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1 **Undrained cylindrical cavity expansion/contraction in stiff**
2 **clays using a two-surface plasticity model**

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26 **Abstract**

27 This paper presents a semi-analytical solution of undrained cylindrical cavity
28 expansion/contraction problem using a two-surface plasticity model for natural stiff clays from the
29 view point of poromechanics. The present solution reduces the classical boundary problem into a
30 group of seven first-order ordinary differential equations via an auxiliary independent variable ζ
31 under undrained conditions. It strictly follows the premise of constant soil mass. Meanwhile, the
32 solution for the constant soil volume problem is used for comparison. It shows that constant soil
33 mass assumption is of paramount importance in developing solutions under high stress condition,
34 while constant soil volume induces non-negligible errors in effective stress and pore pressure
35 distributions. Based on the premise of constant soil mass, extensive parametric studies on cavity
36 expansion/contraction are undertaken to investigate the effects of the model key parameters on the
37 stress components distribution, excess pore pressure distribution and cavity expansion curves. The
38 present solution could be useful while dealing with geotechnical and petroleum engineering
39 problems involving stiff clays or soft claystones. The application in Praclay Gallery excavation in
40 natural Boom clay well demonstrates the validity of the present solution.

41 **Keywords:** stiff clays; cavity expansion/contraction; undrained condition; constant soil mass;
42 elastoplasticity; poromechanics

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55 **1. Introduction**

56 Cavity expansion theory has been widely applied to interpreting the results from some in situ tests
57 such as cone penetration and pressuremeter tests, analyzing the stresses and deformations induced
58 by wellbore drilling, gallery excavation, tunneling and pile installation. Over the past decades,
59 various analytical and semi-analytical solutions [1-13] for cylindrical and spherical cavity
60 expansions have been developed. With the increasing complexity of constitutive models, from
61 elastic models, elastic perfectly plastic models to critical state models, the solutions are
62 increasingly realistic. Among these models, original and modified Cam-clay models have gained
63 large popularity in solving the cavity expansion and contraction problems for their capability of
64 modelling strain hardening/softening behaviors of soils. Carter et al. [14] firstly investigated the
65 cavity expansion problems with original Cam-clay model by means of finite element analysis.
66 Collins and Yu [6] developed a general solution for large strain undrained cavity expansion using
67 both original and modified Cam-clay models with and without zero initial radius assumption. Cao
68 et al. [7], by following Collins and Yu's approach, developed a semi-analytical solution for
69 undrained cavity using modified Cam-clay model. Mo and Yu [8], extended the original Cam-clay
70 model to a unified state parameter model for clays and sands, then developed a novel solution for
71 both cylindrical and spherical cavity problems. Russell and Khalili [29], using the similarity
72 technique, investigated the cavity expansion problem in unsaturated soils by a unified bounding
73 surface model to consider the influence of suction and moisture content. That study demonstrated
74 there was no difficulty in solving cavity expansion problems using two-surface constitutive
75 models. However, it should be noted that the initial stress state is assumed isotropic and the
76 vertical stress is omitted in two stress invariants in that similarity approach [6]. Recently, Chen
77 and Abousleiman [9-11] proposed novel semi-analytical solutions for both drained and undrained
78 cylindrical cavity expansion/contraction problems. Their solutions removed the limitation on the
79 initial condition of isotropic stress state and depicted a more general case of in situ situation by
80 considering the vertical stress around the expanded/contracted cavity. Based on this versatile
81 framework, Chen et al. [12] used the semi-analytical solution to analyze the three-dimensional
82 strength and the anisotropic properties of soils by introducing Spatially Mobilized Plane (SMP)
83 criterion in modified Cam-clay model. More recently, fabric and stress induced anisotropy has

84 also been accounted for using anisotropic elastoplastic models like S-CLAY1 [13] and Dafalias'
85 anisotropic bounding surface model [14]. Although various types of solutions have been proposed,
86 most of them basically treat soil as an original or modified Cam-clay material with an associated
87 flow rule, providing quite low accuracy in modelling heavily over-consolidated clays. In particular,
88 for highly plastic stiff clays or soft claystone, high clay contents, low void ratios, low permeability
89 and high stress states (usually greater than 1 MPa) distinguish them from Cam-clay materials with
90 the following basic hydro-mechanical behaviors:

91 (1) Highly nonlinear behaviors are typically observed on stiff clays at heavily over-consolidated
92 states [16]. These pre-yielding nonlinear behaviors would have significant influence on the
93 variations of pore pressure and displacements which are described by advanced constitutive
94 modelling for stiff clays [16, 17, 18, 22].

95 (2) For highly plastic stiff clays, the yield surface is of a 'tear' shape rather than an 'elliptical'
96 shape. An 'elliptical' shape would overestimate the failure stress on the 'dry side' but
97 underestimate it on the 'wet side'. The yield surface shape variation would lead to misestimating
98 the excess pore pressure under undrained conditions. On the other hand, the plastic flow direction
99 should also be considered for the complex behavior of stiff clays.

100 (3) The pore fluid governs the distribution of excess pore pressure and stress components in
101 expanding process. In relatively low stress states (e.g. 0.1 MPa~1 MPa) under undrained
102 conditions, the volumetric strain due to loading remains almost constant. However, for stiff clays
103 and soft claystone at great depth (e.g. 1MPa~100MPa), the pore water would induce a slight
104 volumetric strain itself, which can significantly influence the excess pore pressure.

105 These three main differences make the existing analytical solutions [6-14] difficult to be used
106 directly in analyzing tunneling and piezocone penetration tests in stiff clays under deep ground.
107 On the other hand, although several two-surface plasticity (bounding/subloading surface) models
108 have already been numerically applied into tunnel/gallery construction, e.g. the ground movement
109 (settlement or uplift) induced by tunneling in stiff London clay[38-39] or Shanghai soft clay[37],
110 the excess pore water pressure distribution in Boom clay[35], the characteristic of load transfer in
111 shanghai clay during tunnel operating period[36], the corresponding novel analytical or
112 semi-analytical solutions are still appealing. Based on the rigorous semi-analytical framework
113 proposed by Chen and Abousleiman [9] and the premise of constant soil mass, the study extends

114 the solution with a two-surface plasticity model named ACC2 [16] to consider the mechanical
115 behavior of natural stiff clays under high stress conditions. After a brief introduction of the basic
116 formulas, the main features and parameters of the two-surface plasticity model, it is extended to
117 undrained conditions by incorporating the mass conservation for the liquid phase in the pore space.
118 By using the incremental stress-strain relationship, the undrained cavity expansion/contraction
119 problem is formulated to solve a system of first-order ordinary differential equations with two
120 different premises, namely constant soil volume and constant soil mass, respectively. A discussion
121 between Constant Soil Volume (CSV) and Constant Soil Mass (CSM) assumptions is included to
122 highlight that undrained condition corresponds to a situation with constant soil mass. Further
123 parametric studies based on four key material parameters about stress paths, stress distribution and
124 cavity expansion/contraction curves are conducted. Lastly, as a precise and convenient tool, the
125 present semi-analytical solution is utilized to predict the pore pressure and stress distribution along
126 the radial distance and the supporting pressure which is required to maintain the Gallery stability
127 in the case of Praclay Gallery in underground research facility HADES-URF built at a depth of
128 223m in natural Boom clay.
129

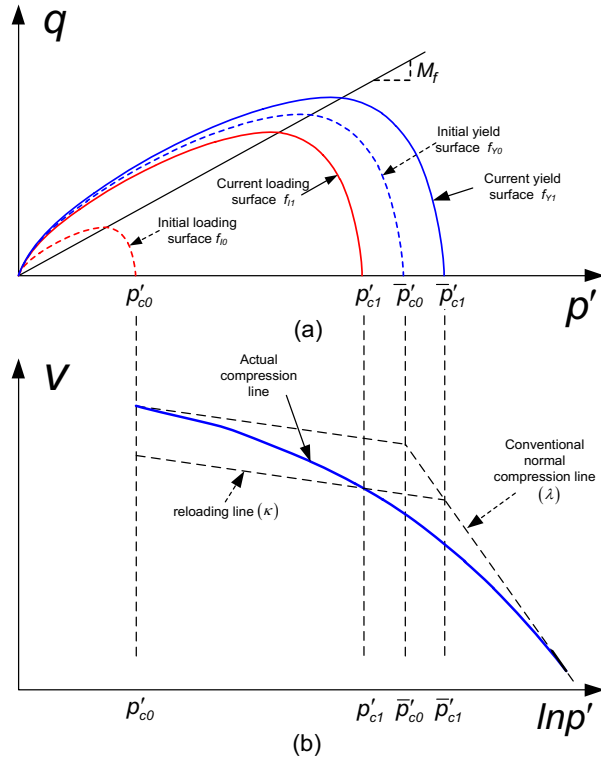
130 **2. Model description**

131 This section gives a brief description of the two-surface plasticity model for stiff clays (e.g.
132 natural Boom clay) named ACC2 [16]. The formulation of ACC2 model is based on the triaxial
133 conditions with three principal stresses (namely, σ'_r , σ'_θ , σ'_z in a cylindrical coordinate
134 system):

$$135 \quad p' = \frac{1}{3}(\sigma'_r + \sigma'_\theta + \sigma'_z) \quad (1)$$

$$136 \quad q = \sqrt{\frac{1}{2}((\sigma'_r - \sigma'_\theta)^2 + (\sigma'_r - \sigma'_z)^2 + (\sigma'_\theta - \sigma'_z)^2)} \quad (2)$$

137



138 Fig.1 Schematic plot of two surface plasticity model ACC2: (a) p' - q plane; (b) v - $\ln p'$ plane

139 Fig.1 gives a schematic illustration of the two surfaces (Loading surface and Yield surface).
 140 These two surfaces are linked through a progressive hardening relationship by a positive scalar r
 141 in v - $\ln p'$ plane. A smooth compression behavior is achieved via the use of two hardening variables
 142 \bar{p}'_c and r before the soil reaches normally consolidated states:

$$143 \quad r = \frac{p'}{\bar{p}'} = \frac{q}{\bar{q}} = \frac{p'_c}{\bar{p}'_c} \quad (3)$$

144 where p'_c is the actual loading yield stress on the Loading surface and \bar{p}'_c is the conventional
 145 pre-consolidation pressure on the conventional Normal Compression Line.

146 Similar to the concept of bounding surface plasticity, the current stress point (p' , q) is
 147 assumed to always lie on the Loading surface which is expressed as:

$$148 \quad f_i(p', q, r, p'_c) = q^2 + \frac{M_f^2}{1 - k_f} \left(\frac{p'}{r\bar{p}'_c} \right)^{k_f} (r\bar{p}'_c)^2 - \frac{M_f^2 p'^2}{1 - k_f} = 0 \quad (4)$$

149 where M_f is the stress ratio at the apex of the two surfaces and k_f is a parameter specifying the
 150 shape of the yield surfaces. Because natural stiff clays (e.g. natural Boom clay) show a unique
 151 yielding behavior, a generalized yield surface ($k_f < 2$) is adopted in this model. While k_f varies

152 from 0.5 to 2, a tear shape surface turns into an elliptical shape more suitable to soft clays in Fig.
 153 2(a). The calibrated yielding behavior of natural Boom clay shows that the closest value of k_f is
 154 0.7.

