



HAL
open science

Determining Bishop's parameter χ based on pore size distribution

Geng Niu, Yu-Jun Cui, Jean-Michel Pereira, Longtan Shao, De'an Sun

► **To cite this version:**

Geng Niu, Yu-Jun Cui, Jean-Michel Pereira, Longtan Shao, De'an Sun. Determining Bishop's parameter χ based on pore size distribution. *Géotechnique Letters*, 2021, 11 (1), pp.1-28. 10.1680/jgele.20.00095 . hal-03171456

HAL Id: hal-03171456

<https://enpc.hal.science/hal-03171456>

Submitted on 16 Mar 2021

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

1 **Determining Bishop's parameter χ based on pore size**
2 **distribution**

3
4 Geng Niu¹, Yu-jun Cui², Jean-Michel Pereira², Longtan Shao¹, De'an Sun³
5
6
7
8

9 ¹Department of Engineering Mechanics, Dalian University of Technology, 116024
10 Dalian, Liaoning, China

11 ²Navier, Ecole des Ponts, Univ Gustave Eiffel, CNRS, Marne-la-Vallée, France

12 ³Department of Civil Engineering, Shanghai University, 200444 Shanghai, China
13
14
15
16
17
18

19 *Corresponding author:

20 Prof. Yu-Jun CUI
21 Ecole des Ponts ParisTech
22 6-8 av. Blaise Pascal, Cité Descartes, Champs-sur-Marne
23 F-77455 Marne-la-Vallée cedex France
24 Telephone: +33 1 64 15 35 50
25 Fax: +33 1 64 15 35 62
26 E-mail: yujun.cui@enpc.fr
27

28 **Abstract**

29 Extension of the effective stress concept to unsaturated soils has been a major concern
30 for decades. Recent studies significantly contributed to the understanding of the
31 fundamentals behind Bishop's parameter χ which is generally used to define the
32 effective stress for unsaturated soils. Examination of the recently proposed methods
33 showed that the contribution of suction to effective stress was often overestimated,
34 especially in high suction range. In this study, considering that soil pores with different
35 sizes contribute differently to the overall hydro-mechanical behaviour, a new method
36 to determine Bishop's parameter χ is proposed. The key variable used in this method
37 is the ratio of the change in water volume to the change in macro-pore void volume
38 due to loading at constant suction. Shear strength data for a weakly expansive clay
39 were used for validation. A good agreement was obtained between prediction and
40 measurement, indicating the validity of the proposed method.

41

42 **Keywords:**

43 constitutive relations; fabric/structure of soils; shear strength; partial saturation;

44 suction

45 **NOTATION**

46	e_{w2}	water ratio caused by change of strain at constant suction
47	e_w^m	microscopic water ratio
48	e^m	microscopic void ratio
49	S_r^e	effective degree of saturation
50	S_r^m	microscopic degree of saturation
51	S_r^{cap}	capillary degree of saturation
52	S_r^{ads}	adsorbed degree of saturation
53	s_m	suction corresponding to the median pore diameter
54	V_{w1}	water volume governed by suction
55	α	ratio of the change in incremental water volume caused by the change of
56		strain under constant suction to total pore volume increment
57	λ	material parameter defining the effective degree of saturation
58	ξ	standard deviation of the log-transformed pore radius
59	φ	fitting parameter related to the theoretical degree of saturation due to
60		adsorption at 1 kPa suction
61	χ	Bishop's effective stress parameter
62		

63 INTRODUCTION

64 The definition of stress variables is essential in developing constitutive models for
65 unsaturated soils. There are two approaches for modeling their mechanical behaviour:
66 the effective stress approach (Bishop, 1959) and the two independent state variables
67 approach (Coleman 1962, Fredlund & Morgenstern, 1977). Currently, using two
68 independent stress variables to describe the behavior of unsaturated soils is widely
69 accepted (Cui and Delage, 1996; Wheeler *et al.*, 2003; Liu *et al.*, 2018). Bishop stress
70 is often considered as one variable in the two independent stresses approach, because
71 it represents a smooth extension of Terzaghi's effective stress (Sheng *et al.*, 2004;
72 Pereira *et al.*, 2005; Zhou *et al.*, 2018). In the Bishop's effective stress concept, the
73 key parameter is χ , as shown in Eq. 1 (Bishop, 1959):

$$74 \quad \sigma' = \sigma - [\chi u_w + (1 - \chi)u_a] \quad (1)$$

75 where σ' is unsaturated effective stress; σ is total stress; χ is the effective stress
76 parameter (Bishop's parameter); u_w and u_a are water and air pressures, respectively.

77 In the past few decades, various criteria were reported for determining χ . In the
78 early contributions (Bishop & Blight, 1963), χ was set equal to degree of saturation S_r :

$$79 \quad \chi = S_r \quad (2)$$

80 which is the simplest, and most used assumption to date. But several studies showed
81 that this choice significantly overestimates the contribution of suction to effective
82 stress, especially in the high suction range (Sheng *et al.*, 2011; Zhou *et al.*, 2016). In
83 a later effort, the overestimation was reduced by using an effective degree of
84 saturation (Vanapalli *et al.*, 1996). To define this latter, attempts have been made to

85 directly relate χ to the total degree of saturation, such as (Vanapalli *et al.*, 1996):

$$86 \quad \chi = f(S_r) = (S_r)^\lambda \quad (3)$$

87 where $\lambda \geq 1$ is a material parameter. For coarse-grained soils, parameter λ is almost
88 equal to unity, while for fine-grained soils λ is larger than unity. Garven & Vanapalli
89 (2006) found that λ is correlated with plasticity index, I_p .

