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**A two-surface thermo-mechanical plasticity model considering  
thermal cyclic behavior**

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## Abstract

In thermal-related engineering such as thermal energy structures and nuclear waste disposal, it is essential to well understand volume change and excess pore water pressure buildup of soils under thermal cycles. However, most existing thermo-mechanical models can merely simulate one heating-cooling cycle and fail in capturing accumulation phenomenon due to multiple thermal cycles. In this study, a two surface elasto-plastic model considering thermal cyclic behavior is proposed. This model is based on the bounding surface plasticity and progressive plasticity by introducing two yield surfaces and two loading yield limits. A dependency law is proposed by linking two loading yield limits with a thermal accumulation parameter  $n_c$ , allowing the thermal cyclic behavior to be taken into account. Parameter  $n_c$  controls the evolution rate of the inner loading yield limit approaching the loading yield limit following a thermal loading path. By extending the thermo-hydro-mechanical equations into the elastic-plastic state, the excess pore water pressure buildup of soil due to thermal cycles is also accounted. Then, thermal cycle tests on four fine-grained soils (natural Boom clay, Geneva clay, Bonny silt and reconstituted Pontida clay) under different OCRs and stresses are simulated and compared. The results show that the proposed model can well describe both strain accumulation phenomenon and excess pore water pressure buildup of fine-grained soils under the effect of thermal cycles.

**Keywords:** fine-grained soils; strain accumulation; thermal cycles; two-surface model; excess pore water pressure buildup

## 1. Introduction

Around energy piles [1-3], nuclear waste canisters [4], buried tunnels and pipes [5], the soils suffer from temperature effects, especially heating-cooling cycles. Observations of different fine-grained soils in the laboratory and in the field [6-11] showed that heating generally induced non-negligible volume change and excess pore water pressure buildup. Several thermo-mechanical models were proposed to describe this thermo-mechanical behavior of fine-grained soils [13-21]. Some of them extended the Cam-clay or modified Cam-clay model to the temperature effect by introducing a shrinking yield surface [13-17]. These models could describe some basic thermo-mechanical behavior, but fail to describe the soil behavior in over-consolidated states. In order to well describe the small thermo-mechanical elastic zone and the shear softening behavior, some advanced models consider two yield surfaces based on the bounding surface theory [18-21]. Hong et al. [22] made a comparison among some of these advanced models, and showed that the multiple plastic hardening mechanism were of paramount importance in modeling complex thermal loading behavior.

Though numerous efforts were made in thermal mechanical modeling, most existing models can only simulate the case of one thermal cycle and fail in describing the soil behavior under multiple heating-cooling cycles. As more attention has been paid to the long-term performance [11,12,25,26,27,40], the volume change and excess pore water pressure accumulation due to temperature variation can act as a significant factor influencing the safety of energy structures. Demars and Charles [23] reported a series

64 of drained thermal cycle tests on high plastic marine clays and found that permanent  
65 reduction in void ratio (a function of over-consolidation ratio) gradually reduced due to  
66 temperature cycles (see Fig.1). Plum et al. [24] conducted an undrained heating-cooling  
67 cycle tests on Newfield clay. After four thermal cycles, the accumulation of excess pore  
68 water pressure was still increasing. Pastén et al. [40] experimentally observed the  
69 thermally driven displacement accumulation with the number of temperature cycles and  
70 developed a thermally induced ratcheting mechanism for a smooth interface. All those  
71 observations show that multiple thermal cycles should be considered in thermal  
72 mechanical modeling of clays. Zhou et al. [28] recently proposed a bounding surface  
73 model with a memory surface to consider the thermal cyclic behavior. However, this  
74 imposed memory surface actually has no effect on the accumulation behavior of  
75 normally consolidated soil because this memory surface coincides with the bounding  
76 surface in this circumstance. Ma et al. [29] proposed a thermal stabilization line in  
77 hypoplastic modeling approach. Though three thermal parameters are introduced, the  
78 proposed hypoplastic model fails to describe the strain accumulation during relatively  
79 small temperature changes because it always simulates an unrealistic elastic behavior  
80 before entering the elasto-plastic zone following a thermal loading path. More recently,  
81 Zymnis et al. [41] proposed a simple calibration procedure for Tsinghua ThermoSoil  
82 (TTS) model and studied the capabilities of simulating the long-term accumulation of  
83 strains due to thermal cycles. However, those existing models for thermal cyclic  
84 behavior merely considered the strain accumulation behavior under drained conditions

though the excess pore water pressure buildup due to temperature effects are at least of equal importance from the best of the authors' knowledge.

In the present work, a two surface thermo-plastic model, namely ACC2-T, is proposed to consider the thermal cyclic behavior based on the previous TEAM model by Hong et al. [21]. The proposed model unifies the loading yield limits and the thermal yield limits. To connect the inner loading yield limit and the loading yield limit, a flexible dependency law is incorporated and combined with isotropic and progressive hardening laws to account for multiple thermal cycles. In the dependency law, a parameter  $n_c$  is introduced to control accumulation rate through the evolution rate of the inner loading yield limit approaching the loading yield limit. Meanwhile, the proposed model is extended to undrained condition by applying the thermo-hydro-mechanical equations by Coussy [30] into the elastic-plastic state. Then, the rationale of ACC2-T model is discussed in terms of elasto-plasticity and thermodynamic theories. Lastly, the thermal cycle test results of four fine-grained soils (natural Boom clay, Geneva clay, Bonny silt and reconstituted Pontida clay) under different OCRs and stresses are used to verify the proposed model.

## **2. Model description**

In this section, the proposed model (ACC2-T) is presented using typical constitutive variables, namely mean effective stress  $p'$ , deviator stress  $q$ , temperature  $T$  and the corresponding increments ( $dp'$ ,  $dq$ ,  $dT$ ). The soil behavior is assumed to be isotropic

and the constitutive model is developed and formulated in the conventional triaxial conditions.

## 2.1 Thermo-elastic behavior

In ACC2-T model, the soil deformation can be divided into thermo-elastic and thermo-plastic parts.

In the thermo-elastic part, the elastic law adopted in many other models [31,36,37] in isothermal conditions are extended into non-isothermal conditions for its simplicity. the total elastic volumetric strain increment associated with mechanical and thermal loading and the elastic shear strain assumed purely mechanical are calculated as follows:

$$d\varepsilon_v^e = \frac{\kappa dp'}{vp'} - \alpha_1 dT, \quad d\varepsilon_s^e = \frac{dq}{3G}, \quad G = \frac{3(1-2\nu)}{2(1+\nu)} \frac{vp'}{\kappa} \quad (1)$$

where  $d\varepsilon_v^e$ ,  $d\varepsilon_s^e$  is the elastic volumetric strain increment ( $d\varepsilon_v^e = d\varepsilon_1^e + d\varepsilon_2^e + d\varepsilon_3^e$ ) and the elastic shear strain increment ( $d\varepsilon_s^e = \frac{2}{3}(d\varepsilon_1^e - d\varepsilon_3^e)$ ).  $\kappa$  is the elastic slope in ( $v$ ,  $\ln p'$ ) plane and  $v$  is the specific volume.  $\alpha_1$  is the thermal elastic compressibility index which assumed a positive constant because heating induces soil expansion.  $G$  is shear modulus and  $\nu$  is the Poisson's ratio.

## 2.2 Thermo-plastic behavior

The thermo-plastic mechanism of ACC2-T model consists of two yield surfaces and plastic potential surface, the dependency law, the isotropic and progressive hardening

laws and the flow rules.

### 2.2.1 Yield surfaces and plastic potential surfaces

In ACC2-T model, two yield surfaces, namely loading surface ( $f_l$ ) and yield surface ( $f_y$ ), with a generalized formulation [31] are implemented to model the yielding of both stiff and soft clays.

$$\text{Loading Surface: } f_l = q^2 + \frac{M_f^2}{1-k_f} \left( \frac{p'}{r\bar{p}'_{cT}} \right)^{\frac{2}{k_f}} (r\bar{p}'_{cT})^2 - \frac{M_f^2 p'^2}{1-k_f} = 0 \quad (2)$$

$$\text{Yield Surface: } f_y = q^2 + \frac{M_f^2}{1-k_f} \left( \frac{p'}{\bar{p}'_{cT}} \right)^{\frac{2}{k_f}} \bar{p}'_{cT}^2 - \frac{M_f^2 p'^2}{1-k_f} = 0 \quad (3)$$

where  $M_f$  is the stress ratio at the apex of two surfaces,  $k_f$  is a parameter specifying the shape of yield surfaces,  $\bar{p}'_{cT}$  is the pre-consolidation pressure at current temperature  $T$  and  $r$  is the inverse of over-consolidation ratio (OCR) at current temperature  $T$ . The conceptual plot of two yield surfaces at various temperature is given in Fig. 2(a). To consider the thermo-mechanical behavior, the two surfaces are extended into three-dimensional plot ( $p'$ ,  $q$ ,  $T$ ) with two loading yield limits. The two loading yield limits, namely the inner loading yield limit (ILY) and the loading yield limit (LY) are linked by the proposed dependency law which is discussed in *Yield limits and dependency law* section.

The plastic potential surface  $g_l$  takes the same form as for loading surface, but with different parameters  $M_g$  and  $k_g$ :



$$g_I = q^2 + \frac{M_g^2}{1-k_g} \left( \frac{p'}{r\beta} \right)^{\frac{2}{k_g}} (r\beta)^2 - \frac{M_g^2 p'^2}{1-k_g} = 0 \quad (4)$$

and it leads to a dilatancy equation in the case of triaxial compression:

$$\frac{d\varepsilon_v^p}{d\varepsilon_s^p} = \frac{\frac{\partial g_I}{\partial p'}}{\frac{\partial g_I}{\partial q}} = \frac{M_g^2 - \eta^2}{k_g \eta} \quad (5)$$

where  $d\varepsilon_v^p$  is the plastic volumetric strain increment,  $d\varepsilon_s^p$  is the plastic shear strain increment,  $\eta$  denotes the stress ratio  $q/p'$ ,  $M_g$  defines the critical stress ratio at failure where no further volumetric strain increment develops,  $k_g$  is a constant parameter which controls the ratio of plastic volumetric strain increment and plastic shear strain increment and  $\beta$  is the size parameter. Typically, an unassociated flow rule is adopted for modeling complex mechanical behaviors. However, when  $M_g = M_f$  and  $k_g = k_f$ , an associated flow rule would be recovered. When  $k_g = k_f = 2$ , elliptical surfaces like modified Cam-clay model are recovered.