155 Two internal variables, namely \bar{p}'_c and r , are used to control the plastic strain hardening or
 156 softening behavior. A cam-clay type volumetric plastic strain isotropic hardening law for $d\bar{p}'_c$ is
 157 adopted:

$$158 \quad d\bar{p}'_c = \frac{v}{\lambda - \kappa} \bar{p}'_c d\varepsilon_v^p \quad (5)$$

159 where v is the specific volume; λ is the slope of conventional normal compression line in v - $\ln p'$
 160 plane; κ is the slope of reloading line in v - $\ln p'$ plane; $d\varepsilon_v^p$ is the volumetric plastic strain
 161 increment.

162 Meanwhile, a progressive hardening law for dr is introduced with a generalized plastic strain
 163 to account for the contribution of both volumetric and shear plastic deformation parts. It should be
 164 noted that different strain types are considered in previous works [30, 31, 32]. Russell and Muir
 165 Wood [28] pointed out that the inclusion of both volumetric and shear parts was of paramount
 166 importance in accurately describing soil behavior. This hardening law reads as follows:

$$167 \quad dr = \frac{v}{\lambda - \kappa} s(1-r) d\varepsilon_d^p = \frac{v}{\lambda - \kappa} s(1-r) (d\varepsilon_v^p + A_d d\varepsilon_s^p) \quad (6)$$

168 where s is the key material parameter ($s > 0$) that determines the rate at which the Loading surface
 169 hardens towards the Yield surface; A_d is a material parameter ($A_d > 0$) which controls the
 170 contribution of plastic shear strain to strain hardening.

171 As plotted in Fig.2(c), s controls the nonlinearity of the isotropic compression line in
 172 over-consolidated states in v - $\ln p'$ plane. As s varies from 4 to 1000, the behavior switches from a
 173 nonlinear trend to a bi-loglinear behavior as in the modified cam-clay model. Calibrated with the
 174 test results of natural Boom clay, a value of 8 is found for s .

175 As plotted in Fig.2(d), parameter A_d ($A_d \geq 0$) can be obtained by fitting stress paths during
 176 drained softening stages in highly over-consolidated states. Parameter A_d describes how the plastic
 177 shear strain affects the hardening process. As A_d varies from 0 to 5, the nonlinearity on the 'dry
 178 side' is increasingly obvious. It is noteworthy that when $A_d = 0$, the plastic shear strain does not
 179 contribute to the hardening process anymore. The most suitable value of A_d for natural Boom clay

180 is 0.1.

181 The stress dilatancy relation proposed by McDowell et al. [19] is adopted to define a
 182 non-associated flow rule for better describing the volumetric strain and excess pore pressure
 183 generation during undrained loadings for natural stiff clays. Assuming a plastic potential having
 184 the same shape as the yield surfaces but a different pair of parameters (M_g, k_g), the stress dilatancy
 185 equation and the non-associated flow rule are expressed as follows:

$$186 \quad \frac{d\varepsilon_v^p}{d\varepsilon_s^p} = \frac{\frac{\partial g_l}{\partial p}}{\frac{\partial g_l}{\partial q}} = \frac{M_g^2 - \eta^2}{k_g \eta} \quad (7)$$

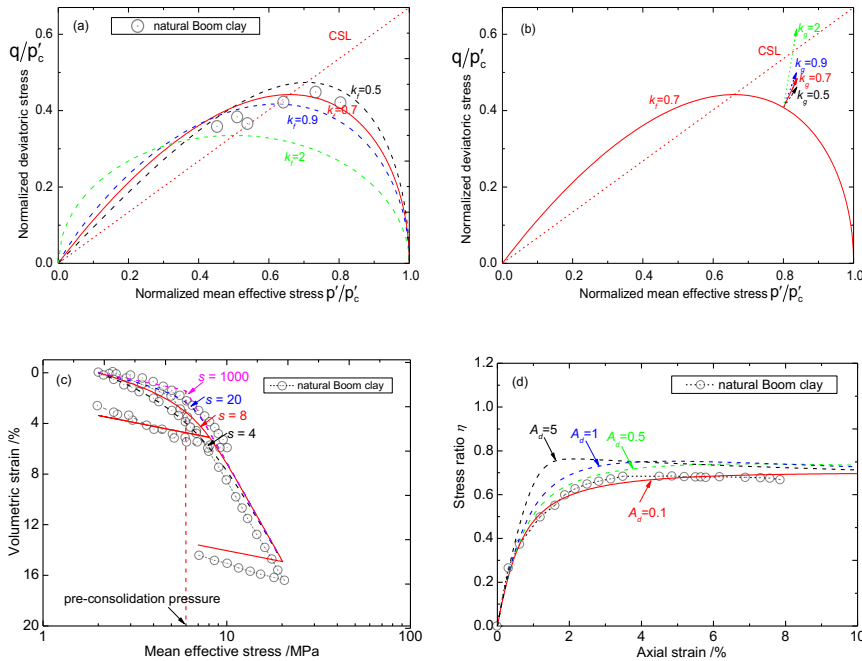
$$187 \quad d\varepsilon_v^p = \Lambda \frac{\partial g_l}{\partial p}, \quad d\varepsilon_s^p = \Lambda \frac{\partial g_l}{\partial q} \quad (8)$$

188 where η denotes the stress ratio q/p' , M_g is the critical state ratio where no further volumetric
 189 strain develops and k_g controls the shape of the plastic potential and the direction of plastic flow.

190 Fig.2(b) depicts the variation of plastic flow direction (k_g) from 0.5 to 2 in p' - q plane. The
 191 calibrated yielding behavior of natural Boom clay shows that the closest value of k_g is 0.9.

192

193



194

195 Fig.2 Variation of parameters of ACC2 model in p' - q plane and v - $\ln p'$ plane: (a) k_f varies from 0.5

196 to 2; (b) k_g varies from 0.5 to 2; (c) s varies from 4 to 1000; (d) A_d varies from 0.1 to 5

197 With eleven material parameters (five of them are the same as MCC model), the two-surface
 198 plasticity model named ACC2, similarly to bounding surface plasticity theory, has been proposed
 199 to consider the mechanical behavior of both normally consolidated and over-consolidated stiff
 200 clays. Five of them (λ , κ , \bar{p}'_c , ν , M_f) are the same as for the modified cam-clay model. r denotes
 201 the inverse of over-consolidation ratio. Two parameters (k_f , k_g) control the shapes of the yield
 202 surface and the plastic potential surface. Two parameters (s , A_d) are used to account for a
 203 progressive hardening mechanism in over-consolidated states. Thus, the following discussion
 204 about ACC2 model is mainly done by considering four principal parameters, namely k_f (shape of
 205 yield surface), k_g (direction of plastic flow), s (isotropic compression nonlinearity) and A_d
 206 (deviatoric nonlinearity).

207

208 **3. Problem statement**

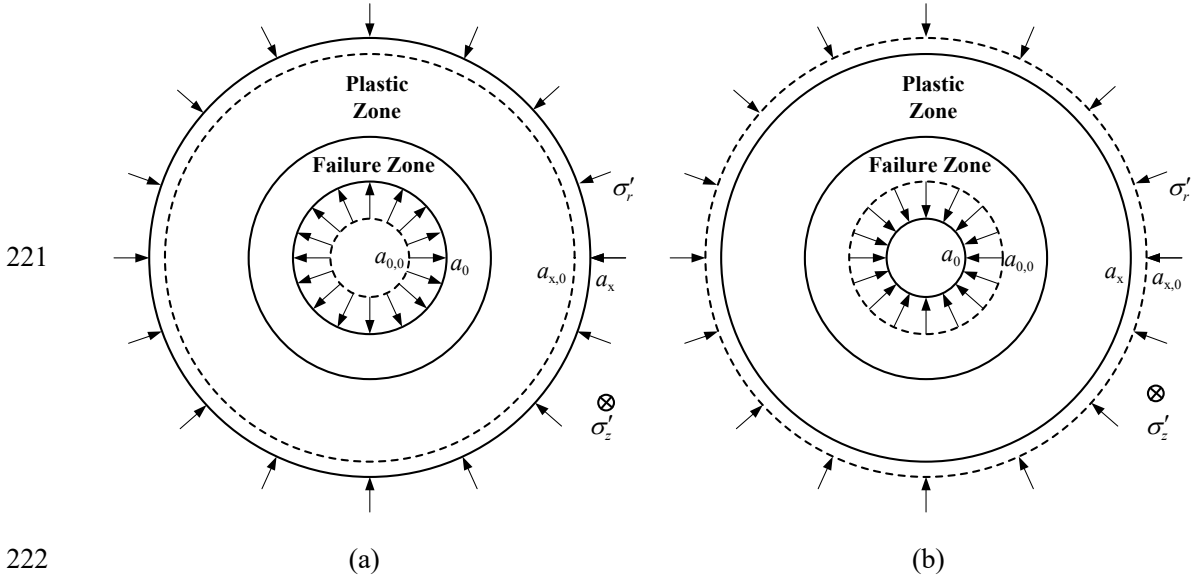
209 **3.1 Undrained cylindrical cavity expansion/contraction**

210 Fig.3 presents schematic plots of cylindrical cavity expansion/contraction problems with initial
 211 radius $a_{0,0}$ in an infinite elasto-plastic soil layer under undrained conditions. As the cavity expands
 212 or shrinks from initial radius $a_{0,0}$ to current radius a_0 , a failure zone would be formed in the
 213 vicinity of the cavity wall. At the same time, the outside zone limited by radius $a_{x,0}$ would be
 214 plastic with stress points always lying on the Loading surface. Beyond that radius, the soil is in the
 215 elastic regime.

216 The radial equilibrium equation for cylindrical cavity expansion/contraction problem is
 217 expressed in terms of the effective stress as:

$$218 \frac{d\sigma'_r}{da_x} + \frac{\sigma'_r - \sigma'_\theta}{a_x} + \frac{du_w}{da_x} = 0 \quad (9)$$

219 where σ'_r , σ'_θ are radial, tangential effective stress of any material point, respectively. u_w stands
 220 for pore pressure.



223 Fig.3 Geometry of cylindrical cavity problem: (a) expansion; (b) contraction

224 Based on the compatibility condition and large deformation theory, a generalized logarithmic
 225 definition of strains in Eulerian description can be adopted in all areas [6]:

226
$$\varepsilon_r = -\ln\left(\frac{da_x}{da_{x0}}\right), \quad \varepsilon_\theta = -\ln\left(\frac{a_x}{a_{x0}}\right), \quad \varepsilon_v = -\ln\left(\frac{v}{v_0}\right) \quad (10)$$

227 where a_x, a_{x0} are the current and initial radii of any material points, respectively. $\varepsilon_r, \varepsilon_\theta, \varepsilon_v$ are the
 228 radial, tangential and volumetric strains, respectively. v, v_0 are the current and initial void ratios,
 229 respectively.

