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# **A two-surface thermo-mechanical plasticity model considering thermal cyclic behavior**

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## 22 **Abstract**

23 In thermal-related engineering such as thermal energy structures and nuclear waste  
24 disposal, it is essential to well understand volume change and excess pore water  
25 pressure buildup of soils under thermal cycles. However, most existing thermo-  
26 mechanical models can merely simulate one heating-cooling cycle and fail in capturing  
27 accumulation phenomenon due to multiple thermal cycles. In this study, a two surface  
28 elasto-plastic model considering thermal cyclic behavior is proposed. This model is  
29 based on the bounding surface plasticity and progressive plasticity by introducing two  
30 yield surfaces and two loading yield limits. A dependency law is proposed by linking  
31 two loading yield limits with a thermal accumulation parameter  $n_c$ , allowing the thermal  
32 cyclic behavior to be taken into account. Parameter  $n_c$  controls the evolution rate of the  
33 inner loading yield limit approaching the loading yield limit following a thermal loading  
34 path. By extending the thermo-hydro-mechanical equations into the elastic-plastic state,  
35 the excess pore water pressure buildup of soil due to thermal cycles is also accounted.  
36 Then, thermal cycle tests on four fine-grained soils (natural Boom clay, Geneva clay,  
37 Bonny silt and reconstituted Pontida clay) under different OCRs and stresses are  
38 simulated and compared. The results show that the proposed model can well describe  
39 both strain accumulation phenomenon and excess pore water pressure buildup of fine-  
40 grained soils under the effect of thermal cycles.

41 **Keywords:** fine-grained soils; strain accumulation; thermal cycles; two-surface model;  
42 excess pore water pressure buildup

## 43 **1. Introduction**

44 Around energy piles [1-3], nuclear waste canisters [4], buried tunnels and pipes [5], the  
45 soils suffer from temperature effects, especially heating-cooling cycles. Observations  
46 of different fine-grained soils in the laboratory and in the field [6-11] showed that  
47 heating generally induced non-negligible volume change and excess pore water  
48 pressure buildup. Several thermo-mechanical models were proposed to describe this  
49 thermo-mechanical behavior of fine-grained soils [13-21]. Some of them extended the  
50 Cam-clay or modified Cam-clay model to the temperature effect by introducing a  
51 shrinking yield surface [13-17]. These models could describe some basic thermo-  
52 mechanical behavior, but fail to describe the soil behavior in over-consolidated states.  
53 In order to well describe the small thermo-mechanical elastic zone and the shear  
54 softening behavior, some advanced models consider two yield surfaces based on the  
55 bounding surface theory [18-21]. Hong et al. [22] made a comparison among some of  
56 these advanced models, and showed that the multiple plastic hardening mechanism  
57 were of paramount importance in modeling complex thermal loading behavior.

58 Though numerous efforts were made in thermal mechanical modeling, most existing  
59 models can only simulate the case of one thermal cycle and fail in describing the soil  
60 behavior under multiple heating-cooling cycles. As more attention has been paid to the  
61 long-term performance [11,12,25,26,27,40], the volume change and excess pore water  
62 pressure accumulation due to temperature variation can act as a significant factor  
63 influencing the safety of energy structures. Demars and Charles [23] reported a series

64 of drained thermal cycle tests on high plastic marine clays and found that permanent  
65 reduction in void ratio (a function of over-consolidation ratio) gradually reduced due to  
66 temperature cycles (see Fig.1). Plum et al. [24] conducted an undrained heating-cooling  
67 cycle tests on Newfield clay. After four thermal cycles, the accumulation of excess pore  
68 water pressure was still increasing. Pastén et al. [40] experimentally observed the  
69 thermally driven displacement accumulation with the number of temperature cycles and  
70 developed a thermally induced ratcheting mechanism for a smooth interface. All those  
71 observations show that multiple thermal cycles should be considered in thermal  
72 mechanical modeling of clays. Zhou et al. [28] recently proposed a bounding surface  
73 model with a memory surface to consider the thermal cyclic behavior. However, this  
74 imposed memory surface actually has no effect on the accumulation behavior of  
75 normally consolidated soil because this memory surface coincides with the bounding  
76 surface in this circumstance. Ma et al. [29] proposed a thermal stabilization line in  
77 hypoplastic modeling approach. Though three thermal parameters are introduced, the  
78 proposed hypoplastic model fails to describe the strain accumulation during relatively  
79 small temperature changes because it always simulates an unrealistic elastic behavior  
80 before entering the elasto-plastic zone following a thermal loading path. More recently,  
81 Zymnis et al. [41] proposed a simple calibration procedure for Tsinghua ThermoSoil  
82 (TTS) model and studied the capabilities of simulating the long-term accumulation of  
83 strains due to thermal cycles. However, those existing models for thermal cyclic  
84 behavior merely considered the strain accumulation behavior under drained conditions

85 though the excess pore water pressure buildup due to temperature effects are at least of  
86 equal importance from the best of the authors' knowledge.

87 In the present work, a two surface thermo-plastic model, namely ACC2-T, is  
88 proposed to consider the thermal cyclic behavior based on the previous TEAM model  
89 by Hong et al. [21]. The proposed model unifies the loading yield limits and the thermal  
90 yield limits. To connect the inner loading yield limit and the loading yield limit, a  
91 flexible dependency law is incorporated and combined with isotropic and progressive  
92 hardening laws to account for multiple thermal cycles. In the dependency law, a  
93 parameter  $n_c$  is introduced to control accumulation rate through the evolution rate of the  
94 inner loading yield limit approaching the loading yield limit. Meanwhile, the proposed  
95 model is extended to undrained condition by applying the thermo-hydro-mechanical  
96 equations by Coussy [30] into the elastic-plastic state. Then, the rationale of ACC2-T  
97 model is discussed in terms of elasto-plasticity and thermodynamic theories. Lastly, the  
98 thermal cycle test results of four fine-grained soils (natural Boom clay, Geneva clay,  
99 Bonny silt and reconstituted Pontida clay) under different OCRs and stresses are used  
100 to verify the proposed model.

101

## 102 **2. Model description**

103 In this section, the proposed model (ACC2-T) is presented using typical constitutive  
104 variables, namely mean effective stress  $p'$ , deviator stress  $q$ , temperature  $T$  and the  
105 corresponding increments ( $dp'$ ,  $dq$ ,  $dT$ ). The soil behavior is assumed to be isotropic

106 and the constitutive model is developed and formulated in the conventional triaxial  
107 conditions.

108

### 109 **2.1 Thermo-elastic behavior**

110 In ACC2-T model, the soil deformation can be divided into thermo-elastic and thermo-  
111 plastic parts.

112 In the thermo-elastic part, the elastic law adopted in many other models [31,36,37]  
113 in isothermal conditions are extended into non-isothermal conditions for its simplicity.  
114 the total elastic volumetric strain increment associated with mechanical and thermal  
115 loading and the elastic shear strain assumed purely mechanical are calculated as follows:

$$116 \quad d\varepsilon_v^e = \frac{\kappa dp'}{\nu p'} - \alpha_1 dT, \quad d\varepsilon_s^e = \frac{dq}{3G}, \quad G = \frac{3(1-2\nu)}{2(1+\nu)} \frac{\nu p'}{\kappa} \quad (1)$$

117 where  $d\varepsilon_v^e$ ,  $d\varepsilon_s^e$  is the elastic volumetric strain increment ( $d\varepsilon_v^e = d\varepsilon_1^e + d\varepsilon_2^e + d\varepsilon_3^e$ )  
118 and the elastic shear strain increment ( $d\varepsilon_s^e = \frac{2}{3}(d\varepsilon_1^e - d\varepsilon_3^e)$ ).  $\kappa$  is the elastic slope in ( $v$ ,  
119  $\ln p'$ ) plane and  $\nu$  is the specific volume.  $\alpha_1$  is the thermal elastic compressibility index  
120 which assumed a positive constant because heating induces soil expansion.  $G$  is shear  
121 modulus and  $\nu$  is the Poisson's ratio.

122

### 123 **2.2 Thermo-plastic behavior**

124 The thermo-plastic mechanism of ACC2-T model consists of two yield surfaces and  
125 plastic potential surface, the dependency law, the isotropic and progressive hardening

126 laws and the flow rules.

127

### 128 2.2.1 Yield surfaces and plastic potential surfaces

129 In ACC2-T model, two yield surfaces, namely loading surface ( $f_l$ ) and yield surface ( $f_Y$ ),

130 with a generalized formulation [31] are implemented to model the yielding of both stiff

131 and soft clays.

132 Loading Surface: 
$$f_l = q^2 + \frac{M_f^2}{1-k_f} \left( \frac{p'}{r\bar{p}'_{cT}} \right)^{k_f} (r\bar{p}'_{cT})^2 - \frac{M_f^2 p'^2}{1-k_f} = 0 \quad (2)$$

133 Yield Surface: 
$$f_Y = q^2 + \frac{M_f^2}{1-k_f} \left( \frac{p'}{\bar{p}'_{cT}} \right)^{k_f} \bar{p}'_{cT}^2 - \frac{M_f^2 p'^2}{1-k_f} = 0 \quad (3)$$

134 where  $M_f$  is the stress ratio at the apex of two surfaces,  $k_f$  is a parameter specifying the

135 shape of yield surfaces,  $\bar{p}'_{cT}$  is the pre-consolidation pressure at current temperature  $T$

136 and  $r$  is the inverse of over-consolidation ratio (OCR) at current temperature  $T$ . The

137 conceptual plot of two yield surfaces at various temperature is given in Fig. 2(a). To

138 consider the thermo-mechanical behavior, the two surfaces are extended into three-

139 dimensional plot ( $p'$ ,  $q$ ,  $T$ ) with two loading yield limits. The two loading yield limits,

140 namely the inner loading yield limit ( $ILY$ ) and the loading yield limit ( $LY$ ) are linked by

141 the proposed dependency law which is discussed in *Yield limits and dependency law*

142 section.

143 The plastic potential surface  $g_l$  takes the same form as for loading surface, but with

144 different parameters  $M_g$  and  $k_g$ :



145 
$$g_I = q^2 + \frac{M_g^2}{1-k_g} \left( \frac{p'}{r\beta} \right)^{\frac{2}{k_g}} (r\beta)^2 - \frac{M_g^2 p'^2}{1-k_g} = 0 \quad (4)$$

146 and it leads to a dilatancy equation in the case of triaxial compression:

147 
$$\frac{d\varepsilon_v^p}{d\varepsilon_s^p} = \frac{\frac{\partial g_I}{\partial p'}}{\frac{\partial g_I}{\partial q}} = \frac{M_g^2 - \eta^2}{k_g \eta} \quad (5)$$

148 where  $d\varepsilon_v^p$  is the plastic volumetric strain increment,  $d\varepsilon_s^p$  is the plastic shear strain  
 149 increment,  $\eta$  denotes the stress ratio  $q/p'$ ,  $M_g$  defines the critical stress ratio at failure  
 150 where no further volumetric strain increment develops,  $k_g$  is a constant parameter which  
 151 controls the ratio of plastic volumetric strain increment and plastic shear strain  
 152 increment and  $\beta$  is the size parameter. Typically, an unassociated flow rule is adopted  
 153 for modeling complex mechanical behaviors. However, when  $M_g = M_f$  and  $k_g = k_f$ , an  
 154 associated flow rule would be recovered. When  $k_g = k_f = 2$ , elliptical surfaces like  
 155 modified Cam-clay model are recovered.