### 2.2.2 Hardening laws

In ACC2-T model, two plastic hardening mechanisms fixed to reference temperature  $T_0$  are adopted to control the evolution of yield surface and loading surface.

In the isotropic hardening rule, as same as in Modified Cam-clay model, the evolution of pre-consolidation pressure  $\bar{p}'_{c0}$  at reference temperature  $T_0$  is linked to the plastic volumetric strain increment:

$$d\bar{p}'_{c0} = \frac{\nu}{\lambda - \kappa} \bar{p}'_{c0} d\varepsilon_v^p \quad (6)$$

where  $\lambda$  is the normal consolidation slope in  $(v, \ln p')$  plane and  $d\bar{p}'_{c0}$  is the increment of pre-consolidation pressure. It means the evolution of yield surface is dominated by the plastic volumetric strain.

The progressive hardening variable  $dr_0$ , at reference temperature  $T_0$ , is defined with a combined volumetric and shear plastic strain as follows:

$$dr_0 = \frac{v}{\lambda - \kappa} s (1 - r_0) (d\varepsilon_v^p + A_d d\varepsilon_s^p) \quad (7)$$

where  $r_0$  is the inverse of over-consolidation ratio (OCR) at current temperature  $T_0$  and  $dr_0$  is the increment.  $s$  is a material constant which controls the hardening rate of loading surface in approaching yield surface.  $A_d$  is a material constant which controls the contribution of shear plastic strain.

In this progressive hardening rule, parameter  $r_0$  is always less than or equal to 1 ( $0 < r_0 \leq 1$ ). When the loading surface is completely inside the yield surface ( $0 < r_0 < 1$ ), it means the soil is over-consolidated. Within the hardening process, plastic strain increases and the loading surface begins to move towards the yield surface ( $dr_0 > 0$ ). When  $r_0 = 1$ , the loading surface and the yield surface coincide. The soil turns into normally consolidated state as the same case of MCC model. Note that the loading surface could not run out of the yield surface ( $r_0 > 1$ ) for the fact that if  $r_0 > 1$ ,  $dr_0 < 0$  holds. With such a combined volumetric-deviatoric hardening law, the inclusion of shear hardening part ( $d\varepsilon_s^p > 0$ ) would lead to a positive plastic modulus ( $h > 0$ ) even when the effective stress state ( $d\varepsilon_v^p = 0$ ) reach the critical state line (CSL). As a consequence, the stress point lying on the loading surface can cross the CSL,

hardening continues to a stress point ( $d\varepsilon_v^p < 0$ ,  $d\varepsilon_s^p > 0$ ,  $h = 0$ ) and then the stress point ( $d\varepsilon_v^p < 0$ ,  $d\varepsilon_s^p > 0$ ,  $h < 0$ ) would move back to the CSL. This approach allows to predict shear softening behavior and it is often the case of highly over-consolidated states of soil.

With these isotropic and progressive hardening rules, the plastic modulus and increment stress-strain relationship of ACC2-T model are obtained in Appendix A.

Under loading and heating processes, two hardening variables  $\bar{p}'_{c0}$  and  $r_0$  are activated with the evolution of plastic strain. However, under unloading and cooling processes, pure elastic strain develops with a fixed value of  $\bar{p}'_{c0}$ . Due to ACC2-T model strictly adhering to the principle that the stress point is always lying on the loading surface, the value of  $r_0$  could be re-calculated by a simple Newton-Raphson Method with unconditional stability.

### 2.2.3 Yield limits and dependency law

In the two surface models, the loading surface and the yield surface are geometrically similar in  $(p', q)$  plane with a corresponding radial mapping rule. In previous TEAM model [21], two yield limit mechanisms, namely thermal yield limits ( $ITY$ ,  $TY$ ) and loading yield limits ( $ILY$ ,  $LY$ ) are adopted to model the thermal behavior. These yield limit mechanisms in TEAM model (see Fig.3(a)) enable the thermal elastic response in reloading path to occur before entering the elasto-plastic zone. Therefore, the TEAM model could not simulate the strain accumulation phenomenon due to temperature

variations. In order to overcome this shortcoming and extend the model to the case of multiple thermal cycles, the loading yield limits ( $ILY$ ,  $LY$ ) are unified to consider both the thermal yield and loading yield mechanisms. Meanwhile, a new dependency law is proposed to connect  $ILY$  and  $LY$ .

In the dependency law, the pre-consolidation pressure  $\bar{p}'_{cT}$  at current temperature  $T$  takes the original form and it would decrease with the increase of temperature:

$$\bar{p}'_{cT} = \bar{p}'_{c0} \exp[-\alpha_0(T - T_0)] \quad (8)$$

where  $\bar{p}'_{c0}$  is pre-consolidation pressure at reference temperature  $T_0$ ;  $\alpha_0$  is a material constant controlling the thermal plastic behavior.

The other part of the dependency law  $r$  is defined as below:

$$r = \frac{1}{OCR} = \frac{p'}{\bar{p}'} = \frac{q}{\bar{q}} = \frac{p'_{cT}}{\bar{p}'_{cT}} \quad (9)$$

where  $(p', q)$ ,  $(\bar{p}', \bar{q})$  is the corresponding stress point in loading surface and yield surface at current temperature  $T$ , respectively.

Unlike TEAM model with the assumption that  $r = r_0$ , ACC2-T model takes the form that would make  $ILY$  not necessarily homologous to  $LY$  any longer (see Fig.3(b)):

$$r = r_0 \exp[-\alpha_0(r_0^{n_c} - 1)(T - T_0)] \quad (10)$$

where  $n_c$  is a thermal parameter which controls the strain accumulation rate under multiple thermal cycles.

The combined dependency law made up by  $\bar{p}'_{cT}$  and  $r$  then is expressed by:

$$r\bar{p}'_{cT} = r_0\bar{p}'_{c0} \exp[-\alpha_0 r_0^{n_c} (T - T_0)] \quad (11)$$

In the proposed law, the shape of  $ILY$  is governed by the key thermal accumulation

parameter  $n_c$ . Parameter  $n_c$  controls the evolution rate of  $ILY$  approaching  $LY$  by the two isotropic and progressive hardening rules. In order to illustrate the plastic mechanism under the thermal path, two cases in the isotropic states, namely  $n_c < 0$  and  $n_c = 0$ , are compared in Fig.4. Both two cases are at initial stress state A, then heated up to stress state B and finally cooled down to stress state A. Under the heating path from A to B, the  $ILY$  without  $n_c$  ( $n_c = 0$ ) would induce a increment of  $\Delta r_1$ . For the case of  $n_c < 0$ , it would induce a increment of  $\Delta r_1 + \Delta r_2 + \Delta r_3$  if the shape of  $ILY$  is fixed. However, with the effect of  $n_c$ , the original  $ILY$  (see Fig.4(b), dotted line I) would gradually turn into solid line II with a actual increment of  $\Delta r_1 + \Delta r_2$ . The larger increment  $\Delta r_1 + \Delta r_2$  in hardening variable  $dr_0$  would mean larger plastic volumetric strain increments than the case of  $n_c = 0$ . Under the cooling path from B to A, the mechanical response is purely elastic and the shape of  $ILY$  would turn from II into I. For the case of  $n_c > 0$ , the increment in hardening variable  $dr_0$  would induce smaller strain increments than  $n_c = 0$ . Under multiple thermal cycles, the value of  $r_0$  would be smaller and smaller.

Under such a unique dependency law, the evolution rate of  $ILY$  is actually the evolution rate of strain accumulation. With a proper selection of parameter  $n_c$  in the dependency law, ACC2-T model allows the strain accumulation under the effect of multiple thermal cycles to be modelled. Some important features about the dependency law and parameter  $n_c$  are summarized below:

1. When the current temperature  $T$  is equal to the reference temperature  $T_0$ ,  $r$  becomes equal to  $r_0$ . It means that  $n_c$  would have no effect on the stress-strain relationship at

reference temperature  $T_0$ .

2. When soil turns into the normal consolidated state ( $r_0 = 1$ ),  $r$  is also equal to 1. In that case,  $ILY$  would coincide with  $LY$ . It means that  $n_c$  would have no effect on the behavior of normal consolidated soils under either isothermal or non-isothermal condition.

3. When  $n_c$  is equal to zero (bold line in Fig. 3(b)),  $r$  becomes equal to  $r_0$ . In that case, a similarity between  $ILY$  and  $LY$  would be recovered. This is just the case of  $ILY$  and  $LY$  used in TEAM model [21].

4. Theoretically, the value of  $n_c$  can be chosen from negative infinity to positive infinity. When  $n_c$  tends to positive infinity,  $ILY$  would tend to be the dotted line in Fig. 3(b). When  $n_c$  tends to negative infinity,  $ILY$  would tend to approach the temperature axis  $T$  in Fig. 3(b). A larger  $n_c$  represents a quicker strain accumulation rate which would induce smaller total volumetric strain accumulation under the effect of thermal cycles.

5. Any possible  $r$  is smaller than 1. During the heating process, the hardening mechanism enables  $r_0$  to approach 1. Thus,  $r$  would also have its limit value of 1 which implies that the over-consolidation ratio of soil is always larger than or equal to 1.

### 3. Loading path independence

After the description of the proposed model extended to thermal cyclic behavior, it is essential to discuss the rationale of ACC2-T model in terms of elasto-plastic theory and thermodynamic theory as Hong et al. [22] did on three existing thermo-mechanical models [19,20,25,28,29]. The verification of loading path independence is made up by

both the thermo-elastic and thermo-plastic part. The schematic plot is given in Fig.5.

Heating is treated as a special isotropic loading. Therefore, merely isotropic condition

( $q=0$ ) is considered during the derivation. The thermo-elastic part is discussed as

below and the derivation in thermo-plastic regime is attached in Appendix B.

In the thermo-elastic mechanism (Eq.3) of ACC2-T model, the bulk modulus

$K = \frac{\nu p'}{\kappa}$  is independent of temperature, and thermal elastic compressibility index  $\alpha_1$

is also independent of mean effective stress. Such elastic volumetric strain in ACC2-T

model is surely loading path independent in ( $p', T$ ) plane. Note that the thermo-elastic

mechanism leads to loading path dependent in ( $p', q$ ) plane since the Maxwell symmetry

relations are not satisfied. A possible way is to extend a hyperelastic law [39] into non-

isothermal conditions. For simplicity of parameter calibration, merely this thermo-

elastic mechanism is considered here.

