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1 **A two-surface thermo-mechanical plasticity model considering**
2 **thermal cyclic behavior**

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22 **Abstract**

23 In thermal-related engineering such as thermal energy structures and nuclear waste
24 disposal, it is essential to well understand volume change and excess pore water
25 pressure buildup of soils under thermal cycles. However, most existing thermo-
26 mechanical models can merely simulate one heating-cooling cycle and fail in capturing
27 accumulation phenomenon due to multiple thermal cycles. In this study, a two surface
28 elasto-plastic model considering thermal cyclic behavior is proposed. This model is
29 based on the bounding surface plasticity and progressive plasticity by introducing two
30 yield surfaces and two loading yield limits. A dependency law is proposed by linking
31 two loading yield limits with a thermal accumulation parameter n_c , allowing the thermal
32 cyclic behavior to be taken into account. Parameter n_c controls the evolution rate of the
33 inner loading yield limit approaching the loading yield limit following a thermal loading
34 path. By extending the thermo-hydro-mechanical equations into the elastic-plastic state,
35 the excess pore water pressure buildup of soil due to thermal cycles is also accounted.
36 Then, thermal cycle tests on four fine-grained soils (natural Boom clay, Geneva clay,
37 Bonny silt and reconstituted Pontida clay) under different OCRs and stresses are
38 simulated and compared. The results show that the proposed model can well describe
39 both strain accumulation phenomenon and excess pore water pressure buildup of fine-
40 grained soils under the effect of thermal cycles.

41 **Keywords:** fine-grained soils; strain accumulation; thermal cycles; two-surface model;
42 excess pore water pressure buildup

43 **1. Introduction**

44 Around energy piles [1-3], nuclear waste canisters [4], buried tunnels and pipes [5], the
45 soils suffer from temperature effects, especially heating-cooling cycles. Observations
46 of different fine-grained soils in the laboratory and in the field [6-11] showed that
47 heating generally induced non-negligible volume change and excess pore water
48 pressure buildup. Several thermo-mechanical models were proposed to describe this
49 thermo-mechanical behavior of fine-grained soils [13-21]. Some of them extended the
50 Cam-clay or modified Cam-clay model to the temperature effect by introducing a
51 shrinking yield surface [13-17]. These models could describe some basic thermo-
52 mechanical behavior, but fail to describe the soil behavior in over-consolidated states.
53 In order to well describe the small thermo-mechanical elastic zone and the shear
54 softening behavior, some advanced models consider two yield surfaces based on the
55 bounding surface theory [18-21]. Hong et al. [22] made a comparison among some of
56 these advanced models, and showed that the multiple plastic hardening mechanism
57 were of paramount importance in modeling complex thermal loading behavior.

58 Though numerous efforts were made in thermal mechanical modeling, most existing
59 models can only simulate the case of one thermal cycle and fail in describing the soil
60 behavior under multiple heating-cooling cycles. As more attention has been paid to the
61 long-term performance [11,12,25,26,27,40], the volume change and excess pore water
62 pressure accumulation due to temperature variation can act as a significant factor
63 influencing the safety of energy structures. Demars and Charles [23] reported a series

64 of drained thermal cycle tests on high plastic marine clays and found that permanent
65 reduction in void ratio (a function of over-consolidation ratio) gradually reduced due to
66 temperature cycles (see Fig.1). Plum et al. [24] conducted an undrained heating-cooling
67 cycle tests on Newfield clay. After four thermal cycles, the accumulation of excess pore
68 water pressure was still increasing. Pastén et al. [40] experimentally observed the
69 thermally driven displacement accumulation with the number of temperature cycles and
70 developed a thermally induced ratcheting mechanism for a smooth interface. All those
71 observations show that multiple thermal cycles should be considered in thermal
72 mechanical modeling of clays. Zhou et al. [28] recently proposed a bounding surface
73 model with a memory surface to consider the thermal cyclic behavior. However, this
74 imposed memory surface actually has no effect on the accumulation behavior of
75 normally consolidated soil because this memory surface coincides with the bounding
76 surface in this circumstance. Ma et al. [29] proposed a thermal stabilization line in
77 hypoplastic modeling approach. Though three thermal parameters are introduced, the
78 proposed hypoplastic model fails to describe the strain accumulation during relatively
79 small temperature changes because it always simulates an unrealistic elastic behavior
80 before entering the elasto-plastic zone following a thermal loading path. More recently,
81 Zymnis et al. [41] proposed a simple calibration procedure for Tsinghua ThermoSoil
82 (TTS) model and studied the capabilities of simulating the long-term accumulation of
83 strains due to thermal cycles. However, those existing models for thermal cyclic
84 behavior merely considered the strain accumulation behavior under drained conditions

85 though the excess pore water pressure buildup due to temperature effects are at least of
86 equal importance from the best of the authors' knowledge.

87 In the present work, a two surface thermo-plastic model, namely ACC2-T, is
88 proposed to consider the thermal cyclic behavior based on the previous TEAM model
89 by Hong et al. [21]. The proposed model unifies the loading yield limits and the thermal
90 yield limits. To connect the inner loading yield limit and the loading yield limit, a
91 flexible dependency law is incorporated and combined with isotropic and progressive
92 hardening laws to account for multiple thermal cycles. In the dependency law, a
93 parameter n_c is introduced to control accumulation rate through the evolution rate of the
94 inner loading yield limit approaching the loading yield limit. Meanwhile, the proposed
95 model is extended to undrained condition by applying the thermo-hydro-mechanical
96 equations by Coussy [30] into the elastic-plastic state. Then, the rationale of ACC2-T
97 model is discussed in terms of elasto-plasticity and thermodynamic theories. Lastly, the
98 thermal cycle test results of four fine-grained soils (natural Boom clay, Geneva clay,
99 Bonny silt and reconstituted Pontida clay) under different OCRs and stresses are used
100 to verify the proposed model.

101

102 **2. Model description**

103 In this section, the proposed model (ACC2-T) is presented using typical constitutive
104 variables, namely mean effective stress p' , deviator stress q , temperature T and the
105 corresponding increments (dp' , dq , dT). The soil behavior is assumed to be isotropic

106 and the constitutive model is developed and formulated in the conventional triaxial
107 conditions.

108

109 **2.1 Thermo-elastic behavior**

110 In ACC2-T model, the soil deformation can be divided into thermo-elastic and thermo-
111 plastic parts.

112 In the thermo-elastic part, the elastic law adopted in many other models [31,36,37]
113 in isothermal conditions are extended into non-isothermal conditions for its simplicity.

114 the total elastic volumetric strain increment associated with mechanical and thermal
115 loading and the elastic shear strain assumed purely mechanical are calculated as follows:

$$116 \quad d\varepsilon_v^e = \frac{\kappa dp'}{\nu p'} - \alpha_1 dT, \quad d\varepsilon_s^e = \frac{dq}{3G}, \quad G = \frac{3(1-2\nu)}{2(1+\nu)} \frac{\nu p'}{\kappa} \quad (1)$$

117 where $d\varepsilon_v^e$, $d\varepsilon_s^e$ is the elastic volumetric strain increment ($d\varepsilon_v^e = d\varepsilon_1^e + d\varepsilon_2^e + d\varepsilon_3^e$)

118 and the elastic shear strain increment ($d\varepsilon_s^e = \frac{2}{3}(d\varepsilon_1^e - d\varepsilon_3^e)$). κ is the elastic slope in (v ,

119 $\ln p'$) plane and ν is the specific volume. α_1 is the thermal elastic compressibility index

120 which assumed a positive constant because heating induces soil expansion. G is shear
121 modulus and ν is the Poisson's ratio.

122

123 **2.2 Thermo-plastic behavior**

124 The thermo-plastic mechanism of ACC2-T model consists of two yield surfaces and
125 plastic potential surface, the dependency law, the isotropic and progressive hardening

126 laws and the flow rules.

127

128 2.2.1 Yield surfaces and plastic potential surfaces

129 In ACC2-T model, two yield surfaces, namely loading surface (f_l) and yield surface (f_Y),

130 with a generalized formulation [31] are implemented to model the yielding of both stiff

131 and soft clays.

132 Loading Surface:
$$f_l = q^2 + \frac{M_f^2}{1-k_f} \left(\frac{p'}{r\bar{p}'_{cT}} \right)^{k_f} (r\bar{p}'_{cT})^2 - \frac{M_f^2 p'^2}{1-k_f} = 0 \quad (2)$$

133 Yield Surface:
$$f_Y = q^2 + \frac{M_f^2}{1-k_f} \left(\frac{p'}{\bar{p}'_{cT}} \right)^{k_f} \bar{p}'_{cT}^2 - \frac{M_f^2 p'^2}{1-k_f} = 0 \quad (3)$$

134 where M_f is the stress ratio at the apex of two surfaces, k_f is a parameter specifying the

135 shape of yield surfaces, \bar{p}'_{cT} is the pre-consolidation pressure at current temperature T

136 and r is the inverse of over-consolidation ratio (OCR) at current temperature T . The

137 conceptual plot of two yield surfaces at various temperature is given in Fig. 2(a). To

138 consider the thermo-mechanical behavior, the two surfaces are extended into three-

139 dimensional plot (p' , q , T) with two loading yield limits. The two loading yield limits,

140 namely the inner loading yield limit (ILY) and the loading yield limit (LY) are linked by

141 the proposed dependency law which is discussed in *Yield limits and dependency law*

142 section.

143 The plastic potential surface g_l takes the same form as for loading surface, but with

144 different parameters M_g and k_g :

145
$$g_I = q^2 + \frac{M_g^2}{1-k_g} \left(\frac{p'}{r\beta} \right)^{\frac{2}{k_g}} (r\beta)^2 - \frac{M_g^2 p'^2}{1-k_g} = 0 \quad (4)$$

146 and it leads to a dilatancy equation in the case of triaxial compression:

147
$$\frac{d\varepsilon_v^p}{d\varepsilon_s^p} = \frac{\frac{\partial g_I}{\partial p'}}{\frac{\partial g_I}{\partial q}} = \frac{M_g^2 - \eta^2}{k_g \eta} \quad (5)$$

148 where $d\varepsilon_v^p$ is the plastic volumetric strain increment, $d\varepsilon_s^p$ is the plastic shear strain
 149 increment, η denotes the stress ratio q/p' , M_g defines the critical stress ratio at failure
 150 where no further volumetric strain increment develops, k_g is a constant parameter which
 151 controls the ratio of plastic volumetric strain increment and plastic shear strain
 152 increment and β is the size parameter. Typically, an unassociated flow rule is adopted
 153 for modeling complex mechanical behaviors. However, when $M_g = M_f$ and $k_g = k_f$, an
 154 associated flow rule would be recovered. When $k_g = k_f = 2$, elliptical surfaces like
 155 modified Cam-clay model are recovered.

