open science

# Imaging non-Brownian particle suspensions with X-ray tomography: application to the microstructure of Newtonian and visco-plastic suspensions 

Stephanie Deboeuf, Nicolas Lenoir, David Hautemayou, Michel Bornert, F. Blanc, Guillaume Ovarlez

## To cite this version:

Stephanie Deboeuf, Nicolas Lenoir, David Hautemayou, Michel Bornert, F. Blanc, et al.. Imaging nonBrownian particle suspensions with X-ray tomography: application to the microstructure of Newtonian and visco-plastic suspensions. Journal of Rheology, 2017, 62 (2), pp.643. 10.1122/1.4994081 . hal02171334

HAL Id: hal-02171334
https://enpc.hal.science/hal-02171334
Submitted on 10 Jul 2019

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Imaging non-Brownian particle suspensions with X-ray tomography: application to the microstructure of Newtonian and visco-plastic suspensions 

Stephanie Deboeuf, Nicolas Lenoir, David Hautemayou, Michel Bornert, Guillaume Ovarlez

## To cite this version:

Stephanie Deboeuf, Nicolas Lenoir, David Hautemayou, Michel Bornert, Guillaume Ovarlez. Imaging non-Brownian particle suspensions with X-ray tomography: application to the microstructure of Newtonian and visco-plastic suspensions. Journal of Rheology, 2017. hal-02171334

HAL Id: hal-02171334
https://hal-enpc.archives-ouvertes.fr/hal-02171334
Submitted on 10 Jul 2019

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Imaging non-Brownian particle suspensions with X-ray tomography: 

## application to the microstructure of Newtonian and visco-plastic suspensions

S. Deboeuf*<br>Sorbonne Universités, UPMC Univ. Paris 06, CNRS, UMR 7190, Institut Jean Le Rond dAlembert, 75005 Paris, France<br>N. Lenoir<br>PLACAMAT, UMS 3626 CNRS-Univ. de Bordeaux, Pessac, France<br>D. Hautemayou and M. Bornert<br>Université Paris-Est, Laboratoire Navier (UMR 8205) CNRS, ENPC, IFSTTAR, F-77420 Marne-la-Vallée, France<br>F. Blanc<br>CNRS-University of Nice, LPMC-UMR 7336, 06108 Nice, France<br>G. Ovarlez<br>Univ. Bordeaux, CNRS, Solvay, LOF, UMR 5258, F-33608 Pessac, France (Dated: September 29, 2017)


#### Abstract

A key element in the understanding of the rheological behaviour of suspensions is their microstructure. Indeed, the spatial distribution of particles is known to depend on flow history in suspensions, which has an impact on their macroscopic properties. These micro-macro couplings appeal for the development of experimental tools allowing for the rheological characterization of a suspension and the imaging of particles. In this paper, we present the technique we developed to image in three dimensions the microstructure of suspensions of non-Brownian particles, using X-ray computed tomography and sub-voxel identification of particle centers. We also give examples of the information we can get in the case of Newtonian and visco-plastic suspensions, referring to Newtonian and visco-plastic suspending fluid. We compute three dimensional pair distribution functions and show that it is possible to get a nearly isotropic microstructure after mixing. Under shear, this microstructure becomes anisotropic in the shear plane (velocity-velocity gradient plane), whereas it is almost isotropic in the two other planes (velocity-vorticity and velocity gradient-vorticity planes). When changing the plane of shear (from a squeeze flow to a rotation flow), the microstructure reorganizes to follow this change of shear plane. It is known that for Newtonian suspensions the anisotropy is independent on the shear rate; we show here that for a visco-plastic suspension it depends on it. Finally, we study the structuration close to boundaries and we evidence some particle alignment along both solid surfaces and, more surprisingly, along free interfaces.


