



HAL
open science

Modelling the suction-and deviator stress-dependent resilient modulus of unsaturated fine/coarse soil mixture by considering soil-water retention curve

Yu Su, Yu-Jun Cui

► **To cite this version:**

Yu Su, Yu-Jun Cui. Modelling the suction-and deviator stress-dependent resilient modulus of unsaturated fine/coarse soil mixture by considering soil-water retention curve. *Transportation Geotechnics*, 2022, 32, pp.100698. 10.1016/j.trgeo.2021.100698 . hal-04155856

HAL Id: hal-04155856

<https://enpc.hal.science/hal-04155856>

Submitted on 7 Jul 2023

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

1 Modelling the suction- and deviator stress-dependent resilient modulus of
2 unsaturated fine/coarse soil mixture by considering soil-water retention curve

3
4 Yu Su^{1,2}, Yu-Jun Cui², Jean-Claude Dupla², Jean Canou²

5
6 1: School of Civil Engineering and Architecture, Nanchang University, Nanchang 330031,
7 China

8 2: Laboratoire Navier/CERMES, Ecole des Ponts ParisTech (ENPC), France

9
10
11
12
13
14
15
16 **Corresponding author**

17 Yu SU

18 1. School of Civil Engineering and Architecture, Nanchang University, Nanchang 330031, China

19 2. Ecole des Ponts ParisTech, Laboratoire Navier/CERMES, 6 – 8 av. Blaise Pascal, Cité Descartes,
20 Champs-sur-Marne, 77455 Marne – la – Vallée cedex 2, France

21 E-mail address: yu.su@enpc.fr

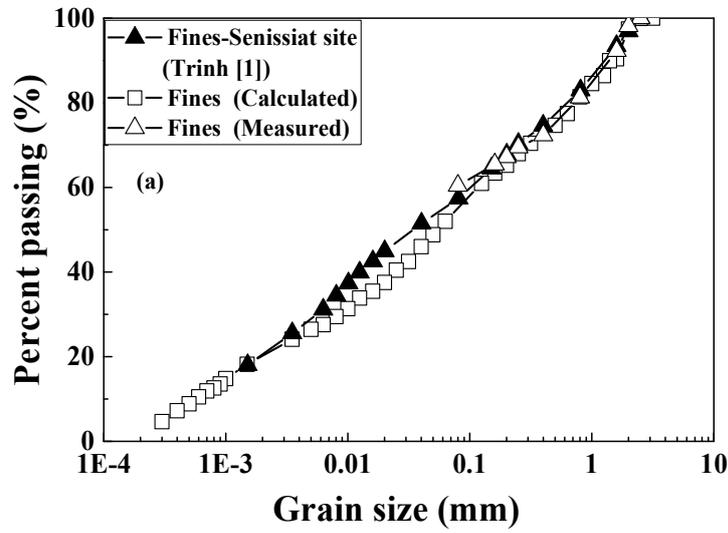
22 **Abstract**

23 Experimental observations have shown that the resilient modulus M_r of fine/coarse soil
24 mixture can be significantly affected by the coarse grain content f_v , deviator stress σ_d and
25 suction ψ . In this study, a constitutive model incorporating the soil-water retention curve
26 (SWRC) was proposed to describe the effects of ψ and σ_d on M_r . This model was then
27 extended to the effect of f_v . The proposed model implied the resilient modulus at saturation
28 condition (M_{r-sat}), the resilient modulus at optimum moisture content (OMC) condition (M_{r-opt}),
29 the suction at OMC (ψ_{opt}) and the parameters related to SWRC. The model was validated
30 using experimental data from five studies reported in literature. Comparisons with three
31 representative existing models showed that the proposed model was capable to well describe
32 the suction-dependent effect of deviator stress in the full range of suction, while the existing
33 models gave satisfactory simulation results only in the low suction range. Indeed,
34 experimental studies revealed that there was a threshold suction ψ_{th} , and with increasing σ_d ,
35 the M_r decreased when $\psi < \psi_{th}$, but increased when $\psi > \psi_{th}$. When $\psi < \psi_{th}$, all models gave
36 good simulations. On the contrary, when $\psi > \psi_{th}$, only the proposed model gave good
37 simulations, in particular when $\psi_{th} > \psi_{opt}$. This showed the performance of the proposed
38 model in describing the variation of resilient modulus of unsaturated fine/coarse soil mixtures
39 with changes in coarse grain content, deviator stress and suction.

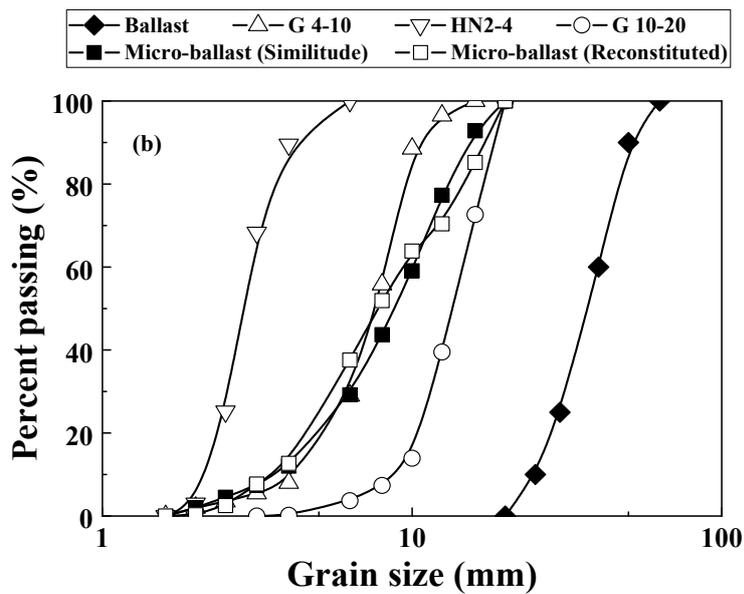
40 **Keywords:** resilient modulus; constitutive model; suction; deviator stress; coarse grain
41 content; soil-water retention curve

42 INTRODUCTION

43 An interlayer soil was naturally created in the French conventional rail tracks, corresponding
44 to a mixture of ballast grains and subgrade fine soil. The in-situ investigation showed a
45 decrease of ballast grain content over the depth of interlayer soil (Trinh [1]). The resilient
46 modulus M_r , defined as the ratio of cyclic deviator stress to resilient strain, was adopted to
47 characterize the stiffness of interlayer soil (Nie et al. [2]). Wang et al. [3 - 5] and Qi et al. [6]
48 studied the effects of coarse grain content f_v (the ratio of the volume of micro-ballast grains to
49 that of mixture) and deviator stress σ_d on M_r of interlayer soil by cyclic triaxial tests under
50 constant suction ψ . Figs. 1(a)-(b) show the grain size distribution curves of fine soil and
51 micro-ballast, among which the micro-ballast was fabricated by a mixture of three coarse
52 grains G 4-10, HN 2-4 and G 10-20 (see more details in Wang et al. [3]). The effect of ψ on
53 the M_r of interlayer soil was further investigated by Su et al. [7] through multi-stage deviator
54 stresses cyclic tests. Those experimental results indicated that the M_r of such fine/coarse soil
55 mixture was significantly affected by coarse grain content f_v , suction ψ and deviator stress σ_d .
56 From a practical point of view, it appears important to develop a constitutive model of M_r for
57 the fine/coarse soil mixture, taking the combined effects of f_v , ψ and σ_d into consideration.



58



59

Fig. 1. Grain size distribution curves of (a) fine soil and (b) micro-ballast (after Wang et al. [3])

60 The effects of f_v , ψ and σ_d on M_r were addressed in numerous experimental studies and
 61 different models were proposed for that. Wang et al. [3] and Cui et al. [8] studied the effect of
 62 f_v on the M_r of fine/coarse soil mixture, and defined a characteristic coarse grain content $f_{v\text{-cha}}$

63 separating two fabric kinds for the mixtures: a fine matrix macrostructure at $f_v \leq f_{v\text{-cha}}$, and a
64 coarse grain skeleton fabric at $f_v \geq f_{v\text{-cha}}$. They found that M_r increased slowly with increasing f_v
65 at $f_v \leq f_{v\text{-cha}}$, while quickly at $f_v \geq f_{v\text{-cha}}$. No constitutive model has been developed for describing
66 this phenomenon.

67 The effects of ψ and σ_d on M_r were generally investigated by multi-stage loadings cyclic
68 triaxial tests (Gupta et al. [9]; Nowamooz et al. [10]). Gu et al. [11] performed a series of
69 large-scale cyclic triaxial tests on unbound granular materials, and found that the increase in
70 both ψ and σ_d led to an increase of M_r . Ng et al. [12] studied the effects of ψ and σ_d on the M_r
71 of unsaturated subgrade soil by suction-controlled cyclic triaxial tests. The results showed that
72 in a narrow range of ψ from 0 to 250 kPa, an increase of ψ induced an increase of M_r , and an
73 increase of σ_d led to a reduction of M_r at a constant ψ . Yang et al. [13] performed suction-
74 controlled tests on residual mudstone soil with various deviator stresses σ_d . They reported that
75 with increasing σ_d , the M_r decreased at low suctions ($\psi = 50$ and 150 kPa), while increased at
76 a high suction ($\psi = 450$ kPa). Similarly, Su et al. [7] studied the effect of w on the M_r of
77 fine/coarse soil mixture through multi-stage loadings cyclic triaxial tests. They found that an
78 increase of w led to a decrease of M_r of soil mixture, due to the effect of ψ . Moreover, in the
79 case of low ψ (smaller than or equal to ψ_{opt} at optimum moisture content OMC), the increase
80 of σ_d resulted in a decrease of M_r , while in the case of high ψ (larger than ψ_{opt}), an opposite
81 trend was observed. Han and Vanapalli [14] investigated the effect of ψ on the M_r of
82 unsaturated subgrade soil, and proposed a constitutive model incorporating the soil-water
83 retention curve (SWRC). Oh et al. [15] and Han and Vanapalli [16 - 17] also consider SWRC

84 while modeling the effect of ψ on M_r . To date, there are no models for the description of M_r
85 variation with changes in ψ and σ_d for unsaturated fine/coarse soil mixtures.

86 In this study, a constitutive model of M_r was proposed for unsaturated fine/coarse soil
87 mixtures, accounting for the effects of ψ , σ_d , and f_v . Experimental data collected from
88 literature including the authors' own data were used to validate the model. Comparisons
89 between the proposed model and three representative existing models showed that the
90 proposed model is capable to well describe the variations of M_r with changes in σ_d and f_v in the
91 full range of ψ .

92

93 MODELLING BACKGROUND

94 Some investigators proposed empirical models to simulate the $M_r - \psi$ relationship. For
95 instance, Sawangsuriya et al. [18] studied the effect of ψ on M_r with four compacted subgrade
96 soils, and proposed Eqs. (1) - (2) by considering two reference M_r values at saturation
97 condition and OMC condition- M_{r-sat} and M_{r-opt} , respectively:

$$98 \quad M_r/M_{r-sat} = -5.61 + 4.54 \log(\psi) \quad (1)$$

$$99 \quad M_r/M_{r-opt} = -0.24 + 0.25 \log(\psi) \quad (2)$$

100 Ba et al. [19] proposed Eq. (3) to estimate the variations of M_r of four compacted granular
101 materials with respect to ψ using a resilient modulus ratio of M_r to the M_{r-opt} :

$$102 \quad M_r/M_{r-opt} = 0.385 + 0.267 \log(\psi) \quad (3)$$

103 These empirical models were simple and their parameters could be easily determined by
 104 regression analysis. However, since they were generally derived from limited experimental
 105 data, it appears difficult to be generalized to other materials.

106 The Mechanistic-Empirical Pavement Design Guide (MEPDG) (ARA, Inc., ERES
 107 Consultants Division. [20]) recommended the commonly used Eq. (4) to describe the variation
 108 of M_r with stress:

$$109 \quad M_r = k_1 p_a \left(\frac{\theta_b}{p_a} \right)^{k_2} \left(\frac{\tau_{\text{oct}}}{p_a} + 1 \right)^{k_3} \quad (4)$$

110 where θ_b is the bulk stress, equal to the sum of the three principal stresses σ_1, σ_2 and σ_3 ; τ_{oct} is
 111 the octahedral shear stress, equal to $\sqrt{2}/3(\sigma_1 - \sigma_3)$ in triaxial condition; p_a is the atmospheric
 112 pressure; k_1, k_2 and k_3 are model parameters.

113 Liang et al. [21] studied the effects of stress state and suction on the M_r of cohesive soil.
 114 They proposed Eq. (5) by incorporating ψ into the bulk stress of Eq. (4) using the Bishop's
 115 effective stress parameter χ :

$$116 \quad M_r = k_4 p_a \left(\frac{\theta_b + \chi \psi}{p_a} \right)^{k_5} \left(\frac{\tau_{\text{oct}}}{p_a} + 1 \right)^{k_6} \quad (5)$$

117 where k_4, k_5 and k_6 are model parameters.

118 Similarly, Heath et al. [22] investigated the M_r of unsaturated granular materials, and
 119 developed Eq. (6) by modifying Eq. (4) using the Bishop's effective stress parameter χ :

$$120 \quad M_r = k_7 p_a \left(\frac{\frac{\theta_b}{3} - u_a + \chi \psi}{p_a} \right)^{k_8} \left(\frac{\sigma_d}{p_a} \right)^{k_9} \quad (6)$$

121 where k_7, k_8 and k_9 are model parameters; u_a is the pore air pressure.

122 Gupta et al. [9] and Khoury et al. [23] modified Eq. (4) by adding ψ as an independent
 123 term, such as $A\psi^B$ in Eqs. (7) and (8) for subgrade soils:

$$124 \quad M_r = k_{10}p_a \left(\frac{\theta_b - 3k_{11}}{p_a} \right)^{k_{12}} \left(\frac{\tau_{oct}}{p_a} + k_{13} \right)^{k_{14}} + A_1 \psi^{B_1} \quad (7)$$

$$125 \quad M_r = k_{15}p_a \left(\frac{\theta_b}{p_a} \right)^{k_{16}} \left(\frac{\tau_{oct}}{p_a} + k_{17} \right)^{k_{18}} + A_2 \psi^{B_2} \quad (8)$$

126 where k_{10} – k_{18} , A_1 , B_1 , A_2 and B_2 are model parameters.