230 For further study, they can also be expressed in incremental form, as follows:

231
$$d\varepsilon_r = -\frac{\partial(da_x)}{\partial a_x}, \quad d\varepsilon_\theta = -\frac{da_x}{a_x}, \quad d\varepsilon_v = -\frac{dv}{v} \quad (11)$$

233 3.2 Elastoplastic analysis under undrained condition

234 The incremental stress-strain relationship for ACC-2 model is expressed in Lagrangian description
 235 (see A-17 in Appendix A):

236
$$\begin{bmatrix} D\sigma'_r \\ D\sigma'_\theta \\ D\sigma'_z \end{bmatrix} = \begin{bmatrix} c_{rr} & c_{r\theta} & c_{rz} \\ c_{\theta r} & c_{\theta\theta} & c_{\theta z} \\ c_{zr} & c_{z\theta} & c_{zz} \end{bmatrix} \begin{bmatrix} D\varepsilon_r \\ D\varepsilon_\theta \\ D\varepsilon_z \end{bmatrix} \quad (12)$$

237 where D denotes the differential of material points in Lagrangian description which should be
 238 distinguished from d in Eulerian form.

239 Under relatively low stress and undrained conditions, most analytical and semi-analytical
240 solutions, including Chen and Abouseleiman's method [9], consider the soil mixture as an
241 incompressible material with the undrained Poisson's ratio being equal to 0.5 and the volumetric
242 strain remaining constant. The volume change of pore water due to external loading is ignored.
243 This is the case referred to as constant soil volume (CSV). Most reasonable analytical and
244 semi-analytical solutions for undrained cavity problems are extensively developed and applied for
245 various geotechnical problems based on this premise [6-9, 11-15]. Appendix B depicts the
246 undrained solution in the form of five first-order ordinary differential equations for the
247 comparative study. However, under high stress and low porosity conditions, the real
248 compressibility of pore fluids (e. g water, oil) would exhibit a significant influence on the excess
249 pore pressure and stress distribution along the cavity. This condition of constant soil volume can
250 actually be relaxed by considering that undrained conditions correspond to a situation with
251 constant soil mass (CSM).

252 The well-known Skempton coefficient B , which indicates the ratio between the change in
253 pore pressure and the change in total applied stress in Appendix C-12, could also be used to
254 illustrate the difference. For CSV premise, the value of B is always 1 in saturated conditions. This
255 situation may be reasonable for soft clayey soils under low stress conditions ($B > 0.95$), but not for
256 stiff clays and soft claystone under high stress conditions (e.g. $B \approx 0.92$ for natural Boom clay [23],
257 $B \approx 0.75$ to 0.84 for Callovo–Oxfordian claystone [24, 34], $B \approx 0.88$ for Opalinus argillite [25]). In
258 such cases, the CSV assumption would lead to unavoidable computational errors for stress and
259 pore pressure distributions, since the effect of pore fluids' compressibility under high stress
260 conditions is not accounted for. Note that the compressibility of the solid particles is still
261 neglected, since it leads to volume changes that are one order of magnitude (at least) lower than
262 the compressibility of the pore fluid.

263

264 **3.3 Undrained cavity expansion/contraction seen as constant soil mass conditions**

265 In the framework of poro-elasto-plasticity theory, pore fluid is typically regarded as a
266 compressible elastic medium with a large bulk modulus (e.g. 2.2×10^3 MPa for water, 1.6×10^3 MPa
267 for oil at room temperature 20°C and isotropic pressure 1 MPa), which is slightly sensitive to

268 temperature and pressure variations [21]. The bulk modulus of pore fluid is usually dozens of
 269 times that of solid grain. For most clays, the compressibility of solid grains can be neglected (e.g.
 270 $C_s=2.0 \times 10^{-5}$ MPa⁻¹ for Illite [33], to be compared to the water compressibility $C_w=45.5 \times 10^{-5}$
 271 MPa⁻¹ for water). Considering infinitesimal transformation and undrained conditions (CSM), and
 272 neglecting solid grains compressibility, pore water pressure change and soil volumetric change are
 273 related using Eulerian porosity n and pore fluid's bulk modulus k_w ($k_w=1/C_w$) as follows:

$$274 \quad du_w = \frac{d\varepsilon_v - C_s dp}{\phi(C_w - C_s)} \approx \frac{d\varepsilon_v}{nC_w} = \frac{k_w d\varepsilon_v}{n} \quad (13)$$

275 It is noteworthy that Eq (13) is deduced in the framework of poromechanics by Coussy [20].
 276 More details are presented in Appendix C.

277 In analyses considering CSM, the soil volumetric strain increment at any material point
 278 cannot be ignored even under undrained condition. Meanwhile, the vertical strain increment is still
 279 null in cylindrical cavity analysis (plane strain conditions). The general incremental solution for
 280 cylindrical cavity problem is expressed in Lagrangian description, as follows:

$$281 \quad D\varepsilon_v = -\frac{D\nu}{\nu}, \quad D\varepsilon_\theta = -\frac{Da_x}{a_x}, \quad D\varepsilon_z = 0 \quad (14)$$

282 An auxiliary independent variable ξ is chosen as the ratio of initial radial position to current
 283 radial position for each specific particle. The following differentiations can be done:

$$284 \quad \xi = \frac{a_{x0}}{a_x}, \quad \frac{D\xi}{Da_x} = -\frac{a_{x0}}{a_x^2}, \quad \frac{D\varepsilon_\theta}{D\xi} = -\frac{Da_x}{a_x D\xi} = \frac{a_x}{a_{x0}} = \frac{1}{\xi} \quad (15)$$

285 Substituting Eq (15) into Eq (12), the incremental stress-strain relationship can be
 286 transformed into three first-order ordinary differential equations with an unknown

287 Lagrangian-form differential $\frac{D\varepsilon_v}{D\xi}$:

$$288 \quad \frac{D\sigma'_r}{D\xi} = c_{rr} \frac{D\varepsilon_v}{D\xi} + \frac{c_{r\theta} - c_{rr}}{\xi} \quad (16)$$

$$289 \quad \frac{D\sigma'_\theta}{D\xi} = c_{\theta r} \frac{D\varepsilon_v}{D\xi} + \frac{c_{\theta\theta} - c_{\theta r}}{\xi} \quad (17)$$

$$290 \quad \frac{D\sigma'_z}{D\xi} = c_{zr} \frac{D\varepsilon_v}{D\xi} + \frac{c_{z\theta} - c_{zr}}{\xi} \quad (18)$$

291 At the same time, the excess pore water pressure can be given in Lagrangian description:

292
$$Du_w = \frac{k_w}{n} D\varepsilon_v \quad (19)$$

293 Thus:

294
$$\frac{Du_w}{D\xi} = \frac{\nu k_w}{\nu-1} \frac{D\varepsilon_v}{D\xi} \quad (20)$$

295 where k_w is the bulk modulus of pore water, n is the current porosity and ν is current specific
296 volume.

297 In order to make the problem solvable, let us consider $\frac{D\varepsilon_v}{D\xi} = f$. Substitute Eq (16) and Eq
298 (20) into the radial equilibrium equation (9) gives:

299
$$\left(\frac{D\sigma'_r}{D\xi} + \frac{Du_w}{D\xi} \right) \frac{d\xi}{da_x} + \frac{\sigma'_r - \sigma'_\theta}{a_x} = 0 \quad (21)$$

300 Unlike $\frac{D\xi}{Da_x}$ in Eq (15), $\frac{d\xi}{da_x}$ corresponds to a Eulerian description:

301
$$\frac{d\xi}{da_x} = \frac{\frac{da_{x0}}{da_x} a_x - a_{x0}}{a_x^2} = \frac{1}{a_x} \left(\frac{da_{x0}}{da_x} - \xi \right) = \frac{1}{a_x} \left(e^{\varepsilon_r} - \xi \right) \quad (22)$$

302 Integrating Eq (10), the full form of radial strain can be obtained:

303
$$\varepsilon_r = \varepsilon_v - \varepsilon_\theta = -\ln\left(\frac{\nu}{\nu_0}\right) + \ln\left(\frac{a_x}{a_{x0}}\right) = \ln\left(\frac{\nu_0}{\nu\xi}\right) \quad (23)$$

304 Then:

305
$$\frac{d\xi}{da_x} = \frac{1}{a_x} \left(\frac{\nu_0}{\nu\xi} - \xi \right) \quad (24)$$

306 Making full use of Eq (16), Eq (20) and Eq (24), the radial equilibrium equation (9) turns into
307 Eq (25):

308
$$\left(c_{rr} f + (c_{r\theta} - c_{rr}) \frac{1}{\xi} + \frac{\nu k_w}{\nu-1} f \right) \times \frac{1}{a_x} \left(\frac{\nu_0}{\nu\xi} - \xi \right) + \frac{\sigma'_r - \sigma'_\theta}{a_x} = 0 \quad (25)$$

309 Then, an equation relating f to the other principal variables σ'_r , σ'_θ , $c_{r\theta}$, c_{rr} , ν and ξ is
310 obtained:

$$311 \quad f = -\frac{\xi(\sigma'_r - \sigma'_\theta) + \left(\frac{\nu_0}{\nu\xi} - \xi\right)(c_{r\theta} - c_{rr})}{\left(\frac{\nu_0}{\nu} - \xi^2\right)\left(c_{rr} + \frac{\nu k_w}{\nu - 1}\right)} \quad (26)$$

312 The previous four first-order ordinary differential equations are finally reduced into a full
313 explicit form with respect to the auxiliary independent variable ξ :

$$314 \quad \frac{D\sigma'_r}{D\xi} = c_{rr}f + \frac{c_{r\theta} - c_{rr}}{\xi} \quad (27)$$

$$315 \quad \frac{D\sigma'_\theta}{D\xi} = c_{\theta r}f + \frac{c_{\theta\theta} - c_{\theta r}}{\xi} \quad (28)$$

$$316 \quad \frac{D\sigma'_z}{D\xi} = c_{zr}f + \frac{c_{z\theta} - c_{zr}}{\xi} \quad (29)$$

$$317 \quad \frac{Du_w}{D\xi} = \frac{\nu k_w}{\nu - 1} f \quad (30)$$

318 The other three variables, namely specific volume ν , pre-consolidation pressure \bar{p}'_c and
319 positive scalar r are also included in the differential equations through equation (26):

$$320 \quad \frac{D\nu}{D\xi} = -\nu f \quad (31)$$

$$321 \quad \frac{D\varepsilon_v^p}{D\xi} = \frac{D\varepsilon_v}{D\xi} - \frac{D\varepsilon_v^e}{D\xi} = f - \frac{\kappa}{\nu(\sigma'_r + \sigma'_\theta + \sigma'_z)} \left(\frac{D\sigma'_r}{D\xi} + \frac{D\sigma'_\theta}{D\xi} + \frac{D\sigma'_z}{D\xi} \right) \quad (32)$$

$$322 \quad \frac{D\varepsilon_s^p}{D\xi} = \frac{D\varepsilon_s^p}{D\varepsilon_v^p} \frac{D\varepsilon_v^p}{D\xi} = \frac{k_g \eta}{M_g^2 - \eta^2} \left[f - \frac{\kappa}{\nu(\sigma'_r + \sigma'_\theta + \sigma'_z)} \left(\frac{D\sigma'_r}{D\xi} + \frac{D\sigma'_\theta}{D\xi} + \frac{D\sigma'_z}{D\xi} \right) \right] \quad (33)$$

$$323 \quad \frac{D\bar{p}'_c}{D\xi} = \frac{\nu}{\lambda - \kappa} \bar{p}'_c \frac{D\varepsilon_v^p}{D\xi} = \frac{\nu}{\lambda - \kappa} \bar{p}'_c \left[f - \frac{\kappa}{\nu(\sigma'_r + \sigma'_\theta + \sigma'_z)} \left((c_{rr} + c_{\theta r} + c_{zr}) \left(f - \frac{1}{\xi} \right) + \frac{c_{r\theta} + c_{\theta\theta} + c_{z\theta}}{\xi} \right) \right] \quad (34)$$

$$324 \quad \begin{aligned} \frac{Dr}{D\xi} &= \frac{\nu}{\lambda - \kappa} s(1-r) \left(\frac{D\varepsilon_v^p}{D\xi} + A_d \frac{D\varepsilon_s^p}{D\xi} \right) \\ &= \frac{\nu}{\lambda - \kappa} s(1-r) \left[1 + A_d \frac{k_g \eta}{M_g^2 - \eta^2} \right] \left[f - \frac{\kappa}{\nu(\sigma'_r + \sigma'_\theta + \sigma'_z)} \left((c_{rr} + c_{\theta r} + c_{zr}) \left(f - \frac{1}{\xi} \right) + \frac{c_{r\theta} + c_{\theta\theta} + c_{z\theta}}{\xi} \right) \right] \end{aligned} \quad (35)$$

