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# A Finite Difference Model for Undefined End Boundary to Analyze the Heat Transfer in Dry Sands

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**Abstract.** Accurate prediction of thermal regime (i.e., the variation of soil temperature,  $\theta$ , and heat flux,  $\phi$ ) to investigate the migration of thermal energy in soil mass, poses challenge to the geoenvironmental engineers while dealing with various thermo-active structures. In this context, several numerical approaches have been attempted to solve the heat transfer equation (HTE) for conduction to predict the thermal regime. However, most of them need accurate boundary condition defined and involve complicated numerical approach which is inconvenient for the practising engineers. Hence, an attempt has been made in the present study to develop a simplified, but numerically efficient, approach based on the finite difference method (FDM) to analyse one dimensional heat transfer in dry sands when the end boundary is not defined. Furthermore, a time dependent initial boundary condition has been applied to this model to simulate similar experimental condition and results have been compared vis-à-vis those obtained from the experiment and COMSOL Multiphysics<sup>®</sup> to validate the proposed approach.

**Keywords:** sands; thermal regime; heat transfer equation; numerical analysis; finite difference model;

## 1. Introduction

Contemporary geoenvironmental engineering and practices deal with design and construction of several thermo-active structures such as furnaces, boiler units, forging units, brick kilns and rocket launching pads, buried conduits and electrical cables, air conditioning ducts (Kadali *et al.* 2013), disposal facilities of waste from the nuclear and thermal power plants (Rao and Singh 1999; Krishnaiah and Singh 2004; Delage *et al.* 2010; Dao *et al.* 2015), underground crude oil storage tanks (Mandal *et al.* 2013; Padmakumar 2013; Usmani *et al.* 2015), oil carrying pipelines (Brandon and Mitchell 1989; Abuel-Naga *et al.* 2008; Lee *et al.* 2010; Manthena and Singh 2001), solar ponds (Velmurugan and Srithar 2008), energy geostructures (Knellwolf *et al.* 2011; Loveridge and Powrie 2013; Salciarini *et al.* 2013; Yavari *et al.* 2014; Di Donna *et al.* 2016; Faizal *et al.* 2016; McCartney *et al.* 2016), which result in conveyance of thermal energy through the soil mass. Moreover, activities like dissociation of gas hydrates by heating (Feng *et al.* 2015; Song *et al.* 2015, 2016) and exploitation of oil sand by steam assisted gravity drainage (SAGD) (Elsayed *et al.* 2015; Lazzaroni *et al.* 2016) and also soil-atmosphere interaction (Heusinkveld *et al.* 2004; Ochsner *et al.* 2007; Cui *et al.* 2013) involve heat migration through soil mass. These applications necessitate studies to understand how heat migrates in the soil mass by means of predicting the thermal regime (i.e., time dependent temperature and thermal flux) in it.

In this context, several studies have been carried out to predict thermal regime of soil mass by solving HTE (Heat Transfer Equation) either numerically or analytically. Some of the broadly used

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approaches are Finite Element Method (FEM) ( Timlin *et al.* 2002; Han and Huang 2002; Bittelli *et al.* 2008.), Finite Volume Method (LeVeque 2002), Finite Difference Method (FDM) (Sharratt *et al.* 1992; Han and Huang 2002; Wu and Sun 2004; Wang *et al.* 2011), Force Restore Method (FRM) ( Liebethal and Foken 2007; Gao *et al.* 2008). Moreover, different analytical methods like Harmonic Method, Laplace Transform Method, and Fourier Transform Method, have been considered by several researchers (Heusinkveld *et al.* 2004; Gao *et al.* 2007; Evett *et al.* 2012; Wang *et al.* 2012) to solve HTE in order to predict the thermal regime in soil mass (Mondal *et al.* 2017). Nonetheless, the major hindrance of some of the abovementioned methods is that they involve complicated mathematics which is not convenient to the practising engineers. Moreover, for any numerical approach, proper boundary conditions are necessary to be defined. On the contrary, many a times while performing experimental investigations, proper boundary conditions are not monitored. This results in a real challenge to adopt a numerical study in order to conduct comparative analysis between experimental and numerical outcomes of any particular problem. Keeping this in view, an effort has been made to develop a computationally efficient and simplified numerical model based on FDM to predict (i) the end boundary and to estimate (ii) the realistic thermal regime in sandy soils, which would be very convenient for the practising geotechnical engineers. Furthermore, the results obtained from FDM and COMSOL Multiphysics®, a commercially available FEM based analysis tool, have been compared vis-à-vis with experimental results, details of which are presented in this paper. The distinctiveness of this model lies in the fact that it is simpler than the complicated analytical solutions yet numerically efficient. This would make the model widely accepted among practicing engineers who are working in the projects of construction of thermo-active structures. Moreover, this FDM can be employed to define the appropriate boundary conditions (in case of comparative analysis between experimental investigations with undefined boundary) to obtain an accurate numerical solution.