#### 4. Analysis and determination of model parameters

The proposed ACC2-T model contain fifteen parameters ( $\lambda, \kappa, \bar{p}'_{c0}, \nu, M_f, k_f, M_g, k_g, s,$

$r_0, A_d, \alpha_0, \alpha_1, T_0, n_c$ ). Five of them ( $\lambda, \kappa, \bar{p}'_{c0}, \nu, M_g$ ) are commonly defined in MCC

model. ( $M_f, k_f$ ), ( $M_g, k_g$ ) control the shapes of yield surfaces and plastic potential

surfaces, respectively. Three parameters ( $s, r_0, A_d$ ) are used to consider a progressive

yielding of two surfaces. Four parameters ( $\alpha_0, \alpha_1, T_0, n_c$ ) are defined to consider the

thermal influence on the soil behavior. The procedure for determining these parameters

is described and key parameters ( $\lambda, \kappa, \bar{p}'_{c0}, s, r_0, \alpha_0, \alpha_1, T_0, n_c$ ) related to thermal

290 accumulation behavior are discussed on the set of model parameters for natural Boom  
291 clay in Fig. 6.

292 1. By an isotropic compression test or oedometer test under reference temperature  $T_0$ ,  
293  $\kappa$ ,  $\lambda$ , the initial  $r_{0,0}$  and  $\bar{p}'_{c0,0}$  are determined in  $(v, \ln p')$  plane in Fig.6(1). Note that  
294 parameter  $s$  controls the hardening rate of loading surface in approaching yield surface.  
295 It can be determined by fitting nonlinearity from over-consolidated states to normal  
296 consolidated state in  $(v, \ln p')$  plane.

297 2. By an isotropic heating-cooling test at normally consolidated states,  $\alpha_1$  is  
298 determined by fitting data of cooling stage and  $\alpha_0$  is determined by the heating test in  
299  $(T, \Delta \varepsilon_v)$  plane in Fig.6(2).

300 3. By a continuous thermal cycle test, the volumetric strain increment by each thermal  
301 cycle  $\Delta \varepsilon_{v,st}$  can be obtained  $(\Delta \varepsilon_{v,st}, n)$  plane in Fig.6(3). The thermal accumulation  
302 parameter  $n_c$ , which controls the strain accumulation rate, is then determined by  
303 calibrating the strain accumulation under each thermal cycle.

304 4.  $M_f$ ,  $k_f$ .  $M_f$  denotes the apex of yield surface and  $k_f$  controls the shape of yield surface.  
305 They are calibrated by fitting the conventional yield points of various stress paths.

306 5.  $M_g$  is the critical state stress ratio and it is calibrated by the effective stress ratio at  
307 critical state.

308 6.  $k_g$  controls plastic flow direction. It is determined by fitting the values of  $d\varepsilon_v^p / d\varepsilon_s^p$   
309 in drained triaxial shear tests.

310 7. The poisson's ratio  $\nu$  is determined by a triaxial tests at low strain level in the  $(\varepsilon_v - \varepsilon_l)$



plane:  $v = (\varepsilon_v - \varepsilon_l)/2$ .

8.  $A_d$  represents the contribution of the plastic shear strain in the hardening process. It is calibrated by fitting the results of triaxial compression tests at over-consolidated states in  $(q, \varepsilon_l)$  or  $(\varepsilon_v, \varepsilon_l)$  plane in Fig.6(4).

## 5. Model Validation and prediction

In this section, four different fine-grained soils, namely natural Boom clay, Geneva clay, Bonny silt and reconstituted Pontida clay, are adopted to verify the capability of ACC2-T model in modeling the thermal cyclic behavior.

### 5.1 Natural Boom clay

One heating and cooling test on natural Boom clay ( $I_p = 31$ ) under isotropic stress states performed by Baldi et al. [8] and two groups of triaxial compression tests conducted by Lê [38] are taken into account.

In the heating and cooling test, specimen TBoom-9 taken from Mol at an in-situ depth of 223 m (initial pre-consolidated pressure of 6 MPa) is tested under a mixed mechanical and thermal loading process: the sample was firstly saturated under a constant mean effective stress of 2 MPa at 21.5 °C, loaded up to 4 MPa, unloaded to 1 MPa, heated up to 95 °C and cooled back to 21.5 °C, loaded up to 3 MPa at 21.5 °C and heated up to 95 °C and cooled back, loaded up to 6 MPa and heated up again and finally cooled back. In the loading process, three thermal cycles of temperature

increment of 73.5 °C at different isotropic stress states, namely mean effective stress of 1 MPa, 3 MPa and 6 MPa, are selected for simulation. For better comparison between the computed and measured results, the increments of volumetric strain are reset at the starting point of each heating-cooling cycle. Moreover, Fig. 7 contains an isotropic compression test of natural Boom clay from 2 MPa to 20 MPa, then back to 6 MPa. Basic parameters for natural Boom clay are listed in Tab.1.

The TEAM model [21] and the proposed ACC2-T model are both used for simulation in Fig.8. It appears that both models can describe the thermal volume change well for normally consolidated or slightly over-consolidated state ( $p' = 6$  MPa) adopting the same thermal elastic parameter  $\alpha_1$  and thermal plastic parameter  $\alpha_0$ . However, for highly over-consolidated states ( $p' = 3$  MPa,  $p' = 1$  MPa), it is observed that ACC2-T model gives a more consistent simulation than TEAM model. One reason is that TEAM model always shows a linear elastic expansion behavior before entering the elastoplastic zone and has no parameters controlling strain evolution. With this shortcoming, TEAM model would fail to respond to thermal cycles and thus to describe strain accumulation behavior, though complicated multiple plastic mechanisms are adopted. However, in ACC2-T model, the stress point is always lying on the loading surface. Combined with the isotropic and progressive hardening rules, the thermal accumulation parameter  $n_c$  in the dependency law allows the ACC2-T model to better describe the nonlinear expansion behavior and strain accumulation under over-consolidated states. Meanwhile, as natural Boom clay is relatively easy to contract under heating, the use

of a negative value of parameter  $n_c$  could facilitate the simulation of such aspect.

In the triaxial compression test, the experimental results of drained triaxial compression tests ( $p'_0 = 2.5$  MPa,  $p'_0 = 4$  MPa) at room temperature 25 °C and drained triaxial compression tests ( $p'_0 = 2.5$  MPa,  $p'_0 = 3.8$  MPa) at high temperature 76 °C [38] are considered in  $(q, \varepsilon_l)$  plane and  $(\varepsilon_v, \varepsilon_l)$  plane in Fig.9. In these moderate over-consolidated states, the shear strength at room temperature is a bit higher than high temperature. However, a larger volumetric strain is generated at high temperature than room temperature as shown in  $(\varepsilon_v, \varepsilon_l)$  plane. At the same time, the smooth predictions by ACC2-T model agree well with the experimental observations. Two values of  $n_c$  ( $n_c = -1, n_c = 0$ ) are selected to simulate the behavior at high temperature 76 °C. It indicates that the value of  $n_c$  has a negligible effect on the prediction of triaxial tests.

A numerical case at high over-consolidated state ( $p'_0 = 0.5$  MPa) under both two temperatures (25 °C, 76 °C) are conducted. A slight shear dilatancy is predicted at axial strain of about 5%. However, the post-peak softening phenomenon in  $(q-\varepsilon_l)$  plane is not very apparent because parameter  $A_d$  ( $A_d = 0.1$ ) for natural Boom clay is very small to simulate behaviors at both high, slightly over-consolidated and normal consolidated states.

## 5.2 Geneva clay

Multiple thermal cycles were applied on Geneva clays ( $I_p = 11.1$ ) in a temperature-controlled oedometer cell under drained condition [25]. A normally consolidated soil

sample with a vertical effective stress of 125 kPa was tested with four temperature cycles from 5 °C to 60 °C, starting at a room temperature of 20 °C. The one-dimension test results are simulated in Fig.7 and the calibrated model parameters are listed in Tab.1.

Fig.10 shows the measured and computed soil behavior under multiple heating and cooling cycles. The computed result of ACC2-T model (Fig.10(a)) shows a good agreement with the measured one. The irreversible volumetric strain accumulates with increasing thermal cycles, but at a decreasing rate (Fig.10(c)). The value of 3.5 is shown appropriate for the thermal accumulation parameter  $n_c$  to control the strain accumulation rate. For the case of TEAM model, the computed result of first thermal cycle is the same as ACC2-T model. However, the response of TEAM model is purely elastic thereafter and the thermal strain accumulation could not be described. In Fig.10(b), a sensitivity analysis of ACC2-T model is conducted to study the effect of parameter  $n_c$  on the strain accumulation. Two other  $n_c$  values of 0, 10 are selected for this purpose. In the first thermal cycle, ACC2-T model gives the same results for all the three  $n_c$  values because the thermal volumetric strain of normally consolidated soil is only dependent on thermal elastic parameter  $\alpha_1$  and thermal plastic parameter  $\alpha_0$ . From the second thermal cycle, the soil becomes over-consolidated. The thermal accumulation parameter  $n_c$  starts to control the strain evolution rate. From Fig.10(c), it appears that parameter  $n_c$  is positively correlated to the strain accumulation rate and a proper selection of parameter  $n_c$  is crucial for total strain accumulation.

### 5.3 *Bonny silt*

Multiple heating and cooling cycles were applied on compacted Bonny silt ( $I_p = 4$ ) under different OCRs in a temperature-controlled oedometer cell [27]. Bonny silts were first mechanically consolidated to a vertical effective stress of approximately 1200 kPa at a constant room temperature of 18 °C. Then, they were unloaded to reach five different OCR values of 1.00, 1.28, 1.80, 7.36 and 30.29. Fig. 7 presents the experimental and calibration results of one-dimensional test of Bonny silt. Four heating and cooling cycles were applied under drained condition with different OCRs in a temperature ranging from 18 °C to 93 °C.

It can be seen from Fig.11 that at slightly over-consolidated states (i.e. OCR = 1, 1.28 and 1.8), the measured and computed results show a good consistency. That means the thermal accumulation parameter  $n_c$  can properly control the strain evolution rate under various over-consolidated states. However, for highly over-consolidated ones (i.e. OCR = 7.36, 30.29) in the literature [27], the volumetric strain accumulation is quite small or even becomes negative, suggesting that the expansion and the contraction of the oedometer cell can be no longer ignored. These two samples are not considered in the simulation.