325 At this stage, based on the premise of CSM, the cylindrical cavity expansion/contraction
326 problem under undrained conditions is finally reduced to seven ordinary differential equations (Eq
327 (27), Eq (28), Eq (29), Eq (30), Eq (31), Eq (34), Eq (35)). These equations can be solved as an

328 initial boundary value problem with ξ starting from 1. It is worth noting that these ordinary
 329 differential equations which are expressed in terms of ξ should finally be converted to the radius
 330 position a_x . From equation (24), it is obtained:

$$331 \quad \frac{da_x}{a_x} = \frac{d\xi}{\frac{v_0}{v\xi} - \xi} \quad (36)$$

332 Integrating the above equation, the following formula is obtained:

$$333 \quad \frac{a_x}{a_0} = \exp \left(\int_{\xi(a_0)}^{\xi} \frac{d\xi}{\frac{v_0}{v\xi} - \xi} \right) \quad (37)$$

334

335 3.4 Initial condition

336 The classical undrained cylindrical cavity expansion/contraction problem is eventually solved
 337 using these seven first-order ordinary differential equations with the corresponding initial stress
 338 conditions. These stress conditions are initially defined as:

$$339 \quad \sigma'_r(a_{x0}) = \sigma'_{r0}, \quad \sigma'_\theta(a_{x0}) = \sigma'_{\theta0}, \quad \sigma'_z(a_{x0}) = \sigma'_{z0}, \quad u_w(a_{x0}) = u_{w0}, \quad v(a_{x0}) = v_0, \quad p'_c(a_{x0}) = p'_{c0},$$

$$340 \quad r(a_{x0}) = r_0$$

341 4. Results and discussion

342 In this section, the results obtained from the semi-analytical solution are presented for the
 343 cylindrical cavity expansion/contraction in soils under undrained conditions. The original 11
 344 material parameters of ACC2 model are calibrated and chosen by fitting the laboratory test
 345 (isotropic compression, drained triaxial compression) results for natural Boom Clay according to
 346 Hong et al. [16] (Table 1 and Table 2): v_s (void ratio in critical state at isotropic stress of 1 MPa) =
 347 1.84, λ (slope of normal consolidation line in v - $\ln p'$ plane) = 0.18, κ (slope of unloading line in
 348 v - $\ln p'$ plane) = 0.02, M_g (critical state ratio) = 0.67, M_f (stress ratio at apex of yield surface) = 0.67,
 349 \bar{p}'_c (pre-consolidated pressure) = 6 MPa. The Poisson ratio ν is assumed to be 0.3. It should be
 350 noted that the calibration process could be improved by an appropriate genetic algorithm which
 351 has been studied in previous literatures [40-45].

352 The undrained shear strength s_u is calculated based on the premise of constant soil volume (Wood,
353 1990):

$$354 \quad s_u = \frac{1}{\sqrt{3}} M_g e^{(v_s - v_0)/\lambda} \quad (38)$$

355 As mentioned previously, under undrained loading process, the soil sample at low void ratio
356 and under high stress would also induce a slight volumetric strain increment which explains the
357 variation of pore pressure. The s_u assumed here is only for the nondimensionalization purpose.

358 This section firstly highlights the effect of the compressibility of pore fluid and the
359 importance of solving the cavity expansion problem using poro-elasto-plasticity under various
360 stress conditions. Representative comparisons are given in Fig. 4-6 between the results for
361 normally consolidated soils based on constant soil volume (CSV) and constant soil mass (CSM)
362 conditions at isotropic effective stresses of 2 MPa, 10 MPa, 50 MPa, respectively. The soil mass is
363 fully saturated with pore water. Moreover, a full range of isotropic stresses from 0.1 MPa to 100
364 MPa is considered to assess the relative errors between these two assumptions under cavity
365 expansion condition ($a_0/a_{0,0} = 2$). Isoerror maps computed for stress components are plotted in Fig.
366 7.

367 Then, based on CSM premises, parametric studies of cavity expansion at normal consolidated
368 stress state ($r=1$) and highly over-consolidated stress state ($r=2.9$) using ACC2 model are
369 conducted. It aims at illustrating the advantages brought by the use of this sophisticated model,
370 when compared to simple models like MCC, for which analytical solutions for the same problem
371 already exist.

372 (1) ACC2 model with different values of yield shape parameter k_f (0.5 to 2) as Case 4 and plastic
373 flow direction parameter k_g (0.5 to 2) as Case 5 at normally consolidated stress state ($r_0=1$) is
374 applied to investigate the effect of the shape of yield surface (the yielding behavior) and the shape
375 of plastic potential surface (the direction of plastic flow).

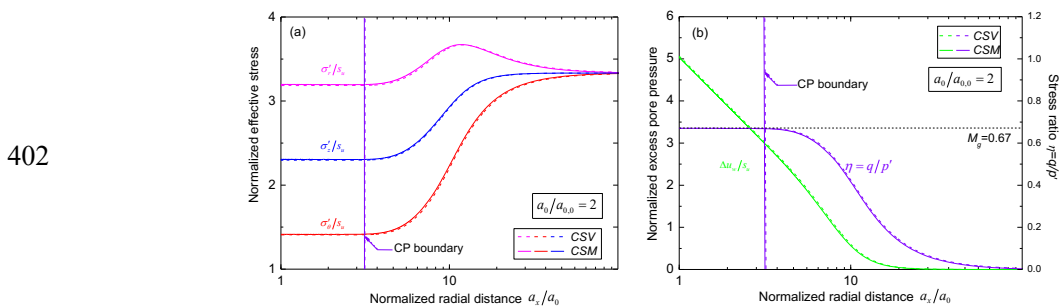
376 (2) ACC2 model with isotropic compression non-linearity parameter s (4 to 1000) as Case 6 and
377 deviatoric non-linearity parameter A_d (0.1 to 5) as Case 7 is applied to investigate the effect of the
378 nonlinearity at a highly over-consolidated stress state ($r_0=2.9$).

379

380 **4.1 Comparison between CSV and CSM conditions**

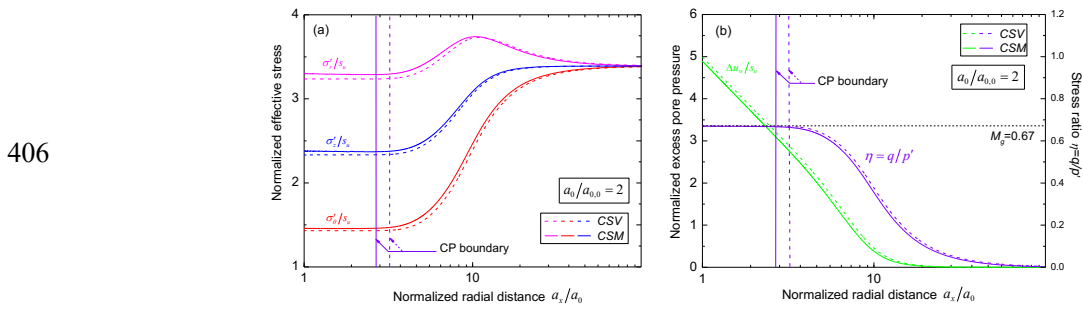
381 The results obtained using CSV and CSM assumptions at three isotropic effective stress states
 382 ($p'=2, 10, 50$ MPa) are compared in Fig. 4 as Case 1, Fig. 5 as Case 2 and Fig. 6 as Case 3. The
 383 soil is initially assumed to be at normally consolidated and isotropic stress state ($r=1$), then rapidly
 384 expanded to a certain cavity radius ($a_0/a_{0,0} = 2$) without considering the rate and seepage effect.
 385 During the expanding process, a critical state zone where the stress state at any point satisfies
 386 $q=M_g p'$ would be formed in vicinity of the cavity. Outside the critical state zone, all material
 387 points reach the plastic state to some extent.

388 There is a clear difference between CSV and CSM in stress and excess pore pressure
 389 distributions along the radius distance in Fig. 4-6. From the normalized effective stress (σ'_r/s_u ,
 390 σ'_θ/s_u , σ'_z/s_u) distributions along the radius distance corresponding to the cavity radius ratio of
 391 $a_0/a_{0,0} = 2$ in Fig. 4(a), Fig. 5(a) and Fig. 6(a), the expanding results obtained based on CSM
 392 appear to be slightly larger than those based on CSV at each corresponding point. The opposite
 393 phenomenon could be observed in the normalized excess pore pressure $\Delta u_w/s_u$ and stress ratio η
 394 distributions along the radius distance in Fig. 4(b), Fig. 5(b) and Fig. 6(b). Meanwhile, the range
 395 of critical state zone (see CP boundary) for CSM is shorter than for CSV. The CP boundary
 396 corresponds to the boundary between critical state zone and plastic zone. It can be determined by
 397 the evolution of current stress ratio ($\eta=q/p'$) along normalized radial distance. With the increase of
 398 initial isotropic stress, the difference between CSV and CSM becomes more and more obvious.
 399 Through the comparison between the two cases, it can be concluded that the premise of CSV
 400 would influence the excess pore pressure and principal effective stress distributions. This confirms
 401 the necessity of considering the compressibility of pore fluid under high stress conditions.

