size distribution; dry density; coarse
38 grain content

39 INTRODUCTION

40 In the French conventional rail track, an interlayer was naturally created in the substructure
41 under the long-term traffic loading, which corresponded to a mixture of ballast grains and
42 subgrade fine soil. As its suction can greatly affect its mechanical behavior (see details in Su
43 et al. 2020a and Wang et al. 2018), it appears important to investigate its water retention
44 property in-depth.

45 In-situ investigations showed a decrease of ballast grain content with the increasing depth
46 of interlayer soil (Trinh 2011, Cui et al. 2013). Globally, the interlayer soil was separated into
47 two parts: the upper part was characterized by a coarse grain skeleton fabric, and the lower
48 part by a fine matrix macrostructure with dispersed coarse grains. In addition, due to the
49 different vertical stresses over depths, the dry density of fine soil ρ_{d-f} was different. Wang et al.
50 (2017, 2018), Cui (2018) and Qi et al. (2020a) studied the effect of coarse grain content f_v (the
51 ratio of the volume of coarse grains V_c to the volume of total sample V) on the mechanical
52 behavior of interlayer soil at a constant $\rho_{d-f} = 1.82 \text{ Mg/m}^3$. They assumed that under a given
53 water content, the matric suction of mixture was the same at a constant $\rho_{d-f} = 1.82 \text{ Mg/m}^3$. Up
54 to now, this point has not been experimentally examined yet. It seems necessary to verify this
55 point by investigating the effects of coarse grain content f_v and dry density of fine soil ρ_{d-f} on
56 the soil-water retention property of fine/coarse soil mixture.

57 The effects of dry density and coarse grain content on soil-water retention property were
58 investigated previously in some studies. The filter paper method (ASTM D5298-10, 2010)
59 was usually adopted for the measurement of suction (Muñoz-Castelblanco et al. 2010; Kim et
60 al. 2015; Jing 2017). It appeared that the effect of dry density on soil-water retention behavior

61 was strongly dependent on the microstructure (Simms and Yanful 2002; Romero et al. 2011).
62 Romero et al. (1999) studied the effect of dry density on water retention and microstructure of
63 Boom clay by vapor equilibrium technique and mercury intrusion porosimetry tests,
64 respectively. The results showed that the soil-water retention curve (SWRC) in the low
65 suction range was governed by the inter-aggregate pores, while that in high suction range was
66 governed by intra-aggregate pores. Similarly, Salager et al. (2013) and Gao and Sun (2017)
67 investigated the water retention capacity of clayey soil and found that the SWRCs for
68 different dry densities was independent of dry density beyond a certain matric suction. It is
69 worth noting that these studies only involved the effect of dry density without the effect of
70 coarse grain content. Fiès et al. (2002) studied the soil-water retention property of fine
71 soil/glass fragments mixture, and found that an increasing glass content led to a reduction of
72 the amount of water stored in the mixture. Baetens et al. (2009) investigated the effect of rock
73 fragments on the water retention property of stony soil, and reported that rock fragments could
74 affect the SWRC when the matric suction was smaller than 30 kPa. Duong et al. (2014)
75 studied the hydraulic behavior of interlayer soil by infiltration column, and observed that
76 increasing coarse grain content resulted in a lower SWRC or a lower water retention capacity.
77 Note that in most of these studies, the effect of coarse grain content on soil-water retention
78 property of mixture was investigated with a large quantity of coarse grains, which
79 corresponded to the coarse grain skeleton structure of mixture, without considering the fine
80 matric macrostructure. In addition, the dry density of mixture ρ_d was taken constant, leading
81 to a decrease of dry density of fine soil ρ_{d-f} with the increase of coarse grain content. That
82 would increase the difficulty of analysis while studying the effect of suction.

83 This study aims to investigate the effects of coarse grain content f_v and dry density of fine
84 soil ρ_{d-f} on soil-water retention property of fine/coarse soil mixture. Three $f_v = 0\%$, 20% and
85 35% were adopted at the same $\rho_{d-f} = 1.82 \text{ Mg/m}^3$ for studying the effect of f_v , and three ρ_{d-f}
86 $= 1.82, 1.67$ and 1.52 Mg/m^3 were adopted at the same $f_v = 0\%$ for studying the effect of ρ_{d-f} .
87 The filter paper method was applied to measure the matric suction of soil mixture. The
88 microstructure of fine soil under varying ρ_{d-f} values was determined by mercury intrusion
89 porosimetry tests. The results obtained allowed the effects of f_v and ρ_{d-f} on soil-water retention
90 property of soil mixture to be clarified.

91

92 MATERIALS AND METHODS

93 *Fine soil and coarse grains*

94 Considering the difficulty of extracting intact interlayer soil from the field, the reconstituted
95 fine soil and coarse grains were fabricated in the laboratory. For the fine soil fraction, to
96 obtain a similar grain size distribution of fines from ‘Senissiat site’ (Trinh 2011) (Fig. 1), nine
97 different mass proportions of commercial soils were mixed (Table 1; see details in Lamas-
98 Lopez 2016). The liquid limit and plasticity index of the reconstituted fine soil were 32% and
99 20%, respectively. Consequently, a good agreement between in-situ fine soil and reconstituted
100 fine soil was observed in terms of grain size distribution, plasticity index and liquid limit (see
101 details in Wang et al. 2017). Fig. 2 presents the standard Proctor compaction curve of the
102 reconstituted fine soil, defining an optimum water content $w_{\text{opt-f}} = 13.7\%$ and a maximum dry
103 density $\rho_{\text{dmax-f}} = 1.82 \text{ Mg/m}^3$. Note that the $\rho_{\text{dmax-f}} = 1.82 \text{ Mg/m}^3$ of reconstituted fine soil was
104 consistent with the $\rho_{d-f} = 1.80 \text{ Mg/m}^3$ of in-situ fine soil measured by Lamas-Lopez (2016).

105 Based on above features, the reconstituted fine soil was considered as representative of the in-
 106 situ fine soil. For the coarse grains fraction, the micro-ballast was adopted to replace the real
 107 ballast by following a parallel gradation method adopted by Wang et al. (2017) and Su et al.
 108 (2020a). The validity of this method was verified by Qi et al. (2020b), who performed
 109 comparisons between micro-ballast and ballast in terms of mechanical behavior under static
 110 and cyclic loadings. Note that the scaled fine/coarse soil mixture was used as representative of
 111 interlayer soil taken from Senissiat (near Lyon, France, Trinh 2011), which was far from the
 112 coastal. Thus, the salt content-related osmotic suction was ignored and only the matric suction
 113 was taken into account in this study.