127 The MEPDG (ARA, Inc., ERES Consultants Division. [20]) adopted Eq. (9) to predict
 128 the variation of M_r with respect to the seasonal variation of water content in the field condition:

$$129 \quad \log \left(\frac{M_r}{M_{r-opt}} \right) = a + \frac{b-a}{1 + \exp \left[\ln \frac{b}{a} + k_m \cdot (S_r - S_{r-opt}) \right]}$$

130 (9)

131 where S_r is the degree of saturation; M_{r-opt} and S_{r-opt} are the resilient modulus and the degree of
 132 saturation at OMC, respectively; a and b are the minimum and maximum values of log
 133 (M_r/M_{r-opt}), respectively; k_m is a regression parameter. For fine-grained soil, $a = -0.5934$, $b =$
 134 0.4 and $k_m = 6.1324$; for coarse-grained soil, $a = -0.3123$, $b = 0.3$ and $k_m = 6.8157$.

135 Han and Vanapalli [14] proposed Eq. (10) for compacted subgrade fine soils,
 136 incorporating SWRC:

$$137 \quad \frac{M_r - M_{r-sat}}{M_{r-opt} - M_{r-sat}} = \frac{\psi}{\psi_{opt}} \left(\frac{S_r}{S_{r-opt}} \right)^\xi \quad (10)$$

138 where ξ is the model parameter.

139 Summarizing, Table 1 presents a summary of model parameters for Eqs. (1) - (10).

140

141 Table 1. A summary of parameters for Eqs. (1) - (10)

Reference	Equation	Model parameters
Sawangsurriya et al. [18]	(1)	ψ and $M_{r\text{-sat}}$
	(2)	ψ and $M_{r\text{-opt}}$
Ba et al. [19]	(3)	ψ and $M_{r\text{-opt}}$
ARA, Inc., ERES Consultants Division. [20]	(4)	$\theta_b, \tau_{\text{oct}}, p_a, k_1, k_2$ and k_3
Liang et al. [21]	(5)	$\theta_b, \tau_{\text{oct}}, p_a, \chi, \psi, k_4, k_5$ and k_6
Heath et al. [22]	(6)	$\theta_b, p_a, \chi, \psi, u_a, \sigma_d, k_7, k_8$ and k_9
Gupta et al. [9]	(7)	$\theta_b, \tau_{\text{oct}}, p_a, \psi, k_{10}, k_{11}, k_{12}, k_{13}, A_1$ and B_1
Khoury et al. [23]	(8)	$\theta_b, \tau_{\text{oct}}, p_a, \psi, k_{15}, k_{16}, k_{17}, k_{18}, A_2$ and B_2
ARA, Inc., ERES Consultants Division. [20]	(9)	$S_r, S_{r\text{-opt}}, M_{r\text{-opt}}, a, b$ and k_m
Han and Vanapalli [14]	(10)	$S_r, S_{r\text{-opt}}, M_{r\text{-opt}}, M_{r\text{-sat}}$ and ξ

142

143

144 PROPOSITION OF A NEW MODEL

145 *Proposing a model accounting for the effects of ψ and σ_d*

146 Han and Vanapalli [17] reviewed the existing constitutive models of M_r with respect to ψ , and
 147 proposed a general form as follows:

$$148 \quad M_r = M_{r\text{-sat}} + f(\psi), \quad f(0) = 0 \quad (11)$$

149 where function $f(\psi)$ represents the contribution of ψ to M_r .

150 Referring to the existing models (e.g. Eq. (5) in Liang et al. [21] and Eq. (6) in Heath et al.
 151 [22]), a factor $\chi\psi$ was adopted to reflect the effect of ψ on the M_r of unsaturated soils. As
 152 stated by Han and Vanapalli [16], using factor $\chi\psi$ induced a change of the role of suction ψ
 153 from a pore-scale stress to a macroscopic stress which contributed to the constitutive stress
 154 and hence the M_r of unsaturated soils. In this study, factor $\chi\psi$ was modified by considering (i)

155 a power relationship of $M_r - \psi$ in Eqs. (7) - (8) (Gupta et al. [9] and Khoury et al. [23]) and (ii)
 156 a parameter χ equal to the effective degree of saturation S_r^e , which was defined as the ratio of
 157 $(S_r - S_{r-r})$ to $(1 - S_{r-r})$, where S_{r-r} is the residual degree of saturation (Alonso et al. [24]; Lu
 158 et al. [25]). Therefore, a new factor $\psi^B S_r^e$ was generated and Eq. (12) was obtained:

$$159 \quad M_r = M_{r-sat} + A \cdot \psi^B \cdot S_r^e \quad (12)$$

160 where A and B are model parameters.

161 Substituting M_{r-opt} and the corresponding ψ_{opt} and S_{r-opt}^e (the effective degree of
 162 saturation at OMC) into Eq. (12) yields Eq. (13):

$$163 \quad M_{r-opt} = M_{r-sat} + A \cdot \psi_{opt}^B \cdot S_{r-opt}^e \quad (13)$$

164 Dividing Eq. (12) by Eq. (13) leads to the normalized Eq. (14) where parameter A
 165 vanishes:

$$166 \quad \frac{M_r - M_{r-sat}}{M_{r-opt} - M_{r-sat}} = \left(\frac{\psi}{\psi_{opt}} \right)^B \cdot \frac{S_r^e}{S_{r-opt}^e} \quad (14)$$

167 Eq. (15) (Moossazadeh and Witczak [26]) was commonly used to characterize the effect
 168 of σ_d on M_r . Based on Eq. (15), Eq. (16) was proposed for relating σ_d to parameter B in Eq.
 169 (14):

$$170 \quad M_r = k_{19} \left(\frac{\sigma_d}{p_a} \right)^{k_{20}} \quad (15)$$

$$171 \quad B = l_1 \cdot \left(\frac{\sigma_d}{p_a} \right)^{l_2} \quad (16)$$

172 where k_{19} , k_{20} , l_1 and l_2 are model parameters.

173 The van Genuchten [27] model was adopted for describing the SWRC:

174
$$S_r^e = \frac{S_r - S_{r-r}}{1 - S_{r-r}} = \left[\frac{1}{1 + (a\psi)^n} \right]^m \quad (17)$$

175 where S_{r-r} is the residual degree of saturation, assumed to be 0 in this study; a , n and m are
 176 model parameters.

177 Substituting Eqs. (16) and (17) into Eq. (14), Eq. (18) was obtained, which allowed
 178 prediction of the variation of M_r under the combined effects of ψ and σ_d :

179
$$\frac{M_r - M_{r-sat}}{M_{r-opt} - M_{r-sat}} = \left(\frac{\psi}{\psi_{opt}} \right)^{l_1 \cdot \left(\frac{\sigma_d}{p_a} \right)^{l_2}} \cdot \left[\frac{1 + (a\psi_{opt})^n}{1 + (a\psi)^n} \right]^m \quad (18)$$

180 Note that when ψ represented the matric suction, ψ_{opt} was the matric suction at optimum water
 181 content (e.g. for soils 1-5 in Table 2 and soils 6-9 in Table 3). On the contrary, when ψ
 182 represented the total suction, ψ_{opt} was the total suction at optimum water content (e.g. for soils
 183 10-12 in Table 3). The matric suction was measured using the filter paper in contact with soil
 184 (for soils 1-8) or a thermal dissipation sensor (for soil 9), while the total suction was measured
 185 using a suction probe (for soil 10) or the filter paper without contact with soil (for soils 11-12).

186 Table 2 shows the properties of soils 1 - 5 tested by Wang et al. [3] and Su et al. [7], with
 187 f_v varying from 0% to 45%. Fig. 2 shows that the same SWRC was obtained for $f_v = 0\%$, 20%
 188 and 35% using the filter paper method (Su et al. [28]). This indicated that an increase of f_v led
 189 to a constant ψ under a given S_r when keeping the dry density of fine soil constant ($\rho_{dmax-f} =$
 190 1.82 Mg/m³ in Table 2), as expected by Wang et al. [3 - 5] and Qi et al. [6]. Fig. 3 shows the
 191 comparisons between the measurements by Wang et al. [3] and Su et al. [7] and the
 192 calculations by Eq. (18) for the variations of M_r with ψ under different deviator stresses σ_d
 193 and five f_v values. It can be observed from Fig. 3(a) that M_r increased with the increase of ψ
 194 under a constant σ_d , and a reasonably good agreement was obtained between the

195 measurements and the calculations. Further examination showed that a threshold suction ψ_{th}
 196 could be identified, corresponding to the intersection of the curves of different deviators
 197 stresses ($\sigma_d = 50, 100$ and 200 kPa). When $\psi < \psi_{th}$, the M_r decreased with increasing σ_d ,
 198 while an opposite trend was observed when $\psi > \psi_{th}$. In addition, the increase of σ_d led to a
 199 decrease of model parameter l_1 , keeping parameter l_2 constant (equal to 1.000). The similar
 200 phenomenon was observed in Figs. 3(b) - (e) for $f_v = 10\% - 45\%$. It seems that the higher the
 201 ψ_{th} , the higher the coefficient of determination R^2 for soils 1-5. Overall, a good agreement was
 202 obtained between measurements and calculations, with the $R^2 \geq 0.90$. Figs. 3(a) - (e) indicate
 203 that an increase of f_v from 0% to 45% resulted in an increase of M_{r-sat} from 11 to 85 MPa.

204

205

Table 2. Soil properties in Wang et al. [3] and Su et al. [7]

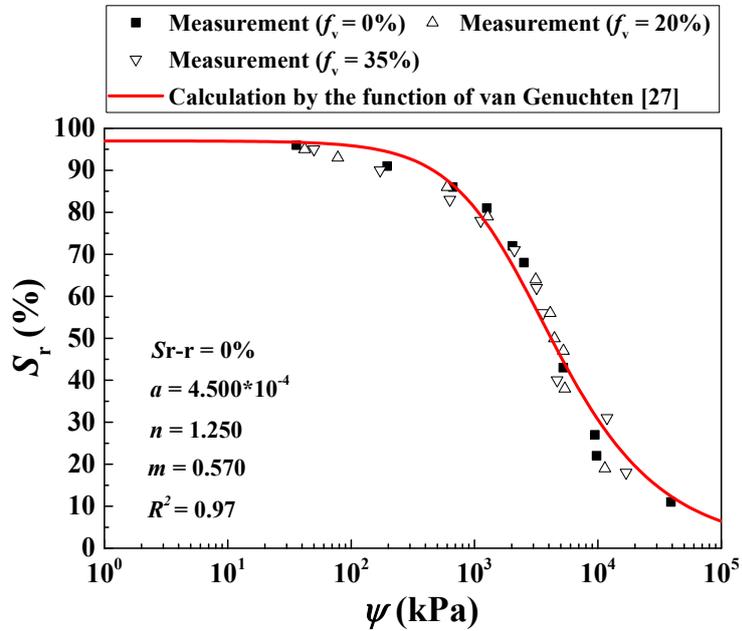
Soil No.	f_v (%)	G_s	Fine soil fraction						USCS classification	Soil mixture	
			w_L (%)	w_p (%)	I_P (%)	w_{opt-f} (%)	ρ_{dmax-f} (Mg/m ³)	S_{r-opt} (%)		Compaction w_{opt-f} (%)	ρ_d (Mg/m ³)
1	0										1.82
2	10										1.91
3	20	2.68	32	12	20	13.7	1.82	78	CL	13.7	1.99
4	35										2.12
5	45										2.21

206

207 Note: f_v represents the ratio of the volume of coarse grains to that of mixture (Su et al. [7]). G_s ,
 208 w_L , I_P , w_{opt-f} and ρ_{dmax-f} represent the specific gravity, liquid limit, plasticity index, optimum
 209 water content and maximum dry density of fine soil, respectively. w_{opt-f} and ρ_{dmax-f} were
 210 determined by standard Proctor compaction tests for soils 1-5. ρ_d represents the dry density of

211 soil mixture sample. USCS refers to the unified soil classification system: CL, low-plasticity
212 clay; CH, high-plasticity clay; MH, high-plasticity silt; ML, low-plasticity silt.