### 5.4 *Reconstituted Pontida clay*

The excess pore water pressure accumulation due to thermal cycles is validated by a heating and cooling test of reconstituted Pontida clay. Hueckel et al [12] reported this measured and computed results on reconstituted Pontida clay. The reconstituted silty

clay sample was firstly loaded to a mean effective stress state of 2.5 MPa. Then it underwent a heating-cooling cycle from 22°C to 65°C and back to 22°C, then a continuous heating to 95°C in a constant total isotropic stress state.

Although some previous models [19] consider the undrained behavior by imposing a constant volume condition which is imprecise under non-isothermal conditions, the undrained computation using ACC2-T model is obtained by extending the energy and mass conservation of thermo-poro-elastic media into elasto-plastic states [30].

In a saturated porous media, the current water mass ( $m_w$ ) per unit volume can be expressed as:

$$m_w = \rho_w \phi \quad (12)$$

where  $\rho_w$  is water density and  $\phi$  is the Lagrangian porosity.

In undrained conditions, water mass is constant and  $dm_w = 0$  holds, then

$$\frac{dm_w}{\rho_w} = d\phi + \phi \frac{d\rho_w}{\rho_w} = d\phi + \phi (C_w du - \alpha_w dT) = 0 \quad (13)$$

where  $\alpha_w$ ,  $C_w$ , is the water thermal expansion coefficient, water compressibility, respectively.

In isotropic stress state and under non-isothermal condition, total mean stress change ( $dp$ ) and Lagrangian porosity change ( $d\phi$ ) can be expressed with Maxwell's symmetry condition:

$$dp = \frac{1}{C_d} d\varepsilon_v + (1 - \frac{C_s}{C_d}) du_w + \frac{\alpha_d}{C_d} dT \quad (14)$$

$$d\phi = -(1 - \frac{C_s}{C_d}) d\varepsilon_v - (\phi - 1 + \frac{C_s}{C_d}) C_s du_w + (\phi - 1 + \frac{C_s}{C_d}) \alpha_s dT \quad (15)$$

where  $C_s$ ,  $C_d$ ,  $\alpha_s$ ,  $\alpha_d$  is the volumetric compressibility coefficient of solid grain, the volumetric compressibility of soil skeleton, the thermal expansion coefficient of solid grain and the thermal expansion coefficient of soil skeleton. It is often the case that  $\alpha_s = \alpha_d$ .

Thus, the expressions for excess pore water pressure can be obtained, as follows:

$$d\varepsilon_v = -(\phi\alpha_w + (1-\phi)\alpha_s)dT + \phi(C_w - C_s)du_w + C_s dp \quad (16)$$

With infinitesimal transformation and clayey soils, the fact ( $C_s \approx 0$ ,  $n \approx \phi$ ) is commonly accepted. Thus

$$du_w = \frac{d\varepsilon_v + (n\alpha_w + (1-n)\alpha_s)dT}{nC_w} \quad (17)$$

where  $n$  is Eulerian porosity.

For reconstituted Pontida clay,  $\alpha_s$  is  $1 \times 10^{-5}$  ( $1/^\circ\text{C}$ ) chosen as same as  $\alpha_l$ , which is calibrated from an undrained cooling process from  $65^\circ\text{C}$  to  $22^\circ\text{C}$ .  $\alpha_w$  and  $C_w$  are calibrated from a heating test of free water in the literature [35].

The computed results of excess pore water pressure by ACC2-T model and TEAM model are compared to the measured data and Hueckel's simulation [12] in Fig.12. It is seen that both predictions of ACC2-T model and TEAM model are more accurate than Hueckel's simulation, especially the existed buildup of pore water pressure in experimental results. During the heating stage from  $22^\circ\text{C}$  to  $65^\circ\text{C}$ , the mean effective stress will decrease with temperature (equivalent to a mechanical loading under undrained condition) and the effective stress-temperature path ( $T - p'$ ) initially enters plasticity. With the plastic hardening of *ILY*, excess pore water pressure is increased. Then during the cooling stage from  $65^\circ\text{C}$  to  $22^\circ\text{C}$ , the response is purely elastic with a decrease of the pore water pressure. Thereby, an irreversible excess pore water pressure is observed after one heating-cooling cycle which is well captured by ACC2-T model and TEAM model. Moreover, though the cooling stage is purely elastic, the *ILY* would shrink to make the stress-temperature state always lying on it. However, during the re-heating process, TEAM model could not build up excess pore water pressure from  $22^\circ\text{C}$  to  $65^\circ\text{C}$  because the response is purely elastic. As for ACC2-T model, the clay sample is initially over-consolidated, then began to build up excess pore water pressure with the control of thermal accumulation parameter  $n_c$  in the whole process from  $22^\circ\text{C}$  to  $95^\circ\text{C}$ .

## 5.5 Discussion

Four fine-grained soils are selected to verify the capability of ACC2-T model on modelling thermal cyclic behaviors under drained and undrained conditions. A thermal accumulation parameter  $n_c$  control the accumulation rate under multiple thermal cycles. Meanwhile, the values of  $n_c$  are ranked as: natural Boom clay< reconstituted Pontida clay<Geneva clay<Bonny silt. It means the clays, containing more clay contents, showing larger plastic index  $I_p$ , are more likely to accumulate volumetric strain and build up excess pore water pressure. Generally, the larger is the plastic index  $I_p$ , the smaller is the value of  $n_c$ . A liner regression function is obtained to evaluate the correlation between parameter  $n_c$  and plastic index  $I_p$  in Fig. 13.

## 6. Concluding remarks

A two-surface plasticity model (ACC2-T) is proposed to describe the thermo-mechanical behavior of fine-grained soils under multiple thermal cycles. Compared with TEAM model, two thermal yield limits are removed and the loading yield limits unify the thermal and loading yielding part. A dependency law with thermal accumulation parameter  $n_c$  is used to link the inner loading yield limit and the loading yield limit. Parameter  $n_c$  controls the evolution rate of the inner loading yield limit approaching the loading yield limit under thermal loading path. With such a unique dependency law and multiple plastic hardening rules, the model could well control the thermal strain accumulation rate under drained conditions and pore water pressure



buildup rate under undrained conditions. Tests on four fine-grained soils are considered to verify the proposed model. Comparison between prediction and experiment shows that ACC2-T model can well capture the thermo-mechanical behavior of soils under multiple thermal cycles.

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## 510 **Notation**

511  $\alpha_0$  = parameter controlling thermal plastic behavior;

512  $\alpha_1$  = parameter controlling thermal elastic behavior;

513  $\beta$  = the size parameter of potential plastic surface;

514  $\lambda, \kappa$  = slope of isotropic loading, reloading line respectively;

515  $\nu, \nu$  = current void ratio, Poisson's ratio respectively;

516  $A_d$  = parameter controlling the contribution of shear plastic strain;

517  $d\boldsymbol{\varepsilon}, d\boldsymbol{\sigma}, dT$  = total strain, stress, temperature increment respectively;

518  $d\boldsymbol{\varepsilon}_\sigma^e, d\boldsymbol{\varepsilon}_T^e$  = stress, thermal induced elastic strain increment respectively;

519  $d\varepsilon_v, d\varepsilon_v^e, d\varepsilon_s^e$  = total volumetric, elastic volumetric, shear strain increment

520 respectively;

521  $d\boldsymbol{\varepsilon}^p, d\varepsilon_v^p, d\varepsilon_s^p$  = plastic, plastic volumetric, plastic shear strain increment

522 respectively;

523  $\Lambda$  = plastic multiplier;

524  $f_l, f_y$  = Loading surface, yield surface respectively;

525  $G, K, h$  = elastic shear, elastic bulk, plastic modulus respectively;

526  $k_f, k_g$  = parameter controlling the shape of yield surface, plastic potential surface

527 respectively;

528  $M_f$  = stress ratio at the apex of yield surface;

529  $M_g, M_{g0}$  = critical stress ratio at current, reference temperature respectively;

530  $p', \bar{p}'$  = current, mapping mean effective stress respectively;

531  $p'_{cT}, p'_{c0}$  = pre-consolidation pressure in loading surface at current, reference  
 532 temperature respectively;  
 533  $\bar{p}'_{cT}, \bar{p}'_{c0}$  = pre-consolidation pressure at current, reference temperature respectively;  
 534  $q, \bar{q}$  = current, mapping deviator stress respectively;  
 535  $r, r_0$  = reverse of OCR at current, reference temperature respectively;  
 536  $s$  = parameter controlling the rate of evolution of the slope of the stress--strain  
 537 relation;  
 538  $T, T_0$  = current, reference temperature respectively;  
 539  $\mathbf{D}_e, \mathbf{D}_{ep}, \mathbf{D}_{et}$  = elastic, elasto-plastic, temperature stiffness matrix respectively;  
 540  $\mathbf{m}$  = unit vector.  
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## 556 **Appendix A**

557 The consistency condition of loading surface in ACC2-T model is adopted herein:

$$558 \left( \frac{\partial f_I}{\partial \boldsymbol{\sigma}'} \right)^t : d\boldsymbol{\sigma}' + \frac{\partial f_I}{\partial T} dT + \frac{\partial f_I}{\partial \bar{p}'_{c0}} d\bar{p}'_{c0} + \frac{\partial f_I}{\partial r_0} dr_0 = 0 \quad (\text{A} - 1)$$

559 The plastic multiplier  $\Lambda$  and the plastic modulus  $h$  are obtained in non-isothermal  
560 condition:

$$561 d\boldsymbol{\varepsilon}^p = \Lambda \frac{\partial \mathbf{g}_I}{\partial \boldsymbol{\sigma}'} \quad (\text{A} - 2)$$

$$562 \Lambda = - \frac{\left( \frac{\partial f_I}{\partial \boldsymbol{\sigma}'} \right)^t : d\boldsymbol{\sigma}' + \frac{\partial f_I}{\partial T} dT}{\frac{\partial f_I}{\partial \bar{p}'_{c0}} \frac{\nu}{\lambda - k} \bar{p}'_{c0} \frac{\partial \mathbf{g}_I}{\partial p'} + \frac{\partial f_I}{\partial r_0} \frac{\nu}{\lambda - k} s(1 - r_0) \left( \frac{\partial \mathbf{g}_I}{\partial p'} + A_d \frac{\partial \mathbf{g}_I}{\partial q} \right)} \quad (\text{A} - 3)$$

$$563 h = \frac{\nu}{\lambda - k} \left( \frac{\partial f_I}{\partial \bar{p}'_{c0}} \bar{p}'_{c0} \frac{\partial \mathbf{g}_I}{\partial p'} + \frac{\partial f_I}{\partial r_0} s(1 - r_0) \left( \frac{\partial \mathbf{g}_I}{\partial p'} + A_d \frac{\partial \mathbf{g}_I}{\partial q} \right) \right) \quad (\text{A} - 4)$$

564 The total strain increment is made up of mechanical elastic strain increment, thermal  
565 elastic strain increment and plastic strain increment.

$$566 d\boldsymbol{\varepsilon} = d\boldsymbol{\varepsilon}_{\sigma'}^e + d\boldsymbol{\varepsilon}_T^e + d\boldsymbol{\varepsilon}^p \quad (\text{A} - 5)$$

567 The thermal elastic strain increment  $d\boldsymbol{\varepsilon}_T^e$  is defined as:

$$568 d\boldsymbol{\varepsilon}_T^e = -\frac{1}{3} \mathbf{m} \alpha_1 dT \quad (\text{A} - 6)$$

569 where  $\mathbf{m}$  is the column vector with 1 at normal stress entries and 0 at shear stress entries.