156

### 157 2.2.2 Hardening laws

158 In ACC2-T model, two plastic hardening mechanisms fixed to reference temperature  
 159  $T_0$  are adopted to control the evolution of yield surface and loading surface.

160 In the isotropic hardening rule, as same as in Modified Cam-clay model, the  
 161 evolution of pre-consolidation pressure  $\bar{p}'_{c0}$  at reference temperature  $T_0$  is linked to the  
 162 plastic volumetric strain increment:

163 
$$d\bar{p}'_{c0} = \frac{\nu}{\lambda - \kappa} \bar{p}'_{c0} d\varepsilon_v^p \quad (6)$$

164 where  $\lambda$  is the normal consolidation slope in  $(v, \ln p')$  plane and  $d\bar{p}'_{c0}$  is the increment  
 165 of pre-consolidation pressure. It means the evolution of yield surface is dominated by  
 166 the plastic volumetric strain.

167 The progressive hardening variable  $dr_0$ , at reference temperature  $T_0$ , is defined with  
 168 a combined volumetric and shear plastic strain as follows:

$$169 \quad dr_0 = \frac{v}{\lambda - \kappa} s (1 - r_0) (d\varepsilon_v^p + A_d d\varepsilon_s^p) \quad (7)$$

170 where  $r_0$  is the inverse of over-consolidation ratio (OCR) at current temperature  $T_0$  and  
 171  $dr_0$  is the increment.  $s$  is a material constant which controls the hardening rate of loading  
 172 surface in approaching yield surface.  $A_d$  is a material constant which controls the  
 173 contribution of shear plastic strain.

174 In this progressive hardening rule, parameter  $r_0$  is always less than or equal to 1 ( $0 <$   
 175  $r_0 \leq 1$ ). When the loading surface is completely inside the yield surface ( $0 < r_0 < 1$ ), it  
 176 means the soil is over-consolidated. Within the hardening process, plastic strain  
 177 increases and the loading surface begins to move towards the yield surface ( $dr_0 > 0$ ).  
 178 When  $r_0 = 1$ , the loading surface and the yield surface coincide. The soil turns into  
 179 normally consolidated state as the same case of MCC model. Note that the loading  
 180 surface could not run out of the yield surface ( $r_0 > 1$ ) for the fact that if  $r_0 > 1$ ,  $dr_0 < 0$   
 181 holds. With such a combined volumetric-deviatoric hardening law, the inclusion of  
 182 shear hardening part ( $d\varepsilon_s^p > 0$ ) would lead to a positive plastic modulus ( $h > 0$ ) even  
 183 when the effective stress state ( $d\varepsilon_v^p = 0$ ) reach the critical state line (CSL). As a  
 184 consequence, the stress point lying on the loading surface can cross the CSL,

185 hardening continues to a stress point ( $d\varepsilon_v^p < 0$ ,  $d\varepsilon_s^p > 0$ ,  $h = 0$ ) and then the stress point  
186 ( $d\varepsilon_v^p < 0$ ,  $d\varepsilon_s^p > 0$ ,  $h < 0$ ) would move back to the CSL. This approach allows to  
187 predict shear softening behavior and it is often the case of highly over-consolidated  
188 states of soil.

189 With these isotropic and progressive hardening rules, the plastic modulus and  
190 increment stress-strain relationship of ACC2-T model are obtained in Appendix A.

191 Under loading and heating processes, two hardening variables  $\bar{p}'_{c0}$  and  $r_0$  are  
192 activated with the evolution of plastic strain. However, under unloading and cooling  
193 processes, pure elastic strain develops with a fixed value of  $\bar{p}'_{c0}$ . Due to ACC2-T model  
194 strictly adhering to the principle that the stress point is always lying on the loading  
195 surface, the value of  $r_0$  could be re-calculated by a simple Newton-Raphson Method  
196 with unconditional stability.

197

### 198 *2.2.3 Yield limits and dependency law*

199 In the two surface models, the loading surface and the yield surface are geometrically  
200 similar in  $(p', q)$  plane with a corresponding radial mapping rule. In previous TEAM  
201 model [21], two yield limit mechanisms, namely thermal yield limits ( $ITY$ ,  $TY$ ) and  
202 loading yield limits ( $ILY$ ,  $LY$ ) are adopted to model the thermal behavior. These yield  
203 limit mechanisms in TEAM model (see Fig.3(a)) enable the thermal elastic response in  
204 reloading path to occur before entering the elasto-plastic zone. Therefore, the TEAM  
205 model could not simulate the strain accumulation phenomenon due to temperature

206 variations. In order to overcome this shortcoming and extend the model to the case of  
 207 multiple thermal cycles, the loading yield limits (*ILY*, *LY*) are unified to consider both  
 208 the thermal yield and loading yield mechanisms. Meanwhile, a new dependency law is  
 209 proposed to connect *ILY* and *LY*.

210 In the dependency law, the pre-consolidation pressure  $\bar{p}'_{cT}$  at current temperature  $T$   
 211 takes the original form and it would decrease with the increase of temperature:

$$212 \quad \bar{p}'_{cT} = \bar{p}'_{c0} \exp[-\alpha_0(T - T_0)] \quad (8)$$

213 where  $\bar{p}'_{c0}$  is pre-consolidation pressure at reference temperature  $T_0$ ;  $\alpha_0$  is a material  
 214 constant controlling the thermal plastic behavior.

215 The other part of the dependency law  $r$  is defined as below:

$$216 \quad r = \frac{1}{OCR} = \frac{p'}{\bar{p}'} = \frac{q}{\bar{q}} = \frac{p'_{cT}}{\bar{p}'_{cT}} \quad (9)$$

217 where  $(p', q)$ ,  $(\bar{p}', \bar{q})$  is the corresponding stress point in loading surface and yield  
 218 surface at current temperature  $T$ , respectively.

219 Unlike TEAM model with the assumption that  $r = r_0$ , ACC2-T model takes the form  
 220 that would make *ILY* not necessarily homologous to *LY* any longer (see Fig.3(b)):

$$221 \quad r = r_0 \exp[-\alpha_0(r_0^{n_c} - 1)(T - T_0)] \quad (10)$$

222 where  $n_c$  is a thermal parameter which controls the strain accumulation rate under  
 223 multiple thermal cycles.

224 The combined dependency law made up by  $\bar{p}'_{cT}$  and  $r$  then is expressed by:

$$225 \quad r\bar{p}'_{cT} = r_0\bar{p}'_{c0} \exp[-\alpha_0 r_0^{n_c} (T - T_0)] \quad (11)$$

226 In the proposed law, the shape of *ILY* is governed by the key thermal accumulation

227 parameter  $n_c$ . Parameter  $n_c$  controls the evolution rate of  $ILY$  approaching  $LY$  by the two  
228 isotropic and progressive hardening rules. In order to illustrate the plastic mechanism  
229 under the thermal path, two cases in the isotropic states, namely  $n_c < 0$  and  $n_c = 0$ , are  
230 compared in Fig.4. Both two cases are at initial stress state A, then heated up to stress  
231 state B and finally cooled down to stress state A. Under the heating path from A to B,  
232 the  $ILY$  without  $n_c$  ( $n_c = 0$ ) would induce a increment of  $\Delta r_1$ . For the case of  $n_c < 0$ , it  
233 would induce a increment of  $\Delta r_1 + \Delta r_2 + \Delta r_3$  if the shape of  $ILY$  is fixed. However, with  
234 the effect of  $n_c$ , the original  $ILY$  (see Fig.4(b), dotted line I) would gradually turn into  
235 solid line II with a actual increment of  $\Delta r_1 + \Delta r_2$ . The larger increment  $\Delta r_1 + \Delta r_2$  in  
236 hardening variable  $dr_0$  would mean larger plastic volumetric strain increments than the  
237 case of  $n_c = 0$ . Under the cooling path from B to A, the mechanical response is purely  
238 elastic and the shape of  $ILY$  would turn from II into I. For the case of  $n_c > 0$ , the  
239 increment in hardening variable  $dr_0$  would induce smaller strain increments than  $n_c = 0$ .  
240 Under multiple thermal cycles, the value of  $r_0$  would be smaller and smaller.

241 Under such a unique dependency law, the evolution rate of  $ILY$  is actually the  
242 evolution rate of strain accumulation. With a proper selection of parameter  $n_c$  in the  
243 dependency law, ACC2-T model allows the strain accumulation under the effect of  
244 multiple thermal cycles to be modelled. Some important features about the dependency  
245 law and parameter  $n_c$  are summarized below:

246 1. When the current temperature  $T$  is equal to the reference temperature  $T_0$ ,  $r$  becomes  
247 equal to  $r_0$ . It means that  $n_c$  would have no effect on the stress-strain relationship at

248 reference temperature  $T_0$ .

249 2. When soil turns into the normal consolidated state ( $r_0 = 1$ ),  $r$  is also equal to 1. In that  
250 case,  $ILY$  would coincide with  $LY$ . It means that  $n_c$  would have no effect on the behavior  
251 of normal consolidated soils under either isothermal or non-isothermal condition.

252 3. When  $n_c$  is equal to zero (bold line in Fig. 3(b)),  $r$  becomes equal to  $r_0$ . In that case,  
253 a similarity between  $ILY$  and  $LY$  would be recovered. This is just the case of  $ILY$  and  $LY$   
254 used in TEAM model [21].

255 4. Theoretically, the value of  $n_c$  can be chosen from negative infinity to positive infinity.  
256 When  $n_c$  tends to positive infinity,  $ILY$  would tend to be the dotted line in Fig. 3(b).  
257 When  $n_c$  tends to negative infinity,  $ILY$  would tend to approach the temperature axis  $T$   
258 in Fig. 3(b). A larger  $n_c$  represents a quicker strain accumulation rate which would  
259 induce smaller total volumetric strain accumulation under the effect of thermal cycles.

260 5. Any possible  $r$  is smaller than 1. During the heating process, the hardening  
261 mechanism enables  $r_0$  to approach 1. Thus,  $r$  would also have its limit value of 1 which  
262 implies that the over-consolidation ratio of soil is always larger than or equal to 1.

263

### 264 **3. Loading path independence**

265 After the description of the proposed model extended to thermal cyclic behavior, it is  
266 essential to discuss the rationale of ACC2-T model in terms of elasto-plastic theory and  
267 thermodynamic theory as Hong et al. [22] did on three existing thermo-mechanical  
268 models [19,20,25,28,29]. The verification of loading path independence is made up by

269 both the thermo-elastic and thermo-plastic part. The schematic plot is given in Fig.5.  
270 Heating is treated as a special isotropic loading. Therefore, merely isotropic condition  
271 ( $q=0$ ) is considered during the derivation. The thermo-elastic part is discussed as  
272 below and the derivation in thermo-plastic regime is attached in Appendix B.

273 In the thermo-elastic mechanism (Eq.3) of ACC2-T model, the bulk modulus  
274  $K = \frac{\nu p'}{\kappa}$  is independent of temperature, and thermal elastic compressibility index  $\alpha_1$   
275 is also independent of mean effective stress. Such elastic volumetric strain in ACC2-T  
276 model is surely loading path independent in  $(p', T)$  plane. Note that the thermo-elastic  
277 mechanism leads to loading path dependent in  $(p', q)$  plane since the Maxwell symmetry  
278 relations are not satisfied. A possible way is to extend a hyperelastic law [39] into non-  
279 isothermal conditions. For simplicity of parameter calibration, merely this thermo-  
280 elastic mechanism is considered here.