156

157 2.2.2 Hardening laws

158 In ACC2-T model, two plastic hardening mechanisms fixed to reference temperature
 159 T_0 are adopted to control the evolution of yield surface and loading surface.

160 In the isotropic hardening rule, as same as in Modified Cam-clay model, the
 161 evolution of pre-consolidation pressure \bar{p}'_{c0} at reference temperature T_0 is linked to the
 162 plastic volumetric strain increment:

163
$$d\bar{p}'_{c0} = \frac{\nu}{\lambda - \kappa} \bar{p}'_{c0} d\varepsilon_v^p \quad (6)$$

164 where λ is the normal consolidation slope in $(v, \ln p')$ plane and $d\bar{p}'_{c0}$ is the increment
 165 of pre-consolidation pressure. It means the evolution of yield surface is dominated by
 166 the plastic volumetric strain.

167 The progressive hardening variable dr_0 , at reference temperature T_0 , is defined with
 168 a combined volumetric and shear plastic strain as follows:

$$169 \quad dr_0 = \frac{v}{\lambda - \kappa} s (1 - r_0) (d\varepsilon_v^p + A_d d\varepsilon_s^p) \quad (7)$$

170 where r_0 is the inverse of over-consolidation ratio (OCR) at current temperature T_0 and
 171 dr_0 is the increment. s is a material constant which controls the hardening rate of loading
 172 surface in approaching yield surface. A_d is a material constant which controls the
 173 contribution of shear plastic strain.

174 In this progressive hardening rule, parameter r_0 is always less than or equal to 1 ($0 <$
 175 $r_0 \leq 1$). When the loading surface is completely inside the yield surface ($0 < r_0 < 1$), it
 176 means the soil is over-consolidated. Within the hardening process, plastic strain
 177 increases and the loading surface begins to move towards the yield surface ($dr_0 > 0$).
 178 When $r_0 = 1$, the loading surface and the yield surface coincide. The soil turns into
 179 normally consolidated state as the same case of MCC model. Note that the loading
 180 surface could not run out of the yield surface ($r_0 > 1$) for the fact that if $r_0 > 1$, $dr_0 < 0$
 181 holds. With such a combined volumetric-deviatoric hardening law, the inclusion of
 182 shear hardening part ($d\varepsilon_s^p > 0$) would lead to a positive plastic modulus ($h > 0$) even
 183 when the effective stress state ($d\varepsilon_v^p = 0$) reach the critical state line (CSL). As a
 184 consequence, the stress point lying on the loading surface can cross the CSL,

185 hardening continues to a stress point ($d\varepsilon_v^p < 0$, $d\varepsilon_s^p > 0$, $h = 0$) and then the stress point
186 ($d\varepsilon_v^p < 0$, $d\varepsilon_s^p > 0$, $h < 0$) would move back to the CSL. This approach allows to
187 predict shear softening behavior and it is often the case of highly over-consolidated
188 states of soil.

189 With these isotropic and progressive hardening rules, the plastic modulus and
190 increment stress-strain relationship of ACC2-T model are obtained in Appendix A.

191 Under loading and heating processes, two hardening variables \bar{p}'_{c0} and r_0 are
192 activated with the evolution of plastic strain. However, under unloading and cooling
193 processes, pure elastic strain develops with a fixed value of \bar{p}'_{c0} . Due to ACC2-T model
194 strictly adhering to the principle that the stress point is always lying on the loading
195 surface, the value of r_0 could be re-calculated by a simple Newton-Raphson Method
196 with unconditional stability.

197

198 *2.2.3 Yield limits and dependency law*

199 In the two surface models, the loading surface and the yield surface are geometrically
200 similar in (p', q) plane with a corresponding radial mapping rule. In previous TEAM
201 model [21], two yield limit mechanisms, namely thermal yield limits (ITY , TY) and
202 loading yield limits (ILY , LY) are adopted to model the thermal behavior. These yield
203 limit mechanisms in TEAM model (see Fig.3(a)) enable the thermal elastic response in
204 reloading path to occur before entering the elasto-plastic zone. Therefore, the TEAM
205 model could not simulate the strain accumulation phenomenon due to temperature

206 variations. In order to overcome this shortcoming and extend the model to the case of
 207 multiple thermal cycles, the loading yield limits (*ILY*, *LY*) are unified to consider both
 208 the thermal yield and loading yield mechanisms. Meanwhile, a new dependency law is
 209 proposed to connect *ILY* and *LY*.

210 In the dependency law, the pre-consolidation pressure \bar{p}'_{cT} at current temperature T
 211 takes the original form and it would decrease with the increase of temperature:

$$212 \quad \bar{p}'_{cT} = \bar{p}'_{c0} \exp[-\alpha_0(T - T_0)] \quad (8)$$

213 where \bar{p}'_{c0} is pre-consolidation pressure at reference temperature T_0 ; α_0 is a material
 214 constant controlling the thermal plastic behavior.

215 The other part of the dependency law r is defined as below:

$$216 \quad r = \frac{1}{OCR} = \frac{p'}{\bar{p}'} = \frac{q}{\bar{q}} = \frac{p'_{cT}}{\bar{p}'_{cT}} \quad (9)$$

217 where (p', q) , (\bar{p}', \bar{q}) is the corresponding stress point in loading surface and yield
 218 surface at current temperature T , respectively.

219 Unlike TEAM model with the assumption that $r = r_0$, ACC2-T model takes the form
 220 that would make *ILY* not necessarily homologous to *LY* any longer (see Fig.3(b)):

$$221 \quad r = r_0 \exp[-\alpha_0(r_0^{n_c} - 1)(T - T_0)] \quad (10)$$

222 where n_c is a thermal parameter which controls the strain accumulation rate under
 223 multiple thermal cycles.

224 The combined dependency law made up by \bar{p}'_{cT} and r then is expressed by:

$$225 \quad r\bar{p}'_{cT} = r_0\bar{p}'_{c0} \exp[-\alpha_0 r_0^{n_c} (T - T_0)] \quad (11)$$

226 In the proposed law, the shape of *ILY* is governed by the key thermal accumulation

227 parameter n_c . Parameter n_c controls the evolution rate of ILY approaching LY by the two
228 isotropic and progressive hardening rules. In order to illustrate the plastic mechanism
229 under the thermal path, two cases in the isotropic states, namely $n_c < 0$ and $n_c = 0$, are
230 compared in Fig.4. Both two cases are at initial stress state A, then heated up to stress
231 state B and finally cooled down to stress state A. Under the heating path from A to B,
232 the ILY without n_c ($n_c = 0$) would induce a increment of Δr_1 . For the case of $n_c < 0$, it
233 would induce a increment of $\Delta r_1 + \Delta r_2 + \Delta r_3$ if the shape of ILY is fixed. However, with
234 the effect of n_c , the original ILY (see Fig.4(b), dotted line I) would gradually turn into
235 solid line II with a actual increment of $\Delta r_1 + \Delta r_2$. The larger increment $\Delta r_1 + \Delta r_2$ in
236 hardening variable dr_0 would mean larger plastic volumetric strain increments than the
237 case of $n_c = 0$. Under the cooling path from B to A, the mechanical response is purely
238 elastic and the shape of ILY would turn from II into I. For the case of $n_c > 0$, the
239 increment in hardening variable dr_0 would induce smaller strain increments than $n_c = 0$.
240 Under multiple thermal cycles, the value of r_0 would be smaller and smaller.

241 Under such a unique dependency law, the evolution rate of ILY is actually the
242 evolution rate of strain accumulation. With a proper selection of parameter n_c in the
243 dependency law, ACC2-T model allows the strain accumulation under the effect of
244 multiple thermal cycles to be modelled. Some important features about the dependency
245 law and parameter n_c are summarized below:

246 1. When the current temperature T is equal to the reference temperature T_0 , r becomes
247 equal to r_0 . It means that n_c would have no effect on the stress-strain relationship at

248 reference temperature T_0 .

249 2. When soil turns into the normal consolidated state ($r_0 = 1$), r is also equal to 1. In that
250 case, ILY would coincide with LY . It means that n_c would have no effect on the behavior
251 of normal consolidated soils under either isothermal or non-isothermal condition.

252 3. When n_c is equal to zero (bold line in Fig. 3(b)), r becomes equal to r_0 . In that case,
253 a similarity between ILY and LY would be recovered. This is just the case of ILY and LY
254 used in TEAM model [21].

255 4. Theoretically, the value of n_c can be chosen from negative infinity to positive infinity.
256 When n_c tends to positive infinity, ILY would tend to be the dotted line in Fig. 3(b).
257 When n_c tends to negative infinity, ILY would tend to approach the temperature axis T
258 in Fig. 3(b). A larger n_c represents a quicker strain accumulation rate which would
259 induce smaller total volumetric strain accumulation under the effect of thermal cycles.

260 5. Any possible r is smaller than 1. During the heating process, the hardening
261 mechanism enables r_0 to approach 1. Thus, r would also have its limit value of 1 which
262 implies that the over-consolidation ratio of soil is always larger than or equal to 1.

263

264 **3. Loading path independence**

265 After the description of the proposed model extended to thermal cyclic behavior, it is
266 essential to discuss the rationale of ACC2-T model in terms of elasto-plastic theory and
267 thermodynamic theory as Hong et al. [22] did on three existing thermo-mechanical
268 models [19,20,25,28,29]. The verification of loading path independence is made up by

269 both the thermo-elastic and thermo-plastic part. The schematic plot is given in Fig.5.
270 Heating is treated as a special isotropic loading. Therefore, merely isotropic condition
271 ($q=0$) is considered during the derivation. The thermo-elastic part is discussed as
272 below and the derivation in thermo-plastic regime is attached in Appendix B.

273 In the thermo-elastic mechanism (Eq.3) of ACC2-T model, the bulk modulus
274 $K = \frac{\nu p'}{\kappa}$ is independent of temperature, and thermal elastic compressibility index α_1
275 is also independent of mean effective stress. Such elastic volumetric strain in ACC2-T
276 model is surely loading path independent in (p', T) plane. Note that the thermo-elastic
277 mechanism leads to loading path dependent in (p', q) plane since the Maxwell symmetry
278 relations are not satisfied. A possible way is to extend a hyperelastic law [39] into non-
279 isothermal conditions. For simplicity of parameter calibration, merely this thermo-
280 elastic mechanism is considered here.