570 By substituting Eq. A-6 into Eq. A-5, the general incremental stress-strain relation  
571 can be expressed:

$$572 d\boldsymbol{\sigma}' = \mathbf{D}_e d\boldsymbol{\varepsilon}_{\sigma'}^e = \mathbf{D}_e \left( d\boldsymbol{\varepsilon} + \frac{1}{3} \mathbf{m} \alpha_1 dT - d\boldsymbol{\varepsilon}^p \right) \quad (\text{A} - 7)$$

573 where  $\mathbf{D}_e$  is the mechanical elastic matrix.

By substituting the plastic multiplier into Eq. A-7, the differential stress–strain equations can be obtained:

$$d\boldsymbol{\sigma}' = \mathbf{D}_{ep} d\boldsymbol{\varepsilon} + \mathbf{D}_{et} dT \quad (\text{A - 8})$$

where  $\mathbf{D}_{ep}$ ,  $\mathbf{D}_{et}$  are the mechanical elasto-plastic matrix and the thermal elasto-plastic matrix, respectively.

$$\mathbf{D}_{ep} = \mathbf{D}_e - \frac{\mathbf{D}_e \frac{\partial \mathbf{g}_I}{\partial \boldsymbol{\sigma}'} \left( \frac{\partial f_I}{\partial \boldsymbol{\sigma}'} \right)^t \mathbf{D}_e}{\left( \frac{\partial f_I}{\partial \boldsymbol{\sigma}'} \right)^t \mathbf{D}_e \frac{\partial \mathbf{g}_I}{\partial \boldsymbol{\sigma}'} + h} \quad (\text{A - 9})$$

$$\mathbf{D}_{et} = \frac{\mathbf{D}_e \frac{\partial \mathbf{g}_I}{\partial \boldsymbol{\sigma}'} \left[ -\frac{1}{3} \left( \frac{\partial f_I}{\partial \boldsymbol{\sigma}'} \right)^t \mathbf{D}_e \mathbf{m} \alpha_1 - \frac{\partial f_I}{\partial T} \right]}{\left( \frac{\partial f_I}{\partial \boldsymbol{\sigma}'} \right)^t \mathbf{D}_e \frac{\partial \mathbf{g}_I}{\partial \boldsymbol{\sigma}'} + h} + \frac{1}{3} \mathbf{D}_e \mathbf{m} \alpha_1 \quad (\text{A - 10})$$

## 596 Appendix B

597 Assume the soil is loaded and heated from Point A to Point B under two different  
 598 loading paths I(  $A \rightarrow C \rightarrow B$  ) and II(  $A \rightarrow D \rightarrow B$  ) in Fig.5. The soil is at the initial stress  
 599 state (  $p'_0, r_0, p'_{c0}, T_1$  ) which automatically satisfies the yield condition:

$$600 \quad f_{I0} = p'_0 - r_0 p'_{c0} \exp[-\alpha_0 r_0^{n_c} (T_1 - T_0)] = 0 \quad (\text{B - 1})$$

601 Under loading path I, the soil element is isotropically loaded to stress point  $C$   
 602 (  $p'_0 + \Delta p'_1, r_1, p'_{c1}, T_1$  ) at constant temperature  $T_1$ .

$$603 \quad f_{I1} = p'_0 + \Delta p'_1 - r_1 p'_{c1} \exp[-\alpha_0 r_1^{n_c} (T_1 - T_0)] = 0 \quad (\text{B - 2})$$

$$604 \quad p'_{c1} = p'_{c0} \exp\left[\frac{\nu}{\lambda - \kappa} \Delta \varepsilon_{v1}^p\right] \quad (\text{B - 3})$$

$$605 \quad r_1 = 1 + (r_0 - 1) \exp\left[-\frac{\nu}{\lambda - \kappa} s \Delta \varepsilon_{v1}^p\right] \quad (\text{B - 4})$$

606 Then the soil element is heated up to stress point  $B$  (  $p'_0 + \Delta p'_1, r_2, p'_{c2}, T_1 + \Delta T_1$  ) at  
 607 constant mean effective stress  $p'_0 + \Delta p'_1$ .

$$608 \quad f_{I2} = p'_0 + \Delta p'_1 - r_2 p'_{c2} \exp[-\alpha_0 r_2^{n_c} (T_1 + \Delta T_1 - T_0)] = 0 \quad (\text{B - 5})$$

$$609 \quad p'_{c2} = p'_{c1} \exp\left[\frac{\nu}{\lambda - \kappa} \Delta \varepsilon_{v2}^p\right] = p'_{c0} \exp\left[\frac{\nu}{\lambda - \kappa} (\Delta \varepsilon_{v1}^p + \Delta \varepsilon_{v2}^p)\right] \quad (\text{B - 6})$$

$$610 \quad r_2 = 1 + (r_1 - 1) \exp\left[-\frac{\nu}{\lambda - \kappa} s \Delta \varepsilon_{v2}^p\right] = 1 + (r_0 - 1) \exp\left[-\frac{\nu}{\lambda - \kappa} s (\Delta \varepsilon_{v1}^p + \Delta \varepsilon_{v2}^p)\right] \quad (\text{B - 7})$$

611 Under loading path II, the soil element is firstly heated up to stress point  $D$   
 612 (  $p'_0, r_3, p'_{c3}, T_1 + \Delta T_1$  ) at constant mean effective stress  $p'_0$ .

$$613 \quad f_{I3} = p'_0 - r_3 p'_{c3} \exp[-\alpha_0 r_3^{n_c} (T_1 + \Delta T_1 - T_0)] = 0 \quad (\text{B - 8})$$

$$614 \quad p'_{c3} = p'_{c0} \exp\left[\frac{\nu}{\lambda - \kappa} \Delta \varepsilon_{v3}^p\right] \quad (\text{B - 9})$$

$$615 \quad r_3 = 1 + (r_0 - 1) \exp\left[-\frac{\nu}{\lambda - \kappa} s \Delta \varepsilon_{v3}^p\right] \quad (\text{B - 10})$$

616 Then, the soil element is isotropically loaded to stress point  $B$  (  $p'_0 + \Delta p'_1, r_4, p'_{c4}, T_1 + \Delta T_1$  )

617 at constant temperature  $T_1 + \Delta T_1$ .

$$618 \quad f_{I4} = p'_0 + \Delta p'_1 - r_4 p'_{c4} \exp\left[-\alpha_0 r_4^{n_c} (T_1 + \Delta T_1 - T_0)\right] = 0 \quad (\text{B - 11})$$

$$619 \quad p'_{c4} = p'_{c3} \exp\left[\frac{\nu}{\lambda - \kappa} \Delta \varepsilon_{v4}^p\right] = p'_{c0} \exp\left[\frac{\nu}{\lambda - \kappa} (\Delta \varepsilon_{v3}^p + \Delta \varepsilon_{v4}^p)\right] \quad (\text{B - 12})$$

$$620 \quad r_4 = 1 + (r_3 - 1) \exp\left[-\frac{\nu}{\lambda - \kappa} s \Delta \varepsilon_{v4}^p\right] = 1 + (r_0 - 1) \exp\left[-\frac{\nu}{\lambda - \kappa} s (\Delta \varepsilon_{v3}^p + \Delta \varepsilon_{v4}^p)\right] \quad (\text{B - 13})$$

621 Compare Eqs.B-5, B-6, B-7 and Eqs.B-11, B-12, B-13, it appears clearly that the two  
622 different loading paths would reach the same final stress state, namely:

$$623 \quad p'_{c2} = p'_{c4} \quad (\text{B - 14})$$

$$624 \quad r_2 = r_4 \quad (\text{B - 15})$$

$$625 \quad \Delta \varepsilon_{v1}^p + \Delta \varepsilon_{v2}^p = \Delta \varepsilon_{v3}^p + \Delta \varepsilon_{v4}^p \quad (\text{B - 16})$$

626 No matter Path I or Path II is followed, the plastic volumetric strain will be equal, which  
627 means that plastic volumetric strain in ACC2-T model is also loading path independent.

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755    **LIST OF TABLES**

756        Table.1 Model parameters of different fine-grained soils

**Table.1 Model parameters of different fine-grained soils**

Parameters	$\kappa$	$\lambda$	$\bar{p}'_{c0,0}$ (kPa)	$r_{0,0}$	$\nu$	$\alpha_0$ ( $^{\circ}C^{-1}$ )	$\alpha_I$ ( $^{\circ}C^{-1}$ )	$s$	$T_0$ ( $^{\circ}C$ )	$n_c$	$M_{g0}$	$M_f$	$A_d$	$k_f$	$k_g$
<b>Boom clay[8]</b>	0.02	0.18	6000	0.33	-	$5 \times 10^{-3}$	$5 \times 10^{-5}$	8	21.5	-1	0.67	0.67	0.1	0.7	0.9
<b>Geneva clay[25]</b>	0.017	0.062	125	1	0.3	$5.2 \times 10^{-3}$	$1.5 \times 10^{-5}$	12	20	3.5	0.94	0.94	0.1	1.1	1.1
<b>Bonny silt[27]</b>	0.011	0.075	1200	*	0.3	$1.6 \times 10^{-3}$	$3.5 \times 10^{-5}$	10	18	4	1.3	1.3	0.2	1.1	1.1
<b>Pontida clay[12 ]</b>	0.016	0.103	2500	1	-	$3.5 \times 10^{-3}$	$5 \times 10^{-5}$	10	23	1	-	-	-	-	-

‘-’ means the parameters are irrelevant to the simulations in this study.