114 Parameter f_v , widely adopted in previous studies (Seif El Dine et al. 2010; Wang et al.
 115 2017, 2018; Su et al. 2020a, 2020b), was considered in this study. Based on the definition of f_v
 116 (Eq.(1)), which only quantified the amount of dry coarse grains in the mixture, all voids and
 117 water were assumed to be contained in the fine soil (Fig. 3), as in Wang et al. (2018) and Su et
 118 al. (2020a). Based on this assumption, the degree of saturation S_r of mixture can be related to
 119 the fine soil fraction. :

$$120 \quad f_v = \frac{V_c}{V} = \frac{V_c}{V_c + V_f} = \frac{V_c}{V_c + V_{s-f} + V_{w-f} + V_{a-f}} \quad (1)$$

121 where V_f is the volume of fine soil; V_{s-f} , V_{w-f} and V_{a-f} are the volume of fine grains, water and
 122 air in the fine soil.

123 For the soil mixture at varying f_v , ρ_{d-f} and water content w_f of fine soil, the mass of coarse
 124 grains m_{s-c} , fine grains m_{s-f} and water content of fine soil m_{w-f} could be determined as follows:

$$125 \quad m_{s-c} = V_c \cdot G_{s-c} \cdot \rho_w = f_v \cdot V \cdot G_{s-c} \cdot \rho_w \quad (2)$$

$$126 \quad m_{s-f} = \rho_{d-f} \cdot V_f = \rho_{d-f} \cdot V \cdot (1 - f_v) \quad (3)$$

127
$$m_{w-f} = w_f \cdot m_{s-f} \quad (4)$$

128 where G_{s-c} is the specific gravity of coarse grains; ρ_w is the water unit mass.

129 For the soil mixture at a given ρ_{d-f} , the corresponding void ratio of fine soil e_f could be
 130 deduced using Eq. (5):

131
$$e_f = \frac{G_{s-f} \rho_w}{\rho_{d-f}} - 1 \quad (5)$$

132 where G_{s-f} is the specific gravity of fine soil.

133 Then, the void ratio e_m of soil mixture can be determined:

134
$$e_m = \frac{V_{v-f}}{V_c + V_{s-f}} = \frac{V_{v-f}}{V - V_{v-f}} \quad (6)$$

135
$$V_{v-f} = e_f \cdot V_{s-f} \quad (7)$$

136
$$V_{s-f} = \frac{m_{s-f}}{G_{s-f} \rho_w} \quad (8)$$

137 where V_{v-f} is the volume of voids in fine soil; V_{s-f} is the volume of fine grains.

138 Combining Eqs. (1), (3) and (5)-(8), the void ratio e_m of soil mixture at varying f_v and ρ_{d-f}
 139 was obtained:

140
$$e_m = -1 + \frac{1}{1 - \frac{(G_{s-f} \rho_w - \rho_{d-f}) \cdot (1 - f_v)}{G_{s-f} \rho_w}} \quad (9)$$

141 Fig. 4 shows the variations of e_f and e_m with f_v at a constant $\rho_{d-f} = 1.82 \text{ Mg/m}^3$. The e_f and
 142 e_m were obtained by substituting $\rho_{d-f} = 1.82 \text{ Mg/m}^3$ and $G_{s-f} = 2.68$ (Duong et al. 2016) into
 143 Eqs. (5) and (9), respectively:

144
$$e_f = 0.47 \quad (10)$$

145
$$e_m = -1 + \frac{1}{1 - 0.32 \cdot (1 - f_v)} \quad (11)$$

146 As mentioned before, all voids and water were assumed to be contained in the fine soil.

147 Thus, the S_r represented both degree of saturation of soil mixture and that of fine soil:

148
$$S_r = \frac{V_{w-f}}{V_{v-f}} \quad (12)$$

149 *Filter paper method*

150 The filter paper method (ASTM D5298-10, 2010) was used to measure the matric suction.
151 Filter paper measurement was generally performed by putting a piece of filter paper between
152 two soil disks to attain suction equilibrium between filter paper and soil disks (ASTM D5298-
153 10 2010; Muñoz-Castelblanco et al. 2010; Kim et al. 2015; Jing 2017). Considering the
154 maximum diameter $d = 20$ mm of coarse grains (Fig. 1), the soil disk was prepared at 100 mm
155 diameter and 100 mm height. As for the preparation of soil disks, the fine soil (Fig. 5(a)) was
156 prepared at a molding water content $w_{\text{opt-f}} = 13.7\%$, then stored in a container for at least 24 h,
157 allowing water homogenization. After that, the fine soil was thoroughly mixed with the coarse
158 grains (Fig. 5(a)) with their pre-determined masses at target f_v and $\rho_{\text{d-f}}$ values (Table 2). Note
159 that a characteristic coarse grain content $f_{\text{v-cha}} \approx 27\%$ identified by Wang et al. (2018)
160 separated the coarse grain skeleton fabric ($f_v > f_{\text{v-cha}}$) and the fine matrix macrostructure ($f_v <$
161 $f_{\text{v-cha}}$) of mixture. Both fabrics were considered in this study with f_v ranging from 0% to 35%.
162 The fine/coarse soil mixture were then compacted in four layers, with an equal amount of fine
163 soil and coarse grains for each layer (Fig. 5(b)). Table 2 presents the variations of e_f , S_r and the
164 dry density of soil mixture ρ_d with f_v and $\rho_{\text{d-f}}$ at a molding water content of fine $w_{\text{opt-f}} = 13.7\%$.
165 After compaction, the soil disks at different f_v and $\rho_{\text{d-f}}$ values were wetted from the molding
166 states to the saturated state. The approach proposed by Su et al. (2020c) was adopted during
167 the wetting process: 10 g water was distributed uniformly on the surface of soil disk by a
168 sprayer each time. The disk was then covered with plastic film for at least 7 h. Fig. 2 shows
169 that the wetting process of soil disk from a molding state to a nearly saturated state induced a
170 slight decrease of $\rho_{\text{d-f}}$ due to the swelling of fine soil, and consequently a slight increase of e_f

171 and slight decrease of ρ_d (Table 2). The measured w_f and the corresponding S_r of soil disks at
172 nearly saturated state were also presented in Table 2.

173 To obtain the drying soil-water retention curve of soil mixture at a given f_v and ρ_{d-f} , 10
174 suction measurements were conducted, corresponding to 10 target w_f values. When a soil disk
175 reached a target w_f value, it was covered with plastic film for at least 24 h prior to measuring
176 its matric suction. A set of three filter paper was prepared, with the middle filter paper
177 (diameter $d = 80\text{mm}$) slightly smaller than the two outer filter paper ($d = 90\text{mm}$) to avoid
178 contamination of the middle one. The set of three filter papers was then placed between two
179 soil disks. The whole set was covered with plastic film, and then sealed with wax. Note that
180 an initial water content of 4.61% of the filter paper was measured, corresponding to a suction
181 of 93 MPa. In this case, the filter paper followed a wetting process during the equilibration
182 process between soil and filter paper. After equilibration, the water content of soil disks and
183 the middle filter paper were measured, with a balance of 1/10000 g accuracy. The
184 corresponding matric suction was then determined for varying f_v , ρ_{d-f} and w_f values. It is worth
185 noting that the volume of soil disks at different w_f values was also measured, by means of a
186 caliper.

