



**HAL**  
open science

## Investigation of the parallel gradation method based on the response of track-bed materials under cyclic loadings

Shuai Qi, Yu Jun Cui, Jean Claude Dupla, Ren-Peng Chen, Han-Lin Wang,  
Yu Su, Francisco Lamas-Lopez, Jean Canou

### ► To cite this version:

Shuai Qi, Yu Jun Cui, Jean Claude Dupla, Ren-Peng Chen, Han-Lin Wang, et al.. Investigation of the parallel gradation method based on the response of track-bed materials under cyclic loadings. *Transportation Geotechnics*, 2020, 24, pp.100360. 10.1016/j.trgeo.2020.100360 . hal-03045872

**HAL Id: hal-03045872**

**<https://enpc.hal.science/hal-03045872v1>**

Submitted on 28 May 2021

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



23 **Abstract**

24 Ballast/fines mixture is often found in railway substructure and the overall behaviour  
25 of tracks is strongly dependent on the mechanical behaviour of this mixture. Since  
26 directly testing such mixture with large grains is limited with common laboratory  
27 equipment, the consideration of a model material at smaller size is advisable. Parallel  
28 gradation method is widely used for this purpose. This study assesses the validity of  
29 this method in the case of ballast/fines mixture. Large-scale cyclic triaxial tests were  
30 carried out on the ballast/fines mixture at six volumetric coarse grain contents. The  
31 results obtained were analysed, together with those obtained previously from  
32 small-scale triaxial tests on a microballast/fines mixture whose microballast grain size  
33 distribution was determined by applying the parallel gradation method. The cyclic  
34 parameters (permanent strain and resilient modulus) were obtained for the two types  
35 of mixture. Results show that for all mixtures two distinct soil fabrics can be  
36 identified according to the variations of permanent strain and resilient modulus with  
37 the volumetric content of coarse grains  $f_v$ : a fine-fine contact structure for  $f_v \leq 20\%$   
38 and a grain-grain contact structure for  $f_v \geq 35\%$ . In the case of fine-fine contact  
39 structure, the permanent strains and resilient modulus values of the ballast specimens  
40 are consistent with those of the microballast specimens, evidencing the validity of the  
41 parallel gradation method. In the case of grain-grain contact structure, the permanent  
42 strains and resilient modulus values are found to coincide globally at the two scales,  
43 also justifying the validity of the parallel gradation method, the slight differences  
44 between the two scales being attributed to the irregular grain sliding and the

45 distribution of the fines soils.

46

47 **Keywords:** interlayer soil; cyclic triaxial tests; permanent deformation; resilient

48 modulus; parallel gradation method

49 **1. Introduction**

50 Ballast/fines mixture is often found in railway substructure [10,17]. For instance, in  
51 the French conventional railway lines (representing 94% of the total railway network),  
52 there is an interlayer which was formed by interpenetration of ballast and subgrade  
53 soils under the effect of train circulation [10]. During the renewable program for  
54 conventional lines, this layer was maintained since the dry density of the interlayer  
55 was as high as  $2.4 \text{ Mg/m}^3$  and thus a favourable mechanical behaviour was expected  
56 [7]. According to the in-situ investigation, the soil fabric of the interlayer is found to  
57 be greatly dependent on the content of ballast grains [37,38]: the ballast grains are in  
58 contact with each other for the upper part with high ballast content, but floated in the  
59 fines matrix for the lower part with lower ballast content. Since the ballast/fines  
60 mixture plays an important role in the overall behaviour of rail tracks, it is important  
61 to investigate the mechanical response of such mixture under cyclic loading in terms  
62 of permanent deformation and resilient modulus. Considering the large size of ballast  
63 grains, large-scale triaxial apparatus is normally needed for this purpose. However,  
64 since the large testing system is complex and costly, scaling the ballast grains down to  
65 a model ballast material with smaller size and performing tests using standard  
66 apparatus appear quite promoting.

67 Three methods are commonly used for the scaling purpose: the scalping method  
68 [50] which consists in discarding the grains larger than a specified size from the  
69 prototype materials, the parallel gradation method [24] which consists in creating a  
70 model material at smaller size with its grain size distribution parallel to that of the

71 prototype material, and the replacement method [14] which consists in replacing the  
72 grains from a specified size to the maximum size with the grains from this specified  
73 size to a smaller size. Among them, the parallel gradation method is regarded to be the  
74 most appropriate because it can consider a large range of prototype gradation shape;  
75 thus, it has been adopted mostly [1,16,29,39,42,43,44,47]. The validity of this method  
76 under cyclic loadings was assessed by Sevi and Ge [34] based on the response of the  
77 prototype and model grains of ballast grains. Their results in terms of resilient  
78 modulus, permanent axial strain and permanent volumetric strain were unfortunately  
79 not conclusive for validating the parallel gradation method. In addition, only pure  
80 ballast grains were tested without fines fraction.

81 In this study, the parameters characterizing the cyclic response (permanent  
82 strain and resilient modulus) of ballast/fines mixtures representing the interlayer soils  
83 in the conventional French railway track-beds were determined by conducting  
84 large-scale cyclic triaxial tests. Six volumetric ballast contents (0%, 5%, 10%, 20%,  
85 35%, 45%) were considered. The results obtained were analysed together with those  
86 obtained previously by Wang et al. [42,43] from small-scale cyclic triaxial tests on  
87 microballast/fines mixture with microballast prepared by applying the parallel  
88 gradation method. Comparison of the mechanical parameters obtained at two scales  
89 allowed the validity of the parallel gradation method to be assessed on coarse  
90 grain/fines mixtures.

91

92 **2. Materials**

93 The ballast grains having the same grain size distribution as the interlayer soil in  
94 “Sénissiat site” were used [37], as shown in Fig. 1. The minimum grain size  $D_{\min}^b$   
95 and the maximum grain size  $D_{\max}^b$  are 20 mm and 63 mm, respectively. The  
96 microballast was tested previously by Wang et al. [42,43]; its maximum grain size  
97  $D_{\max}^m$  was chosen as 20 mm to adapt to a small-scale triaxial cell of 100 mm with a  
98 value of 5 as the ratio of specimen to maximum grain size. This is consistent with the  
99 recommendation of Fagnoul and Bonnechere [12], Nitchiporovitch [25] and Pedro  
100 [28]: a ratio larger than 5 must be adopted to minimize the specimen size effect. The  
101 grain size distribution of the microballast is transited from that of the ballast by  
102 following the parallel gradation method. In this method, the correlative grain sizes of  
103 ballast and microballast share the same percentage of grain passing. The grain sizes of  
104 these two materials are correlated according to the following equation:

105 (1) 
$$\frac{D^b - D_{\min}^b}{D^m - D_{\min}^m} = \frac{D_{\max}^b - D_{\min}^b}{D_{\max}^m - D_{\min}^m} = A$$

106 where  $D_{\max}$ ,  $D_{\min}$  and  $D$  are the maximum grain size, the minimum grain size, and a  
107 given grain size, respectively. The superscripts  $b$  and  $m$  stand for ballast and  
108 microballast, respectively.  $A$  is a constant.

109 To prepare the microballast, three angular-shaped commercial coarse grains  
110 were used. Visually, the grain shape of the chosen commercial grains was similar to  
111 that of the field ballast from “Sénissiat site”. The minimum grain size of the  
112 commercial grains  $D_{\min}^m$  was 1.6 mm, defining a value of 2.337 for parameter  $A$ . For  
113 a given grain size of microballast  $D^m$ , 10.0 mm for instance, equation (1) gives a

114 correlated ballast grain size  $D^b$  of 39.6 mm. As the ballast grain passing percentage  
115 is 59.1 % for  $D^b = 39.6$  mm, according to the parallel gradation method, the  
116 microballast grain passing percentage is also 59.1 % for  $D^m = 10$  mm. Following this  
117 procedure, the complete grain size distribution of the microballast was obtained, as  
118 shown in Fig. 1. This target (parallel gradation) curve was verified by measurement  
119 and it is observed that the measured grain size distribution and the target one show a  
120 good agreement (Fig. 1).

121 For the fines soils, since obtaining enough quantity from the field was difficult,  
122 they were fabricated in the laboratory. Nine commercial soils were mixed to simulate  
123 the fines soils from “Sénissiat site”, which included five kinds of medium sands, two  
124 kinds of fine sands and two kinds of clay soils (kaolinite and bentonite). Table 1 listed  
125 the properties of these soils. The mass proportion of each constituting soil was  
126 calculated according to the grain-size distributions of the fines from “Sénissiat site”  
127 (Fig. 2). The calculated results were also shown in Table 1. With the proportion and  
128 the grain size distribution of each soil, the grain size distribution of the fabricated  
129 fines can be calculated, which was plotted in Fig. 2 as well. During the mixing  
130 procedure, the materials to be mixed were carefully added into the mixer following  
131 the sequence of medium sands, water, fine sands, kaolinite and bentonite. The mass of  
132 the added water corresponded to a small water content of 4% of the total mixed soils.  
133 The aim of water addition was to prevent the small particles loss during the mixing,  
134 occurring when fine sands, kaolinite and bentonite were dry mixed with others.  
135 Bentonite was added last since it was the most reactive soil with water. With this

136 sequence, most kaolinite and bentonite particles were expected to be adhered to the  
137 surface of sands. Using this procedure, a homogeneous mixture can be obtained [21].  
138 Wang et al. [42,43] also adopted this method when preparing specimens of  
139 microballast and fines mixture. After preparation, the fabricated fines soil was  
140 compared with the field one in terms of grain size distribution and plasticity. As can  
141 be observed from Fig. 2, the grain size distribution of the fabricated fines showed a  
142 good agreement with that of the in situ one. In addition, the liquid limit and the  
143 plasticity index of the fabricated fines, which were 32% and 20%, respectively,  
144 matched the target liquid limit (35.5%) and plasticity index (17.3%) well. This soil is  
145 classified as lean clay (CL) according to ASTM D2487-11 [3]. Standard Proctor  
146 compaction tests were carried out on the fines material following ASTM D698-12 [4].  
147 An optimum water content  $w_{\text{opt-f}} = 13.70\%$  and a maximum dry density  $\rho_{\text{dmax-f}} = 1.82$   
148  $\text{Mg/m}^3$  were determined.

149

### 150 **3. Experimental methods**

#### 151 **3.1. Specimen preparation**

152 In order to quantify the coarse grains in a specimen, a parameter namely volumetric  
153 content of coarse grains  $f_v$  was adopted, which was defined as the ratio of the total  
154 volume of coarse grains to the total specimen volume [28,29,32,42,43,44]. The total  
155 specimen volume consisted of two parts: the coarse grain volume and the fines  
156 volume. All pores were assumed to be included in the fines soils. For the upper part of  
157 the natural interlayer soils at “Sénissiat site”, the  $f_v$  value was 52.9% [37]. In this

158 study, six  $f_v$  values (0%, 5%, 10%, 20%, 35%, 45%) were considered for the  
159 specimens prepared with both ballast and microballast. For convenience, in further  
160 analysis, the specimens prepared with ballast and microballast are referred to as  
161 ballast specimens and microballast specimens, respectively. The dimensions of the  
162 ballast specimens are 300 mm diameter and 600 mm height, while the dimensions of  
163 microballast specimens are 100 mm diameter and 200 mm height.