‘\*’ means different data are mentioned in this study.

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$I_p$

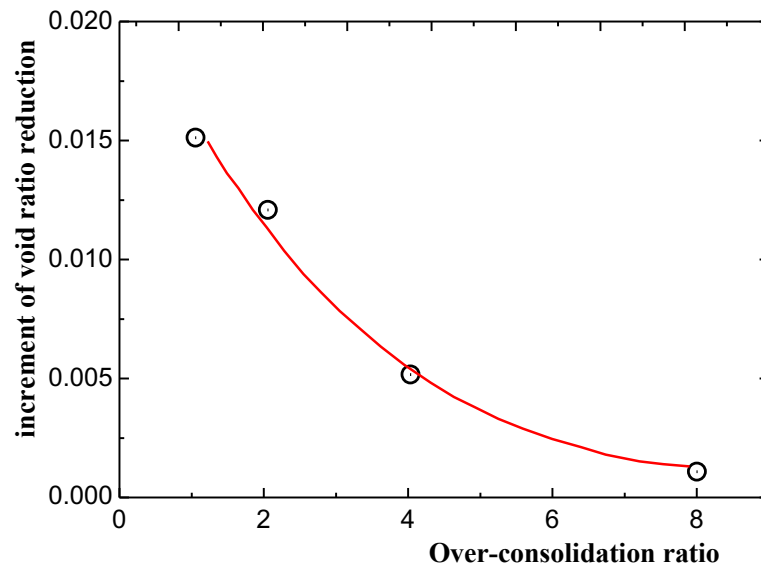
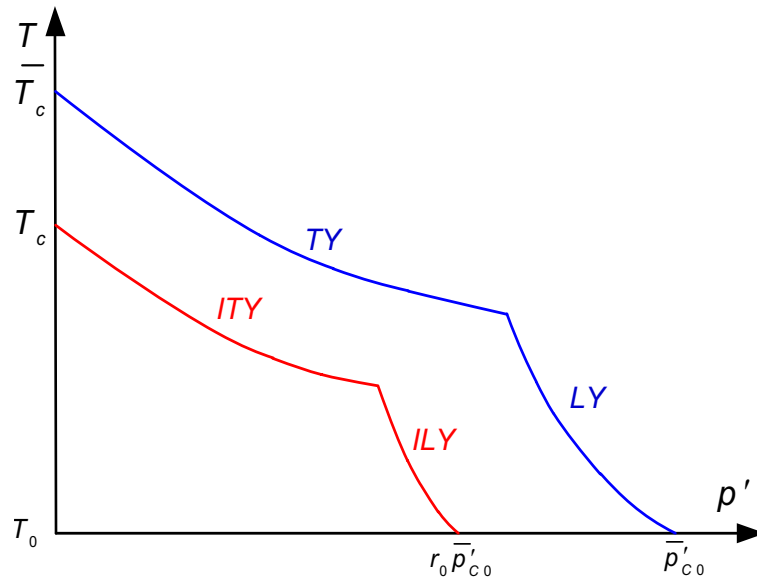


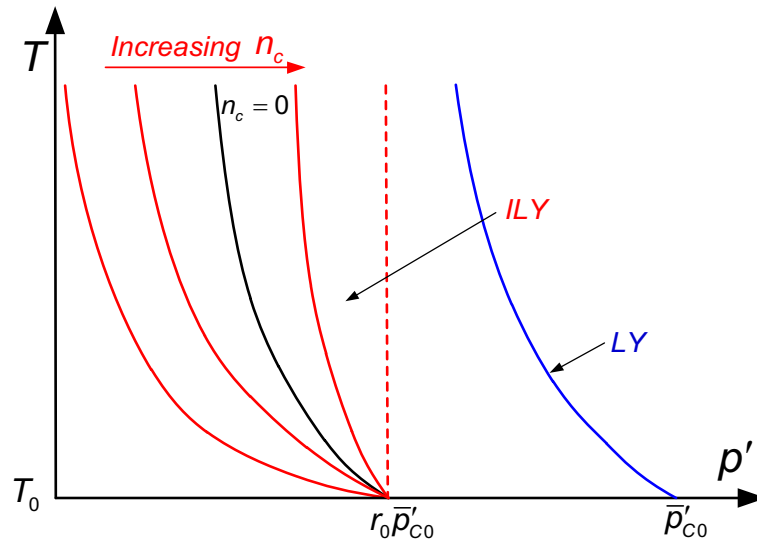
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(a)



(b)

Fig.3 Yield limits in  $(p', T)$  plane: (a) TEAM Model (b) ACC2-T Model

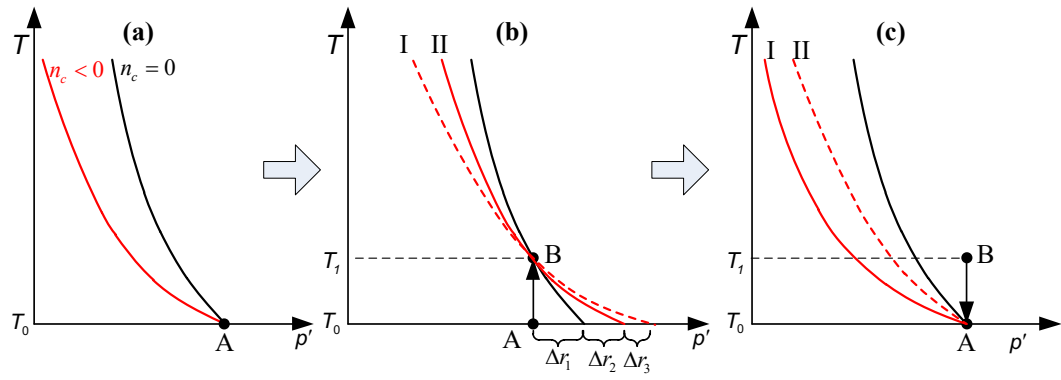


Fig.4 Evolution of yield limits in  $(p', T)$  plane during one heating and cooling cycle

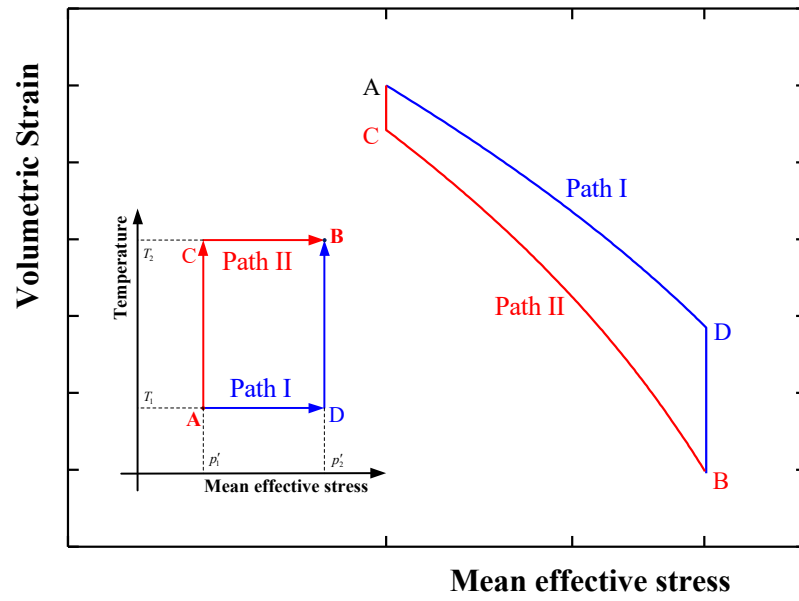
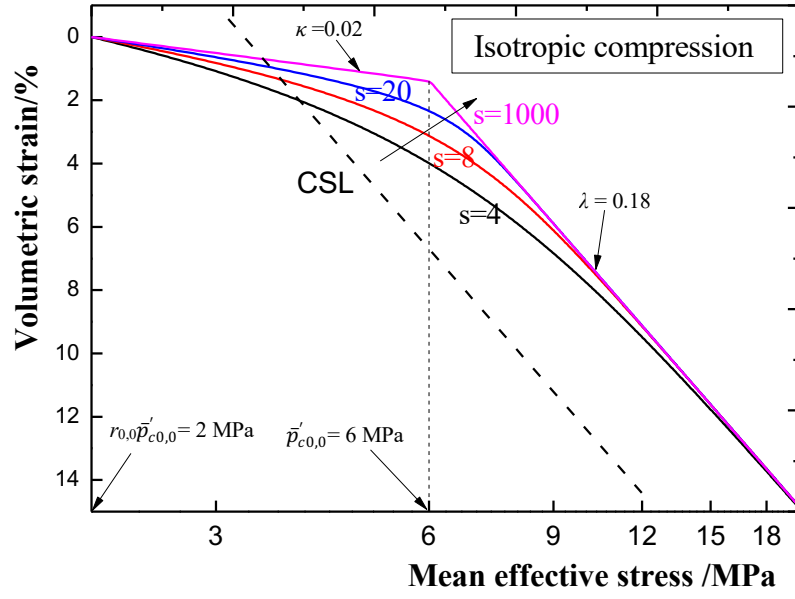
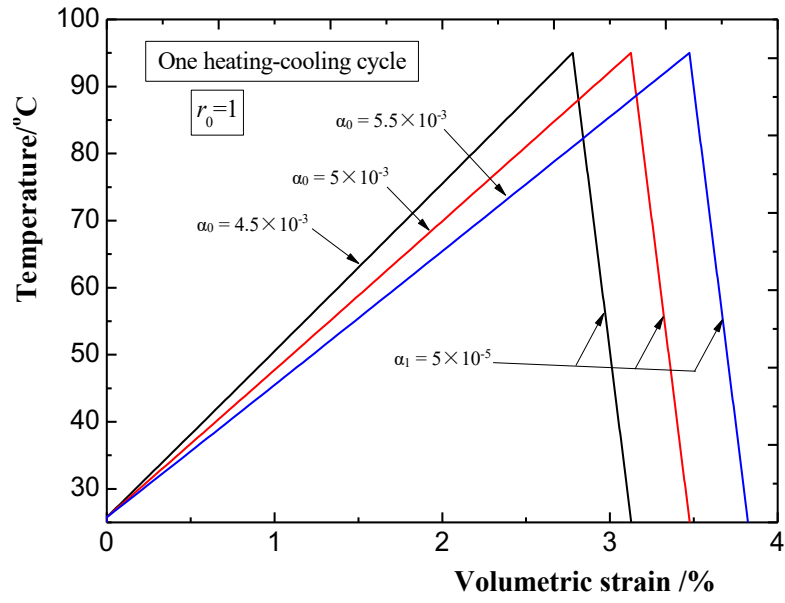


