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# Scale invariant relationship between rainfall kinetic energy and intensity in Paris region: An evaluation using universal multifractal framework

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## Abstract

To calculate the effect of rainfall in detaching particles and initiating soil erosion, or in eroding wind turbine leading edge, it is important to measure recorded drop size distributions (DSD) and fall velocity over long period. Commonly used relationships between kinetic energy ( $KE$ ) and rainfall rate ( $R$ ) exhibit strong dependence on the temporal resolution at which the analysis is carried out. Here we aim at developing a new scale invariant relationship relying on the framework of Universal Multifractals (UM), which is widely used to analyze and characterize geophysical fields that exhibit extreme variability across wide range of scales.

Rainfall data is collected using three optical disdrometers working on different underlying technologies (one Campbell Scientific PWS100 and two OTT Parsivel<sup>2</sup> instruments) and operated by the Hydrology Meteorology and Complexity laboratory of École des Ponts ParisTech in Paris area (France). They provide access to the size and velocity of drops falling through sampling areas of few tens of  $\text{cm}^2$ . Such data enables estimation of rainfall DSD,  $R$  and  $KE$  at various resolutions. The temporal variations of this geophysical data over wide range of scales are then characterized in the UM framework, which was never done for  $KE$ . A new power law relation is developed and tested against the theoretical

24 framework assuming gamma DSD for describing the dependence between  $KE$  and  $R$ . The  
25 developed equation using scale invariant features of UM does not rely on gamma DSD as-  
26 sumption, performs as well as the existing tools, and is valid not only at a single scale, but  
27 also across scales.

28 *Keywords:*

29 rainfall intensity, rainfall kinetic energy, disdrometer, multifractal, scale invariance,

30 gamma DSD

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## 31 **1. Introduction**

### 32 *1.1. On the importance of rainfall and kinetic energy*

33 Understanding the relation between rainfall rate ( $R$ ) and kinetic energy ( $KE$ ) is essen-  
34 tial for accurate determination of various rainfall parameters and understanding their after  
35 effects on surrounding ecosystem (Karlen et al., 2003). It has been well established that  
36 onsite erosion of soil - splash and runoff - depends on drop size distribution (DSD) and fall  
37 velocity of the spectrum (Ellison, 1944; Fernández-Raga et al., 2017). Hence,  $KE$  and  $R$   
38 are primarily used to quantify rainfall erosivity and to estimate erosion rates in universal  
39 models towards sustainable land use planning (Angulo-Martínez et al., 2012; Shojaei et al.,  
40 2020; Mohamadi and Kavian, 2015). Erosion heavily affects agricultural sector: On on-site  
41 level, it impoverishes the top soil off nutrients and organic matter along with their water  
42 holding capacity; this in turn increases the use of fertilizers and hence causes pollution at  
43 the recipient end of off-site erosion. Further, on off-site level, soil transport by erosion can  
44 trigger flood events through silting up of basins and rivers (Pimentel, 2006; Enne et al.,  
45 2000). Rainfall erosivity is a key parameter in various erosion models such as US based  
46 Universal Soil Loss Equation (USLE, Smith and Wischmeier, 1962) and its revised ver-  
47 sion (RUSLE, Renard et al., 1997), South Korean based SEMMA models for calculating  
48 soil loss (Deog Park et al., 2012), and in European models such as EUROSEM, WaTEM  
49 etc. that account for sediment transportation along soil loss (Morgan et al., 1998; Van Oost  
50 et al., 2000; Kirkby et al., 2008, an advance on USLE). Most of these frameworks use  $KE$   
51 as the major quantifying factor for estimating erosivity at spatial scales as rainfall  $KE$  rep-  
52 resents the total energy available for detachment and transport of soil on surface of impact.  
53 As direct measurement of  $KE$  is limited to specific geographical locations possessing re-  
54 quired instrumental capabilities, understanding the relationship between  $KE$  and the more

55 commonly available rainfall parameter,  $R$ , is important in estimating the former in more  
56 places.

57 Accurate estimation of rainfall  $KE$  is also important in understanding and mitigating  
58 leading-edge erosion (LEE) on wind turbine blades. Erosion damage reduces aerodynamic  
59 performance of blades resulting in reduced annual energy production and increased down-  
60 time (Keegan et al., 2013). Though LEE involves a multitude of atmospheric factors,  
61 impact velocity and amount of precipitation have been established as some of the major  
62 external factors in erosion (Herring et al., 2019). As in soil erosion, larger drops with  
63 greater mass and vertical terminal velocity causes a disproportionate amount of erosion in  
64 LEE also. Rainfall  $KE$  helps to quantify the impact of droplets hitting perpendicular to the  
65 surface and hence rainfall erosion on blades which is the accumulated aggregate of multi-  
66 ple impacts stochastically distributed over the surface of the coated laminate (Bech et al.,  
67 2018). In Whirling arm rain erosion test (WA-RET) (ASTM-G73-10, 2017; Liersch and  
68 Michael, 2014; DNVGL-RP-0171, 2014), the industrial standard for measuring durability  
69 of leading-edge structures, specimens are subjected in controlled velocity and rain condi-  
70 tions to assess the damage caused by droplet impacts (Bech et al., 2018). Considering the  
71 rapid growth of offshore wind industry as sustainable clean energy solution in the Amer-  
72 icas and Asia with monsoon seasons, proper representation of  $KE$  and  $R$  is important in  
73 quantifying LEE in wind turbines.

#### 74 *1.2. Review of existing relations and need for a scale invariant representation*

75 Application of erosion frameworks such as USLE or RUSLE poses uncertainties be-  
76 cause of their empirical basis which has single or limited measurement locations and spe-  
77 cific methods of data collection. These frameworks take  $KE$  as the major quantifying  
78 factor for erosivity, and employ various corrections to mitigate overestimation at low inten-

79 sity rainfall as smaller droplets are less effective in soil detachment (van Dijk et al., 2002).  
80 Another commonly used erosivity index is rainfall momentum, but it has been shown that  
81 for natural rainfall they exhibit similar relationship with rainfall intensity (Hudson, 1971).  
82 These models are based on traditional two parameter exponential rain drop size distribu-  
83 tion models developed from smaller sample collection methods such as flour pellets or stain  
84 paper (Laws and Parsons, 1943; Marshall and Palmer, 1948). Later studies using multi pa-  
85 rameter radiometry and disdrometers have shown improvements in measurement if DSD is  
86 assumed to be a three parameter gamma distribution (Ulbrich, 1983). Gamma distribution  
87 assumes fewer large drops in rain and represents a narrower DSD than the exponential one,  
88 and thereby reduces the overestimation of  $KE$  by the latter. For recording DSD and in turn  
89 obtaining empirical  $KE$ , disdrometers are commonly used in meteorological campaigns,  
90 where fall velocity is either directly measured or estimated from empirical relations (Gunn  
91 and Kinzer, 1949; Atlas et al., 1973).

92 There has been various studies towards accurate representation of  $KE - R$  relationship.  
93 Fox (2004) demonstrates the impact of formulation of DSD in calculation of  $KE$  and hence  
94 on erosion. Smith and Wischmeier (1962) proposed a logarithmic function based on DSD  
95 formulation by Laws and Parsons (1943) and terminal velocity by Gunn and Kinzer (1949).  
96 It was used in modelling erosion in USLE, which was later replaced by continuous expo-  
97 nential functions in revised USLE approach (Renard et al., 1997; Brown and Foster, 1987).  
98 Other proposed forms of  $KE - R$  equations were linear (Kinnell, 1981; Sempere-Torres  
99 et al., 1998), polynomial (Carter, 1979) and power-law (Park et al., 1982). Critical litera-  
100 ture appraisal by van Dijk et al. (2002) on various  $KE - R$  relations lists measurement tech-  
101 niques and procedures, sampling biases, interpretation methods and storm types as reasons  
102 for discrepancies and suggests another exponential based predictive equation. More recent

103 reviews such as Angulo-Martínez and Barros (2015), Wilken et al. (2018), Mineo et al.  
104 (2019) shows lack of universality among various  $KE - R$  relationships (logarithmic, linear,  
105 power law and exponential) using disdrometer measurements and modelling at different  
106 geographic stations, meteorologic conditions and time aggregation used for calibration.

107 Most of the empirical formulations express  $KE$  as volume specific - kinetic energy  
108 per unit area and mm of rainfall or  $KE_{mm}$  ( $\text{Jm}^{-2}\text{mm}^{-1}$ ) due to prevalence of non auto-  
109 mated measurements and lack of accuracy in determining exposure time. But expression  
110 of volume specific kinetic energy creates a statistical artefact (spurious ratio correlation) in  
111  $KE - R$  relationship due to the inclusion of  $R$  ( $\text{mm h}^{-1}$ ) in the  $KE_{mm}$  expression. Salles et al.  
112 (2002) suggests usage of more consistent time specific  $KE$  or  $KE_{time}$  ( $\text{Jm}^{-2}\text{h}^{-1}$ ) that has  
113 been shown to produce less heteroscedasticity than corresponding  $KE_{mm} - R$  scatter plots.  
114 For representing erosion,  $KE_{time}$  has been expressed as the rate of expenditure of rain-  
115 fall kinetic energy (Kinnell, 1981), rainfall or kinetic power (Smith and De Veaux, 1992)  
116 and rainfall kinetic energy flux density (Steiner and Smith, 2000). The two expressions  
117 are related to each other through rain intensity and hydrological studies usually harmonize  
118  $KE - R$  relationships with  $KE_{time}$ .

$$KE_{time} = R \times KE_{mm} \quad (1)$$

119 In this paper the expressions for  $KE$  are derived and analyzed in the form of  $KE_{time}$  unless  
120 otherwise specified.

121 Due to variations in methodologies used and DSD characteristics during measurement,  
122 different functional forms show different behaviour towards  $KE - R$  estimation. Though  
123 commonly preferred, exponential relations have been shown to underestimate  $KE_{time}$  for  
124 lower intensities of rainfall (Carollo and Ferro, 2015). Power-laws that predict kinetic

125 energy well at lower intensities tend to overestimate the same for higher intensities. Log-  
126 arithmic curves are limited in their usage though they fit low and high intensity  $KE$  rather  
127 decently (van Dijk et al., 2002). Further due to the empirical formulation, when it comes to  
128 expressing  $KE_{time}$  vs  $R$ , most of these mathematical equations fail to have much physical  
129 justification. Using generalized scaling formulation that expresses various existing DSD  
130 models as it's special cases (Torres et al., 1994; Sempere-Torres et al., 1998), Salles et al.  
131 (2002) found that power law is the most suitable function to relate  $KE_{time}$  and  $R$  from a  
132 microphysical point of view. The parameters (prefactor and exponent) of power law are  
133 related to rain type, geographical location and measuring technique. Shin et al. (2016) has  
134 proposed a representative power law based on the ideal assumption that the drop-size is  
135 uniformly distributed under the constant rainfall intensity.

### 136 1.3. Purpose of paper

137 Current literature on  $KE$  and  $R$  lacks a common consensus on the usage of expres-  
138 sion that is valid across various scales of measurement, hydro-meteorological regimes or  
139 observation techniques. There is an increased focus on research characterizing rainfall mi-  
140 crophysics at local and regional scales (Petan et al., 2010). Here we examine the variation  
141 of  $KE$  and  $R$  for over 7 years in Paris region using continuous data from three disdrom-  
142 eters from two different manufacturers. Using the framework of Universal Multifractal  
143 (UM) (Schertzer and Lovejoy, 1987), efforts were made to characterize the variability of  
144  $KE_{time}$  and  $R$  on event based and year based analysis, and to formulate a scale invariant  
145 relation based on power law relationships. Multifractals allow characterization of com-  
146 plex geophysical fields with a limited number of scale invariant exponents (see Schertzer  
147 and Tchiguirinskaia, 2020, for a recent review). Multifractal behaviour of measured rain-  
148 fall has been abundantly studied previously (see Gupta and Waymire, 1990; Kumar and



149 Fofoula-Georgiou, 1993; Deidda et al., 1999; Olsson and Niemczynowicz, 1996; García-  
150 Marín et al., 2008; Langousis et al., 2009; Emmanouil et al., 2020, for some examples  
151 among others). Wolfensberger et al. (2017) & Schertzer and Lovejoy (2011) used UM in  
152 climatological analysis of precipitation - modelled and actual - in relation to external geo-  
153 graphical and meteorological descriptors. Checking the validity of UM framework on  $KE$ ,  
154 which was never done to the knowledge of the authors, is a first goal of the paper. The  
155 main goal of this paper is to explore the possibility of establishing a physically based scale  
156 invariant power law relationship between  $KE$  and  $R$  using the UM framework, without hav-  
157 ing to rely on strong assumptions on DSD shape; and compare its performance with more  
158 classical approach.

159 This article is structured as follows. In section 2 the methodology used is detailed.  
160 First, the commonly used framework of gamma distributed DSD which yields a power-law  
161 relation is reminded along with the associated parameters' estimation techniques. Then, the  
162 process to identify power-law relation in the UM framework is explained after a required  
163 reminder on the theoretical underlying basis. Similarly, the associated parameters' estima-  
164 tion techniques, which will be implemented, are described. Section 3 includes details of  
165 data collection and quality control. Results are discussed in section 4 where validity of  
166 formulated relation is tested and contrasted over different types of rain events. The final  
167 section, section 5, concludes the study and summarizes the main observations.