164 For the preparation of the ballast specimen, the total volume was first calculated  
165 with the specimen dimensions (300 mm diameter and 600 mm height). For a given  $f_v$   
166 value, the volumes of both ballast grains and fines can be obtained based on this total  
167 specimen volume. After that, the mass of ballast was obtained using the dry unit mass  
168 of ballast (2.68 Mg/m<sup>3</sup>). The fines in all specimens were kept at the same state defined  
169 by the optimum water content  $w_{\text{opt-f}} = 13.70\%$  and the maximum dry density  $\rho_{\text{dmax-f}} =$   
170 1.82 Mg/m<sup>3</sup>. The optimum state was selected aiming to simulate the heavily compact  
171 state in the field condition [8,21,38]. Accordingly, the masses of dry fines and water  
172 can be determined. Water was firstly added into the fines by spray to reach the  
173 optimum water content  $w_{\text{opt-f}} = 13.7\%$ . Then, the wetted fines soil was stored in  
174 hermetic containers for 24 h for moisture homogenization. Afterwards, the  
175 homogenized wetted fines soil was equally divided into eight parts (by weight) and so  
176 were the oven-dried ballast grains, after which they were mixed separately. Finally,  
177 the ballast specimen was compacted using a layered compaction method, which was  
178 widely adopted by many researchers to ensure reasonable uniformity of the specimen  
179 [5,22,35,46]. The mixed materials were compacted in eight layers using a vibration

180 hammer, each with a thickness of 75 mm.

181 Similar protocol was applied for the preparation of the microballast specimen  
182 (100 mm diameter and 200 mm height). The masses of the dry microballast, dry fines  
183 soil and water were firstly determined. Afterwards, the fines soil was wetted with the  
184 sprayed water to reach the optimum water content of 13.7%, stored in hermetic  
185 container for 24 h and then mixed with the microballast. Finally, this mixture was  
186 compacted dynamically to reach the target height, also following the layered  
187 compaction method to ensure reasonable uniformity of the specimen. More details can  
188 be found in Wang et al. [42,43]. Note that the test results at five  $f_v$  values (5%, 10%,  
189 20%, 35%, 45%) obtained by Wang et al. [42,43] were taken for comparison with the  
190 corresponding ones of the ballast samples in this study.

191

### 192 **3.2. Cyclic triaxial tests**

193 For the microballast specimen with 100 mm diameter and 200 mm height, a  
194 small-scale cyclic triaxial apparatus was used. Details about this device can be found  
195 in Wang et al. [43]. For testing a ballast specimen (300 mm diameter and 600 mm  
196 height), a large-scale cyclic triaxial apparatus developed by Dupla et al. [11] was  
197 adopted. The loading procedures adopted are the same for the ballast and the  
198 microballast specimens. No saturation process was applied and the specimen water  
199 content was kept constant during the test. In addition, the drainage valve was kept  
200 open during the loading process. All the tests were performed at a confining pressure  
201 of 30 kPa, which corresponded to the average horizontal stress estimated for the field

202 condition, considering the effects of the train wheel load, the interlayer depth and the  
203 Poisson ratio [9,42,43]. The applied cyclic loads followed a sine-shaped pattern (Fig.  
204 3). The deviator stress amplitude  $\Delta q$  is defined as the difference value between the  
205 maximum deviator stress  $q_{max}$  and the minimum deviator stress  $q_{min}$  [40,41,42,43]. A  
206 frequency of 1.78 Hz corresponding to a low train speed of 50 km/h was chosen,  
207 aiming to guarantee the quality of controlling the pre-defined loading shape. Note that  
208 this low frequency did not reflect the high train speed normally encountered in the  
209 field. This selection was based on the hypothesis that the train speed does not  
210 influence the assessment of the validity of the parallel gradation. During the loading  
211 procedure, axial stress and strain were monitored.

212         The application of cyclic loads included two consecutive stages. The loading  
213 stage 1 was focused on the aspect of the permanent deformation under a large number  
214 of loading cycles. The loading stage 2 was focused on the aspect of the resilient  
215 modulus or stiffness property under a wide range of stress amplitudes, including some  
216 large amplitudes possibly applied on the interlayer soils. In stage 1, a multi-step  
217 loading procedure proposed by Gidel et al. [13] was adopted (Fig. 4a): the deviator  
218 stress amplitude  $\Delta q = q_{max} - q_{min}$  was increased stepwise from 10 kPa to 30 kPa with  
219 an increment of 5 kPa. At each amplitude, 90,000 cycles were applied, which were  
220 thought to be enough for the stabilization of the permanent strain [10,13,21,38]. This  
221 multi-stage loading procedure allowed the application of several stress amplitudes on  
222 the same specimen, with which not only the number of tests could be reduced, but  
223 also the effect of the variability of soil specimens on the testing results can be

224 minimised. Thus, this procedure has been widely used in the study of the permanent  
225 deformation behaviour [6,10,19,21,36,38,43]. These loading amplitudes were selected  
226 based on the vertical stress measured at the equivalent depth of interlayer in the field  
227 [20,21]. In stage 2 (Fig. 4b), a procedure proposed by Lamas-Lopez [23] was adopted.  
228 The deviator stress amplitude  $\Delta q$  was firstly increased following a  $\Delta q$  sequence (10  
229 kPa, 30 kPa and 50 kPa) and then decreased following the reverse sequence.  
230 Afterwards,  $\Delta q$  was increased following a sequence from 10 kPa to 100 kPa and then  
231 decreased to 10 kPa. Finally,  $\Delta q$  was increased in steps to 200 kPa and then decreased  
232 to 10 kPa. At each stress level, 100 cycles were applied. The aim of firstly increasing  
233  $\Delta q$  and then decreasing it was to obtain the resilient modulus at both elastoplastic  
234 stage and pure elastic stage. Note that the large stress amplitudes (50 kPa, 100 kPa  
235 and 200 kPa) were defined according to the vertical stress of 40-90 kPa at similar  
236 depth of the interlayer observed by Selig and Waters [33], Jain and Keshav [18] and  
237 Yang et al. [49], the vertical stress of 120-138 kPa applied by heavier wagons in some  
238 countries [15,18,23] and the contingent maximum vertical stress at the top of railway  
239 structure as high as 200 kPa [21].

240

## 241 **4. Results and discussions**

### 242 **4.1. Variations of permanent deformation with $f_v$**

243 Fig. 5 plots the deviator stress  $q$  versus the axial strain  $\varepsilon_1$  during the first 90,000  
244 cycles of loading stage 1 for the ballast specimen at  $f_v = 35\%$ . In a loading-unloading  
245 cycle, the permanent axial strain  $\varepsilon_{1p}$  represented the irreversible part of the total

246 axial strain. It can be observed that the  $\varepsilon_{1p}$  value of the first cycle was particularly  
247 large. This phenomenon was also observed for other ballast specimens and the  
248 microballast specimens. This large permanent strain can be attributed to the  
249 adaptation of the specimen to the loading system, in particular in terms of contact  
250 between the specimen and the loading piston. As the initial specimen-piston contact  
251 was not identical for all the specimens, this  $\varepsilon_{1p}$  value would make impossible the  
252 relevant comparison of permanent strain  $\varepsilon_{1p}$  between different specimens. Thus, in further  
253 analysis, the first cycle was not accounted for the cumulative permanent strain.

254 Fig. 6 presents the permanent strain  $\varepsilon_{1p}$  against the number of cycles  $N$  for six  
255 ballast specimens, with the corresponding stress levels. The evolutions of permanent  
256 strain with number of cycles  $N$  were the same as the ones of microballast specimens  
257 [43]. It can be observed from Fig. 6 that for each  $f_v$  value,  $\varepsilon_{1p}$  increased quickly at the  
258 start of each loading level and then tended to stabilize with the increasing cycles.  
259 According to Werkmeister et al. [45], the large deformation in the beginning resulted  
260 from a significant rearrangement of the particles. As the cycle number increased, this  
261 particle rearrangement became more and more limited, and correspondingly, the  
262 permanent strain tended to become stable. As can also be seen from Fig. 6, a  
263 smaller  $f_v$  value led to a larger  $\varepsilon_{1p}$ , which can be attributed to the involvement of less  
264 coarse grains.

265 Due to the multi-step loading procedure adopted, the  $\varepsilon_{1p}$  at each stress level  
266 was greatly influenced by the previous loadings (Fig. 6). If this loading history is  
267 eliminated (as if the specimen has been subjected to only one loading level), the effect

268 of the ballast contents can be better evaluated. For this purpose, a permanent strain  
269 estimation method proposed by Gidel et al. [13] and then used by Lamas-Lopez [21]  
270 and Wang et al. [43] was adopted. The application of this method is explained in Fig.  
271 7 by considering two successive loading levels (Loading level  $M$  and Loading level  $M$   
272  $+ 1$ ). For the estimated permanent strain  $\varepsilon_{1p}^{M+1}$  of the Loading level  $M + 1$  at the first  
273 90,000 cycles, it is determined as:

$$274 \quad (2) \quad \varepsilon_{1p}^{M+1} = \varepsilon_{1p}^M + \delta\varepsilon_{1p}^{M+1}$$

275 where  $\varepsilon_{1p}^M$  is the measured permanent strain of the Loading Level  $M$ ;  $\delta\varepsilon_{1p}^{M+1}$  is the  
276 translated permanent strain determined by resetting both the initial strain value and  
277 the first cycle of the measured permanent strain of the Loading level  $M + 1$  to 0.

278 For estimating the permanent strain of the Loading level  $M + 1$  after 90,000  
279 cycles, a linear increasing trend between permanent strain and number of cycles is  
280 considered, with its slope kept at the same value as the one determined at the last  
281 cycles of Loading level  $M + 1$ . More details can be found in Wang et al. [43].

282 The estimated permanent strain curves for  $f_v = 0\%$  of the ballast specimen are  
283 presented in Fig. 8, together with the measured results. It can be observed that at the  
284 last cycle of each stress level, the estimated strain was larger than the measured one.  
285 For example, for  $\Delta q = 20$  kPa, the measured permanent strain at the last cycle  $N =$   
286 270,000 was 0.182%, while the corresponding estimated one was 0.226%. This  
287 phenomenon can be attributed to the different loading histories: for the estimated  
288 strain, the specimen was assumed to undergo one higher stress level of 20 kPa. When  
289 it came to the measured strain, successive lower stress levels of 10kPa, 15 kPa and 20

290 kPa were applied, leading to a smaller strain value.