213



214

215

216 Fig. 2. Measured and calculated soil-water retention curves at varying f_v values for soils 1-5
217 (after Su et al. [28])

218

219

220

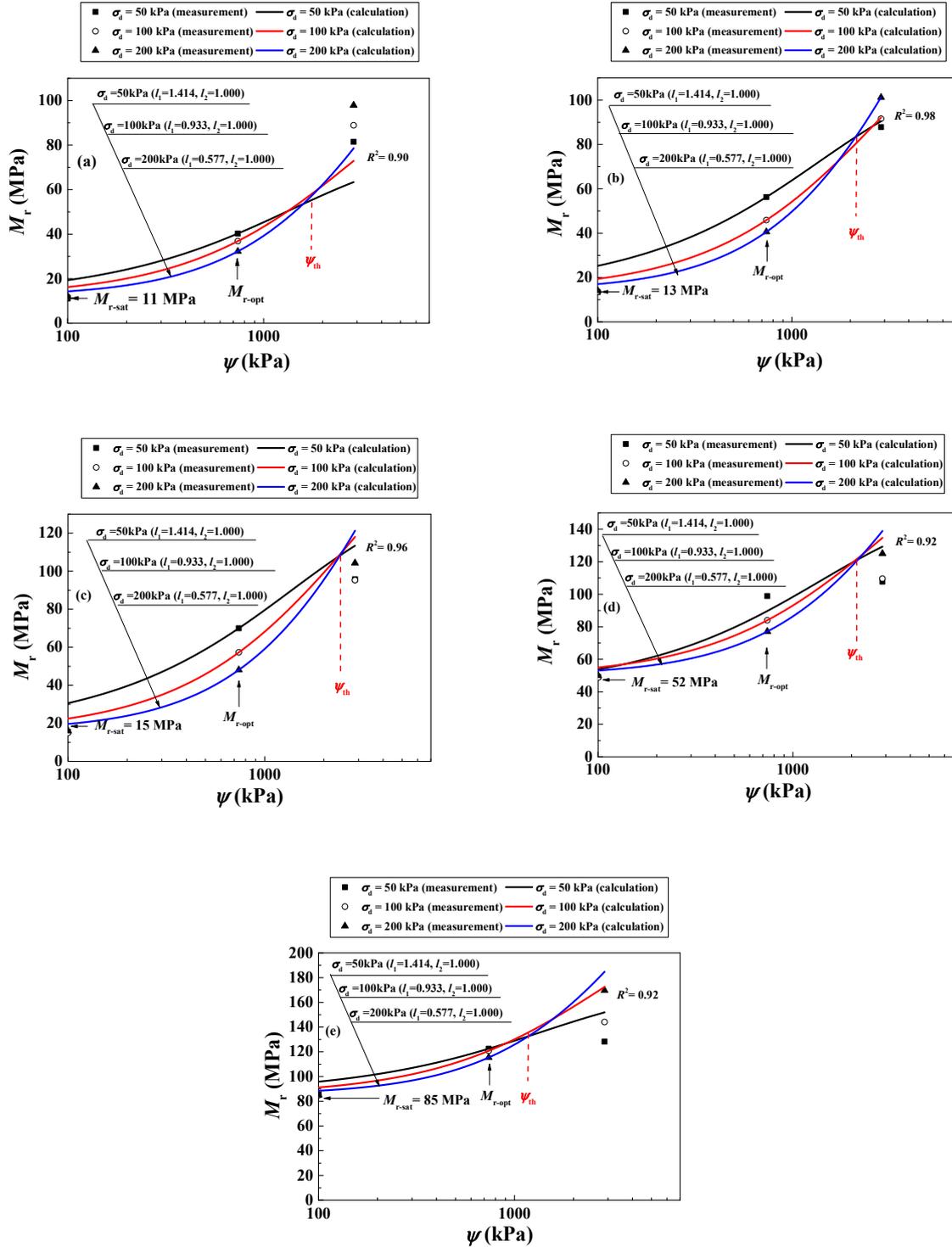
221

222

223

224

225



226

227 Fig. 3. Measured and calculated variations of M_r with ψ under varying σ_d for soils 1-5: (a) soil
 228 1 at $f_v = 0\%$; (b) soil 2 at $f_v = 10\%$; (c) soil 3 at $f_v = 20\%$; (d) soil 4 at $f_v = 35\%$; (e) soil 5 at f_v
 229 $= 45\%$ (data from Wang et al. [3] and Su et al. [7])

230

231 The $M_r - \sigma_d$ relationship depended on the combined effects of the soil hardening in the
232 loading process and the rebounding in the unloading process. When $\psi < \psi_{th}$, a matrix
233 structure of fine soil was expected due to the effect of water hydration (Su et al. [29]). On the
234 contrary, the high ψ induced an aggregated fine soil microstructure (Cui and Delage [30]; Ng
235 et al. [31]). Upon loading, an increase of σ_d contributed to the compression of fine matrix and
236 the rearrangement of fine aggregates (Werkmeister et al. [32]). Thus, an increase of M_r is
237 expected for both fabrics due to the hardening phenomenon. Conversely, in the unloading
238 process, owing to the rebounding effect, the resilient strain increased, which resulted in a
239 decrease of M_r . For the fine matrix fabric ($\psi < \psi_{th}$), due to its larger deformability, the effect
240 of rebounding on M_r appeared to be more significant than the effect of hardening, leading to a
241 decrease of M_r with increasing σ_d . By contrast, for the aggregated fabric ($\psi > \psi_{th}$), owing to
242 its lower deformability, the rebounding effect was not as significant as the hardening effect. In
243 this case, the M_r increased with increasing σ_d . This indicated that ψ_{th} could be considered as
244 the threshold value between the fine matrix fabric (at $\psi < \psi_{th}$) and the fine aggregate fabric (at
245 $\psi > \psi_{th}$). It appeared that ψ_{th} was slightly affected by the coarse grain content f_v . At $f_v = 0\% -$
246 20% , a fine matrix macrostructure was obtained for soils 1 - 3, while at $f_v = 35\% - 45\%$ the
247 coarse grains were dominant for soils 4 - 5. With increasing f_v , the transition of these two
248 fabrics contributed to a slight decrease of ψ_{th} , for that less fine soil was needed to be
249 transferred from the fine matrix fabric to the fine aggregate fabric.

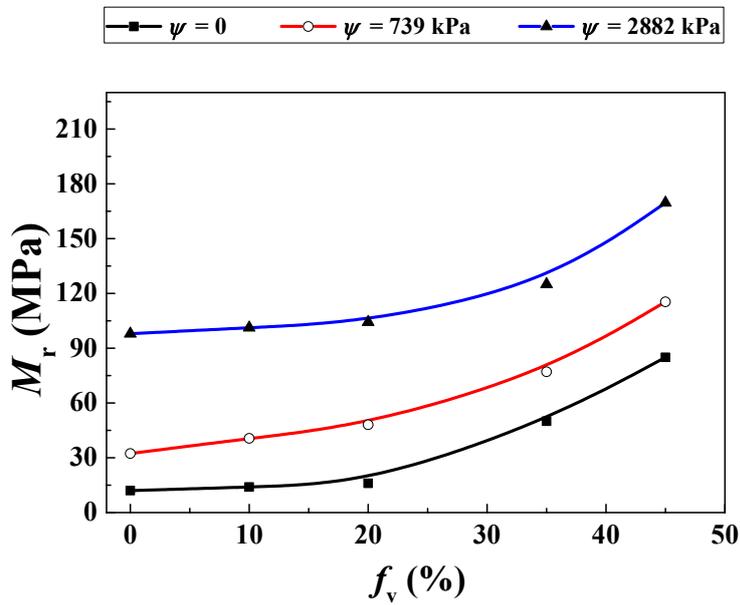
250 Fig. 4 shows the variations of M_r with f_v under varying ψ and a constant $\sigma_d = 200$ kPa for
251 soils 1-5. Fig. 5 shows a linear variation of parameter l_1 with $\log(\sigma_d/p_a)$, leading to Eq. (19):

252
$$l_1 = \alpha_1 \log\left(\frac{\sigma_d}{p_a}\right) + \beta_1 \quad (19)$$

253 where α_1 and β_1 are model parameters. Eq. (19) allows the determination of the two
 254 parameters ($\alpha_1 = -1.390$, $\beta_1 = 0.967$) with a regression coefficient $R^2 = 0.99$.

255 Substituting Eq. (19) into Eq. (18) yields Eq. (20):

256
$$\frac{M_r - M_{r-sat}}{M_{r-opt} - M_{r-sat}} = \left(\frac{\psi}{\psi_{opt}}\right)^{[\alpha_1 \log\left(\frac{\sigma_d}{p_a}\right) + \beta_1] \cdot \left(\frac{\sigma_d}{p_a}\right)} \cdot \left[\frac{1 + (\alpha\psi_{opt})^n}{1 + (\alpha\psi)^n}\right]^m \quad (20)$$



257
 258 Fig. 4 Variations of M_r with f_v under varying ψ and a constant $\sigma_d = 200$ kPa for soils 1-5 (data
 259 from Wang et al. [3] and Su et al. [7])

260

261 *Extending the model to the effect of f_v*

262 Fig. 6 shows the variation of M_{r-sat} with f_v , measured by Su et al. [7] and Duong et al. [33].

263 Note that the same fine soil fraction was adopted by Su et al. [7] and Duong et al. [33], while

264 the micro-ballast was adopted by Su et al. [7] as a substitute of ballast adopted by Duong et al.

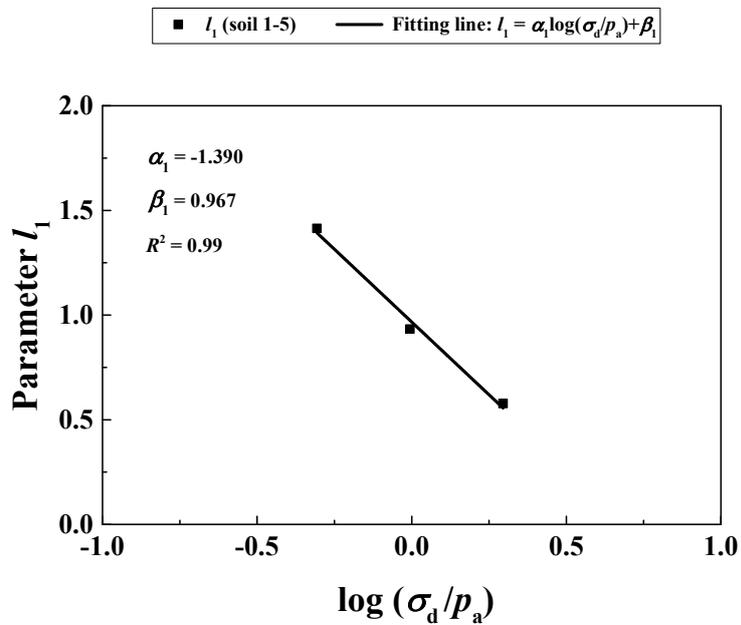
265 [33] following the parallel gradation method. The validity of this method was verified by Qi et
 266 al. [34]. Since an increase of f_v led to a constant ψ under a given S_r (Fig. 2), the term $f(\psi)$ was
 267 independent of f_v in Eq. (11). Similarly, the whole term on the right-hand side of Eq. (20) was
 268 also independent of f_v . The effect f_v was reflected on the term M_{r-sat} in Eqs. (11) and (20). It
 269 was found from Figs. 3(a) - (e) (Wang et al. [3] and Su et al. [7]) that an increase of f_v from
 270 0% to 45% resulted in an increase of M_{r-sat} from 11 to 85 MPa. This $M_{r-sat} - f_v$ relationship was
 271 expressed by Eq. (21):

$$272 \quad M_{r-sat} = M_0 + \frac{M_1 - M_0}{1 + e^{kf_v + l}} \quad (21)$$

273 where M_0 and M_1 are the values of M_{r-sat} at $f_v = 0\%$ and 100% , respectively; k and l are model
 274 parameters. Eq. (21) provides good simulations of $M_{r-sat} - f_v$ relationship measured by Duong
 275 et al. [33] and Su et al. [7] with $R^2 = 0.97$, using parameters $M_0 = 11$ MPa, $M_1 = 200$ MPa, $k =$
 276 -0.163 and $l = 7.514$.

277 It appears from Fig. 6 that the $M_{r-sat} - f_v$ curve could be divided into three zones with two
 278 critical f_v values: a fine matrix macrostructure zone at $f_v < f_{v1}$, a transition zone at $f_{v1} < f_v < f_{v2}$
 279 and a coarse grain skeleton zone at $f_v > f_{v2}$. Vallejo and Mawby [35] studied the stiffness and
 280 shear strength of sand and clay mixture, and found $f_{v1} \approx 26\%$ and $f_{v2} \approx 56\%$. Cui et al. [8],
 281 Wang et al. [4] and Su et al. [36] investigated the mechanical behavior of fine/coarse soil
 282 mixture subjected to monotonic and cyclic loadings, and defined a characteristic coarse grain
 283 content $f_{v-cha} \approx 25\% \sim 33\%$. They found that a fine matrix macrostructure was identified at $f_v \leq$
 284 f_{v-cha} . Obviously, the f_{v-cha} identified corresponded to f_{v1} .

285 Summarizing, Eqs. (20) - (21) allow the determination of the variation of M_r under the
286 combined effects of ψ , σ_d and f_v for the fine/coarse soil mixtures.
287

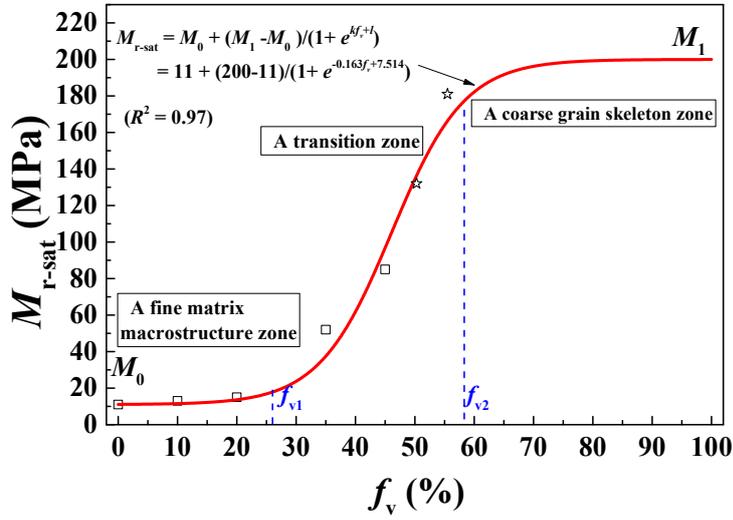
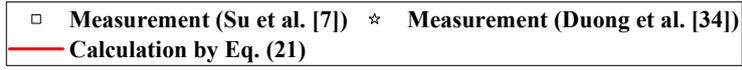


288

289 Fig. 5. Variation of parameter l_1 with $\log(\sigma_d/p_a)$ for soils 1-5

290

291



292

293

Fig. 6. Measured and calculated variation of M_{r-sat} with f_v

294

295 VALIDATION OF THE PROPOSED MODEL

296 Table 3 shows the properties of soils 6 - 12 compiled from five different studies. As $f_v = 0\%$ in

297 these studies, Eq. (21) was simplified to Eq. (22):

298
$$M_{r-sat} = M_0 \tag{22}$$

299 Soils 6 - 12 were divided into two categories. Category I included soils 6 - 8 (Zaman and

300 Khoury [37]), soil 9 (Gupta et al. [9]) and soil 10 (Ng et al. [12]), for which M_r decreased with

301 increasing σ_d under a given ψ . The ψ values considered were supposed to be smaller than

302 ψ_{th} . Category II included soils 11 - 12 (Yang et al. [13], [38]), for which a value of ψ_{th} existed

303 separating the ψ into two zones: with increasing σ_d , the M_r decreased at $\psi < \psi_{th}$ but increased

304 at $\psi > \psi_{th}$. It is worth noting that ψ_{th} appeared soil type – dependent: smaller than ψ_{opt} for

305 soils 11 - 12 (Yang et al. [13], [38]) and larger than ψ_{opt} for soils 1 - 5 (Wang et al. [3]; Su et
306 al. [7]). The plastic limit w_p can be helpful in explaining this phenomenon. With respect to the
307 findings of Delage et al. [39], w_p was the threshold value between the fine matrix fabric and
308 the fine aggregate fabric. For soils 1-5, the plastic limit $w_p = 12\%$ was smaller than $w_{opt-f} =$
309 13.7% (Table 2), leading to $\psi_{th} > \psi_{opt}$. On the contrary, for soils 11-12, the $w_p = 34\%$ and 22%
310 were larger than $w_{opt-f} = 18.1\%$ and 16.8% respectively (Table 3), leading to $\psi_{th} < \psi_{opt}$.