90 Soil water consists in capillary water and adsorbed one. Since adsorbed water is
91 strongly bonded to soil particles (Lu *et al.*, 2010; Konrad & Lebeau, 2015; Zhou *et*
92 *al.*, 2016; Gao *et al.*, 2018), it does not contribute to the contact stress and thus to the
93 effective stress. Thereby, Zhou *et al.* (2016) related the Bishop's parameter χ to the
94 capillary degree of saturation S_r^{cap} which is the difference between the total degree
95 of saturation and the adsorbed degree of saturation S_r^{ads} :

$$96 \quad \chi = S_r^{\text{cap}} = \frac{C(s) - \varphi C(s)A(s)}{1 - \varphi C(s)A(s)} \quad (4)$$

$$97 \quad \text{with } C(s) = \frac{1}{2} \operatorname{erfc} \left[\frac{\ln(s/s_m)}{\sqrt{2}\xi} \right], \quad A(s) = 1 - \frac{\ln(s)}{\ln(s_d)}$$

98 where s is matric suction; φ and ξ are fitting parameters determined by soil water
99 retention curve (SWRC); s_m is the suction which corresponds to the median pore
100 diameter; s_d is the suction at extreme dry state, assumed equal to 10^6 kPa. However,
101 the latest investigation showed that using capillary degree of saturation
102 underestimates the contribution of suction to Bishop's effective stress for most soils
103 (Gao *et al.*, 2020). It has been well documented that for compacted soils, the
104 microstructure is characterised by two pore populations: i) macro-pores where
105 capillary effects dominate, and ii) micro-pores where the hygroscopic effect
106 dominates (Romero & Vaunat, 2000; Alonso *et al.*, 2010). It has been admitted that

107 the contribution of suction to Bishop's effective stress is only related to the water
 108 trapped in macro- or inter-aggregate pores (Alonso *et al.*, 2010; Zhai *et al.*, 2019).
 109 This was confirmed by some experimental results which showed that loading and
 110 drying paths predominantly influenced the macropores, while micropores remain
 111 almost undisturbed (Monroy *et al.*, 2010; Mašín, 2013). Therefore, Bishop's
 112 parameter χ can be considered as the effective degree of saturation of macro-pores S_r^e
 113 (Alonso *et al.*, 2010):

$$114 \quad \chi = S_r^e = \frac{S_r - S_r^m}{1 - S_r^m} \quad (5)$$

115 where S_r^m the degree of saturation of micro-pores. This method can improve the
 116 prediction of effective stress, but still overestimates the contribution of suction in
 117 some cases (Sheng *et al.*, 2011; Zhai *et al.*, 2019).

118 In this study, a new method for determining χ using the macro-PSD is proposed,
 119 which satisfies Houlsby's power equation.

120 **DETERMINING BISHOP'S PARAMETER χ BASED ON MACROPOROSITY**

121 Fig. 1 shows a conceptual sketch of water retention curves at two different void ratios
 122 (Vaunat & Casini, 2017). The incremental change in water ratio δe_w (path OB) caused
 123 either by suction change or mechanical loading can be split into δe_{w1} (path OA) and
 124 δe_{w2} (path AB). The change of water content along path OA is caused by the increment
 125 of suction at constant volume, while the incremental component of path AB is caused
 126 by the change of void ratio under constant suction (net stress effect). This latter can
 127 be expressed as a proportion of total volume change (Vaunat & Casini, 2017). The
 128 total incremental work input per unit volume in unsaturated soils δw (Houlsby, 1997)

129 can be expressed as:

$$\begin{aligned} 130 \quad \delta w &= -[\sigma - \alpha u_w - (1 - \alpha)u_a] \frac{\delta V_V}{V} - (u_a - u_w) \frac{\delta V_{w1}}{V} \\ 131 \quad &= -[\sigma - \alpha u_w - (1 - \alpha)u_a] \delta \varepsilon_V - (u_a - u_w) \frac{\delta V_{w1}}{V} \end{aligned} \quad (6)$$

132 where σ is the total mean stress; V is the total soil volume; V_V is the volume of voids;
133 δV_{w1} is the change of volume occupied by water due to suction; α is the ratio of the
134 increment of volume occupied by water caused by the incremental volumetric strain
135 under constant suction to the pore volume increment. As the first term between
136 brackets in Eq. (6) expresses Bishop's effective stress, Vaunat & Casini (2017)
137 proposed:

$$138 \quad \chi = \alpha = \frac{\delta V_{w2}}{\delta V_V} = \frac{\delta e_{w2}}{\delta e} \quad (7)$$

139 where δe_{w2} is the increment of water ratio caused by the change of volumetric strain
140 under constant suction; δe is the increment of void ratio.

141 Fig. 2 shows the SWRCs under different void ratios for several soils (Li *et al.*,
142 2007; Salager *et al.*, 2013; Seiphoori *et al.*, 2014; Gao & Sun, 2017). It appears that
143 in low suction range, the water retention curve is dependent on void ratio. By contrast,
144 in higher suction range, the SWRCs are almost the same (Fig. 3). The separating point
145 corresponds to the delimitation between the range of capillary water in macro-pores
146 and the range of adsorbed water in micro-pores (Romero, 1999). Mařín (2013) also
147 discussed the structure evolution with mechanical loading and assumed that loading
148 (compaction) influenced predominantly the macro-pores, while micro-pores remained
149 untouched (Fig. 3). Thus, as the volume change is mainly due to macro-pores changes,
150 Bishop's parameter χ is solely related to the low suction range (Fig. 3). Thereby, Eq.

151 (7) is modified as:

$$152 \quad \chi = \frac{\delta e_{w2} - \delta e_w^m}{\delta e - \delta e^m} \quad (8)$$

153 where δe_w^m is the incremental microscopic water ratio; δe^m is the incremental
154 microscopic void ratio. In Fig. 3, two SWRCs at different void ratios should be
155 provided and the void ratio of every SWRC should be kept constant in the whole
156 suction range. However, in a SWRC test, the void ratio changes with suction changes.
157 As the pore size distribution (PSD) obtained by Mercury Intrusion Porosimetry (MIP)
158 test can be used to determine the SWRC at a constant void ratio (Zhang *et al.*, 2018),
159 the parameters in Eq. (8) can be obtained using two different PSDs, as illustrated in
160 Fig. 4.

161 **EVALUATION OF THE PROPOSED METHOD THROUGH SHEAR** 162 **STRENGTH**

163 Bishop & Blight (1963) described the shear strength of unsaturated soils by:

$$164 \quad \tau_f = c' + [(\sigma_n - u_a) + \chi(u_a - u_w)] \tan \theta' \quad (9)$$

165 where τ_f is the shear strength; c' is the effective cohesion at saturated state; σ_n is
166 the normal stress; θ' is the internal friction angle. Using Eq. 9 and considering
167 triaxial conditions, parameter χ can be back-calculated, as follows:

$$168 \quad \chi = \frac{\frac{q_f - c'}{3 - \sin \theta'} \frac{6 \cos \theta'}{M} \bar{p}}{s} \quad (10)$$

169 where q_f is the deviator stress at failure; $M = 6 \sin \phi' / (3 - \sin \phi')$ is the slope of critical
170 state line; $\bar{p} = [\sigma_1 + 2\sigma_3] / 3 - u_a$ is the net mean stress.