## 2. Statement of the Problem

This section deals with the details of the problem investigated in the present study, which essentially has been derived from the experiments conducted by Mondal *et al.* (2016). A 20 cm long (L) column of the Indian standard sands, designated as SI (refer Table 1 for the thermal properties of the sands) was used in the experiments. A temperature of 60 ° C was applied at the top surface of this column, as depicted in Fig1a. Furthermore, as depicted in Fig1b, the flux sensors, and thermocouples were embedded at 5 cm and 9 cm depths, respectively, in the sample (Mondal *et al.* 2017). The heat gun was kept at a height of 400mm which ensured uniform heat application at the top surface of the column. Since the lateral boundary is maintained adiabatic as explained in Mondal *et al.* (2016) and the base of the column is made up of aluminium, which has very high thermal conductivity as compared to that of sand, the bottom boundary of this column can be considered as the ‘open end boundary’ through which thermal flux could exit. As such, the heat transfer through this column can be assumed to be one-dimensional, and after a specific time, the steady-state condition could be achieved in it. Hence, the novelty of the present study is that it’s an attempt to establish a simple numerical approach, based on finite difference method (FDM), to identify the correct end boundary conditions, which would facilitate determination of thermal regime for one-dimensional heat conduction in dry sands.

Table 1. Thermal Properties of Sand used in this study

Sand	Dry density ( $\gamma_d$ ) (g/cm <sup>3</sup> )	Thermal Conductivity (k) (W/°C/m)	Specific Heat ( $C_p$ ) (J/kg /°C)
SI	1.556	0.315	171

## 3. The Proposed Methodology

95 The one-dimensional heat conduction equation can be written as:

$$96 \quad \frac{\partial \theta}{\partial t} = \alpha \frac{\partial^2 \theta}{\partial z^2} \quad (1)$$

97 where  $\theta(z,t)$  is the temperature in the soil mass, at the depth of  $z$  and corresponding to time,  $t$ , and  
98  $\alpha$  is the thermal diffusivity of the soil mass which can be determined if  $\kappa$ , thermal conductivity  
99 and  $C$ , Specific heat for a particular bulk density,  $\rho$ , of the soil mass is known (refer Eq. 2).

$$100 \quad \alpha = \frac{\kappa}{\rho C} \quad (2)$$

101 It should be noted that the conduction is the predominant mechanism of heat transfer through  
102 solids (Jackson and Taylor 1986; Mondal *et al.* 2018) and hence, equation of heat conduction is  
103 considered.

### 104 3.1 The model based on the FDM

105

106 The one-dimensional HTE, Eq. 1, is discretised in time and space using simple difference  
107 equations and ignoring the truncation error, which is negligibly small. Subsequently, the explicit  
108 solution has been obtained as:

$$109 \quad \theta_{i+1}^n = \frac{\alpha \Delta t}{(\Delta z)^2} (\theta_i^{n+1} - 2\theta_i^n + \theta_i^{n-1}) + \theta_i^n \quad (3)$$

110 It is worth mentioning that the condition of stability for this explicit solution is  $\frac{\alpha \Delta t}{(\Delta z)^2} \leq \frac{1}{2}$ . As  
111 such, in order to maintain the stability, a time step of 30 seconds which corresponds to  
112 characteristic length of 1 cm, has been chosen.

113

### 114 3.2 Determination of the End boundary

115 The purpose of this simple numerical method is to analyse the experimental data where the  
116 end boundary condition is not defined. As described by Mondal *et al.* (2016) neither the  
117 temperature nor the thermal flux was measured at the end boundary while performing the  
118 experiment. Hence, it became mandatory to devise a methodology that could be utilized to define  
119 the end boundary. With this in view, beyond the considered depth of the model,  $L$  (=20 cm), an  
120 additional depth of the soil mass of length,  $\Delta L$ , has been chosen (refer Fig. 2). This additional  
121 depth of the soil mass is beyond the zone of influence of the temperature applied at the top  
122 boundary, for a particular time, and is constrained by the bottom boundary. Hence, changes in the  
123 applied temperature at the top surface of the model would not be felt by the end boundary which  
124 is considered as the end point of the additional depth. As a result, at this end boundary the  
125 prevailing temperature could be considered as ambient when numerical simulation is being  
126 conducted. The obtained result, for different values of  $\Delta L$  were compared with the analytical  
127 solution of HTE (refer Eq. 4) for establishing the most suitable depth of end boundary.

$$128 \quad \theta(z, t) = (\theta_j - \theta_0) \operatorname{erf}\left(\frac{z}{2\sqrt{\alpha t}}\right) + \theta_0 \quad (4)$$

129 where  $\theta_j$  and  $\theta_0$  are initial and applied temperature respectively.

130

### 131 3.3 Time dependent boundary condition

132 As described by Mondal *et al.* (2016), the top surface of the sand column takes some time to  
133 achieve the applied temperature (i.e., the temperature at which the heat gun is set). While, in case  
134 of the FDM, corresponding to the second time step, the first node is considered to have achieved  
135 the maximum applied temperature, which results in higher rate of heat transfer. Hence, to  
136 simulate

137 the true experimental conditions, a step thermal loading of  $0.4^\circ \text{C}/\text{min}$  has been imposed at the  
138 second time step of the FDM, until the first node achieves the maximum applied temperature.