311

312

Table 3. Soil properties in five different studies

Reference	Soil No.	f_v (%)	Fine soil fraction							Soil mixture		
			G_s	w_L (%)	w_p (%)	I_P (%)	w_{opt-f} (%)	ρ_{dmax-f} (Mg/m ³)	S_{r-opt} (%)	USCS classification	Compaction w (%)	ρ_d (Mg/m ³)
Zaman and Khoury [37]	6										19.5	
	7		N/A	55	25	30	23.5	1.53	N/A	CH	23.5	1.53
	8										27.5	
Gupta et al. [9]	9	0	2.75	28	17	11	13.5	1.79	70	CL	13.5	1.79
Ng et al. [12]	10		2.73	43	29	14	16.3	1.76	80	ML	16.3	1.76
Yang et al. [38]	11		2.71	54	34	20	18.1	1.76	91	MH	18.1	1.76
Yang et al. [13]	12		2.67	37	22	15	16.8	1.77	88	CL	16.8	1.77

313

314 Note: $w_{\text{opt-f}}$ and $\rho_{\text{dmax-f}}$ were determined by standard Proctor compaction tests for soils 6-11,
 315 while modified Proctor compaction test for soils 12; N/A means data not available in the
 316 reference.

317 Zaman and Khoury [37] studied the effects of compaction water content and suction on
 318 M_r for Burlison soil (soils 6, 7 and 8 in Table 3). In their study, the soils were compacted at w
 319 = 19.5% (optimum water content of fine soil $w_{\text{opt-f}} - 4\%$), 23.5% ($w_{\text{opt-f}}$) and 27.5% ($w_{\text{opt-f}} + 4\%$)
 320 respectively and at the same maximum dry density of soil $\rho_{\text{dmax-f}} = 1.53 \text{ Mg/m}^3$. Cyclic triaxial
 321 tests were performed under a constant deviator stress $\sigma_d = 28 \text{ kPa}$ and a constant confining
 322 pressure $\sigma_c = 41 \text{ kPa}$. The filter paper method was adopted to measure the suction ψ after
 323 completion of the cyclic tests. Fig. 7(a) shows the measured and calculated SWRCs for soils 6,
 324 7 and 8 using Eq. (17) with the parameters shown in Table 4. The different SWRCs for soils 6,
 325 7 and 8 were the consequences of their varying compaction water contents, as reported by
 326 Delage et al. [39] and Vanapalli et al. [40]. Figs. 7(b) - (d) depict the variations of M_r with ψ
 327 under $\sigma_d = 28 \text{ kPa}$ for soils 6, 7 and 8 respectively. Eq. (20) calculated the variations of M_r
 328 with ψ for these three soils using parameter $\alpha_1 = 0$ and $\beta_1 = 2.891$. Comparisons between the
 329 measurements and calculations showed a good agreement ($R^2 = 0.95$).

330
 331

332 Table 4. Parameters of SWRCs for soils 1-12

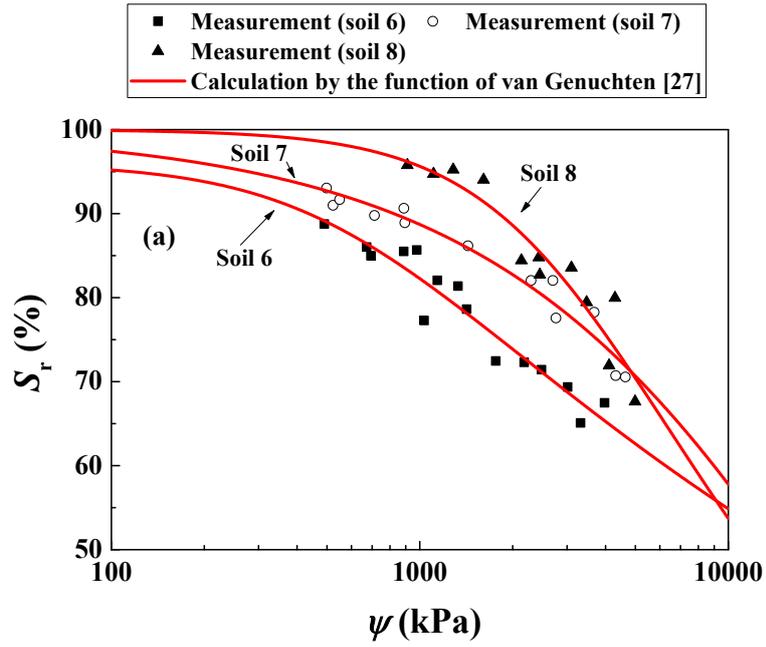
Soil No.	$S_{r-r}(\%)$	a	n	m	R^2	SWRC type
1						
2						
3	0	$4.500 \cdot 10^{-4}$	1.250	0.570	0.97	Drying SWRC
4						
5						
6						
		$1.770 \cdot 10^{-3}$	1.540	0.126	0.91	Apparent

7	6.500×10^{-8}	0.662	70.750	0.95	SWRC
8	3.200×10^{-4}	1.624	0.306	0.89	
9	1.656×10^{-4}	0.769	1.049	0.99	
10	1.120×10^{-6}	0.688	243.330	0.99	Drying SWRC
11	0.034	2.747	0.021	0.98	
12	0.139	0.779	0.197	0.98	

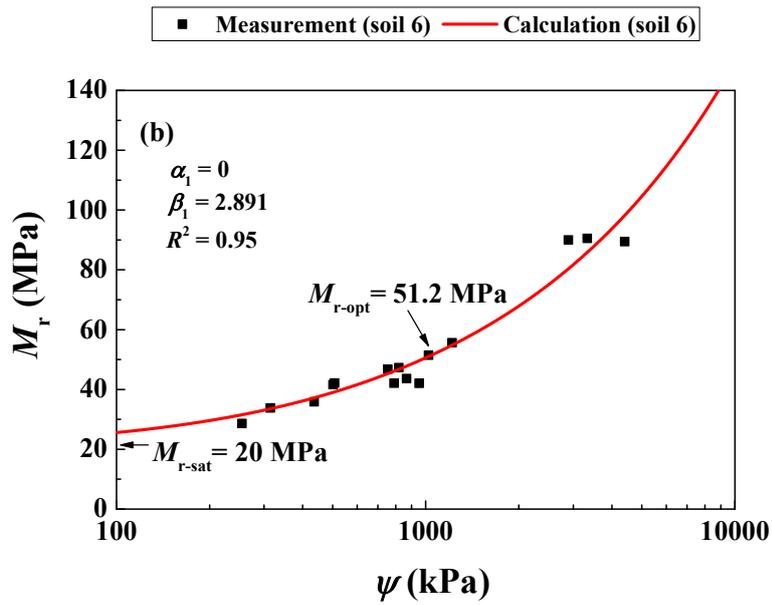
333 Note: Drying SWRC was obtained by following a desaturation path; Apparent SWRC was
334 obtained by the $S_r - \psi$ relationships determined after cyclic tests.

335

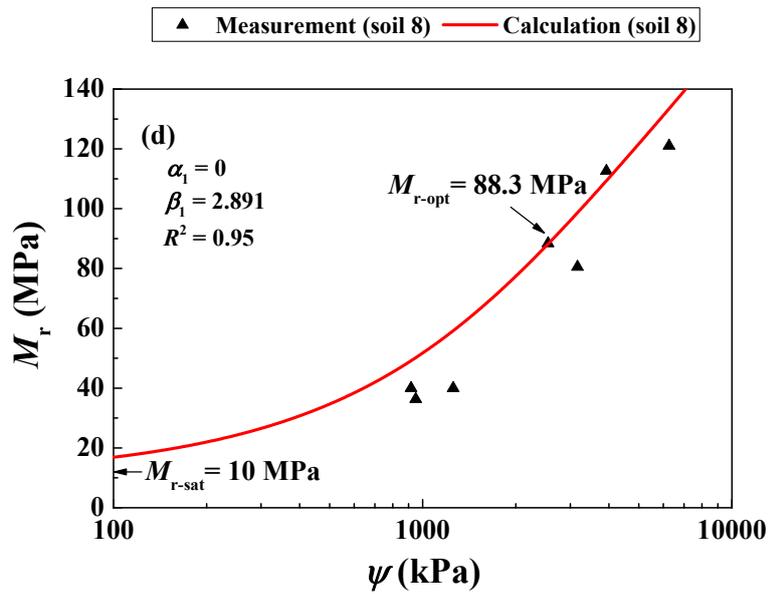
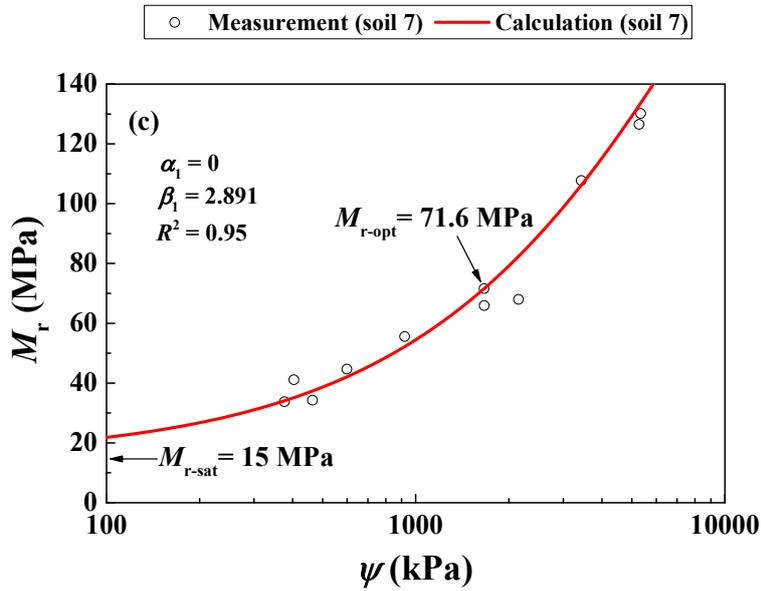
336



337



338

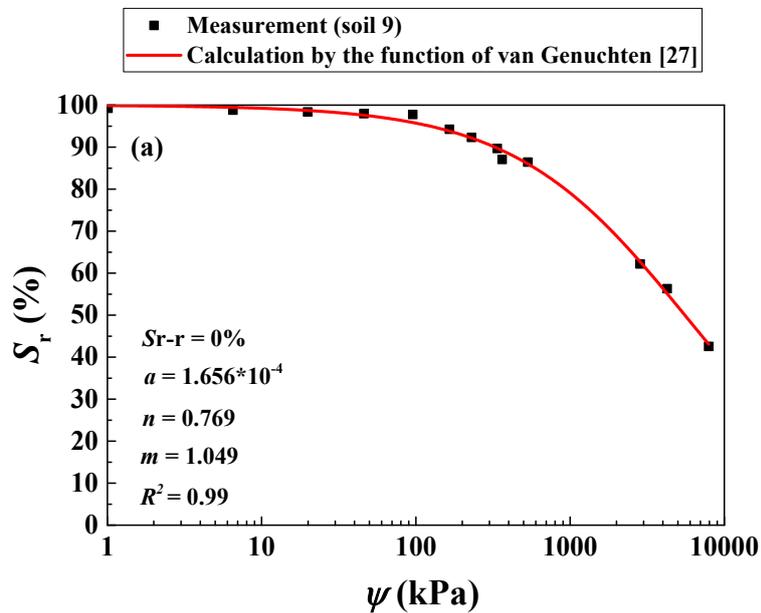


342 Fig. 7. Measured and calculated (a) SWRCs and (b)-(d) variations of M_r with ψ for soils 6-8
 343 (data from Zaman and Khoury [37])

345 Gupta et al. [9] investigated the effects of ψ and σ_d on M_r of Duluth slope soil (soil 9 in

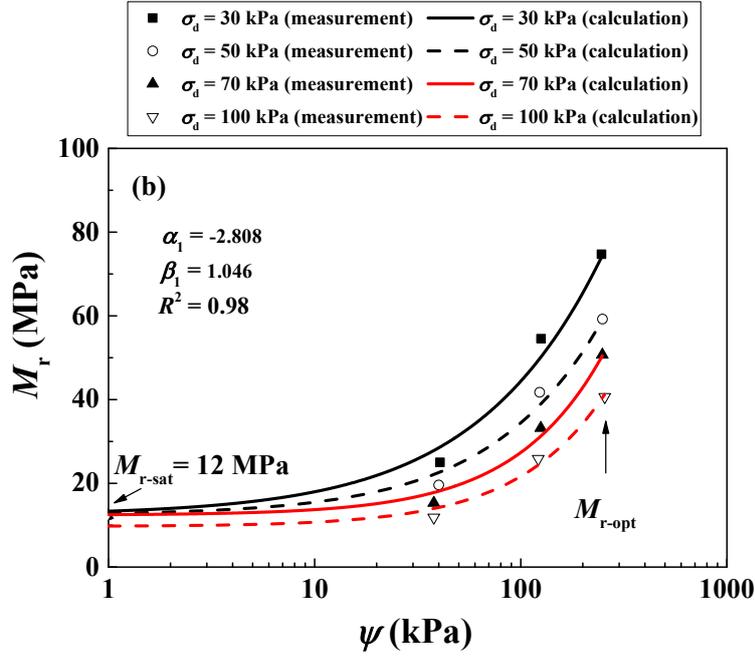
346 Table 3). Soil 9 was compacted at $w_{opt-f} = 13.5\%$ and $\rho_{dmax-f} = 1.79 \text{ Mg/m}^3$, defining a suction

347 $\psi_{\text{opt}} = 245$ kPa, which was measured using a thermal dissipation sensor. The compacted soil
 348 was then subjected to a saturation process, followed by a drying process to different target
 349 suctions. Cyclic triaxial tests were performed for the M_r measurement, with axis- translation
 350 technique and a thermal dissipation sensor adopted for suction control and measurement. A
 351 multi-stage loading with $\sigma_d = 30, 50, 70$ and 100 kPa was adopted under a constant $\sigma_c = 14.5$
 352 kPa. Fig. 8(a) shows the measured and calculated SWRCs for soil 9 using Eq. (17). It appears
 353 from Fig. 8(b) that M_r increased with increasing ψ under a constant σ_d , and the increase of σ_d
 354 led to a decrease of M_r in the full measured ψ range. Eq. (20) provides the calculated results
 355 using parameters $\alpha_1 = -2.808$ and $\beta_1 = 1.046$. Comparisons between the measurements and
 356 calculations showed a good agreement ($R^2 = 0.98$).