171 The results from the tests on Nanyang expansive clay were used to validate the

172 proposed method for Bishop's parameter χ determination (Fig. 8). Table 1 summarises
173 the physical property indexes and triaxial shear parameters for this clay. Triaxial shear
174 tests were conducted using an unsaturated triaxial testing apparatus supplied by GDS
175 company. In order to obtain the molded samples only in drying process, all triaxial
176 specimens were prepared by static compaction at initial water content of about 0.215
177 (suction is about 170 kPa) and initial dry density around 1.25 Mg/m³. Table 2 gives a
178 summary of the test conditions and stress paths. For tests No.1-3, suction was applied
179 using axis-translation technique in triaxial cell, while for tests No.4 and No.5 the
180 suctions (0.8 and 2.5 MPa, respectively) were applied through controlling the water
181 contents ($w=14.8\%$ and $w=12.55\%$, respectively) by referring to the SWRC in drying
182 process. For higher suctions, *i.e.* tests No.6-8, the vapor equilibrium method was
183 employed. When the samples of tests No.4-8 reached the target respective suctions,
184 they were put into triaxial cell for further consolidation and shear tests following the
185 stress and suction paths shown in Figs. 5 (a) and (b). The net confining pressure was
186 100 kPa, and the suctions were 0.05, 0.2, 0.4, 0.8, 2.5, 3.29, 38 and 368 MPa,
187 respectively. In the triaxial shear tests, q_f and \bar{p} were measured for every suction state
188 and given in Table 3. As illustrated in Fig. 2, when the suction is higher than a specific
189 value, the water retention curve is independent of void ratio. In addition, the modulus
190 of soils with high suction are normally very large and the changes of void during shear
191 tests are thus small. Thereby, shear tests under constant water content are often
192 considered as under constant suction in high suction range (Gao *et al.*, 2019; Zhang
193 *et al.*, 2020). Substitution of q_f , \bar{p} , c' θ' and s (see Tables 1, 2 and 3) into Eq. (10)

194 allows the calculation of Bishop's parameter χ for every suction (or degree of
195 saturation), as illustrated in Fig. 8.

196 As the void ratio of this sample with 3.29 MPa in shear test is 0.99, the PSD
197 curves of two samples compacted at different void ratios (0.81 and 0.92) at the same
198 constant suction of 3.29 MPa are considered. The results are shown in Fig. 6.
199 According to Lloret *et al.* (2003), the macro- and micro-pores are separated by the
200 point where the PSD becomes independent of loading. For bimodal structure, the
201 diameter at the valley bottom between the two pore size families can be considered as
202 the delimiting diameter between micro- and macro-pores. It appears from Fig. 6 that
203 the PSD curves are not affected by the compaction load when the pore diameter is
204 smaller than 4900 nm. Thereby, this diameter was taken as the boundary between
205 macro- and micro-pores. Eq. (8) can then be applied to determine Bishop's parameter
206 χ using the macro-PSDs. Fig. 7 shows the obtained results. For comparison, the
207 parameters in Table 1 are substituted into Eq. (10), and several other methods are
208 applied including Bishop's method with $\chi = S_r$ (Eq. 2), Vanapalli *et al.*'s method (Eq.
209 3), Zhou *et al.*'s method (Eq. 4), Alonso *et al.*'s method (Eq. 5), and Vaunat & Casini's
210 method (Eq. 7). Results are included in Fig. 8. It is observed that when the global
211 degree of saturation (Eq. 2) is considered as Bishop's parameter χ , significant
212 overestimation of suction contribution is obtained, in agreement with the observation
213 of Alonso *et al.* (2010) and Zhou *et al.* (2016). In Zhou *et al.*'s method, the capillary
214 water is separated from the adsorbed water. Then the capillary degree of saturation
215 (Eq. 4) is used to determine Bishop's parameter χ . Fig. 7 shows the fitted SWRCs by

216 Zhou *et al.*' SWRC method (Zhou *et al.*, 2016) and the corresponding parameters. The
217 relationship between capillary degree of saturation and degree of saturation is plotted
218 in Fig. 8. It appears that this method underestimates the suction contribution to
219 Bishop's effective stress over a wide suction range. In Alonso *et al.*'s method, S_r^m is
220 the degree of saturation of the micro-pores with diameter smaller than 4900 nm. It
221 was found to be about 20% by calculation based on the PSD in Fig. 6. The effective
222 degree of saturation (Eq. 5) is used to determine χ . It appears that the prediction is
223 improved, but the suction contribution is overestimated. Similarly, Vaunat & Casini's
224 method can improve the prediction in the low suction range as Alonso *et al.*'s method,
225 but remains unsatisfactory in the higher suction range. In particular, a small bump on
226 the curve obtained by Vaunat & Casini's method appears in the range of degree of
227 saturation from 5% to 40%, owing to the contribution of micro-pores to the calculation
228 of χ . In Vanapalli *et al.*'s method, a value of 1.6 was fitted for exponent parameter λ .
229 It is observed that the prediction matches the test data well in the low suction range,
230 but gives higher χ values in higher suction range. Interestingly, the proposed method
231 (Eq. 8) shows a good agreement with the experimental results in the full suction range.

232 CONCLUSIONS

233 The hydro-mechanical behavior of unsaturated soils is strongly related to the water
234 distribution in pores. As loading and drying predominantly affect the macro-pores,
235 with the micro-pores remaining almost undisturbed, the pore size below which the
236 PSD curves become independent of loading or drying processes can be considered as
237 the delimiting diameter between macro-and micro-pores. This also implies that the

238 contribution of suction to Bishop's effective stress is only related to the water in
239 macro-pores.

240 Based on a method proposed by Vaunat & Casini (2017) for Bishop's parameter
241 χ determination, a modified method was proposed considering the contribution of
242 capillary water in macro-pores. Bishop's parameter χ was defined as the ratio of the
243 change in -water volume to the change in macro-pores during a loading process at
244 constant suction. When the degree of saturation of macro-pores is zero, χ becomes
245 zero too. The proposed method was evaluated using experimental shear strength data
246 and compared with different methods reported in literature. It appeared that the
247 proposed method allows good agreement between the prediction and measurement
248 over a wide suction range, as opposed to other methods which either overestimate or
249 underestimate the suction contribution to the effective stress. It is however worth
250 noting that more test results are needed to further validate the proposed method.