291 Gidel et al. [13] examined this permanent strain estimation method through  
292 cyclic triaxial tests, concluding that the permanent strain estimated for a single stress  
293 level agreed well with the measured one for the loading cycles applied at each step  
294 (90,000 cycles in this study). In other words, in this study, the estimated permanent  
295 strain at the first 90,000 cycles was relatively accurate. Fig. 9 depicts the estimated  
296 permanent strain evolutions at the first 90,000 cycles under different stress levels for  
297 ballast specimen at  $f_v = 0\%$ . As can be observed, a larger stress level led to a larger  
298 strain value. In addition, the permanent strain gradually stabilized with the increase of  
299 the cycle number. To further illustrate the effect of ballast content  $f_v$ , the estimated  
300 permanent strains for cycle  $N = 90,000$  (end-stage cycle in Fig. 9) at all stress levels  
301 were taken and compared (Fig. 10). It can be observed that at each stress level, the  
302 permanent strain decreased with the increase of  $f_v$ . Moreover, a bi-linear decreasing  
303 trend could be identified. When  $f_v \leq 20\%$ , the permanent strain decreased rapidly. By  
304 contrast, when  $f_v \geq 35\%$ , this decreasing rate became smaller. The same observations  
305 were made by Wang et al. [43] for the microballast specimens.

306 In the study of Wang et al. [43], X-ray microcomputed tomography ( $\mu$ CT) scans  
307 were carried out on the as-compacted microballast specimens. Two soil fabrics were  
308 identified at different  $f_v$  values: a fine-fine contact structure at  $f_v = 0\%$ - $20\%$  and a  
309 distinct grain-grain contact structure at  $f_v = 35\%$ - $45\%$ . It is worth noting that at  $f_v =$   
310  $20\%$ , the coarse grains started to be partially connected without forming a whole  
311 skeleton. Unfortunately, this  $\mu$ CT scan could not be conducted on ballast specimens

312 due to their large size. However, basically, it seems plausible to consider the same  
313 feature of fabrics for the ballast specimens: also a fine-fine contact structure at low  $f_v$   
314 and a grain-grain contact structure at high  $f_v$ . For fine-fine contact structure, the  
315 permanent strain results from the compression of the fines, which decreases rapidly  
316 with increasing  $f_v$ . By contrast, for grain-grain contact structure, the rearrangement of  
317 the grains plays a dominant role and the permanent strain decreases slightly when  $f_v$   
318 increases.

319

#### 320 **4.2. Variations of resilient modulus with $f_v$**

321 Typical hysteresis loops at the first cycles of different stress levels are shown in Fig.  
322 11 for the ballast specimen at  $f_v = 20\%$ . The loops for stress levels of  $\Delta q = 10$  kPa,  $\Delta q$   
323  $= 30$  kPa,  $\Delta q = 50$  kPa,  $\Delta q = 100$  kPa and  $\Delta q = 200$  kPa corresponded to cycle  
324 numbers of  $N = 1$ ,  $N = 101$ ,  $N = 201$ ,  $N = 701$  and  $N = 1401$ , respectively. It appears  
325 that for the low stress levels of 10 kPa and 30 kPa, the hysteresis loops were pretty  
326 small and closed. This was due to the application of 450,000 loading cycles under  $\Delta q$   
327  $\leq 30$  kPa in loading stage 1. By contrast, for stress levels larger than 30 kPa,  
328 permanent strain developed, leading to larger and unclosed hysteresis loops, in  
329 particular under stress levels as high as  $\Delta q = 100$  kPa and 200 kPa.

330 The evolution of hysteresis loop with number of cycles is illustrated in Fig. 12  
331 by plotting the loops for 100 cycles (from  $N = 701$  to  $N = 800$ ) at  $\Delta q = 100$  kPa for  
332 the ballast specimen with  $f_v = 20\%$ . A large and unclosed hysteresis loop was observed  
333 for the first cycle, due to the stress level of 100 kPa which was larger than all the

334 previous applied stress levels. In addition, with the increase of the cycle number, the  
335 hysteresis loop became smaller and smaller and at the end of loading, the loops  
336 became closed and relatively unchanged, suggesting a pure elastic behaviour of soil.

337 As shown in Fig. 13, the resilient modulus  $M_r$  is defined as [30,31]:

338 (3) 
$$M_r = \Delta q / \varepsilon_{1r}$$

339 where  $\Delta q$  is the deviator stress amplitude, and  $\varepsilon_{1r}$  is the resilient strain.

340 The evolution of  $M_r$  with the number of cycles  $N$  at varying stress levels is  
341 presented in Fig. 14 for the ballast specimen at  $f_v = 20\%$ . The results of other ballast  
342 specimens and microballast specimens obey the same rule and are not shown for  
343 clarity. It can be observed that when the stress amplitude increased, the  $M_r$  value  
344 immediately decreased and then tended to stabilize. By contrast, with the stress  
345 amplitude decreasing, the  $M_r$  value increased instantly and then tended to reach a  
346 stabilization state. The decrease of  $M_r$  with the increase of stress amplitude can be  
347 explained by the strain increasing due to the increase of stress amplitude [2].

348 In order to illustrate the effect of  $f_v$  on resilient modulus, the  $M_r$  values of the  
349 last (end-stage) cycles of all stress levels were taken and presented as a function of  $f_v$   
350 for different stress levels (Fig. 15). The data were named according to the number of  
351 cycles  $N$  and the corresponding  $\Delta q$  values. It appears that an increasing trend of  $M_r$   
352 with  $f_v$  can be identified for all stress levels. Moreover, this increasing trend followed  
353 a bi-linear pattern with the slope at  $f_v \geq 35\%$  larger than the one at  $f_v \leq 20\%$ ,  
354 evidencing that the soil fabrics of the ballast specimens at these two  $f_v$  ranges were  
355 different: fine-fine contact structure at low  $f_v$  and grain-grain contact structure at

356 higher  $f_v$ . Same bi-linear variation trend of  $M_r$  was identified by Wang et al. [42] for  
357 the microballast specimens, similarly, evidencing that the soil fabrics of microballast  
358 specimens at  $f_v \leq 20\%$  and  $f_v \geq 35\%$  were separately dominated by fine-fine contact  
359 structure and grain-grain contact structure.

360

### 361 **4.3. Comparison of permanent strain between ballast and microballast** 362 **specimens**

363 The estimated permanent strains for cycle  $N = 90,000$  (end-stage cycle) at six  $f_v$  values  
364 are presented in Fig. 16 for the ballast and the microballast specimens, for five stress  
365 levels (10, 15, 20, 25 and 30 kPa). It can be observed that in the case of  $f_v = 0\%$ , at  
366 each stress level, the  $\varepsilon_{1p}$  value of the ballast specimen coincides well with that of the  
367 microballast specimen. For the fine-fine contact structure at  $f_v = 5\%$  and  $10\%$ , on the  
368 whole, the  $\varepsilon_{1p}$  values of the corresponding different scale specimens are found to be  
369 similar. This phenomenon can be explained by the fact that in this case the fines  
370 matrix constitutes the soil skeleton and the permanent strain was dominated by the  
371 compression of the fines, giving rise to the permanent strains which were independent  
372 of coarse grain size. For the grain-grain contact structure at  $f_v = 35\%$  and  $45\%$ , it can  
373 be observed that the permanent strains of the ballast specimens and the microballast  
374 specimens were almost the same (Fig. 16), in particular at  $\Delta q = 10$  kPa (Fig. 16a).  
375 Note that although the case at  $f_v = 20\%$  was categorized into the group of fine-fine  
376 contact structure, the influence of coarse grain size on the permanent strain followed  
377 the same pattern. This can be explained by the fact that at  $f_v = 20\%$ , part of the coarse

378 grains started to be in contact with each other. Further examination shows that at  $\Delta q$   
379 higher than 10 kPa, the strains of microballast specimens were slightly larger than  
380 those of the ballast specimens. This phenomenon can be explained as follows: upon  
381 loadings, the grains would slide into the pores nearby, giving rise to permanent strain.  
382 If the difficulty of sliding into large pores for the large grains was the same as that for  
383 the smaller grains to slide into smaller pores, identical permanent strains can be  
384 expected. It is the case for the ballast specimen and the microballast specimen at  $\Delta q =$   
385 10 kPa. Apparently, the grains sliding depended on the stress level and the grain sizes;  
386 in the microballast specimen, the grains with smaller sizes would slide into the  
387 adjacent pores more easily, leading to a slightly larger strain.

388 Another possible explanation for the slightly larger strains in the microballast  
389 specimen is related to the effect of the fines soils. As shown in Fig. 17, the fines soils  
390 can be roughly considered to be distributed at two locations: the pores near the coarse  
391 grain and the coarse grain intervals along the stress chain (supposed to be along the  
392 specimen height). The applied load was transmitted along the stress chain by  
393 compressing the fines between grains. When the load reached a certain level, grain  
394 rearrangement occurred [26,27]. It can be inferred that the fines in the pores inhibited  
395 the coarse grain sliding, while those between grains promoted it. At a given  $f_v$  value,  
396 for the microballast specimen, more grain-fines-grain contacts can be expected along  
397 the stress chain, leading to more distribution of fines soils. As a result, relatively  
398 larger permanent strain can be expected.

399 As the difference of permanent strain between the two scale specimens appears

400 quite small (Figure. 16), it can be reasonably concluded that the parallel gradation  
401 method is valid for the ballast/fines mixture in terms of permanent strain.

402

#### 403 **4.4. Comparison of resilient modulus between ballast and microballast specimens**

404 The comparison of  $M_r$  values between the microballast and ballast specimens with  
405 fine-fine contact structure is shown in Fig. 18. At  $f_v = 0\%$ , the  $M_r$  values of the ballast  
406 specimen were comparable with those of the microballast specimen. This was normal  
407 because the scaling question was not involved in that case. For higher  $f_v$  values, except  
408  $f_v = 10\%$ , the  $M_r$  values of the ballast and the microballast specimens coincided with  
409 each other. The singular case at  $f_v = 10\%$  was probably due to a technical problem.

410 The  $M_r$  values of the ballast specimens at grain-grain contact structure were  
411 compared with those of the microballast specimens in Fig. 19 (Fig. 19a for  $f_v = 35\%$   
412 and Fig. 19b for  $f_v = 45\%$ ). It can be observed that almost the same  $M_r$  values were  
413 obtained for the two specimens, indicating that the resilient modulus was independent  
414 of grain size. Further investigation shows that on the whole the  $M_r$  values of the  
415 microballast specimens were slightly smaller than those of the ballast specimens.  
416 These phenomena can be explained based on the analysis of Yang and Gu [48]. They  
417 derived the resilient modulus of a simple cubic array of identical spheres under a  
418 certain confining pressure by applying a small shear stress onto it. In the analysis, the  
419 induced resilient shear strain of the cubic array was determined by adding up the  
420 tangential strains between every two spheres along the vertical stress chain. This  
421 tangential strain was calculated using the Hertz-Mindlin contact law, which was a

422 function of the applied shear stress. The expression for the resilient modulus (defined  
423 as the ratio of the shear stress to the shear strain) was obtained, showing no  
424 dependence on sphere radius. Note that this analysis did not consider any contribution  
425 from fine soils, thus not fully reflecting the case of this study. Indeed, as mentioned  
426 previously, in the case of mixture, the resilient modulus is affected by the fine soils  
427 along the stress chain. For the microballast specimens, as stated before, there are more  
428 fines soils along it. Thus, relatively smaller resilient modulus values are expected.