187

188 *Mercury intrusion porosimetry test (MIP)*

189 For the MIP tests, three soil disks were prepared at varying $\rho_{d-f} = 1.82, 1.67$ and 1.52 Mg/m^3
190 with the same $f_v = 0\%$. The freeze-drying method was adopted: fine soil was cut into small
191 pieces of around 1.5 g each, and then immersed into nitrogen under vacuum; afterwards, the
192 frozen soil was transferred to the chamber of a freeze dryer for lyophilizing. This method

193 minimized the microstructure disturbance of fine soil, which was widely used in previous
194 studies (Cui et al. 2002; Delage et al. 2006; Wang et al. 2014).

195

196 RESULTS

197 To determine the equilibration time of samples with the filter paper method, three samples
198 were prepared at the same $f_v = 0\%$, $\rho_{d-f} = 1.82 \text{ Mg/m}^3$ and $w_{\text{opt-f}} = 13.7\%$, with 5, 7 and 9 days
199 waited, respectively. Fig. 6 depicts the variations of matric suction and water content of filter
200 paper with time. It appears that with the increase of time from 0 to 5, 7 and 9 days, the water
201 content of filter paper increased from 4.61% to 30.59%, 32.02% and 32.11%, and the
202 corresponding matric suction decreased from 93 MPa to 879, 679 and 670 kPa, respectively.
203 This indicated that at least a time of 7 days was needed for the suction equilibration between
204 soil and filter paper. Thereby, a duration of 7 days was adopted for all tests.

205 Fig. 7 shows the drying SWRC expressed in terms of w_f and S_r with ψ and the variations
206 of e_f with ψ for different f_v values (0%, 20% and 35%) and different initial ρ_{d-f} values (1.82,
207 1.67 and 1.52 Mg/m^3). The retention curves were fitted with the van Genuchten model (1980).
208 The legends were defined following the rule: ' f_v 0%- ρ_{d-f} 1.82' refers to the $f_v = 0\%$ and the
209 initial $\rho_{d-f} = 1.82 \text{ Mg/m}^3$. Fig. 7(a) depicts the variations of w_f with ψ for various f_v and ρ_{d-f}
210 values. It appears clearly that the water retention curves were only dependent on the dry
211 density of fines ρ_{d-f} , and independent of the coarse grain content f_v . In addition, the gaps
212 between the three curves for different ρ_{d-f} values decreased with the increase of matric suction
213 ψ . The curves converged to the same one beyond a threshold suction $\psi = 715 \text{ kPa}$. Thus, the
214 SWRC in low suction range ($\psi < 715 \text{ kPa}$) was sensitive to the variation of ρ_{d-f} , while

215 independent of ρ_{d-f} in high suction range ($\psi > 715$ kPa).

216 Fig. 7(b) depicts the variations of void ratio of fine soil e_f with ψ for the fine/coarse soil
217 mixture at varying f_v and ρ_{d-f} values. It appears that such variations were also only dependent
218 on ρ_{d-f} and independent of f_v . In addition, the lower the ρ_{d-f} value, the larger the decrease of e_f
219 with increasing ψ , showing a larger volume change under the effect of suction for the case of
220 lower ρ_{d-f} .

221 Fig.7(c) presents the variations of degree of saturation S_r with ψ for the fine/coarse soil
222 mixture at varying f_v and ρ_{d-f} values. The curves were found to be independent of f_v , which
223 agreed with those in Fig.7 (a)-(b). In addition, the larger the ρ_{d-f} value the higher the water
224 retention capacity. With the decrease of ρ_{d-f} from 1.82 to 1.67 and 1.52 Mg/m³, the air entry
225 value (AEV) decreased from 550 to 96 and 36 kPa respectively.

226 Fig. 8 shows the pore size distributions (PSD) of fine soil at $\rho_{d-f} = 1.82, 1.67$ and 1.52
227 Mg/m³. It can be observed from Fig. 8(a) that a decrease of ρ_{d-f} resulted in an increase of
228 intruded mercury void ratio e_M , which was a little smaller than the corresponding global e_f .
229 Fig. 8(b) presents typical bi-modal porosity of fine soil, with a delimiting diameter $d = 0.65$
230 μm for micro- and macro-pores. For compacted soils, the micro-pores were generally within
231 aggregates (intra-aggregate pores), while the macro-pores were between aggregates (inter-
232 aggregate pores) (Delage et al. 1996). With the increase of ρ_{d-f} , the volume of inter-aggregate
233 pores was observed to decrease, while that of intra-aggregate pores was almost constant,
234 suggesting that the compaction process only affected the macro-pores, in agreement with
235 Wang et al. (2014).

236

237 DISCUSSIONS

238 *Effect of microstructure of fine soil on SWRC at varying ρ_{d-f}*

239 Fig. 7(a) shows that ρ_{d-f} affected the SWRC only for $\psi < 715$ kPa. Correspondingly, Fig. 8(b)
240 indicates that a decrease of ρ_{d-f} led to an increase of the volume of inter-aggregate pores ($d >$
241 $0.65 \mu\text{m}$) without modifying the volume of intra-aggregate pores ($d < 0.65 \mu\text{m}$). The
242 delimiting $d = 0.65 \mu\text{m}$ could be associated with an equivalent matric suction based on
243 Laplace's law under the assumption of cylindrical pore shape:

244
$$\Psi = \frac{4T_s \cdot \cos \theta}{d} \quad (13)$$

245 where T_s is the surface tension of water, equal to 0.073 N/m at temperature of $20 \text{ }^\circ\text{C}$; θ is the
246 contact angle between the liquid-air interface and the solid, taken equal to 0° in this study.

247 Substituting $d = 0.65 \mu\text{m}$ into Eq. (13), the corresponding ψ was obtained:

248
$$\psi = 449 \text{ kPa} \quad (14)$$

249 It was found that this value was close to the AEV of 550 kPa for the mixture at $\rho_{d-f} = 1.82$
250 Mg/m^3 (Fig. 7(c)), which was consistent with the findings of Zhang et al. (2018). This value
251 was smaller than the threshold $\psi = 715 \text{ kPa}$ in Fig. 7(a). This difference could be explained as
252 follows: the fine soil with varying ρ_{d-f} values for MIP tests was compacted at a molding water
253 content $w_{\text{opt-f}} = 13.7\%$, while the fine soil corresponding to the threshold point in Fig. 7(a) was
254 subjected to a saturation process from the molding water content $w_{\text{opt-f}} = 13.7\%$, followed by a
255 drying process. Li and Zhang (2009) studied the effect of wetting-drying history on bi-modal
256 porosity of soil, and found that the drying process induced shrinkage of soil, leading to
257 smaller intra-aggregate pores. Similarly, Sun and Cui (2020) investigated the soil-water
258 retention curve of reconstituted silt, and reported that the drying process led to a shrinkage of