139

## 140 4. Results

141 To define the bottom end boundary for the FDM, a methodology has been adopted as described  
142 in Section 3.2 and the results are compared vis-a-vis the analytical solution (refer Eq. 4), as  
143 depicted in Fig 3. It can be observed from Figs. 3 (a) and (b) that the results obtained from the  
144 FDM, for an additional depth ( $\Delta L$ ) of 20 cm, exhibit the best agreement with the results obtained  
145 from analytical solution. Based on this finding, the end boundary condition for the FDM was  
146 considered as an optimum additional depth of 20 cm when there is no significant improvement for  
147 further increment of  $\Delta L$ . Moreover, the experimental data has been compared vis-à-vis the results  
148 obtained from the FDM and COMSOL Multiphysics<sup>®</sup>. It is worth mentioning here that the lateral  
149 boundaries of the COMSOL model have been assumed to be ‘adiabatic’ and the bottom boundary  
150 has been assigned the ambient temperature like the FDM. Subsequently, a comparison between  
151 the results obtained from the experiments, and FDM and COMSOL Multiphysics<sup>®</sup> has been  
152 depicted in Fig 4 and Fig 5, respectively.

153 It can be noticed from Fig 4 that there is a good match between the experimental and FDM  
154 results. However, it can be noticed from Fig 5 that the COMSOL Multiphysics<sup>®</sup> predicts lower  
155 temperatures as compared to experimentally obtained results. Furthermore, it can be observed  
156 from Fig 6 that the temperature profile obtained from the FDM matches very well with the  
157 experimentally obtained temperature profile. However, Fig 7 exhibits not so good match between  
158 the temperature profiles obtained from COMSOL Multiphysics<sup>®</sup> simulation and the experiments.  
159

## 160 5. Concluding Remarks

161 Prediction of realistic thermal regime in soil mass by means of mathematical approach enforces  
162 challenges to geoenvironmental practitioners. In this context, a simple FDM has been developed  
163 to solve one-dimensional HTE equation in order to establish thermal regime in sandy soil mass. A  
164 simple mathematical approach has been proposed to define end boundary for the FDM, as in case  
165 of the experiment the temperature at the end boundary could not be monitored. It is worth  
166 mentioning that the proposed numerical approach can be used for any dry sandy soil to define the  
167 end boundary and to determine the thermal regime. Moreover, experimentally obtained thermal  
168 regime has been compared vis-à-vis that obtained from the FDM and COMSOL Multiphysics<sup>®</sup>  
169 and it has been demonstrated that the FDM results match satisfactorily with experimental results  
170 as compared to the COMSOL Multiphysics<sup>®</sup>. Hence, it is believed that the proposed FDM would  
171 be an efficient tool to estimate thermal regime in sandy soils. However, the applicability of the  
172 proposed model should be checked in clays to enhance its versatility.  
173