## 168 2. Methodology

### 169 2.1. Theoretical relation assuming gamma distributed DSD

#### 170 2.1.1. Rainfall microstructure and commonly used gamma distribution

171 Rainfall is measured and represented as distribution of raindrops in different diame-  
172 ter classes or drop size distribution (DSD) and their corresponding terminal fall velocity  
173 in stable air. For meteorological purposes, size distribution of raindrops is represented as  
174 mean number of drops per unit volume in a particular diameter (more precisely equivo-  
175 lumatic diameter, i.e. the diameter of a drop with the same volume but a spherical shape)  
176 range between  $D$  and  $D + dD$ ,  $N_v(D)$  ( $\text{m}^{-3} \text{mm}^{-1}$ ). However, in hydrological studies, DSD  
177 measured by ground based devices such as disdrometers or optical spectrometers are rep-  
178 resented as mean number of raindrops in a particular diameter range arriving at a surface  
179 per unit area per unit time,  $N_A(D)$  ( $\text{m}^{-2} \text{mm}^{-1} \text{s}^{-1}$ ). If effects of wind, turbulence and  
180 raindrop interactions are neglected,  $N_v(D)$  and  $N_A(D)$  are related as follows (Uijlenhoet  
181 and Stricker, 1999) :

$$N_A(D) = v(D)N_v(D) \quad (2)$$

182 where  $v(D)$  represents the terminal fall velocity ( $\text{ms}^{-1}$ ) as a function of the equivalent  
183 spherical diameter of raindrop  $D$  (mm).

184 Traditional mathematical expressions describing  $N_v(D)$  such as exponential (Marshall  
185 and Palmer, 1948), weibull (Best, 1950), gamma (Ulbrich, 1983) and lognormal (Feingold  
186 and Levin, 1986) can be expressed as particular case of general formulation proposed by  
187 Sempere-Torres et al. (1998). For the scope of this paper, gamma distribution of DSD,  
188 which has been recognized to better represent natural rain, will be considered (Ulbrich,

189 1983). In this framework :

$$N_v(D) = N_0 D^\mu e^{-(\Lambda D)} \quad (3)$$

190 where  $N_v(D)$  is in  $\text{m}^{-3} \text{mm}^{-1}$ ,  $D$  in  $\text{mm}$ ,  $N_0$  (in  $\text{m}^{-3} \text{mm}^{-1-\mu}$ ),  $\mu$  and  $\Lambda$  (in  $\text{mm}^{-1}$ ) are  
191 distribution parameters measuring raindrop concentration, mean size and shape of spectrum  
192 respectively.  $\Lambda$  is usually expressed with the help of the median volume diameter  $D_0$  as  
193  $\Lambda = (3.67 + \mu)/D_0$ .

#### 194 2.1.2. Theoretical power law relation between $R$ and $KE$

195 Rainfall intensity or rain rate ( $R$ , in  $\text{mm h}^{-1}$ ) can be calculated from  $N_A(D)$  using fol-  
196 lowing expression (Steiner and Smith, 2000) :

$$R = 3.6 \times 10^{-3} \frac{\pi}{6} \int_0^\infty D^3 N_A(D) dD \quad (4)$$

197 Kinetic energy per unit area per unit time ( $KE_{time}$  in  $\text{J m}^{-2} \text{h}^{-1}$ ) of falling drops can also  
198 be expressed in terms of measured  $N_A(D)$  as follows (Steiner and Smith, 2000) :

$$KE = 3.6 \times 10^{-6} \frac{\pi \rho}{12} \int_0^\infty D^3 v^2(D) N_A(D) dD \quad (5)$$

199 where  $\rho$  is density of water in standard conditions in ( $\text{kg m}^{-3}$ ).

200 Though there are more sophisticated equations proposed in the literature for  $v(D)$ , for  
201 simplicity in calculation, here we are following the widely used power law formulation by  
202 Atlas and Ulbrich (1977). Uijlenhoet (2001) demonstrated it has the only functional form  
203 consistent with power law relationship between rainfall related parameters :

$$v(D) = cD^g \quad (6)$$

204  $c = 3.78 \text{ m s}^{-1} \text{ mm}^{-g}$  and  $g = 0.67$  (with  $v$  in  $\text{m s}^{-1}$  and  $D$  in  $\text{mm}$ ).

205 Most of the rainfall parameters can be approximated as moments of the DSD; and when  
 206 DSD follows gamma model, the  $n^{\text{th}}$  moment,  $M_n$  can be computed as (Atlas and Ulbrich,  
 207 1977) :

$$M_n = \int_0^{\infty} D^n N(D) d(D) = N_0 \Lambda^{-(\mu+n+1)} \Gamma(\mu+n+1) \quad (7)$$

208 where  $\Gamma(\alpha)$  is the complete gamma function

$$\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x} dx, \quad \text{where } \alpha > 0. \quad (8)$$

209 Substituting equations 2, 3, 6 and 7, and solving the integral gives reduced expressions  
 210 for  $KE$  and  $R$  in terms of gamma function;

$$R = 6 \times 10^{-4} \pi c N_0 \frac{\Gamma(4+g+\mu)}{\Lambda^{(4+g+\mu)}} \quad (9)$$

$$KE = 3.6 \times 10^{-6} \frac{\rho \pi c^3}{12} N_0 \frac{\Gamma(4+3g+\mu)}{\Lambda^{(4+3g+\mu)}} \quad (10)$$

211 This reduction using gamma function enables representation of  $KE$  and  $R$  in the form  
 212 of a power law as follows :

$$KE = bR^a \quad (11)$$

213 where

$$\begin{aligned} a &= \frac{4 + 3g + \mu}{4 + g + \mu} \\ b &= 5 \times 10^{-4} \rho c^2 [6\pi c N_0 \times 10^{-4}]^{1-a} \frac{\Gamma(4 + 3g + \mu)}{(\Gamma(4 + g + \mu))^a} \end{aligned} \quad (12)$$

214 The final expression follows the same pattern as that by Salles et al. (2002) based on  
215 generalized DSD and that of Uijlenhoet and Stricker (1999) based on exponential DSD.  
216 When  $\mu = 0$  gamma distribution for DSD becomes a simple Marshall and Palmer negative  
217 exponential parameterization representation of DSD (Marshall and Palmer, 1948) with Eq.  
218 11 becoming  $KE = 8.539R^{1.287}$ . Same values were obtained for exponent 'a' and pre-factor  
219 'b' using the general function proposed by Salles et al. (2002) for Marshall and Palmer ap-  
220 proximation. Closer value of exponential coefficient were also reported by Uijlenhoet and  
221 Stricker (1999) in a power law formulation based on Marshall and Palmer (1948) approxi-  
222 mation of DSD and power law dependence of rain drop terminal velocity, Eq. 6 (Atlas and  
223 Ulbrich, 1977).

224 Many simplifications were followed in this formulation of  $KE-R$  relation as a power  
225 law (Eq. 11). Gamma DSD is notably assumed as well as a power law form for terminal  
226 fall velocity as function of diameter. This approximation shows limitations at higher val-  
227 ues of diameter ( $D > 5$  mm) and doesn't account for atmospheric turbulence or updrafts  
228 and downdrafts (Adirosi et al., 2016). Effect of truncation errors in measurement is not  
229 considered. The formulation also ignores the effect of horizontal wind velocity and sur-  
230 face impact angle of rain drops. Some authors tend to consider normalized spectra of DSD  
231 (Testud, 2001) that requires only two parameters to describe the DSD; it was also not con-  
232 sidered here for the sake of simplicity in deriving a theoretical relation between  $KE$  and  $R$ .  
233 It should be noted that although widely accepted as the best representation of natural DSD,

234 gamma function also has associated errors that exists outside experimental methodology  
235 and sampling (Adirosi et al., 2014).

### 236 2.1.3. Estimation of power law coefficients from gamma DSD parameters

237 As mentioned before, most rainfall parameters can be expressed as moments of the DSD  
238 according to Eq. 7. The right hand side of the equation is specific to the non normalized  
239 three parameter gamma distribution considered here, as shown in Eq. 3. In practice DSD is  
240 not measured continuously but for discrete diameters  $D_i$ . As a consequence, the estimated  
241 moments  $\hat{M}_n$  of order  $n$  are computed thanks to the following discrete sum rather than  
242 previous integral :

$$\hat{M}_n = \sum_{i=1}^{N_{class}} D_i^n N_i(D_i) \Delta D_i \quad [\text{mm}^n \text{m}^{-3}] \quad (13)$$

243 where  $D_i$  is the diameter of droplet in class  $i$ ,  $N_i(D_i)$  is the drop size distribution (estimation  
244 discussed later in Eq. 26),  $\Delta D_i$  is the width of diameter class  $i$  and  $N_{class}$  the total number  
245 of diameter classes.

246 Investigation of various DSD parameter estimating methods by Cao and Zhang (2009)  
247 highlighted the risk in usage of maximum likelihood and L-moment estimators for pro-  
248 cessing data with truncation in lower end of DSD spectra. As all disdrometers are limited  
249 by some minimum value of measurable diameter  $D_{min} (> 0)$ , it is advisable to use tradi-  
250 tional method of moments (Brawn and Upton, 2008). Although the choice of moments for  
251 proper parameters estimation with the method of moments is a relevant topic, it is outside  
252 the scope of this paper. Hence, authors relied on the existing literature. More precisely,  
253 although higher DSD moments are considered to be associated with higher errors, the ten-  
254 dency of lower errors with middle order moments was shown later in Smith et al. (2009)

255 (using radar measurements) and Cao et al. (2008) (using joint disdrometer-radar observa-  
256 tions). Cao and Zhang (2009) evaluated the performance of various moment estimators  
257 using simulations of gamma DSD (with a more realistic estimation of errors) and found  
258 that second, third and fourth moments (M234) as the best overall performer for estimating  
259  $R$ . Konwar et al. (2014) (M234) and Huang et al. (2021) (even higher order moments - 2,3  
260 and 6) are some recent examples of using middle order moments while analysing data from  
261 same disdrometer make as the one in current study (OTT Parsivel<sup>2</sup>). Hence, it was chosen  
262 to employ these moments for computing gamma DSD parameters:  $\mu$ ,  $N_0$  and  $\Lambda$  (refer Eq.  
263 7). Specific moment equations are provided in the appendix (Appendix B). These parame-  
264 ters are then introduced in Eq. 12 to derive the expected power law parameters assuming a  
265 gamma DSD.

266 Variation of  $a$  and  $b$  with DSD parameters is examined with available data set in section  
267 4 for different type of rainfall events.

## 268 *2.2. Theoretical relation between multifractal fields in UM framework*

### 269 *2.2.1. Overview of the theoretical framework*

270 The framework of Universal Multifractals (UM) enables to characterize the extreme  
271 variability of geophysical fields across scales, with the help of a limited number of parame-  
272 ters with physical meaning (Schertzer and Lovejoy, 1987). It relies on the assumption that  
273 these fields are generated through an underlying multiplicative cascade. Such behaviour  
274 is assumed to be inherited from the scale invariant features of the Navier-Stokes equa-  
275 tions, but has not yet been formally showed. Consequently, statistical properties of fields  
276 are conserved in all scales. In this study, time specific kinetic energy and rainfall intensity  
277 from disdrometer measurements are analyzed using UM. Review by Schertzer and Lovejoy  
278 (2011) discusses the techniques and methodologies employed in more detail.