429         As for the permanent strain, globally, the resilient modulus also showed a good  
430 agreement between the two scale specimens albeit the slight difference discussed  
431 above, which validates the parallel gradation method in terms of resilient modulus.

432

## 433 **5. Conclusions**

434 The parallel gradation method was assessed on coarse grain/fines mixture by  
435 comparing the mechanical parameters (permanent strain and resilient modulus) of the  
436 two different scale specimens. Six volumetric coarse grain contents were considered  
437 for each kind of specimens. Based on the results obtained, the following conclusions  
438 can be drawn.

439         Two soil fabrics could be defined as fine-fine contact structure at  $f_v = 0-20\%$   
440 and grain-grain contact structure at  $f_v = 35-45\%$ . It was observed that for the ballast  
441 and microballast specimens, with the increase of  $f_v$  value, the permanent strain  
442 decreased rapidly when  $f_v \leq 20\%$  and this decreasing rate slowed down when  $f_v \geq 35\%$ ,  
443 while the resilient modulus increased slightly when  $f_v \leq 20\%$  and increased

444 significantly when  $f_v \geq 35\%$ .

445         The consistency between the large-scale and small-scale triaxial tests were  
446 found to be satisfactory: at  $f_v = 0\%$ , the permanent strain as well as the resilient  
447 modulus values obtained from the two scales showed a good correspondence. In the  
448 case of fine-fine contact structure, the parallel gradation method was found to be valid.  
449 The permanent strain and resilient modulus values of the ballast specimens were  
450 consistent with the respective ones of the microballast specimens. This could be  
451 explained by the fact that in this case the fines matrix governed the soil structure.

452         In the case of grain-grain contact structure, on the whole, the permanent strains  
453 of the ballast specimens coincided with those of the microballast specimens  
454 (especially at  $\Delta q = 10$  kPa). This coincidence was also observed in terms of resilient  
455 modulus. For the permanent strain, it could be explained by the fact that the  
456 difficulties of the sliding of large ballast grains into the adjacent large pores and the  
457 sliding of the small microballast grains into the smaller pores were expected to be the  
458 same. For the resilient modulus, it could be explained by the analytical results of Yang  
459 and Gu (2013), showing that the resilient modulus is independent of grain size.  
460 Further examination showed that there were slight differences of permanent strain and  
461 resilient modulus between two different scale specimens, which could be attributed to  
462 the irregular grain sliding and the fines distribution.

463

#### 464 **Acknowledgements**

465 The support from the Chinese Scholar Council (CSC) is greatly acknowledged.

467 **Notations**

$A$	constant in the similitude equation from ballast to microballast
$D^b$	given grain size of ballast
$D_{\max}^b$	maximum grain size of ballast
$D_{\min}^b$	minimum grain size of ballast
$D^m$	given grain size of microballast
$D_{\max}^m$	maximum grain size of microballast
$D_{\min}^m$	minimum grain size of microballast
$f_v$	volumetric content of coarse grains
$M_r$	resilient modulus
$N$	number of cycles
$q$	deviator stress
$q_{\max}$	maximum deviator stress
$q_{\min}$	minimum deviator stress
$w_{\text{opt-f}}$	optimum water content of fines
$\Delta q$	deviator stress amplitude
$\delta\varepsilon_{1p}^{M+1}$	translated permanent strain from the measured curve of Loading level M+1
$\varepsilon_1$	axial strain

$\varepsilon_{1p}$	permanent axial strain
$\varepsilon_{1r}$	resilient strain
$\varepsilon_{1p}^M$	measured permanent strain of the Loading Level $M$
$\varepsilon_{1p}^{M+1}$	estimated permanent strain of Loading level $M+1$
$\rho_{\text{dmax-f}}$	maximum dry density of fines

468

469 **References**

- 470 [1] Aingaran, S. 2014. Experimental investigation of static and cyclic behavior of  
471 scaled railway ballast and the effect of stress reversal. Ph.D. thesis, University of  
472 Southampton.
- 473 [2] Atkinson, J.H. 2000. Non-linear soil stiffness in routine design. *Géotechnique*,  
474 50(5): 487-507.
- 475 [3] ASTM. 2011. D2487-11: Standard practice for classification of soils for  
476 engineering purposes (unified soil classification system). ASTM International,  
477 West Conshohocken, PA, USA.
- 478 [4] ASTM. 2012. D698-12: Standard test methods for laboratory compaction  
479 characteristics of soil using standard effort. ASTM International, West  
480 Conshohocken, PA, USA.
- 481 [5] Cai, Y.Q., Chen, J.Y., Cao, Z.G., Gu, C., and Wang, J. 2018. Influence of grain  
482 gradation on permanent strain of unbound granular materials under low confining  
483 pressure and high-cycle loading. *International Journal of Geomechanics*, 18(3):  
484 04017156.

- 485 [6] Cao, Z.G., Chen, J.Y., Cai, Y.Q., Zhao, L., Gu, C., and Wang, J. 2018. Long-term  
486 behavior of clay-fouled unbound granular materials subjected to cyclic loadings  
487 with different frequencies. *Engineering Geology*, 243: 118-127.
- 488 [7] Cui, Y.J., Duong, T.V., Tang, A.M., Dupla, J.C., Calon, N., and Robinet, A. 2013.  
489 Investigation of the hydro-mechanical behavior of fouled ballast. *Journal of*  
490 *Zhejiang University-Science A (Applied Physics & Engineering)*, 14(4): 244-255.
- 491 [8] Duong, T.V. 2013. Investigation of the hydro-mechanical behavior of ancient  
492 railway platforms in scope to reinforcement by soil-mixing. Ph.D. thesis, Ecole  
493 Nationale des Ponts et Chaussées, Université Paris-Est.
- 494 [9] Duong, T.V., Cui, Y.J., Tang, A.M., Dupla, J.C., Canou, J., Calon, N., and Robinet,  
495 A. 2016. Effects of water and fines contents on the resilient modulus of the  
496 interlayer soil of railway substructure. *Acta Geotechnica*, 11(1): 51-59.
- 497 [10] Duong, T.V., Tang, A.M., Cui, Y.J., Trinh, V.N., Dupla, J.C., Calon, N., Canou, J.,  
498 and Robinet, A. 2013. Effects of fines and water contents on the mechanical  
499 behavior of interlayer soil in ancient railway sub-structure. *Soils and Foundations*,  
500 53(6): 868-878.
- 501 [11] Dupla, J.C., Pedro, L.S., Canou, J., and Dormieux, L. 2007. Mechanical behavior  
502 of coarse grained soils reference. *Bulletin de Liaison des Laboratoires des Ponts*  
503 *et Chaussées*, 268-269: 31-58.
- 504 [12] Fagnoul, A., and Bonnechere, F. 1969. Shear strength of porphyry materials. *In*  
505 *Proceedings of the 7th International Conference on Soil Mechanics and*  
506 *Foundation Engineering, Mexico*, pp. 61-65.

- 507 [13] Gidel, G., Hornych, P., Chauvin, J.J., Breysse, D., and Denis, A. 2001. A new  
508 approach for investigating the permanent deformation behavior of unbound  
509 granular material using the repeated load triaxial apparatus. *Bulletin des*  
510 *Laboratoires des Ponts et Chaussées*, 233: 5-21.
- 511 [14] Frost, R.J. 1973. Some testing experiences and characteristics of boulder-gravel  
512 fills in earth dams. *In* Evaluation of relative density and its role in geotechnical  
513 projects involving cohesionless soils. STP 523. ASTM Standards and  
514 Publications.
- 515 [15] Grabe, P., and Clayton, C. 2009. Effects of principal stress rotation on permanent  
516 deformation in rail track foundations. *Journal of Geotechnical and*  
517 *Geoenvironmental Engineering*, 135(4): 555–565.
- 518 [16] Indraratna, B., Wijewardena, L.S.S., and Balasubramaniam, A.S. 1993.  
519 Large-scale triaxial testing of greywacke rockfill. *Géotechnique*, 43: 37-51.
- 520 [17] Indraratna, B., Su, L., and Rujikiatkamjorn, C. 2011. A new parameter for  
521 classification and evaluation of railway ballast fouling. *Canadian Geotechnical*  
522 *Journal*, 48(2): 322-326.
- 523 [18] Jain, V., and Keshav, K. 1999. Stress distribution in railway formation - a  
524 simulated study. *In* Proceeding of the 2<sup>nd</sup> International Symposium on Pre-Failure  
525 Deformation Characteristics of Geomaterials-IS Torino.
- 526 [19] Jing, P., Nowamooz, H., and Chazallon, C. 2016. Permanent deformation  
527 behaviour of a granular material used in low-traffic pavements. *Road Materials*  
528 *and Pavement Design*, 19(2): 289-314.

- 529 [20] Lamas-Lopez, F., Costa D'Aguiar, S., Robinet, A., Cui, Y.J., Calon, N., Canou, J.,  
530 Dupla, J.C., and Tang, A.M. 2015. In-situ investigation of the behaviour of a  
531 French conventional railway platform. *In* Proceedings of Transportation Research  
532 Board TRB 2015, Washington, DC.
- 533 [21] Lamas-Lopez, F. 2016. Field and laboratory investigation on the dynamic  
534 behaviour of conventional railway track-bed materials in the context of traffic  
535 upgrade. Ph.D. thesis, Ecole Nationale des Ponts et Chaussées, Université  
536 Paris-Est.
- 537 [22] Lenart, S., Koseki, J., Miyashita, Y., and Sato, T. 2014. Large-scale triaxial tests  
538 of dense gravel material at low confining pressures. *Soils and Foundation*, 54(1):  
539 45-55.
- 540 [23] Li, D., and Selig, E. 1998. Method for railroad track foundation design. I:  
541 Development. *Journal of Geotechnical and Geoenvironmental Engineering*,  
542 124(4): 316–322.
- 543 [24] Lowe, J. 1964. Shear strength of coarse embankment dam materials. *In*  
544 *Proceedings of the 8th International Congress on Large Dams 3*, Edinburgh, U.K.,  
545 pp. 745-761.
- 546 [25] Nitchiporovitch, A.A. 1969. Shearing strength of coarse shell materials. *In*  
547 *Proceedings of the 7th International Conference on Soil Mechanics and*  
548 *Foundation Engineering*, Mexico, pp. 211-216.
- 549 [26] Oda, M., Konishi, J., and Nasser, N.S. 1982. Experimental micromechanical  
550 evaluation of strength of granular materials: effects of particle rolling. *Mechanics*