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181

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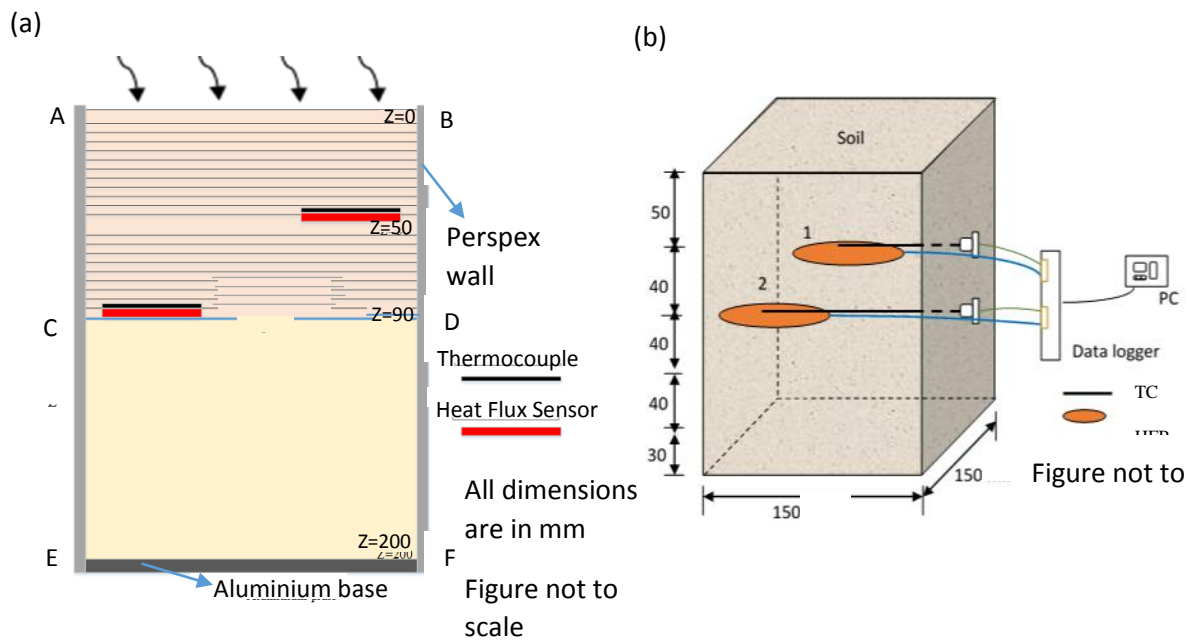
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