251

252 **ACKNOWLEDGEMENTS**

253 The authors are grateful to the China Scholarship Council (CSC grant No.
254 201806060075), Ecole des Ponts ParisTech and Guilin University of Technology.

255

256 **References**

- 257 Alonso, E. E., Pereira, J. M., Vaunat, J. & Olivella, S. (2010). A microstructurally-
258 based effective stress for unsaturated soils. *Géotechnique* **60**, No. 12, 913-925.
- 259 Bishop, A. W. (1959). The principle of effective stress. *Tek. Ukebl.***106**, No. 39, 859-
260 863.
- 261 Bishop, A. W. & Blight, G. E. (1963). Some aspects of effective stress in saturated
262 and partly saturated soils. *Géotechnique* **13**, No. 3, 177-197.
- 263 Coleman J.D.1962. Stress strain relations for partly saturated soil. Correspondence,
264 *Géotechnique* **12**, No. 4, 348-350.
- 265 Cui Y. J. & Delage P. (1996). Yielding and plastic behaviour of an unsaturated
266 compacted silt. *Géotechnique* **46**, No. 2, 291-311.
- 267 Fredlund, D. G. & Morgenstern, N. R. (1977). Stress state variables for unsaturated
268 soils. *J. Geotech. Geoenviron. Engng* **103** (ASCE 12919).
- 269 Gao, Y. & Sun, D. A. (2017). Soil-water retention behavior of compacted soil with
270 different densities over a wide suction range and its prediction. *Comput. Geotech.*
271 **91**, 17-26.
- 272 Gao, Y., Sun, D., Zhou, A. & Li, J. (2018). Effect of stress state on soil-water retention
273 and its application on the strength prediction. *Géotech. Lett.* **8**, No. 4, 324-329.
- 274 Gao, Y., Sun, D. A., Zhu, Z., & Xu, Y. (2019). Hydromechanical behavior of
275 unsaturated soil with different initial densities over a wide suction range. *Acta*
276 *Geotech.*, 14(2), 417-428.
- 277 Gao, Y., Sun, D. A., Zhou, A., & Li, J. (2020). Predicting Shear Strength of

278 Unsaturated Soils over Wide Suction Range. *Int. J. Geomech.* 20, No.2, 04019175.

279 Garven, E. A. & Vanapalli, S. K. (2006). Evaluation of empirical procedures for
280 predicting the shear strength of unsaturated soils. In *Unsaturated Soils 2006* (pp.
281 2570-2592).

282 Houlsby, G. T. (1997). The work input to an unsaturated granular material.
283 *Géotechnique* 47, No. 1, 193-6.

284 Konrad, J. M. & Lebeau, M. (2015). Capillary-based effective stress formulation for
285 predicting shear strength of unsaturated soils. *Can. Geotech. J.* 52, No. 12, 2067-
286 2076.

287 Li, J., Sun, D. A., Sheng, D. C., Sloan, S. & Fredlund, D. G. (2007). Preliminary study
288 on soil water characteristics of Maryland clay. In *Proceedings of the 3rd Asian*
289 *Conference on Unsaturated Soils*, Nanjing, China, 569-574.

290 Liu, Y., Wei, C. F., Zhao, C. G., Fang, Q. & Li, J. (2018). Work input for unsaturated
291 soils considering interfacial effects. *Int. J. Numer. Anal Methods in Geomech.* 42,
292 No. 9, 1078-1094.

293 Lloret, A., Villar, M. V., Sanchez, M., Gens, A., Pintado, X. & Alonso, E. E. (2003).
294 Mechanical behaviour of heavily compacted bentonite under high suction changes.
295 *Géotechnique* 53, No. 1, 27-40.

296 Lu, N., Godt, J. W. & Wu, D. T. (2010). A closed-form equation for effective stress
297 in unsaturated soil. *Water Resour. Res.* 46, No. 5.

298 Mašín, D. (2013). Double structure hydromechanical coupling formalism and a model
299 for unsaturated expansive clays. *Engng Geol.* 165, 73-88.

300 Monroy, R., Zdravkovic, L. & Ridley, A. (2010). Evolution of microstructure in
301 compacted London Clay during wetting and loading. *Géotechnique* **60**, No. 2, 105-
302 119.

303 Pereira, J. M., Wong, H., Dubujet, P., & Dangla, P. (2005). Adaptation of existing
304 behaviour models to unsaturated states: application to CJS model. *Int. J. Numer.*
305 *Anal. Methods Geomech.*, **29**, No. 11, 1127-1155.

306 Romero Morales, E. E. (1999). Characterisation and thermo-hydro-mechanical
307 behaviour of unsaturated Boom clay: an experimental study. Ph.D. thesis.
308 Universitat Politècnica de Catalunya.

309 Salager, S., Nuth, M., Ferrari, A. & Laloui, L. (2013). Investigation into water
310 retention behaviour of deformable soils. *Can. Geotech. J.* **50**, No. 2, 200-208.

311 Seiphoori, A., Ferrari, A. & Laloui, L. (2014). Water retention behaviour and
312 microstructural evolution of MX-80 bentonite during wetting and drying cycles.
313 *Géotechnique* **64**, No. 9, 721-734.

314 Sheng, D., Sloan, S. W. & Gens, A. (2004). A constitutive model for unsaturated soils:
315 thermomechanical and computational aspects. *Comput. Mech.* **33**, No. 6, 453-465.

316 Sheng, D., Zhou, A. & Fredlund, D. G. (2011). Shear strength criteria for unsaturated
317 soils. *Geotech. Geol. Eng.* **29**, No. 2, 145-159.

318 Vanapalli, S. K., Fredlund, D. G., Pufahl, D. E. & Clifton, A. W. (1996). Model for
319 the prediction of shear strength with respect to soil suction. *Can. Geotech. J.* **33**,
320 No. 3, 379-392.

321 Vaunat, J. & Casini, F. (2017). A procedure for the direct determination of Bishop's

322 chi parameter from changes in pore size distribution. *Géotechnique* **67**, No. 7, 631-
323 636.