259 soil and a smaller value of diameter of voids. It could be thus inferred that the larger threshold
260 $\psi = 715$ kPa in Fig. 7(a) was the consequence of pore size decrease due to soil shrinkage
261 shown in Fig. 8(b). It could be thus deduced that in low suction range ($\psi < 715$ kPa) the
262 SWRC was governed by inter-aggregate pores, while in high suction range ($\psi > 715$ kPa) the
263 SWRC was governed by intra-aggregate pores. The similar phenomenon was reported by
264 Salager et al. (2013) while studying the water retention property of clayey soil. They
265 identified a threshold suction $\psi = 5000$ kPa separating the inter-aggregate governing suction
266 from intra-aggregate governing suction. With the decrease of ρ_{d-f} , the volume of inter-
267 aggregate pores increased (Fig. 8(b)), resulting in an increase of water volume in inter-
268 aggregate pores; thereby, an increase of w_f was observed (Fig. 7(a)). However, such decrease
269 of ρ_{d-f} did not affect the volume of intra-aggregate pores (Fig. 8(b)); thus, a constant w_f was
270 observed in high suction range (Fig. 7(a)). This also confirmed that the compaction effort
271 greatly affected the inter-aggregate pores without touching the intra-aggregate pores (Delage
272 et al. 1996).

273 Two categories of fine soil in the mixture at $f_v = 35\%$ were reported by Su et al. (2020c,
274 2021): a relatively dense fine soil in between coarse grains and a relatively loose fine soil in
275 macro-pores among coarse grains. In other words, while compacted to $\rho_{d-f} = 1.82$ Mg/m³, the
276 dense fines had a higher ρ_{d-f} and the loose fines had a lower ρ_{d-f} , with the global ρ_{d-f} being 1.82
277 Mg/m³. Thereby, a higher SWRC was expected for the dense fine soil in between coarse
278 grains and a lower SWRC for the loose fine soil in macro-pores among coarse grains.
279 However, as the SWRC at $f_v = 35\%$ was the same as that at $f_v = 0\%$ and 20% for $\rho_{d-f} = 1.82$
280 Mg/m³ (Fig. 7(c)), it could be inferred that in spite of the inhomogeneous distribution of fine

281 soil in the mixture, the SWRC appeared to be controlled by the global dry density of fine soil
282 ρ_{d-f} only. Similarly, Zeng et al. (2020) studied the axial swelling property of compacted
283 bentonite/claystone mixture, and found that this behavior was mainly dependent on the global
284 ρ_d of mixture, irrespective of its heterogeneity.

285

286 *Comparison of present study at constant ρ_{d-f} with previous study at constant ρ_d*

287 In present study, the different $f_v = 0\%$, 20% and 35% corresponded to the same SWRC under
288 the constant $\rho_{d-f} = 1.82 \text{ Mg/m}^3$ (Fig. 7(c)). On the contrary, Duong et al. (2014) investigated
289 the hydraulic behavior of the upper part interlayer soil with two different $f_v = 50.3\%$ and
290 55.5% and a constant dry density of mixture $\rho_d = 2.01 \text{ Mg/m}^3$ by infiltration column (Table 3).
291 Fig. 9 shows that an increase of f_v from 50.3% to 55.5% led to a lower SWRC under the
292 constant $\rho_d = 2.01 \text{ Mg/m}^3$. This phenomenon could be attributed to the effect of ρ_{d-f} on SWRC.
293 Fig. 10 shows a constant $e_m = 0.33$ (corresponding to $\rho_d = 2.01 \text{ Mg/m}^3$) for $f_v = 50.3\%$ and
294 55.5% in Duong et al. (2014), which was different from that in Fig. 4 of present study. While
295 increasing f_v from 50.3% to 55.5%, the e_f was increased from 1.01 to 1.28; thereby, a decrease
296 of ρ_{d-f} from 1.33 to 1.17 Mg/m^3 (Table 3). As a result, the increase of f_v from 50.3% to 55.5%
297 led to a lower SWRC (Fig. 9).

298

299 *The maximum degrees of saturation S_r of samples*

300 It appears from Fig. 7(c) and Fig. 9 that the SWRCs all started at a degree of saturation S_r
301 lower than 100%. Moreover, the values in Fig. 9 ($S_r = 91\%$ and 78% for $f_v = 50.3\%$ and 55.5%
302 respectively) were smaller than the values around $S_r = 95\%$ for varying $f_v = 0\%$, 20% and 35%

303 at $\rho_{d-f} = 1.82 \text{ Mg/m}^3$ in Fig. 7(c), showing that the maximum S_r value reached during the
304 saturation process decreased with the increase of f_v . Saba et al. (2014) worked on a
305 compacted sand/bentonite mixture and found that the fine grains of larger sizes (e.g. the
306 maximum $d = 2 \text{ mm}$) was preferentially arranged with a large face side in the horizontal
307 direction. In addition, due to the restriction of mould wall, more macro-pores were formed in
308 the side part of sample, as evidenced by Zeng et al. (2020) on a compacted
309 bentonite/claystone mixture. Such preferential presence of macro-pores in the side part was
310 further confirmed on the mixture of fine soil and micro-ballast by Wang et al. (2018) and Qi et
311 al. (2020a) through observation at X-ray μCT . As these macro-pores could not retain water
312 under the effect of gravity, a maximum value of $S_r = 95\%$ was obtained. With the increase of f_v ,
313 the addition of coarse grains increased the mould wall restriction effect, generating thus more
314 macro-pores. This led to a decrease of the initial S_r value of the SWRC with f_v . It is worth
315 noting that Duong et al. (2014) adopted the ballast grains of maximum $d = 60 \text{ mm}$ to prepare
316 the sample of $d = 300 \text{ mm}$ and $h = 600 \text{ mm}$, while in this study the micro-ballast of maximum
317 $d = 20 \text{ mm}$ (Fig. 1) was adopted to prepare soil disk of $d = 100 \text{ mm}$ and $h = 100 \text{ mm}$. Much
318 larger mould wall restriction effect was thus expected in the case of Duong et al. (2014) due to
319 the larger dimensions of the grains and the sample. Consequently, much lower maximum
320 degrees of saturation ($S_r = 91\%$ and 78% for $f_v = 50.3\%$ and 55.5%) were obtained in their
321 case.

322

323 CONCLUSIONS

324 To investigate the role of fine soil on the soil-water retention property of fine/coarse soil

325 mixture, three $f_v = 0\%$, 20% and 35% at the same $\rho_{d-f} = 1.82 \text{ Mg/m}^3$ and three $\rho_{d-f} = 1.82, 1.67$
326 and 1.52 Mg/m^3 at the same $f_v = 0\%$ were considered. A filter paper method was applied to
327 measure the matric suction of soil mixture at different water contents. Mercury intrusion
328 porosimetry tests were performed for microstructure observation of fine soil at varying ρ_{d-f} .
329 The results obtained allowed the following conclusions to be drawn.

