

# DAMAGE ANALYSIS OF MULTILAYERED PLATES USING THE SCLS1 LAYERWISE MODEL

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To overcome the inaccurate estimations of local response near free-edges of equivalent single-layer theories, a collection of layerwise models have been developed by the Navier laboratory in which the interlaminar shear stress is one of the generalized forces [1, 2]. The latest one (SCLS1) with first-order membrane stresses is considered in this work. The laminated plate is considered as a superposition of Reissner-Mindlin plates coupled by shear and normal interface stresses. The model ensures an accurate estimation of the shear stress and respect the free edge boundary conditions.

First the equations of the SCLS1 model are derived from the 3D Cauchy model under the assumption of piecewise linear membrane stress. The model is then built by satisfying the 3D equilibrium equations. The problem is numerically solved by a standard displacement finite element methods. The generalized displacements are the Lagrange multipliers of each equilibrium equations. A variational damage formulation is then used to predict the initialization and the propagation of the delamination between two related plies. The delamination is driven by  $2n$  scalar variables which lower the interfacial stress to zero. In order to manage the evolution of these scalar variables, we postulate that the potential energy is divided between the strain energy and an interfacial fracture energy  $E_{\text{pot}}(\underline{u}, \underline{d}) = E_{\text{el}}^{\Omega}(\underline{u}) + E_{\text{F}}^{\Gamma}(\underline{d}) - \mathcal{W}_{\text{ext}}$ . The pair displacement-damage  $(\underline{u}, \underline{d})$  is found using an incremental strategy  $\Rightarrow$  stationarity of the potential energy under the constraint of strict growth of the damage variables:

$$(\underline{u}, \underline{d}) = \arg \min_{\underline{d}; \underline{d} \geq \underline{d}_n} E_{\text{pot}}(\underline{u}, \underline{d})$$

A double cantilever beam problem is considered to evaluate the accuracy of such models. An analytical solution given by linear fracture mechanics is used to validate our approach. All the simulations have been implemented using the open source FEniCS platform for solving partial differential equations.

## REFERENCES

- [1] Rawad Baroud, Karam Sab, Jean-François Caron, and Fouad Kaddah. A statically compatible layerwise stress model for the analysis of multilayered plates. *International Journal of Solids and Structures*, 96 :11–24, 2016.
- [2] Rui Pedro Carreira, Jean-François Caron, and Alberto Diaz Diaz. Model of multilayered materials for interface stresses estimation and validation by finite element calculations. *Mechanics of materials*, 34(4) :217–230, 2002.