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Atmospheric dispersion using a Lagrangian stochastic approach: application to an idealized urban area under neutral and stable meteorological conditions

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8 Abstract

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We present an adaptation of the Lagrangian stochastic dispersion model of the com-9 putational fluid dynamics (CFD) open source code Code_Saturne to simulate atmospheric 10 dispersion of pollutants in complex urban geometries or around industrial plants. The wind 11 is modeled within the same code with an Eulerian RANS (Reynolds-averaged Navier-Stokes 12 equations) approach and thus involves the solution for the ensemble-mean velocity field and 13 turbulent moments, using eddy viscosity or Reynolds stress turbulence models adapted to 14 the atmosphere and complex geometries. The Lagrangian stochastic model used for the 15 dispersion of the particles within this flow field is the simplified Langevin model, which 16 pertains to the approaches referred to as PDF (Probability Density Function) methods. 17 This formulation of model has not been widely used in atmospheric applications, despite 18 interesting theoretical and computational benefits. Therefore, its usage must be validated 19 on different atmospheric cases. In this paper, we present the validation of the model with 20 a field experiment, considering atmospheric stratification and buildings: the MUST (Mock 21 Urban Setting Test) campaign, conducted in Utah's desert, USA. 22

Keywords: Atmospheric dispersion, Lagrangian models, Eulerian models, Wind flow
 modeling, MUST experiment

25 1. Introduction

An atmospheric dispersion model is a tool that can be used to simulate the atmospheric phenomena involved in the turbulent pollutant dispersion process. The differences between the many existing models to date are mainly in terms of the number of atmospheric processes considered, their degree of complexity, their field of application and, in particular, the methods used to solve the equations governing them. We can distinguish mainly three types of models:

• Gaussian models, based on the analytical resolution of the so-called advection-diffusion equation (on a scalar corresponding to the concentration of pollutant) coupled with semi-empirical parameterizations of the main physical phenomena;

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• Eulerian models, based on the resolution of the discretized advection-diffusion equa-35 tion in time and space on a mesh;

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• Lagrangian models, based on the computation of particle trajectories.

Eulerian models, when used through Computational Fluid Dynamics (CFD) methods, 38 rely on the resolution of the advection-diffusion equation of a scalar on a mesh. This 39 equation implies the knowledge of the velocity and turbulent fields. Therefore, the first step 40 is the resolution of the Eulerian Navier-Stokes momentum equation, in order to compute 41 the flow field over which the dispersion will be calculated. Generally, the resolution of the 42 Navier-Stokes momentum and scalar transport equations are carried out within the same 43 model. This therefore supposes to have a model that provides a solution for the dynamical 44 fields of high enough quality, since it strongly influences the solution obtained for the 45 concentration field calculated through the advection-diffusion equation. In particular, this 46 highlights the crucial importance of a correct modeling of flow turbulence. This topic 47 has been addressed in various papers as atmospheric simulations using CFD have been 48 increasingly used in the past few decades (Franke et al., 2004; Blocken, 2014), especially 49 at local scale (with, for instance, studies of wind flow patterns at pedestrian level around 50 buildings, see Yoshie et al. (2007); Mochida et al. (2008); Tominaga et al. (2008); Blocken 51 and Stathopoulos (2013)). 52

On the other hand, Lagrangian models consist of calculating and following the trajec-53 tories of particles in a turbulent flow. Thus, the frame of reference is not fixed but follows 54 the cloud described by a large number of particles emitted into the atmosphere. For each 55 of these particles, a stochastic differential equation of a Langevin type is written on their 56 velocity. By integrating over time, we are thus able to get the position of each and then 57 deduce the concentration field over the computational domain. The main strength of La-58 grangian models is that they treat convection without any approximation. In particular, 59 they can treat without approximation local source terms when they are provided as known 60 expressions of the variables associated with the particles, such as chemical source terms 61 (Minier, 2015). Also, Lagrangian models are grid-free, which makes them accurate to cap-62 ture the different turbulent structures in a statistical sense and avoid numerical diffusion 63 problems that can be encountered within Eulerian models – especially near the source. 64 However, one must keep in mind that they usually still depend on a grid in two ways: 65

• the stochastic differential equation that governs the velocity evolution of the particles 66 usually involves fluid mean quantities that are in practice provided by a grid-based 67 meteorological pre-processor or a CFD calculation; 68

• the concept of 'concentration' is by definition mesh-based. 69

Both CFD Eulerian and Lagrangian models are well-suited for atmospheric dispersion 70 studies in urban neighborhoods or around industrial plants, in the sense that they are 71 capable of capturing the complex interactions between the air flow and the buildings for 72 different meteorological conditions. However, these two types of models have often been 73

compared ignoring the level of turbulence closure used for each. Loosely speaking, a com-74 mon belief is that 'Eulerian models do not work well near the source'. In reality, a more 75 correct affirmation would be: 'Eulerian models that are based on a gradient-diffusion hy-76 pothesis do not work well near the source', since this region corresponds to the short-time 77 limit where the fully diffusive regime has not been reached yet (Taylor, 1921). In fact, in the 78 atmospheric dispersion field, Lagrangian models are commonly referred to models simulat-79 ing the particle **velocities** as stochastic diffusion processes, which by construction makes 80 them second-order. On the other hand, a Lagrangian model simulating the **positions** as 81 stochastic diffusion processes is equivalent to an Eulerian model using a gradient-diffusion 82 hypothesis. More details can be found on that subject in Minier (2016). To sum up, the 83 Eulerian/Lagrangian comparisons should not be about the approaches in themselves but 84 rather about the level of closure that is considered. Throughout this paper, we will be 85 coming back on this point of significant importance when it comes to rigorously comparing 86 the accuracy of the results given by both approaches. 87