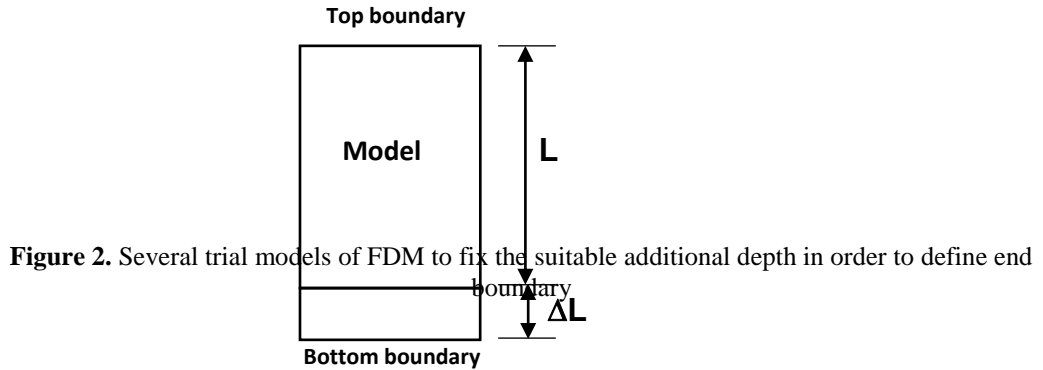
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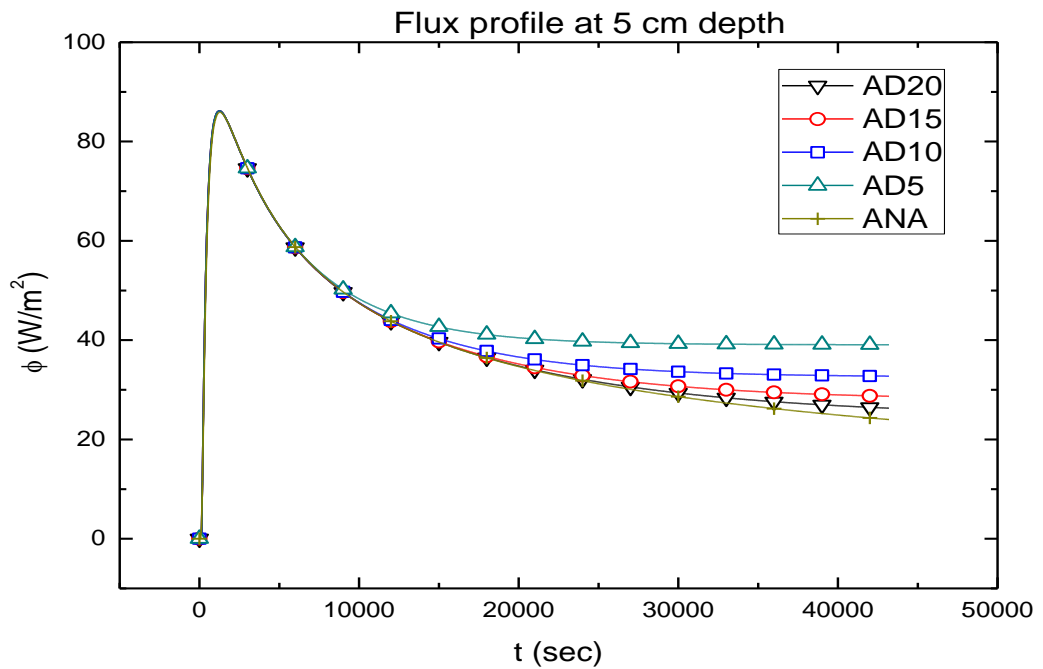
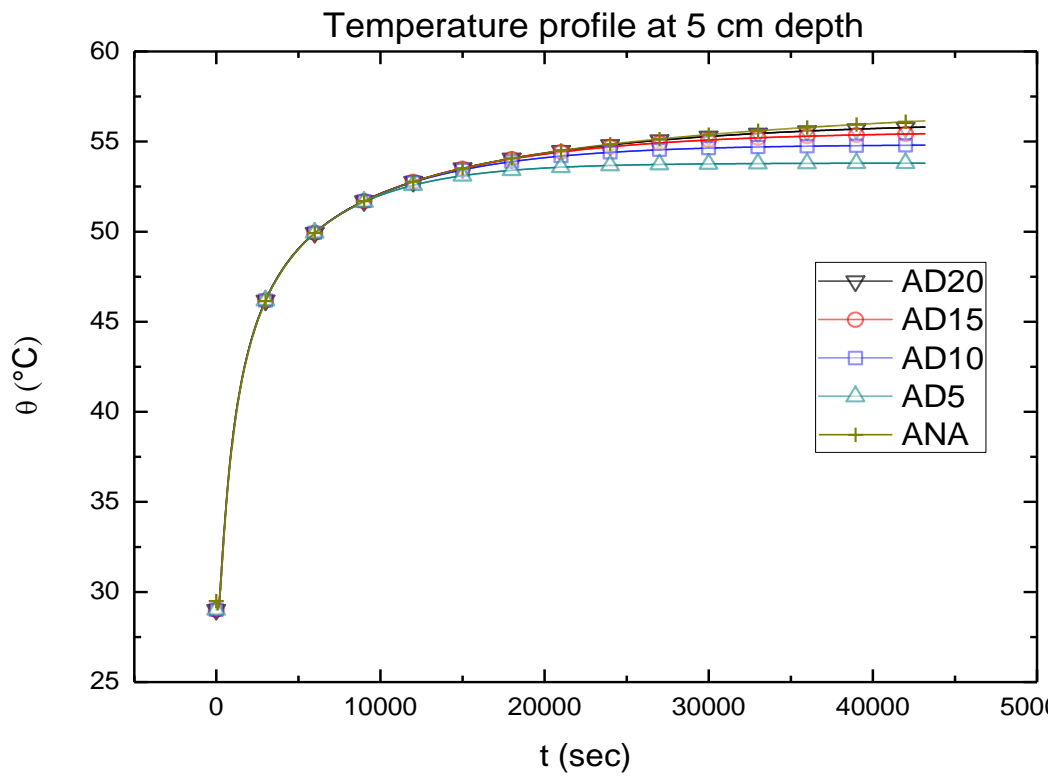