357

358

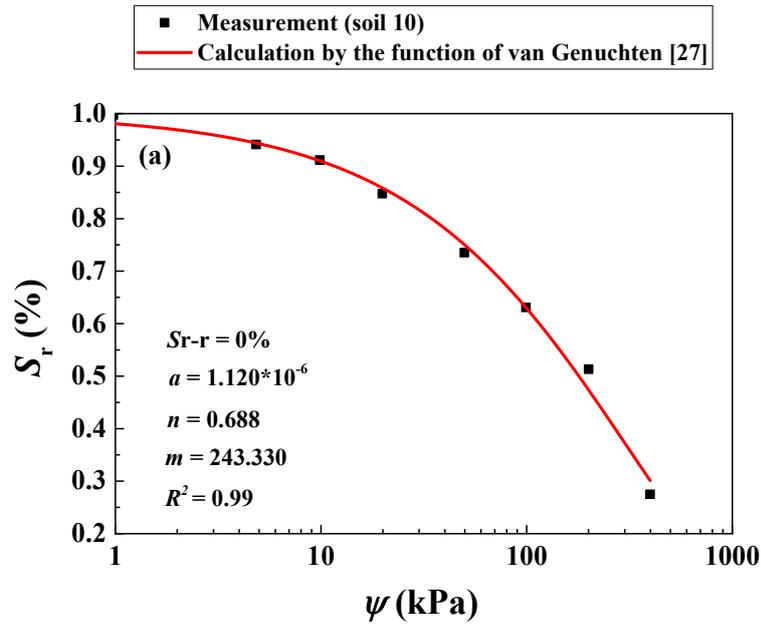


359

360 Fig. 8. Measured and calculated (a) SWRC and (b) variations of M_r with ψ for soil 9 (data
 361 from Gupta et al. [9])

362

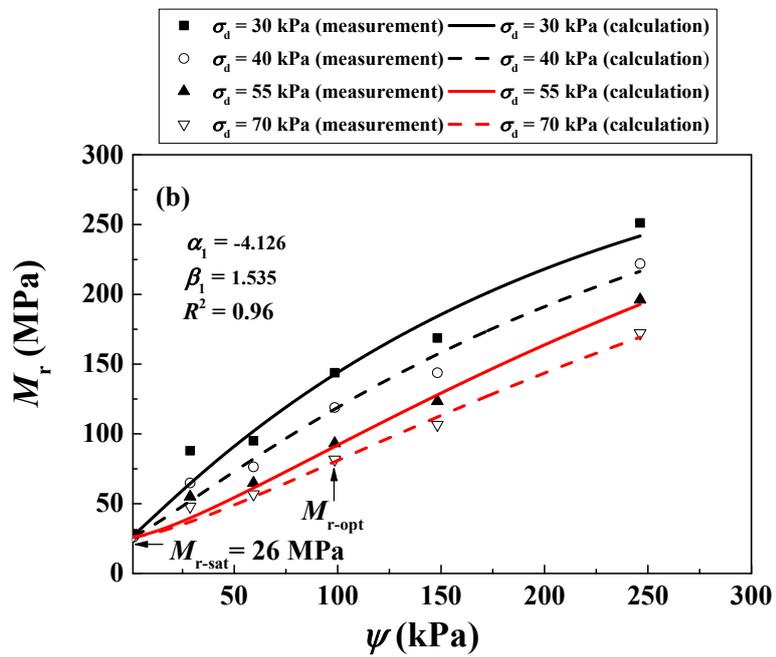
363 Ng et al. [12] investigated the variations of M_r with ψ under varying σ_d for a completely
 364 decomposed tuff (soil 10 in Table 3). Soil 10 was compacted at $w_{opt-f} = 16.3\%$ and $\rho_{dmax-f} =$
 365 1.76 Mg/m^3 , and its corresponding $\psi_{opt} = 95 \text{ kPa}$ was measured by a suction probe. The
 366 compacted soil was then wetted or dried to different target suctions. Suction-controlled cyclic
 367 triaxial tests were performed under varying $\sigma_d = 30, 40, 55$ and 70 kPa and a constant $\sigma_c = 30$
 368 kPa . Fig. 9(a) presents the measured and fitted SWRCs using Eq. (17). Fig. 9(b) depicts the
 369 variations of M_r with ψ under varying σ_d measured by Ng et al. [12] and those calculated by
 370 Eq. (20) using $\alpha_1 = -4.126$ and $\beta_1 = 1.535$. A good agreement was observed between the
 371 measurements and the calculations ($R^2 = 0.96$).



372

373

374



375

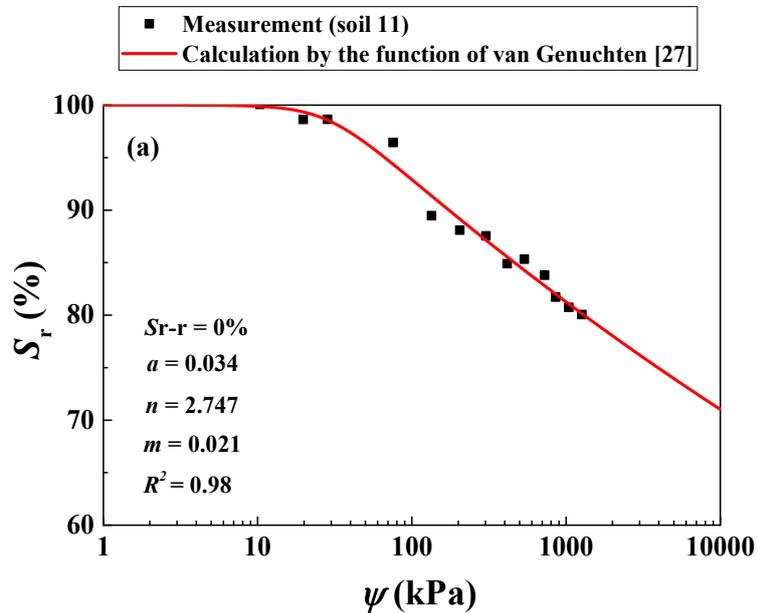
376

377

378

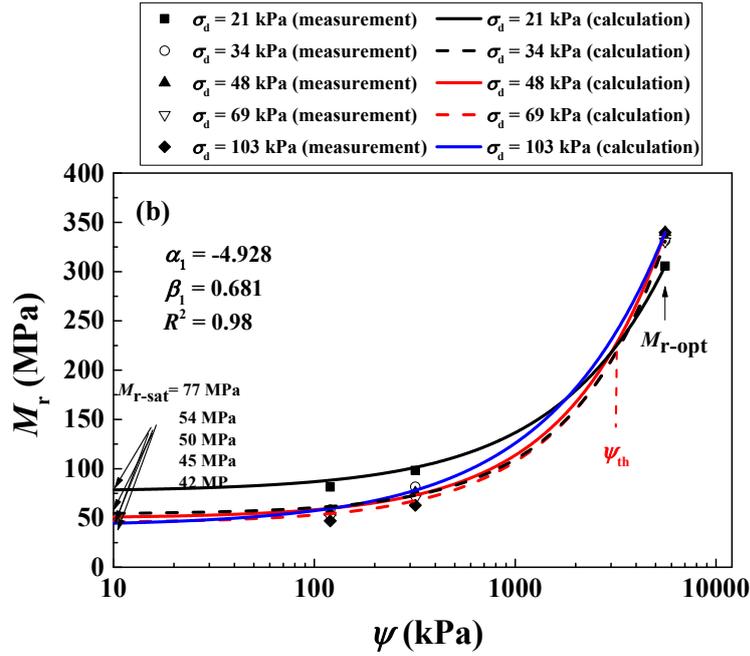
Fig. 9. Measured and calculated (a) SWRC and (b) variations of M_r with ψ for soil 10 (data from Ng et al. [12])

379 Yang et al. [38] studied the variations of M_r with ψ under varying σ_d for a compacted
380 subgrade soil (soil 11 in Table 3). Soil 11 was compacted at $w_{opt-f} = 18.1\%$ and $\rho_{dmax-f} = 1.76$
381 Mg/m^3 , with a suction $\psi_{opt} = 5580$ kPa measured using the filter paper method. The
382 compacted soil was then wetted to higher water contents (lower suctions). Cyclic triaxial tests
383 were performed under varying $\sigma_d = 21, 34, 48, 69, 103$ kPa and a constant $\sigma_c = 21$ kPa. After
384 completion of the tests, the filter paper method was applied to measure the suction ψ . Fig.
385 10(a) shows the SWRCs measured by Yang et al. [41] and fitted by Eq. (17). Fig. 10(b)
386 compares the measured M_r at different ψ and σ_d values and the corresponding calculated M_r
387 by Eq. (20) with parameters $\alpha_1 = -4.928$ and $\beta_1 = 0.681$. A threshold suction ψ_{th} could be
388 identified, separating the ψ into two zones with different effects of σ_d .



389

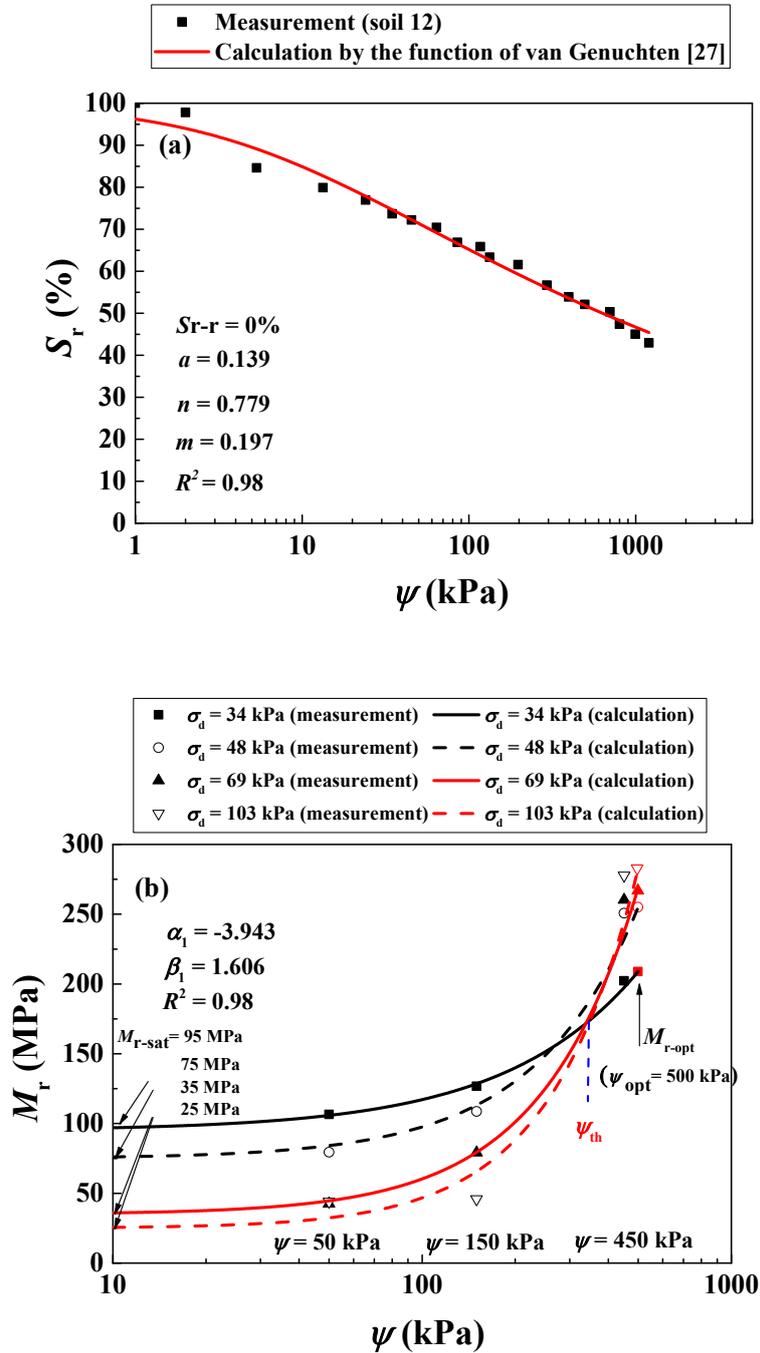
390



391
 392 Fig. 10. Measured and calculated (a) SWRC (data from Yang et al. [41]) and (b) variations of
 393 M_r with ψ for soil 11 (data from Yang et al. [38])

394
 395 Yang et al. [13] investigated the variations of M_r with ψ under varying σ_d for a
 396 compacted mudstone soil (soil 12 in Table 3). Soil 12 was compacted at $w_{opt-f} = 16.8\%$ and
 397 $\rho_{dmax-f} = 1.77 \text{ Mg/m}^3$, defining a suction $\psi_{opt} = 500 \text{ kPa}$ which was measured by the filter paper
 398 method. It was then wetted to higher water contents (19.1%, 20.2% and 23.2%) to reach
 399 various suctions (450, 150 and 50 kPa respectively). Cyclic triaxial tests were performed with
 400 suction controlled by the axis-translation technique. Four deviator stresses $\sigma_d = 34, 48, 69$ and
 401 103 kPa were applied in sequence under a constant $\sigma_c = 21 \text{ kPa}$. Fig. 11(a) shows the
 402 measured and fitted SWRCs by Eq. (17). Fig. 11(b) depicts the variations of M_r with ψ under
 403 varying σ_d measured by Yang et al. [13] and those calculated by Eq. (20) with parameters α_1

404 = -3.943 and $\beta_1 = 1.606$. A ψ_{th} was identified from Fig. 11(b), separating the ψ into two zones
 405 with different effects of σ_d , which was consistent with the observation in Fig. 10(b).