In the past few years, the constantly increasing computing power has enabled (and 88 above all made easier) the use of Lagrangian stochastic methods for atmospheric pur-89 poses – see for example Franzese (2003), Stohl et al. (2005), Cassiani et al. (2005a,b), 90 Bernardin et al. (2009), Alessandrini and Ferrero (2009), Tinarelli et al. (2013), etc. For 91 our work, a simulation tool using a Lagrangian PDF (Probability Density Function) to 92 carry out pollutant dispersion studies has been developed in the three-dimensional CFD 93 code Code_Saturne (http://code-saturne.org/, see Archambeau et al. (2004) for more 94 details). It has been validated on simple academic cases and shown to satisfactorily re-95 spect the well-mixed condition (see Bahlali et al. (2018a,b)). This paper is concerned with 96 validating the model in real conditions by studying continuous point source dispersion of 97 a non-reactive pollutant in an idealized urban area, as such a case is typical of industrial 98 emissions or an accidental release. 99

The present work will focus on the Mock Urban Setting Test (MUST) campaign, which 100 has been widely studied in the literature. For previous numerical simulations of the ex-101 periment, the reader may for example refer to Hanna et al. (2004), Camelli et al. (2005), 102 Donnelly et al. (2009), Antonioni et al. (2012), Kumar et al. (2015). Comparisons between 103 different modeling systems have also been studied: Santiago et al. (2010) and Dejoan 104 et al. (2010) compared large-eddy simulations (LES) to Reynolds-averaged Navier-Stokes 105 (RANS) computations, and Castelli et al. (2017) performed comparisons between different 106 atmospheric Eulerian and Lagrangian modeling approaches. Experimental comparisons 107 have been conducted as well: Leitl et al. (2007) worked on the Hamburg wind tunnel 108 experiment, and Yee et al. (2006) compared experimental wind-tunnel and water-channel 109 simulations. 110

In the CFD code *Code_Saturne*, previous numerical simulations of the MUST campaign have also already been performed by Milliez and Carissimo (2007, 2008) and used an Eulerian $k - \epsilon$ model of turbulent dispersion. The present work aims at studying the same cases using the Lagrangian stochastic model. The objective is twofold: • first, we would like to assess the accuracy of the results provided by the Lagrangian model for this specific industrial application;

second, we would like, in the same CFD simulations, to compare the Lagrangian results to the ones obtained with several Eulerian turbulence models, and above all explain the differences to provide a better understanding of the different modeling options.

A specificity of this work is that the wind is modeled within the same code as the 121 dispersion, with an Eulerian RANS approach. It thus involves the solution for the ensemble-122 mean velocity field and turbulent moments, using $k - \epsilon$ or second-moment $R_{ij} - \epsilon$ turbulence 123 closures adapted to the atmosphere and complex geometries. In the aforementioned nota-124 tions, k is the turbulent kinetic energy (TKE), ϵ the turbulent dissipation rate and R_{ij} the 125 **Reynolds stress tensor**. Note that Milliez and Carissimo (2007, 2008) only simulated the 126 wind dynamical mean fields using a $k - \epsilon$ model. Therefore, in addition to the Lagrangian 127 results, this paper will also expose new results on the Eulerian approach through the use 128 of the second-order $R_{ij} - \epsilon$ model for the wind mean quantities. 129

This paper will be organized as follows. First, both the Eulerian and Lagrangian model equations used in this work will be introduced. Second, the experiment characteristics will be exposed and the two cases chosen for this study will be described. Finally, results for both these cases will be shown and the different modeling approaches will be discussed.

¹³⁴ 2. Model equations

¹³⁵ The methodology for atmospheric dispersion calculations in *Code_Saturne* consists of ¹³⁶ two simulations:

• the first simulation solves the mean Navier-Stokes equations for the flow field;

the second simulation restarts from the previous frozen flow field (velocity, turbulence and temperature) and computes the dispersion.

In the following subsections, the different modeling options in Eulerian and Lagrangian
 approaches are presented.

142 2.1. The Eulerian approach

Eulerian models, as explained in the introduction, are based on the resolution of the mean advection-diffusion equation of a given Reynolds-averaged scalar $\langle c \rangle$ through its discretization in time and space on a mesh. This equation is written as:

$$\frac{\partial \langle c \rangle}{\partial t} + \langle U_{f,j} \rangle \frac{\partial \langle c \rangle}{\partial x_j} = \frac{\partial}{\partial x_j} \left(D \frac{\partial \langle c \rangle}{\partial x_j} - \langle U'_{f,j} c' \rangle \right) + \langle S \rangle + \langle R \rangle , \qquad (1)$$

where $U_{f,j}$ is the fluid velocity along the *j* axis, *D* the molecular diffusivity, *S* and *R* respectively the source and reactive terms. Also, the notation $\langle \rangle$ stands for Reynolds average and ' its deviation part.

This equation involves the unknown term $\langle U'_{f,j}c'\rangle$. For practical applications, two main families of closures are considered:

• eddy diffusivity models, which directly provide a local expression for $\langle U'_{f,i}c' \rangle$;

• second-order models, which consist of a complete transport of the turbulent scalar fluxes $\langle U'_{f,i}c' \rangle$.

¹⁵⁴ Eddy diffusivity models are widely used in the atmospheric dispersion literature, usually ¹⁵⁵ through the following simple gradient-diffusion hypothesis:

$$\langle U'_{f,j}c'\rangle = -D_t \frac{\partial \langle c\rangle}{\partial x_j} , \qquad (2)$$

where $D_t = \nu_t / Sc_t$, Sc_t being the turbulent Schmidt number, usually ranging between 0.7 and 1 for air.