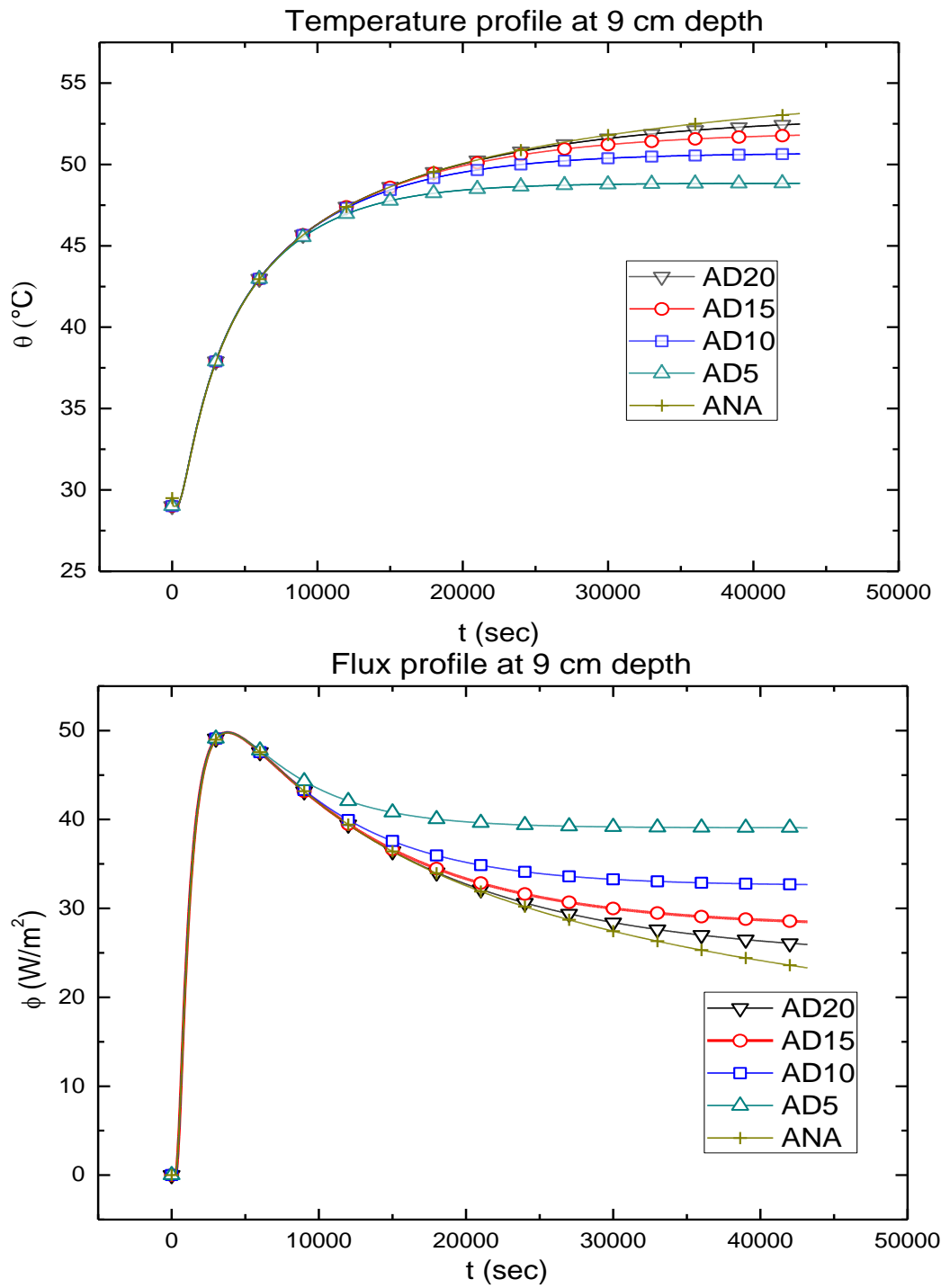
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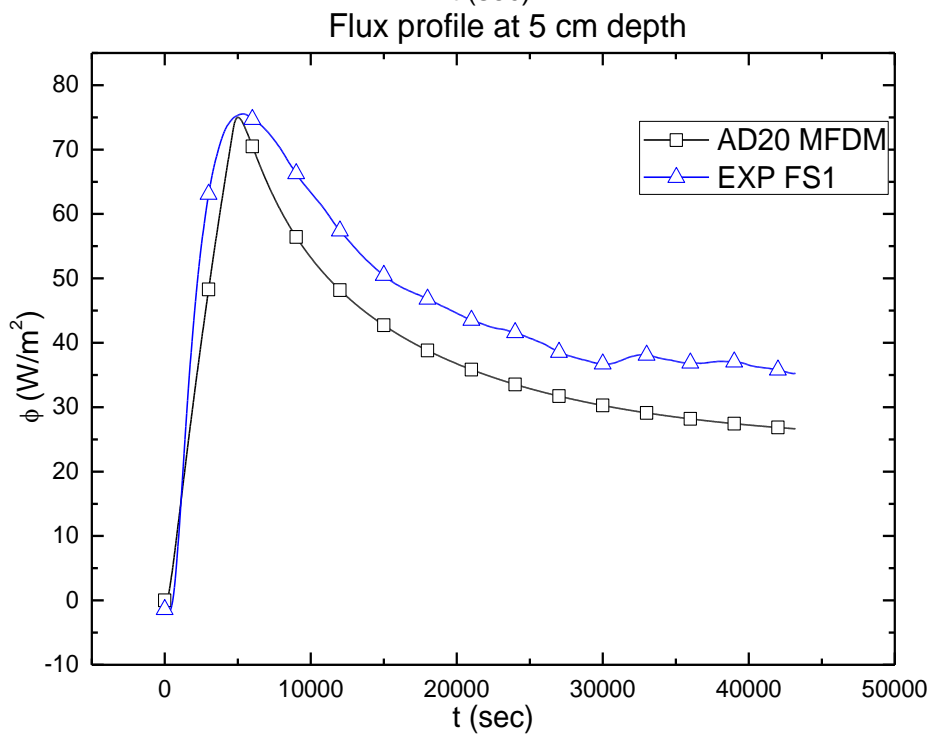
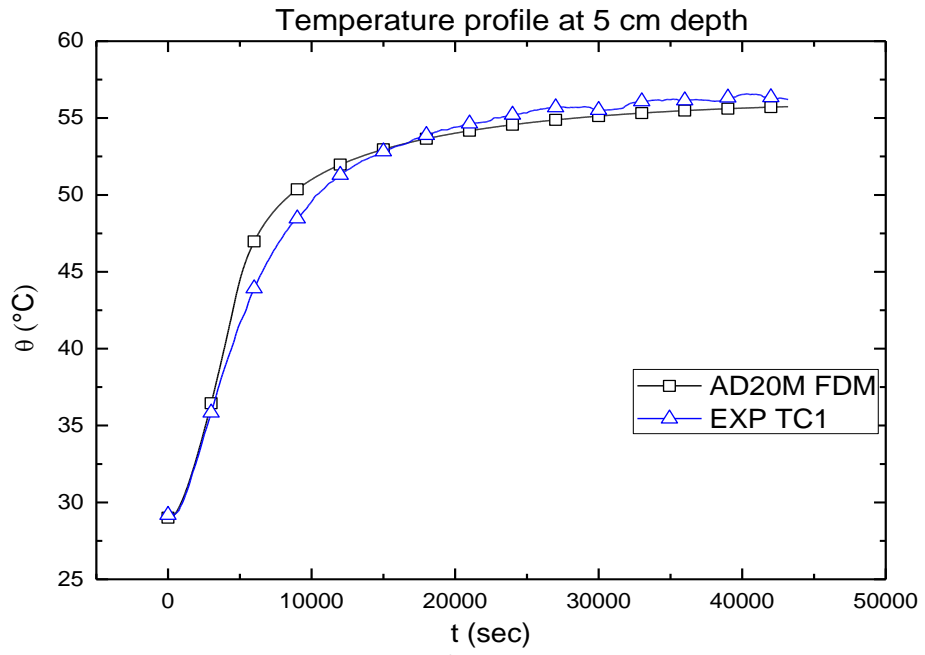