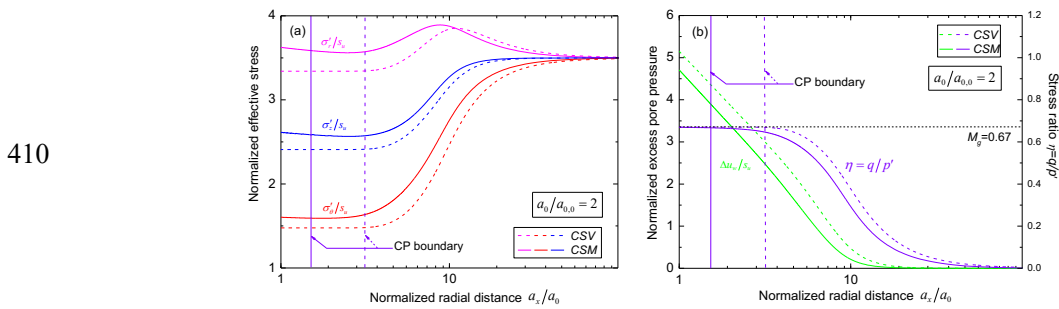


403 Fig. 4 Undrained cavity expansion at normally consolidated and isotropic stress state ($p'=2$ MPa):

404 (a) normalized principal stress distributions; (b) normalized excess pore pressure and stress ratio
 405 distribution



407 Fig. 5 Undrained cavity expansion at normally consolidated and isotropic stress state ($p' = 10$ MPa):
 408 (a) normalized principal stress distributions; (b) normalized excess pore pressure and stress ratio
 409 distribution

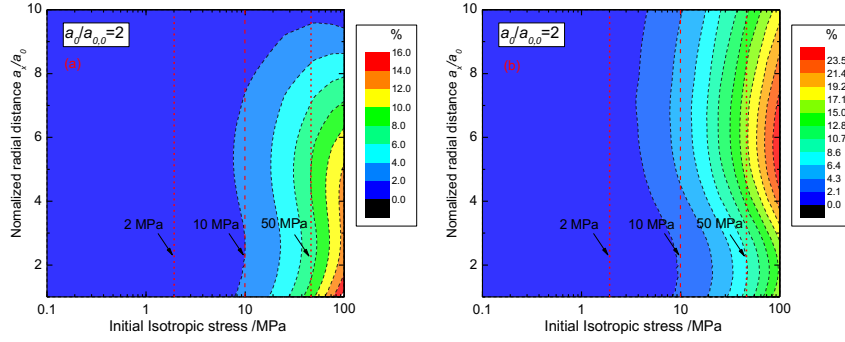


411 Fig. 6 Undrained cavity expansion at normally consolidated and isotropic stress state ($p' = 50$ MPa):
 412 (a) normalized principal stress distributions; (b) normalized excess pore pressure and stress ratio
 413 distribution

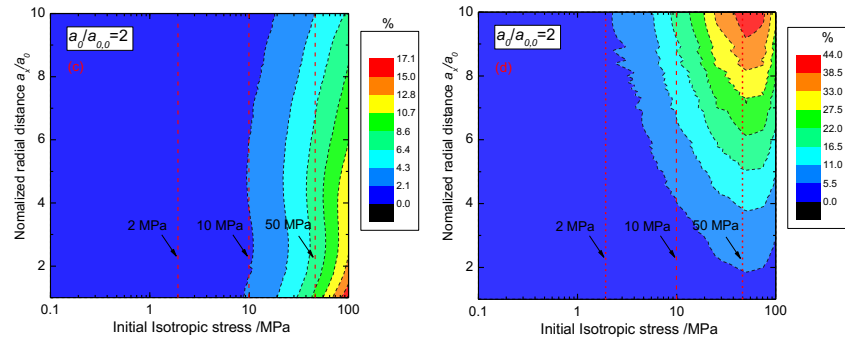
414
 415 In order to estimate the relative errors based on the two premises, a full range of isotropic
 416 stress from 0.1 MPa to 100 MPa is considered in cavity expansion process. The relative errors (Re)
 417 for three principal stresses and pore pressure are defined as: $Re = |V_{CSM} - V_{CSV}|/|V_{CSM}|$. In the
 418 formula, V_{CSM} , V_{CSV} represent the corresponding results of each material point based on CSM and
 419 CSV, respectively.

420

421



422



423 Fig. 7 Isoerror maps plotted for constant soil mass (CSM) and constant soil volume (CSV): (a)
424 radial effective stress; (b) tangential effective stress; (c) vertical effective stress; (d) excess pore
425 pressure

426 Figs. 7(a)-(c) plot isoerror maps for the three principal effective stress distributions from a_x/a_0
427 = 1 to $a_x/a_0 = 10$ in mean effective stress range from 0.1 MPa to 100 MPa when the cavity expands
428 to $a_0/a_{0,0} = 2$. Case 1 of $p'=2$ MPa, case 2 of $p'=10$ MPa, case 3 of $p'=50$ MPa are specially pointed
429 out. In the range from 0.1 MPa to 1 MPa (low stress conditions), the value of Re is less than 1%,
430 suggesting that both cases (CSM, CSV) would get reliable results in principal stress distributions.
431 However, when the isotropic stress is larger than 10 MPa (relatively high stress conditions), the
432 value of Re would increase rapidly. Meanwhile, the Re value for excess pore pressure distribution
433 in Fig. 7(d) is much more sensitive to pressure change than the effective stress distribution at both
434 low and high stress conditions.

435 Thus, for the applications focusing on the evolution of excess pore pressure, CSM would be a
436 preferential choice for solving the cavity expansion problems even under low stress conditions.

437

438 **4.2 Results with variations of yield surface parameter**

439 A parametric study of cavity expansion is presented as Case 4 with variable k_f and fixed k_g ($k_g =$
 440 0.9) for $r=1$ (normally consolidated state) based on CSM. Four different values of k_f (0.5, 0.7, 0.9,
 441 2), representing tear shape to elliptical shape, are considered to investigate the impact of the shape
 442 of yield surface on the stress and pore pressure distributions around the cavity.

443 Fig. 8(a) shows the effective stress paths normalized by the undrained shear strength s_u from
 444 $a_0/a_{0,0} = 1$ to $a_0/a_{0,0} = 2$ at the cavity wall. With the change of k_f from 0.5 to 2, the shape of yield
 445 surface varies and the intersection with the critical state line decreases. Figs. 8(b), 8(d) depict the
 446 normalized effective stresses (σ'_r/s_u , σ'_θ/s_u , σ'_z/s_u) and pore pressure $\Delta u_w/s_u$ distributions
 447 along radius position. The radius position a_x is normalized by cavity radius a_0 and plotted in the
 448 scale of logarithm. It can be seen that in the vicinity of cavity wall the three principal stresses
 449 remain almost unchanged, indicating that the soil in this range has reached the critical state.

450 Further investigation on the shape variation of yield surface shows that positive increment of
 451 k_f would sharply decrease the values of the three principal stresses in the critical state range (Fig.
 452 8(a)), but have negligible effects on the location of CP boundary. Fig. 8(c) presents the cylindrical
 453 cavity expansion curve from $a_0/a_{0,0} = 1$ to $a_0/a_{0,0} = 10$ at the cavity wall. The cavity pressure and
 454 excess pore pressure increase rapidly to a limit and then stay constant with cavity expanding. With
 455 the change of k_f from 0.5 to 2, the present solution predicts a decreasing cavity pressure and excess
 456 pore pressure limit around cavity wall. In Fig. 8(d), the normalized excess pore pressure would
 457 decrease monotonously along the radial distance. In the vicinity of the cavity wall, the stress point
 458 would reaches the critical state where three principal stresses (σ'_r , σ'_θ , σ'_z) remain almost

459 constant ($\frac{d\sigma'_r}{da_x} = \frac{d\sigma'_\theta}{da_x} = \frac{d\sigma'_z}{da_x} = 0$) and the radial balance equation ($\frac{d\sigma'_r}{da_x} + \frac{\sigma'_r - \sigma'_\theta}{a_x} + \frac{du_w}{da_x} = 0$)

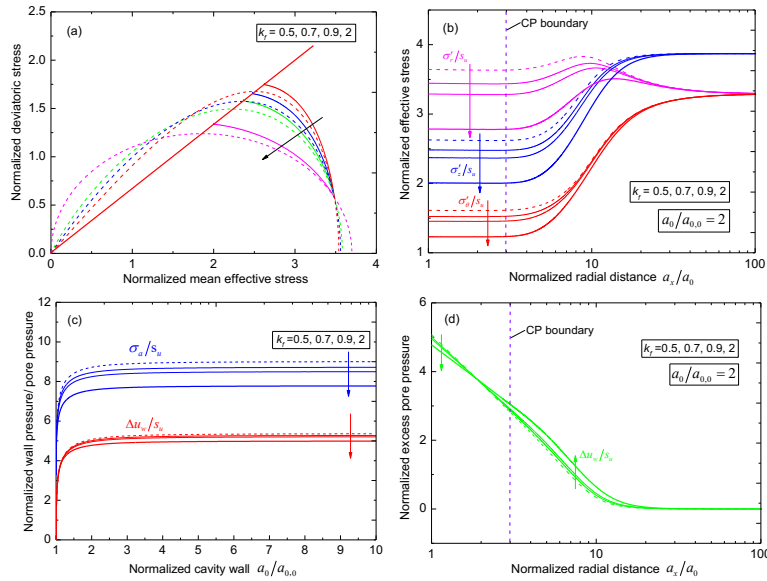
460 would reduce into $\frac{\sigma'_{rc} - \sigma'_{\theta c}}{a_x} + \frac{du_w}{da_x} = 0$. σ'_{rc} , $\sigma'_{\theta c}$ is the corresponding radial, tangential

461 effective stress at the critical state zone, respectively. So the solution of pore water pressure at this
 462 zone would be $u_w = -(\sigma'_{rc} - \sigma'_{\theta c}) \ln a_x + C$ (C is a constant). For the case of cavity expansion,

463 σ'_{rc} is larger than $\sigma'_{\theta c}$ and the excess pore pressure would decrease in the vicinity of the cavity
 464 wall (critical state zone). Meanwhile, under normal consolidated state, the stress would directly

465 turn into shear contraction state before reaching the critical state line. So the excess pore pressure
 466 would directly decrease from CP boundary to $a_x/a_0 = 100$.

467



468

469 Fig. 8 Undrained cavity expansion results with change of k_f from 0.5 to 2: (a) normalized stress
 470 paths at cavity wall; (b) normalized principal stress distributions; (c) normalized cavity expansion
 471 curve; (d) normalized excess pore pressure distribution

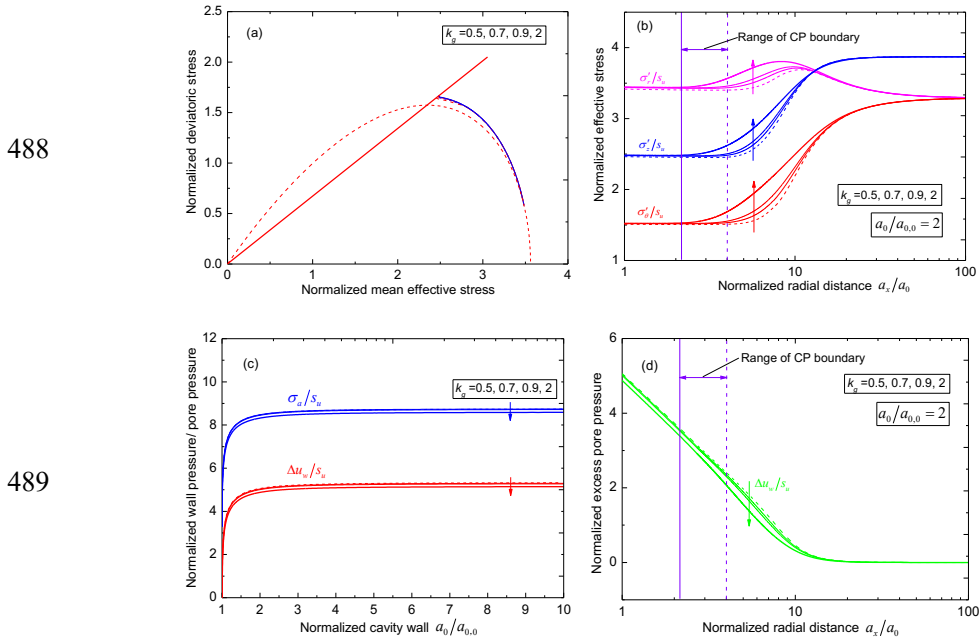
472

473 4.3 Results with variations of plastic potential surface parameter

474 A parametric study is presented as Case 5 with variable k_g and fixed k_f ($k_f = 0.7$) for $r=1$ and \bar{p}'_c
 475 (normally consolidated state) to investigate the impact of the direction of plastic flow on the stress
 476 and pore pressure distributions around the cavity.