330 The drying SWRC of mixture was found to be only dependent on ρ_{d-f} and independent of
331 f_v . A typical bi-modal microstructure of fine soil was identified for varying ρ_{d-f} values,
332 defining a micro-pore and a macro-pore populations. When expressed in terms of w_f with ψ ,
333 the SWRC was found to be significantly affected by ρ_{d-f} for the matric suction ψ lower than
334 715 kPa. By contrast, when ψ was higher than 715 kPa, the SWRC kept the same,
335 independent of ρ_{d-f} . Interestingly, a delimiting pore diameter $d = 0.65 \text{ }\mu\text{m}$ was identified,
336 separating micro-pores (or intra-aggregate pores) from macro-pores (or inter-aggregate pores).
337 This delimiting diameter corresponded to a matric suction of 449 kPa, which was smaller than
338 the threshold $\psi = 715 \text{ kPa}$ due to the effect of volume change experienced in the course of soil
339 wetting/drying from the remolded state ($w_{\text{opt-f}} = 13.7\%$). Thus, the SWRC in low suction range
340 ($\psi < 715 \text{ kPa}$) was governed by macro-pores, while in high suction range ($\psi > 715 \text{ kPa}$) by
341 micro-pores. In addition, the SWRC of mixture appeared to be controlled by the global fine
342 soil dry density ρ_{d-f} only. The effect of the possible heterogeneity of fine soil distribution
343 inside the mixture seemed to be negligible.

344 When expressed in terms of S_r with ψ , the SWRC appeared to be significantly affected by
345 ρ_{d-f} , while unaffected by f_v . This helped better understand the observation of Duong et al.
346 (2014) – the water retention capacity was decreased by the increase of f_v : in the study of

347 Duong et al. (2014), a constant $\rho_d = 2.01 \text{ Mg/m}^3$ was adopted instead of a constant ρ_{d-f} . In this
348 case, an increase of f_v resulted in a decrease of ρ_{d-f} , thereby a decrease of water retention
349 capacity.

350 The initial S_r of the SWRC appeared to decrease with the increase of f_v . At $f_v = 0\%$, due to
351 the effect of mould wall restriction, macro-pores were formed in the side part of sample. As
352 these macro-pores could not retain water under the effect of gravity, the maximum value of S_r
353 = 95% smaller than 100% was obtained in the saturation process. With the increase of f_v , the
354 addition of coarse grains increased the mould wall restriction effect, generating more macro-
355 pores and thus lower maximum S_r . This phenomenon was expected to be more pronounced in
356 the case of larger grains and larger sample dimensions.

357 From a practical point of view, these findings suggest that when investigating the water-
358 retention property of interlayer soil, the scaled coarse grains at small size can be used as a
359 substitute for real ballast grains at large size. With the increasing depth of interlayer soil, the
360 decrease of f_v induced no changes on water-retention capacity of interlayer soil, provided that
361 the ρ_{d-f} of fine soil kept constant. When the water retention property is determined, it can be
362 incorporated in the mechanical models to better describe the variation of permanent strain of
363 unsaturated interlayer soil under the effect of cyclic loadings. In addition, it is worth noting
364 that to some extent these findings can be helpful in evaluating the effects of ρ_{d-f} and f_v on the
365 water retention property of mixtures with varying types of coarse and fine soil.

366

367 ACKNOWLEDGEMENTS

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369 ParisTech.

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371

372 NOTATIONS

e	void ratio
e_f	void ratio of fine soil
e_m	void ratio of fine/coarse soil mixture
f_v	coarse grain content
$f_{v\text{-cha}}$	characteristic coarse grain content
$G_{s\text{-c}}$	specific gravity of coarse grains
$m_{a\text{-f}}$	mass of air in fine soil
$m_{w\text{-f}}$	mass of water in fine soil
$m_{s\text{-f}}$	mass of fine grains
$m_{s\text{-c}}$	mass of coarse grains
ρ_d	dry density of sample
$\rho_{d\text{-f}}$	dry density of fine soil
$\rho_{d\text{max-f}}$	maximum dry density of fine soil
ρ_w	water unit mass
S_r	degree of saturation
V	volume of fine/coarse soil mixture
V_f	volume of fine soil
$V_{v\text{-f}}$	volume of voids in fine soil
$V_{a\text{-f}}$	volume of air in fine soil
$V_{w\text{-f}}$	volume of water in fine soil
$V_{s\text{-f}}$	volume of fine grains
V_c	volume of coarse grains
$w_{\text{opt-f}}$	optimum water content of fine soil

w_f water content of fine soil

ψ matric suction

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Table 1. Constitution of fine soil

Soil classification	Commercial Soil	Mass proportion (%)	Range of grain size (mm)
Sand	C4	16.7	0.0009 – 0.50
	C10	20	0.0009 – 0.25
	HN34	3.3	0.063 - 0.50
	HN31	3.3	0.16 - 0.63
	HN0.4-0.8	6.7	0.25 - 1
	HN0.6-1.6	6.7	0.32 - 2
	HN1-2.5	13.3	0.32 – 3.20
Clay	Speswhite	23.3	0.0003 – 0.01
	Bentonite	6.7	0.001 – 0.01

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Table 2. As-compacted and saturated states of fine/coarse soil mixture

f_v (%)	As-compacted state					Saturated state				
	ρ_{d-f} (Mg/m ³)	e_f	w_{opt-f} (%)	S_r (%)	ρ_d (Mg/m ³)	ρ_{d-f} (Mg/m ³)	e_f	Measured w_f (%)	Measured S_r (%)	ρ_d (Mg/m ³)
	1.82	0.47		78	1.82	1.81	0.48	17.2	95	1.81
0	1.67	0.60		61	1.67	1.65	0.62	22.0	95	1.65
	1.52	0.76	13.7	48	1.52	1.50	0.78	27.3	93	1.50
20	1.82	0.47		78	1.99	1.80	0.49	17.3	95	1.97
35	1.82	0.47		78	2.12	1.80	0.48	17.2	95	2.10

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Note: f_v represents the volumetric ratio of coarse grains to the fine/coarse soil mixture. ρ_{d-f} , e_f , w_f , and w_{opt-f} represent the dry density, void ratio, water content and optimum water content of fine soil, respectively. S_r represents the degree of saturation of fine soil, which is also the degree of saturation of the mixture. ρ_d represents the dry density of soil mixture.

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Table 3. Soil properties of Duong et al. (2014)

f_v (%)	ρ_d (Mg/m ³)	e_m	ρ_{d-f} (Mg/m ³)	e_f
50.3	2.01	0.33	1.33	1.01
55.5			1.17	1.28

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Note: e_m represents the void ratio of soil mixture.