In this expression, $\nu_t = C_{\mu}k^2/\epsilon$ is the fluid turbulent viscosity, $k = (1/2) R_{ii}$ where by definition $R_{ij} = \langle U'_{f,i}U'_{f,j} \rangle$. In $k - \epsilon$ models, R_{ij} is constructed as follows:

$$R_{ij} = (2/3) \ k - 2 \ \nu_t \ S_{ij} \tag{3}$$

where $S_{ij} = (\partial U'_{f,i}/\partial x_j + \partial U'_{f,j}/\partial x_i)/2$, so that R_{ij} is symmetric and $\operatorname{Tr}(R_{ij}) = 2k$.

An important point to recall here is that if a turbulent-viscosity model is used for the 161 resolution of the mean dynamical fields, then obviously, closure of $\langle U'_{f,i}c'\rangle$ will be performed 162 through a 'turbulent-diffusivity' model. On the other hand, one can use a second-order 163 $R_{ij} - \epsilon$ model to compute the mean dynamical fields and still use a turbulent-diffusivity 164 model for the turbulent scalar fluxes closure (*i.e.*, here, the model of Eq. (2)). The present 165 work will provide new elements on that topic, by comparing, using either a $k - \epsilon$ or a 166 $R_{ij} - \epsilon$ model to compute the mean dynamical fields, the results obtained through a tur-167 bulent-diffusivity model for the scalar dispersion. As for second-order scalar fluctuations 168 models for meteorological applications, they are in fact still an open and difficult research 169 problem, and the subject of further investigations. 170

171 2.2. The Lagrangian approach

Let \mathbf{X}_p be the position of a particle included in the air flow and \mathbf{U}_p its velocity. Both variables are driven by the following system:

$$dX_{p,i} = U_{p,i}(t)dt , \qquad (4a)$$

$$dU_{p,i} = -\frac{1}{\rho} \frac{\partial \langle P \rangle}{\partial x_i} dt + (\langle U_{p,i} \rangle - \langle U_{f,i} \rangle) \frac{\partial \langle U_{f,i} \rangle}{\partial x_i} dt - \frac{U_{p,i} - \langle U_{f,i} \rangle}{T_L} dt + \sqrt{C_0 \epsilon} dW_j , \quad (4b)$$

where dW_i are independent Wiener processes. Briefly speaking, a Wiener process is a Gaus-174 sian process with independent increments of zero mean and with a variance equal to dt175 (more details can be found in Gardiner (1985); Öttinger (1996)). Also, $T_L = \frac{1}{\frac{1}{2} + \frac{3}{4}C_0} \frac{k}{\epsilon}$ is 176 the Lagrangian integral timescale and C_0 is a constant given by Kolmogorov's theory. 177 In Eq. (4b), the sum of the first, second and third terms on the right-hand-side is 178 known as the drift term. More precisely, the drift term is the sum of a mean component 179 (the mean pressure gradient $-\frac{1}{\rho} \frac{\partial \langle P \rangle}{\partial x_i} dt$ and the term accounting for the crossing-trajectory 180 effect $\langle U_{p,i} \rangle - \langle U_{f,i} \rangle \frac{\partial \langle U_{f,i} \rangle}{\partial x_i} dt$, more detailed in the following paragraph) and a fluctuating 181 component (the return-to-equilibrium term $-\frac{U_{p,i}-\langle U_{f,i}\rangle}{T_L}dt$). On the other hand, the fourth 182 term on the right-hand-side of Eq. (4b) is known as the diffusion term and is defined 183 following Kolmogorov's similarity theory. 184

The stochastic differential equation governing the evolution of \mathbf{U}_p is a model inspired by 185 the two-phase flow formulation of Minier and Peirano (2001) and the Simplified Langevin 186 Model (SLM) of Pope (2000) – it actually stands between the two of them, the difference 187 lying in the $(\langle U_{p,i} \rangle - \langle U_{f,i} \rangle) \frac{\partial \langle U_{f,i} \rangle}{\partial x_i} dt$ term. The formulation with this term makes sense since even though we are dealing with fluid particles (putting us in the single-phase flow 188 189 situation), particles dispersing from a point source can be seen as a subset of the whole 190 simulated flow. Therefore, in each cell of the computational domain, their mean velocity 191 has no reason to be equal to the fluid velocity, hence the non-null production term on the 192 second term of the right-hand-side of Eq. (4b). In fact, for a better understanding, let us 193 assume that the whole flow is represented by particles uniformly distributed in the domain 194 and affect a scalar α to each of them: 195

¹⁹⁶
$$\begin{cases} \alpha = 1 \text{ if the particle comes from the source;} \\ \alpha = 0 \text{ if it does not come from the source.} \end{cases}$$

¹⁹⁷ Therefore, the condition that the mean particle velocity field needs to observe is: ¹⁹⁸ $\operatorname{div}(\alpha \langle \mathbf{U}_p \rangle) = \mathbf{0}$, which is a completely different condition than the one the fluid velocity ¹⁹⁹ field has to meet, *i.e.*, $\operatorname{div}(\langle \mathbf{U}_f \rangle) = \mathbf{0}$.