324 Wheeler, S. J., Sharma, R. S. & Buisson, M. S. R. (2003). Coupling of hydraulic
325 hysteresis and stress–strain behaviour in unsaturated soils. *Géotechnique* **53**, No.
326 1, 41-54.

327 Zhai, Q. Rahardjo, H., Satyanaga, A., & Dai, G. (2019). Estimation of unsaturated
328 shear strength from soil-water characteristic curve. *Acta Geotech.* **14**, 1977-1990.

329 Zhang, F., Cui, Y., Zeng, L., Robinet, J. C., Conil, N. & Talandier, J. (2018). Effect
330 of degree of saturation on the unconfined compressive strength of natural stiff
331 clays with consideration of air entry value. *Engng Geol.* **237**, 140-148.

332 Zhang, J., Niu, G., Li, X. & Sun, D. A. (2020). Hydro-mechanical behavior of
333 expansive soils with different dry densities over a wide suction range. *Acta*
334 *Geotech.* **15**, No. 1, 265-278.

335 Zhou, A., Huang, R. & Sheng, D. (2016). Capillary water retention curve and shear
336 strength of unsaturated soils. *Can. Geotech. J.* **53**, No. 6, 974-987.

337 Zhou, A., Wu, S., Li, J. & Sheng, D. (2018). Including degree of capillary saturation
338 into constitutive modelling of unsaturated soils. *Comput Geotech.* **95**, 82-98.

339 **Table captions.**

340 Table 1 Physical property indexes and shear strength parameters of Nanyang weakly
341 expansive soil

342 Table 2 Relevant state variables in triaxial tests

343 Table 3 Stress state at triaxial shear failure

344 **Figure captions.**

345 Fig. 1 Partition of total water volume change into components due to suction and
346 deformation only for a path going from the water retention curve at e to $e+\delta e$ (after
347 Vaunat & Casini, 2017)

348 Fig. 2. SWRCs under different void ratios over a wide suction range for soils: (a) a
349 clayey silty sand (data from after Salager et al., 2013); (b) MX-80 granular bentonite
350 (data from Seiphoori et al., 2014); (c) Maryland clay (data from Li et al., 2007); (d)
351 Pearl clay (data from Gao & Sun, 2017)

352 Fig. 3 Sketches of soil-water retention behavior of specimens with different densities
353 over a wide suction range

354 Fig. 4 PSDs at two different void ratios (compacted at different void ratios at a
355 constant suction)

356 Fig. 5 Stress and suction paths for tests: (a) tests in the lower suction range; (b) tests
357 in the higher suction range

358 Fig. 6 The PSD of Nanyang weakly expansive soil and the criterion adopted to
359 discriminate macro- and micro-pores

360 Fig. 7 Measured SWRC and fitted curves from Zhou et al.'s SWRC method (2016) for
361 Nanyang weakly expansive soil

362 Fig. 8 Comparison of measured Bishop's parameter χ with the predictions obtained
363 by various methods for Nanyang weakly expansive soil

364

365 **Table 1 Physical property indexes and shear strength parameters of Nanyang weakly expansive**
 366 **soil**

Specific gravity	Liquid limit (%)	Plastic limit (%)	Plasticity index	Shrinkage limit (%)	Maximum dry density (g/cm ³)	Optimum water content (%)	Free swelling ratio (%)	Effective cohesion (kPa)	Effective friction angle (°)
2.74	38.8	17.2	21.6	10.5	1.69	18.2	53.8	10.4	20.8

367
368

369 **Table 2 Relevant state variables in triaxial tests**

Test No.	Molding state			Before triaxial shearing			Suction (MPa)	Control suction method	Net cell pressure (kPa)
	w_0	$S_{r0}(\%)$	e_0	w	$S_r(\%)$	e			
1	21.50	48.45	1.216	24.14	79.13	0.826	0.05	Axis-translation technique	100
2	22.01	50.13	1.203	19.12	52.79	0.982	0.20		
3	21.50	48.97	1.203	17.07	45.30	1.02	0.40		
4	21.64	49.54	1.197	14.80	38.44	1.04	0.80	Air-drying	
5	21.78	49.57	1.204	12.55	35.52	0.957	2.50		
6	21.86	49.95	1.199	11.90	32.53	0.996	3.29	Vapor equilibrium technique	
7	21.72	49.51	1.202	6.60	19.57	0.924	38.0		
8	21.76	49.52	1.204	4.10	12.20	0.907	367.5		

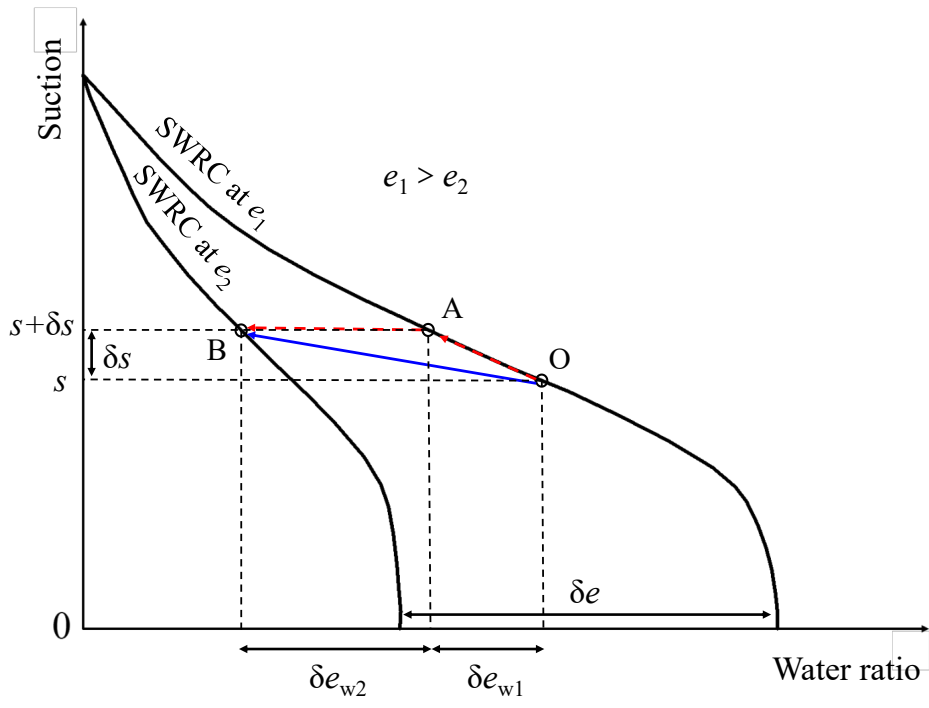
370

371 **Table 3 Stress state at triaxial shear failure**

Test No.	Control suction method	Net cell pressure (kPa)	Deviator stress q_f (kPa)	Net mean stress \bar{p} (kPa)
1	Axis-translation technique	100	140	146
2			210	160
3			280	196
4	Air-drying		320	210
5			600	300
6	Vapor equilibrium technique		660	326
7			1290	550
8			1680	660