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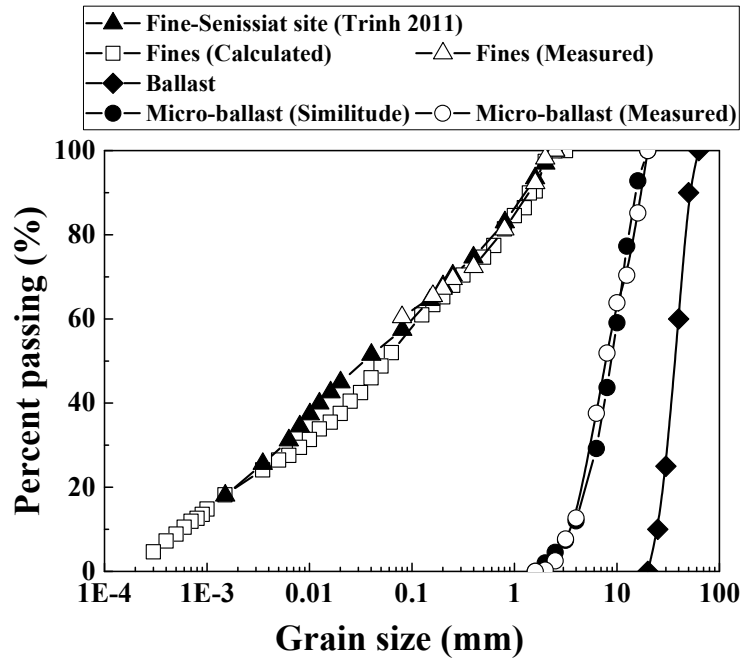
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Fig. 1. Grain size distribution curves of fine soil and micro-ballast (after Wang et al. 2017)

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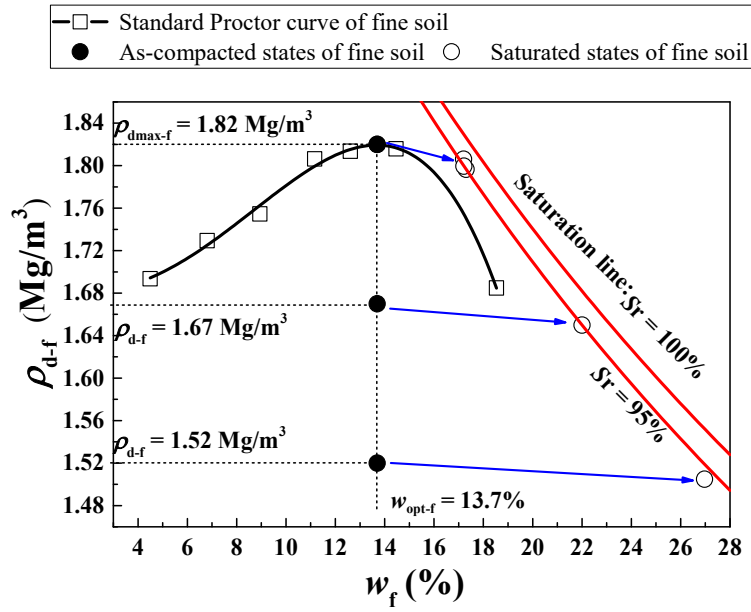
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537 Fig. 2. As-compacted and saturated states of fine soil with respect to its standard Proctor
538 compaction curve

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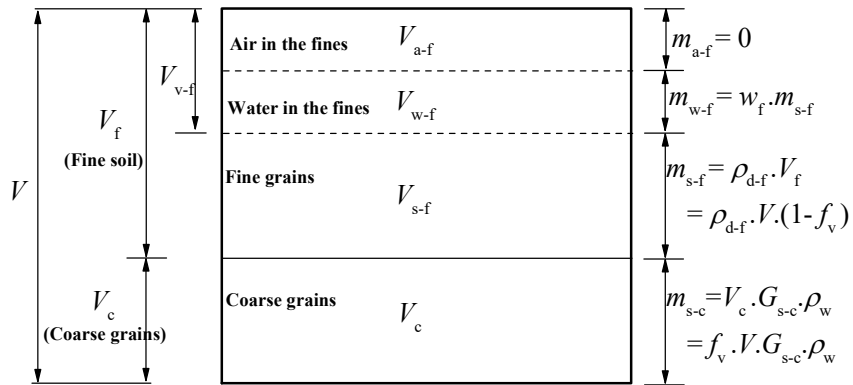
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Fig. 3. Constitution of fine/coarse soil mixture

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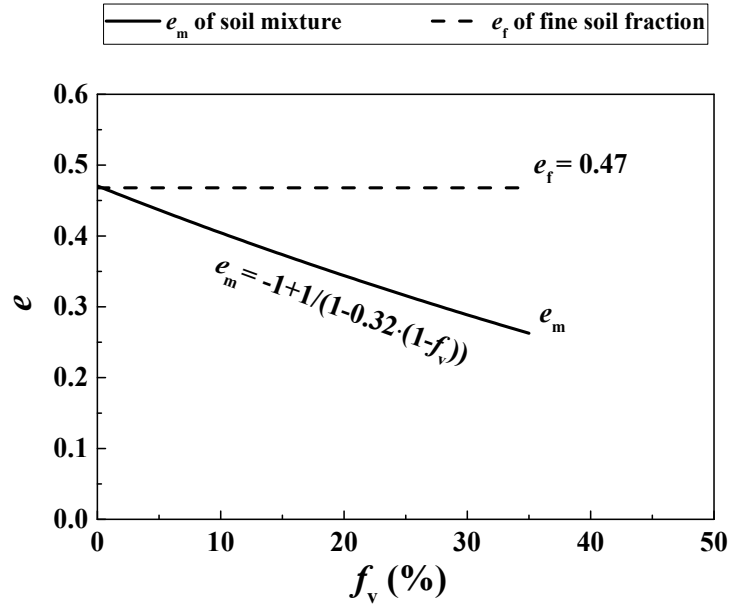
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576 Fig. 4. Variations of void ratios with f_v at a constant $\rho_{d-f} = 1.82 \text{ Mg/m}^3$