281

## 282 **4. Analysis and determination of model parameters**

283 The proposed ACC2-T model contain fifteen parameters ( $\lambda, \kappa, \bar{p}'_{c0}, \nu, M_f, k_f, M_g, k_g, s,$   
284  $r_0, A_d, \alpha_0, \alpha_1, T_0, n_c$ ). Five of them ( $\lambda, \kappa, \bar{p}'_{c0}, \nu, M_g$ ) are commonly defined in MCC  
285 model.  $(M_f, k_f), (M_g, k_g)$  control the shapes of yield surfaces and plastic potential  
286 surfaces, respectively. Three parameters ( $s, r_0, A_d$ ) are used to consider a progressive  
287 yielding of two surfaces. Four parameters ( $\alpha_0, \alpha_1, T_0, n_c$ ) are defined to consider the  
288 thermal influence on the soil behavior. The procedure for determining these parameters  
289 is described and key parameters ( $\lambda, \kappa, \bar{p}'_{c0}, s, r_0, \alpha_0, \alpha_1, T_0, n_c$ ) related to thermal

290 accumulation behavior are discussed on the set of model parameters for natural Boom  
291 clay in Fig. 6.

292 1. By an isotropic compression test or oedometer test under reference temperature  $T_0$ ,  
293  $\kappa$ ,  $\lambda$ , the initial  $r_{0,0}$  and  $\bar{p}'_{c0,0}$  are determined in  $(v, \ln p')$  plane in Fig.6(1). Note that  
294 parameter  $s$  controls the hardening rate of loading surface in approaching yield surface.  
295 It can be determined by fitting nonlinearity from over-consolidated states to normal  
296 consolidated state in  $(v, \ln p')$  plane.

297 2. By an isotropic heating-cooling test at normally consolidated states,  $\alpha_1$  is  
298 determined by fitting data of cooling stage and  $\alpha_0$  is determined by the heating test in  
299  $(T, \Delta\varepsilon_v)$  plane in Fig.6(2).

300 3. By a continuous thermal cycle test, the volumetric strain increment by each thermal  
301 cycle  $\Delta\varepsilon_{v,st}$  can be obtained  $(\Delta\varepsilon_{v,st}, n)$  plane in Fig.6(3). The thermal accumulation  
302 parameter  $n_c$ , which controls the strain accumulation rate, is then determined by  
303 calibrating the strain accumulation under each thermal cycle.

304 4.  $M_f$ ,  $k_f$ .  $M_f$  denotes the apex of yield surface and  $k_f$  controls the shape of yield surface.  
305 They are calibrated by fitting the conventional yield points of various stress paths.

306 5.  $M_g$  is the critical state stress ratio and it is calibrated by the effective stress ratio at  
307 critical state.

308 6.  $k_g$  controls plastic flow direction. It is determined by fitting the values of  $d\varepsilon_v^p/d\varepsilon_s^p$   
309 in drained triaxial shear tests.

310 7. The poisson's ratio  $\nu$  is determined by a triaxial tests at low strain level in the  $(\varepsilon_v-\varepsilon_l)$



311 plane:  $v = (\varepsilon_v - \varepsilon_l)/2$ .

312 8.  $A_d$  represents the contribution of the plastic shear strain in the hardening process. It  
313 is calibrated by fitting the results of triaxial compression tests at over-consolidated  
314 states in  $(q, \varepsilon_l)$  or  $(\varepsilon_v, \varepsilon_l)$  plane in Fig.6(4).

315

## 316 **5. Model Validation and prediction**

317 In this section, four different fine-grained soils, namely natural Boom clay, Geneva clay,  
318 Bonny silt and reconstituted Pontida clay, are adopted to verify the capability of ACC2-  
319 T model in modeling the thermal cyclic behavior.

320

### 321 **5.1 Natural Boom clay**

322 One heating and cooling test on natural Boom clay ( $I_p = 31$ ) under isotropic stress  
323 states performed by Baldi et al. [8] and two groups of triaxial compression tests  
324 conducted by Lê [38] are taken into account.

325 In the heating and cooling test, specimen TBoom-9 taken from Mol at an in-situ depth  
326 of 223 m (initial pre-consolidated pressure of 6 MPa) is tested under a mixed  
327 mechanical and thermal loading process: the sample was firstly saturated under a  
328 constant mean effective stress of 2 MPa at 21.5 °C, loaded up to 4 MPa, unloaded to 1  
329 MPa, heated up to 95 °C and cooled back to 21.5 °C, loaded up to 3 MPa at 21.5 °C  
330 and heated up to 95 °C and cooled back, loaded up to 6 MPa and heated up again and  
331 finally cooled back. In the loading process, three thermal cycles of temperature

332 increment of 73.5 °C at different isotropic stress states, namely mean effective stress of  
333 1 MPa, 3 MPa and 6 MPa, are selected for simulation. For better comparison between  
334 the computed and measured results, the increments of volumetric strain are reset at the  
335 starting point of each heating-cooling cycle. Moreover, Fig. 7 contains an isotropic  
336 compression test of natural Boom clay from 2 MPa to 20 MPa, then back to 6 MPa.  
337 Basic parameters for natural Boom clay are listed in Tab.1.

338 The TEAM model [21] and the proposed ACC2-T model are both used for simulation  
339 in Fig.8. It appears that both models can describe the thermal volume change well for  
340 normally consolidated or slightly over-consolidated state ( $p' = 6$  MPa) adopting the  
341 same thermal elastic parameter  $\alpha_1$  and thermal plastic parameter  $\alpha_0$ . However, for  
342 highly over-consolidated states ( $p' = 3$  MPa,  $p' = 1$  MPa), it is observed that ACC2-T  
343 model gives a more consistent simulation than TEAM model. One reason is that TEAM  
344 model always shows a linear elastic expansion behavior before entering the elasto-  
345 plastic zone and has no parameters controlling strain evolution. With this shortcoming,  
346 TEAM model would fail to respond to thermal cycles and thus to describe strain  
347 accumulation behavior, though complicated multiple plastic mechanisms are adopted.  
348 However, in ACC2-T model, the stress point is always lying on the loading surface.  
349 Combined with the isotropic and progressive hardening rules, the thermal accumulation  
350 parameter  $n_c$  in the dependency law allows the ACC2-T model to better describe the  
351 nonlinear expansion behavior and strain accumulation under over-consolidated states.  
352 Meanwhile, as natural Boom clay is relatively easy to contract under heating, the use

353 of a negative value of parameter  $n_c$  could facilitate the simulation of such aspect.

354 In the triaxial compression test, the experimental results of drained triaxial  
355 compression tests ( $p'_0 = 2.5$  MPa,  $p'_0 = 4$  MPa) at room temperature 25 °C and drained  
356 triaxial compression tests ( $p'_0 = 2.5$  MPa,  $p'_0 = 3.8$  MPa) at high temperature 76 °C  
357 [38] are considered in  $(q, \varepsilon_l)$  plane and  $(\varepsilon_v, \varepsilon_l)$  plane in Fig.9. In these moderate over-  
358 consolidated states, the shear strength at room temperature is a bit higher than high  
359 temperature. However, a larger volumetric strain is generated at high temperature than  
360 room temperature as shown in  $(\varepsilon_v, \varepsilon_l)$  plane. At the same time, the smooth predictions  
361 by ACC2-T model agree well with the experimental observations. Two values of  $n_c$  ( $n_c$   
362 = -1,  $n_c = 0$ ) are selected to simulate the behavior at high temperature 76 °C. It indicates  
363 that the value of  $n_c$  has a negligible effect on the prediction of triaxial tests.

364 A numerical case at high over-consolidated state ( $p'_0 = 0.5$  MPa) under both two  
365 temperatures (25 °C, 76 °C) are conducted. A slight shear dilatancy is predicted at axial  
366 strain of about 5%. However, the post-peak softening phenomenon in  $(q-\varepsilon_l)$  plane is not  
367 very apparent because parameter  $A_d$  ( $A_d = 0.1$ ) for natural Boom clay is very small to  
368 simulate behaviors at both high, slightly over-consolidated and normal consolidated  
369 states.

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## 371 **5.2 Geneva clay**

372 Multiple thermal cycles were applied on Geneva clays ( $I_p = 11.1$ ) in a temperature-  
373 controlled oedometer cell under drained condition [25]. A normally consolidated soil

374 sample with a vertical effective stress of 125 kPa was tested with four temperature  
375 cycles from 5 °C from 60 °C, starting at a room temperature of 20 °C. The one-  
376 dimension test results are simulated in Fig.7 and the calibrated model parameters are  
377 listed in Tab.1.

378 Fig.10 shows the measured and computed soil behavior under multiple heating and  
379 cooling cycles. The computed result of ACC2-T model (Fig.10(a)) shows a good  
380 agreement with the measured one. The irreversible volumetric strain accumulates with  
381 increasing thermal cycles, but at a decreasing rate (Fig.10(c)). The value of 3.5 is shown  
382 appropriate for the thermal accumulation parameter  $n_c$  to control the strain  
383 accumulation rate. For the case of TEAM model, the computed result of first thermal  
384 cycle is the same as ACC2-T model. However, the response of TEAM model is purely  
385 elastic thereafter and the thermal strain accumulation could not be described. In  
386 Fig.10(b), a sensitivity analysis of ACC2-T model is conducted to study the effect of  
387 parameter  $n_c$  on the strain accumulation. Two other  $n_c$  values of 0, 10 are selected for  
388 this purpose. In the first thermal cycle, ACC2-T model gives the same results for all the  
389 three  $n_c$  values because the thermal volumetric strain of normally consolidated soil is  
390 only dependent on thermal elastic parameter  $\alpha_1$  and thermal plastic parameter  $\alpha_0$ .  
391 From the second thermal cycle, the soil becomes over-consolidated. The thermal  
392 accumulation parameter  $n_c$  starts to control the strain evolution rate. From Fig.10(c), it  
393 appears that parameter  $n_c$  is positively correlated to the strain accumulation rate and a  
394 proper selection of parameter  $n_c$  is crucial for total strain accumulation.

### 395 **5.3 Bonny silt**

396 Multiple heating and cooling cycles were applied on compacted Bonny silt ( $I_p = 4$ )  
397 under different OCRs in a temperature-controlled oedometer cell [27]. Bonny silts were  
398 first mechanically consolidated to a vertical effective stress of approximately 1200 kPa  
399 at a constant room temperature of 18 °C. Then, they were unloaded to reach five  
400 different OCR values of 1.00, 1.28, 1.80, 7.36 and 30.29. Fig. 7 presents the  
401 experimental and calibration results of one-dimensional test of Bonny silt. Four heating  
402 and cooling cycles were applied under drained condition with different OCRs in a  
403 temperature ranging from 18 °C to 93 °C.

404 It can be seen from Fig.11 that at slightly over-consolidated states (i.e. OCR = 1, 1.28  
405 and 1.8), the measured and computed results show a good consistency. That means the  
406 thermal accumulation parameter  $n_c$  can properly control the strain evolution rate under  
407 various over-consolidated states. However, for highly over-consolidated ones (i.e. OCR  
408 = 7.36, 30.29) in the literature [27], the volumetric strain accumulation is quite small or  
409 even becomes negative, suggesting that the expansion and the contraction of the  
410 oedometer cell can be no longer ignored. These two samples are not considered in the  
411 simulation.