- 551 of Materials, 1(4): 269-283.
- 552 [27] Oda, M., and Kazama, H. 1998. Microstructure of shear band and its relation to  
553 the mechanisms of dilatancy and failure of dense granular soils. *Géotechnique*,  
554 48(4): 465-481.
- 555 [28] Pedro, L. 2004. De l'étude du comportement mécanique de sols hétérogènes  
556 modèles à son application au cas des sols naturels. Ph.D. thesis, Ecole Nationale  
557 des Ponts et Chaussées, Université Paris-Est.
- 558 [29] Qi, S., Cui, Y.J., Chen, R.P., Wang, H.L., Lamas-Lopez, F., Aïmediou, P., Dupla,  
559 J.C., Canou, J., and Saussine, G. 2019. Influence of grain size distribution of  
560 inclusions on the mechanical behaviors of track-bed materials. *Géotechnique in*  
561 *press*. doi : 10.1680/jgeot.18.P.047.
- 562 [30] Rollins, K.M., Evans, M.D., Diehl, N.B., and Daily III, W.D. 1998. Shear  
563 modulus and damping relationships for gravels. *Journal of Geotechnical and*  
564 *Geoenvironmental Engineering*, 124(5): 396-405.
- 565 [31] Seed, H.B., Wong, R.T., Idriss, I.M., and Tokimatsu, K. 1986. Moduli and  
566 damping factors for dynamic analyses of cohesionless soils. *Journal of*  
567 *Geotechnical Engineering*, 112(11): 1016-1032.
- 568 [32] Seif El Dine, B., Dupla, J.C., Frank, R., Canou, J., and Kazan, Y. 2010.  
569 Mechanical characterization of matrix coarse-grained soils with a large-sized  
570 triaxial device. *Canadian Geotechnical Journal*, 47: 425-438.
- 571 [33] Selig, E., and Waters, J. 1994. Track geotechnology and sub-structure  
572 management. Thomas Telford, London, U.K.

- 573 [34] Sevi, A., and Ge, L. 2012. Cyclic behaviors of railroad ballast within the parallel  
574 gradation scaling framework. *Journal of Materials in Civil Engineering*, 24(7):  
575 797-804.
- 576 [35] Suiker, A.S.J., Selig, E.T., and Frenkel, R. 2005. Static and cyclic triaxial testing  
577 of ballast and subballast. *Journal of Geotechnical and Geoenvironmental*  
578 *Engineering*, 131: 771-782.
- 579 [36] Tang, L., Yan, M.H., Ling, X.Z., and Tian, S. 2017. Dynamic behaviours of  
580 railway's base course materials subjected to long-term low-level cyclic loading:  
581 experimental study and empirical model. *Géotechnique*, 67(6): 537-545.
- 582 [37] Trinh, V.N. 2011. Comportement hydromécanique des matériaux constitutifs de  
583 plateformes ferroviaires anciennes. Ph.D. thesis, Ecole Nationale des Ponts et  
584 Chaussées, Université Paris-Est.
- 585 [38] Trinh, V.N., Tang, A.M., Cui, Y.J., Dupla, J.C., Canou, J., Calon, N., Lambert, L.,  
586 Robinet, A., and Schoen, O. 2012. Mechanical characterisation of the fouled  
587 ballast in ancient railway track sub-structure by large-scale triaxial tests. *Soils*  
588 *and Foundations*, 52(3): 511-523.
- 589 [39] Varadarajan, A., Sharma, K.G., Venkatachalam, K., and Gupta, A.K. 2003.  
590 Testing and modeling two rockfill materials. *Journal of Geotechnical and*  
591 *Geoenvironmental Engineering*, 129(3): 206-218.
- 592 [40] Wang, H.L., and Chen, R.P. 2019. Estimating static and dynamic stresses in  
593 geosynthetic-reinforced pile-supported track-bed under train moving loads.  
594 *Journal of Geotechnical and Geoenvironmental Engineering*, 145(7): 04019029.

- 595 [41] Wang, H.L., Chen, R.P., Cheng, W., Qi, S., and Cui, Y.J. 2019. Full-scale model  
596 study on variations of soil stress in geosynthetic-reinforced pile-supported  
597 track-bed with water level change and cyclic loading. *Canadian Geotechnical*  
598 *Journal*, 56(1): 60–68.
- 599 [42] Wang, H.L., Cui, Y.J., Lamas-Lopez, F., Dupla, J.C., Canou, J., Calon, N.,  
600 Saussine, G., Aïmediou, P., and Chen, R.P. 2017. Effects of inclusion contents on  
601 resilient modulus and damping ratio of unsaturated track-bed materials. *Canadian*  
602 *Geotechnical Journal*, 54: 1672-1681.
- 603 [43] Wang, H.L., Cui, Y.J., Lamas-Lopez, F., Dupla, J.C., Canou, J., Calon, N.,  
604 Saussine, G., Aïmediou, P., and Chen, R.P. 2018a. Permanent deformation of  
605 track-bed materials at various inclusion contents under large number of loading  
606 cycles. *Journal of Geotechnical and Geoenvironmental Engineering*, 144(8):  
607 04018044.
- 608 [44] Wang, H.L., Cui, Y.J., Lamas-Lopez, F., Calon, N., Saussine, G., Dupla, J.C.,  
609 Canou, J., Aïmediou, P., and Chen, R.P. 2018b. Investigation on the mechanical  
610 behavior of track-bed materials at various contents of coarse grains. *Construction*  
611 *and Building Materials journal*, 164: 228-237.
- 612 [45] Werkmeister, S., Dawson, A.R., and Wellner, F. 2004. Pavement design model for  
613 unbound granular materials. *Journal of Transportation Engineering*, 130:  
614 665-674.
- 615 [46] Wichtmann, T., Rondon, H.A., Niemunis, A., Triantafyllidis, Th., and Lizcano, A.  
616 2010. Prediction of permanent deformations in pavements using a high-cycle

617 accumulation model. *Journal of Geotechnical and Geoenvironmental Engineering*,  
618 136: 728-740.

619 [47] Xiao, Y., Liu, H.L., Chen, Y.M., and Jiang, J.S. 2014. Bounding surface model  
620 for rockfill materials dependent on density and pressure under triaxial stress  
621 conditions. *Journal of Engineering Mechanics*, 140: 04014002.

622 [48] Yang, J., and Gu, X.Q. 2013. Shear stiffness of granular material at small strains:  
623 does it depend on grain size?. *Géotechnique*, 63(2): 165-179.

624 [49] Yang, L., Powrie, W., and Priest, J. 2009. Dynamic stress analysis of a ballasted  
625 railway track bed during train passage. *Journal of Geotechnical and*  
626 *Geoenvironmental Engineering*, 135(5): 680–689.

627 [50] Zeller, J., and Wullimann, R. 1957. The shear strength of the shell materials for  
628 the Göschenalp Dam, Switzerland. *In Proceedings of the 4th International*  
629 *Conference on Soil Mechanics and Foundation Engineering*, London, pp.  
630 399-404.

## List of tables

Table 1. Mass proportions and grain size ranges of nine commercial soils

## List of Figures

Fig. 1. Grain size distributions of the ballast and the microballast (modified from Wang et al. 2018a)

Fig. 2. Grain size distributions of fines (modified from Wang et al. 2018a)

Fig. 3. Typical sine-shaped signals applied in the cyclic loadings

Fig. 4. Loading paths in two stages: (a) stage 1 for permanent strain investigation; (b) stage 2 for resilient modulus investigation

Fig. 5. Deviator stress  $q$  versus axial strain  $\varepsilon_1$  for the first 90,000 cycles of loading stage 1 (ballast specimen at  $f_v = 35\%$ )

Fig. 6. Permanent strain evolutions with number of cycles at different  $f_v$  values (ballast specimens)

Fig. 7. Illustration of the method to eliminate the loading history effect

Fig. 8. Measured and estimated permanent strain curves for ballast specimen at  $f_v = 0\%$

Fig. 9. Estimated permanent strain evolutions at the first 90,000 cycles for ballast specimen at  $f_v = 0\%$

Fig. 10. Variations of estimated end-stage permanent strains with  $f_v$  at different stress levels (ballast specimens)

Fig. 11. Hysteresis loops at first cycles of different stress levels for ballast specimen at

$f_v = 20\%$ :  $\Delta q = 10$  kPa (N = 1),  $\Delta q = 30$  kPa (N = 101),  $\Delta q = 50$  kPa (N = 201),  $\Delta q = 100$  kPa (N = 701) and  $\Delta q = 200$  kPa (N = 1401)

Fig. 12. Hysteresis loops during 100 cycles (N = 701 to 800) under  $\Delta q = 100$  kPa for ballast specimen at  $f_v = 20\%$

Fig. 13. Determination of  $M_r$

Fig. 14. Evolution of  $M_r$  with number of cycles at different stress levels for ballast specimen at  $f_v = 20\%$

Fig. 15. Evolutions of  $M_r$  at end stage with  $f_v$  for ballast specimens at: (a)  $\Delta q = 10$  kPa; (b)  $\Delta q = 30$  kPa; (c)  $\Delta q = 50$  kPa; (d)  $\Delta q = 100$  kPa and 200 kPa

Fig. 16. Variations of estimated end-stage permanent strains with  $f_v$  for ballast and microballast specimens at: (a)  $\Delta q = 10$  kPa; (b)  $\Delta q = 15$  kPa; (c)  $\Delta q = 20$  kPa; (d)  $\Delta q = 25$  kPa and 30 kPa

Fig. 17. Schematic illustration of the distribution of fines soils in case of grain-grain contact structure

Fig. 18. Comparison of  $M_r$  value between ballast specimens and microballast specimens in case of fine-fine contact structure for: (a)  $f_v = 0\%$ ; (b)  $f_v = 5\%$ ; (c)  $f_v = 10\%$ ; (d)  $f_v = 20\%$

Fig. 19. Comparison of  $M_r$  value between ballast specimens and microballast specimens in case of grain-grain contact structure for (a)  $f_v = 35\%$  and (b)  $f_v = 45\%$

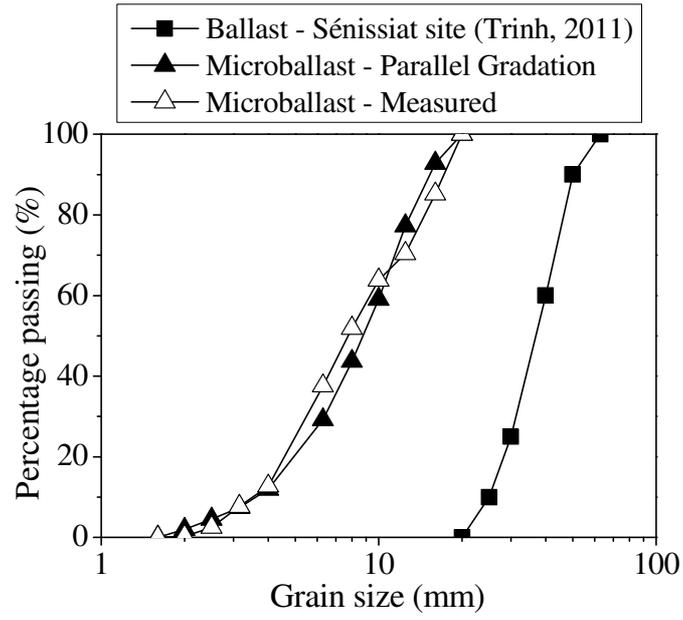


Fig. 1. Grain size distributions of the ballast and the microballast (modified from Wang et al. 2018a)