406

407

408 Fig. 11. Measured and calculated (a) SWRC and (b) variations of M_r with ψ for soil 12 (data
 409 from Yang et al. [13])

410

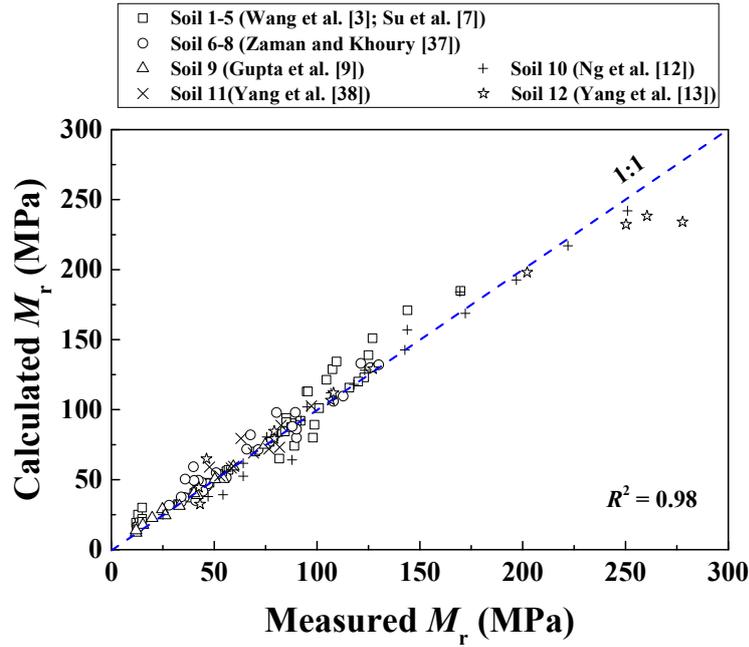
411 Table 5 presents the values of model parameters α_1 and β_1 in the proposed Eq. (20) for
 412 soils 1-12, indicating that these parameters are dependent of soil studied. Fig. 12 shows the
 413 comparison between the measured and the calculated M_r values using Eq. (20) for soils 1-12
 414 listed in Tables 2 and 3, illustrating that the proposed Eq. (20) provides satisfactory
 415 simulations with $R^2 = 0.98$.

416 It is worth noting that as the coarse grain content $f_v = 0\%$ was obtained for soils 6-12, the
 417 proposed model (Eq. (21)) could not be used to demonstrate the effect of f_v on M_r . It should
 418 be mentioned that previous studies were difficult to be used for validating Eq. (21), because
 419 Eq. (21) was proposed based on the variations of M_{r-sat} with f_v for soils 1-5 (Fig. 6), where the
 420 fine soil fraction was kept constant at $\rho_{dmax-f} = 1.82 \text{ Mg/m}^3$ (leading to a constant ψ for
 421 mixture), whatever the f_v values (as shown in Fig. 2). However, in previous studies, the ρ_d of
 422 mixture was generally kept constant. In that case, an increase of f_v led to a decrease of the dry
 423 density of fine soil fraction ρ_{d-f} and thus a decrease of ψ .

424 Table 5. Values of model parameters in the proposed Eq. (20) for soils 1-12

Reference	Soil. No	α_1	β_1	R^2
Wang et al. [3] and Su et al. [7]	1			0.90
	2			0.98
	3	-1.390	0.967	0.96
	4			0.92
	5			0.92
	6			0.95
Zaman and Khoury [37]	7	0	2.891	0.95
	8			0.95
Gupta et al. [9]	9	-2.808	1.046	0.98
Ng et al. [12]	10	-4.126	1.535	0.96
Yang et al. [38]	11	-4.928	0.681	0.98
Yang et al. [13]	12	-3.943	1.606	0.98

425



426

427 Fig. 12. Comparison between measured and calculated M_r values for soils 1-12

428

429 COMPARISON BETWEEN THE PROPOSED MODEL AND REPRESENTATIVE
 430 EXISTING MODELS

431 To better illustrate the performance of the proposed model, a comparison was made with some
 432 representative existing models. According to Han and Vanapalli [17], the constitutive models
 433 of M_r could be categorized into three groups: group A - empirical models (e.g. Eqs. (1) - (4)
 434 and (9)); group B - models incorporating ψ into deviator or mean stresses (e.g. Eqs. (5) - (6));
 435 group C- models considering ψ as an independent term (e.g. Eqs. (7) - (8) and (10)). Three
 436 representative models were selected from these three groups: Eq. (9) (ARA, Inc., ERES
 437 Consultants Division. [18]), Eq. (5) (Liang et al. [19]) and Eq. (10) (Han and Vanapalli [14])
 438 for groups A, B and C, respectively. Note that Eqs. (9) - (10) incorporated SWRC using Eq.
 439 (17). Three studies were adopted for the comparison among (Table 6): soil 10 (Ng et al. [12])

440 of category I ($\psi < \psi_{th}$), soil 12 (Yang et al. [13]) of category II ($\psi > \psi_{th}$ and $\psi_{th} < \psi_{opt}$) and
 441 soil 5 (Wang et al. [3]; Su et al. [7]) of category II ($\psi > \psi_{th}$ and $\psi_{th} > \psi_{opt}$).

442

443

Table 6. Model parameters for three representative existing models

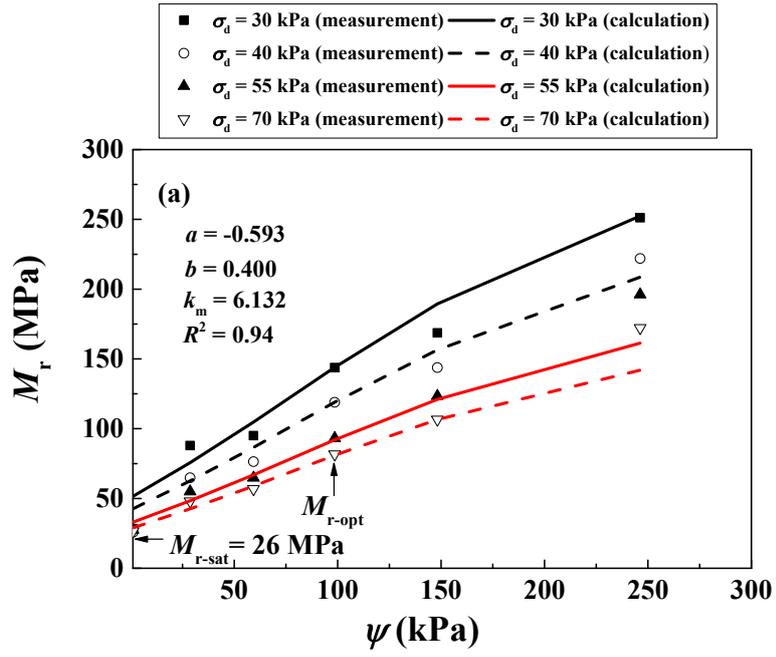
Equation/Parameter	Category I: $\psi < \psi_{th}$		Category II: $\psi > \psi_{th}$				
	Soil 10	$\psi_{th} < \psi_{opt}$	$\psi_{th} > \psi_{opt}$				
		Soil 12	Soil 1	Soil 2	Soil 3	Soil 4	Soil 5
Eq. (9)							
a	-0.593	-0.593					
b	0.400	0.400					
k_m	6.132	6.132			—		
R^2	0.94	0.72					
Eq. (5)							
k_4	0.453	0.513					
k_5	3.287	1.337					
k_6	-6.887	-1.303			—		
R^2	0.97	0.75					
Eq. (10)							
ξ	1.092	0.983	1.105	0.846	0.714	0.637	0.423
R^2	0.92	0.94	0.88	0.85	0.82	0.68	0.67

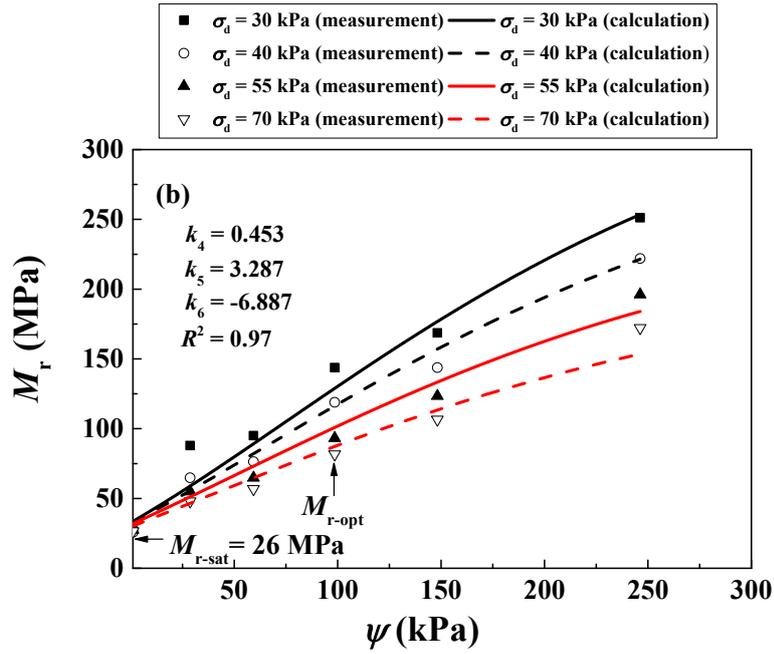
444

445

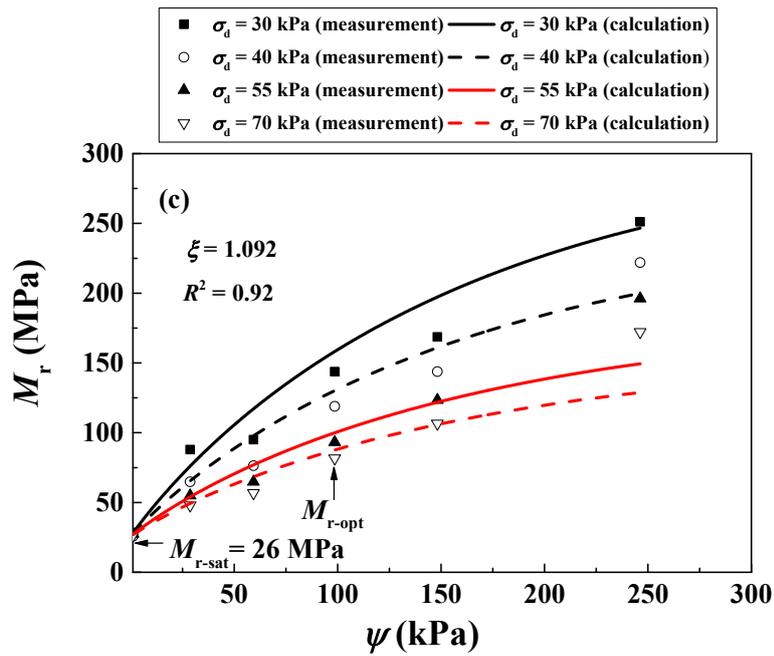
446 Fig. 13(a) shows the M_r measured by Ng et al. [12] and that calculated by Eq. (9) using
 447 parameters $a = -0.593$, $b = 0.400$ and $k_m = 6.132$. A reasonably good agreement was obtained
 448 between the measurements and the calculations ($R^2 = 0.94$). The similar phenomenon was
 449 observed in Figs. 13(b) - (c): Eqs. (5) and (10) provided satisfactory simulations with
 450 parameters presented in Table 6 ($R^2 = 0.97$ and 0.92 , respectively). This indicates that in the
 451 case of category I ($\psi < \psi_{th}$), all the three representative existing models as well as the
 452 proposed Eq. (20) can be used to describe the M_r variations.