279 For analysis in UM framework, the field in consideration is normalized (divided by  
 280 its mean value) and its total size in terms of numerical values is restricted to be a power  
 281 of two. The field can be one or two dimensional; since current study involves only time  
 282 series we will be limiting our discussion to single dimension. Let us consider a normalized  
 283 conservative field (an additional parameter for non-conservative fields is introduced later)  
 284  $\varepsilon_\lambda$  at resolution  $\lambda$ . Resolution  $\lambda$  is the ratio of  $L$ , the outer scale, to  $l$ , the observational  
 285 scale. Let us first consider the occurrence pattern of the field, i.e. the same field with 1 for  
 286 strictly positive values and 0 otherwise. If the field is fractal, then the probability  $p$  that  
 287 a segment of length  $l$  intersects the field ( $p$  is the probability of occurrence of non zero  
 288 elements in the total binary field) scales with resolution as follows:

$$p = \frac{\lambda^{D_f}}{\lambda^D} = \lambda^{-c_f} \quad (14)$$

289 where  $c_f = D - D_f$ , the fractal co-dimension of the field. The fractal dimension  $D_f$  indi-  
 290 cates how the binary field (rain and no rain in this case) fills the available space in a scale  
 291 invariant way. As  $D$  is constant, in order to fully characterize the field a value of  $D_f$  for  
 292 each threshold is required. This is the intuitive notion of multifractality. In order to be  
 293 mathematically consistent, this characterization should actually be done with the help, not  
 294 of a direct threshold at the maximum resolution, but with a scale-invariant threshold called  
 295 singularity  $\gamma$ , and their corresponding codimension function  $c(\gamma)$  as :

$$p(\varepsilon_\lambda \geq \lambda^\gamma) \approx \lambda^{-c(\gamma)} \quad (15)$$

296 Above equation implies that statistical moments  $q$  of the field scale with resolution  
 297 (Schertzer and Lovejoy, 1987, 1988) are :



$$\langle \varepsilon_\lambda^q \rangle \approx \lambda^{K(q)} \quad (16)$$

298 where  $K(q)$  is the moment scaling function related to  $c(\gamma)$  by Legendre transform (Parisi  
 299 and Frisch, 1985). For a conservative field in UM framework,  $K_c(q)$  can be fully character-  
 300 ized with only two parameters, multi-fractality index  $\alpha$  and mean intermittency codimen-  
 301 sion  $C_1$ .

$$K_c(q) = \frac{C_1}{\alpha - 1} (q^\alpha - q) \quad (17)$$

302  $C_1$  measures clustering of average intensity across scales; when the field is homogeneous  
 303  $C_1$  is equal to zero.  $\alpha$  measures how this clustering changes with respect to intensity levels;  
 304  $\alpha \in [0, 2]$ . Larger values of both corresponds to stronger extremes. Simulations of such  
 305 fields can be obtained by generating a levy noise with parameter  $\alpha$ , 'coloring' it to introduce  
 306  $C_1$  and then taking the exponential which yields solely non-negative values.

307 For a non conservative field  $\phi_\lambda$ , i.e. a field whose average ( $\langle \phi_\lambda \rangle$ ) changes with scales, a  
 308 non-conservative parameter  $H$  is used in expression of scaling:

$$\phi_\lambda = \varepsilon_\lambda \lambda^{-H} \quad (18)$$

309 where  $\varepsilon$  is a conservative field characterized with  $C_1$  and  $\alpha$ .

310 Positive  $H$  represents a smoother field which needs to be fractionally differentiated for  
 311 obtaining a conservative field. Conversely negative  $H$  represents a field in need of fractional  
 312 integration for retrieving conservative field.  $H$  is related to the spectral slope  $\beta$ , which gives  
 313 a measurement of the correlation range. Larger  $\beta$  means higher contribution of large-scale  
 314 phenomenon in variability of data.

$$\beta = 1 + 2H - K_c(2) \quad (19)$$

315 *2.2.2. Estimation of scaling behaviour and UM parameters*

316 The quality of scaling of multifractal fields can be examined using trace moment (TM)  
 317 where log-log plot of upscaled fields against resolution  $\lambda$  is taken for each moment  $q$  (Eq.  
 318 16). For multifractal fields, the plot is a straight line with moment scaling function  $K(q)$  as  
 319 slope. Quality of the scaling is given by the estimate  $R^2$  of the linear regression. The value  
 320 for  $q = 1.5$  is used as reference. Double trace moment (DTM) is a more robust version of  
 321 TM tailored for UM fields where the moment scaling function  $K(q, \eta)$  of the field  $\varepsilon_\lambda^{(\eta)}$   
 322 (obtained in practice by raising  $\varepsilon$  to power  $\eta$  at the maximum resolution and then upscaling  
 323 it) is expressed as a function of multifractality index  $\alpha$  (Lavallée et al., 1993) :

$$\langle (\varepsilon_\lambda^{(\eta)})^q \rangle \approx \lambda^{K(q, \eta)} = \lambda^{\eta^\alpha K(q)} \quad (20)$$

324 From the above equation, value of  $\alpha$  can be obtained as the slope of the linear part when  
 325  $K(q, \eta)$  is represented for a given  $q$  as a function of  $\eta$  in log-log plot.

326 *2.2.3. Power law relations in UM framework and practical implementation*

327 If a field is UM, then a power law relation of it is also a UM field with coefficients  
 328 depending upon initial UM parameters. The reasoning for power law comes from the DTM  
 329 analysis in UM framework (Tessier et al., 1993; Lovejoy et al., 2008).

330 Consider two multifractal fields ( $\varepsilon_1$  and  $\varepsilon_2$ ) that are power law related by an exponent  
 331  $a$  and a prefactor  $b$ , as below :

$$\varepsilon_1 = b\varepsilon_2^a \quad (21)$$

332 From exponents in Eq. 20,  $K(q)$  of  $\varepsilon_1$  can be expressed as follows

$$\begin{aligned} \langle (\varepsilon_1)^q \rangle &\approx \langle (\varepsilon_2^{(a)})^q \rangle \approx \lambda^{K_{\varepsilon_2}(q,a)} \\ K_{\varepsilon_1}(q) &= K_{\varepsilon_2}(q,a) = a^{\alpha_{\varepsilon_2}} K_{\varepsilon_2}(q) \end{aligned} \quad (22)$$

333 Expanding  $K(q)$  with UM parameters (as in Eq. 17)

$$\frac{C_{1,\varepsilon_1}}{\alpha_{\varepsilon_1} - 1} (q^{\alpha_{\varepsilon_1}} - q) = a^{\alpha_{\varepsilon_2}} \frac{C_{1,\varepsilon_2}}{\alpha_{\varepsilon_2} - 1} (q^{\alpha_{\varepsilon_2}} - q) \quad (23)$$

334 From above equation, we can deduce that for if a power relation exists, the UM param-  
335 eters are related as follow :

$$\begin{aligned} \alpha_{\varepsilon_1} &= \alpha_{\varepsilon_2} = \alpha \\ C_{1,\varepsilon_1} &= a^\alpha C_{1,\varepsilon_2} \end{aligned} \quad (24)$$

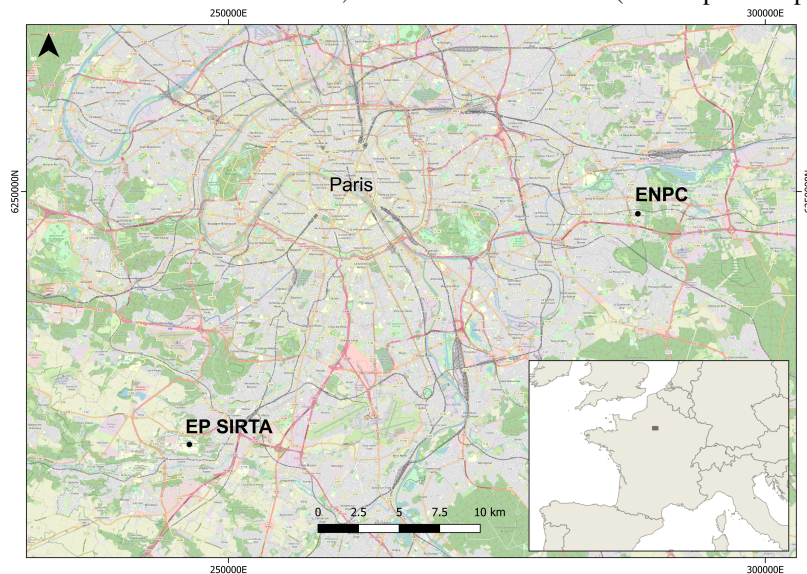
336 In this study, both  $KE$  and  $R$  time series were analysed using UM framework. As the  
337 UM model discussed here is solely non negative, it removes all possibility of having non-  
338 negative values for both fields in analysis. Using their UM parameters in Eq. 24, it is  
339 possible to formulate power law relationship in the format of Eq. 21. It has the same shape  
340 as the theoretical power law obtained using moments of the DSD in section 2.1, in Eq. 11.  
341 Details of disdrometer data collection and subsequent multifractal analysis are covered in  
342 upcoming sections.

### 343 **3. Data collection and instrumentation**

#### 344 *3.1. Measurement campaigns*

345 For this study, continuously monitored data in natural conditions was obtained from  
346 three optical disdrometers operating with two different principles. The devices are part  
347 of the TARANIS observatory (exTreme and multi-scAle RAiNdrop parIS observatory,  
348 Gires et al., 2018) of the Fresnel Platform of École des Ponts ParisTech (<https://hmco.enpc.fr/Page/Fresnel-Platform/en>); and are operated by Hydrology Meteorology  
349 and Complexity laboratory of École des Ponts ParisTech (HM&Co-ENPC). Table 1 gives  
350 a brief outline of measurement campaigns. From November 2016 to September 2017  
351 the instruments were moved to École Polytechnique (EP) on SIRTa (Site Instrumenté de  
352 Recherche par Télédétection Atmosphérique) for a joint intensive measurement campaign  
353 over the Ile-de-France region. The aforementioned location is about 38 km away from  
354 ENPC towards south west of Paris. To summarise, the measurements presented in the fol-  
355 lowing sections involve two different types of optical disdrometers and data collected from  
356 two different locations of Paris region.  
357

Figure 1: Location of disdrometers in Paris area, at ENPC and EP SIRTA (basemap from openstreetmap.org)



location	start time	end time
ENPC (1)	18 Jun 2013	10 Nov 2016
EP SIRTA	14 Nov 2016	20 Sep 2017
ENPC (2)	27 Dec 2017	31 Dec 2019

Table 1: Short description of the precipitation measurement campaign selected

358 *3.2. Overview of instrument functioning and outputs*

359 The three optical disdrometers used here are two OTT Parsivel<sup>2</sup> (see Battaglia et al.,  
 360 2010 or the device documentation OTT, 2014) and one PWS 100 (see Ellis et al., 2006 or  
 361 the device documentation Campbell-Scientific-Ltd, 2012).

362 The OTT Parsivel<sup>2</sup> are occlusion based devices with a transmitter that creates laser sheet  
 363 and an intercepting receiver directly aligned with it. The size (equivolumic diameter) and  
 364 fall velocity are assessed from changes (decrease in amplitude of intensity and duration  
 365 of the decrease) in received laser intensity due to the passing of rainfall drops through a  
 366 sampling area of 54 cm<sup>2</sup>. An ellipsoidal shape model with a standard relation between the

367 axis ratio and the equivolumic diameter are assumed for drops in the process.

368 The PWS100 consists of a transmitter that generates four horizontal parallel laser sheets,  
369 and two receptors which are not aligned with the transmitter. The signal received by each  
370 receptor corresponds to the light refracted by drops and contains four consecutive peaks  
371 associated with each laser sheet. From the delay between those peaks, fall velocity and  
372 diameter of the drops are estimated. PWS100 has a sampling area of 40 cm<sup>2</sup>. Computa-  
373 tions assuming spherical shape of droplets are performed here with a later correction for  
374 oblateness before final result generation (Gires et al., 2017).

375 Both disdrometers have a collection time step of 30 s and provide main output as a  
376 matrix containing the number of drops ( $n_{i,j}$ ) recorded during the time step  $\Delta t$  according  
377 to classes of equivolumic diameter (index  $i$  defined by a centre  $D_i$  and a width  $\Delta D_i$  both  
378 expressed in mm) and fall velocity (index  $j$  and defined by a centre  $v_j$  and a width  $\Delta v_j$ ,  
379 both expressed in ms<sup>-1</sup>). For Parsivel<sup>2</sup> there are 32 classes of  $\Delta D_i$  from 0.062 mm to 24.5  
380 mm and  $\Delta v_j$  from 0.05 ms<sup>-1</sup> to 20.8 ms<sup>-1</sup>. And for PWS the same ranges from 0.05 to  
381 27.2 ( $\Delta D_i$ , mm) and 0.05 to 27.2 ( $\Delta v_j$ , ms<sup>-1</sup>) in 34 classes. Width of diameter and velocity  
382 classes are not similar for all classes, as they are designed to be more and more refined  
383 towards smaller values.

384 From the raw matrix, the studied rainfall parameters - rain rate, drop size distribution  
385 and time specific kinetic energy were obtained using following expression for each time  
386 steps.

$$R = \frac{\pi}{6\Delta t} \sum_{i,j} \frac{n_{i,j} D_i^3}{S_{eff}(D_i)} \quad (25)$$

$$N(D_i) = \frac{1}{S_{eff}(D_i)\Delta D_i\Delta t} \sum_j \frac{n_{i,j}}{v_j} \quad (26)$$

$$KE = \frac{\rho_{wat}\pi}{6\Delta t} \sum_{i,j} \frac{n_{i,j}D_i^3v_j^2}{S_{eff}(D_i)} \quad (27)$$

387 where  $S_{eff}(D_i)$  is the sampling area of disdrometer in  $\text{mm}^2$ ,  $\Delta t$  is the time step duration  
 388 in hr and  $\rho_{wat}$  is the volumic mass of water ( $10^3 \text{ kg m}^{-3}$ ).  $N(D_i)\Delta D_i$  gives the number of  
 389 drops with a diameter in the class  $i$  per unit volume (in  $\text{m}^{-3}$ ). Details of the devices, their  
 390 functioning and data collection can be found in Gires et al. (2018).