Loosely speaking, this formulation stands in a philosophical line that is close to the 200 LRR-IP (Launder, Reece, Rodi - Isotropization of Production) model of Pope (2000), 201 except that the production term here is related to the mean particle velocity instead of the 202 instantaneous one. This production term actually makes significant physical sense, since 203 it adds more anisotropy to the dispersion of the particles. Furthermore, if one studies the 204 limiting case of particles modeling the whole flow (i.e., the 'fluid limit'), then it yields 205 $\langle U_{p,i} \rangle = \langle U_{f,i} \rangle$ and the SLM is retrieved. More precisely, Bahlali et al. (2018b) have shown 206 that the SLM fully respects the well-mixed criterion (as defined in Thomson (1987)) and 207 that it is completely consistent with a second-order $R_{ij} - \epsilon$ (Rotta) turbulence model for the 208 fluid phase. Both these conditions are in fact essential for any Lagrangian stochastic model 209 of a Langevin type to be regarded as acceptable (see Minier et al. (2014)). In conclusion, 210 the model defined in Eq. (4), since it relaxes to the SLM for the fluid limit, is well-mixed 211

and able to reproduce transport equations for the first two moments of the velocity field: it can thus reasonably be used to simulate point source dispersion in non-homogeneous flows such as the one studied in this work. However, one must keep in mind that this model has been developed for neutral conditions. It is possible to use it also for buoyancy-driven flows, as we did in the present work, but some improvements can be made for these cases and are the subject of further investigations (cf. conclusion).

It should be noted that this Lagrangian model alone does not take into account molecu-218 lar diffusion. Indeed, if a scalar c is added to the state vector associated with each particle, 219 then: dc/dt = 0 (conservative particles are studied, with a constant concentration along 220 their trajectories). In order to represent the molecular diffusion phenomenon, a so-called 221 'micro-mixing model' can be used (see, for example, Villermaux and Devillon (1972); Pope 222 (2000); Sawford (2004); Luhar and Sawford (2005); Amicarelli et al. (2012); Cassiani et al. 223 (2015)). This type of model is often used in the case of reactive pollutants, since in this 224 context molecular diffusion plays an important role. In the case of high Reynolds numbers, 225 molecular diffusion does not affect the concentration mean (Pope, 1998), and is therefore 226 generally neglected compared to turbulent diffusion. 227

228 3. The Mock Urban Setting Test experiment

229 3.1. Description of the site

The experimental program this paper focuses on is the Mock Urban Setting Test campaign, conducted in Utah's desert, USA, by the US Defense Threat Reduction Agency (DTRA). It consists of the release of a pollutant in an idealized urban environment represented by several rows of containers. The details of the experiment as well as the results are described in Biltoft (2001) and Yee and Biltoft (2004).

It is worth noting that pollutant dispersion simulations in urban geometries using CFD 235 have been widely studied in the past few years, for instance within street canyons (Tom-236 inaga and Stathopoulos, 2011; Salim et al., 2011) or around high-rise buildings (Yoshie 237 et al., 2011). In this work, the objective is to reproduce some of the simulations of the 238 MUST campaign that have already been performed in the past by Milliez and Carissimo 239 (2007, 2008) in Mercure (former name of the atmospheric module of Code_Saturne), but 240 completing them with new results using the Lagrangian stochastic model previously in-241 troduced and the second-order $R_{ij} - \epsilon$ model for the fluid phase computation. Milliez 242 and Carissimo (2007) described in detail the characteristics of the campaign, which are 243 summarized below: 244

• 63 emissions of a neutral gas (propylene C_3H_6), among which 58 are continuous and 5 are puff releases;

- flat terrain with some bushes from 50 cm to 1 m high;
- presence of obstacles through a regular alignment of containers of dimensions 12.2 m (length) × 2.42 m (width) × 2.54 m (height);



Figure 1: Some views of the MUST experiment (after Milliez (2006)).

- different wind conditions through varying angles of incidence, wind velocities, turbulence, temperature, stability conditions;
- different release heights: 0.15 m, 1.3 m, 1.8 m, 2.6 m and 5.2 m.

Figure 1 shows two photos of the MUST experiment. The regular alignment of the 253 containers is supposed to represent an idealized city and the objective of the experiment 254 is to observe the point source dispersion of the propylene continuously released in this 255 environment. We are interested in checking how the Lagrangian model behaves in the 256 presence of obstacles in a real situation. Indeed, when assessing the respect of the well-257 mixed criterion for the SLM, Bahlali et al. (2018b) studied the case of a non-homogeneous 258 turbulent flow around an obstacle within a boundary layer and observed notable differences 259 in the behavior of the particles depending on the turbulence model that was used to 260 compute the fluid phase. 261

The experimental devices used in the campaign are illustrated in Figure 2. Wind and temperature measurements were carried out using sonic anemometers (30-m mast S just upstream of the canopy, 32-m central tower T and 8-m masts A, B, C and D inside the canopy). As for pollutant concentrations, they were measured by photoionization detectors (PIDs), positioned on four horizontal lines (*'line 1, 2, 3, 4'* in Figure 2). These four lines were located at height 1.6 m. PIDs were also placed at 6 levels on the masts A, B, C, D and at 8 levels on the tower T.

Finally, it should be noted that the tracer releases occured at dusk or dawn, thus under meteorological conditions ranging from stable to neutral. The duration of each release was 15 min and for the analysis of the results, periods of 200 s were extracted by Yee and Biltoft (2004). These periods were indeed quasi-steady in terms of wind speed and direction and also remained greater than the plume travel time.

274 3.2. Description of the cases studied

As previously mentioned, the MUST experiment consisted of 63 tracer releases and in the work of Milliez and Carissimo (2007, 2008), twenty cases were simulated. In the present



Figure 2: Schematic diagram of the MUST experiment and location of the experimental devices (after Milliez and Carissimo (2008)).