372

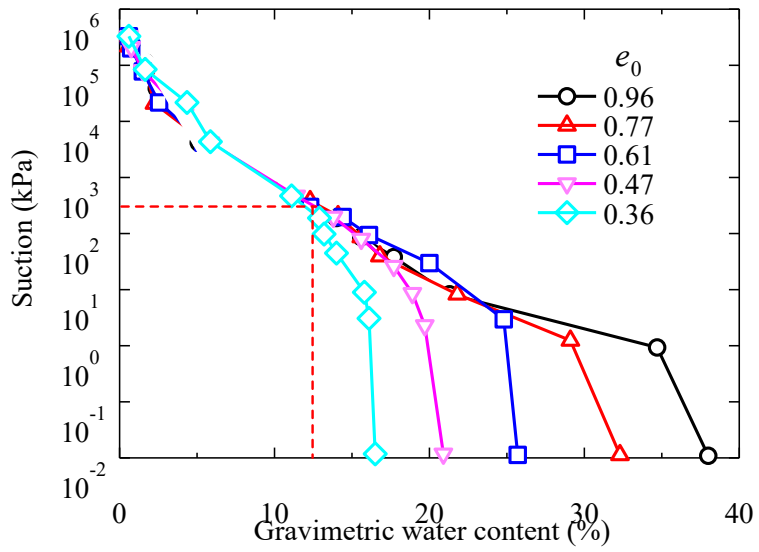
373



374

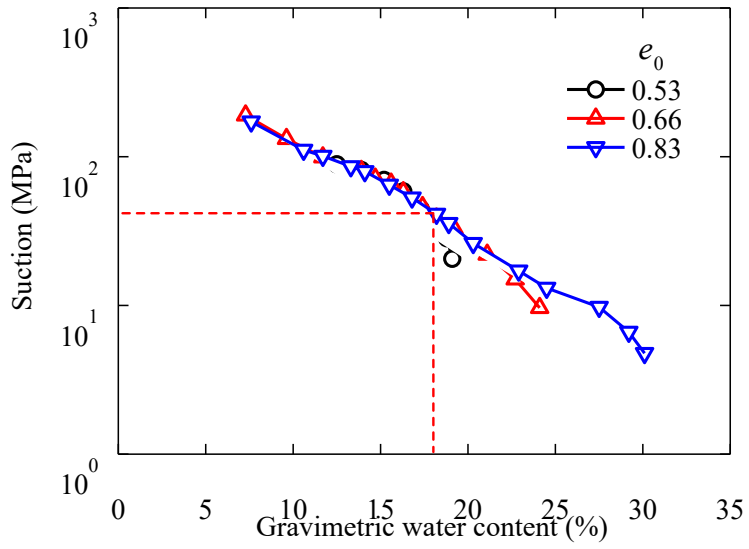
375 Fig. 1. Partition of total water volume change into components due to suction and
 376 deformation only for a path going from the water retention curve at e to $e + \delta e$ (after
 377 Vaunat & Casini, 2017)

378



379

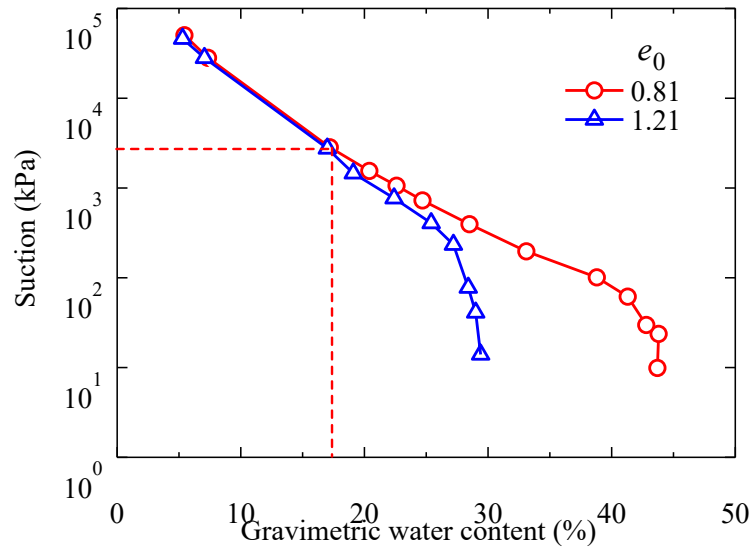
(a)



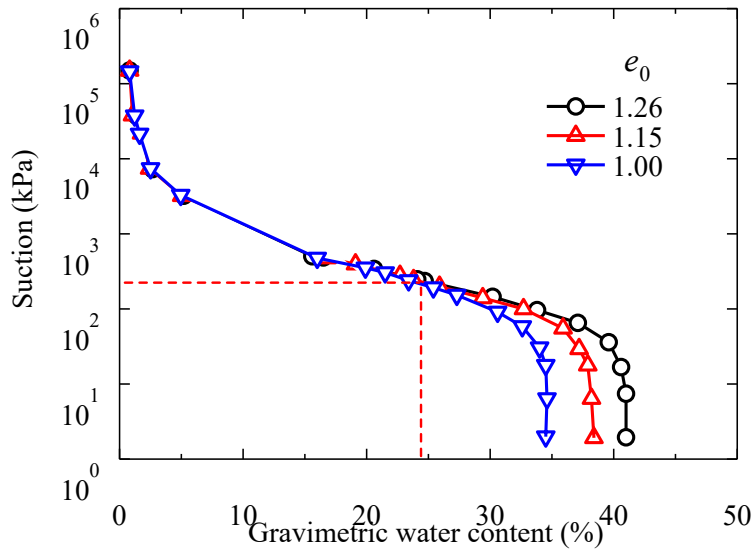
380

(b)