### 391 3.3. Data quality and filtering

392 For the data presented in this paper, filters suggested by various authors (Kruger and  
 393 Krajewski, 2002; Thurai and Bringi, 2005; Jaffrain and Berne, 2012; Gires et al., 2018)  
 394 were used to remove possible non-meteorological measurements (from environmental fac-  
 395 tors such as splashing, horizontal wind etc.) on the basis of size and velocity of drops.  
 396 Using the disdrometer data set, two series of Multifractal analysis were performed - event  
 397 based and year based. For event based analysis, individual rainfall events were identified  
 398 with following criteria in rain intensity time series - rainfall events with a cumulative depth  
 399 greater than 0.7 mm and separated by at least 15 minutes of dry weather before and after.  
 400 From all measured events with this condition, rain rate ( $R$ ), drop size distribution (DSD)  
 401 and kinetic energy ( $KE$ ) were calculated for the three disdrometers (denoted Pars 1, Pars 2  
 402 and PWS hereafter). From the results further filtering was done to remove events having a  
 403 percentage of *nan* values (not a number - blank /missing data)  $> 1 \%$  and  $R^2$  value  $< 0.9$ ,  
 404 for both  $KE$  and  $R$ . Remaining *nans* were then replaced with 0. In year based methodology,  
 405 continuous time series from Jan 2016 to Dec 2019 - including rain and no rain conditions

406 - were used for multifractal analysis, and results were compiled according to the year of  
407 measurement.

408 There were minor lapses in continuity of data measurement from Jun 2013 to Dec 2015;  
409 for that reason those time periods were not considered in continuous year based analysis  
410 to avoid possible measurement biases. However, since selection of individual events are  
411 not affected by that, event based analysis involves data from Jun 2013 to Dec 2019. Lesser  
412 number of events can be observed in the excluded years for year based analysis in Table  
413 2. Between 2016 and 2019, there were also a few days of maintenance where data was not  
414 recorded - 27 Sep 2017 to 26 Dec 2017, 01 to 07 Jan 2018 and 12 to 14 Apr 2019. After  
415 quality control for each disdrometer, a total of 214,665 time steps were analysed in 556 rain  
416 events (total 1610 events counting all three disdrometers; data was not always available for  
417 all the disdrometers, hence the lesser number) from 2013 to 2019 for event based analysis.  
418 In year based analysis, a total of 3,919,680 time steps were considered from 2016 to 2019;  
419 percentages of rainy time steps were about 7%, 6.6% and 5.7% for Pars 1, Pars 2 and PWS  
420 respectively.

## 421 **4. Results and discussions**

### 422 *4.1. Overview of analysis*

423 For analysing the *KE* and *R* time series using UM framework, as explained in previous  
424 sections, two strategies were followed - event based analysis and year based analysis. For  
425 event based analysis, we identified a total of 556 rainfall events between 28 Sep 2013 and  
426 31 Dec 2019 of which 493 events were common among all three disdrometers. For UM  
427 analysis, each time series was resized to the highest power of two in such a way that the  
428 trimmed series accommodated maximum rainfall cumulative depth and then the field was



429 normalized. After resizing for UM analysis, length of individual events ranged from 64  
430 to 2048 time steps, where each time step corresponds to 30 s, i.e. the recording time step  
431 of disdrometers used.  $KE$  and  $R$  from each events were analyzed as separate fields in UM  
432 framework. For year based methodology, similar procedure was followed for UM analysis  
433 on year long continuous time series, for the years from 2016 to 2019.

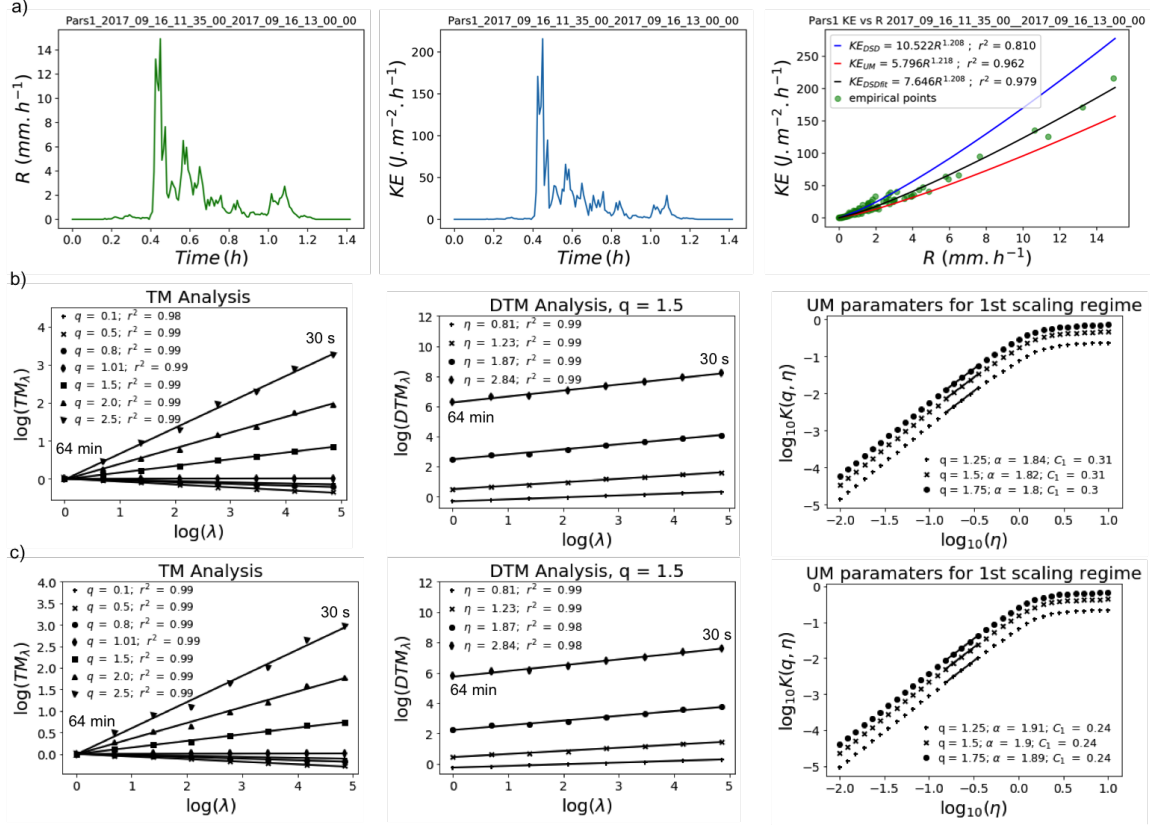
434 In coming subsections, estimation of UM parameters and power law relations are il-  
435 lustrated using one event data for event based analysis and one year data for year based  
436 analysis. Power law coefficients were also estimated using theoretical framework with  
437 DSD parameters as mentioned in section 2.1. Variation and correspondence among coeffi-  
438 cients determined by UM and DSD parameters are discussed thereafter and validated with  
439 data.

#### 440 *4.2. Multifractal analysis of events*

441 For illustration of the analysis carried out, one event from 2017 for Pars 1 disdrometer,  
442 that occurred on 16 September between 11:35:00 and 13:00:00 (local time) is presented  
443 here. Figure 2a displays the time series of  $R$  and  $KE$  for this event, as well as  $KE$  vs.  $R$   
444 plots. The latter also shows power law fits with coefficients from UM analysis and DSD  
445 parameters (explained later in this section). For this event, 171 time steps were trimmed  
446 to 128 time steps along region of maximum rain occurrence of which 124 were rainy data  
447 points. Trimmed and normalized  $KE$  and  $R$  were then subjected to analysis using UM  
448 framework discussed in section 2.2. Initial analysis indicated values of non-conservative  
449 parameter  $H$  greater than 0.5 among many events. Hence to retrieve a conservative field on  
450 which the UM analysis can be implemented without bias, fluctuations of  $KE$  and  $R$  time  
451 series were used (Lavallée et al. 1993). Characterization of variability in  $KE$  field for the  
452 event considered here can be seen in Figure 2b with TM (Eq. 16 in log-log plot), DTM

453 graphs (Eq. 20 in log-log plot) and UM parameter values. As shown in TM and DTM  
454 graphs, the field exhibits a very good multifractal behaviour with a single scaling regime  
455 from 30 s to 64 min. For example, TM coefficient of determination  $r^2$  for  $q = 1.5$  was  
456 greater than 0.99. Values of UM parameters  $\alpha$ ,  $C_1$ , and  $H$  for  $KE$  of this particular event  
457 were 1.820, 0.311 and 0.547 respectively. It should be mentioned that  $H$  computed on the  
458 fluctuations of  $KE$  was found to be equal to 0.189, meaning the taking the fluctuations  
459 indeed enabled retrieval of a conservative field from the original smoother field.  $R$  also  
460 exhibited excellent scaling behavior and corresponding values of UM parameters  $\alpha$ ,  $C_1$ ,  
461 and  $H$  for this event were 1.655, 0.229, and 0.100 respectively.

Figure 2: a) Time series of  $R$ , time series of  $KE$ ,  $KE$  vs  $R$  graph b) and c) MF analysis graphs with  $KE$  and  $R$  (log-log plot of Eq. 16 and Eq. 20 for TM and DTM analysis respectively and log-log plot of exponents in Eq. 20 for UM parameters) for Pars 1 event 16 September 2017 11:35:00 to 13:00:00



462  $KE$  and  $R$  analyzed from every event exhibited similar multifractal characteristics with  
463 a unique scaling regime. The quality of scaling was examined using coefficient of deter-  
464 mination,  $R^2$  for  $q = 1.5$  in TM analysis, and as previously mentioned events with values  
465  $< 0.9$  were discarded. 9.8 % percentage of total events were rejected on this basis. For  
466 the events with good scaling behaviour, robust retrieval of multifractal parameters  $\alpha$ ,  $C_1$ ,  
467  $\beta$  and  $H$  was possible. For example, the assessed values of  $\alpha$  and  $C_1$  for both  $KE$  and  $R$   
468 time series, exhibited a maximum standard deviation (using various values of  $q$  in DTM  
469 analysis) of 0.0584 and 0.0670 for measurement at EP-SIRTA and 0.0446 and 0.0443 for

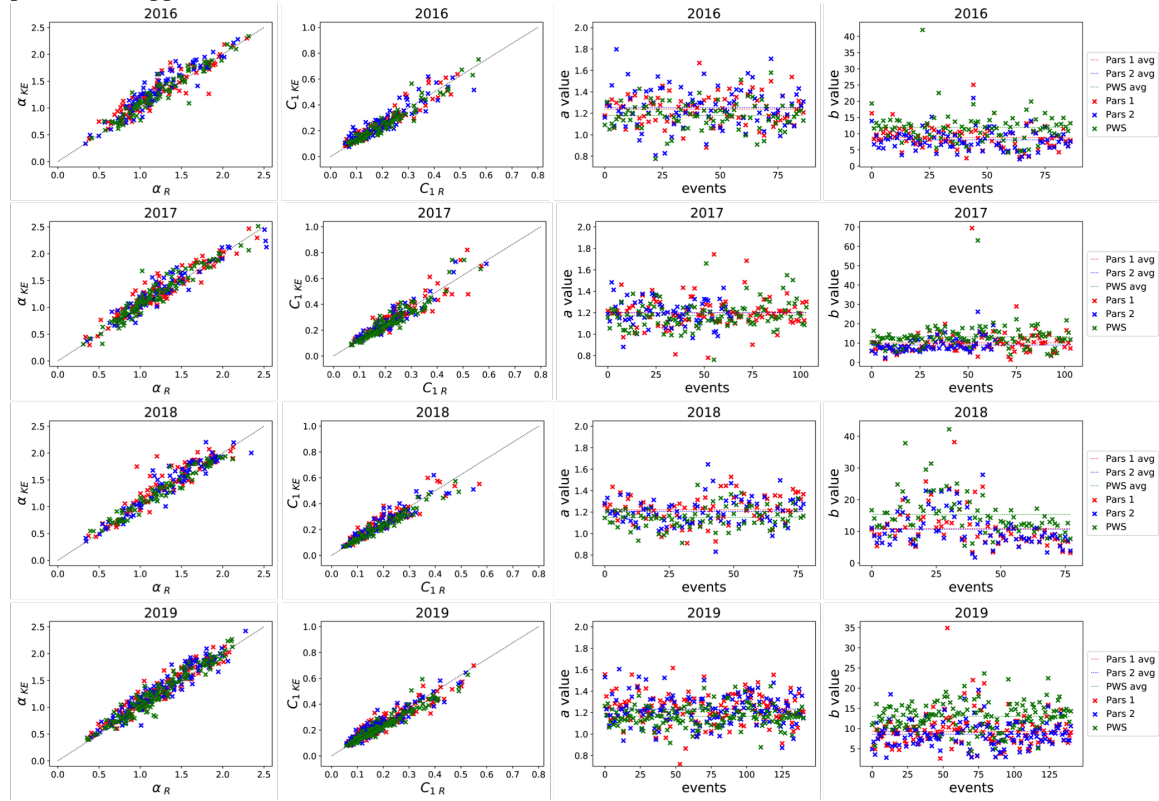
470 measurements at ENPC.