477 From the same initial stress state, the four effective stress paths with different k_g values (0.5,
 478 0.7, 0.9, 2) normalized by the undrained shear strength s_u from $a_0/a_{0,0} = 1$ to $a_0/a_{0,0} = 2$ at the
 479 cavity wall are found to be almost overlapped in Fig. 9(a). This suggests that k_g merely control the
 480 direction of plastic flow and portion of shear plastic strain in loading process. Fig. 9(b), 9(d)
 481 depict the normalized effective stresses (σ'_r/s_u , σ'_θ/s_u , σ'_z/s_u) and pore pressure $\Delta u_w/s_u$
 482 distributions with the variation of k_g . The effective stresses in the vicinity of cavity wall typically
 483 remain constant. As opposed to changes of k_f , the direction variation of plastic flow shows that an
 484 positive increment of k_g induces more deviatoric plastic strains, leading to a sharp decrease of the

485 radius of critical state zone, but has negligible effects on the critical values of the three principal
 486 stresses in the critical state zone. It also shows a slight reduction of excess pore pressure around
 487 the cavity wall when an increasing plastic shear strain portion is produced.



490 Fig. 9 Undrained cavity expansion results with the variation of k_g from 0.5 to 2: (a) normalized
 491 stress paths at cavity wall; (b) normalized principal stress distributions; (c) normalized cavity
 492 expansion curve; (d) normalized excess pore pressure distribution

493

494 4.4 Results with variations of isotropic nonlinear parameter

495 In highly over-consolidated states, stiff clays typically exhibit a non-linear behavior under loading
 496 process without pure elastic deformations. A parametric study of cavity expansion in
 497 over-consolidated states with different isotropic nonlinear parameter s (4, 8, 20, 1000) is presented
 498 as Case 6. Note that s controls the evolution rate of Loading surface in approaching Yield surface,
 499 namely the nonlinearity of isotropic compression from over-consolidated state to normally
 500 consolidated state. A smaller s represents a clear nonlinearity of isotropic loading in v - $\ln p'$ plane,
 501 while a larger s means that the compression line in v - $\ln p'$ plane is closer to bi-linearity.

502 The expanding cavity is defined by an initial in-situ stress of $\sigma'_r=1.91$ MPa, $\sigma'_\theta=1.91$ MPa
 503 and $\sigma'_z=2.25$ MPa with a pre-consolidation pressure of $\bar{p}'_c=6$ MPa and over-consolidation ratio

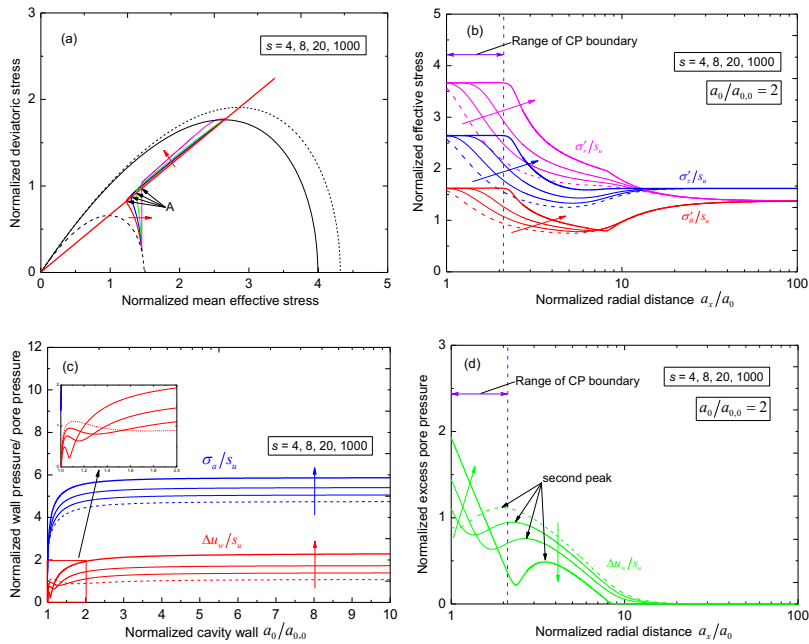
504 of $r=2.9$ as listed in Table.2. Figs. 10(a)-(d) depict the normalized effective stress paths at cavity
 505 wall, normalized cavity expansion curves, the normalized effective stresses and excess pore
 506 pressure distributions with change of s . Under the expanding process from $a_0/a_{0,0} = 1$ to $a_0/a_{0,0} = 2$,
 507 the normalized effective stresses follow different stress paths with s varying from 4 to 1000 in Fig.
 508 10(a). The direction of arrow means the increasing trend of s . When s reaches the value of 1000,
 509 the initial mechanical behavior is more elastic which explains the stress path being upward at the
 510 beginning of undrained loading process.

511 In Fig. 10(b), the range of critical state zone is sharply dependent on s . But the critical values
 512 of normalized effective stresses are independent of s . It is worth noting that with the decreasing of
 513 s (the increasing nonlinearity of isotropic compression), the radius of critical state zone would
 514 decrease to zero where no pure critical state zone exists. With the increasing of s (the decreasing
 515 nonlinearity of isotropic compression), the radius of critical state zone would increase to a limit
 516 where no further critical state zone develops. For an extremely large value of s (e.g. 1000), the
 517 analytical results are similar to the case of over-consolidated states expressed in modified
 518 cam-clay model [9]. Meanwhile, the increase in nonlinearity of isotropic compression would
 519 eventually results in the decrease of effective stress in plastic zone.

520 In Figs. 10(c)-(d), the normalized wall pressure at cavity wall is positive, in agreement with
 521 the value of s . A sophisticated phenomenon is observed on the normalized excess pore pressure. At
 522 the beginning of expanding process, the normalized excess pore pressure would firstly increase,
 523 decrease and then increase to a limited value. With the increasing of s , the increasing and
 524 decreasing phenomenon would be more obvious. At the expanded cavity of $a_0/a_{0,0} = 2$, the
 525 normalized excess pore pressure at cavity wall would increase to a limit value with increasing s . In
 526 the critical state zone, the normalized excess pore pressure would decrease along the normalized
 527 radial distance. Unlike the results by modified cam-clay model [9], a ‘second peak’ of normalized
 528 excess pore pressure is observed, which is owing to the soil’s shear dilatancy behavior under
 529 highly over-consolidated states. From $a_x/a_0 = 100$ to the ‘second peak’, the excess pore pressure
 530 would increase (shear contraction); from the ‘second peak’ to the CP boundary, the excess pore
 531 pressure would decrease (shear dilatancy) and the value of excess pore pressure would be
 532 $-(\sigma'_{rc} - \sigma'_{\theta c}) \ln a_x + C$ (C is a constant) in the range of critical state zone. The position of the

533 'second peak' is also consistent with the value of s . In the far end of expanded cavity, excess pore
 534 pressure would vanish.

535



536

537 Fig. 10 Undrained cavity expansion results with different values of s from 4 to 1000: (a)
 538 normalized stress paths at cavity wall; (b) normalized principal stress distributions; (c) normalized
 539 cavity expansion curve; (d) normalized excess pore pressure distribution

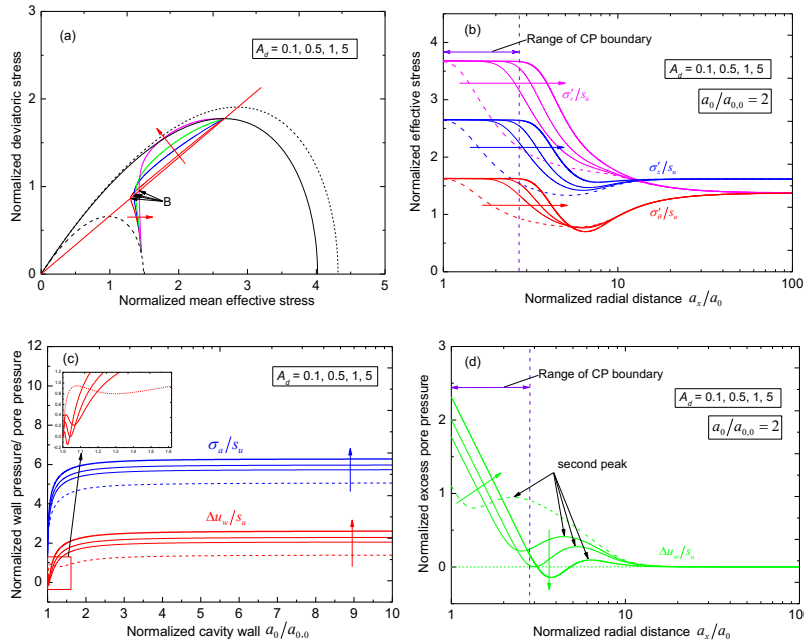
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541 4.5 Results with the variations of deviatoric nonlinear parameter

542 In over-consolidated states, parameter A_d describes how the plastic shear strain affects the
 543 hardening process. It can be calibrated from the stress paths under high over-consolidated states in
 544 undrained conditions. A parametric study of cavity expansion in over-consolidated states with
 545 different values of deviatoric nonlinear parameter A_d (0.1, 0.5, 1, 5) is presented as Case 7.

546 An in-situ stress of $\sigma'_r=1.91$ MPa, $\sigma'_\theta=1.91$ MPa and $\sigma'_z=2.25$ MPa with a
 547 pre-consolidation pressure of $\bar{p}'_c=6$ MPa and over-consolidation ratio of $r=2.9$ is considered to
 548 evaluate the impact of A_d on the stress and pore pressure distributions around the cavity.

549



550

551 Fig. 11 Undrained cavity expansion results with different values of A_d from 0.1 to 5: (a)
 552 normalized stress paths at cavity wall; (b) normalized principal stress distributions; (c) normalized
 553 cavity expansion curve; (d) normalized excess pore pressure distribution

554 Figs. 11(a)-(d) depict the results with different A_d values. Similar to nonlinear parameter s ,
 555 the radius of critical state zone is sharply dependent on A_d . But the critical values of normalized
 556 effective stress are independent of A_d . With the decreasing of A_d , the radius of critical state zone
 557 would decrease to zero where no pure critical state zone exists. At the expanded cavity of $a_0/a_{0,0} =$
 558 2, the normalized excess pore pressure at the cavity wall and the range of critical state zone would
 559 increase with increasing A_d . One should also note that the increasing of A_d means the increasing
 560 portion of plastic shear strain and the decreasing portion of plastic volumetric strain in hardening
 561 process. With the increasing of A_d , the stress path would be more likely upward at the beginning of
 562 undrained loading process. In Fig. 11(d), a 'second peak' of normalized excess pore pressure is
 563 also observed and the peak gradually vanish with the increasing of A_d .