381

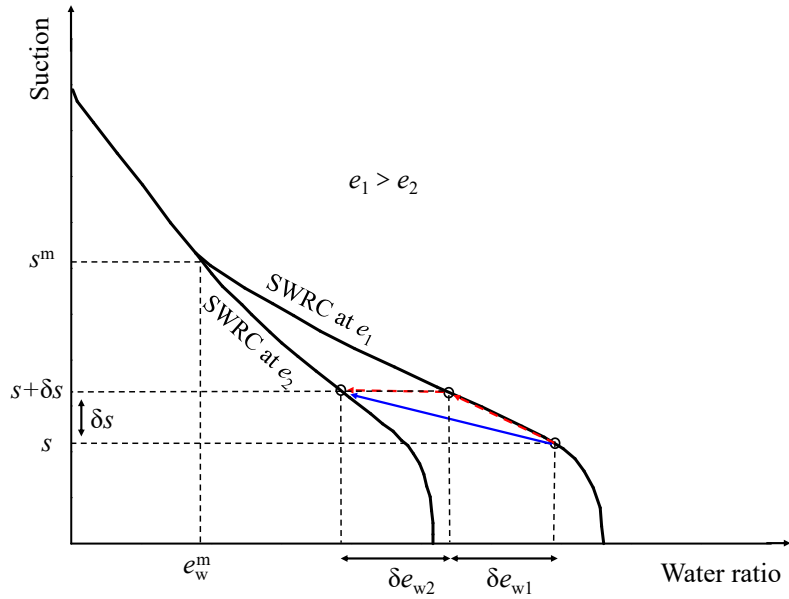


382



383

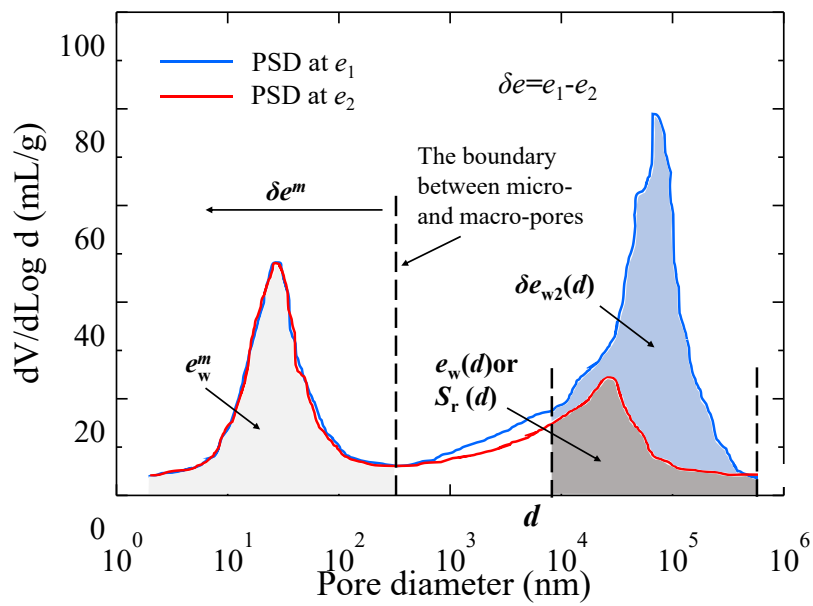
384 Fig. 2. SWRCs under different void ratios over a wide suction range for soils: (a) a
 385 clayey silty sand (data from after Salager *et al.*, 2013); (b) MX-80 granular bentonite
 386 (data from Seiphoori *et al.*, 2014); (c) Maryland clay (data from Li *et al.*, 2007); (d)
 387 Pearl clay (data from Gao & Sun, 2017)



388

389 Fig. 3. Sketches of soil-water retention behavior of specimens with different densities

390 over a wide suction range

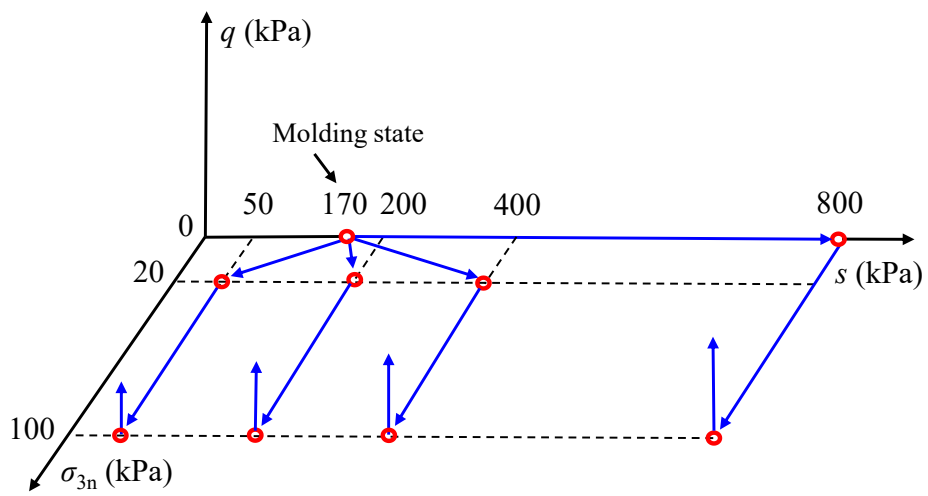


391

392 Fig. 4. PSDs at two different void ratios (compacted at different void ratios at a

393 constant suction)

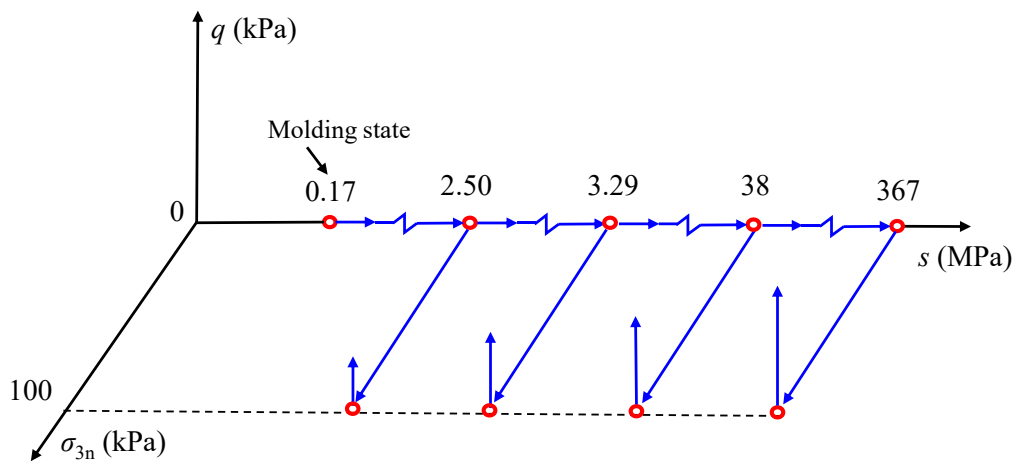
394



(a)