281

282 **4. Analysis and determination of model parameters**

283 The proposed ACC2-T model contain fifteen parameters ($\lambda, \kappa, \bar{p}'_{c0}, \nu, M_f, k_f, M_g, k_g, s,$
284 $r_0, A_d, \alpha_0, \alpha_1, T_0, n_c$). Five of them ($\lambda, \kappa, \bar{p}'_{c0}, \nu, M_g$) are commonly defined in MCC
285 model. $(M_f, k_f), (M_g, k_g)$ control the shapes of yield surfaces and plastic potential
286 surfaces, respectively. Three parameters (s, r_0, A_d) are used to consider a progressive
287 yielding of two surfaces. Four parameters ($\alpha_0, \alpha_1, T_0, n_c$) are defined to consider the
288 thermal influence on the soil behavior. The procedure for determining these parameters
289 is described and key parameters ($\lambda, \kappa, \bar{p}'_{c0}, s, r_0, \alpha_0, \alpha_1, T_0, n_c$) related to thermal

290 accumulation behavior are discussed on the set of model parameters for natural Boom
291 clay in Fig. 6.

292 1. By an isotropic compression test or oedometer test under reference temperature T_0 ,
293 κ , λ , the initial $r_{0,0}$ and $\bar{p}'_{c0,0}$ are determined in $(v, \ln p')$ plane in Fig.6(1). Note that
294 parameter s controls the hardening rate of loading surface in approaching yield surface.
295 It can be determined by fitting nonlinearity from over-consolidated states to normal
296 consolidated state in $(v, \ln p')$ plane.

297 2. By an isotropic heating-cooling test at normally consolidated states, α_1 is
298 determined by fitting data of cooling stage and α_0 is determined by the heating test in
299 $(T, \Delta\varepsilon_v)$ plane in Fig.6(2).

300 3. By a continuous thermal cycle test, the volumetric strain increment by each thermal
301 cycle $\Delta\varepsilon_{v,st}$ can be obtained $(\Delta\varepsilon_{v,st}, n)$ plane in Fig.6(3). The thermal accumulation
302 parameter n_c , which controls the strain accumulation rate, is then determined by
303 calibrating the strain accumulation under each thermal cycle.

304 4. M_f , k_f . M_f denotes the apex of yield surface and k_f controls the shape of yield surface.
305 They are calibrated by fitting the conventional yield points of various stress paths.

306 5. M_g is the critical state stress ratio and it is calibrated by the effective stress ratio at
307 critical state.

308 6. k_g controls plastic flow direction. It is determined by fitting the values of $d\varepsilon_v^p/d\varepsilon_s^p$
309 in drained triaxial shear tests.

310 7. The poisson's ratio ν is determined by a triaxial tests at low strain level in the $(\varepsilon_v-\varepsilon_l)$

311 plane: $v = (\varepsilon_v - \varepsilon_l)/2$.

312 8. A_d represents the contribution of the plastic shear strain in the hardening process. It
313 is calibrated by fitting the results of triaxial compression tests at over-consolidated
314 states in (q, ε_l) or $(\varepsilon_v, \varepsilon_l)$ plane in Fig.6(4).

315

316 **5. Model Validation and prediction**

317 In this section, four different fine-grained soils, namely natural Boom clay, Geneva clay,
318 Bonny silt and reconstituted Pontida clay, are adopted to verify the capability of ACC2-
319 T model in modeling the thermal cyclic behavior.

320

321 **5.1 Natural Boom clay**

322 One heating and cooling test on natural Boom clay ($I_p = 31$) under isotropic stress
323 states performed by Baldi et al. [8] and two groups of triaxial compression tests
324 conducted by Lê [38] are taken into account.

325 In the heating and cooling test, specimen TBoom-9 taken from Mol at an in-situ depth
326 of 223 m (initial pre-consolidated pressure of 6 MPa) is tested under a mixed
327 mechanical and thermal loading process: the sample was firstly saturated under a
328 constant mean effective stress of 2 MPa at 21.5 °C, loaded up to 4 MPa, unloaded to 1
329 MPa, heated up to 95 °C and cooled back to 21.5 °C, loaded up to 3 MPa at 21.5 °C
330 and heated up to 95 °C and cooled back, loaded up to 6 MPa and heated up again and
331 finally cooled back. In the loading process, three thermal cycles of temperature

332 increment of 73.5 °C at different isotropic stress states, namely mean effective stress of
333 1 MPa, 3 MPa and 6 MPa, are selected for simulation. For better comparison between
334 the computed and measured results, the increments of volumetric strain are reset at the
335 starting point of each heating-cooling cycle. Moreover, Fig. 7 contains an isotropic
336 compression test of natural Boom clay from 2 MPa to 20 MPa, then back to 6 MPa.
337 Basic parameters for natural Boom clay are listed in Tab.1.

338 The TEAM model [21] and the proposed ACC2-T model are both used for simulation
339 in Fig.8. It appears that both models can describe the thermal volume change well for
340 normally consolidated or slightly over-consolidated state ($p' = 6$ MPa) adopting the
341 same thermal elastic parameter α_1 and thermal plastic parameter α_0 . However, for
342 highly over-consolidated states ($p' = 3$ MPa, $p' = 1$ MPa), it is observed that ACC2-T
343 model gives a more consistent simulation than TEAM model. One reason is that TEAM
344 model always shows a linear elastic expansion behavior before entering the elasto-
345 plastic zone and has no parameters controlling strain evolution. With this shortcoming,
346 TEAM model would fail to respond to thermal cycles and thus to describe strain
347 accumulation behavior, though complicated multiple plastic mechanisms are adopted.
348 However, in ACC2-T model, the stress point is always lying on the loading surface.
349 Combined with the isotropic and progressive hardening rules, the thermal accumulation
350 parameter n_c in the dependency law allows the ACC2-T model to better describe the
351 nonlinear expansion behavior and strain accumulation under over-consolidated states.
352 Meanwhile, as natural Boom clay is relatively easy to contract under heating, the use

353 of a negative value of parameter n_c could facilitate the simulation of such aspect.

354 In the triaxial compression test, the experimental results of drained triaxial
355 compression tests ($p'_0 = 2.5$ MPa, $p'_0 = 4$ MPa) at room temperature 25 °C and drained
356 triaxial compression tests ($p'_0 = 2.5$ MPa, $p'_0 = 3.8$ MPa) at high temperature 76 °C
357 [38] are considered in (q, ε_l) plane and $(\varepsilon_v, \varepsilon_l)$ plane in Fig.9. In these moderate over-
358 consolidated states, the shear strength at room temperature is a bit higher than high
359 temperature. However, a larger volumetric strain is generated at high temperature than
360 room temperature as shown in $(\varepsilon_v, \varepsilon_l)$ plane. At the same time, the smooth predictions
361 by ACC2-T model agree well with the experimental observations. Two values of n_c (n_c
362 = -1, $n_c = 0$) are selected to simulate the behavior at high temperature 76 °C. It indicates
363 that the value of n_c has a negligible effect on the prediction of triaxial tests.

364 A numerical case at high over-consolidated state ($p'_0 = 0.5$ MPa) under both two
365 temperatures (25 °C, 76 °C) are conducted. A slight shear dilatancy is predicted at axial
366 strain of about 5%. However, the post-peak softening phenomenon in $(q-\varepsilon_l)$ plane is not
367 very apparent because parameter A_d ($A_d = 0.1$) for natural Boom clay is very small to
368 simulate behaviors at both high, slightly over-consolidated and normal consolidated
369 states.

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371 **5.2 Geneva clay**

372 Multiple thermal cycles were applied on Geneva clays ($I_p = 11.1$) in a temperature-
373 controlled oedometer cell under drained condition [25]. A normally consolidated soil

374 sample with a vertical effective stress of 125 kPa was tested with four temperature
375 cycles from 5 °C from 60 °C, starting at a room temperature of 20 °C. The one-
376 dimension test results are simulated in Fig.7 and the calibrated model parameters are
377 listed in Tab.1.

378 Fig.10 shows the measured and computed soil behavior under multiple heating and
379 cooling cycles. The computed result of ACC2-T model (Fig.10(a)) shows a good
380 agreement with the measured one. The irreversible volumetric strain accumulates with
381 increasing thermal cycles, but at a decreasing rate (Fig.10(c)). The value of 3.5 is shown
382 appropriate for the thermal accumulation parameter n_c to control the strain
383 accumulation rate. For the case of TEAM model, the computed result of first thermal
384 cycle is the same as ACC2-T model. However, the response of TEAM model is purely
385 elastic thereafter and the thermal strain accumulation could not be described. In
386 Fig.10(b), a sensitivity analysis of ACC2-T model is conducted to study the effect of
387 parameter n_c on the strain accumulation. Two other n_c values of 0, 10 are selected for
388 this purpose. In the first thermal cycle, ACC2-T model gives the same results for all the
389 three n_c values because the thermal volumetric strain of normally consolidated soil is
390 only dependent on thermal elastic parameter α_1 and thermal plastic parameter α_0 .
391 From the second thermal cycle, the soil becomes over-consolidated. The thermal
392 accumulation parameter n_c starts to control the strain evolution rate. From Fig.10(c), it
393 appears that parameter n_c is positively correlated to the strain accumulation rate and a
394 proper selection of parameter n_c is crucial for total strain accumulation.

395 **5.3 Bonny silt**

396 Multiple heating and cooling cycles were applied on compacted Bonny silt ($I_p = 4$)
397 under different OCRs in a temperature-controlled oedometer cell [27]. Bonny silts were
398 first mechanically consolidated to a vertical effective stress of approximately 1200 kPa
399 at a constant room temperature of 18 °C. Then, they were unloaded to reach five
400 different OCR values of 1.00, 1.28, 1.80, 7.36 and 30.29. Fig. 7 presents the
401 experimental and calibration results of one-dimensional test of Bonny silt. Four heating
402 and cooling cycles were applied under drained condition with different OCRs in a
403 temperature ranging from 18 °C to 93 °C.

404 It can be seen from Fig.11 that at slightly over-consolidated states (i.e. OCR = 1, 1.28
405 and 1.8), the measured and computed results show a good consistency. That means the
406 thermal accumulation parameter n_c can properly control the strain evolution rate under
407 various over-consolidated states. However, for highly over-consolidated ones (i.e. OCR
408 = 7.36, 30.29) in the literature [27], the volumetric strain accumulation is quite small or
409 even becomes negative, suggesting that the expansion and the contraction of the
410 oedometer cell can be no longer ignored. These two samples are not considered in the
411 simulation.

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413 **5.4 Reconstituted Pontida clay**

414 The excess pore water pressure accumulation due to thermal cycles is validated by a
415 heating and cooling test of reconstituted Pontida clay. Hueckel et al [12] reported this
416 measured and computed results on reconstituted Pontida clay. The reconstituted silty

417 clay sample was firstly loaded to a mean effective stress state of 2.5 MPa. Then it
 418 underwent a heating-cooling cycle from 22°C to 65°C and back to 22°C, then a
 419 continuous heating to 95°C in a constant total isotropic stress state.