2681829 -41 7.93 4.263 28000 225 29 2602157 43 2.08 0.510 130 225 29	$ \begin{array}{cccc} 29 & 1.8 \\ 36 & 2.6 \end{array} $

Table 1: Characteristics of the two selected trials: S_4 and α_4 are respectively the mean wind horizontal speed and direction at the 4-m level of mast S, k_4 is the turbulent kinetic energy and L the Monin-Obukhov length at the 4-m level of tower T, Q is the tracer release rate at the source, 'Source location' is the position of the source and z_s is the height of the source (after Milliez and Carissimo (2007)).

paper, trials 2681829 and 2092157 are studied, respectively corresponding to situations of neutral and stable atmospheres. The characteristics of these trials are summarized in Table 1, and the characteristics of all cases can be found in Milliez and Carissimo (2007).

²⁸⁰ More precisely, the MUST simulations made in different meteorological conditions by ²⁸¹ Milliez and Carissimo (2007) have been reused. These simulations only used the Eulerian ²⁸² approach with a $k - \epsilon$ turbulence closure. In this paper, new $R_{ij} - \epsilon$ Eulerian and La-²⁸³ grangian dispersion simulations are provided within the same calculations so that they can ²⁸⁴ be compared with the same Eulerian dynamical field, including the stratification effects. To ²⁸⁵ take into account these effects, the code solves for potential temperature, as fully explained ²⁸⁶ in Milliez and Carissimo (2007)).

287



Figure 3: Mesh of the computational domain. From left to right and top to bottom: general view, horizontal cross-section, vertical cross-section.

288 4. Numerical simulations

In this section, the methodology and results of the numerical simulations of trials 289 2681829 and 2692157 in *Code_Saturne* are presented.

291 4.1. Simulation domain and mesh

The simulation domain is of dimensions 240 m (North-South) \times 240 m (East-West) \times 100 m (vertical direction). The corresponding mesh is displayed in Figure 3. It is refined near the ground and the obstacles, the horizontal resolution varying from 4 m to 0.6 m. The vertical resolution of the mesh increases gradually from 0.2 m near the ground until it reaches 4 m at the top of the domain. In total, the mesh contains 1 426 010 cells.

297 4.2. Numerical setup

298 4.2.1. Fluid phase

For the flow, the models $k - \epsilon$ et $R_{ij} - \epsilon$ (with simple gradient-diffusion hypothesis for the scalars) are used and will be further compared. The imposed boundary conditions are as follows:

• Inlet condition of Dirichlet type, with dynamical profiles derived from experimental measurements. Indeed, it has been shown in Milliez and Carissimo (2007) that the use of analytical profiles in equilibrium can induce an underestimation of turbulence. They found better results using experimental profiles (when available), which is why we have chosen to follow the same road. More precisely, the TKE profiles are estimated from the available measurements and then the turbulent dissipation rate profiles are deduced from the relations from Monin-Obukhov theory as given in Duynkerke (1988) and reused in Milliez and Carissimo (2007):

$$k = \frac{u_L^2}{\sqrt{C_u}},\tag{5a}$$

$$\epsilon = \frac{u_L^3}{\kappa} \left(\frac{1}{z} + \frac{4}{L} \right), \tag{5b}$$

where u_L is the local stress equal to $u_*(1-z/h)$ where h is the height of the boundary 310 layer (Stull, 1988) and u_* the surface stress. Also, κ is the von Karman constant and 311 L is the Obukhov length. 312 In the case of the $R_{ij} - \epsilon$ model, the R_{ij} tensor at the boundary is given as follows: 313 the diagonal terms are constructed isotropically from the turbulent kinetic energy 314 and the other terms are taken as zero. These are reasonable hypotheses that can be 315 expected in operational use (and similar for both $k - \epsilon$ and $R_{ij} - \epsilon$ models). We have 316 verified elsewhere that the model rapidly adjusts to a more realistic R_{ij} tensor, well 317 before reaching the obstacles. 318

• Outlet condition: free outflow.

• Ground and containers: rough wall, with a roughness length of 0.04 m. A logarithmic wall function is used. A Dirichlet condition is applied for k, such that $k = u_*^2/\sqrt{C_{\mu}}$ except in the viscous sub-layer where k = 0. A Neumann condition is applied for ϵ , such that $\epsilon = u_*^3/(\kappa z)$, except in the viscous sublayer where a zero-flux is imposed.

324 4.2.2. Dispersed phase

With the Eulerian approach, the pollutant is injected through a scalar source term. At the injection cell, a pollutant flow rate of 225 L/min has been imposed for both neutral 2681829 and stable 2692157 trials (cf. Table 1). With the Lagrangian approach, at the same injection cell, 2000 particles per time step have been injected, with the same flow rate.

330 4.3. Results for neutral trial 2681829

331 4.3.1. Simulation of the fluid phase

As exposed in the previous section, two simulations of the fluid phase have been performed, corresponding respectively to the use of the $k - \epsilon$ or second-order $R_{ij} - \epsilon$ turbulence closures. Figure 4 shows the mean velocity and TKE fields at height z = 4 m for both

turbulence models. This height stands above the roofs $(z_{roofs} = 2.54 \text{ m})$ and we can ob-335 serve the influence of the row of containers on the flow. Naturally, a slowdown of the flow 336 and an increase in turbulent kinetic energy can be observed. It can also be seen that $k - \epsilon$ 337 model tends to predict higher levels of turbulent kinetic energy upstream of the obstacles, 338 which is a well-known result in turbulence modeling. However, when comparing to the 339 measurements, it can be seen that the TKE values are always underestimated, with both 340 $k - \epsilon$ and $R_{ii} - \epsilon$ models. Figure 5 shows the wind field in the source area, at 1.6 m height: 341 the recirculation zones between the containers are well-captured and explain the previously 342 mentioned decrease of velocity above the roofs at 4 m height. The recirculation zones are 343 also wider and more pronounced with the $R_{ij} - \epsilon$ model. Finally, the mean velocity and 344 TKE vertical profiles extracted from the masts have been plotted in Figure 6 when data 345 was available. Once again, turbulence production is more pronounced with $k - \epsilon$ model. 346 Velocity profiles are on the other hand not much affected by the turbulence model (as is 347 also seen in the velocity magnitude field of Figure 4) and stand in good agreement with 348 the measurements. 349