453





455



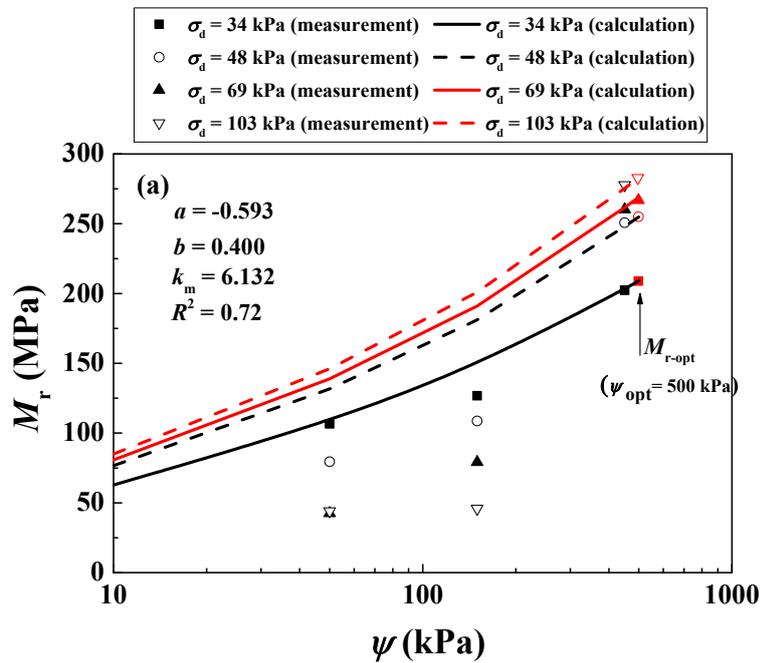
456

457 Fig. 13. Comparisons between the variations of M_r with ψ for soil 10 measured by Ng et al.
 458 [12] and those calculated by: (a) Eq. (9) (ARA, Inc., ERES Consultants Division. [20]); (b) Eq.
 459 (5) (Liang et al. [21]); (c) Eq. (10) (Han and Vanapalli [14])

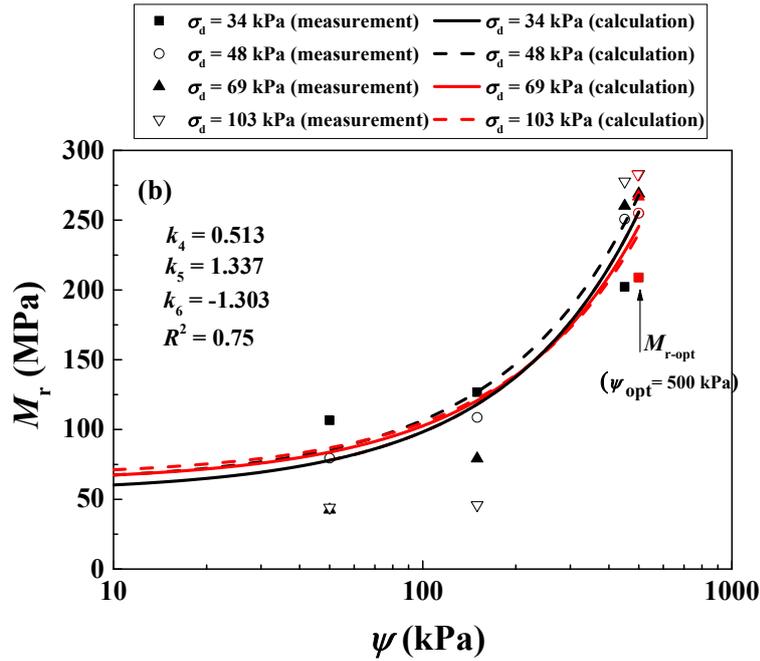
460

461 Fig. 14(a) shows the M_r measured by Yang et al. [13] and that calculated by Eq. (9).
 462 Compared with Eq. (20) (the proposed model), Eq. (9) provided less satisfactory simulations:
 463 $R^2 = 0.72$ for Eq. (9) against $R^2 = 0.98$ for Eq. (20) (see Fig. 11(b)). More importantly, the ψ_{th}
 464 cannot be reproduced by Eq. (9). The similar observations were made from Fig. 14(b) for Eq.
 465 (5). On the contrary, Fig. 14(c) shows good simulations by Eq. (10) with $R^2 = 0.94$ using
 466 parameter $\xi = 0.983$. Moreover, the ψ_{th} ($< \psi_{opt} = 500$ kPa in Yang et al. [13]) was identified,
 467 separating the ψ into two zones with different effects of σ_d . This indicates that in the case of
 468 category II ($\psi > \psi_{th}$ and $\psi_{th} < \psi_{opt}$), among the three representative models, only Eq. (10) can
 469 be used describe the M_r variations.

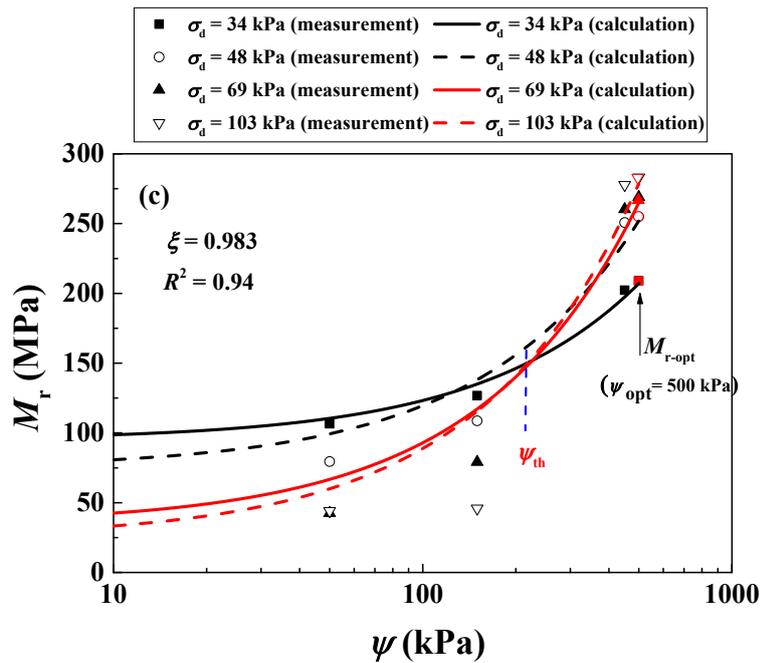
470



471



472



473

474 Fig. 14. Comparisons between the variations of M_r with ψ for soil 12 measured by Yang et al.
 475 [13] and those calculated by: (a) Eq. (9) (ARA, Inc., ERES Consultants Division. [20]); (b) Eq.
 476 (5) (Liang et al. [21]); (c) Eq. (10) (Han and Vanapalli [14])

477

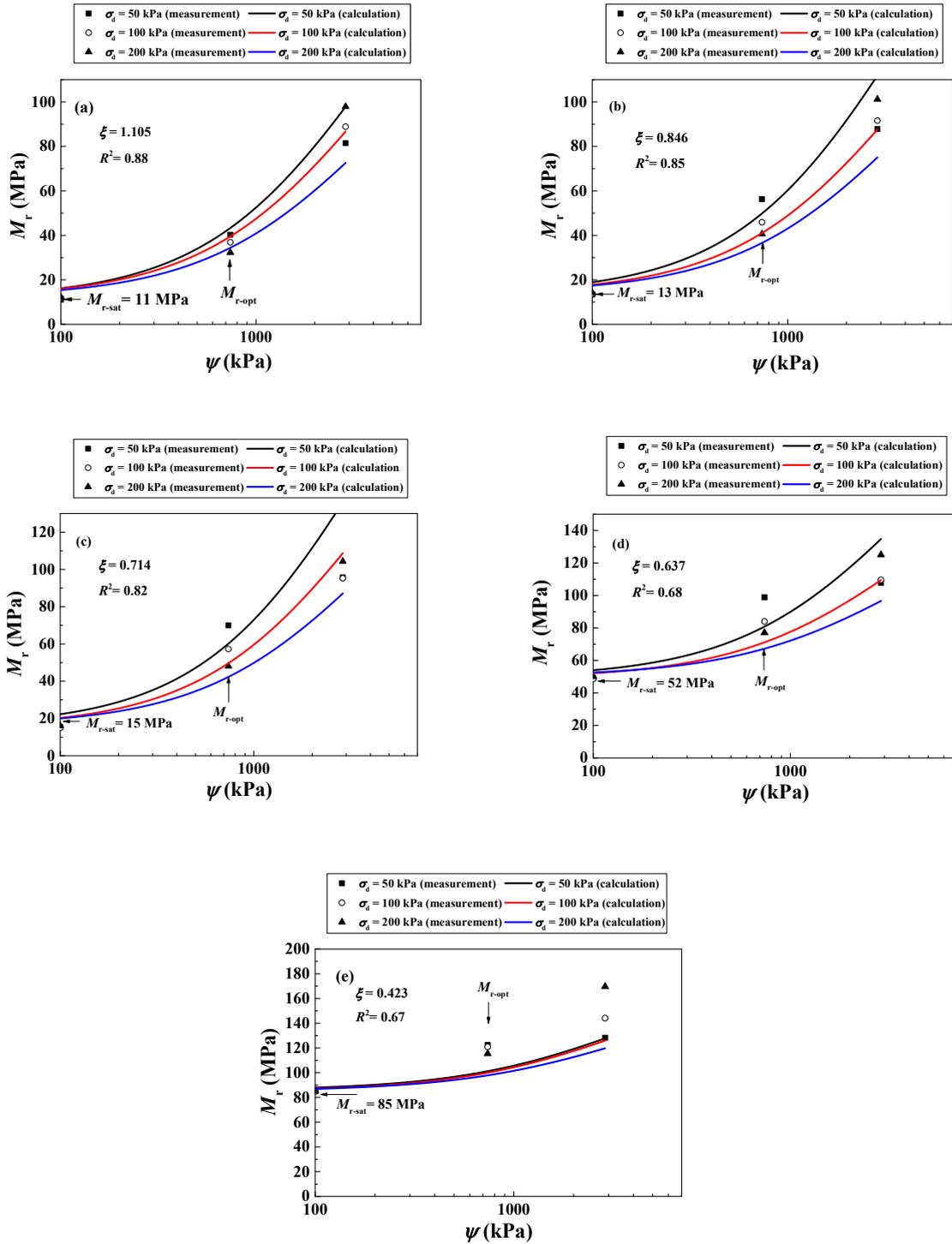
478 Fig. 15 shows the comparisons between the M_r measured by Wang et al. [3] and Su et al.
479 [7] and that calculated by Eq. (10) for soils 1-5. Since Eqs. (5) and (9) cannot fit well for soil
480 12 of category II ($\psi_{th} < \psi_{opt}$, see details in Figs. 14(a) - (b)), they were excluded for soils 1-5
481 of category II ($\psi_{th} > \psi_{opt}$). Figs. 15(a) - (e) show that Eq. (10) provides simulation results with
482 $R^2 = 0.88, 0.85, 0.82, 0.68$ and 0.67 , smaller than $R^2 = 0.90, 0.98, 0.96, 0.92$ and 0.92 by the
483 proposed Eq. (20) for soils 1-5 respectively. Moreover, the increase of σ_d led to a decrease of
484 calculated M_r by Eq. (10) in the full ψ range, without identifying ψ_{th} . This indicated that Eq.
485 (10) is not suitable for the case of category II ($\psi_{th} > \psi_{opt}$). This can be explained by the
486 consideration of two reference M_r values- M_{r-sat} and M_{r-opt} in Eq. (10). Indeed, through such
487 consideration, the variation of M_r from the saturated condition to the OMC condition (ψ varies
488 from 0 to ψ_{opt}) is expected to be well depicted (see Fig. 14 (c)). However, when $\psi_{th} > \psi_{opt}$, the
489 good description of Eq. (10) is no longer guaranteed (see Fig. 15).

490

491

492

493



494

495 Fig. 15. Comparisons between the variations of M_r with ψ for soils 1 - 5 measured by Wang et
 496 al. [3] and Su et al. [7] and those calculated by Eq. (10) (Han and Vanapalli [14]): (a) soil 1 at
 497 $f_v = 0\%$; (b) soil 2 at $f_v = 10\%$; (c) soil 3 at $f_v = 20\%$; (d) soil 4 at $f_v = 35\%$; (e) soil 5 at $f_v =$
 498 45%

499 The comparison of different models indicates that the variation of M_r can be well
500 described by Eqs. (9), (5) and (10) from group A, B and C for category I ($\psi < \psi_{th}$). For
501 category II ($\psi_{th} < \psi_{opt}$), only Eq. (10) among the three representative models can give
502 satisfactory description, while for category II ($\psi_{th} > \psi_{opt}$), even Eq. (10) fails. This leads to
503 the conclusion that only the proposed Eq. (20) which incorporates the combined effects of σ_d
504 and ψ on the M_r in the full range of suction can provide good description for both $\psi_{th} < \psi_{opt}$
505 and $\psi_{th} > \psi_{opt}$.

506 It is worth noting that as the existing models incorporating SWRC, such as Eq. (10) (Han
507 and Vanapalli [14]), the hysteresis effect has not been accounted for. Further studies are
508 needed to extend the proposed model to the hysteresis effect.

509

510 CONCLUSIONS

511 A constitutive model was proposed to describe the variation of resilient modulus M_r with
512 suction ψ and deviator stress σ_d . This model was then extended to the effect of coarse grain
513 content f_v based on the experimental data from Wang et al. [3] and Su et al. [7] to describe the
514 variation of M_r for unsaturated fine/coarse soil mixtures. The model incorporates soil-water
515 retention curve (SWRC). The key parameters are the resilient modulus at saturated state M_{r-sat} ,
516 the resilient modulus at optimum state M_{r-opt} , the suction at optimum state ψ_{opt} , soil parameters
517 α_1 and β_1 , as well as the parameters related to SWRC. The proposed model was validated
518 using five different studies. A comparative study was also conducted between the proposed

519 model and three representative existing models from literature. The following conclusions can
520 be drawn:

521 The effect of σ_d on M_r was found to be highly dependent on ψ , with a threshold suction
522 ψ_{th} separating ψ into two zones: with an increasing σ_d , the M_r decreased at $\psi < \psi_{th}$, while
523 increased at $\psi > \psi_{th}$. Using the threshold suction ψ_{th} , previous studies from literature could
524 be divided into two categories: category I ($\psi < \psi_{th}$) and category II ($\psi > \psi_{th}$). For category I
525 ($\psi < \psi_{th}$), the variation of M_r could be satisfactorily described by the three representative
526 models. For category II ($\psi > \psi_{th}$), only Eq. (10) among the three representative models
527 provided satisfactory simulations in the case of $\psi_{th} > \psi_{opt}$. However, in the case of $\psi_{th} < \psi_{opt}$,
528 Eq. (10) failed also. Unlike the three representative models, the proposed model Eq. (20)
529 could give good results in the full ψ range. In addition, the effect of f_v was well incorporated
530 using Eq. (21). Thus, the proposed model constitutes a helpful tool for describing the variation
531 of resilient modulus of unsaturated fine/coarse soil mixtures under the combined effects of
532 deviator stress and suction.

533

534 ACKNOWLEDGEMENTS

535 This work was supported by China Scholarship Council (CSC) and Ecole des Ponts ParisTech.

536

537 NOTATIONS

f_v	coarse grain content
$f_{v\text{-cha}}$	characteristic coarse grain content

G_s	specific gravity
I_p	plasticity index
M_r	resilient modulus
M_{r-sat}	resilient modulus at saturation condition
M_{r-opt}	resilient modulus at optimum moisture content condition
M_0	M_{r-sat} at $f_v = 0\%$
M_1	M_{r-sat} at $f_v = 100\%$
ρ_d	dry density of soil mixture
ρ_{dmax-f}	maximum dry density of fine soil
S_r	degree of saturation
S_{r-r}	residual degree of saturation
S_{r-opt}	degree of saturation at optimum moisture content condition
S_r^e	effective degree of saturation
S_{r-opt}^e	effective degree of saturation at optimum moisture content condition
w	water content of soil mixture
w_{opt-f}	optimum water content of fine soil
w_f	water content of fine soil
w_L	liquid limit
ψ	suction
ψ_{opt}	suction at optimum water content condition
ψ_{th}	threshold suction
θ_b	bulk stress
τ_{oct}	octahedral shear stress
χ	Bishop's effective stress parameter
σ_d	deviator stress

538
539
540
541
542
543
544
545
546
547
548
549
550
551
552
553
554
555
556
557
558
559

REFERENCES

[1] Trinh, V. N. (2011). Comportement hydromécanique des matériaux constitutifs de plateformes ferroviaires anciennes. PhD Thesis, Ecole Nationale des Ponts et Chaussées, Université Paris-Est.