420 Although some previous models [19] consider the undrained behavior by imposing
 421 a constant volume condition which is imprecise under non-isothermal conditions, the
 422 undrained computation using ACC2-T model is obtained by extending the energy and
 423 mass conservation of thermo-poro-elastic media into elasto-plastic states [30].

424 In a saturated porous media, the current water mass (m_w) per unit volume can be
 425 expressed as:

$$426 \quad m_w = \rho_w \phi \quad (12)$$

427 where ρ_w is water density and ϕ is the Lagrangian porosity.

428 In undrained conditions, water mass is constant and $dm_w = 0$ holds, then

$$429 \quad \frac{dm_w}{\rho_w} = d\phi + \phi \frac{d\rho_w}{\rho_w} = d\phi + \phi (C_w du - \alpha_w dT) = 0 \quad (13)$$

430 where α_w , C_w , is the water thermal expansion coefficient, water compressibility,
 431 respectively.

432 In isotropic stress state and under non-isothermal condition, total mean stress change
 433 (dp) and Lagrangian porosity change ($d\phi$) can be expressed with Maxwell's symmetry
 434 condition:

$$435 \quad dp = \frac{I}{C_d} d\varepsilon_v + (I - \frac{C_s}{C_d}) du_w + \frac{\alpha_d}{C_d} dT \quad (14)$$

$$436 \quad d\phi = -(I - \frac{C_s}{C_d}) d\varepsilon_v - (\phi - I + \frac{C_s}{C_d}) C_s du_w + (\phi - I + \frac{C_s}{C_d}) \alpha_s dT \quad (15)$$

437 where C_s , C_d , α_s , α_d is the volumetric compressibility coefficient of solid grain, the
 438 volumetric compressibility of soil skeleton, the thermal expansion coefficient of solid
 439 grain and the thermal expansion coefficient of soil skeleton. It is often the case that α_s
 440 = α_d .

441 Thus, the expressions for excess pore water pressure can be obtained, as follows:

442
$$d\varepsilon_v = -(\phi\alpha_w + (1-\phi)\alpha_s)dT + \phi(C_w - C_s)du_w + C_s dp \quad (16)$$

443 With infinitesimal transformation and clayey soils, the fact ($C_s \approx 0$, $n \approx \phi$) is
 444 commonly accepted. Thus

445
$$du_w = \frac{d\varepsilon_v + (n\alpha_w + (1-n)\alpha_s)dT}{nC_w} \quad (17)$$

446 where n is Eulerian porosity.

447 For reconstituted Pontida clay, α_s is 1×10^{-5} ($1/^\circ\text{C}$) chosen as same as α_l , which is
 448 calibrated from an undrained cooling process from 65°C to 22°C . α_w and C_w are
 449 calibrated from a heating test of free water in the literature [35].

450 The computed results of excess pore water pressure by ACC2-T model and TEAM
 451 model are compared to the measured data and Hueckel's simulation [12] in Fig.12. It is
 452 seen that both predictions of ACC2-T model and TEAM model are more accurate than
 453 Hueckel's simulation, especially the existed buildup of pore water pressure in
 454 experimental results. During the heating stage from 22°C to 65°C , the mean effective
 455 stress will decrease with temperature (equivalent to a mechanical loading under
 456 undrained condition) and the effective stress-temperature path ($T - p'$) initially enters
 457 plasticity. With the plastic hardening of *ILY*, excess pore water pressure is increased.
 458 Then during the cooling stage from 65°C to 22°C , the response is purely elastic with a
 459 decrease of the pore water pressure. Thereby, an irreversible excess pore water pressure
 460 is observed after one heating-cooling cycle which is well captured by ACC2-T model
 461 and TEAM model. Moreover, though the cooling stage is purely elastic, the *ILY* would
 462 shrink to make the stress-temperature state always lying on it. However, during the re-
 463 heating process, TEAM model could not build up excess pore water pressure from 22°C
 464 to 65°C because the response is purely elastic. As for ACC2-T model, the clay sample
 465 is initially over-consolidated, then began to build up excess pore water pressure with
 466 the control of thermal accumulation parameter n_c in the whole process from 22°C to
 467 95°C .

468 **5.5 Discussion**

469 Four fine-grained soils are selected to verify the capability of ACC2-T model on
470 modelling thermal cyclic behaviors under drained and undrained conditions. A thermal
471 accumulation parameter n_c control the accumulation rate under multiple thermal cycles.
472 Meanwhile, the values of n_c are ranked as: natural Boom clay < reconstituted Pontida
473 clay < Geneva clay < Bonny silt. It means the clays, containing more clay contents,
474 showing larger plastic index I_p , are more likely to accumulate volumetric strain and
475 build up excess pore water pressure. Generally, the larger is the plastic index I_p , the
476 smaller is the value of n_c . A liner regression function is obtained to evaluate the
477 correlation between parameter n_c and plastic index I_p in Fig. 13.

478

479 **6. Concluding remarks**

480 A two-surface plasticity model (ACC2-T) is proposed to describe the thermo-
481 mechanical behavior of fine-grained soils under multiple thermal cycles. Compared
482 with TEAM model, two thermal yield limits are removed and the loading yield limits
483 unify the thermal and loading yielding part. A dependency law with thermal
484 accumulation parameter n_c is used to link the inner loading yield limit and the loading
485 yield limit. Parameter n_c controls the evolution rate of the inner loading yield limit
486 approaching the loading yield limit under thermal loading path. With such a unique
487 dependency law and multiple plastic hardening rules, the model could well control the
488 thermal strain accumulation rate under drained conditions and pore water pressure

489 buildup rate under undrained conditions. Tests on four fine-grained soils are considered
490 to verify the proposed model. Comparison between prediction and experiment shows
491 that ACC2-T model can well capture the thermo-mechanical behavior of soils under
492 multiple thermal cycles.

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510 **Notation**

511 α_0 = parameter controlling thermal plastic behavior;

512 α_1 = parameter controlling thermal elastic behavior;

513 β = the size parameter of potential plastic surface;

514 λ, κ = slope of isotropic loading, reloading line respectively;

515 ν, ν = current void ratio, Poisson's ratio respectively;

516 A_d = parameter controlling the contribution of shear plastic strain;

517 $d\boldsymbol{\varepsilon}, d\boldsymbol{\sigma}, dT$ = total strain, stress, temperature increment respectively;

518 $d\boldsymbol{\varepsilon}_\sigma^e, d\boldsymbol{\varepsilon}_T^e$ = stress, thermal induced elastic strain increment respectively;

519 $d\varepsilon_\nu^e, d\varepsilon_\nu^e, d\varepsilon_s^e$ = total volumetric, elastic volumetric, shear strain increment

520 respectively;

521 $d\boldsymbol{\varepsilon}^p, d\varepsilon_\nu^p, d\varepsilon_s^p$ = plastic, plastic volumetric, plastic shear strain increment

522 respectively;

523 Λ = plastic multiplier;

524 f_l, f_Y = Loading surface, yield surface respectively;

525 G, K, h = elastic shear, elastic bulk, plastic modulus respectively;

526 k_f, k_g = parameter controlling the shape of yield surface, plastic potential surface

527 respectively;

528 M_f = stress ratio at the apex of yield surface;

529 M_g, M_{g0} = critical stress ratio at current, reference temperature respectively;

530 p', \bar{p}' = current, mapping mean effective stress respectively;

531 p'_{cT}, p'_{c0} = pre-consolidation pressure in loading surface at current, reference
532 temperature respectively;

533 $\bar{p}'_{cT}, \bar{p}'_{c0}$ = pre-consolidation pressure at current, reference temperature respectively;

534 q, \bar{q} = current, mapping deviator stress respectively;

535 r, r_0 = reverse of OCR at current, reference temperature respectively;

536 s = parameter controlling the rate of evolution of the slope of the stress--strain
537 relation;

538 T, T_0 = current, reference temperature respectively;

539 $\mathbf{D}_e, \mathbf{D}_{ep}, \mathbf{D}_{et}$ = elastic, elasto-plastic, temperature stiffness matrix respectively;

540 \mathbf{m} = unit vector.

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556 **Appendix A**

557 The consistency condition of loading surface in ACC2-T model is adopted herein:

$$558 \left(\frac{\partial f_I}{\partial \boldsymbol{\sigma}'} \right)^t : d\boldsymbol{\sigma}' + \frac{\partial f_I}{\partial T} dT + \frac{\partial f_I}{\partial \bar{p}'_{c0}} d\bar{p}'_{c0} + \frac{\partial f_I}{\partial r_0} dr_0 = 0 \quad (\text{A - 1})$$

559 The plastic multiplier Λ and the plastic modulus h are obtained in non-isothermal
560 condition:

$$561 d\boldsymbol{\varepsilon}^p = \Lambda \frac{\partial \mathbf{g}_I}{\partial \boldsymbol{\sigma}'} \quad (\text{A - 2})$$

$$562 \Lambda = - \frac{\left(\frac{\partial f_I}{\partial \boldsymbol{\sigma}'} \right)^t : d\boldsymbol{\sigma}' + \frac{\partial f_I}{\partial T} dT}{\frac{\partial f_I}{\partial \bar{p}'_{c0}} \frac{\nu}{\lambda - k} \bar{p}'_{c0} \frac{\partial \mathbf{g}_I}{\partial p'} + \frac{\partial f_I}{\partial r_0} \frac{\nu}{\lambda - k} s(1 - r_0) \left(\frac{\partial \mathbf{g}_I}{\partial p'} + A_d \frac{\partial \mathbf{g}_I}{\partial q} \right)} \quad (\text{A - 3})$$

$$563 h = \frac{\nu}{\lambda - k} \left(\frac{\partial f_I}{\partial \bar{p}'_{c0}} \bar{p}'_{c0} \frac{\partial \mathbf{g}_I}{\partial p'} + \frac{\partial f_I}{\partial r_0} s(1 - r_0) \left(\frac{\partial \mathbf{g}_I}{\partial p'} + A_d \frac{\partial \mathbf{g}_I}{\partial q} \right) \right) \quad (\text{A - 4})$$

564 The total strain increment is made up of mechanical elastic strain increment, thermal
565 elastic strain increment and plastic strain increment.

$$566 d\boldsymbol{\varepsilon} = d\boldsymbol{\varepsilon}_{\sigma'}^e + d\boldsymbol{\varepsilon}_T^e + d\boldsymbol{\varepsilon}^p \quad (\text{A - 5})$$

567 The thermal elastic strain increment $d\boldsymbol{\varepsilon}_T^e$ is defined as:

$$568 d\boldsymbol{\varepsilon}_T^e = -\frac{1}{3} \mathbf{m} \alpha_1 dT \quad (\text{A - 6})$$

569 where \mathbf{m} is the column vector with 1 at normal stress entries and 0 at shear stress entries.