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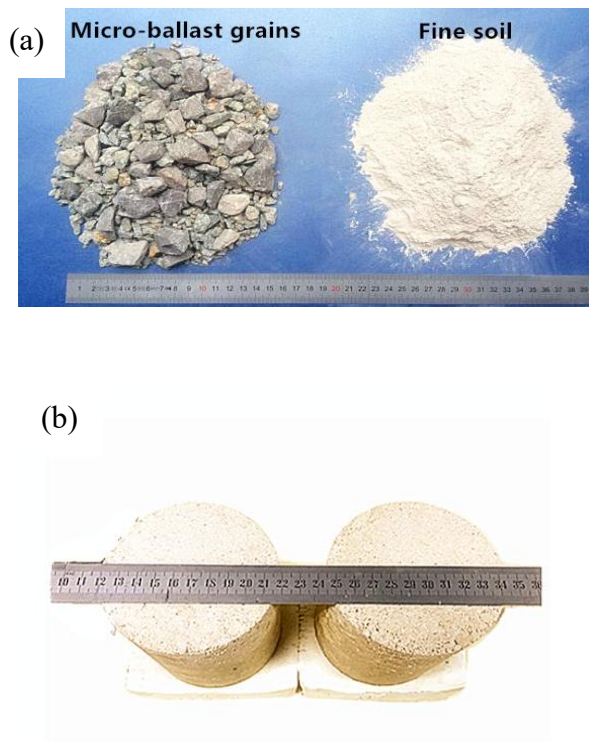
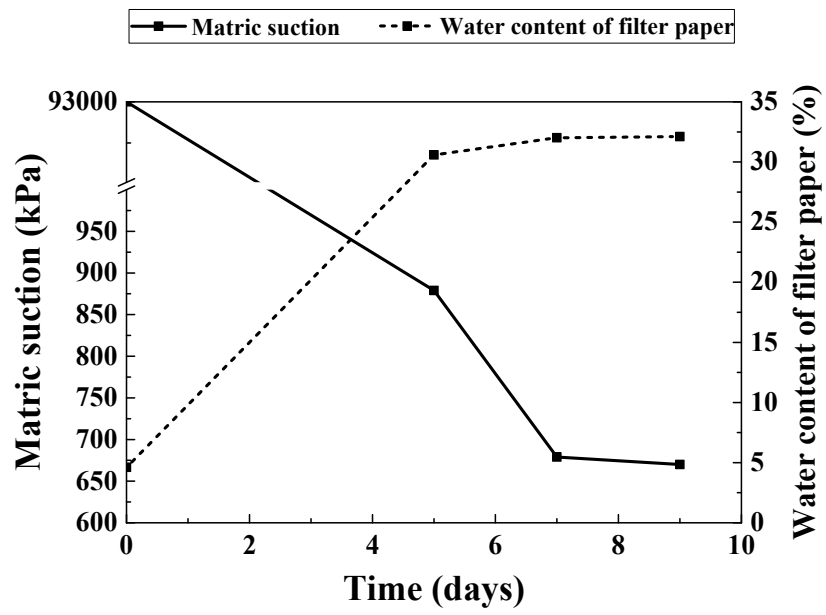


Fig. 5. (a) Preparation of fine soil and micro-ballast and (b) compaction of two soil disks



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Fig. 6. Determination of equilibration time by filter paper method

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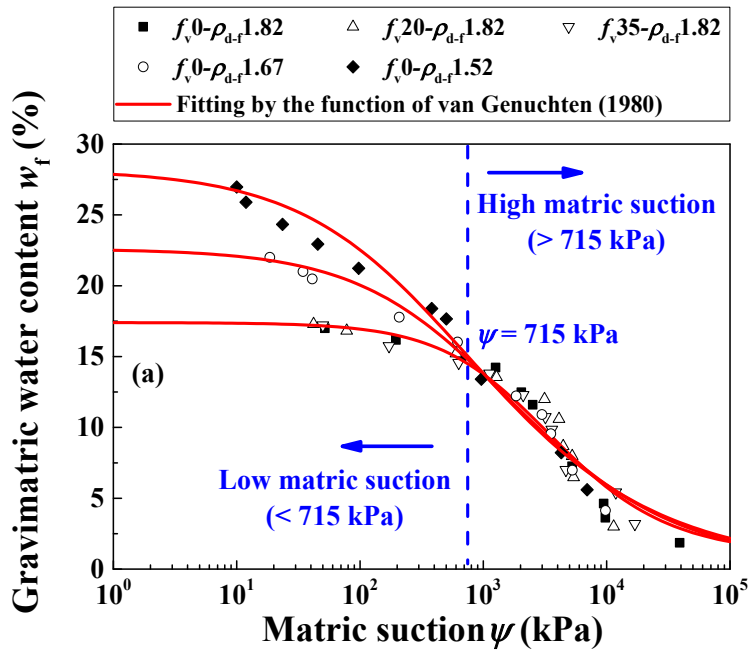
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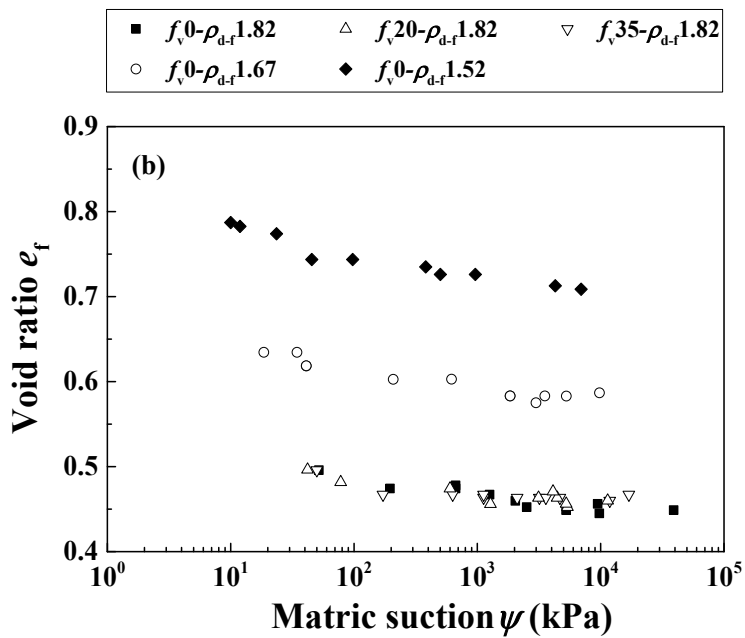
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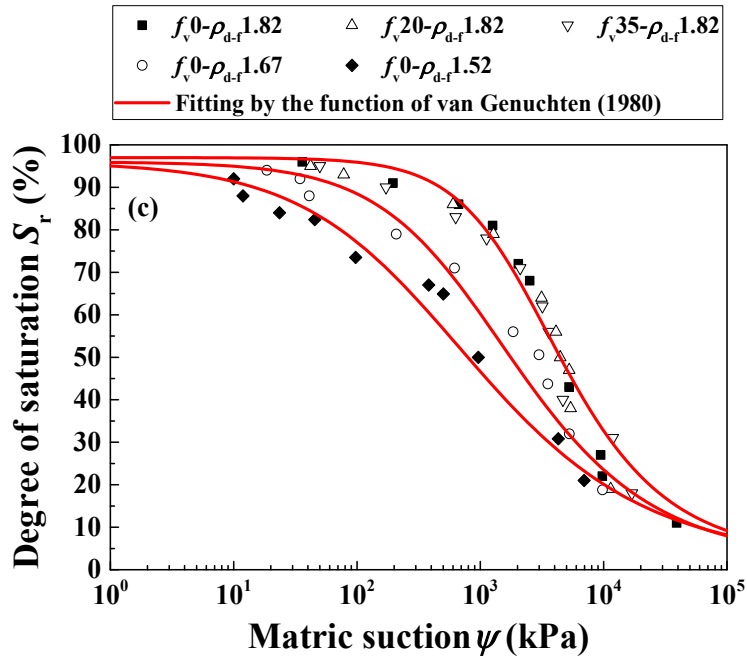