350 4.3.2. Simulation of the dispersion

Figure 7 shows the concentration fields at height 1.6 m, for both $k - \epsilon$ and $R_{ij} - \epsilon$ models. Whether it be through the Eulerian or the Lagrangian approach, it can be observed that the choice of the turbulence model plays an important role on the concentration patterns. In particular, it can be seen that the plume is wider using the $R_{ij} - \epsilon$ model.

One can also observe that there is a deflection of the plume centerline compared to 355 the -41° wind direction. Note also that the plume deflection is more pronounced when 356 using the $R_{ij} - \epsilon$ model, for both Eulerian and Lagrangian approaches. This deviation 357 phenomenon has also been observed in Carissimo and Macdonald (2004) and Milliez and 358 Carissimo (2007). Milliez and Carissimo (2007) explained it by the fact the pollutant is 359 channeled into the streets perpendicular to the obstacle array axis, as is also observed in 360 the experiment. Castelli et al. (2017), who also performed the same trial Eulerian $k - \epsilon$ 361 and Lagrangian calculations within their models RAMS6.0-mod and MicroRMS, noticed 362 a lesser pronounced deflection with the Lagrangian modeling. They partly explained it by 363 pointing out that their Lagrangian code did not account for the cross-correlation terms 364 between the different components of wind velocity fluctuations. This is not the case with 365 the model used in the present work since it is based on Pope's SLM, which implies that 366 the cross-correlations of the wind velocity fluctuations are included in the mean-pressure 367 gradient term of Eq. (4b). This is actually one of the main advantages of this formulation, 368 see Bahlali et al. (2018b) for more theoretical details. This may be why, when roughly 369 comparing the concentration field obtained by Castelli et al. (2017) to the one displayed 370 in Figure 7, a more pronounced deflection is found with the SLM than in Castelli et al. 371 (2017)'s results. 372

For a more precise analysis of the concentration field, the concentration horizontal profiles on lines 1, 2, 3, 4 are shown in Figure 8, and the vertical profiles on masts B, C, D and tower T are displayed in Figure 9. It can be observed that, in particular, the lines 1 and



Figure 4: Comparison of mean velocity and TKE fields at z = 4 m computed by $k - \epsilon$ or $R_{ij} - \epsilon$ turbulence models, for neutral trial 2681829.



Figure 5: Comparison of wind fields around the source computed by $k - \epsilon$ or $R_{ij} - \epsilon$ turbulence models, for neutral trial 2681829.

2 show a shift in the plumes obtained by respectively the $k - \epsilon$ and the $R_{ij} - \epsilon$ models, which 376 again highlights the observed more pronounced deflection with the $R_{ij} - \epsilon$ closure. When 377 using the $R_{ij} - \epsilon$ closure, the effect of the obstacles is clearly visible through the 'steps' 378 corresponding to the regular spacing between the containers. If one compares Eulerian 379 to Lagrangian results, it is interesting to have a look at the concentration evolution from 380 line 1 (closest to the source) to line 4. Both models seem to have approximately the same 381 evolution of diffusion. This is better seen in Figure 10, where the maximum concentration 382 value over each line against the distance from the source has been plotted. It is well-383 known that Langevin-like Lagrangian models show rapid diffusion near the source and 384 then tend to follow the diffusive law in the far-field. On the other hand, Eulerian models 385 based on a simple gradient diffusion hypothesis portray the whole dispersion process by 386 the diffusive law (which is actually a shortcoming of this kind of models since the gradient-387 diffusion hypothesis is no longer valid near the source). One then might wonder why, here 388 in Figure 10, both Eulerian and Lagrangian models seem to show the same evolution of 389 diffusion, independently from the distance to the source. In fact, one important thing to 390 point out here is that line 1 is already located in the so-called 'far-field region'. Indeed, 391 when speaking about near and far fields, it is always in comparison to the value of the 392 Lagrangian integral timescale T_L . For small diffusion times with respect to the value of 393 T_L , diffusion should be evolving proportionally to time, while for higher diffusion times, it 394 should be evolving as square-root of time (Taylor, 1921). In the present case, the value of 395 T_L at the injection cell is 0.93 s, and the velocity norm is 4.5 m.s⁻¹, which yields a 'near-field 396 region' of approximately $0.93 \times 4.5 = 4.2$ m. The far field is thus quickly reached. As the 397 maximum concentration value on line 1 is located at approximately 60 m ($\gg 4.2$ m) from 398 the source, we deduce by extension that all the lines are already in the well-established far-399 field region. In consequence, it is only logical that both the Lagrangian and the Eulerian 400 models show the same evolution of diffusion. 401



Figure 6: Comparison of vertical velocity and TKE profiles computed by $k - \epsilon$ or $R_{ij} - \epsilon$ turbulence models, for neutral trial 2681829.



Figure 7: Comparison of mean concentration (kg/kg) fields at z = 1.6 m computed by both Eulerian and Lagrangian models, through $k - \epsilon$ or $R_{ij} - \epsilon$ turbulence closures, for neutral trial 2681829.