570 By substituting Eq. A-6 into Eq. A-5, the general incremental stress-strain relation
571 can be expressed:

$$572 d\boldsymbol{\sigma}' = \mathbf{D}_e d\boldsymbol{\varepsilon}_{\sigma'}^e = \mathbf{D}_e \left(d\boldsymbol{\varepsilon} + \frac{1}{3} \mathbf{m} \alpha_1 dT - d\boldsymbol{\varepsilon}^p \right) \quad (\text{A - 7})$$

573 where \mathbf{D}_e is the mechanical elastic matrix.

574 By substituting the plastic multiplier into Eq. A-7, the differential stress–strain
 575 equations can be obtained:

$$576 \quad d\boldsymbol{\sigma}' = \mathbf{D}_{ep} d\boldsymbol{\varepsilon} + \mathbf{D}_{et} dT \quad (\text{A - 8})$$

577 where \mathbf{D}_{ep} , \mathbf{D}_{et} are the mechanical elasto-plastic matrix and the thermal elasto-plastic
 578 matrix, respectively.

$$579 \quad \mathbf{D}_{ep} = \mathbf{D}_e - \frac{\mathbf{D}_e \frac{\partial \mathbf{g}_I}{\partial \boldsymbol{\sigma}'} \left(\frac{\partial f_I}{\partial \boldsymbol{\sigma}'} \right)^t \mathbf{D}_e}{\left(\frac{\partial f_I}{\partial \boldsymbol{\sigma}'} \right)^t \mathbf{D}_e \frac{\partial \mathbf{g}_I}{\partial \boldsymbol{\sigma}'} + h} \quad (\text{A - 9})$$

$$580 \quad \mathbf{D}_{et} = \frac{\mathbf{D}_e \frac{\partial \mathbf{g}_I}{\partial \boldsymbol{\sigma}'} \left[-\frac{1}{3} \left(\frac{\partial f_I}{\partial \boldsymbol{\sigma}'} \right)^t \mathbf{D}_e \mathbf{m} \alpha_1 - \frac{\partial f_I}{\partial T} \right]}{\left(\frac{\partial f_I}{\partial \boldsymbol{\sigma}'} \right)^t \mathbf{D}_e \frac{\partial \mathbf{g}_I}{\partial \boldsymbol{\sigma}'} + h} + \frac{1}{3} \mathbf{D}_e \mathbf{m} \alpha_1 \quad (\text{A - 10})$$

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596 **Appendix B**

597 Assume the soil is loaded and heated from Point A to Point B under two different
 598 loading paths I($A \rightarrow C \rightarrow B$) and II($A \rightarrow D \rightarrow B$) in Fig.5. The soil is at the initial stress
 599 state (p'_0, r_0, p'_{c0}, T_1) which automatically satisfies the yield condition:

$$600 \quad f_{I0} = p'_0 - r_0 p'_{c0} \exp[-\alpha_0 r_0^{n_c} (T_1 - T_0)] = 0 \quad (\text{B - 1})$$

601 Under loading path I, the soil element is isotropically loaded to stress point C
 602 ($p'_0 + \Delta p'_1, r_1, p'_{c1}, T_1$) at constant temperature T_1 .

$$603 \quad f_{I1} = p'_0 + \Delta p'_1 - r_1 p'_{c1} \exp[-\alpha_0 r_1^{n_c} (T_1 - T_0)] = 0 \quad (\text{B - 2})$$

$$604 \quad p'_{c1} = p'_{c0} \exp\left[\frac{\nu}{\lambda - \kappa} \Delta \varepsilon_{v1}^p\right] \quad (\text{B - 3})$$

$$605 \quad r_1 = 1 + (r_0 - 1) \exp\left[-\frac{\nu}{\lambda - \kappa} s \Delta \varepsilon_{v1}^p\right] \quad (\text{B - 4})$$

606 Then the soil element is heated up to stress point B ($p'_0 + \Delta p'_1, r_2, p'_{c2}, T_1 + \Delta T_1$) at
 607 constant mean effective stress $p'_0 + \Delta p'_1$.

$$608 \quad f_{I2} = p'_0 + \Delta p'_1 - r_2 p'_{c2} \exp[-\alpha_0 r_2^{n_c} (T_1 + \Delta T_1 - T_0)] = 0 \quad (\text{B - 5})$$

$$609 \quad p'_{c2} = p'_{c1} \exp\left[\frac{\nu}{\lambda - \kappa} \Delta \varepsilon_{v2}^p\right] = p'_{c0} \exp\left[\frac{\nu}{\lambda - \kappa} (\Delta \varepsilon_{v1}^p + \Delta \varepsilon_{v2}^p)\right] \quad (\text{B - 6})$$

$$610 \quad r_2 = 1 + (r_1 - 1) \exp\left[-\frac{\nu}{\lambda - \kappa} s \Delta \varepsilon_{v2}^p\right] = 1 + (r_0 - 1) \exp\left[-\frac{\nu}{\lambda - \kappa} s (\Delta \varepsilon_{v1}^p + \Delta \varepsilon_{v2}^p)\right] \quad (\text{B - 7})$$

611 Under loading path II, the soil element is firstly heated up to stress point D
 612 ($p'_0, r_3, p'_{c3}, T_1 + \Delta T_1$) at constant mean effective stress p'_0 .

$$613 \quad f_{I3} = p'_0 - r_3 p'_{c3} \exp[-\alpha_0 r_3^{n_c} (T_1 + \Delta T_1 - T_0)] = 0 \quad (\text{B - 8})$$

$$614 \quad p'_{c3} = p'_{c0} \exp\left[\frac{\nu}{\lambda - \kappa} \Delta \varepsilon_{v3}^p\right] \quad (\text{B - 9})$$

$$615 \quad r_3 = 1 + (r_0 - 1) \exp\left[-\frac{\nu}{\lambda - \kappa} s \Delta \varepsilon_{v3}^p\right] \quad (\text{B - 10})$$

616 Then, the soil element is isotropically loaded to stress point B ($p'_0 + \Delta p'_1, r_4, p'_{c4}, T_1 + \Delta T_1$)

617 at constant temperature $T_1 + \Delta T_1$.

$$618 \quad f_{I4} = p'_0 + \Delta p'_1 - r_4 p'_{c4} \exp\left[-\alpha_0 r_4^{n_c} (T_1 + \Delta T_1 - T_0)\right] = 0 \quad (\text{B - 11})$$

$$619 \quad p'_{c4} = p'_{c3} \exp\left[\frac{\nu}{\lambda - \kappa} \Delta \varepsilon_{v4}^p\right] = p'_{c0} \exp\left[\frac{\nu}{\lambda - \kappa} (\Delta \varepsilon_{v3}^p + \Delta \varepsilon_{v4}^p)\right] \quad (\text{B - 12})$$

$$620 \quad r_4 = 1 + (r_3 - 1) \exp\left[-\frac{\nu}{\lambda - \kappa} s \Delta \varepsilon_{v4}^p\right] = 1 + (r_0 - 1) \exp\left[-\frac{\nu}{\lambda - \kappa} s (\Delta \varepsilon_{v3}^p + \Delta \varepsilon_{v4}^p)\right] \quad (\text{B - 13})$$

621 Compare Eqs.B-5, B-6, B-7 and Eqs.B-11, B-12, B-13, it appears clearly that the two
622 different loading paths would reach the same final stress state, namely:

$$623 \quad p'_{c2} = p'_{c4} \quad (\text{B - 14})$$

$$624 \quad r_2 = r_4 \quad (\text{B - 15})$$

$$625 \quad \Delta \varepsilon_{v1}^p + \Delta \varepsilon_{v2}^p = \Delta \varepsilon_{v3}^p + \Delta \varepsilon_{v4}^p \quad (\text{B - 16})$$

626 No matter Path I or Path II is followed, the plastic volumetric strain will be equal, which
627 means that plastic volumetric strain in ACC2-T model is also loading path independent.

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755 **LIST OF TABLES**

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Table.1 Model parameters of different fine-grained soils

Parameters	κ	λ	$\bar{p}'_{c0,0}$ (kPa)	$r_{0,0}$	ν	α_0 ($^{\circ}C^{-1}$)	α_1 ($^{\circ}C^{-1}$)	s	T_0 ($^{\circ}C$)	n_c	M_{g0}	M_f	A_d	k_f	k_g
Boom clay[8]	0.02	0.18	6000	0.33	-	5×10^{-3}	5×10^{-5}	8	21.5	-1	0.67	0.67	0.1	0.7	0.9
Geneva clay[25]	0.017	0.062	125	1	0.3	5.2×10^{-3}	1.5×10^{-5}	12	20	3.5	0.94	0.94	0.1	1.1	1.1
Bonny silt[27]	0.011	0.075	1200	*	0.3	1.6×10^{-3}	3.5×10^{-5}	10	18	4	1.3	1.3	0.2	1.1	1.1
Pontida clay[12]	0.016	0.103	2500	1	-	3.5×10^{-3}	5×10^{-5}	10	23	1	-	-	-	-	-

‘-’ means the parameters are irrelevant to the simulations in this study.

‘*’ means different data are mentioned in this study.