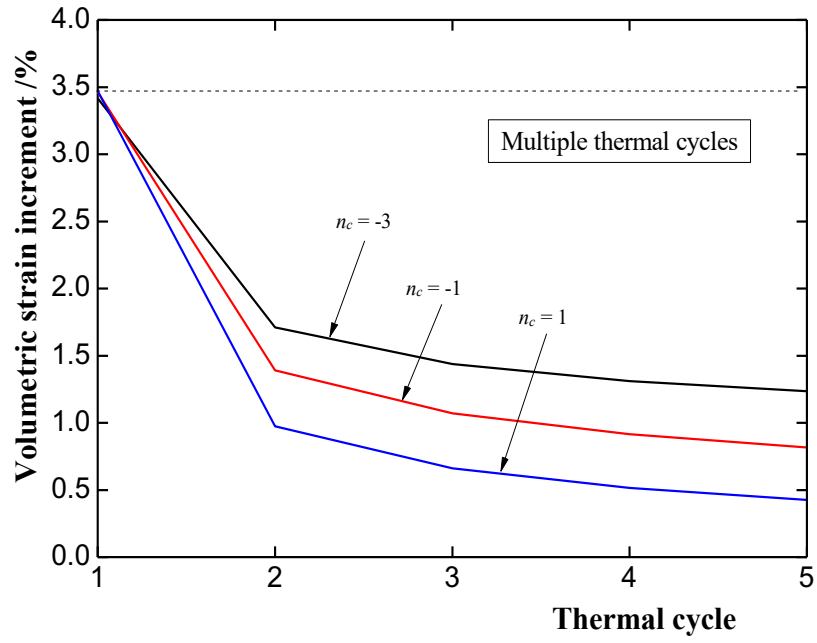
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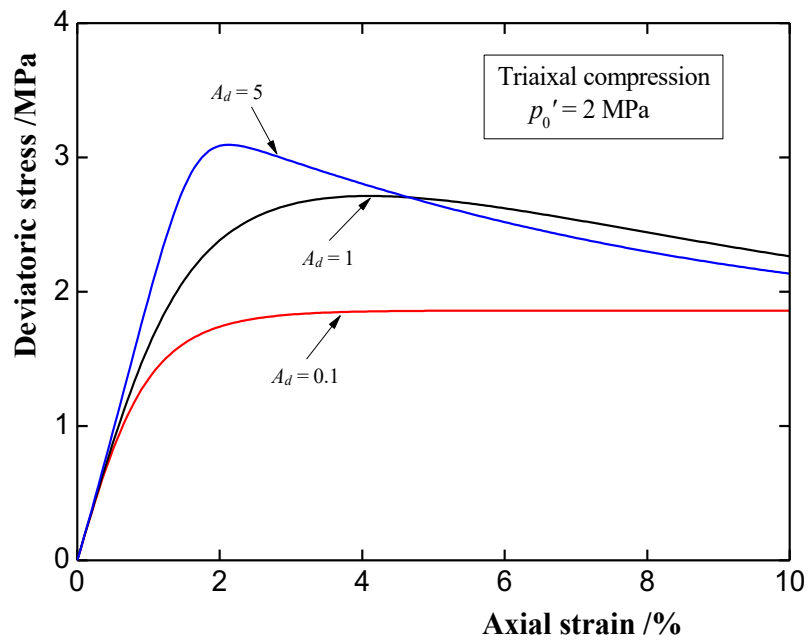
(a)



(b)



(c)



(d)

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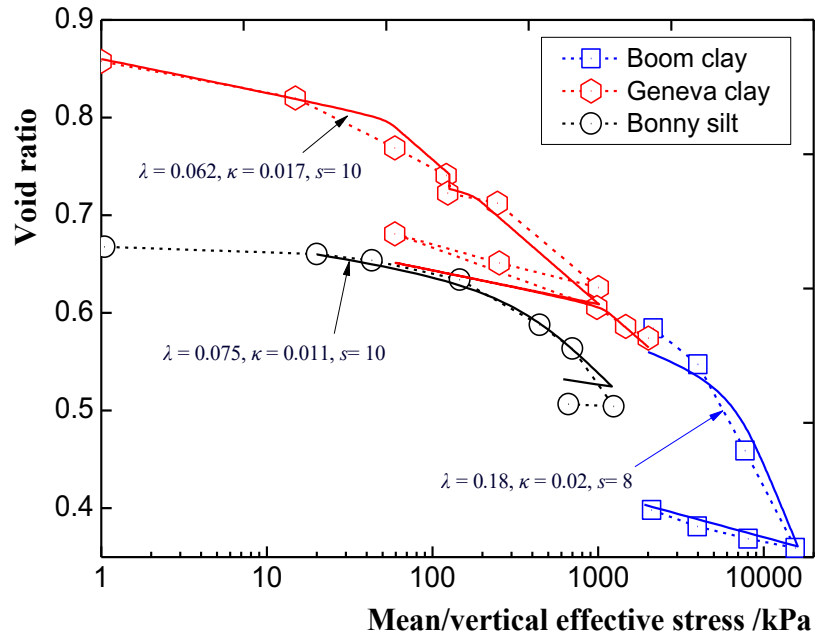


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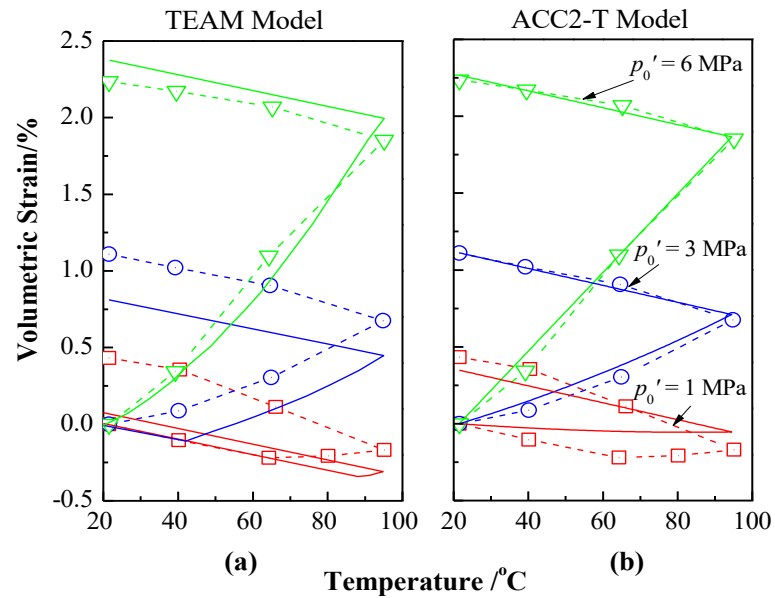
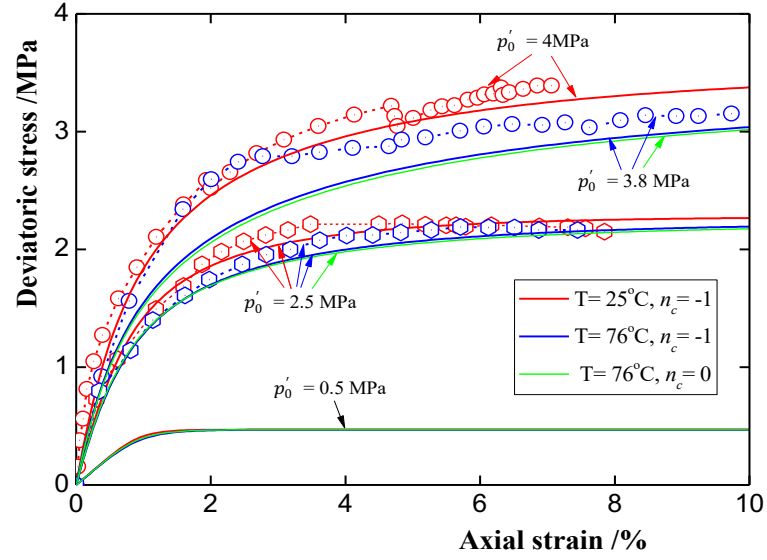
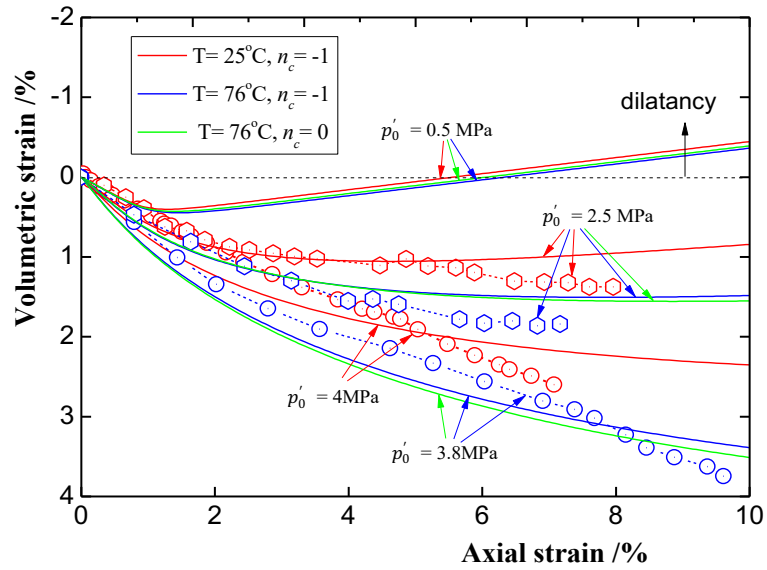


Fig.8 Volumetric strain change of Boom clay due to one heating and cooling cycle with different mean effective stresses: (a) TEAM model (b) ACC2-T model