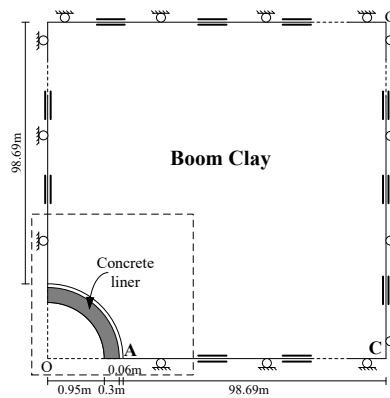
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565 5. Application to Praclay Gallery excavation

566 This section examines the application of the present solution into the excavation of Praclay
 567 Gallery in underground research facility HADES-URF built at a depth of 223m in natural Boom

568 clay and located in Mol, Belgium [26, 27]. The natural Boom clay is a deposit of
 569 over-consolidated stiff clay which have been specifically investigated in the previous studies [16].
 570 Meanwhile, the natural Boom clay is initially considered as homogeneous, at an isotropic effective
 571 stress of 2.3 MPa and fully saturated with an initial pore pressure of 2.2 MPa.

572 Praclay Gallery is excavated 45 meters long in horizontal plane, so it is regarded as a plane
 573 strain problem as plotted in Fig. 12. The inner radius of Praclay Gallery and the thickness of the
 574 concrete liner is equal to 0.95 m, 0.3m, respectively. During the construction of Praclay Gallery, a
 575 total over-excavation of 0.06m is applied. Since it is finished in a short period (less than 1 day)
 576 and the intrinsic permeability of natural Boom clay is extremely low ($2\sim4\times 10^{-19} \text{ m}^2$), the
 577 excavation of Praclay Gallery can also be regarded as an undrained cavity contraction process.



578

579 Fig. 12 Excavation plane of Praclay Gallery in natural Boom clay

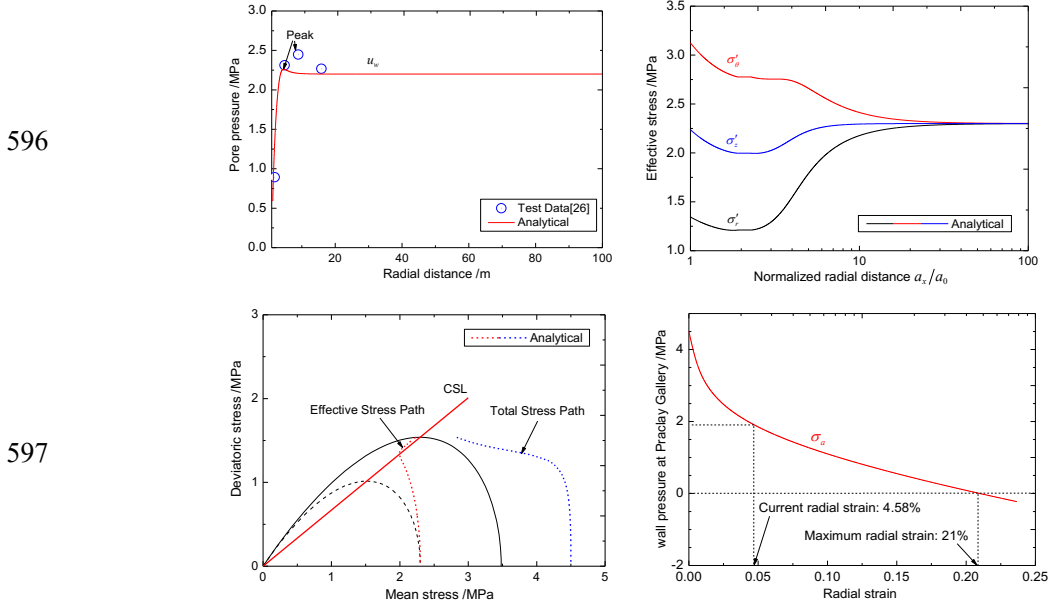
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581 Fig. 13(a) presents the analytical results by cavity contraction and the test values from four
 582 pore pressure transducers placed at different nominal radial distances from the center of Praclay
 583 Gallery [26]. It shows that the present solution could basically describe the pore pressure
 584 distribution along radial distances. In the vicinity of cavity wall, pore pressure would reduce with
 585 radial distance decreasing. Then a peak value shows up around the radial distance of 5~10m. After
 586 that peak, the pore pressure would gradually reduce to its normal value of 2.2 MPa.

587 Fig. 13(b) depicts three principal stress distributions along radial distance by the solution of
 588 cavity contraction. It shows that the vicinity of cavity wall has not reached the critical state yet.
 589 The effective stress path of point A in Fig. 13(c) can also be preferential for studying the stress
 590 states around the contracting wall.

591 In the cavity contraction curve (Fig. 13(d)), the wall pressure at Praclay Gallery would

592 gradually reduce with the increasing radial strain. In the present case, the radial strain is equal to
 593 4.58% (radial displacement is 0.06m) and a wall pressure of 1.93 MPa is formed. However, when
 594 the radial strain reaches 21%, the wall pressure would be zero. It indicates that no more supporting
 595 pressure needs to be added on the cavity wall.



598 Fig. 13 The excavation of Praclay Gallery: (a) pore pressure distribution along radial distance; (b)
 599 principal effective stress distributions along normalized radial distance; (c) total and effective
 600 stress paths of Point A; (d) cavity contraction curve

601

602 **6. Conclusion**

603 This paper presents a semi-analytical solution for undrained cylindrical cavity
 604 expansion/contraction problems using a two-surface plasticity model, which can well reproduce
 605 the yielding behaviors, unassociated plastic flowing directions and high nonlinearity of stiff clays.
 606 To develop the new solution, the premise of constant soil mass based on poromechanics is recalled.
 607 A set of seven governing differential equations is obtained and solved as an initial boundary value
 608 problem with an auxiliary independent variable ζ . The new solution based on constant soil mass is
 609 then compared with the one based on constant soil volume, showing its relevance under high
 610 stress conditions. Then, based on the constant soil mass premise, the variations of yield surface
 611 parameter, plastic flow direction parameter, isotropic nonlinear parameter and deviatoric nonlinear

612 parameter are investigated in a parametric study focusing on the stress paths, stress and excess
613 pore pressure distributions and cavity expansion/contraction curves. The results obtained allows
614 the following conclusion to be drawn:

615 (1) The premise of constant soil mass is important in solving cavity expansion/contraction
616 problems especially under high stress, low porosity and multi-fluid situations. Indeed, under
617 relatively low stress conditions (e.g. $p' < 1$ MPa), the solutions based on constant soil mass and
618 constant soil volume are found to be similar. However, with the increasing of stress, the
619 compressibility of pore fluid becomes crucial and the volumetric strain could be no longer
620 regarded as constant. The premise of constant soil mass become preferential.

621 (2) In normally consolidated states, the shape variation of yield surface (k_f) would change the three
622 principal stresses in critical state zone but have no effect on the range of critical state zone. On the
623 contrary, the variation of plastic flow direction (k_g) would change the range of critical state zone
624 but have no effect on the three principal stresses in the critical state zone.

625 (3) The high nonlinearity means no pure elastic zones in over-consolidated states. The decreasing
626 of both nonlinearity parameter s and A_d decreases the range of critical state zone even to zero
627 where no pure critical state zone exists. Meanwhile, a ‘second peak’ value of normalized excess
628 pore pressure along cavity radius is observed and the peak would gradually approach the cavity
629 wall with the decreasing of s and A_d .

630 (4) The present semi-analytical solution can be directly applied to explore the pore pressure, stress
631 distributions and wall pressure around the expanding/contracting cavity (e.g. Praclay Gallery
632 excavation).

633

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639

640 **Appendix A**

641 The effective stress and total strain increments are defined with three principal components in a
642 cylindrical coordinate system:

$$643 \quad \mathbf{d}\boldsymbol{\sigma}' = [d\sigma'_r \quad d\sigma'_\theta \quad d\sigma'_z]^T \quad (\text{A - 1})$$

$$644 \quad \mathbf{d}\boldsymbol{\varepsilon} = [d\varepsilon_r \quad d\varepsilon_\theta \quad d\varepsilon_z]^T \quad (\text{A - 2})$$

645 The total strain increment is split into elastic and plastic parts:

$$646 \quad \mathbf{d}\boldsymbol{\varepsilon} = \mathbf{d}\boldsymbol{\varepsilon}^e + \mathbf{d}\boldsymbol{\varepsilon}^p \quad (\text{A - 3})$$

$$647 \quad \mathbf{d}\boldsymbol{\varepsilon}^e = [d\varepsilon_r^e \quad d\varepsilon_\theta^e \quad d\varepsilon_z^e]^T \quad (\text{A - 4})$$

$$648 \quad \mathbf{d}\boldsymbol{\varepsilon}^p = [d\varepsilon_r^p \quad d\varepsilon_\theta^p \quad d\varepsilon_z^p]^T \quad (\text{A - 5})$$

$$649 \quad \boldsymbol{\alpha} = [a_r \quad a_\theta \quad a_z]^T \quad (\text{A - 6})$$

$$650 \quad a_r = \frac{\partial f_l}{\partial \sigma'_r} = \frac{M_f^2}{3(1-k_f)} \left(\frac{2}{k_f} p'^{\frac{2}{k_f}-1} (rp'_c)^{2-\frac{2}{k_f}} - 2p' \right) + 6(\sigma'_r - p') \quad (\text{A - 7})$$

$$651 \quad a_\theta = \frac{\partial f_l}{\partial \sigma'_\theta} = \frac{M_f^2}{3(1-k_f)} \left(\frac{2}{k_f} p'^{\frac{2}{k_f}-1} (rp'_c)^{2-\frac{2}{k_f}} - 2p' \right) + 6(\sigma'_\theta - p') \quad (\text{A - 8})$$

$$652 \quad a_z = \frac{\partial f_l}{\partial \sigma'_z} = \frac{M_f^2}{3(1-k_f)} \left(\frac{2}{k_f} p'^{\frac{2}{k_f}-1} (rp'_c)^{2-\frac{2}{k_f}} - 2p' \right) + 6(\sigma'_z - p') \quad (\text{A - 9})$$

$$653 \quad \mathbf{b} = [b_r \quad b_\theta \quad b_z]^T \quad (\text{A - 10})$$

$$654 \quad b_r = \left(\frac{\partial g}{\partial p'} / \frac{\partial g}{\partial q} \times \frac{\partial p'}{\partial \sigma'_r} + \frac{\partial q}{\partial \sigma'_r} \right) \times k_g \eta = \frac{1}{3} (M_g^2 - \eta^2) + \frac{3}{q} k_g \eta (\sigma'_r - p') \quad (\text{A - 11})$$