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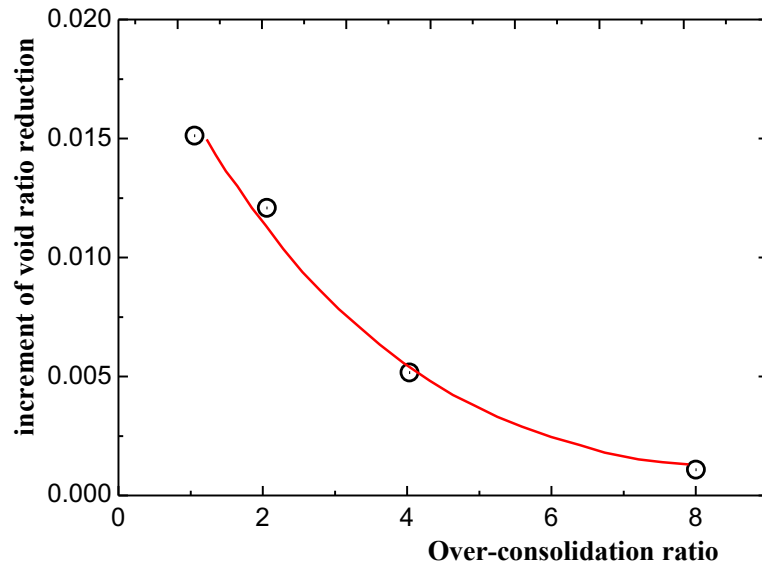


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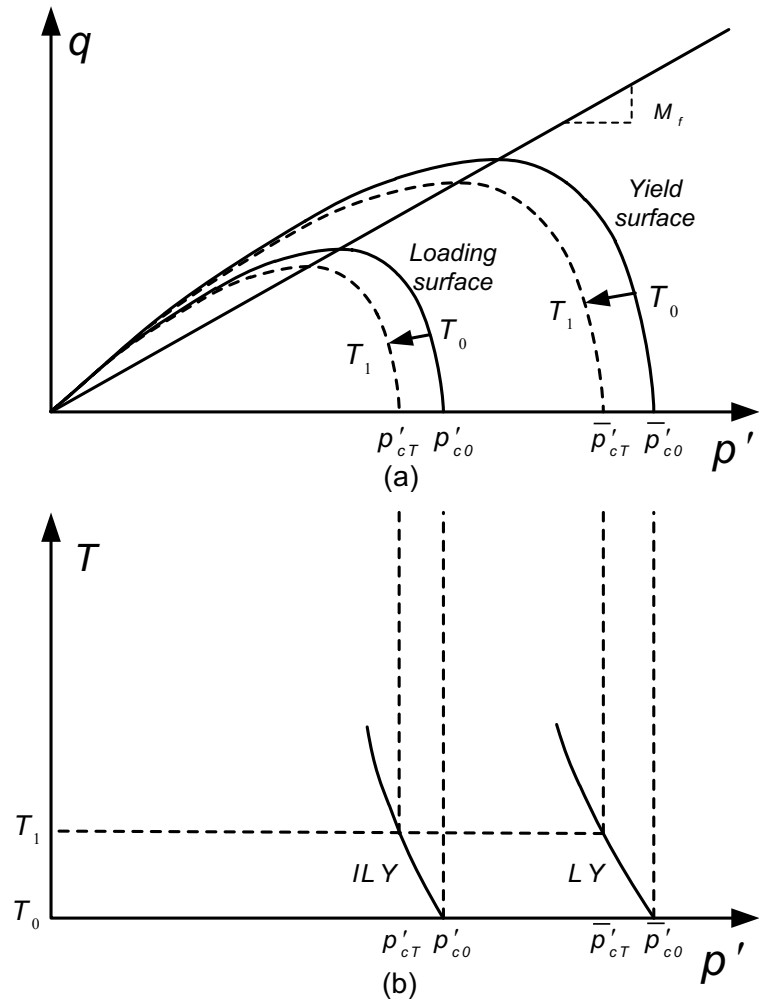
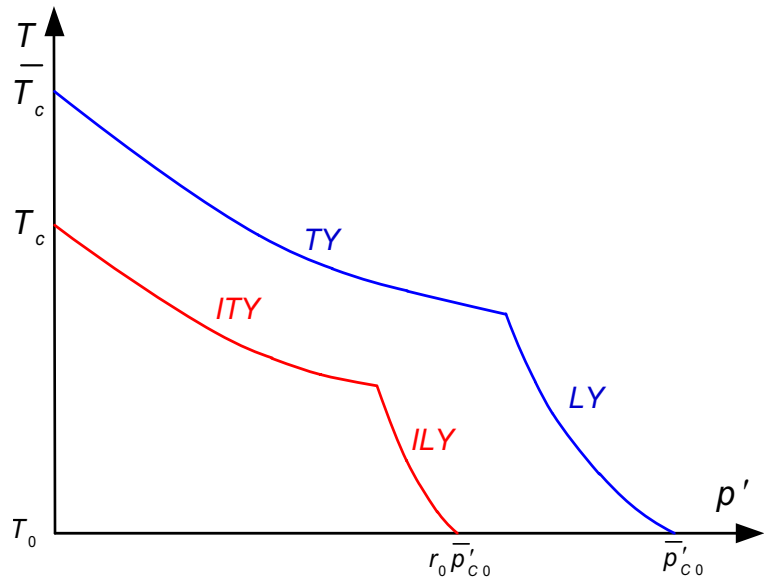
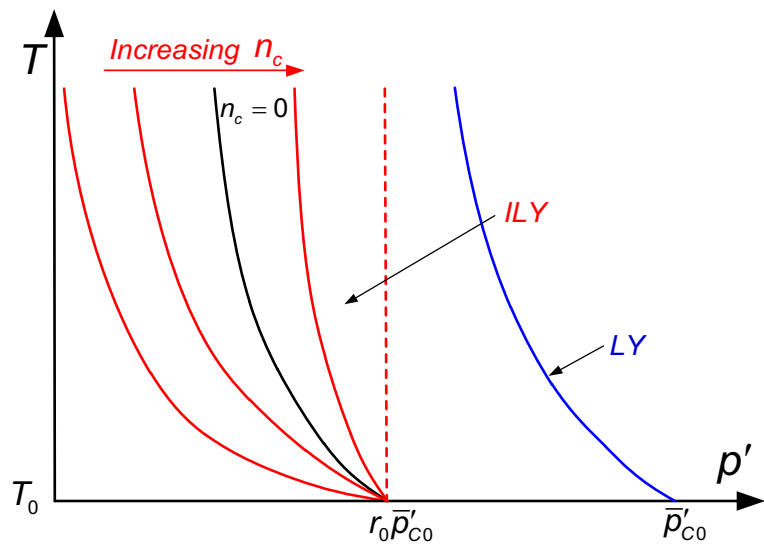


Fig.2 Yield locus in (a): (p', q) plane at two different temperatures (with $T_1 > T_0$) and (b): two loading yield limits ILY and LY in $(p'-T)$ plane.



(a)



(b)

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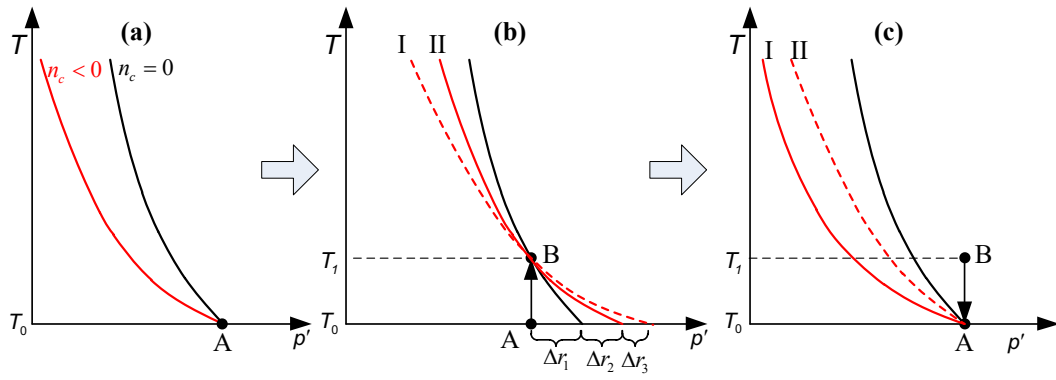


Fig.4 Evolution of yield limits in (p', T) plane during one heating and cooling cycle

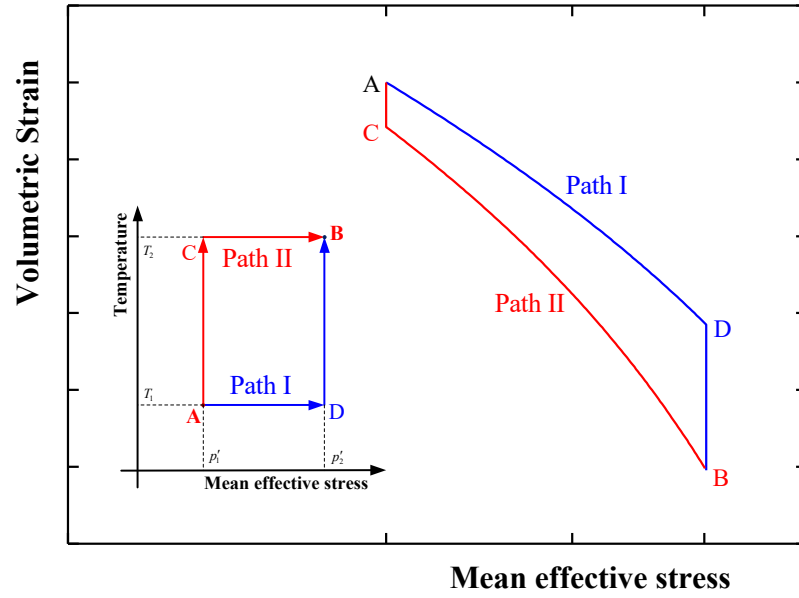
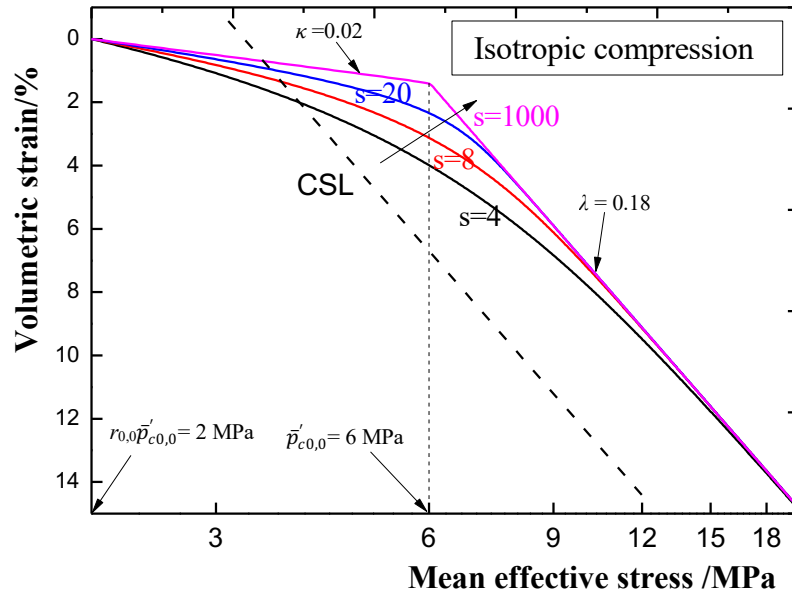
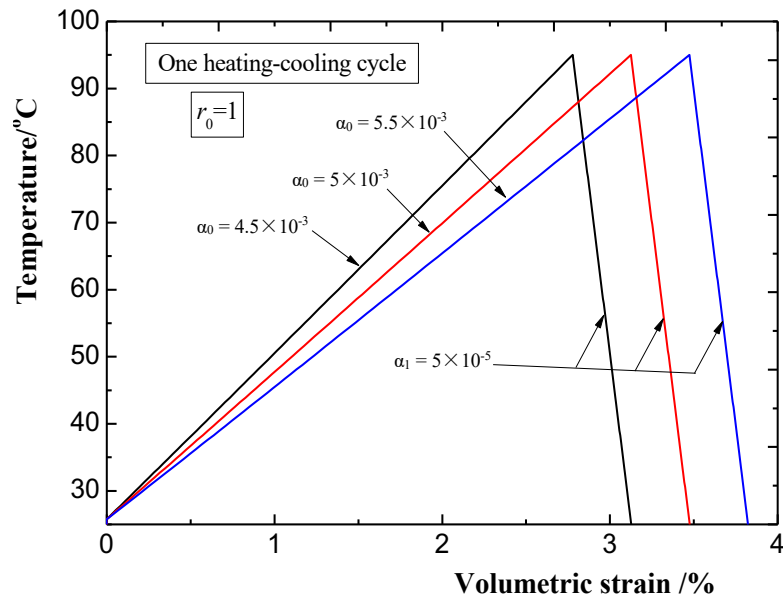