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### 413 **5.4 Reconstituted Pontida clay**

414 The excess pore water pressure accumulation due to thermal cycles is validated by a  
415 heating and cooling test of reconstituted Pontida clay. Hueckel et al [12] reported this  
416 measured and computed results on reconstituted Pontida clay. The reconstituted silty

417 clay sample was firstly loaded to a mean effective stress state of 2.5 MPa. Then it  
 418 underwent a heating-cooling cycle from 22°C to 65°C and back to 22°C, then a  
 419 continuous heating to 95°C in a constant total isotropic stress state.

420 Although some previous models [19] consider the undrained behavior by imposing  
 421 a constant volume condition which is imprecise under non-isothermal conditions, the  
 422 undrained computation using ACC2-T model is obtained by extending the energy and  
 423 mass conservation of thermo-poro-elastic media into elasto-plastic states [30].

424 In a saturated porous media, the current water mass ( $m_w$ ) per unit volume can be  
 425 expressed as:

$$426 \quad m_w = \rho_w \phi \quad (12)$$

427 where  $\rho_w$  is water density and  $\phi$  is the Lagrangian porosity.

428 In undrained conditions, water mass is constant and  $dm_w = 0$  holds, then

$$429 \quad \frac{dm_w}{\rho_w} = d\phi + \phi \frac{d\rho_w}{\rho_w} = d\phi + \phi (C_w du - \alpha_w dT) = 0 \quad (13)$$

430 where  $\alpha_w$ ,  $C_w$ , is the water thermal expansion coefficient, water compressibility,  
 431 respectively.

432 In isotropic stress state and under non-isothermal condition, total mean stress change  
 433 ( $dp$ ) and Lagrangian porosity change ( $d\phi$ ) can be expressed with Maxwell's symmetry  
 434 condition:

$$435 \quad dp = \frac{I}{C_d} d\varepsilon_v + (I - \frac{C_s}{C_d}) du_w + \frac{\alpha_d}{C_d} dT \quad (14)$$

$$436 \quad d\phi = -(I - \frac{C_s}{C_d}) d\varepsilon_v - (\phi - I + \frac{C_s}{C_d}) C_s du_w + (\phi - I + \frac{C_s}{C_d}) \alpha_s dT \quad (15)$$

437 where  $C_s$ ,  $C_d$ ,  $\alpha_s$ ,  $\alpha_d$  is the volumetric compressibility coefficient of solid grain, the  
 438 volumetric compressibility of soil skeleton, the thermal expansion coefficient of solid  
 439 grain and the thermal expansion coefficient of soil skeleton. It is often the case that  $\alpha_s$   
 440 =  $\alpha_d$ .

441 Thus, the expressions for excess pore water pressure can be obtained, as follows:

442 
$$d\varepsilon_v = -(\phi\alpha_w + (1-\phi)\alpha_s)dT + \phi(C_w - C_s)du_w + C_s dp \quad (16)$$

443 With infinitesimal transformation and clayey soils, the fact ( $C_s \approx 0$ ,  $n \approx \phi$ ) is  
 444 commonly accepted. Thus

445 
$$du_w = \frac{d\varepsilon_v + (n\alpha_w + (1-n)\alpha_s)dT}{nC_w} \quad (17)$$

446 where  $n$  is Eulerian porosity.

447 For reconstituted Pontida clay,  $\alpha_s$  is  $1 \times 10^{-5}$  ( $1/^\circ\text{C}$ ) chosen as same as  $\alpha_l$ , which is  
 448 calibrated from an undrained cooling process from  $65^\circ\text{C}$  to  $22^\circ\text{C}$ .  $\alpha_w$  and  $C_w$  are  
 449 calibrated from a heating test of free water in the literature [35].

450 The computed results of excess pore water pressure by ACC2-T model and TEAM  
 451 model are compared to the measured data and Hueckel's simulation [12] in Fig.12. It is  
 452 seen that both predictions of ACC2-T model and TEAM model are more accurate than  
 453 Hueckel's simulation, especially the existed buildup of pore water pressure in  
 454 experimental results. During the heating stage from  $22^\circ\text{C}$  to  $65^\circ\text{C}$ , the mean effective  
 455 stress will decrease with temperature (equivalent to a mechanical loading under  
 456 undrained condition) and the effective stress-temperature path ( $T - p'$ ) initially enters  
 457 plasticity. With the plastic hardening of *ILY*, excess pore water pressure is increased.  
 458 Then during the cooling stage from  $65^\circ\text{C}$  to  $22^\circ\text{C}$ , the response is purely elastic with a  
 459 decrease of the pore water pressure. Thereby, an irreversible excess pore water pressure  
 460 is observed after one heating-cooling cycle which is well captured by ACC2-T model  
 461 and TEAM model. Moreover, though the cooling stage is purely elastic, the *ILY* would  
 462 shrink to make the stress-temperature state always lying on it. However, during the re-  
 463 heating process, TEAM model could not build up excess pore water pressure from  $22^\circ\text{C}$   
 464 to  $65^\circ\text{C}$  because the response is purely elastic. As for ACC2-T model, the clay sample  
 465 is initially over-consolidated, then began to build up excess pore water pressure with  
 466 the control of thermal accumulation parameter  $n_c$  in the whole process from  $22^\circ\text{C}$  to  
 467  $95^\circ\text{C}$ .

## 468 **5.5 Discussion**

469 Four fine-grained soils are selected to verify the capability of ACC2-T model on  
470 modelling thermal cyclic behaviors under drained and undrained conditions. A thermal  
471 accumulation parameter  $n_c$  control the accumulation rate under multiple thermal cycles.  
472 Meanwhile, the values of  $n_c$  are ranked as: natural Boom clay < reconstituted Pontida  
473 clay < Geneva clay < Bonny silt. It means the clays, containing more clay contents,  
474 showing larger plastic index  $I_p$ , are more likely to accumulate volumetric strain and  
475 build up excess pore water pressure. Generally, the larger is the plastic index  $I_p$ , the  
476 smaller is the value of  $n_c$ . A liner regression function is obtained to evaluate the  
477 correlation between parameter  $n_c$  and plastic index  $I_p$  in Fig. 13.

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## 479 **6. Concluding remarks**

480 A two-surface plasticity model (ACC2-T) is proposed to describe the thermo-  
481 mechanical behavior of fine-grained soils under multiple thermal cycles. Compared  
482 with TEAM model, two thermal yield limits are removed and the loading yield limits  
483 unify the thermal and loading yielding part. A dependency law with thermal  
484 accumulation parameter  $n_c$  is used to link the inner loading yield limit and the loading  
485 yield limit. Parameter  $n_c$  controls the evolution rate of the inner loading yield limit  
486 approaching the loading yield limit under thermal loading path. With such a unique  
487 dependency law and multiple plastic hardening rules, the model could well control the  
488 thermal strain accumulation rate under drained conditions and pore water pressure



489 buildup rate under undrained conditions. Tests on four fine-grained soils are considered  
490 to verify the proposed model. Comparison between prediction and experiment shows  
491 that ACC2-T model can well capture the thermo-mechanical behavior of soils under  
492 multiple thermal cycles.

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510 **Notation**

511  $\alpha_0$  = parameter controlling thermal plastic behavior;

512  $\alpha_1$  = parameter controlling thermal elastic behavior;

513  $\beta$  = the size parameter of potential plastic surface;

514  $\lambda, \kappa$  = slope of isotropic loading, reloading line respectively;

515  $\nu, \nu$  = current void ratio, Poisson's ratio respectively;

516  $A_d$  = parameter controlling the contribution of shear plastic strain;

517  $d\boldsymbol{\varepsilon}, d\boldsymbol{\sigma}, dT$  = total strain, stress, temperature increment respectively;

518  $d\boldsymbol{\varepsilon}_\sigma^e, d\boldsymbol{\varepsilon}_T^e$  = stress, thermal induced elastic strain increment respectively;

519  $d\varepsilon_\nu^e, d\varepsilon_\nu^e, d\varepsilon_s^e$  = total volumetric, elastic volumetric, shear strain increment

520 respectively;

521  $d\boldsymbol{\varepsilon}^p, d\varepsilon_\nu^p, d\varepsilon_s^p$  = plastic, plastic volumetric, plastic shear strain increment

522 respectively;

523  $\Lambda$  = plastic multiplier;

524  $f_l, f_Y$  = Loading surface, yield surface respectively;

525  $G, K, h$  = elastic shear, elastic bulk, plastic modulus respectively;

526  $k_f, k_g$  = parameter controlling the shape of yield surface, plastic potential surface

527 respectively;

528  $M_f$  = stress ratio at the apex of yield surface;

529  $M_g, M_{g0}$  = critical stress ratio at current, reference temperature respectively;

530  $p', \bar{p}'$  = current, mapping mean effective stress respectively;

531  $p'_{cT}, p'_{c0}$  = pre-consolidation pressure in loading surface at current, reference  
532 temperature respectively;  
533  $\bar{p}'_{cT}, \bar{p}'_{c0}$  = pre-consolidation pressure at current, reference temperature respectively;  
534  $q, \bar{q}$  = current, mapping deviator stress respectively;  
535  $r, r_0$  = reverse of OCR at current, reference temperature respectively;  
536  $s$  = parameter controlling the rate of evolution of the slope of the stress--strain  
537 relation;  
538  $T, T_0$  = current, reference temperature respectively;  
539  $\mathbf{D}_e, \mathbf{D}_{ep}, \mathbf{D}_{et}$  = elastic, elasto-plastic, temperature stiffness matrix respectively;  
540  $\mathbf{m}$  = unit vector.  
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556 **Appendix A**

557 The consistency condition of loading surface in ACC2-T model is adopted herein:

$$558 \left( \frac{\partial f_I}{\partial \boldsymbol{\sigma}'} \right)^t : d\boldsymbol{\sigma}' + \frac{\partial f_I}{\partial T} dT + \frac{\partial f_I}{\partial \bar{p}'_{c0}} d\bar{p}'_{c0} + \frac{\partial f_I}{\partial r_0} dr_0 = 0 \quad (\text{A - 1})$$

559 The plastic multiplier  $\Lambda$  and the plastic modulus  $h$  are obtained in non-isothermal  
560 condition:

$$561 d\boldsymbol{\varepsilon}^p = \Lambda \frac{\partial \mathbf{g}_I}{\partial \boldsymbol{\sigma}'} \quad (\text{A - 2})$$

$$562 \Lambda = - \frac{\left( \frac{\partial f_I}{\partial \boldsymbol{\sigma}'} \right)^t : d\boldsymbol{\sigma}' + \frac{\partial f_I}{\partial T} dT}{\frac{\partial f_I}{\partial \bar{p}'_{c0}} \frac{\nu}{\lambda - k} \bar{p}'_{c0} \frac{\partial \mathbf{g}_I}{\partial p'} + \frac{\partial f_I}{\partial r_0} \frac{\nu}{\lambda - k} s(1 - r_0) \left( \frac{\partial \mathbf{g}_I}{\partial p'} + A_d \frac{\partial \mathbf{g}_I}{\partial q} \right)} \quad (\text{A - 3})$$

$$563 h = \frac{\nu}{\lambda - k} \left( \frac{\partial f_I}{\partial \bar{p}'_{c0}} \bar{p}'_{c0} \frac{\partial \mathbf{g}_I}{\partial p'} + \frac{\partial f_I}{\partial r_0} s(1 - r_0) \left( \frac{\partial \mathbf{g}_I}{\partial p'} + A_d \frac{\partial \mathbf{g}_I}{\partial q} \right) \right) \quad (\text{A - 4})$$

564 The total strain increment is made up of mechanical elastic strain increment, thermal  
565 elastic strain increment and plastic strain increment.

$$566 d\boldsymbol{\varepsilon} = d\boldsymbol{\varepsilon}_{\sigma'}^e + d\boldsymbol{\varepsilon}_T^e + d\boldsymbol{\varepsilon}^p \quad (\text{A - 5})$$

567 The thermal elastic strain increment  $d\boldsymbol{\varepsilon}_T^e$  is defined as:

$$568 d\boldsymbol{\varepsilon}_T^e = -\frac{1}{3} \mathbf{m} \alpha_1 dT \quad (\text{A - 6})$$

569 where  $\mathbf{m}$  is the column vector with 1 at normal stress entries and 0 at shear stress entries.