[2] Nie, R., Li, Y., Leng, W., Mei, H., Dong, J., & Chen, X. (2020). Deformation characteristics of fine-grained soil under cyclic loading with intermittence. *Acta Geotechnica*, 1-14.

[3] Wang, H. L., Cui, Y. J., Lamas-Lopez, F., Dupla, J. C., Canou, J., Calon, N., & Chen, R. P. (2017). Effects of inclusion contents on resilient modulus and damping ratio of unsaturated track-bed materials. *Canadian Geotechnical Journal*, 54(12), 1672-1681.

[4] Wang, H.L., Cui, Y.J., Lamas-Lopez, F., Calon, N., Saussine, G., Dupla, J.C., Canou, J., Aïmedieu, P. and Chen, R.P., (2018a). Investigation on the mechanical behavior of track-bed materials at various contents of coarse grains. *Construction and Building Materials*, 164, pp.228-237.

[5] Wang, H.L., Cui, Y.J., Lamas-Lopez, F., Dupla, J.C., Canou, J., Calon, N., Saussine, G., Aïmedieu, P. and Chen, R.P., (2018b). Permanent deformation of track-bed materials at various inclusion contents under large number of loading cycles. *Journal of Geotechnical and Geoenvironmental Engineering*, 144(8), p.04018044.

- 560 [6] Qi, S., Cui, Y.J., Chen, R.P., Wang, H.L., Lamas-Lopez, F., Aïmedieu, P., Dupla, J.C.,
561 Canou, J. and Saussine, G., (2020). Influence of grain size distribution of inclusions on
562 the mechanical behaviours of track-bed materials. *Géotechnique*, 70(3), pp.238-247.
- 563 [7] Su, Y., Cui, Y. J., Dupla, J. C., & Canou, J. (2021a). Effect of water content on
564 resilient modulus and damping ratio of fine/coarse soil mixture with varying coarse
565 grain contents. *Transportation Geotechnics*, 100452.
- 566 [8] Cui, Y. J. (2018). Mechanical behaviour of coarse grains/fines mixture under
567 monotonic and cyclic loadings. *Transportation Geotechnics*, 17, 91-97.
- 568 [9] Gupta, S., Ranaivoson, A., Edil, T., Benson, C., & Sawangsuriya, A. (2007).
569 Pavement design using unsaturated soil technology.
- 570 [10] Nowamooz, H., Chazallon, C., Arsenie, M. I., Hornyh, P., & Masrouri, F. (2011).
571 Unsaturated resilient behavior of a natural compacted sand. *Computers and*
572 *Geotechnics*, 38(4), 491-503.
- 573 [11] Gu, C., Zhan, Y., Wang, J., Cai, Y., Cao, Z., & Zhang, Q. (2020). Resilient and
574 permanent deformation of unsaturated unbound granular materials under cyclic
575 loading by the large-scale triaxial tests. *Acta Geotechnica*, 15(12), 3343-3356.
- 576
- 577 [12] Ng, C. W. W., Zhou, C., Yuan, Q., & Xu, J. (2013). Resilient modulus of unsaturated
578 subgrade soil: experimental and theoretical investigations. *Canadian Geotechnical*
579 *Journal*, 50(2), 223-232.

- 580 [13] Yang, S. R., Lin, H. D., Kung, J. H., & Huang, W. H. (2008). Suction-controlled
581 laboratory test on resilient modulus of unsaturated compacted subgrade soils. *Journal*
582 *of Geotechnical and Geoenvironmental Engineering*, 134(9), 1375-1384.
- 583 [14] Han, Z., & Vanapalli, S. K. (2015). Model for predicting resilient modulus of
584 unsaturated subgrade soil using soil-water characteristic curve. *Canadian Geotechnical*
585 *Journal*, 52(10), 1605-1619.
- 586 [15] Oh, W. T., Vanapalli, S. K., & Puppala, A. J. (2009). Semi-empirical model for the
587 prediction of modulus of elasticity for unsaturated soils. *Canadian Geotechnical*
588 *Journal*, 46(8), 903-914.
- 589 [16] Han, Z., & Vanapalli, S. K. (2016a). Stiffness and shear strength of unsaturated soils
590 in relation to soil-water characteristic curve. *Géotechnique*, 66(8), 627-647.
- 591 [17] Han, Z., & Vanapalli, S. K. (2016b). State-of-the-Art: Prediction of resilient modulus
592 of unsaturated subgrade soils. *International Journal of Geomechanics*, 16(4), 04015104.
- 593 [18] Sawangsurriya, A., Edil, T. B., & Benson, C. H. (2009). Effect of suction on resilient
594 modulus of compacted fine-grained subgrade soils. *Transportation research record*,
595 2101(1), 82-87.
- 596 [19] Ba, M., Nokkaew, K., Fall, M., & Tinjum, J. M. (2013). Effect of matric suction on
597 resilient modulus of compacted aggregate base courses. *Geotechnical and Geological*
598 *Engineering*, 31(5), 1497-1510.
- 599 [20] ARA, Inc., ERES Consultants Division. (2004). Guide for mechanistic - empirical
600 design of new and rehabilitated pavement structures. Final report, NCHRP Project 1-
601 37A. Transportation Research Board, Washington, D.C.

- 602 [21] Liang, R. Y., Rabab'ah, S., & Khasawneh, M. (2008). Predicting moisture-dependent
603 resilient modulus of cohesive soils using soil suction concept. *Journal of*
604 *Transportation Engineering*, 134(1), 34-40.
- 605 [22] Heath, A. C., Pestana, J. M., Harvey, J. T., & Bejerano, M. O. (2004). Normalizing
606 behavior of unsaturated granular pavement materials. *Journal of Geotechnical and*
607 *Geoenvironmental Engineering*, 130(9), 896-904.
- 608 [23] Khoury, N., Brooks, R., Boeni, S. Y., & Yada, D. (2013). Variation of resilient
609 modulus, strength, and modulus of elasticity of stabilized soils with postcompaction
610 moisture contents. *Journal of Materials in Civil Engineering*, 25(2), 160-166.
- 611 [24] Alonso, E. E., Pereira, J. M., Vaunat, J., & Olivella, S. (2010). A microstructurally
612 based effective stress for unsaturated soils. *Géotechnique*, 60(12), 913-925.
- 613 [25] Lu, N., Godt, J. W., & Wu, D. T. (2010). A closed-form equation for effective stress
614 in unsaturated soil. *Water Resources Research*, 46(5).
- 615 [26] Moossazadeh, J., & Witczak, M. W. (1981). Prediction of subgrade moduli for soil
616 that exhibits nonlinear behavior. *Transportation Research Record*, (810).
- 617 [27] Van Genuchten, M. T. (1980). A closed-form equation for predicting the hydraulic
618 conductivity of unsaturated soils. *Soil science society of America journal*, 44(5), 892-
619 898.
- 620 [28] Su, Y., Cui, Y. J., Dupla, J. C., & Canou, J. (2021). Soil-water retention behaviour of
621 fine/coarse soil mixture with varying coarse grain contents and fine soil dry densities.
622 *Canadian Geotechnical Journal*, (ja).

- 623 [29] Su, Y., Cui, Y.J., Dupla, J.C., Canou, J., Qi, S., (2021b). Developing a sample
624 preparation approach to study the mechanical behavior of unsaturated fine/coarse soil
625 mixture. *Geotechnical Testing Journal*. <https://doi.org/10.1520/GTJ20190450>.
- 626 [30] Cui, Y. J., & Delage, P. (1996). Yielding and plastic behaviour of an unsaturated
627 compacted silt. *Géotechnique*, 46(2), 291-311.
- 628 [31] Ng, C. W. W., Baghbanrezvan, S., Sadeghi, H., Zhou, C., & Jafarzadeh, F. (2017).
629 Effect of specimen preparation techniques on dynamic properties of unsaturated fine-
630 grained soil at high suctions. *Canadian Geotechnical Journal*, 54(9), 1310-1319.
- 631 [32] Werkmeister, S., Dawson, A. R., & Wellner, F. (2004). Pavement design model for
632 unbound granular materials. *Journal of Transportation Engineering*, 130(5), 665-674.
- 633 [33] Duong, T. V., Cui, Y. J., Tang, A. M., Dupla, J. C., Canou, J., Calon, N., & Robinet, A.
634 (2016). Effects of water and fines contents on the resilient modulus of the interlayer
635 soil of railway substructure. *Acta Geotechnica*, 11(1), 51-59.
- 636 [34] Qi, S., Cui, Y. J., Dupla, J. C., Chen, R. P., Wang, H. L., Su, Y., ... & Canou, J. (2020).
637 Investigation of the parallel gradation method based on the response of track-bed
638 materials under cyclic loadings. *Transportation Geotechnics*, 24, 100360.
- 639 [35] Vallejo, L. E., & Mawby, R. (2000). Porosity influence on the shear strength of
640 granular material–clay mixtures. *Engineering Geology*, 58(2), 125-136.
- 641 [36] Su, Y., Cui, Y. J., Dupla, J. C., & Canou, J. (2020). Investigation of the effect of water
642 content on the mechanical behavior of track-bed materials under various coarse grain
643 contents. *Construction and Building Materials*, 263, 120206.

- 644 [37] Zaman, M., & Khoury, N. (2007). Effect of soil suction and moisture on resilient
645 modulus of subgrade soils in Oklahoma (No. ORA 125-6662).
- 646 [38] Yang, S. R., Huang, W. H., & Tai, Y. T. (2005). Variation of resilient modulus with
647 soil suction for compacted subgrade soils. *Transportation Research Record*, 1913(1),
648 99-106.
- 649 [39] Delage, P., Audiguier, M., Cui, Y.J. and Howat, M.D., (1996). Microstructure of a
650 compacted silt. *Canadian Geotechnical Journal*, 33(1), pp.150-158.
- 651 [40] Vanapalli, S. K., Fredlund, D. G., & Pufahl, D. E. (1999). The influence of soil
652 structure and stress history on the soil–water characteristics of a compacted till.
653 *Geotechnique*, 49(2), 143-159.
- 654 [41] Yang, S. R., Kung, J. H., Huang, W. H., & Lin, H. D. (2006). Resilient modulus of
655 unsaturated cohesive subgrade soils. *Yantu Gongcheng Xuebao*(Chinese Journal of
656 *Geotechnical Engineering*), 28(2), 225-229.

657

658 LIST OF TABLES

- Table 1. A summary of model parameters for Eqs. (1) - (10)
- Table 2. Soil properties in Wang et al. [3] and Su et al. [7]
- Table 3. Soil properties in five different studies
- Table 4. Parameters of SWRCs for soils 1-12
- Table 5. Values of model parameters in the proposed Eq. (20) for soils 1-12
- Table 6. Model parameters for three representative existing models

659

LIST OF FIGURES

- Fig. 1. Grain size distribution curves of fine soil and micro-ballast (after Wang et al. [3])
- Fig. 2. Measured and calculated soil-water retention curves at varying f_v values for soils 1-5 (after Su et al. [28])
- Fig. 3. Measured and calculated variations of M_r with ψ under varying σ_d for soils 1-5: (a) soil 1 at $f_v = 0\%$; (b) soil 2 at $f_v = 10\%$; (c) soil 3 at $f_v = 20\%$; (d) soil 4 at $f_v = 35\%$; (e) soil 5 at $f_v = 45\%$ (data from Wang et al. [3] and Su et al. [7])
- Fig. 4. Variations of M_r with f_v under varying ψ and a constant $\sigma_d = 200$ kPa for soils 1-5 (data from Wang et al. [3] and Su et al. [7])
- Fig. 5. Variation of parameter l_1 with $\log(\sigma_d/p_a)$ for soils 1-5
- Fig. 6. Measured and calculated variations of M_{r-sat} with f_v
- Fig. 7. Measured and calculated (a) SWRCs and (b)-(d) variations of M_r with ψ for soils 6-8 (data from Zaman and Khoury [38])
- Fig. 8. Measured and calculated (a) SWRC and (b) variations of M_r with ψ for soil 9 (data from Gupta et al. [9])
- Fig. 9. Measured and calculated (a) SWRC and (b) variations of M_r with ψ for soil 10 (data from Ng et al. [12])
- Fig. 10. Measured and calculated (a) SWRC (data from Yang et al. [42]) and (b) variations of M_r with ψ for soil 11 (data from Yang et al. [39])
- Fig. 11. Measured and calculated (a) SWRC and (b) variations of M_r with ψ for soil 12 (data from Yang et al. [13])
- Fig. 12. Comparison between measured and calculated M_r values for soils 1-12
- Fig. 13. Comparisons between the variations of M_r with ψ for soil 10 measured by Ng et al. [12] and those calculated by: (a) Eq. (9) (ARA, Inc., ERES Consultants Division. [20]); (b) Eq. (5) (Liang et al. [21]); (c) Eq. (10) (Han and Vanapalli [14])
- Fig. 14. Comparisons between the variations of M_r with ψ for soil 12 measured by Yang et al. [13] and those calculated by: (a) Eq. (9) (ARA, Inc., ERES Consultants Division. [20]); (b) Eq. (5) (Liang et al. [21]); (c) Eq. (10) (Han and Vanapalli [14])
- Fig. 15. Comparisons between the variations of M_r with ψ for soils 1 - 5 measured by Wang et al. [3] and Su et al. [7] and those calculated by Eq. (10) (Han and Vanapalli [14]): (a) soil 1 at $f_v = 0\%$; (b) soil 2 at $f_v = 10\%$; (c) soil 3 at $f_v = 20\%$; (d) soil 4 at $f_v = 35\%$; (e) soil 5 at $f_v = 45\%$

660

661