Figure 8: Comparison of concentration profiles on horizontal line samplers computed by both Eulerian and Lagrangian models, through $k - \epsilon$ or $R_{ij} - \epsilon$ turbulence closures, for neutral trial 2681829.

Apart from the previous remarks, and still analyzing the lines, the agreement between simulations and measurements is overall satisfactory and both Eulerian and Lagrangian models provide a quite acceptable representation of the spread of the plume, although the Lagrangian model seems to slightly underestimate the concentrations on all the lines.

If we focus now on vertical profiles (see Figure 9), it can be seen that the use of the 406 $R_{ij} - \epsilon$ model for the fluid phase tends to reduce the maxima of concentrations on mast 407 B (closest to the source), especially with the Lagrangian approach, making the results in 408 better agreement with the measurements. On tower T, the maxima of concentrations are 409 also lower and are due to the fact that the plume deflection is more pronounced using the 410 $R_{ij} - \epsilon$ model for both Eulerian and Lagrangian approaches, making the plume go towards 411 the South direction with more intensity and leaving lower concentration values on tower T. 412 On mast C, the concentrations are also slightly reduced when using the $R_{ij} - \epsilon$ model for 413



Figure 9: Comparison of vertical concentration profiles computed by both Eulerian and Lagrangian models, through $k - \epsilon$ or $R_{ij} - \epsilon$ turbulence closures, for neutral trial 2681829.



Figure 10: Maximum concentration value over each line against the distance from the source, for neutral trial 2681829.

the same reason as for tower T. It is interesting to note that on this mast, the Lagrangian 414 approach captures the change of sign in the vertical gradient of concentration near the 415 ground, while the Eulerian model does not. This is even more marked with the use of the 416 $R_{ij} - \epsilon$ model. Finally, on mast D, concentrations are raised up when using the $R_{ij} - \epsilon$ 417 model, which can also be explained by the plume deflection towards the South direction 418 more pronounced with this model. On this mast, the Lagrangian approach provides better 419 agreement with the measurements. However, note that one must be cautious in giving 420 definite conclusions for masts C and D, since the concentrations are lower and can then 421 imply more significant errors and uncertainties. 422

423 4.4. Results for stable trial 2692157

424 4.4.1. Simulation of the fluid phase

Analogously to neutral trial 2681829, we show in Figure 11 the mean velocity and TKE vertical profiles on masts A, B, C and tower T for both $k - \epsilon$ and $R_{ij} - \epsilon$ turbulence models. Velocities and TKE values are much lower than for trial 2681829, which is typical of stable stratification meteorological conditions. As in case 2681829, $k - \epsilon$ model tends to predict higher values of TKE than does $R_{ij} - \epsilon$ model. Velocity profiles are in good agreement with the measurements. Once again, they are not much affected by the choice of the turbulence model.

432 4.4.2. Simulation of the dispersion

Figure 12 displays the different concentration fields at height 1.6 m. It can be seen that as in trial 2681829, the plume is also wider when $R_{ij} - \epsilon$ model is used. The difference is even more pronounced through the Lagrangian approach. In addition, a plume deflection (compared to the 43° wind direction) can also be observed in both Eulerian and Lagrangian



Figure 11: Comparison of vertical velocity and TKE profiles computed by $k - \epsilon$ or $R_{ij} - \epsilon$ turbulence models, for stable trial 2692157.

results, more significant when using the $R_{ij} - \epsilon$ than the $k - \epsilon$ model for the fluid phase computation.

The horizontal concentration profiles on lines 1, 2, 3 and 4 are shown in Figure 13. It 439 can be seen that the use of the $R_{ij} - \epsilon$ model leads to an overprediction of concentrations 440 on line 1, for both Eulerian and Lagrangian approaches, even more marked on the Eulerian 441 results. Such an overprediction may be due to the fact that TKE is underestimated with 442 the $R_{ij} - \epsilon$ model as seen in Figure 11. Nevertheless, the $R_{ij} - \epsilon$ model shows again the 443 advantage of better capturing the presence of the obstacles through the visible regular 444 concentration 'steps'. Regardless of the turbulence model, one can also notice the strong 445 decrease of concentrations between line 1 and line 2 with both Lagrangian and Eulerian 446 models, less marked when moving to the next lines: the evolution of diffusion induced by 447 both models look similar. In this trial, at the injection cell, $T_L = 0.73$ s and the velocity 448 equals 1.7 m.s⁻¹: in consequence, the region corresponding to the 'near field' corresponds to 449 distances below $0.73 \times 1.7 = 1.2$ m. The far field is thus immediately reached. Finally, one 450 may notice that the larger the distance from the source, the closer Eulerian and Lagragian 451 curves approach each other. Also, the agreement with experimental measurements is much 452 better. 453

The vertical concentration profiles on masts A, B, D and tower T are displayed in 454 Figure 14. On mast D, which is the closest to the source, both Lagrangian and Eulerian 455 models overpredict the concentrations when the $R_{ij} - \epsilon$ model is used. Nevertheless, when 456 using the $k - \epsilon$ closure, both models provides very accurate results. Same goes for tower 457 T, where the use of the $R_{ij} - \epsilon$ turbulence closure leads to an overprediction of concen-458 tration especially near the ground. On this mast, it is interesting to point out that even 459 though the Lagrangian model used with the $k - \epsilon$ closure underpredicts the concentration 460 values, it captures the change in the sign of the vertical concentration gradient near the 461 ground. Stepping even further from the source and looking at mast A, one can notice a 462 stronger vertical diffusion in the Lagrangian results. In this case again, concentrations are 463 overpredicted. For this trial, the study of masts D, T and A is interesting as the 'D-T-A' 464 parametrical line is roughly aligned with the centerline of the plume. One general conclu-465 sion from these three masts is that the use of the $R_{ij} - \epsilon$ model leads to an overprediction 466 of concentrations especially near the ground. As for the concentrations on mast B, they 467 are influenced by the plume deviation. Indeed, since the $R_{ij} - \epsilon$ model involves a more 468 pronounced deflection of the plume than the $k-\epsilon$ model towards the West direction, where 469 mast B is located, then concentrations are expected to be higher, which is indeed what 470 is observed in the simulation results. However, and once again, better accuracy is found 471 when the $k - \epsilon$ model is used for both Eulerian and Lagrangian approaches (except on 472 mast B for the Lagrangian results). 473