570 By substituting Eq. A-6 into Eq. A-5, the general incremental stress-strain relation  
571 can be expressed:

$$572 d\boldsymbol{\sigma}' = \mathbf{D}_e d\boldsymbol{\varepsilon}_{\sigma'}^e = \mathbf{D}_e \left( d\boldsymbol{\varepsilon} + \frac{1}{3} \mathbf{m} \alpha_1 dT - d\boldsymbol{\varepsilon}^p \right) \quad (\text{A - 7})$$

573 where  $\mathbf{D}_e$  is the mechanical elastic matrix.

574 By substituting the plastic multiplier into Eq. A-7, the differential stress–strain  
 575 equations can be obtained:

$$576 \quad d\boldsymbol{\sigma}' = \mathbf{D}_{ep} d\boldsymbol{\varepsilon} + \mathbf{D}_{et} dT \quad (\text{A - 8})$$

577 where  $\mathbf{D}_{ep}$ ,  $\mathbf{D}_{et}$  are the mechanical elasto-plastic matrix and the thermal elasto-plastic  
 578 matrix, respectively.

$$579 \quad \mathbf{D}_{ep} = \mathbf{D}_e - \frac{\mathbf{D}_e \frac{\partial \mathbf{g}_I}{\partial \boldsymbol{\sigma}'} \left( \frac{\partial f_I}{\partial \boldsymbol{\sigma}'} \right)^t \mathbf{D}_e}{\left( \frac{\partial f_I}{\partial \boldsymbol{\sigma}'} \right)^t \mathbf{D}_e \frac{\partial \mathbf{g}_I}{\partial \boldsymbol{\sigma}'} + h} \quad (\text{A - 9})$$

$$580 \quad \mathbf{D}_{et} = \frac{\mathbf{D}_e \frac{\partial \mathbf{g}_I}{\partial \boldsymbol{\sigma}'} \left[ -\frac{1}{3} \left( \frac{\partial f_I}{\partial \boldsymbol{\sigma}'} \right)^t \mathbf{D}_e \mathbf{m} \alpha_1 - \frac{\partial f_I}{\partial T} \right]}{\left( \frac{\partial f_I}{\partial \boldsymbol{\sigma}'} \right)^t \mathbf{D}_e \frac{\partial \mathbf{g}_I}{\partial \boldsymbol{\sigma}'} + h} + \frac{1}{3} \mathbf{D}_e \mathbf{m} \alpha_1 \quad (\text{A - 10})$$

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596 **Appendix B**

597 Assume the soil is loaded and heated from Point A to Point B under two different  
 598 loading paths I(  $A \rightarrow C \rightarrow B$  ) and II(  $A \rightarrow D \rightarrow B$  ) in Fig.5. The soil is at the initial stress  
 599 state (  $p'_0, r_0, p'_{c0}, T_1$  ) which automatically satisfies the yield condition:

$$600 \quad f_{I0} = p'_0 - r_0 p'_{c0} \exp[-\alpha_0 r_0^{n_c} (T_1 - T_0)] = 0 \quad (\text{B - 1})$$

601 Under loading path I, the soil element is isotropically loaded to stress point  $C$   
 602 (  $p'_0 + \Delta p'_1, r_1, p'_{c1}, T_1$  ) at constant temperature  $T_1$ .

$$603 \quad f_{I1} = p'_0 + \Delta p'_1 - r_1 p'_{c1} \exp[-\alpha_0 r_1^{n_c} (T_1 - T_0)] = 0 \quad (\text{B - 2})$$

$$604 \quad p'_{c1} = p'_{c0} \exp\left[\frac{\nu}{\lambda - \kappa} \Delta \varepsilon_{v1}^p\right] \quad (\text{B - 3})$$

$$605 \quad r_1 = 1 + (r_0 - 1) \exp\left[-\frac{\nu}{\lambda - \kappa} s \Delta \varepsilon_{v1}^p\right] \quad (\text{B - 4})$$

606 Then the soil element is heated up to stress point  $B$  (  $p'_0 + \Delta p'_1, r_2, p'_{c2}, T_1 + \Delta T_1$  ) at  
 607 constant mean effective stress  $p'_0 + \Delta p'_1$ .

$$608 \quad f_{I2} = p'_0 + \Delta p'_1 - r_2 p'_{c2} \exp[-\alpha_0 r_2^{n_c} (T_1 + \Delta T_1 - T_0)] = 0 \quad (\text{B - 5})$$

$$609 \quad p'_{c2} = p'_{c1} \exp\left[\frac{\nu}{\lambda - \kappa} \Delta \varepsilon_{v2}^p\right] = p'_{c0} \exp\left[\frac{\nu}{\lambda - \kappa} (\Delta \varepsilon_{v1}^p + \Delta \varepsilon_{v2}^p)\right] \quad (\text{B - 6})$$

$$610 \quad r_2 = 1 + (r_1 - 1) \exp\left[-\frac{\nu}{\lambda - \kappa} s \Delta \varepsilon_{v2}^p\right] = 1 + (r_0 - 1) \exp\left[-\frac{\nu}{\lambda - \kappa} s (\Delta \varepsilon_{v1}^p + \Delta \varepsilon_{v2}^p)\right] \quad (\text{B - 7})$$

611 Under loading path II, the soil element is firstly heated up to stress point  $D$   
 612 (  $p'_0, r_3, p'_{c3}, T_1 + \Delta T_1$  ) at constant mean effective stress  $p'_0$ .

$$613 \quad f_{I3} = p'_0 - r_3 p'_{c3} \exp[-\alpha_0 r_3^{n_c} (T_1 + \Delta T_1 - T_0)] = 0 \quad (\text{B - 8})$$

$$614 \quad p'_{c3} = p'_{c0} \exp\left[\frac{\nu}{\lambda - \kappa} \Delta \varepsilon_{v3}^p\right] \quad (\text{B - 9})$$

$$615 \quad r_3 = 1 + (r_0 - 1) \exp\left[-\frac{\nu}{\lambda - \kappa} s \Delta \varepsilon_{v3}^p\right] \quad (\text{B - 10})$$

616 Then, the soil element is isotropically loaded to stress point  $B$  (  $p'_0 + \Delta p'_1, r_4, p'_{c4}, T_1 + \Delta T_1$  )

617 at constant temperature  $T_1 + \Delta T_1$ .

$$618 \quad f_{I4} = p'_0 + \Delta p'_1 - r_4 p'_{c4} \exp\left[-\alpha_0 r_4^{n_c} (T_1 + \Delta T_1 - T_0)\right] = 0 \quad (\text{B - 11})$$

$$619 \quad p'_{c4} = p'_{c3} \exp\left[\frac{\nu}{\lambda - \kappa} \Delta \varepsilon_{v4}^p\right] = p'_{c0} \exp\left[\frac{\nu}{\lambda - \kappa} (\Delta \varepsilon_{v3}^p + \Delta \varepsilon_{v4}^p)\right] \quad (\text{B - 12})$$

$$620 \quad r_4 = 1 + (r_3 - 1) \exp\left[-\frac{\nu}{\lambda - \kappa} s \Delta \varepsilon_{v4}^p\right] = 1 + (r_0 - 1) \exp\left[-\frac{\nu}{\lambda - \kappa} s (\Delta \varepsilon_{v3}^p + \Delta \varepsilon_{v4}^p)\right] \quad (\text{B - 13})$$

621 Compare Eqs.B-5, B-6, B-7 and Eqs.B-11, B-12, B-13, it appears clearly that the two  
622 different loading paths would reach the same final stress state, namely:

$$623 \quad p'_{c2} = p'_{c4} \quad (\text{B - 14})$$

$$624 \quad r_2 = r_4 \quad (\text{B - 15})$$

$$625 \quad \Delta \varepsilon_{v1}^p + \Delta \varepsilon_{v2}^p = \Delta \varepsilon_{v3}^p + \Delta \varepsilon_{v4}^p \quad (\text{B - 16})$$

626 No matter Path I or Path II is followed, the plastic volumetric strain will be equal, which  
627 means that plastic volumetric strain in ACC2-T model is also loading path independent.

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755 **LIST OF TABLES**

756 Table.1 Model parameters of different fine-grained soils

**Table.1 Model parameters of different fine-grained soils**

Parameters	$\kappa$	$\lambda$	$\bar{p}'_{c0,0}$ (kPa)	$r_{0,0}$	$\nu$	$\alpha_0$ ( $^{\circ}C^{-1}$ )	$\alpha_1$ ( $^{\circ}C^{-1}$ )	$s$	$T_0$ ( $^{\circ}C$ )	$n_c$	$M_{g0}$	$M_f$	$A_d$	$k_f$	$k_g$
<b>Boom clay[8]</b>	0.02	0.18	6000	0.33	-	$5 \times 10^{-3}$	$5 \times 10^{-5}$	8	21.5	-1	0.67	0.67	0.1	0.7	0.9
<b>Geneva clay[25]</b>	0.017	0.062	125	1	0.3	$5.2 \times 10^{-3}$	$1.5 \times 10^{-5}$	12	20	3.5	0.94	0.94	0.1	1.1	1.1
<b>Bonny silt[27]</b>	0.011	0.075	1200	*	0.3	$1.6 \times 10^{-3}$	$3.5 \times 10^{-5}$	10	18	4	1.3	1.3	0.2	1.1	1.1
<b>Pontida clay[12 ]</b>	0.016	0.103	2500	1	-	$3.5 \times 10^{-3}$	$5 \times 10^{-5}$	10	23	1	-	-	-	-	-

‘-’ means the parameters are irrelevant to the simulations in this study.

‘\*’ means different data are mentioned in this study.