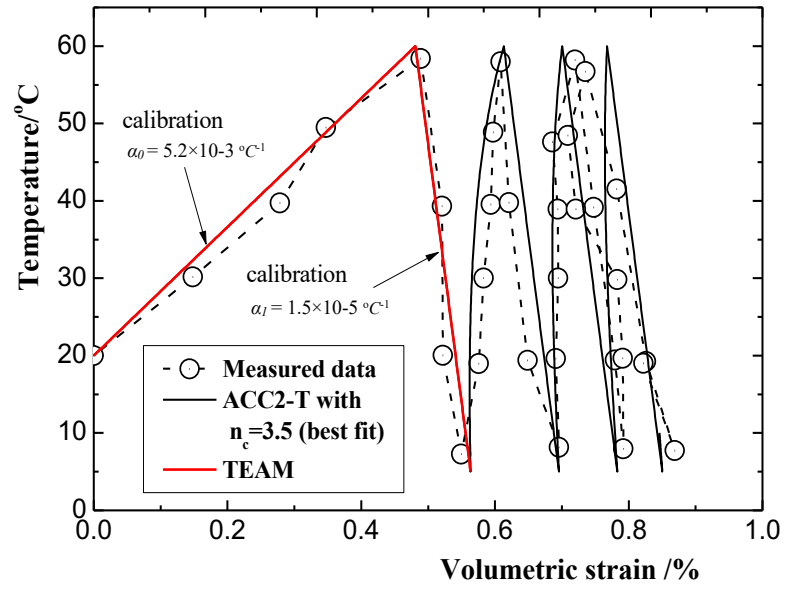
(a)



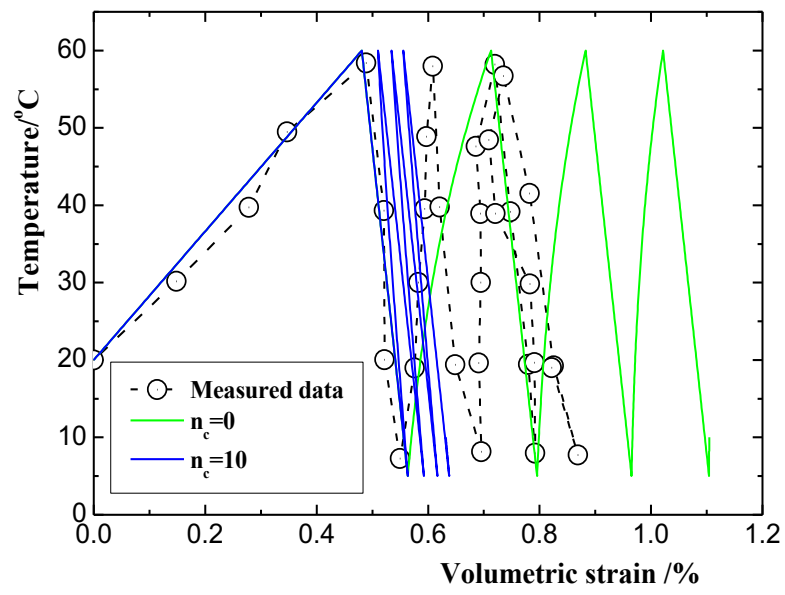
(b)

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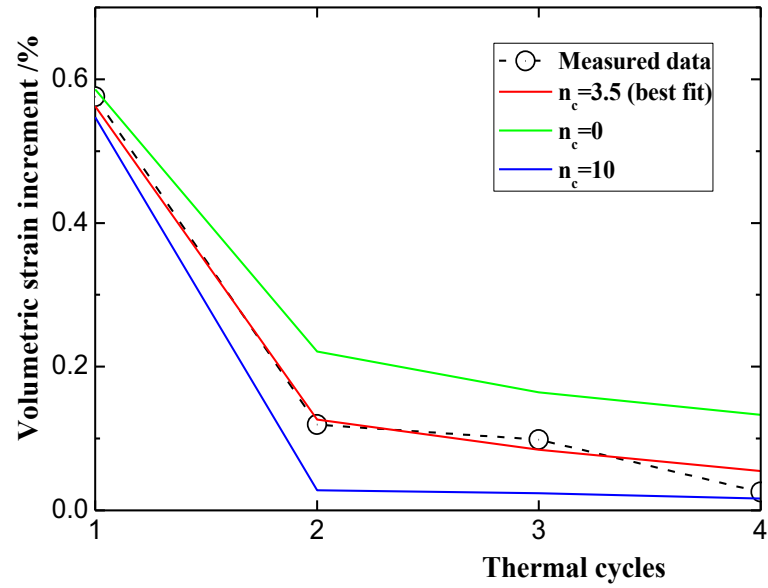




(a)



(b)



(c)

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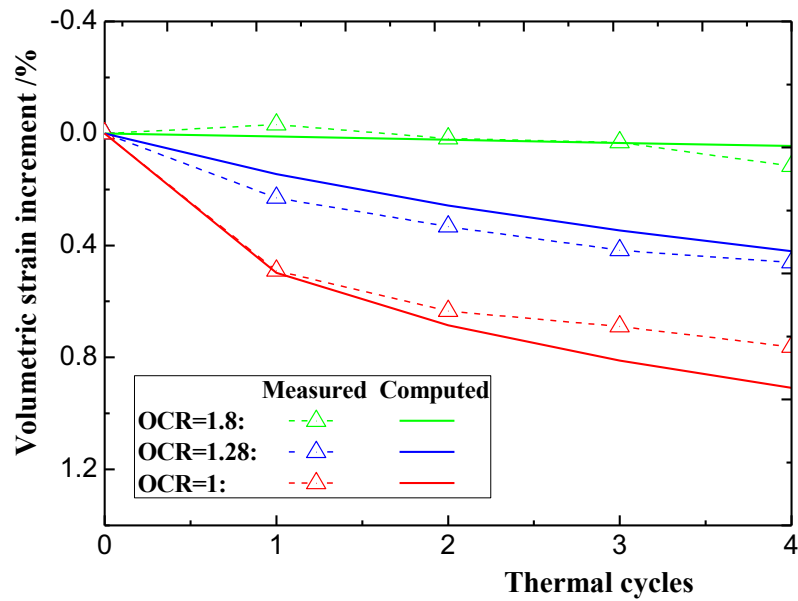
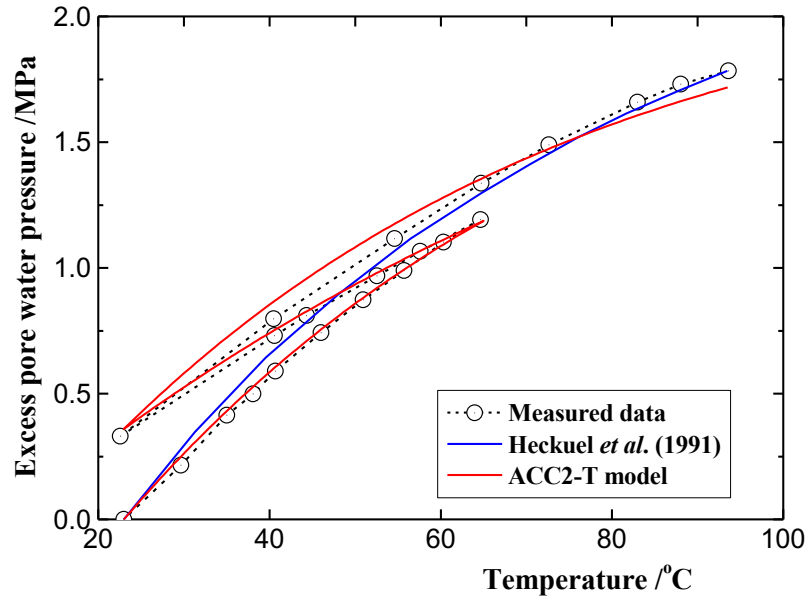
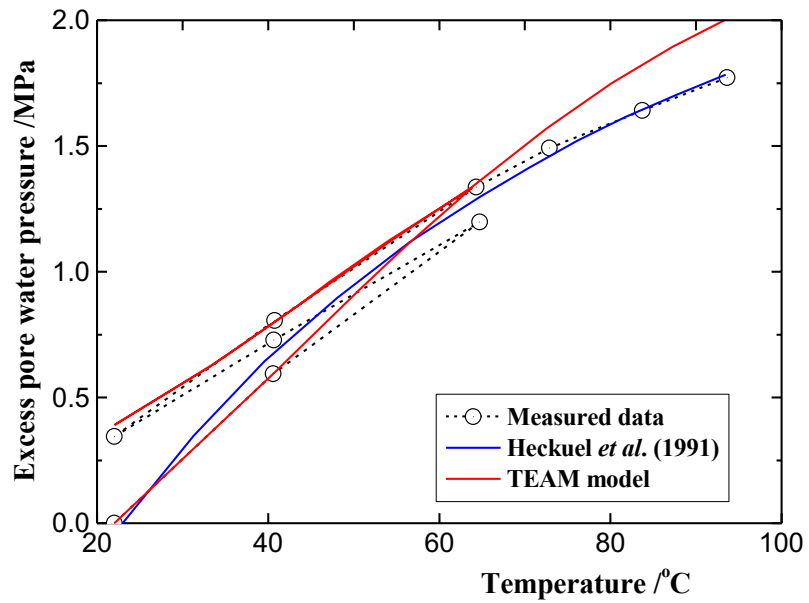


Fig.11 Accumulated strain behavior of Bonny Silt with different OCRs under drained condition



(a)



(b)

Fig.12 Excess pore water pressure accumulation of Pontida clay due to thermal cycles

under undrained condition: (a) ACC2-T model; (b) TEAM model

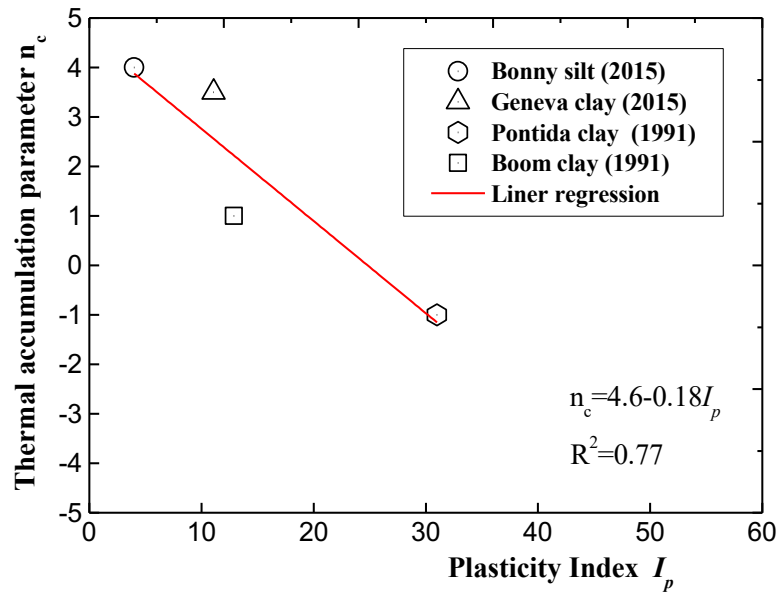


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