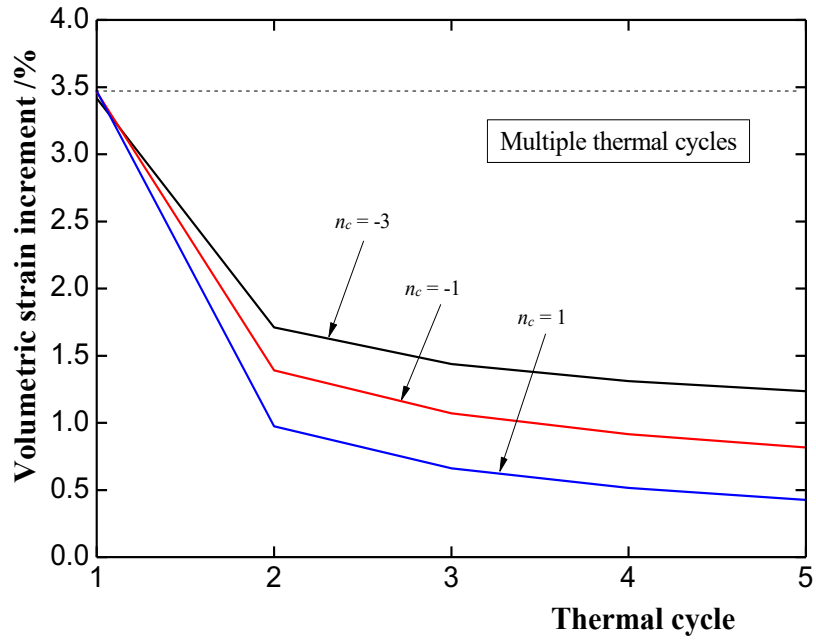
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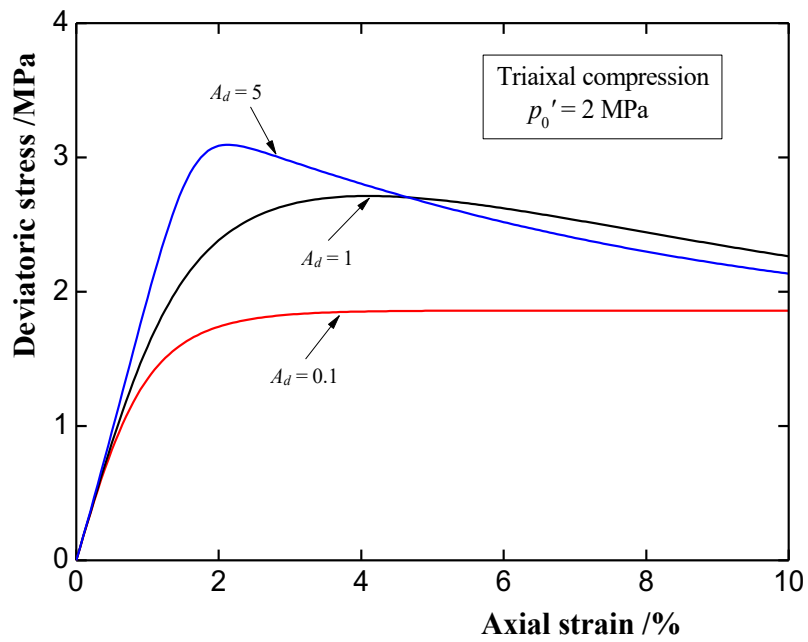
(a)



(b)



(c)



(d)

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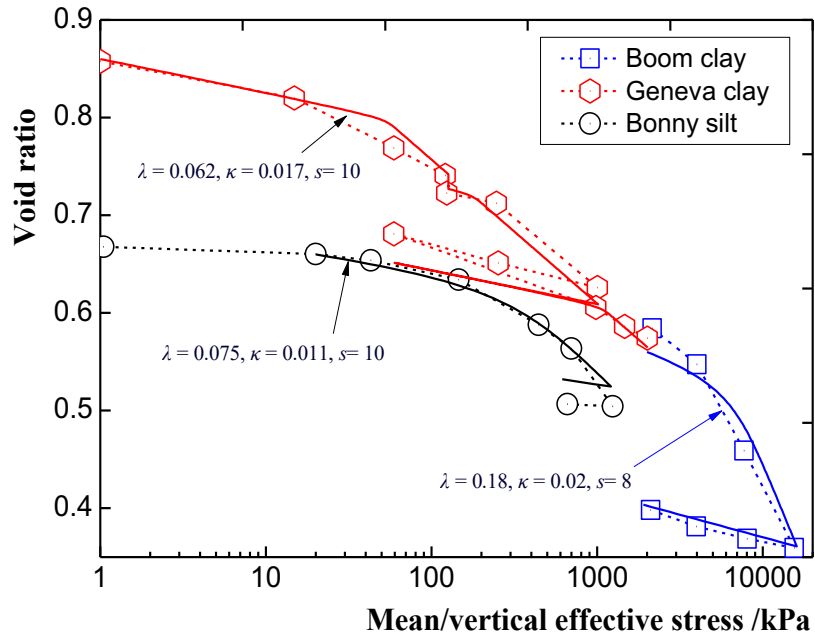


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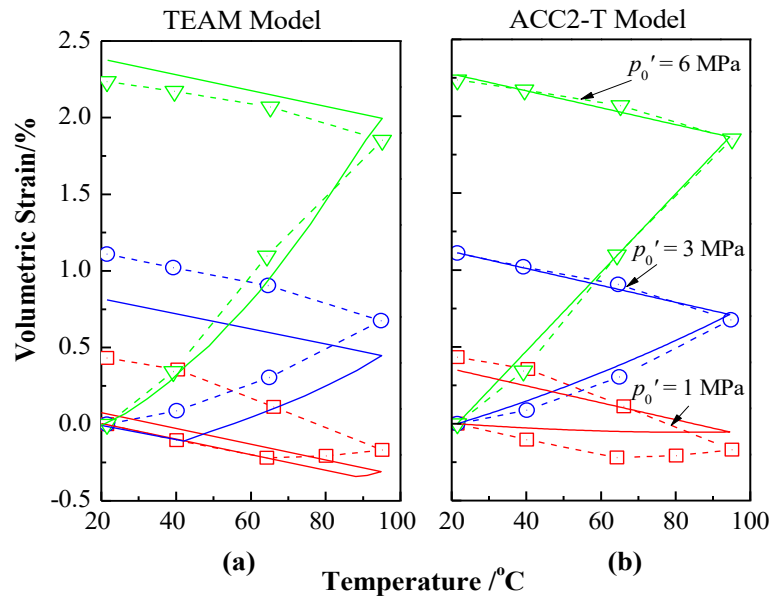
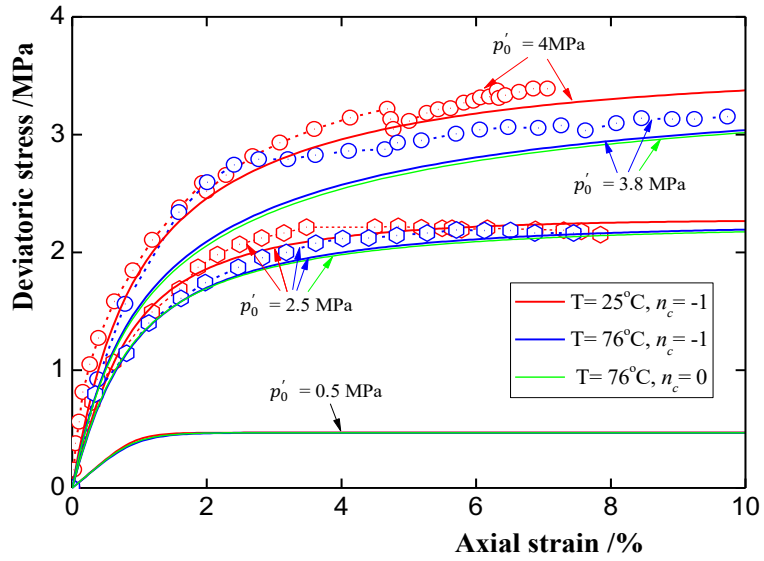
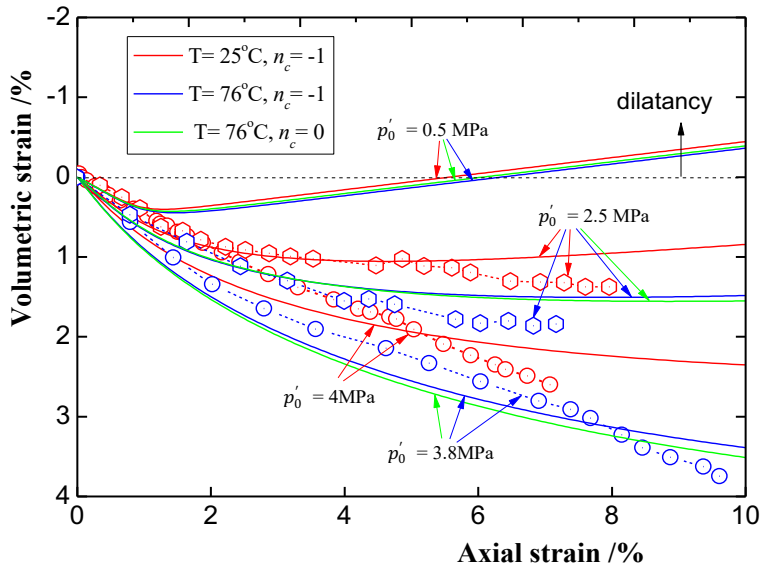