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643 Fig. 7. Drying soil-water retention curves and variations of void ratios of fine soil with matric
 644 suction for varying f_v and ρ_{d-f} values: (a) gravimetric water content of fine soil versus matric
 645 suction; (b) void ratio of fine soil versus matric suction; (c) degree of saturation versus matric
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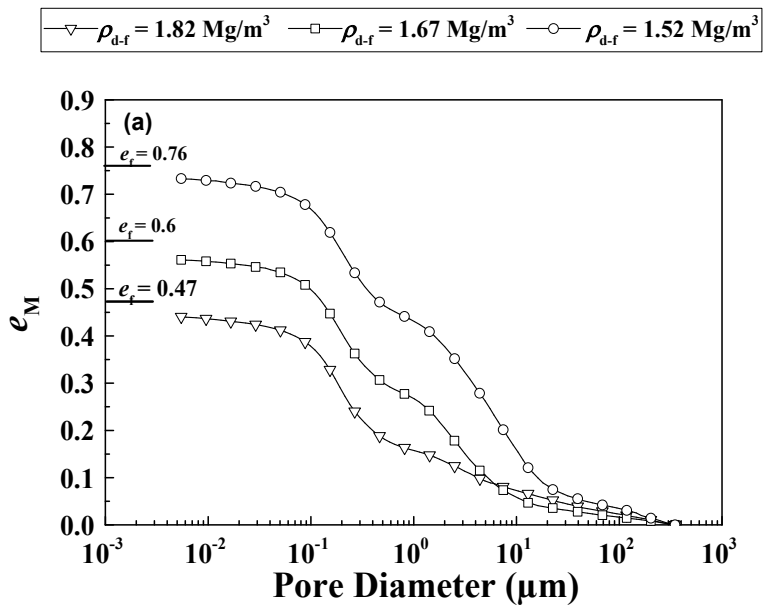
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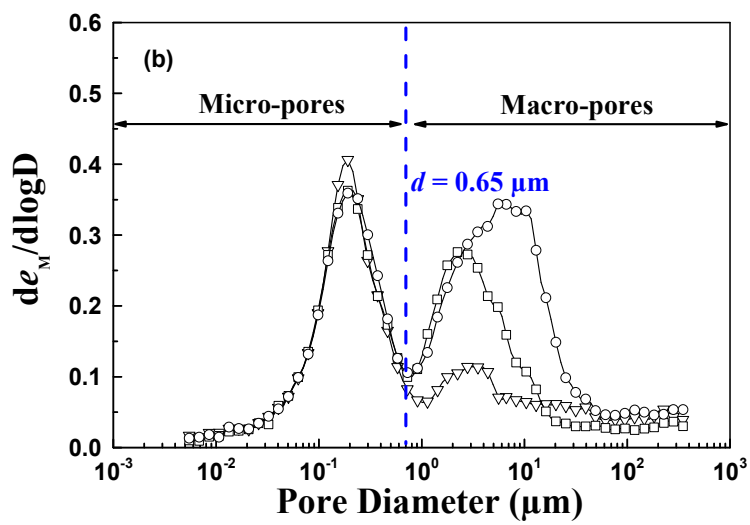
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662 Fig. 8. Pore size distributions of fine soil at $f_v = 0\%$ and different ρ_{d-f} values: (a) cumulative

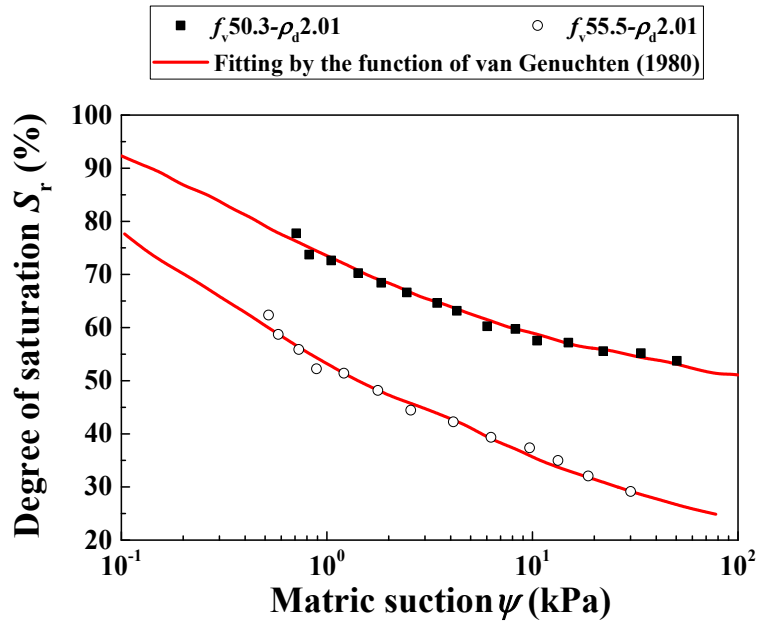
663 density function curves; (b) density function curves

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Fig. 9. Drying soil-water retention curves in the study of Duong et al. (2014)

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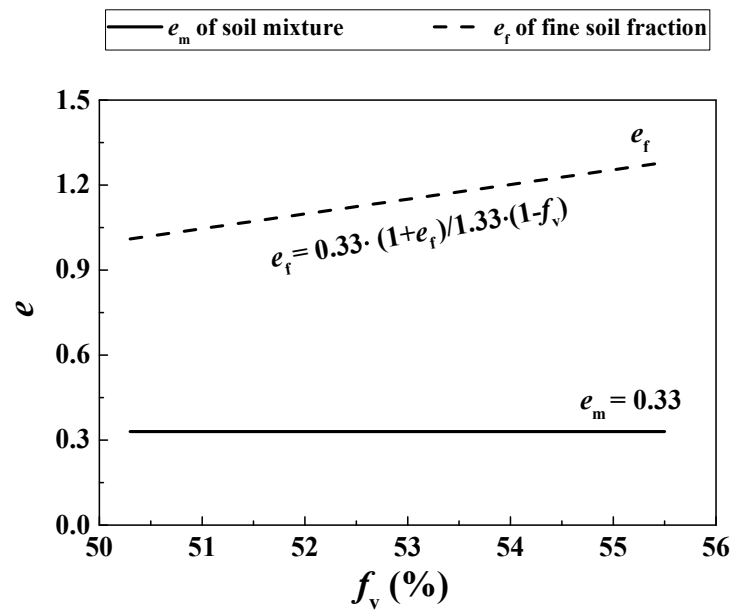
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Fig. 10. Variations of void ratio with f_v at $\rho_d = 2.01 \text{ Mg/m}^3$ in Duong et al. (2014)

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