471 Figure 3 displays the values of multifractality index  $\alpha$  and mean intermittency  $C_1$  for  
472 all the studied events for both fields. It appears that the values of  $\alpha$  for  $KE$  and  $R$  are rather  
473 well distributed along the bisector. This pattern suggests a power law relation between  
474 these two quantities as discussed in section 2.2 (Eq. 21) where  $\varepsilon_1$  and  $\varepsilon_2$  are  $KE$  and  $R$   
475 respectively (i.e.  $KE = bR^a$  as in Eq. 11). The exponent of the power law  $a$  was deduced  
476 from corresponding  $\alpha$  and  $C_1$  values of UM fields ( $KE$  and  $R$ ) for every event subjected to  
477 UM analysis, using the expected relations for power law related UM fields, i.e. Eq. 24. The  
478  $\alpha$  used is the average of  $\alpha_{KE}$  and  $\alpha_R$  (which were anyway similar). Value of prefactor  $b$  was  
479 estimated by fitting Eq. 11 at maximum resolution with estimated values of  $a$  on event's  
480  $KE - R$  graph. For the event used as illustration, we found  $a = 1.083$  and  $b = 11.493$ . This  
481 power law fit from UM parameters is displayed in Figure. 2a as  $KE_{UM}$  in the  $KE$  vs.  $R$   
482 graph.

483 Variation of computed power law exponent  $a$  and prefactor  $b$  are also shown in Fig. 3 as  
484 time series of events for each year. Graphs of remaining years are provided in appendix A.  
485 For 3.5 % of total events filtered, estimates of  $\alpha$  was found to be greater than the theoretical  
486 maximum ( $> 2$ ); however  $a$  values for those events were found to be consistent with the  
487 overall average. Despite appreciable variability in UM parameters across events, values of  
488  $a$  and  $b$  showed overall stability in the short range of values specified in graphs, suggesting  
489 robustness of the estimates. Year wise average values over the events for  $a$  and  $b$ , and  
490 number of events for all three disdrometers are given in Table 2. Average values of power  
491 law parameters from the events that were common between the three disdrometers are also  
492 shown in the same table. Comparable values of  $a$  and  $b$  were observed in both cases. A  
493 clear range of variation can be observed between the two types of disdrometers. For both

494 Parsivel<sup>2</sup> the average  $a$  and  $b$  were similar and around 1.22 and 8.17 respectively, while for  
495 PWS100  $a$  values were consistently lower than that of Parsivel<sup>2</sup> at around 1.17 and  $b$  values  
496 greater at 13.02. PWS generally registered slightly higher intensities than Pars 1 and Pars 2.  
497 It should be mentioned that  $a$  and  $b$  seem to show a very rough correlation where values of  
498  $b$  decreases with increase in values of  $a$ . But between make of instruments, irrespective of  
499 the type and number of events, values of  $a$  remains rather constant. Differences in values  
500 obtained between Parsivel<sup>2</sup> and PWS is expected due to operational differences between  
501 disdrometers. Such effects are also reported in Johannsen et al. (2020a) who showed biases  
502 in measurement and subsequent  $R - KE$  relation due to difference in type of sensors used  
503 (three optical disdrometer were used). Angulo-Martínez and Barros (2015) also highlighted  
504 some differences among various Parsivel<sup>2</sup>.

Figure 3: Year wise  $\alpha$  and  $C_1$  variation on event based analysis from 2016 to 2019 (remaining years are provided in appendix)



year	location	disdrometer	total events			common events		
			# events	avg $a$	avg $b$	# events	avg $a$	avg $b$
2013	ENPC	Pars 1	29	1.248	9.224		1.246	9.224
		Pars 2	11	1.254	9.093	11	1.254	9.093
		PWS	29	1.186	11.912		1.193	11.748
2014	ENPC	Pars 1	84	1.207	11.548		1.207	11.548
		Pars 2	84	1.186	11.065	84	1.186	11.065
		PWS	84	1.159	14.065		1.159	14.065
2015	ENPC	Pars 1	38	1.235	9.459		1.235	9.459
		Pars 2	38	1.207	8.532	38	1.207	8.532
		PWS	38	1.190	11.712		1.190	11.712
2016	ENPC	Pars 1	87	1.238	8.802		1.238	8.802
		Pars 2	87	1.253	8.069	87	1.253	8.069
		PWS	87	1.180	11.939		1.180	11.939
2017	EP-SIRTA*	Pars 1	102	1.197	10.126		1.193	10.013
		Pars 2	65	1.202	8.944	65	1.202	8.944
		PWS	104	1.176	13.238		1.151	15.282
2018	ENPC	Pars 1	78	1.221	10.832		1.221	10.832
		Pars 2	78	1.205	10.647	78	1.205	10.647
		PWS	78	1.151	15.282		1.151	15.282
2019	ENPC	Pars 1	138	1.231	9.254		1.238	8.913
		Pars 2	130	1.231	8.550	130	1.231	8.55
		PWS	141	1.165	13.024		1.649	12.960

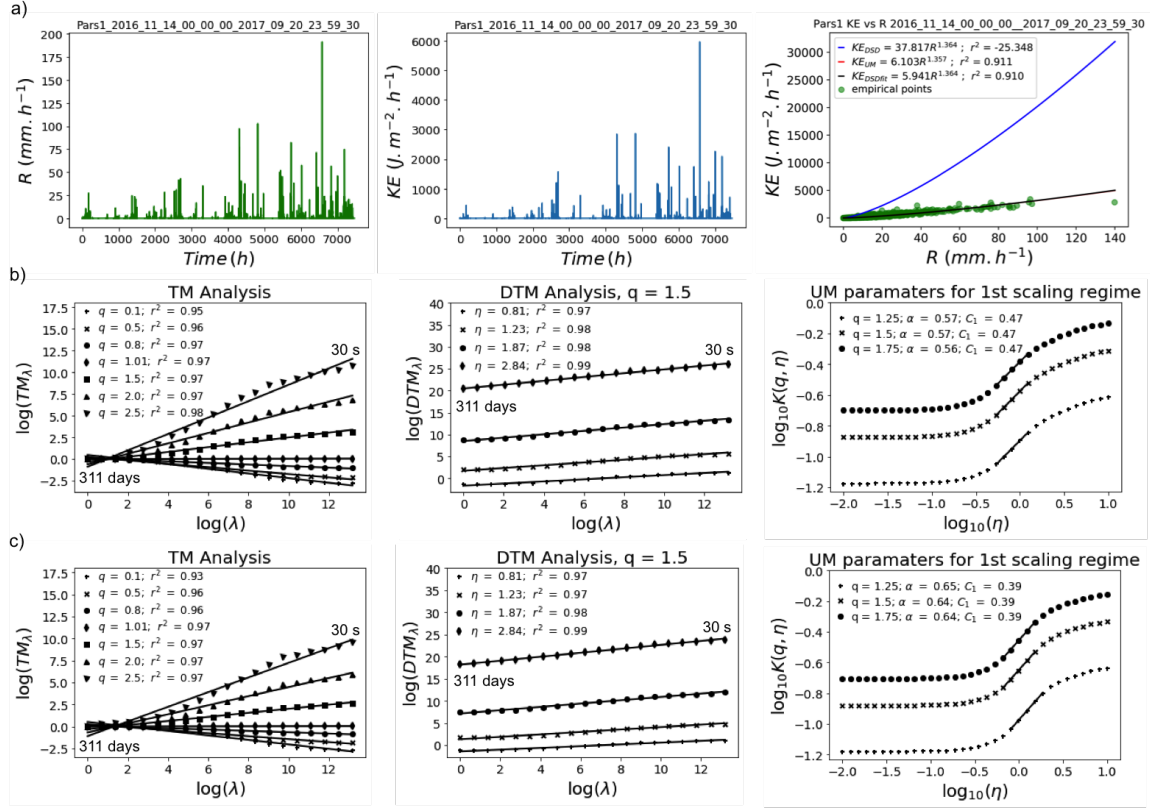
Table 2:  $a$ ,  $b$  and no. of events analysed between 2013 and 2019 according to location of measurement and disdrometer used.

\* From Nov 2016 to Sep 2017 as shown in Table 1; preceding and succeeding years are adjusted accordingly, refer Table 1.

#### 505 4.3. Multifractal analysis of continuous data (year based analysis)

506 To illustrate year based analysis, continuous time series and UM analysis for Pars 1  
507 disdrometer for the year 2017 is shown in Fig. 4. Both  $KE$  and  $R$  fields showed similar  
508 multifractal features. Scaling behaviour of  $KE$  is shown in Fig. 4b. A unique scaling  
509 regime from 30 s to 311 days was considered.

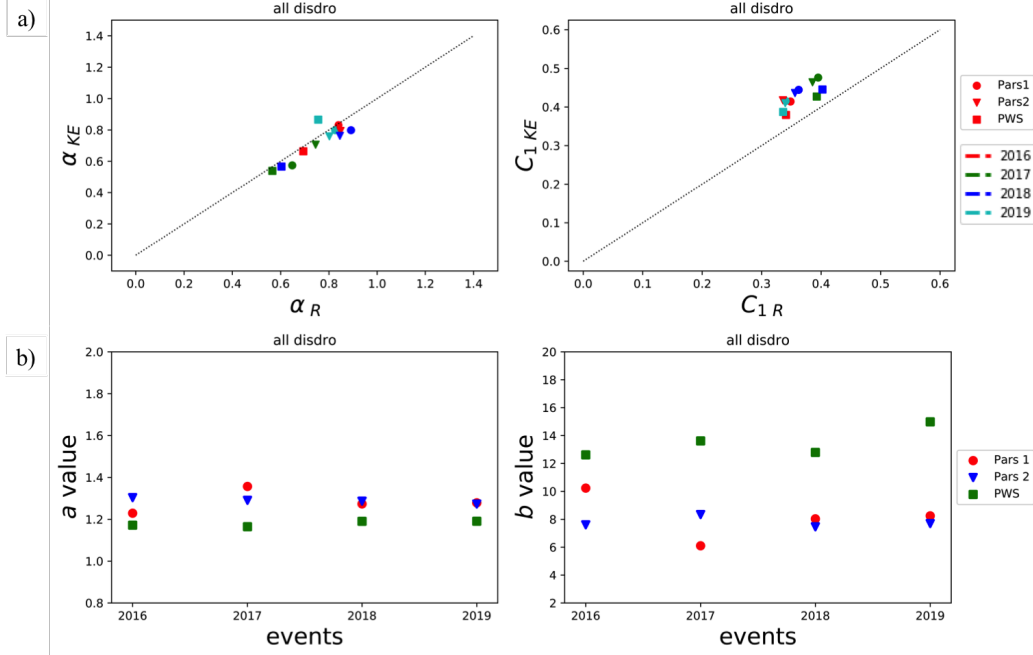
Figure 4: a) Time series of  $R$ , time series of  $KE$ ,  $KE$  vs  $R$  graph b) and c) Multifractal analysis graphs with  $KE$  and  $R$  (log-log plot of Eq. 16 and Eq. 20 for TM and DTM analysis respectively and log-log plot of exponents in Eq. 20 for UM parameters) using the year based analysis, for Pars 1, for 2017



510 Variation of UM parameters  $\alpha$  and  $C_1$  between  $KE$  and  $R$  fields as well as values of  
 511 power law exponent  $a$  and prefactor  $b$  (computed by fitting the relation at maximum res-  
 512 olution) are displayed in Fig. 5. Precise values of  $a$  and  $b$  according to the year of mea-  
 513 surement are given in Table 3 in annexes. The curve  $KE_{UM}$  in the  $KE$  vs  $R$  graph of Fig.  
 514 4a, shows the power law fit using UM parameters for continuous data of 2016, for Pars 1.  
 515 Values and trend of variation are similar to that observed for the event based analysis. This  
 516 confirms the robustness of the discussed power law, which is valid and retrieved not only  
 517 at the event scale, but also at the year scale.



Figure 5: a) Results from multifractal analysis on continuous year wise data set: a)  $\alpha$  and  $C_1$  (DTM); b)  $a$  and  $b$  values



Disdrometer	coefficient	2016	2017*	2018	2019
Pars 1	$a$	1.229	1.356	1.273	1.278
	$b$	10.232	6.102	8.033	8.247
Pars 2	$a$	1.303	1.290	1.286	1.273
	$b$	7.605	8.338	7.463	7.695
PWS	$a$	1.171	1.164	1.190	1.190
	$b$	12.616	13.614	12.792	14.981

Table 3:  $a$  and  $b$  from year based analysis, from 2014 to 2019 according to disdrometer used for measurement.