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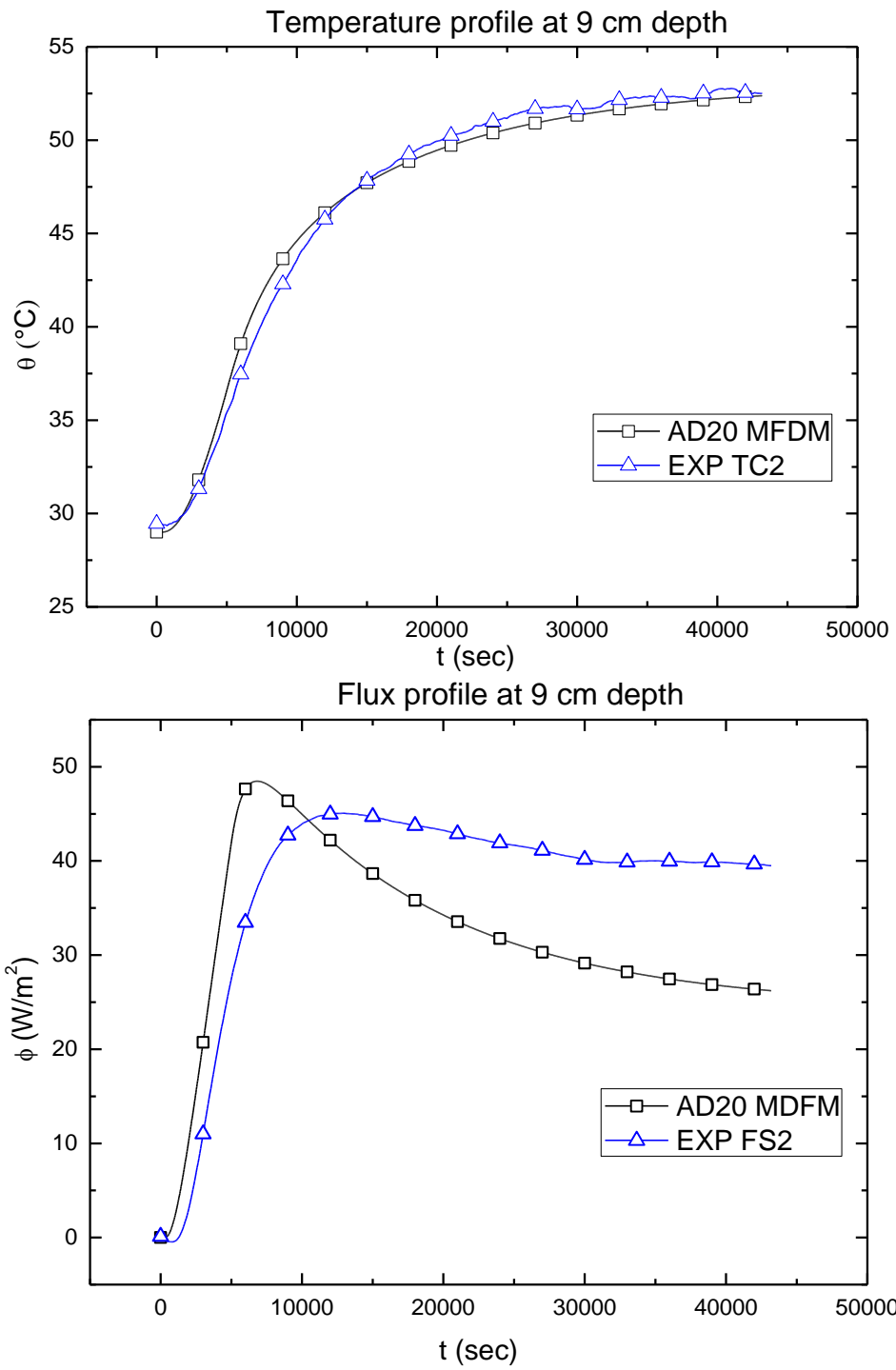
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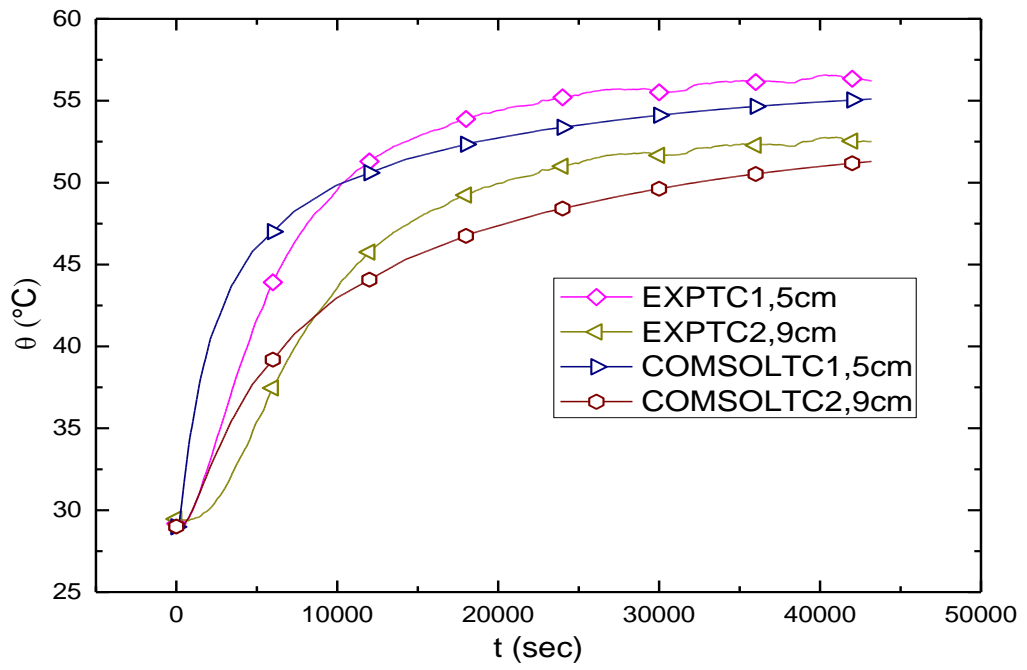
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