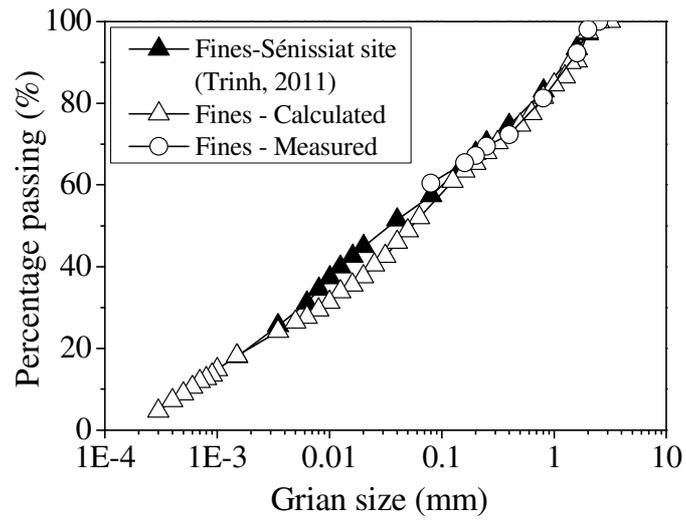


Fig. 2. Grain size distributions of fines (modified from Wang et al. 2018a)

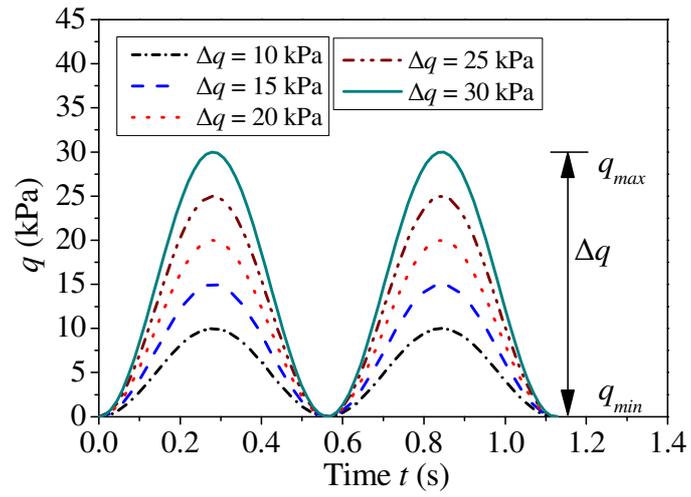


Fig. 3. Typical sine-shaped signals applied in the cyclic loadings

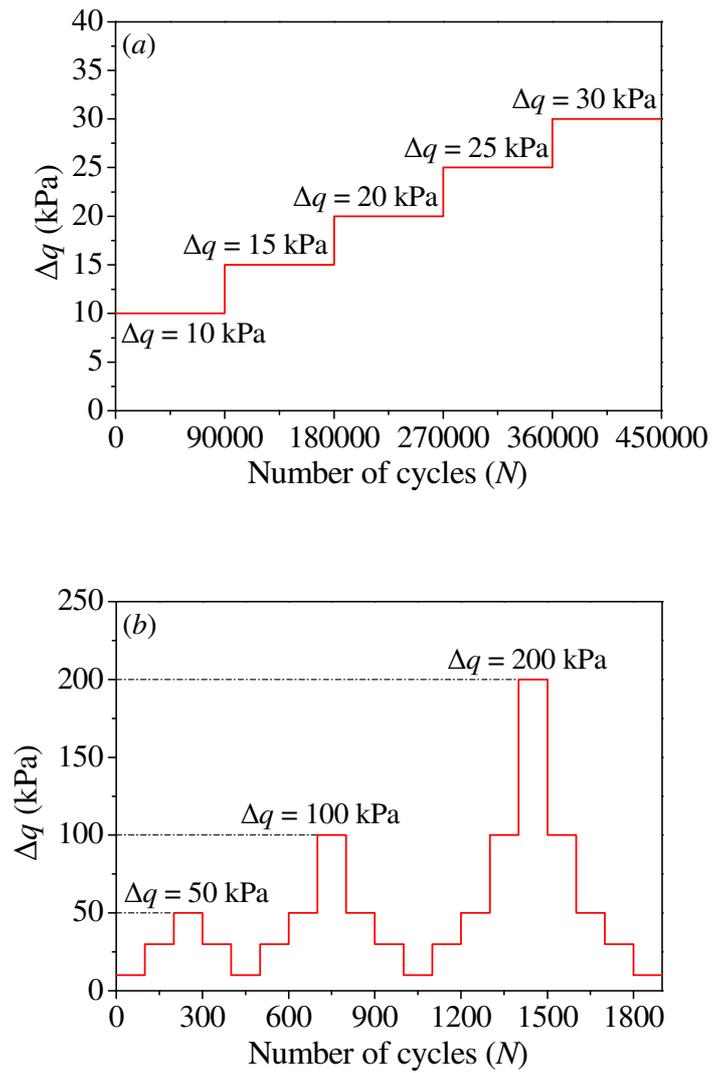


Fig. 4. Loading paths in two stages: (a) stage 1 for permanent strain investigation; (b) stage 2 for resilient modulus investigation

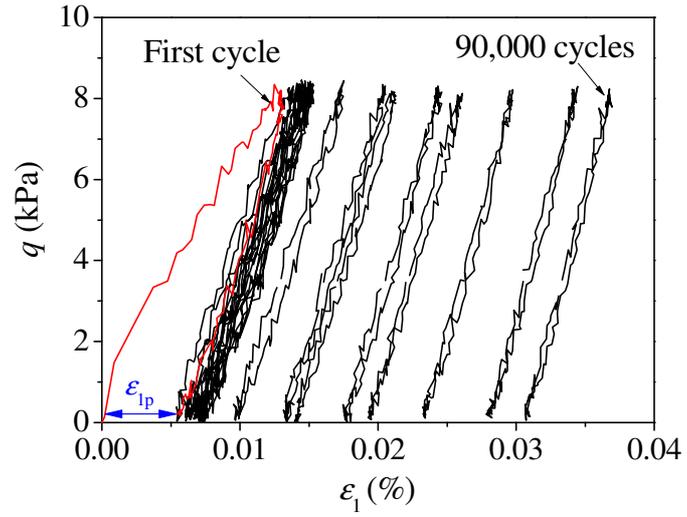


Fig. 5. Deviator stress  $q$  versus axial strain  $\varepsilon_1$  for the first 90,000 cycles of loading stage 1 (ballast specimen at  $f_v = 35\%$ )

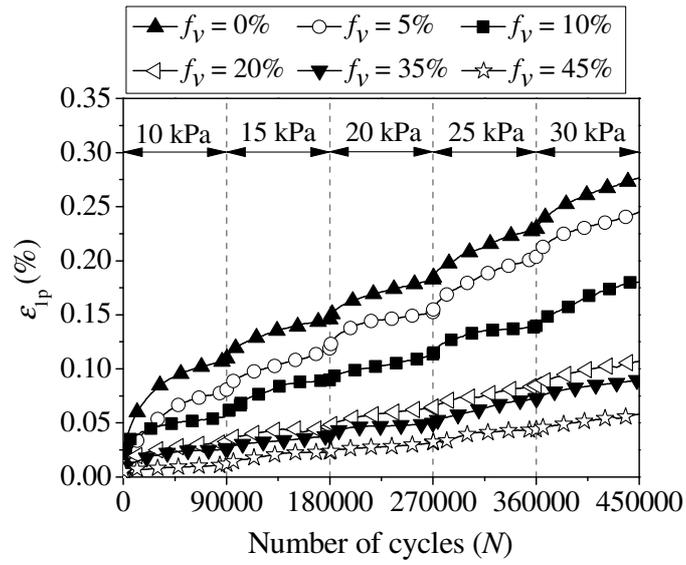


Fig. 6. Permanent strain evolutions with number of cycles at different  $f_v$  values  
(ballast specimens)

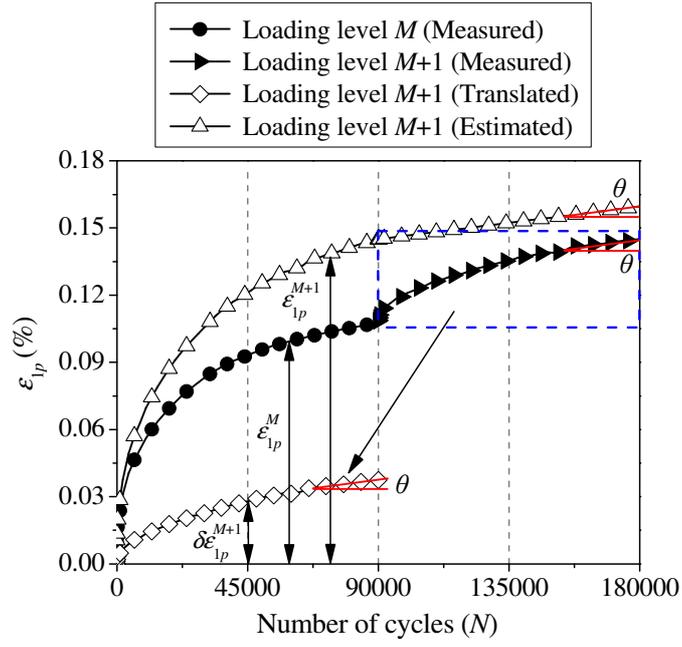


Fig. 7. Illustration of the method to eliminate the loading history effect

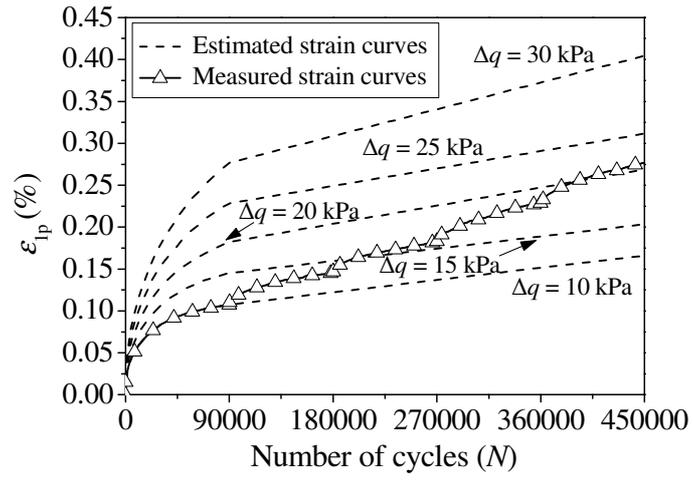


Fig. 8. Measured and estimated permanent strain curves for ballast specimen at  $f_v =$