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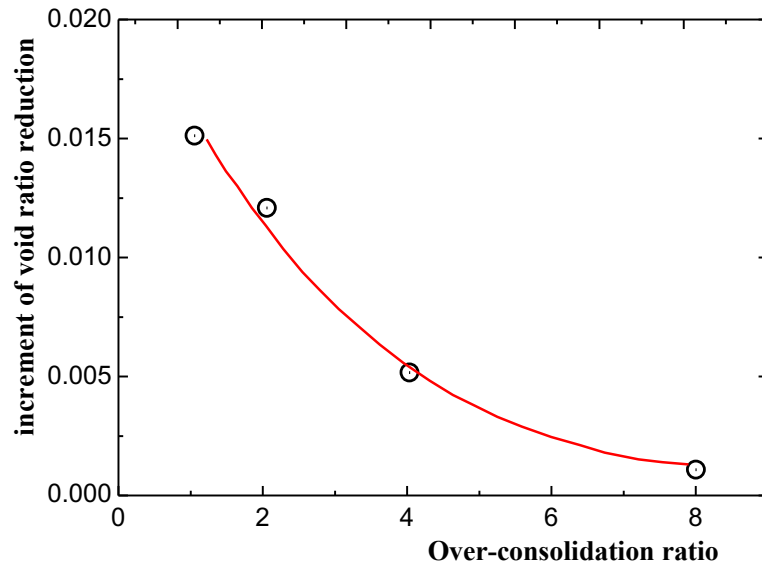


Fig.1 Permanent reduction in void ratio due to temperature cycles versus over-consolidation ratio (replotted from Demars and Charles [23])



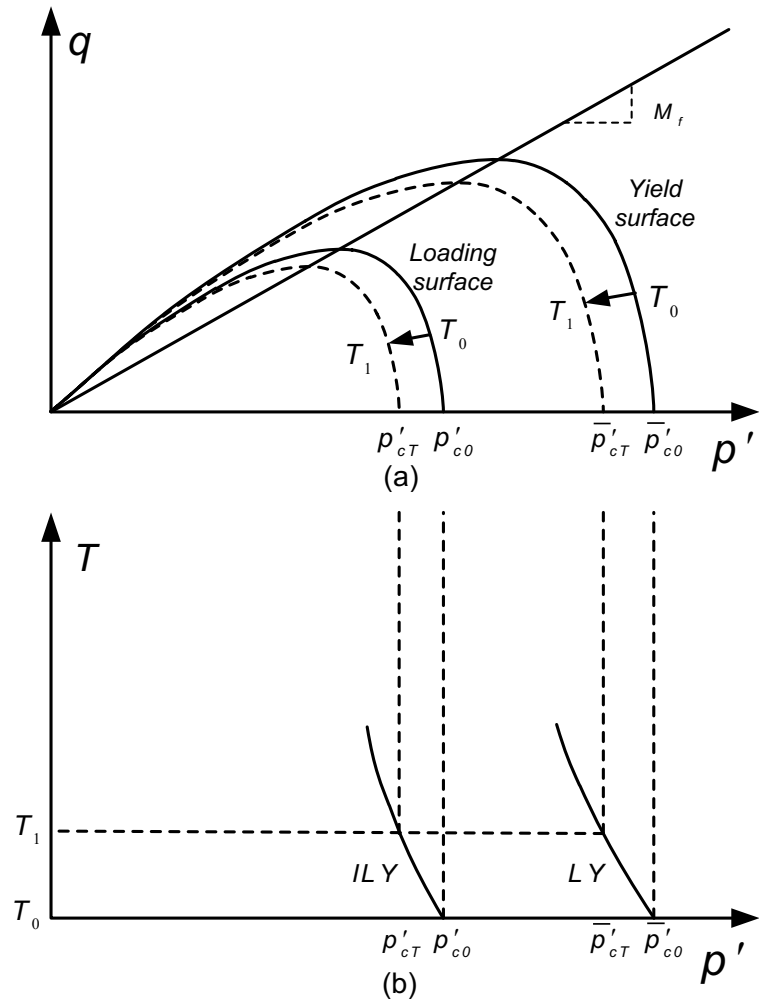
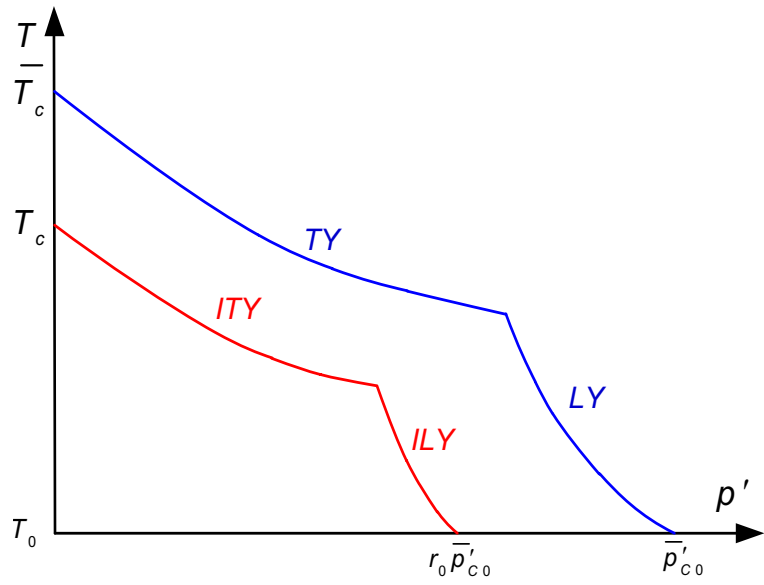
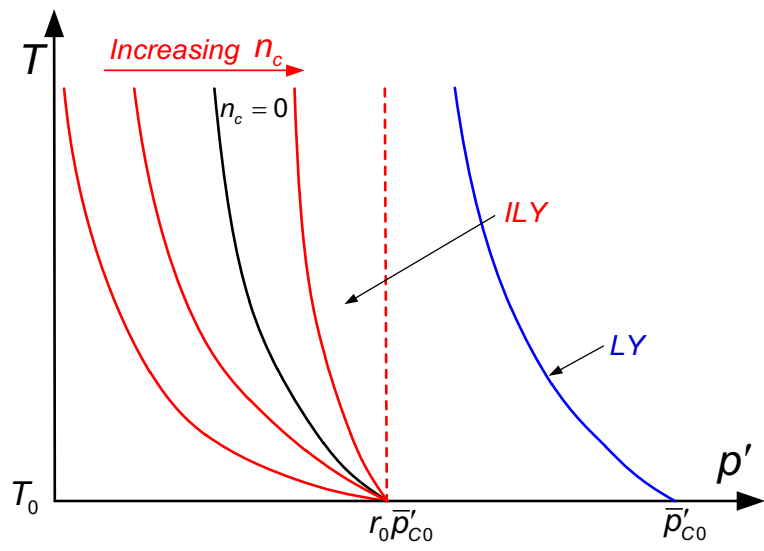


Fig.2 Yield locus in (a):  $(p', q)$  plane at two different temperatures (with  $T_1 > T_0$ ) and (b): two loading yield limits  $ILY$  and  $LY$  in  $(p'-T)$  plane.



(a)



(b)

Fig.3 Yield limits in  $(p', T)$  plane: (a) TEAM Model (b) ACC2-T Model

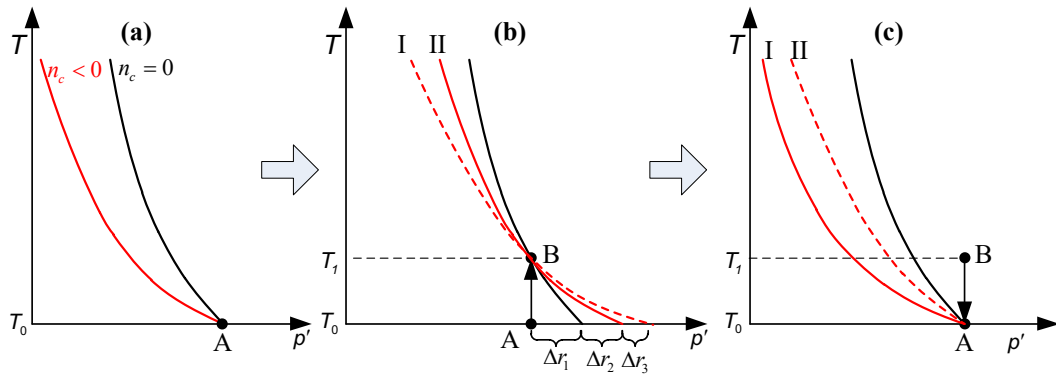


Fig.4 Evolution of yield limits in  $(p', T)$  plane during one heating and cooling cycle

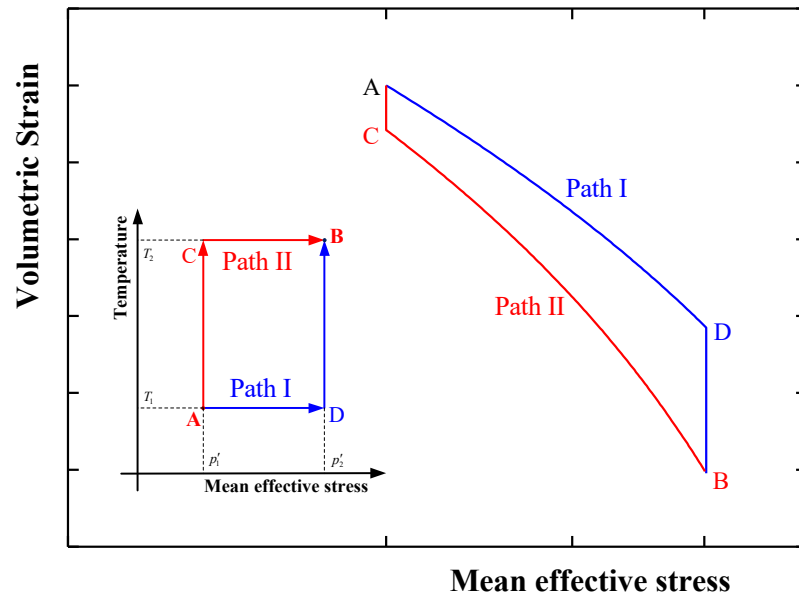
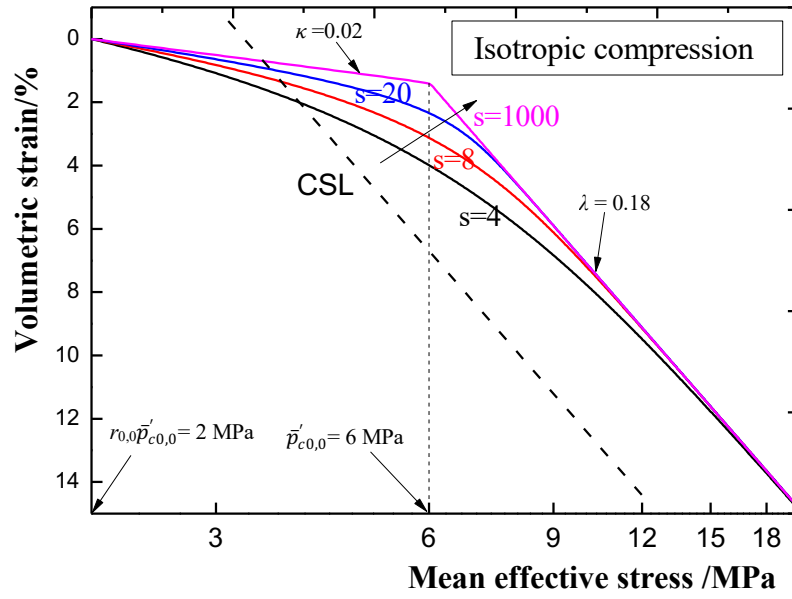
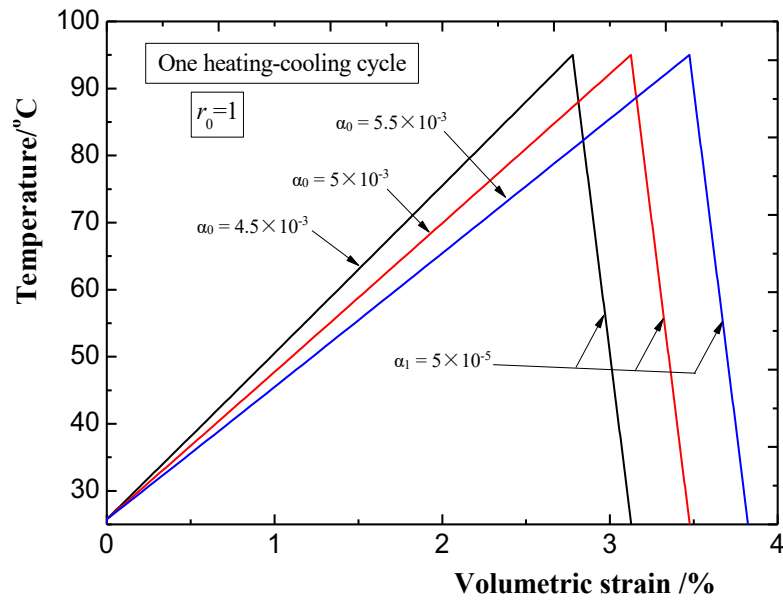