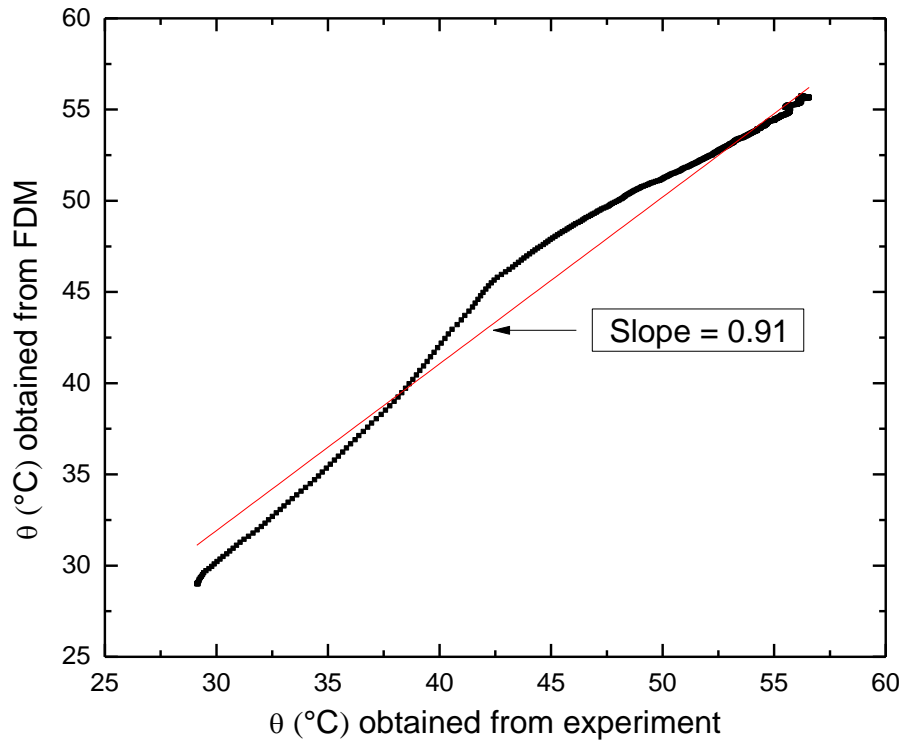
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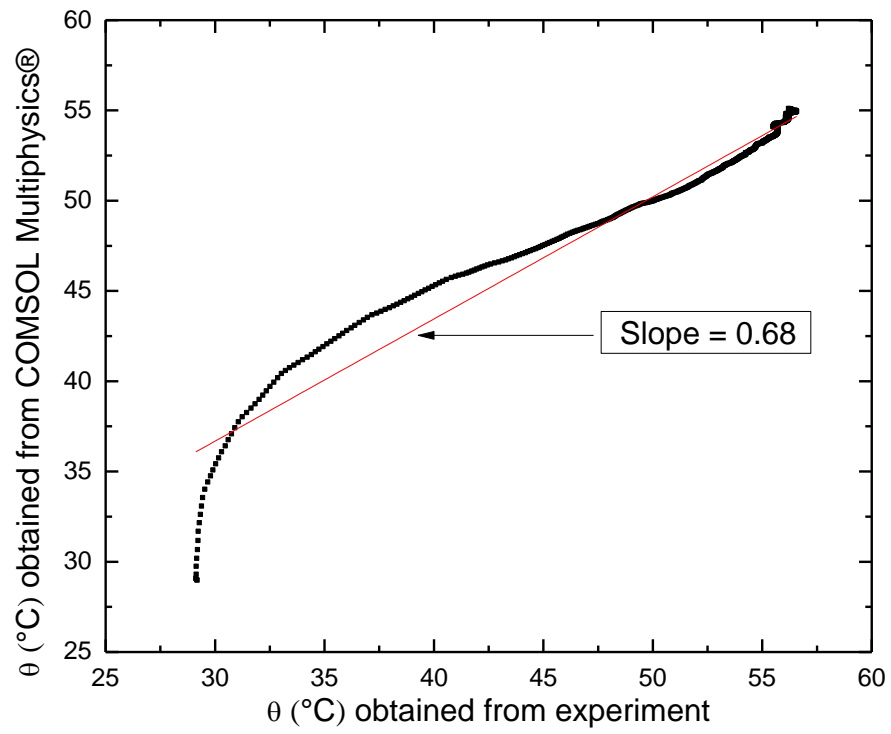
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