0%

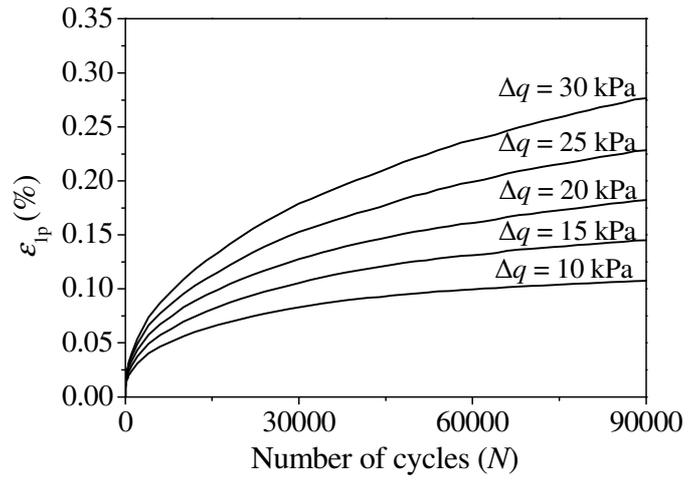


Fig. 9. Estimated permanent strain evolutions at the first 90,000 cycles for ballast specimen at  $f_v = 0\%$

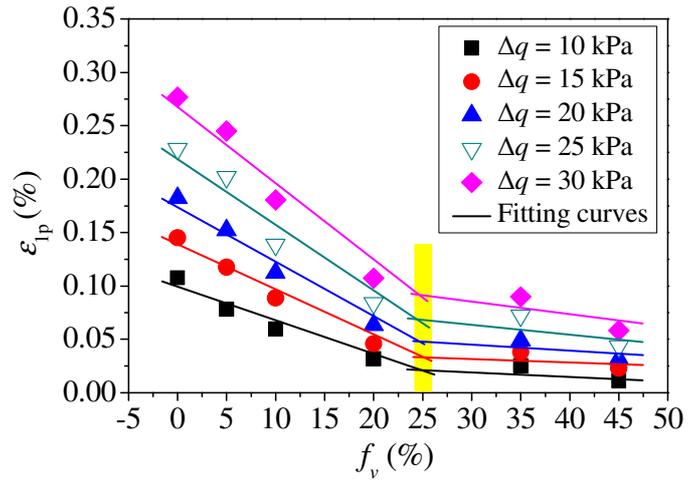


Fig. 10. Variations of estimated end-stage permanent strains with  $f_v$  at different stress levels (ballast specimens)

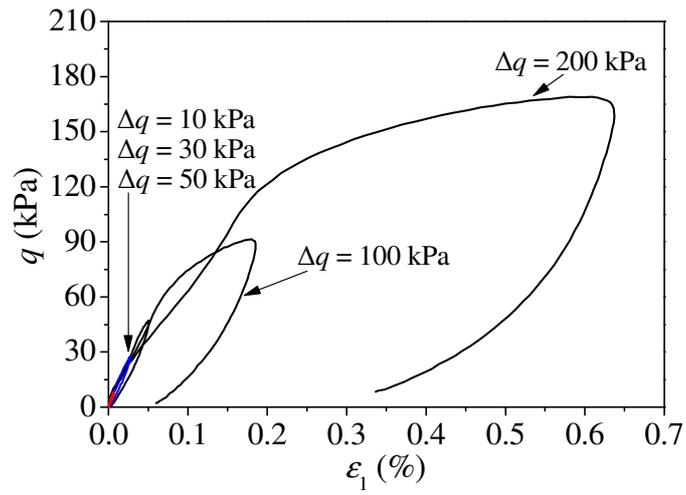


Fig. 11. Hysteresis loops at the first cycles of different stress levels for ballast specimen at  $f_v = 20\%$ :  $\Delta q = 10$  kPa ( $N = 1$ ),  $\Delta q = 30$  kPa ( $N = 101$ ),  $\Delta q = 50$  kPa ( $N = 201$ ),  $\Delta q = 100$  kPa ( $N = 701$ ) and  $\Delta q = 200$  kPa ( $N = 1401$ )

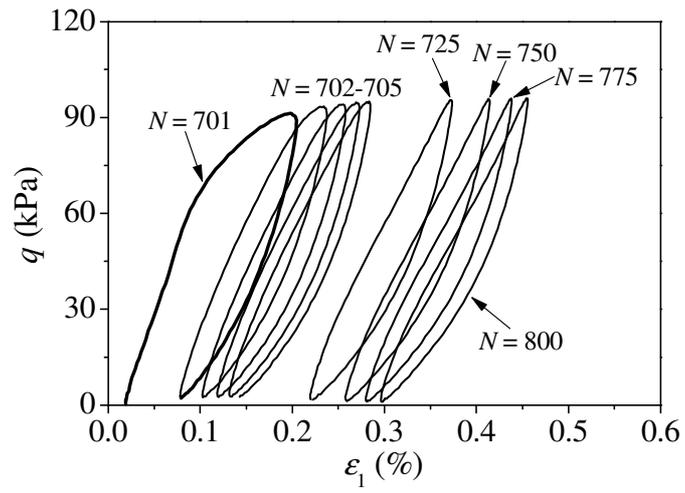


Fig. 12. Hysteresis loops during 100 cycles ( $N = 701$  to 800) under  $\Delta q = 100$  kPa for ballast specimen at  $f_v = 20\%$

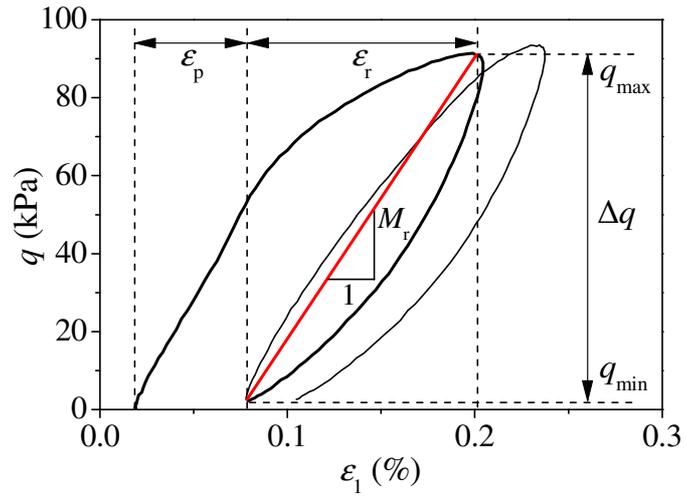


Fig. 13. Determination of  $M_r$

Note: The two hysteresis loops shown are the testing results of ballast specimen at  $f_v = 20\%$  under  $\Delta q = 100$  kPa at cycle numbers of  $N = 701$  and  $702$ .

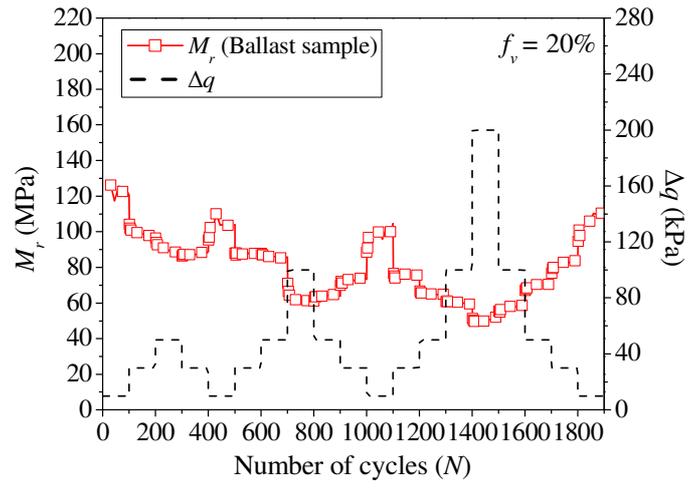


Fig. 14. Evolution of  $M_r$  with number of cycles at different stress levels for ballast specimen at  $f_v = 20\%$

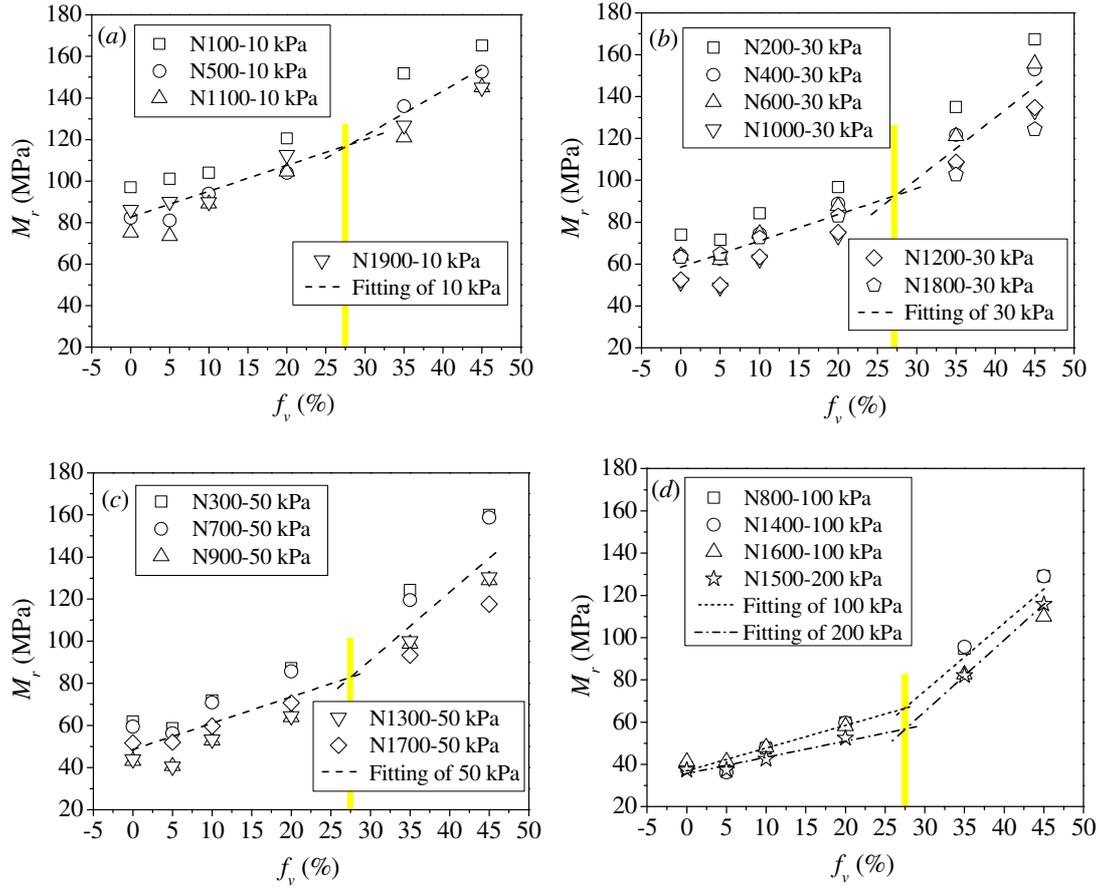


Fig. 15. Evolutions of  $M_r$  at end stage with  $f_v$  for ballast specimens at: (a)  $\Delta q = 10$  kPa;