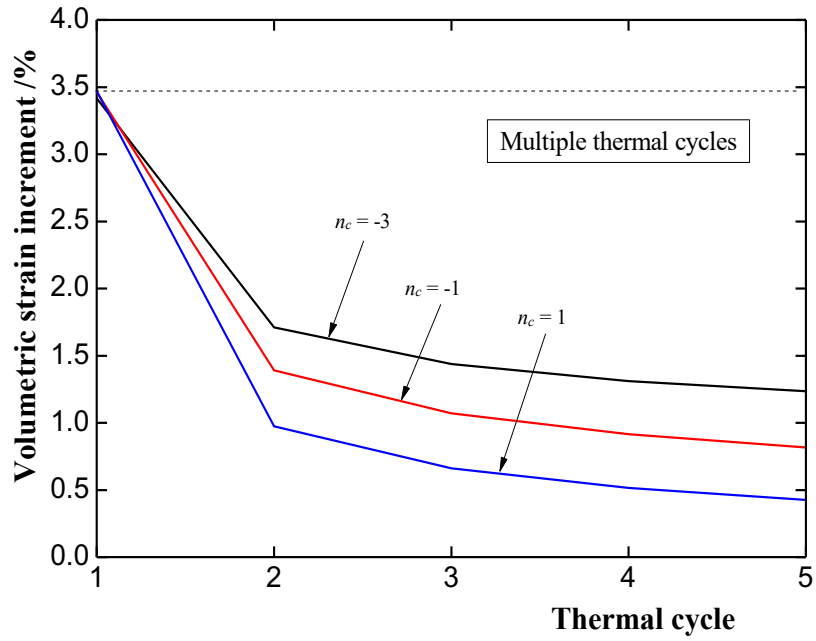
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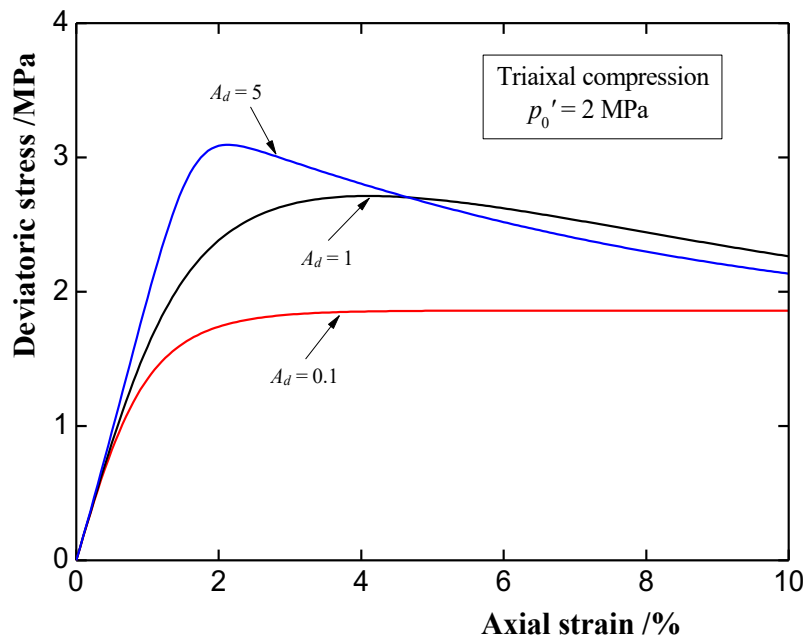
(a)



(b)



(c)



(d)

Fig.6 Calibration process of ACC2-T model on set of material parameters for Boom clay: (a) isotropic compression test; (b) one heating-cooling cycle at normal consolidated state; (c) multiple thermal cycles; (d) triaxial compression test at over-consolidated state

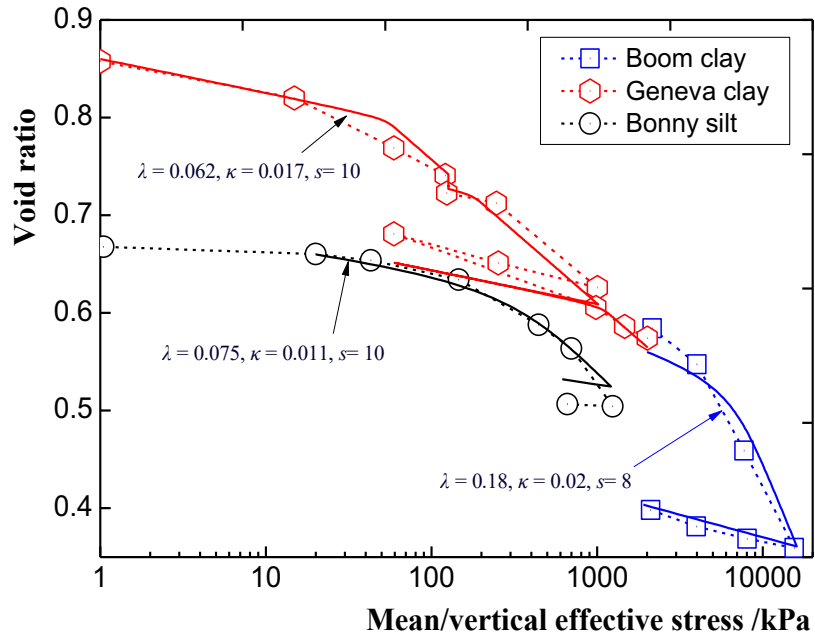


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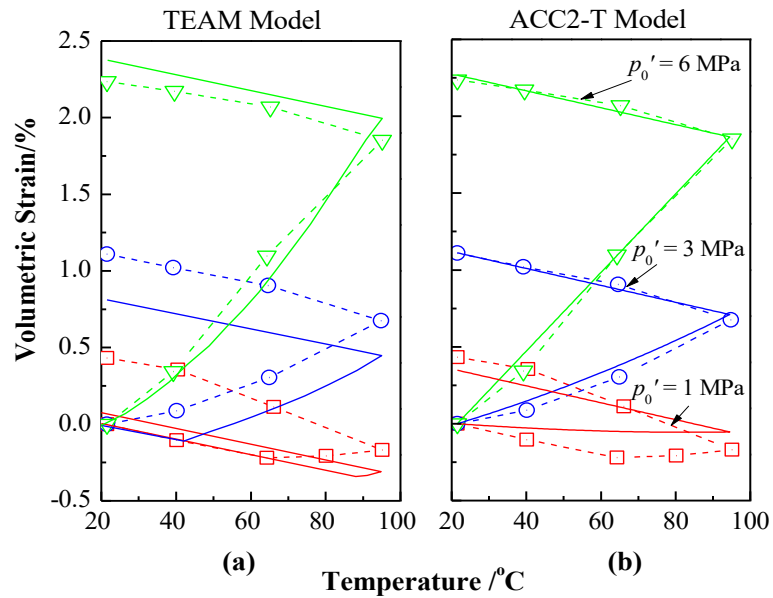
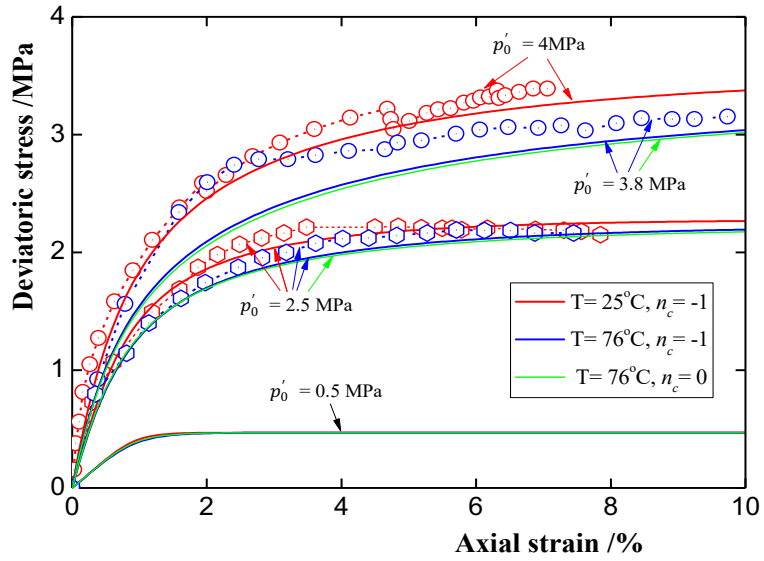
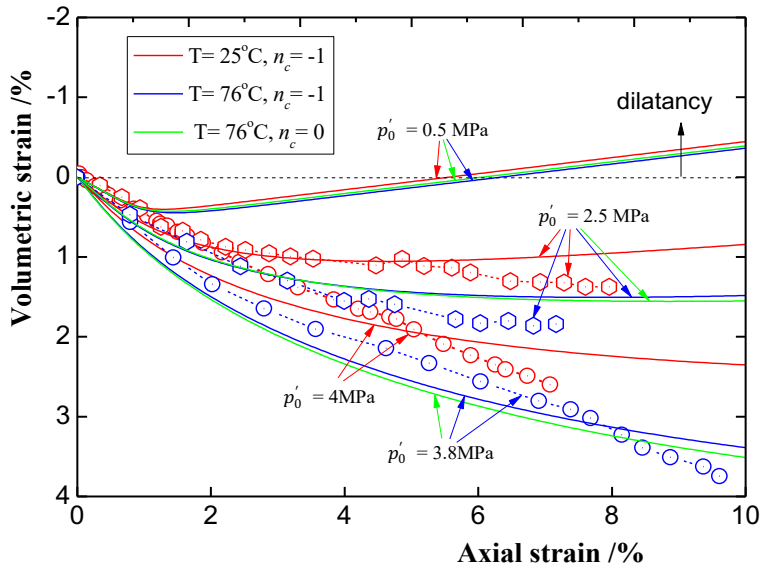


Fig.8 Volumetric strain change of Boom clay due to one heating and cooling cycle with different mean effective stresses: (a) TEAM model (b) ACC2-T model