\* From Nov 2016 to Sep 2017 as shown in Table 1; preceding and succeeding years are adjusted accordingly

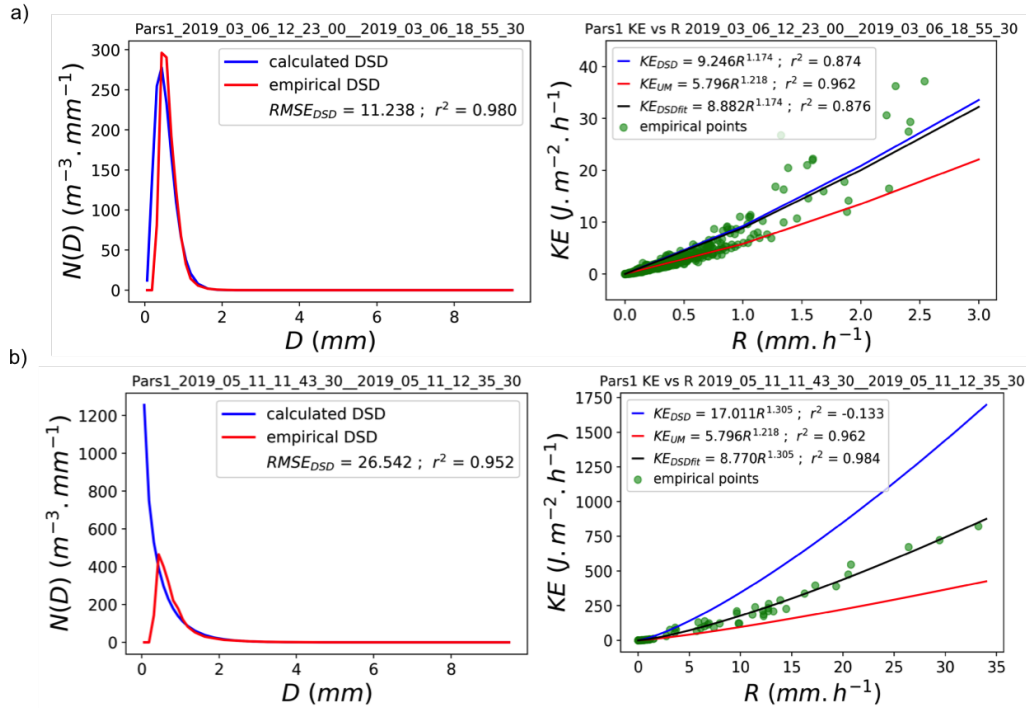
518 4.4. Power law coefficients from DSD parameters

519 To understand previous findings further, values of  $a$  and  $b$  were computed from theo-  
 520 retical framework discussed in section 2.1. Rainfall DSD was assumed to follow gamma  
 521 distribution (Eq. 3) and theoretical values of  $a$  and  $b$  (denoted as  $a_{DSD}$  and  $b_{DSD}$  from here

522 on for clarity) were computed as per Eq. 12 after estimating values of gamma DSD param-  
523 eters ( $\mu$ ,  $N_0$  and  $\Lambda$ ) with the help of method of moments. The variation of rain rate  $R$  was  
524 accurately reproduced using M234 moment estimators (Fig. 8d for example). Theoretical  
525 power law relation ( $KE_{DSD} = b_{DSD}R^{a_{DSD}}$ ) was then compared with the one estimated using  
526 UM analysis (represented from here on as  $KE_{UM}$  with coefficients  $a_{UM}$  and  $b_{UM}$  for clarity)  
527 for every events. Average values of DSD parameters, estimation error and corresponding  
528  $a_{DSD}$  and  $b_{DSD}$  for Pars 1 events are given in Table 4 according to event's rain type (defini-  
529 tion introduced later).  $RMSE_{DSD}$  is the root mean square error (RMSE) between empirical  
530 DSD and theoretical gamma DSD with fitted parameters (using the value for the center of  
531 each diameter class  $D_i$ ). It is used here as an indicator of the quality of fit of the assumed  
532 gamma DSD distribution and the empirical one. For  $RMSE$  estimation only the portion of  
533 DSD above 0.5 mm diameter class was considered as smaller drops have lesser contribution  
534 in overall  $KE$  and  $R$ , and are associated with greater uncertainties in measurement. Also,  
535 higher size drops (above 9 mm diameter class) were not observed during the events and are  
536 hence not shown in DSD figures discussed after (Fig. 6 and Fig. 8).

Figure 6:

a) 2019 Pars 1 event where empirical DSD corresponds with gamma distribution and b) where it doesn't follow gamma distribution (DSD displayed only till diameter class around 9 mm as higher drops were not observed during the events)

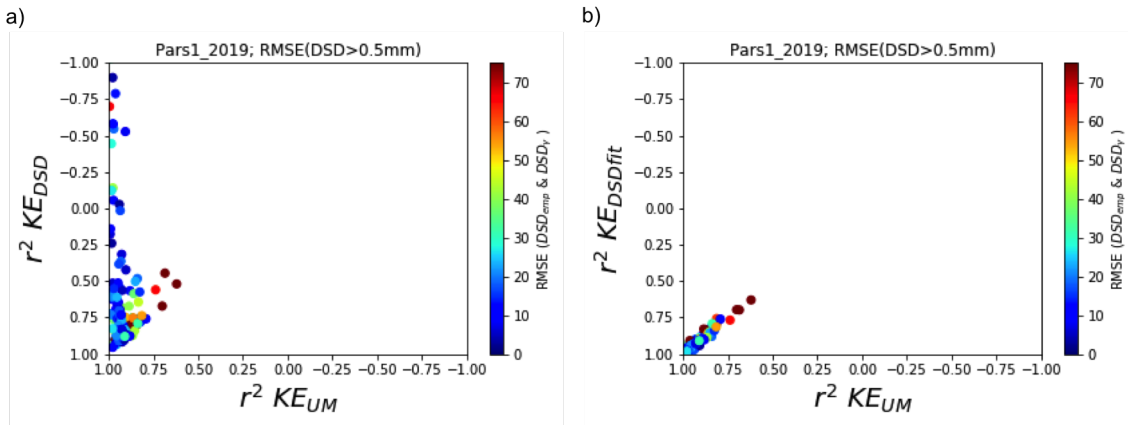


537 Figure 6 represents two extremes cases among events from year 2019 for Pars 1: one  
 538 event (Fig. 6a) where empirical DSD corresponds with a gamma distribution and one  
 539 event (Fig. 6b) where it does not. The DSD fitting as well as the  $KE$  vs.  $R$  plots (fitted  
 540 with power law relation from UM analysis and DSD extraction) are also displayed. For  
 541 the event with empirical DSD closer to estimated gamma DSD, both power law relations  
 542 ( $KE_{UM}$  &  $KE_{DSD}$ ) are similar and show good fit. For the other event, power law relation  
 543 from DSD approach shows a considerable deviation from actual values of  $KE$ , with a strong  
 544 overestimation. Power law from UM analysis on the other hand still provides a close fit. To  
 545 see if there is such a trend through all the events, coefficient of determination of both  $KE$  –  
 546  $R$  fits were plotted against each other ( $r_{UM}^2$  vs  $r_{DSD}^2$ ) and compared using corresponding

547 values of  $RMSE_{DSD}$  (Fig. 7a). Negative value of coefficient of determination for DSD  
 548 ( $r^2_{DSD}$ ) is due to the high difference between  $KE_{DSD}$  and empirical  $KE$  in certain events.  
 549 From the plot it is evident that a generalized conclusion - theoretical values of  $a$  and  $b$   
 550 ( $a_{DSD}$  &  $b_{DSD}$ ) works well in cases where empirical DSD coincides with gamma DSD -  
 551 is not possible. However, there were many events with good DSD correspondence with  
 552 gamma, where  $KE_{DSD}$  fitted data better.

Figure 7:

a)  $r^2$  values between  $KE_{UM}$  and  $KE_{DSD}$  for Pars1, 2019; b)  $r^2$  values between  $KE_{UM}$  and  $KE_{DSDfit}$  ( $b$  from fitting of data) for Pars1, 2019 (RMSE calculated by considering only parts of DSD where drop diameter > 0.5 mm)

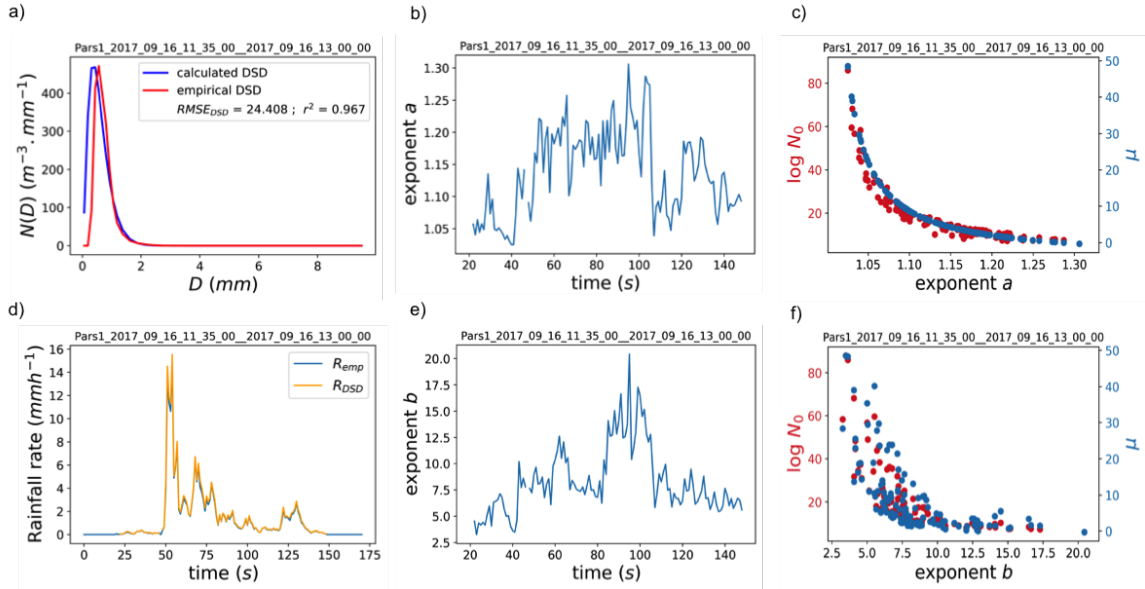


553 To investigate further this issue and given that the DSD is available for all time steps,  
 554  $a_{DSD}$  and  $b_{DSD}$  were computed for each time step to study their variations within a rainfall  
 555 event. Figure 8 displays their temporal evolution for the event in Fig. 2. Values of pa-  
 556 rameters, especially  $b_{DSD}$ , shows considerable variation within an event. These variations  
 557 basically come from variations in DSD parameters  $\mu$  and  $N_0$  as it can be seen on Fig. 8c  
 558 and 8f (also evident from Eq. 12), which are reflecting physical variations in the rainfall  
 559 process. It should be mentioned that during this event (and other events), gamma DSD  
 560 parameters were able to properly reproduce observed rain rate (Fig. 8d), meaning that the

561 assumption of gamma DSD distribution and the M234 moment estimator approach remains  
562 valid throughout the event(s). This suggests that the variability of  $a_{DSD}$  and  $b_{DSD}$  observed  
563 at event scale is also valid within events at much smaller scales. This could explain some  
564 of the bias previously observed with DSD approach developed in this paper.

565 As there were still considerable variation in  $r^2$  values of  $r_{KE_{DSD}}^2$  (Fig. 7a), especially if  
566 we compare with  $KE_{UM}$  which shows better  $r_{KE_{UM}}^2$  regardless the type of event, a question  
567 of possible bias arises due to difference in methods of estimation of power law coefficients.  
568 Unlike  $KE_{DSD}$  for which both coefficients are obtained from theoretical relation involving  
569 DSD parameters, for  $KE_{UM}$  only coefficient  $a$  is fully estimated from UM analysis. Pref-  
570 actor  $b_{UM}$  is obtained by fitting the data at highest available resolution (30 s) using UM  
571 estimated  $a_{UM}$ . Such discrepancy may introduce a bias in the comparison between DSD  
572 and UM approach to retrieving a power law. Hence, to understand this further and to make  
573 a fair comparison, a new power law was considered where only  $a_{DSD}$  is obtained using Eq.  
574 12 while  $b_{DSD}$  is calculated from fitting of data (denoted hereafter  $b_{DSDfit}$ ) -  $KE_{DSDfit}$ . Fig-  
575 ure 7b shows  $r^2$  between  $KE_{DSDfit}$  and  $KE_{UM}$ ; and it can be seen that comparable fits are  
576 obtained between UM and DSD power laws regardless the nature of DSD, thus illustrating  
577 the presence of bias. This can also be observed in previous  $KE - R$  plots - Fig. 2a, Fig. 4a,  
578 Fig. 6a and Fig. 6b.