395

396



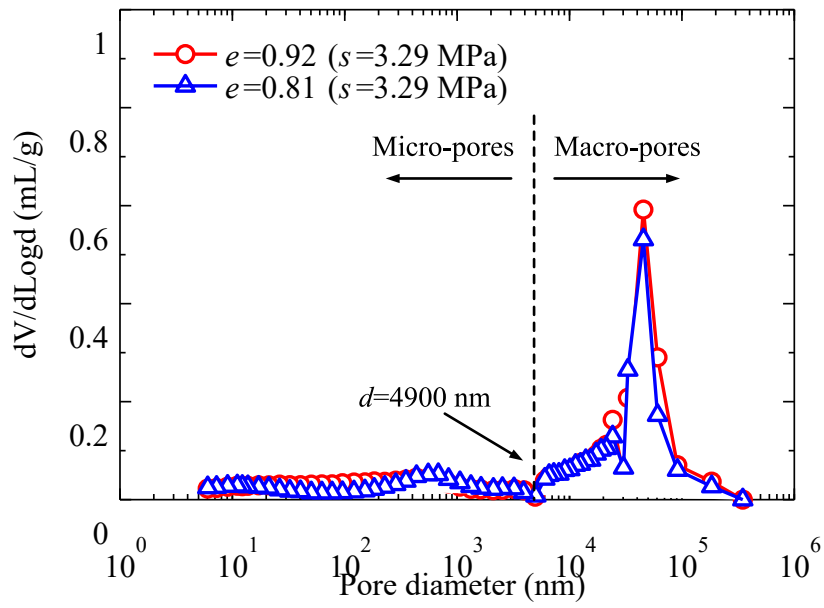
(b)

397

398 Fig. 5. Stress and suction paths for tests: (a) tests in the lower suction range; (b) tests

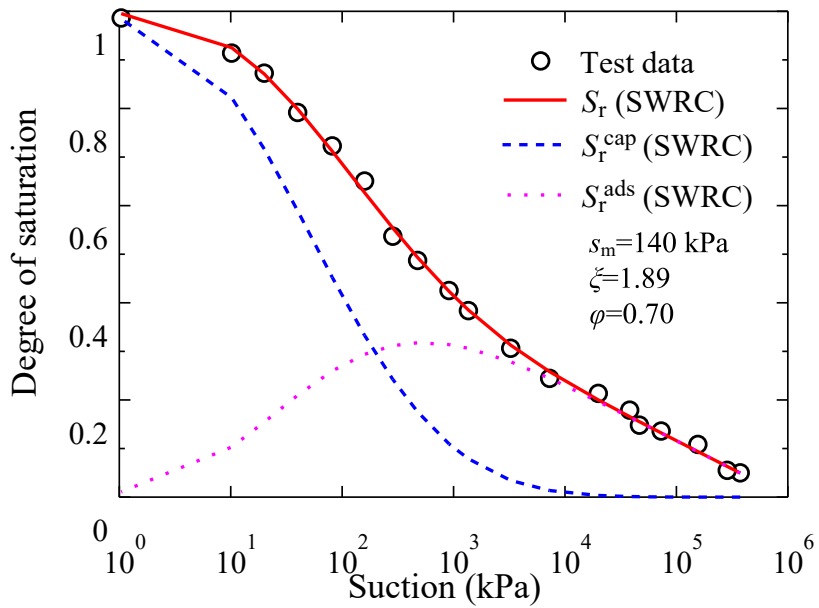
399 in the higher suction range

400



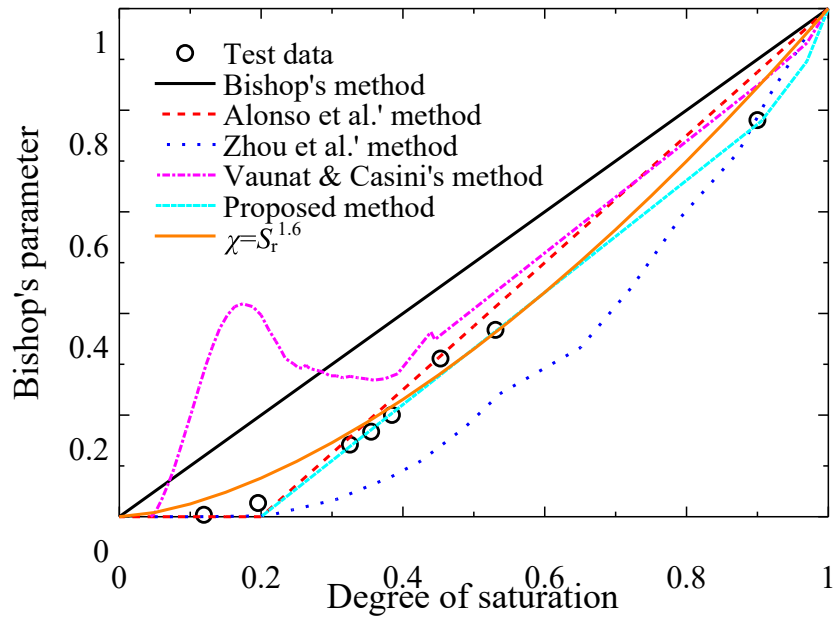
401

402 Fig. 6. The PSDs of Nanyang weakly expansive soil and the criterion adopted to
403 discriminate macro- and micro-pores



404

405 Fig. 7 Measured SWRC and fitted curves from Zhou *et al.*' SWRC method (2016)
406 for Nanyang weakly expansive soil



407

408 Fig. 8. Comparison of measured Bishop's parameters χ with the predictions obtained
 409 by various methods for Nanyang weakly expansive soil