(b)  $\Delta q = 30$  kPa; (c)  $\Delta q = 50$  kPa; (d)  $\Delta q = 100$  kPa and 200 kPa

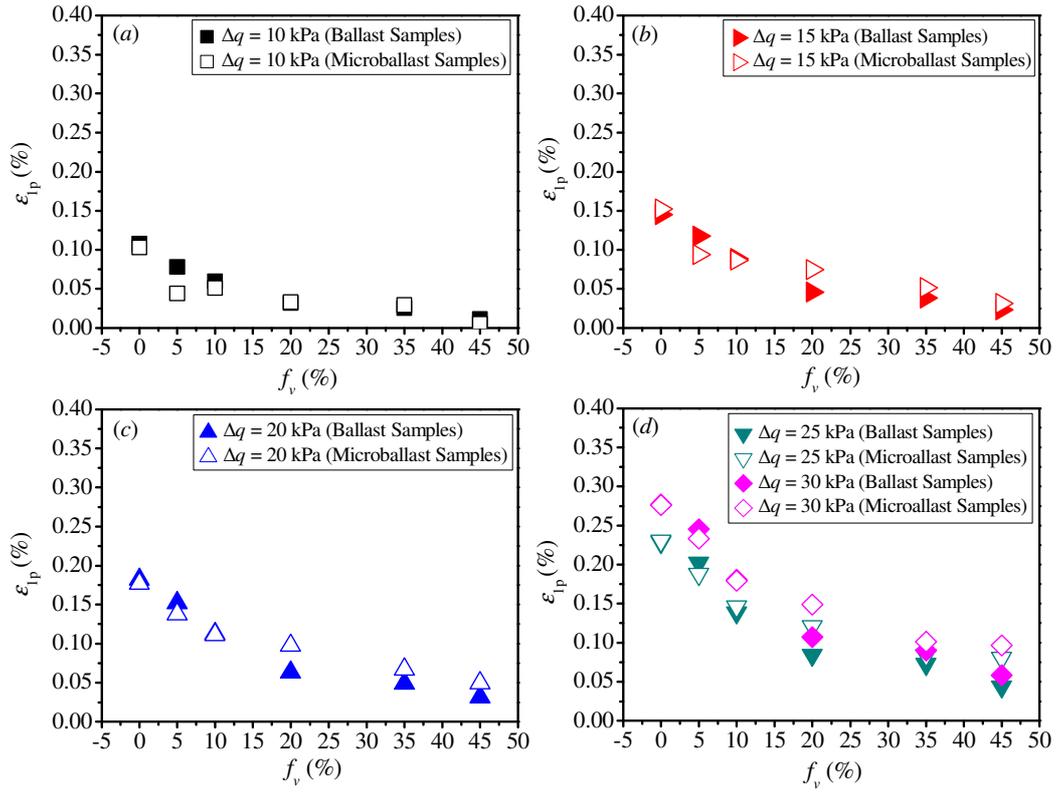


Fig. 16. Variations of estimated end-stage permanent strains with  $f_v$  for ballast and microballast specimens at: (a)  $\Delta q = 10$  kPa; (b)  $\Delta q = 15$  kPa; (c)  $\Delta q = 20$  kPa; (d)  $\Delta q = 25$  kPa and  $30$  kPa

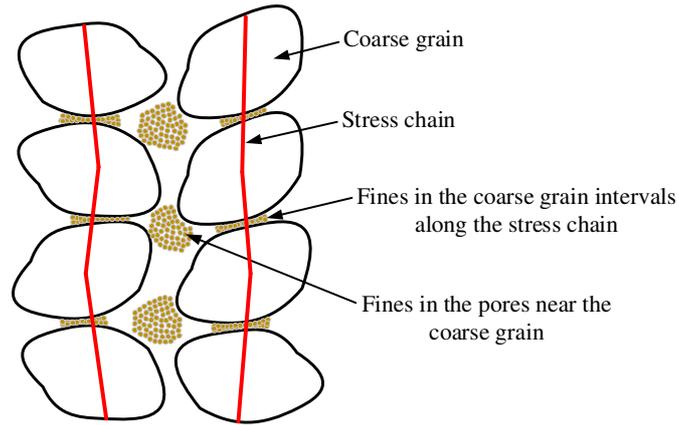


Fig. 17. Schematic illustration of the distribution of fines soils in case of grain-grain contact structure

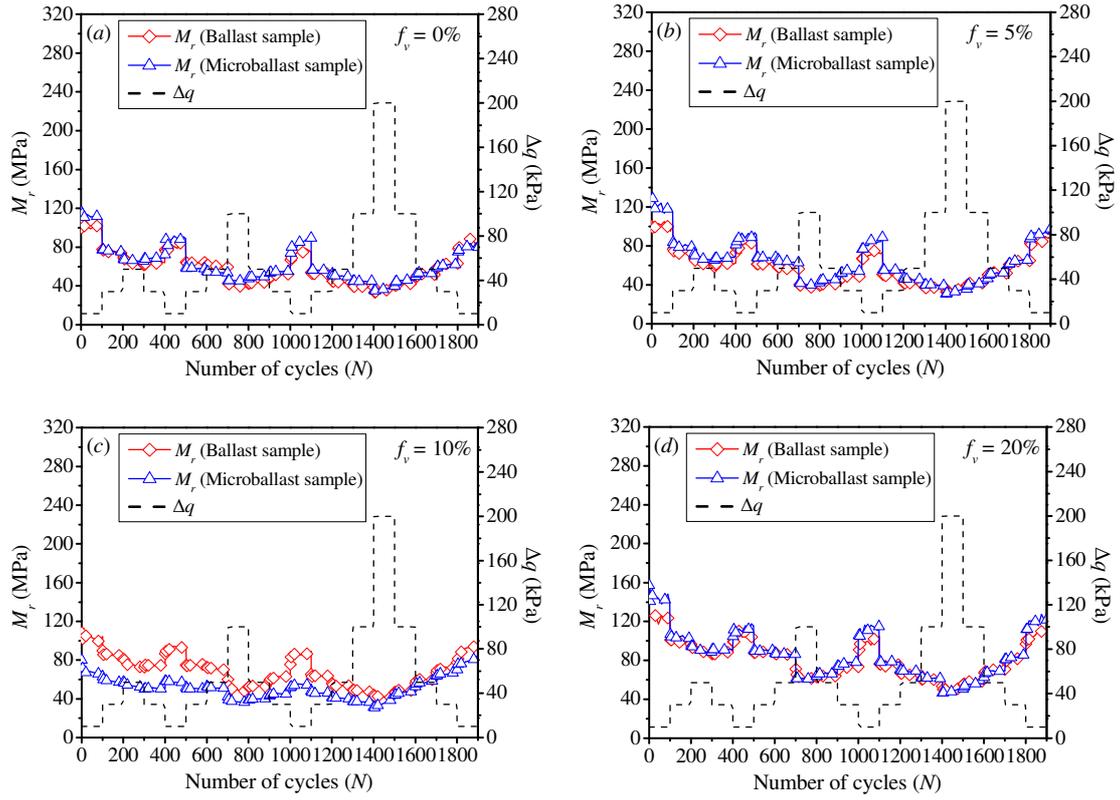


Fig. 18. Comparison of  $M_r$  value between ballast specimens and microballast specimens in case of fine-fine contact structure for: (a)  $f_v = 0\%$ ; (b)  $f_v = 5\%$ ; (c)  $f_v = 10\%$ ; (d)  $f_v = 20\%$

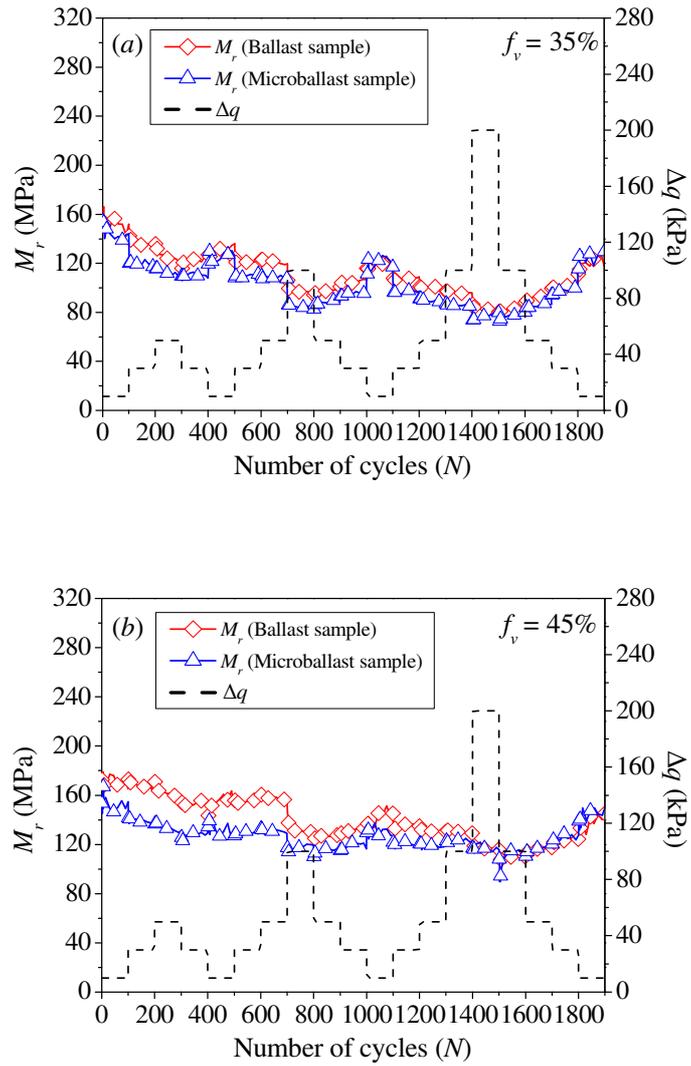


Fig. 19. Comparison of  $M_r$  value between ballast specimens and microballast specimens in case of grain-grain contact structure for (a)  $f_v = 35\%$  and (b)  $f_v = 45\%$

Table 1. Mass proportions and grain size ranges of nine commercial soils

---

0.001 - 0.01 (20% of the particles)

0.0003 - 0.01 (80% of the particles)

0.0009 - 0.25

0.0009 - 0.50

0.063 - 0.50

0.16 - 0.63

0.25 - 1

0.32 - 2

0.32 - 3.20

---