Figure 8: Variation of  $a_{DSD}$  and  $b_{DSD}$  for each time steps in an event (same event discussed in Fig. 2)  
a) Correspondence between empirical DSD (red) and calculated DSD (blue); d) Correspondence between empirical rainfall rate ( $R_{emp}$ ) and that calculated from DSD moments ( $R_{DSD}$ )  
b) & e) Variation of  $a_{DSD}$  and  $b_{DSD}$  within the event  
c) & f) Variation of  $a_{DSD}$  and  $b_{DSD}$  with DSD parameters  $N_0$  and  $\mu$

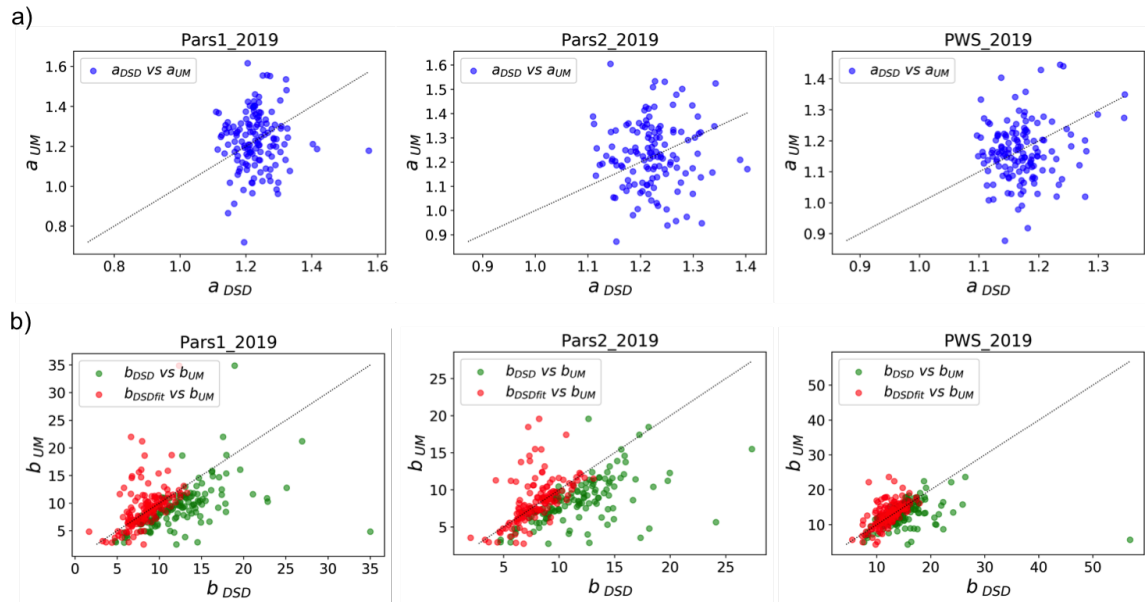


#### 579 4.5. Comparison between $KE - R$ relations

580 Though the power law obtained is similar from UM and DSD analysis, they fit the data  
581 differently due to difference in values of corresponding parameters.  $KE_{DSD}$  in  $KE$  vs.  $R$   
582 graph of Fig. 2a and Fig. 4a shows the power law fit using theoretical  $a$  and  $b$  ( $a_{DSD}$   
583 &  $b_{DSD}$ ), from DSD parameters for illustrated examples in event based and year based  
584 analysis analysis. The difference in fit between two calculations of power law,  $KE_{UM}$  and  
585  $KE_{DSD}$  can be observed there. It can be seen from Fig. 7a that  $KE_{UM}$  gives rather good  
586 fit regardless the event specific DSD shape, while  $r^2$  value of  $KE_{DSD}$  fluctuates. However,  
587 there is a limited tendency for good fit towards empirical DSD following gamma distribu-  
588 tion (for few events like Fig. 6). Figure 9 shows variation of  $a$  and  $b$  estimates from DSD  
589 and UM for Pars 1, Pars 2 and PWS for all events in year 2019. Values of  $a_{UM}$  and  $a_{DSD}$

590 are mostly clustered around the approx. 1.2 for all three disdrometers with no clear relation  
 591 between them. On the other hand, values of  $b_{UM}$  and  $b_{DSD}$  are more spread out, ranging ap-  
 592 proximately from 2 to 25. Such spreading for  $b_{DSD}$  is not caused by the computation issues  
 593 previously mentioned since it is visible on both  $b_{DSD}$  and  $b_{DSDfit}$ . This observed scattering,  
 594 which is comparable regardless the estimation techniques and device, suggests that even if  
 595 the power relation between  $KE$  and  $R$  remains relevant for all events, its parameters exhibit  
 596 strong variability between events.

Figure 9: Variation of power law coefficients from UM and DSD calculation, for events in year 2019 (similar variation for other years also)



597 The constants of the power law relations (in literature) between  $KE$  and  $R$  are not uni-  
 598 versal in application and need tweaking as per the rainfall type, measurement location as  
 599 well as techniques. Based on assumed dependence between DSD parameters and rain rate  
 600 in formulation exponents; Salles et al. (2002) suggest four range of values for exponent  
 601 values of the universal power law, and Uijlenhoet and Stricker (1999) propose six differ-

ent relationships in their research. Hence, in order to refine the analysis of this observed variability between events and to examine possible dependence of  $a$  and  $b$  on type of rain, events were sorted according to rain types. Table 4 shows, for Pars 1, averaged values of gamma DSD parameters, indicator of the quality of the fitting, and power law coefficients from both DSD and UM estimations across events sorted according to type of rainfall. Tables for Pars 2 and PWS are given in appendix - (Table B1). For defining types of rainfall from light to extreme, a classification based on intensity (Tokay and Short, 1996) was employed.  $\mu$ ,  $\lambda$  and  $N_0$  tend to decrease with heavier events, with a stronger trend for  $N_0$ . It should be mentioned that an opposite trend is reported in reference used; this could be due to instrumental bias in DSD measurement as the impact disdrometer used in reference is known to under-represents smaller drops in intense rainfall. It should also be noticed that the capacity of the gamma distribution to model observed DSD diminishes with heavier rainfall (this is more visible on RMSE estimation over whole range of DSD and less prominent in displayed estimate here involving only higher drop sizes), suggesting a limit in validity for gamma distribution assumption. Also, an increase in mean diameter is noticed (not shown here). These findings are in agreement with previous studies reported in literature (Carollo Francesco Giuseppe and Ferro Vito, 2015). When it comes to power law coefficients, both  $a_{DSD}$  and  $b_{DSD}$  show increase with increasing rain rate, and this is more pronounced for  $b_{DSD}$ . Given the observed decrease in validity of gamma distribution of DSD, these trends should be taken carefully because they are likely to be mere artifacts and not representative of the actual process at stake. In the case of UM estimated parameters,  $b_{UM}$  follows similar trend with stronger magnitude (going from 8 to 33) while  $a_{UM}$  shows slight reduction in value with increasing rain rate (from 1.25 to 1.05). Since the average rainfall criteria used for classification here is a somehow arbitrary and more biased towards



626 lesser rainfalls, another classification relying on the maximum of 10 minute moving av-  
627 erage was also employed. The results are tabulated in Table B2 (Appendix B). With this  
628 criteria which is more biased towards larger rainfall events, consistent and similar results  
629 are retrieved when it comes to values and variation of power law coefficients  $a$  and  $b$ , from  
630 DSD as well as UM, suggesting robustness of obtained trends.

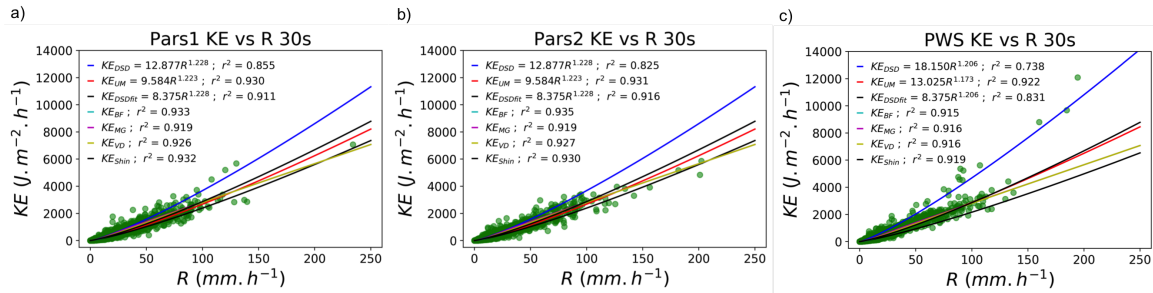
		from DSD moments							from UM	
		# events	$\mu$	$N_0$	$\Lambda$	$RMSE_{DSD}$	$a_{DSD}$	$b_{DSD}$	$a_{UM}$	$b_{UM}$
			$(m^{-3}mm^{-1-\mu})$	$(mm^{-1})$	$(m^{-3}mm^{-1})$			$(Jm^{-2}mm^{-a}h^{a-1})$		$(Jm^{-2}mm^{-a}h^{a-1})$
very light	$R < 1$	188	2.53	$1.05 \times 10^7$	6.34	19.32	1.196	11.056	1.253	8.554
light	$1 \leq R < 2$	173	1.75	$1.71 \times 10^9$	4.81	24.53	1.222	12.694	1.224	9.334
moderate	$2 \leq R < 5$	141	1.01	$2.51 \times 10^7$	3.42	21.25	1.256	15.580	1.214	10.216
heavy	$5 \leq R < 10$	36	0.44	$1.40 \times 10^4$	2.25	20.45	1.275	17.259	1.210	11.336
very heavy	$10 \leq R < 20$	14	-0.05	$1.17 \times 10^3$	1.63	21.14	1.300	18.419	1.119	19.670
extreme	$R \geq 20$	4	-0.85	$5.48 \times 10^2$	1.04	44.82	1.353	44.82	1.052	33.250

Table 4: Variation of DSD parameters and power law coefficients according to the type of rainfall for Pars 1 ( $R$  = average rain rate for rainy time steps)

631 To evaluate the performance of established power law relationship across scales of  
632 measurement,  $KE$  was calculated from empirical  $R$  using  $KE = bR^a$  with average val-  
633 ues of  $a$  and  $b$  and compared with existing relations in literature. For this purpose, ex-  
634 ponential equation used in RUSLE ( $KE_{BF} = 29[1 - 0.72\exp(-0.05R)]$ , Brown and Fos-  
635 ter, 1987), exponential equation used in RUSLE2 ( $KE_{MG} = 29[1 - 0.72\exp(-0.082R)]$ ,  
636 C. McGregor et al., 1995), universal exponential law proposed by van Dijk et al. (2002)  
637 ( $KE_{VD} = 28.3[1 - 0.52\exp(-0.0421R)]$ ) and ideal power law proposed by Shin et al.  
638 (2016) ( $KE_{Shin} = 10.3R^{11/9}$ ) were used alongside  $KE_{UM}$  and  $KE_{DSD}$  derived here (power  
639 law where fitted values of  $b$  were used - $KE_{DSDfit}$ - is also included). For UM and DSD  
640 power laws in this paper, average values among all calculated events were used for repre-  
641 sentation according to make of disdrometers. Average values of coefficient  $a$  and prefactor  
642  $b$  were close for both Pars 1 and Pars 2 disdrometers and were taken common for the

643 Parsivel<sup>2</sup> make (for both UM and DSD). It is also logical to keep values of power law coef-  
 644 ficients separate between different disdrometers as varying  $KE$  and  $R$  estimation has been  
 645 reported across types of disdrometers used (Angulo-Martínez and Barros, 2015; Angulo-  
 646 Martínez et al., 2018; Johannsen et al., 2020b). Figure 10 shows  $KE$  and  $R$  variation fitted  
 647 with above mentioned equations for maximum resolution, 30 seconds. As expected  $KE_{UM}$   
 648 provides better fit than  $KE_{DSD}$  (and  $KE_{DSDfit}$  shows closer fits). With respect to empirical  
 649 data, UM power laws exhibit - for all three devices - slightly better or comparable coef-  
 650 ficient of determination ( $r^2$ ) with regards to commonly used relations. The exponential  
 651 equations from literature appear very close to each other. The relatively lower values of co-  
 652 efficient of determination than that during event based fits are likely to be due to the effect  
 653 of using average values instead of event specific values of  $a$  and  $b$ .

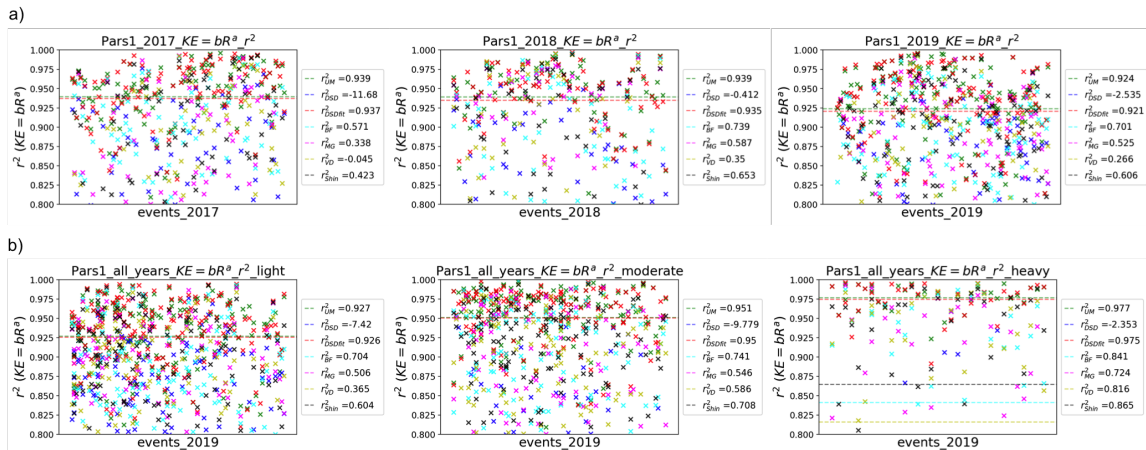
Figure 10: Fitting of empirical  $KE - R$  using power laws from UM and DSD, and popular expressions from literature for a) and b) Parsivel<sup>2</sup>, and c) PWS



654 To understand the performance of various equations further,  $r^2$  was examined across  
 655 various time periods and also across different type of rains. Results are displayed in Fig.  
 656 11. It appears that regardless of the duration as well as type of rainfall considered, tuned re-  
 657 lations  $KE_{DSDfit}$  and  $KE_{UM}$  perform significantly better than the other fixed ones, yielding  
 658  $r^2$  values greater than 0.9. In most cases,  $KE_{UM}$  exhibits slightly better performances with  
 659 a difference not significant. The analysis also confirmed the poor performance of  $KE_{DSD}$ ,

660 i.e. the inability of the DSD approach to properly fit the prefactor 'b'. It should be stated  
661 that the power law obtained using UM analysis is not providing significant performance en-  
662 hancement compared to that obtained from gamma DSD while the prefactor  $b$  is estimated  
663 from fitting of the data ( $KE_{DSDfit}$ ). However, with UM analysis, it is possible to discard  
664 all assumptions of DSD following a gamma distribution thus eliminating the known inad-  
665 equacy of gamma model and its sensitivity to sampling resolution (Adirosi et al., 2013,  
666 2014; Ignaccolo and De Michele, 2014; Adirosi et al., 2016; Gatidis et al., 2020). The in-  
667 adequacy was observed in current analysis as well, as previously discussed and illustrated  
668 in Fig. 6 and Fig. 7.