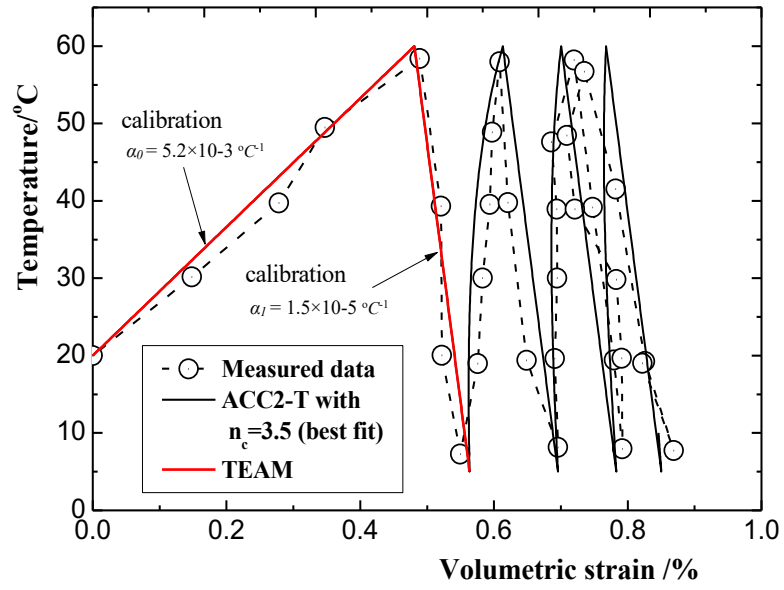
(a)



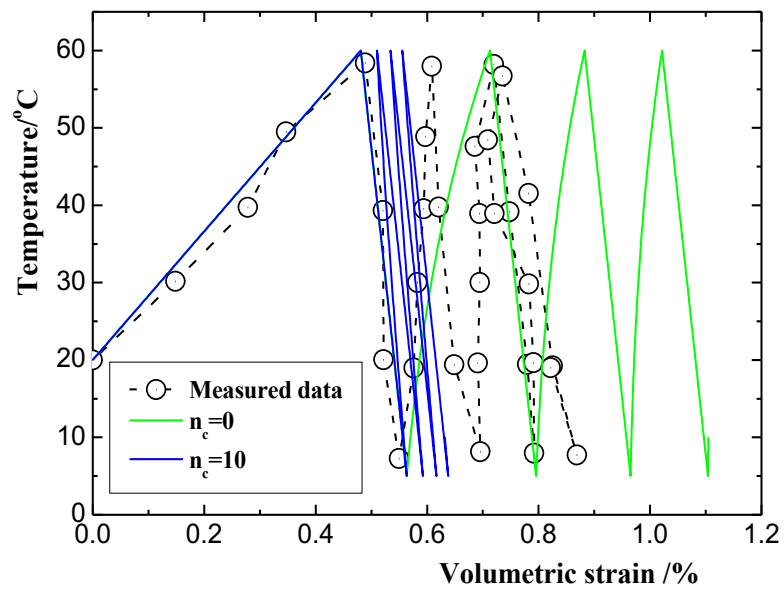
(b)

Fig. 9 Triaxial drained behaviors of Boom clay at room temperature  $25^{\circ}\text{C}$  and high temperature  $76^{\circ}\text{C}$ : (a) deviatoric stress vs axial strain; (b) volumetric strain vs axial strain;

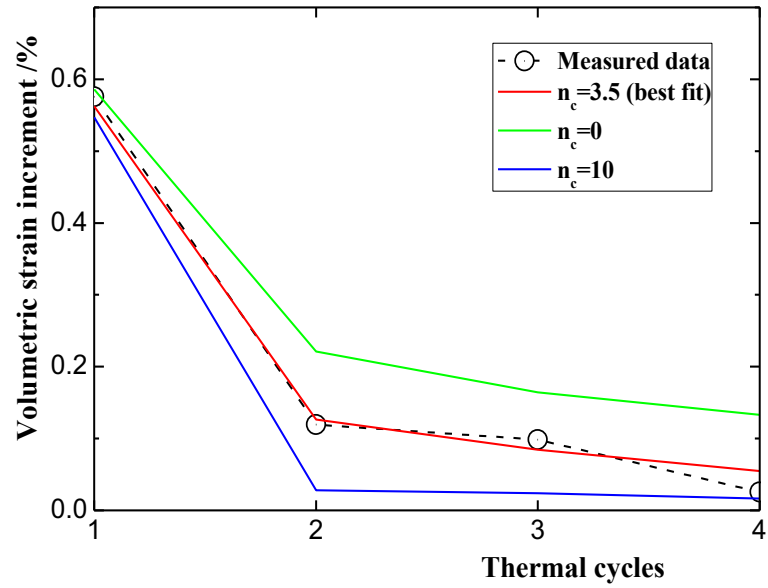




(a)



(b)



(c)

Fig.10 Thermal cyclic behaviors of normal consolidated Geneva clay under drained condition: (a) comparison between ACC2-T model with  $n_c=3.5$  (best fit) and TEAM model; (b) ACC2-T model with  $n_c=0$  and  $n_c=10$ ; (c) volumetric strain increment due to each thermal cycle by ACC2-T model

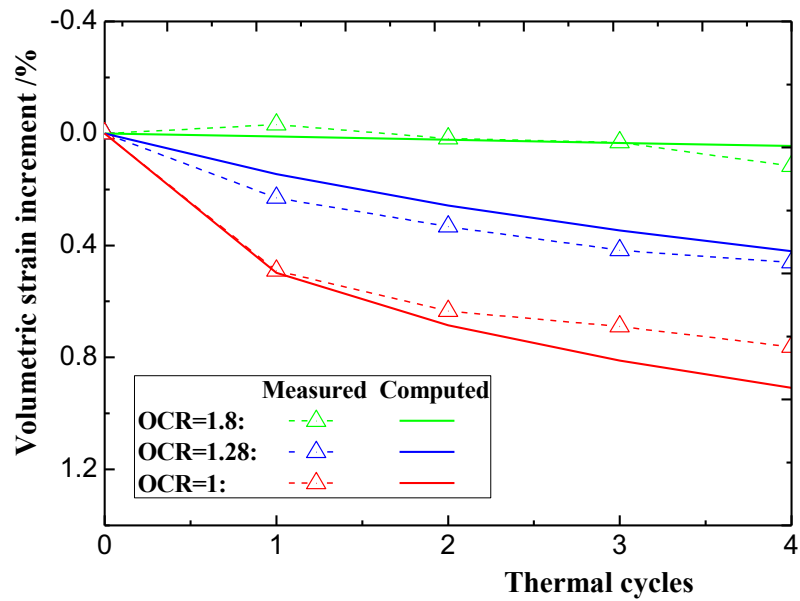
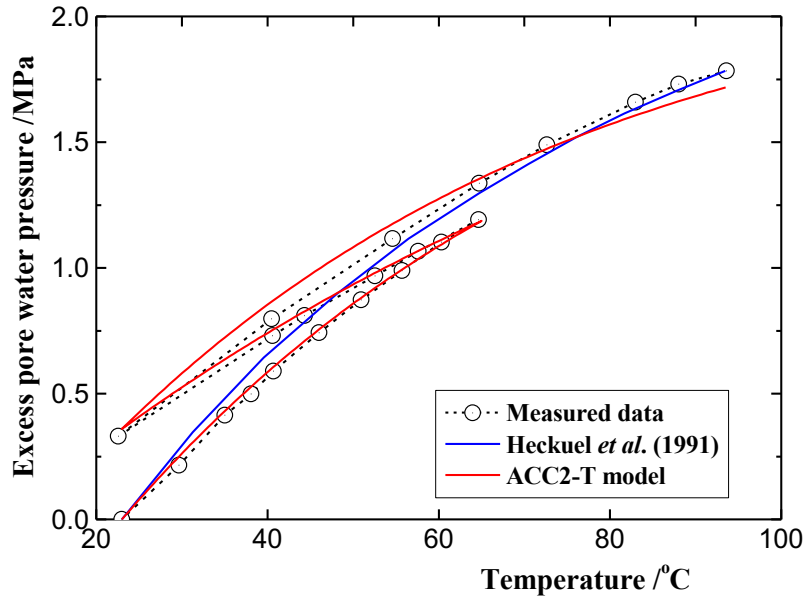
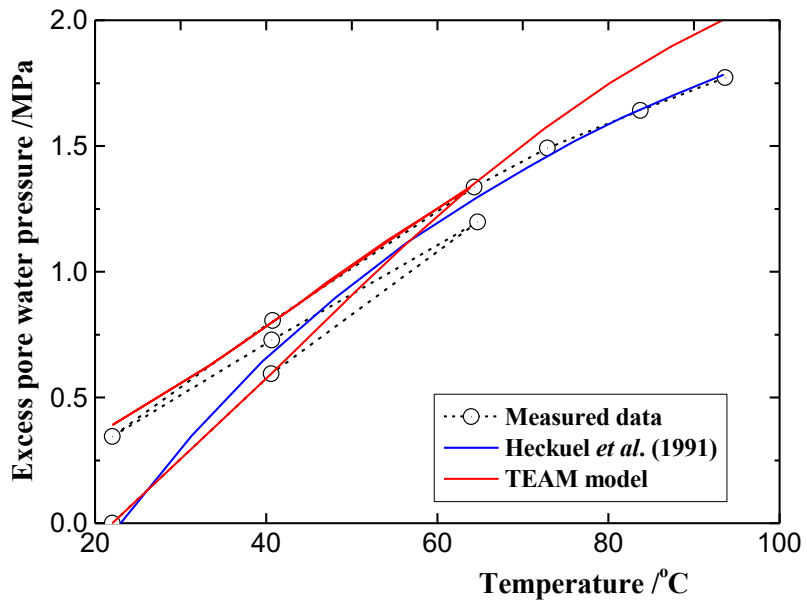


Fig.11 Accumulated strain behavior of Bonny Silt with different OCRs under drained condition



(a)



(b)

Fig.12 Excess pore water pressure accumulation of Pontida clay due to thermal cycles

under undrained condition: (a) ACC2-T model; (b) TEAM model

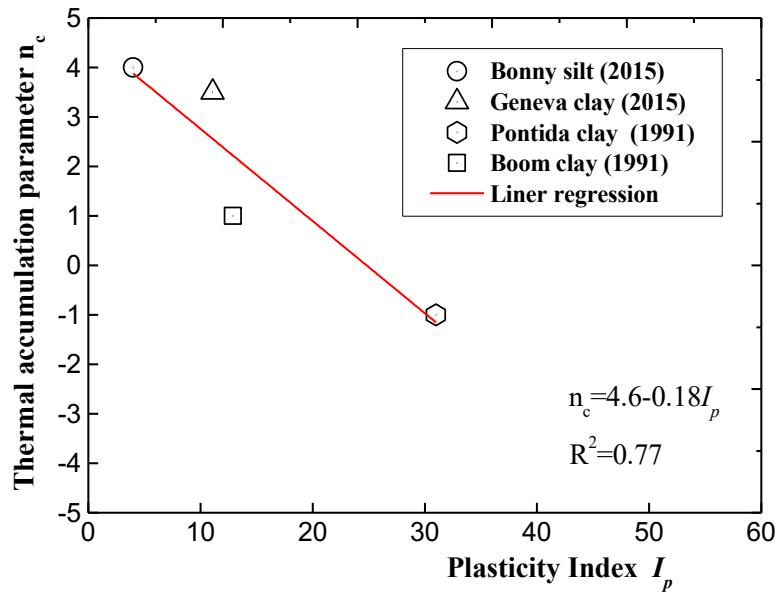


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index  $I_p$