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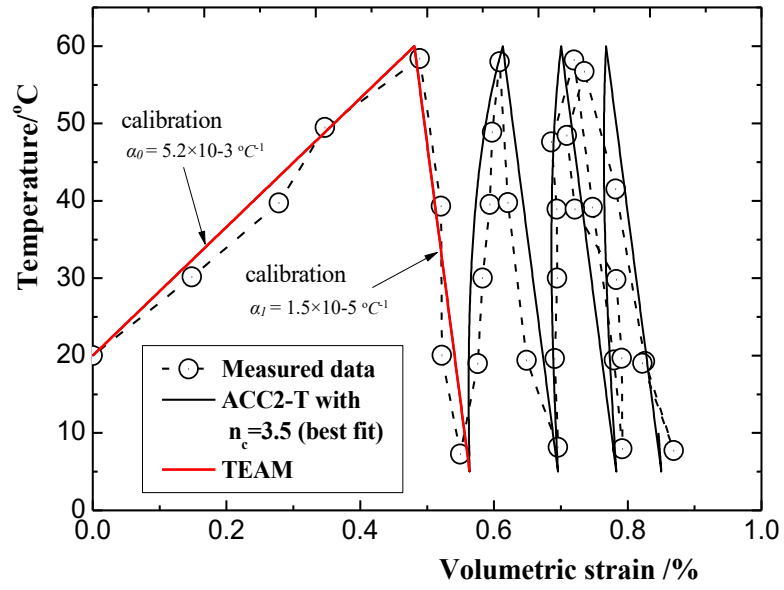


(a)

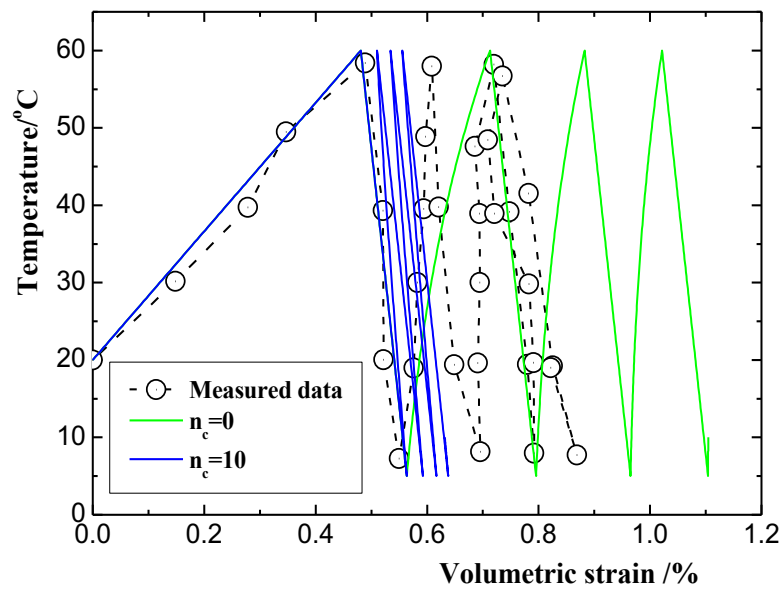


(b)

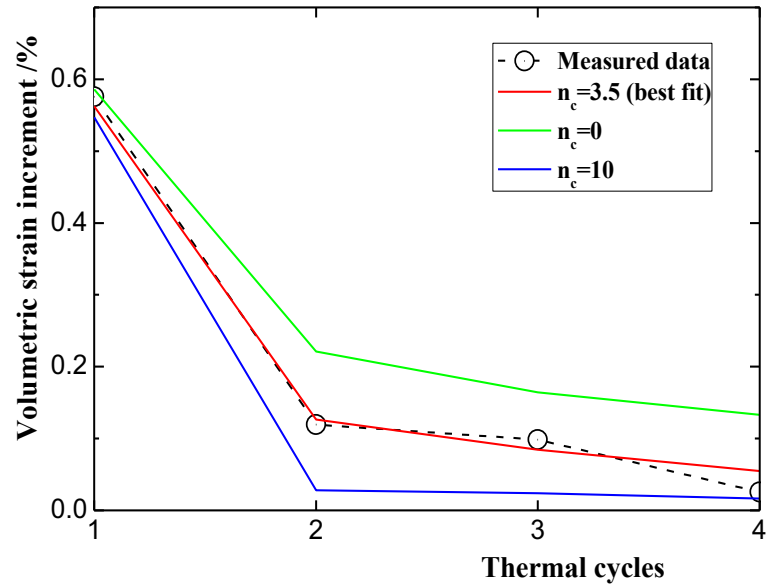
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(a)



(b)



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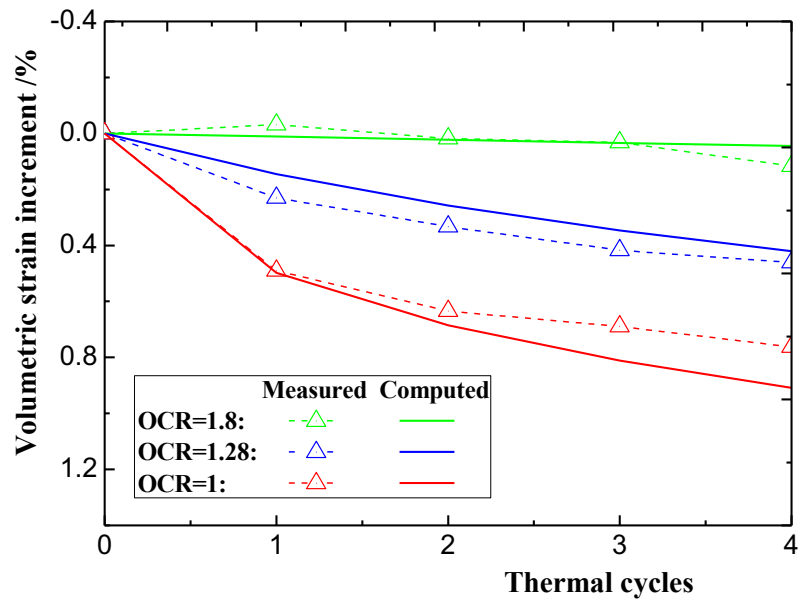
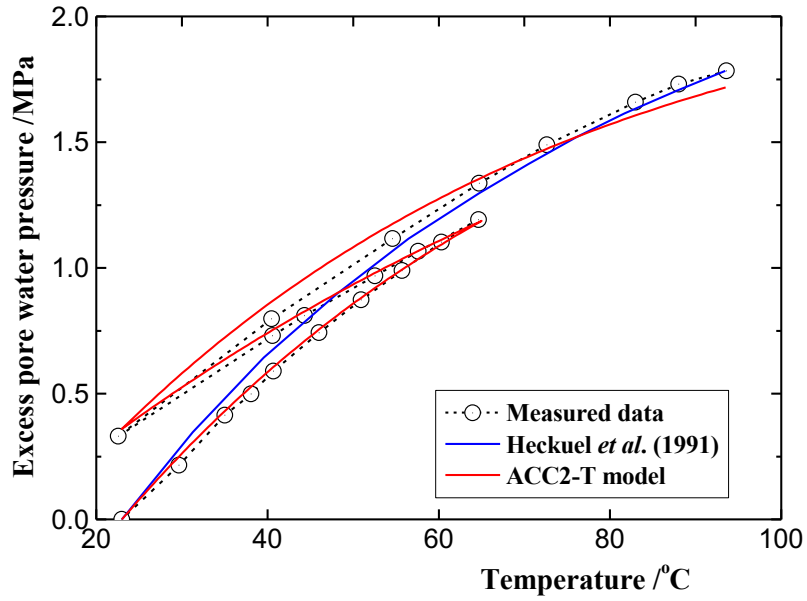
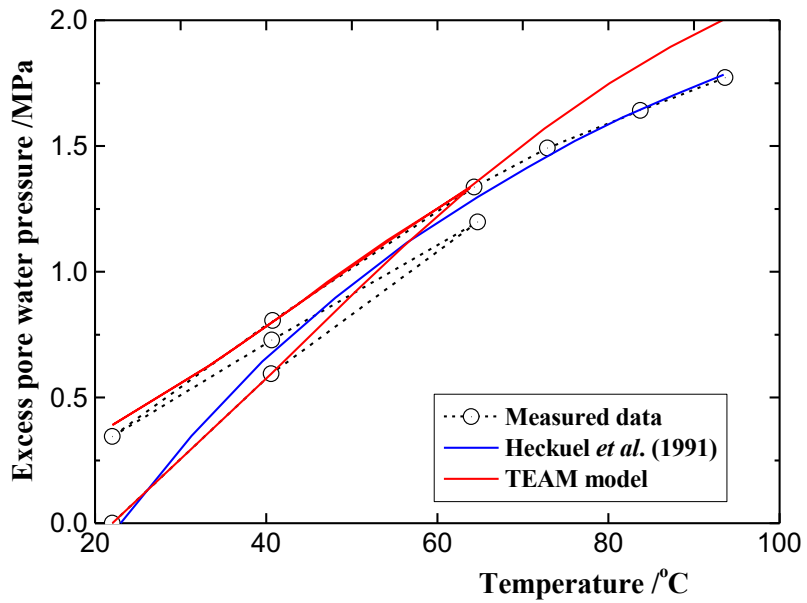


Fig.11 Accumulated strain behavior of Bonny Silt with different OCRs under drained condition



(a)



(b)

Fig.12 Excess pore water pressure accumulation of Pontida clay due to thermal cycles

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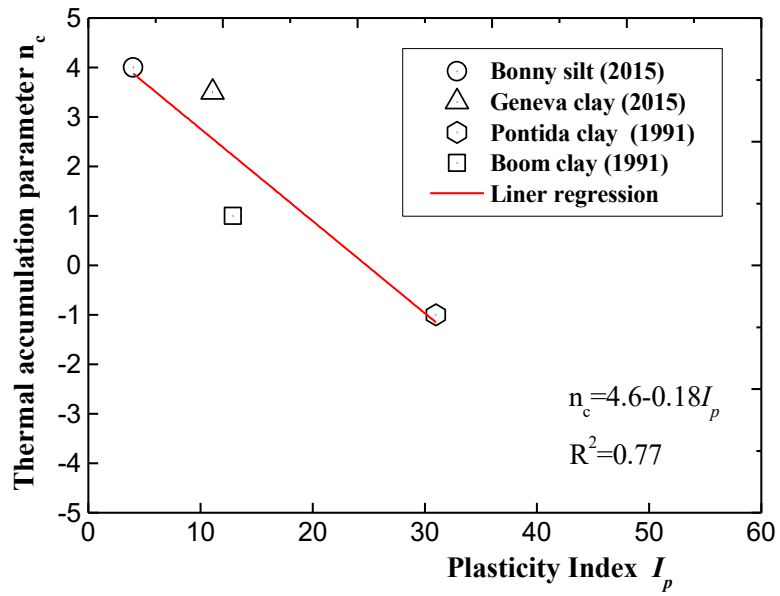


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