Figure 11:  $r^2$  values of various KE-R relations discussed for a) events analysed in 2017, 2018 and 2019 (for Pars 1),  
b) all events grouped according to type of rain - light, moderate and heavy  
Average value for each year is given in legends, with  $KE_{UM}$  and  $KE_{DSDfit}$  displayed in dotted lines



## 669 5. Conclusion

670 We examined the relationship between rainfall intensity  $R$  and time specific kinetic  
671 energy  $KE$  using high resolution (30 s) optical disdrometer data from the past 7 years in  
672 Paris region. The variability across scales of both parameters was characterized using the

673 framework of Universal Multifractals. Analyzed  $KE$  and  $R$  times series were found to  
674 convey excellent multifractal behaviour (which is novel for  $KE$ ), with multifractality index  
675  $\alpha$  and mean intermittency  $C_1$  suggesting power law relation between them; it can be written  
676 as  $KE = bR^a$ . Such power law was found to be valid across analyzed data, i.e. independent  
677 of the event, on whether they are computed on event or yearly basis, and of the underlying  
678 corresponding drop size distribution. Some variability in the value of the exponent  $a$  and  
679 prefactor  $b$  is reported according the event and disdrometer type.

680 As shown by previous results, similar power-law can be theoretically obtained when  
681 relying on the common assumption of a gamma distribution for the DSD, and a power-  
682 law relation between fall velocity and equivolumic drop diameter.  $KE$ - $R$  relation obtained  
683 through UM analysis was compared with results found using this common framework and  
684 biases were acknowledged. Despite some exceptions, in most cases when a gamma DSD  
685 approximation was relevant, estimations of power law parameters from the two approaches  
686 were found to be consistent. When not, UM approach provided slightly better fit in general  
687 but not in a significant manner, keeping in mind that the pre-factor needs to fitted to data in  
688 both cases to ensure a fair comparison. Thus, the newly discussed power law relationship  
689 between  $KE$  and  $R$  retrieved with the help of UM framework generalizes previous results  
690 and theoretical formulations without having to rely on the ad-hoc assumption of a gamma  
691 DSD. The main underlying assumption of UM framework, i.e. that there is an underlying  
692 multiplicative process, is actually physically based in the sense that comes from the scale  
693 invariance features of the Navier-Stokes equations. Here, a UM analysis confirmed the  
694 validity of this assumption. Deriving the power-law relation in a multifractal framework  
695 opens the path to new approaches for simulating  $KE$  from simple  $R$  measurements. Given  
696 that complete  $KE$  measurement is much less available than  $R$  one, this impact will be

697 investigated further in future work.

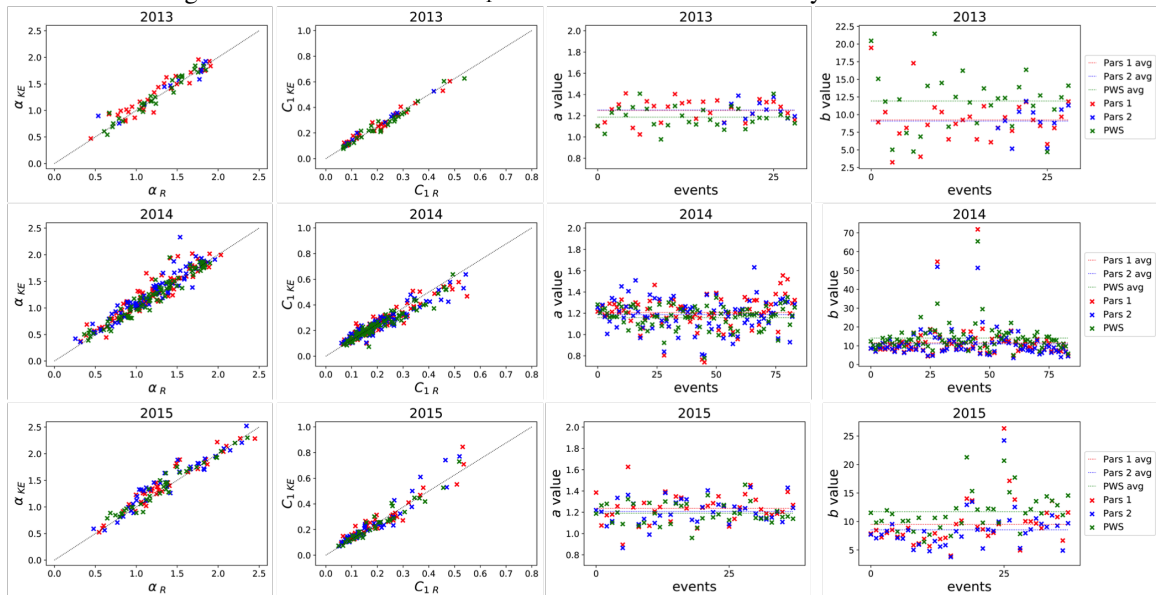
698 For the future, it would hence be interesting to expand the data set across geographi-  
699 cal and meteorological conditions to reduce the biases that might have accumulated from  
700 region of measurement.

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705 France region RadX@IdF Project, and the ANR JCJC RW-Turb project (ANR-19-CE05-  
706 0022-01)

## 707 Appendix A: Multifractal events year wise

Figure A1: Year wise  $\alpha$  and  $C_1$  variation on event based analysis 2013 to 2015



708 **Appendix B: Method of moments**

709 **The formulation of moment estimator**

710 Gamma distribution parameters were estimated from second, fourth and sixth moments  
711 using following relations:

712

713 M234:

$$D_m = M_4/M_3 \quad (\text{B.1})$$

$$\eta = \frac{(M_3^2)}{(M_2M_4)} \quad (\text{B.2})$$

$$\mu = \frac{1}{(1 - \eta - 1)} - 4 \quad (\text{B.3})$$

$$\Lambda = \frac{M_2}{M_3}(\mu + 3) \quad (\text{B.4})$$

$$N_0 = \frac{M_2\Lambda^{(\mu+3)}}{\Gamma(\mu + 4)} \quad (\text{B.5})$$

714

715

716 **DSD parameters and power-law coefficients**

		from DSD moments						from UM			
		# events	$\mu$	$N_0$	$\Lambda$	$RMSE_{DSD}$	$a_{DSD}$	$b_{DSD}$	$a_{UM}$	$b_{UM}$	
			$(m^{-3}mm^{-1-\mu})$	$(mm^{-1})$	$(m^{-3}mm^{-1})$		$(Jm^{-2}mm^{-a}h^{a-1})$		$(Jm^{-2}mm^{-a}h^{a-1})$		
Pars 2	very light	$R < 1$	190	2.53	$1.36 \times 10^7$	6.59	19.78	1.196	10.180	1.246	8.097
	light	$1 \leq R < 2$	160	1.61	$1.20 \times 10^8$	4.70	24.81	1.229	12.630	1.202	9.262
	moderate	$2 \leq R < 5$	101	0.82	$1.66 \times 10^6$	3.19	20.93	1.260	15.026	1.208	9.911
	heavy	$5 \leq R < 10$	28	0.26	$2.72 \times 10^4$	2.14	23.85	1.281	17.171	1.247	10.140
	very heavy	$10 \leq R < 20$	8	-0.38	$5.81 \times 10^2$	1.36	21.20	1.317	19.951	1.098	19.997
	extreme	$R \geq 20$	6	-0.94	$6.16 \times 10^2$	1.07	57.99	1.363	15.593	1.180	19.040
PWS	very light	$R < 1$	163	2.79	$7.82 \times 10^6$	4.73	10.95	1.187	10.95	1.194	12.070
	light	$1 \leq R < 2$	176	2.65	$2.64 \times 10^7$	4.15	14.30	1.195	14.30	1.154	12.703
	moderate	$2 \leq R < 5$	152	1.87	$4.73 \times 10^3$	2.85	14.98	1.216	20.024	1.169	13.136
	heavy	$5 \leq R < 10$	46	1.16	$2.82 \times 10^3$	1.96	18.90	1.253	24.439	1.193	13.933
	very heavy	$10 \leq R < 20$	18	0.59	$7.07 \times 10^2$	1.51	19.24	1.266	20.834	1.133	18.983
	extreme	$R \geq 20$	6	0.53	$4.09 \times 10^2$	1.28	20.63	1.26	21.031	1.165	18.016

Table B1: Variation of DSD parameters and power law coefficients according to the type of rainfall (R = average of rainy data points)

		from DSD moments						from UM			
		# events	$\mu$	$N_0$	$\Lambda$	$RMSE_{DSD}$	$a_{DSD}$	$b_{DSD}$	$a_{UM}$	$b_{UM}$	
			$(m^{-3}mm^{-1-\mu})$	$(mm^{-1})$	$(m^{-3}mm^{-1})$		$(Jm^{-2}mm^{-a}h^{a-1})$		$(Jm^{-2}mm^{-a}h^{a-1})$		
Pars 1	very light	$R < 1$	13	3.82	$1.30 \times 10^7$	8.19	8.19	1.161	9.471	1.245	8.396
	light	$1 \leq R < 2$	82	2.63	$7.76 \times 10^6$	6.39	15.96	1.191	10.962	1.271	8.747
	moderate	$2 \leq R < 5$	235	2.04	$1.10 \times 10^5$	5.32	21.77	1.213	12.528	1.227	9.119
	heavy	$5 \leq R < 10$	134	1.44	$2.22 \times 10^7$	4.19	24.79	1.235	13.826	1.225	9.369
	very heavy	$10 \leq R < 20$	54	0.51	$4.55 \times 10^3$	2.67	20.75	1.269	16.214	1.197	11.697
	extreme	$R \geq 20$	38	-0.22	$1.77 \times 10^3$	1.67	26.84	1.314	26.84	1.147	16.642
Pars 2	very light	$R < 1$	13	4.27	$2.02 \times 10^7$	8.94	6.75	1.154	9.097	1.217	7.833
	light	$1 \leq R < 2$	82	2.97	$2.96 \times 10^6$	7.04	15.74	1.183	10.165	1.253	8.245
	moderate	$2 \leq R < 5$	215	1.99	$9.38 \times 10^7$	5.43	22.15	1.214	11.857	1.212	8.810
	heavy	$5 \leq R < 10$	112	1.17	$3.20 \times 10^6$	3.94	24.33	1.239	12.969	1.222	9.280
	very heavy	$10 \leq R < 20$	42	0.39	$5.92 \times 10^3$	2.74	25.49	1.276	15.369	1.196	10.696
	extreme	$R \geq 20$	29	-0.63	$1.78 \times 10^3$	1.55	35.88	1.343	18.133	1.192	13.445
PWS	very light	$R < 1$	6	3.84	$3.39 \times 10^4$	6.15	6.78	1.16	13.92	1.223	13.322
	light	$1 \leq R < 2$	61	3.26	$5.29 \times 10^4$	5.27	10.13	1.175	14.687	1.187	11.997
	moderate	$2 \leq R < 5$	201	2.59	$1.91 \times 10^7$	4.20	12.56	1.195	17.258	1.176	12.602
	heavy	$5 \leq R < 10$	168	2.32	$2.78 \times 10^6$	3.50	14.65	1.202	19.013	1.168	12.702
	very heavy	$10 \leq R < 20$	75	1.72	$3.31 \times 10^3$	2.65	16.90	1.223	20.846	1.155	14.401
	extreme	$R \geq 20$	50	0.67	$1.11 \times 10^3$	1.69	19.48	1.264	20.879	1.156	15.430

Table B2: Variation of DSD parameters and power law coefficients according to the type of rainfall (R = maximum value of 10 minute moving average)

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