$$655 \quad b_\theta = \left(\frac{\partial g}{\partial p'} / \frac{\partial g}{\partial q} \times \frac{\partial p'}{\partial \sigma'_\theta} + \frac{\partial q}{\partial \sigma'_\theta} \right) \times k_g \eta = \frac{1}{3} (M_g^2 - \eta^2) + \frac{3}{q} k_g \eta (\sigma'_\theta - p') \quad (\text{A - 12})$$

$$656 \quad b_z = \left(\frac{\partial g}{\partial p'} / \frac{\partial g}{\partial q} \times \frac{\partial p'}{\partial \sigma'_z} + \frac{\partial q}{\partial \sigma'_z} \right) \times k_g \eta = \frac{1}{3} (M_g^2 - \eta^2) + \frac{3}{q} k_g \eta (\sigma'_z - p') \quad (\text{A - 13})$$

$$657 \quad h = -\frac{\partial f}{\partial p'_c} \frac{\partial p'_c}{\partial \varepsilon_v^p} \frac{\partial g}{\partial p'} - \frac{\partial f}{\partial r} \left(\frac{\partial r}{\partial \varepsilon_v^p} \frac{\partial g}{\partial p'} + \frac{\partial r}{\partial \varepsilon_s^p} \frac{\partial g}{\partial q} \right) \quad (\text{A - 14})$$

$$= \frac{\partial f}{\partial p'_c} \frac{v_0}{\lambda - \kappa} (M_g^2 - \eta^2) + \frac{\partial f}{\partial r} \frac{v_0}{\lambda - \kappa} s(1-r) \left((M_g^2 - \eta^2) + A_d k_g \eta \right)$$

$$658 \quad \frac{\partial f}{\partial p'_c} = \frac{M_f^2}{1-k_f} p'^{\frac{2}{k_f}} \left(2 - \frac{2}{k_f}\right) p_c^{1-\frac{2}{k_f}} r^{2-\frac{2}{k_f}} \quad (\text{A - 15})$$

$$659 \quad \frac{\partial f}{\partial r} = \frac{M_f^2}{1-k_f} p'^{\frac{2}{k_f}} \left(2 - \frac{2}{k_f}\right) r^{1-\frac{2}{k_f}} p_c^{2-\frac{2}{k_f}} \quad (\text{A - 16})$$

$$660 \quad d\boldsymbol{\sigma}' = \mathbf{D}_{ep} d\boldsymbol{\varepsilon} = \left(\mathbf{D}_e - \frac{\mathbf{D}_e \mathbf{b} \boldsymbol{\alpha}^T \mathbf{D}_e}{\boldsymbol{\alpha}^T \mathbf{D}_e \mathbf{b} + h} \right) d\boldsymbol{\varepsilon} \quad (\text{A - 17})$$

$$661 \quad \mathbf{D}_e = \frac{3K}{1+\nu} \begin{bmatrix} 1-\nu & \nu & \nu \\ \nu & 1-\nu & \nu \\ \nu & \nu & 1-\nu \end{bmatrix} \quad (\text{A - 18})$$

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676 **Appendix B**

677 In the constant soil volume assumption where pore water is regarded as an incompressible fluid,
 678 the undrained cylindrical cavity expansion/contraction problem is a simple plane strain
 679 deformation problem with five first-order ordinary differential equations for all material points,
 680 namely:

$$681 \quad \frac{D\sigma'_r}{Da_x} = \frac{c_{rr} - c_{r\theta}}{a_x} \quad (\text{B - 1})$$

$$682 \quad \frac{D\sigma'_\theta}{Da_x} = \frac{c_{\theta r} - c_{\theta\theta}}{a_x} \quad (\text{B - 2})$$

$$683 \quad \frac{D\sigma'_z}{Da_x} = \frac{c_{zr} - c_{z\theta}}{a_x} \quad (\text{B - 3})$$

$$684 \quad \frac{Dp'_c}{Da_x} = -\frac{\kappa p'_c}{\lambda - \kappa} \frac{c_{rr} + c_{\theta r} + c_{zr} - c_{r\theta} - c_{\theta\theta} - c_{z\theta}}{a_x (\sigma'_r + \sigma'_\theta + \sigma'_z)} \quad (\text{B - 4})$$

$$685 \quad \frac{Dr}{Da_x} = -\frac{\kappa}{\lambda - \kappa} s(1-r) \left(1 + \frac{A_d k_g \eta}{M_g^2 - \eta^2} \right) \frac{c_{rr} + c_{\theta r} + c_{zr} - c_{r\theta} - c_{\theta\theta} - c_{z\theta}}{a_x (\sigma'_r + \sigma'_\theta + \sigma'_z)} \quad (\text{B - 5})$$

686 where D/Da_x denotes the material derivative taken along the particle motion path (Lagrangian
 687 description), and η denotes the current stress ratio q/p' .

688 Meanwhile, the constant soil volume assumption would induce a constraint following the equation
 689 regarding the current position of any material point a_x and the expanded cavity radius a_0 :

$$690 \quad a_x^2 - a_{x0}^2 = a_0^2 - a_{00}^2 \quad (\text{B - 6})$$

691 where a_{x0} is the initial position of the material point and a_{00} is the initial cavity radius.

692 The distribution of pore water pressure $u_w(a_x)$ can be obtained by integrating the radial equilibrium
 693 equation:

$$694 \quad u_w(a_x) = \sigma'_{r0} + u_{w0} - \sigma'_r(a_x) - \int_{\infty}^{a_x} \frac{\sigma'_r - \sigma'_\theta}{a_x} da_x \quad (\text{B - 7})$$

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698 **Appendix C**

699 In isotropic stress state, the total mean stress change (dp) and lagrangian porosity change ($d\phi$)

700 can be expressed by changes of volumetric strain ($d\varepsilon_v$) and pore water pressure (du_w), as follows:

701
$$dp = \frac{I}{C_d} d\varepsilon_v + b du_w \quad (C - 1)$$

702
$$d\phi = -bd\varepsilon_v - \frac{I}{N} du_w \quad (C - 2)$$

703 with the Maxwell's symmetry condition given by:

704
$$b = \left(\frac{\partial p}{\partial u_w} \right)_{\varepsilon_v} = - \left(\frac{\partial \phi}{\partial \varepsilon_v} \right)_{u_w} \quad (C - 3)$$

705 The volumetric compressibility of soil skeleton C_d , the Biot coefficient b appearing in Eq (C - 1)

706 are defined as:

707
$$\frac{I}{C_d} = \left(\frac{\partial p}{\partial \varepsilon_v} \right)_{u_w}, \quad b = \left(\frac{\partial p}{\partial u_w} \right)_{\varepsilon_v} \quad (C - 4)$$

708 The Biot coefficient b , Biot modulus N appearing in Eq (C - 2) are defined as:

709
$$b = - \left(\frac{\partial \phi}{\partial \varepsilon_v} \right)_{u_w}, \quad \frac{I}{N} = - \left(\frac{\partial \phi}{\partial u_w} \right)_{\varepsilon_v} \quad (C - 5)$$

710 Eqs (C - 4, C - 5) are partial differential equations and the subscripts indicate that the

711 corresponding variables are kept constant.

712 Considering the relations between the skeleton and mineral solid properties, the compatibility

713 relations can be obtained:

714
$$b = 1 - \frac{C_s}{C_d}, \quad \frac{I}{N} = (\phi - b)C_s \quad (C - 6)$$

715 where C_s is the volumetric compressibility coefficient of mineral solid.

716 In a fully saturated porous medium, the current fluid mass (m_w) per unit volume can be expressed

717 as:

718
$$m_w = \rho_w \phi \quad (C - 7)$$

719 Differentiating Eq (C - 7) gives:

720
$$\frac{dm_w}{\rho_w} = d\phi + \phi \frac{d\rho_w}{\rho_w} \quad (C - 8)$$

721 The change of fluid density reads as follows:

722
$$\frac{d\rho_w}{\rho_w} = C_w du_w \quad (\text{C - 9})$$

723 where C_w is the volumetric compressibility of pore fluid.

724 From Eqs (C - 1, C - 2, C - 6, C - 8, C - 9), the following equation can be obtained:

725
$$\frac{dm_w}{\rho_w} = -(C_d - C_s)dp - ((\phi C_s - \phi C_w) - (C_d - C_s))du_w \quad (\text{C - 10})$$

726 In undrained conditions, the mass of the fluid phase is constant and $dm_w = 0$ holds. Thus, the
727 expressions for incremental pore water pressure can be obtained:

728
$$du_w = \frac{C_d - C_s}{C_d - C_s + \phi(C_w - C_s)} dp = \frac{d\varepsilon_v - C_s dp}{\phi(C_w - C_s)} \quad (\text{C - 11})$$

729 The well-known Skempton coefficient B which indicates the ratio between pore pressure
730 increment and total isotropic stress increment is thus given by:

731
$$B = \frac{C_d - C_s}{C_d - C_s + \phi(C_w - C_s)} \quad (\text{C - 12})$$

732 The compressibility of pore fluid is usually dozens of times that of solid grain. For most clays, the
733 compressibility of solid grains can be neglected ($C_s \approx 0$). With infinitesimal transformation, Eq (C
734 - 11) can be written using Eulerian porosity n and pore fluid's bulk modulus k_w ($k_w = 1/C_w$).

735
$$du_w = \frac{d\varepsilon_v - C_s dp}{\phi(C_w - C_s)} \approx \frac{d\varepsilon_v}{nC_w} = \frac{k_w d\varepsilon_v}{n} \quad (\text{C - 13})$$

736

737 **Data availability statement**

738 The data used to support the findings of this study are available from the corresponding author

739 upon reasonable request.

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871 **LIST OF TABLES**

872 Table.1 The selected parameters of ACC2 model for natural Boom clay

873 Table 2. Summary of material properties of natural Boom clay

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Table 1. The selected parameters of ACC2 model for natural Boom clay

λ	κ	ν	\bar{p}'_c : MPa	M_f	k_f
0.18	0.02	0.3	6	0.67	0.7
M_g	k_g	r_0	s	A_d	
0.67	0.9	-	8	0.1	

Table 2. Summary of material properties of natural Boom clay

Case	OCR	σ'_r : MPa	σ'_θ : MPa	σ'_z : MPa	k_f	k_g	s	A_d	r_0	\bar{p}'_c : MPa	K_0	ν_0	s_u : MPa
1	1	2	2	2	0.7	0.9	8	0.1	1	2	1	1.76	0.60
2	1	10	10	10	0.7	0.9	8	0.1	1	10	1	1.47	2.95
3	1	50	50	50	0.7	0.9	8	0.1	1	50	1	1.19	14.29
4	1	5.1	5.1	6	*	0.9	8	0.1	1	*	0.85	1.59	1.55
5	1	5.1	5.1	6	0.7	*	8	0.1	1	5.51	0.85	1.59	1.55
6	2.9	1.91	1.91	2.25	0.7	0.9	*	0.1	0.34	6	0.85	1.61	1.39
7	2.9	1.91	1.91	2.25	0.7	0.9	8	*	0.34	6	0.85	1.61	1.39

$\nu_s=1.84$, $M_g=0.67$, '*' means different values are taken.