From all these results on stable stratification, one may think that the tendency of the $k - \epsilon$ model to compute high TKE values can be seen as an 'advantage' for stable conditions, since this strengthens diffusion processes and thus leads to lower concentration values (that are, in general, and in particular looking at the lines, in better agreement with the measurements). In fact, one should remember here that even when using a $R_{ij} - \epsilon$



Figure 12: Comparison of mean concentration (kg/kg) fields at z = 1.6 m computed by both Eulerian and Lagrangian models, through $k - \epsilon$ or $R_{ij} - \epsilon$ turbulence closures, for stable trial 2692157.



Figure 13: Comparison of concentration profiles on horizontal line samplers computed by both Eulerian and Lagrangian models, through $k - \epsilon$ or $R_{ij} - \epsilon$ turbulence closures, for stable trial 2692157.



Figure 14: Comparison of vertical concentration profiles computed by both Eulerian and Lagrangian models, through $k - \epsilon$ or $R_{ij} - \epsilon$ turbulence closures, for stable trial 2692157.

model, the closure on potential temperature is still local, through the following Generalized
Gradient Diffusion Hypothesis (GGDH):

$$\langle U'_{f,j}\theta'\rangle = -C_{\theta}\frac{k}{\epsilon} \left(\langle U'_{f,j}U'_{f,k}\rangle \frac{\partial \langle \theta \rangle}{\partial x_k} \right) , \qquad (6)$$

481 where $C_{\theta} = 0.3$.

An improvement, still under investigation, could be the use of a second-order model on potential temperature.

484 5. Conclusions

This work aimed at assessing the ability of a Lagrangian stochastic model to perform point source dispersion in an idealized urban area, for neutral and stable meteorological conditions. The Lagrangian model used was a model based on Pope (2000)'s SLM and Minier and Peirano (2001)'s two-phase flow formulation. Pope's PDF models have gone quite unnoticed in the atmospheric dispersion community, despite presenting some nonnegligible theoretical and numerical advantages (see Bahlali et al. (2018a,b)).

As used within a hybrid Eulerian/Lagrangian approach, the Lagrangian solver was 491 fed by the mean dynamical fields provided by the Eulerian solver of the same CFD code 492 Code_Saturne. These fields were computed either through a $k - \epsilon$ model or a second-order 493 $R_{ii} - \epsilon$ model. The Lagrangian results were compared to the Eulerian ones, for both 494 turbulence models. The Eulerian turbulence closure for the scalar fluctuations was based 495 on the gradient-diffusion hypothesis, which implied constant diffusivity. We showed that 496 since the region considered for concentration measurements is already located in the far 497 field (*i.e.*, diffusion times higher than the Lagrangian integral timescale), both Eulerian 498 and Lagrangian show the same evolution of diffusion. 499

Then, we showed that the obstacle array induced a deflection of the plume, as already 500 observed by Milliez and Carissimo (2007) in the same code and Castelli et al. (2017) in 501 another methodology. These works modeled turbulence only through a $k - \epsilon$ closure. More 502 precisely, we showed that this plume deflection was more pronounced using a $R_{ij} - \epsilon$ 503 model. This was the case with both Eulerian and Lagrangian approaches. For neutral 504 stratification, the agreement between results and measurements is quite satisfactory. In 505 particular, we can validate the accuracy of the results of the Lagrangian model. We showed 506 that the use of the $R_{ij} - \epsilon$ model made it possible to have a more physical representation 507 of the obstacles' influence on the concentration profiles, through the visible regular 'steps' 508 corresponding to the spacing between the containers. 509

For stable stratification, we showed that the results obtained by both approaches were more accurate when using a $k - \epsilon$ closure for the fluid phase. Not as satisfactory results as for neutral trial were found with the use of the $R_{ij} - \epsilon$ closure for stable stratification. The Lagrangian model, which is dependent on the quality of the flow calculated by the Eulerian solver, also see its results affected by the use of the $R_{ij} - \epsilon$ model. Deeper investigations are needed on this subject, as it should be noted that the temperature and scalar fluctuations closures are still local even when using the $R_{ij} - \epsilon$ model.

It is worth noting that the differences between the various results shown in this paper can be interpreted in different ways. In a practical context where order of magnitude differences are expected, they can be seen as small. On the other hand, they can also be looked at as an estimate we can have on the uncertainty effects of adopting either a certain turbulence closure $(k - \epsilon \text{ or } R_{ij} - \epsilon)$ or one of the two Lagrangian or Eulerian approaches.

For further investigations, an idea would be to explore full second-moment closure modeling for both temperature and scalar fluctuations. In the Lagrangian modeling, another research path would be to add a temperature scalar to the state vector of the particles, driven by a new stochastic differential equation (in the spirit of the works of Das and Durbin (2005) or Bossy et al. (2018) for instance), in order to better account for buoyancy-induced turbulent patterns.

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