[^0]
## I. INTRODUCTION

 ${ }_{21}$ raw materials produced in industry (concrete, batter, ...) are mixtures of dispersed solids in 22 a liquid. These systems are composed of a wide range of materials of various sizes, which ${ }_{23}$ makes them complex to understand. In order to predict the behaviour and optimize the use 24 of these suspensions, a useful step is to identify model systems capable of reproducing the ${ }_{25}$ phenomenology of interest and to understand the rheology of controlled suspensions. When ${ }_{26}$ length scales between the coarser and the finer dispersed particles of a suspension are well ${ }_{77}$ separated, a possible model system is a visco-plastic suspension of solid particles, referring to ${ }_{28}$ a visco-plastic suspending fluid, i.e. of nonlinear viscosity with a yield stress. Indeed, from a 29 rheological point of view, a fresh concrete can be considered as coarse particles (sand, gravel) ${ }_{30}$ suspended in a visco-plastic fluid (cement paste) [1, 2]. Even more simply, one can consider ${ }_{31}$ a Newtonian suspension of solid particles as a model system, referring to a Newtonian 32 suspending fluid [3]. A lot can indeed be learnt from such materials: studying first the $з_{з}$ interactions of multiple particles in a linear (Newtonian) fluid is crucial to understand their ${ }_{34}$ interaction in a nonlinear medium [4, 5]. This knowledge can then be applied to materials ${ }_{35}$ closer to applications such as cementitious materials $[2,6]$.A key element in the understanding of the rheological behaviour of suspensions is their microstructure $[7,8]$. Indeed, the spatial distribution of particles in suspensions is known to \& depend on the flow history - an anisotropic distribution is induced by shear - , which has an э impact on their properties [7, 9-14] - an anisotropic microstructure is at the origin of normal 40 stress differences and of viscosity variations during transient regimes. These micro-macro ${ }_{1}$ couplings between microstructure and flow appeal for the development of experimental and 42 numerical tools allowing for the characterization of the rheology of a suspension and the ${ }_{43}$ imaging of its microstructure. A lot of numerical studies used Stokesian dynamics equations ${ }^{4}$ [8, 15-18]. The existing experimental techniques efficient for non-Brownian suspensions are ${ }_{45}$ all based on optics. Non-Brownian particles are indeed too large to use scattering techniques ${ }_{66}$ such as X-ray scattering or coherent diffraction imaging. For non transparent particles in a ${ }_{7}$ transparent fluid, it is nevertheless possible to image the microstructure only near the free ${ }_{48}$ surface [10] or near a transparent boundary. As we will demonstrate here, the proximity of a ${ }_{49}$ boundary (a solid wall or a free interface) influences strongly the microstructure and makes
$5_{0}$ it different from the one in the bulk. The bulk microstructure can be obtained only for 51 specific suspensions, where particles and the fluid have the same optical index; a laser sheet 52 can then be used to light a plane embedding a fluorescent stain so as to visualize and identify ${ }_{3}$ particles in two dimensions (2D) [11, 12]. Obtaining a three dimensional microstructure 54 then implies to displace the position of the laser sheet. Confocal microscopy can also be used ${ }_{55}$ for micron-sized particles $[18,19]$, submitted to Brownian diffusion, that is in competition 6 with shear flow, quantified by the Peclet number (being the ratio of hydrodynamic over ${ }_{7}$ thermal forces). Here we choose to use non intrusive X-ray techniques as a promising tool s to image in the bulk suspensions in three dimensions (3D).

In this paper, we present the technique we developed to image in 3D the microstructure so of suspensions of non-Brownian particles, using X-ray computed tomography (CT) and then ${ }_{1}$ identifying by image analysis the particle centers with a sub-voxel resolution. All the details $\sigma_{2}$ to duplicate our technique are given. We give examples of the information we can get in the ${ }_{3}$ case of Newtonian and visco-plastic suspensions. We compute 3D pair distribution functions ${ }^{6}$ (pdf) and show that it is possible to get a nearly isotropic microstructure after mixing. Under ${ }_{55}$ shear, this microstructure becomes anisotropic in the shear plane (velocity-velocity gradient ${ }_{6}$ plane), whereas it is nearly isotropic in the two other planes (velocity-vorticity and velocity ${ }_{7}$ gradient-vorticity planes). When changing the plane of shear (from a squeeze flow to a 8 rotation flow), the microstructure reorganizes to follow this change of shear plane. While it 9 is known that for Newtonian suspensions the anisotropy is independent on the shear rate, we o show that for a visco-plastic suspension it depends on it. Finally, we study the structuration ${ }_{1}$ close to boundaries and we evidence particle alignment along both solid surfaces and more 22 surprisingly along free interfaces.

In Sec II, we present the principles of our study and we provide the details on the studied ${ }_{74}$ suspensions and the set-up. Sec III is devoted to the X-ray tomography technique and to the ${ }_{75}$ method we developed to identify particles center and compute pair distribution functions. ${ }_{76}$ In Sec IV, we present various examples of the microstructure of Newtonian and visco-plastic 7 suspensions.


FIG. 1. X-ray computed micro-tomography set-up (a and b) and dedicated parallel plates set-up (c). The sketch (a) shows the X-ray source (in white), the imaging device (in green), the rotation (in red) and translation (in blue) stages and the parallel plates set-up (in yellow).

## II. MATERIALS AND METHODS

## A. Goals and general principles

Our goal is to get enough information at the microstructure scale to guide developments ${ }_{81}$ of micro-mechanical models for complex suspensions rheology. Thus, we need to image ${ }_{82}$ particles in a flow cell and to characterize their spatial distribution for various flow histories.

Our system made of a suspension in a rheometrical set-up should allow for the investiga${ }_{84}$ tion from small to large space scales, from the length of the dispersed particles to the whole ${ }_{85}$ suspension. The flow cell should be large enough to be representative of a macroscopic shear ${ }_{86}$ flow, to ensure that the material exhibits a bulk behaviour not influenced by finite size or ${ }_{87}$ non local effects [20], and to get enough statistics for studying the microstructure. This ${ }_{88}$ leads us to choose a rheometer equipped with a parallel plates geometry (Fig. 1c). In the ${ }_{89}$ same time as doing rheology, we want to image the microstructure in 3D, even in the case 90 of non matched optical index materials. This leads us to use X-ray CT imaging, which is ${ }_{91}$ efficient for many suspensions and geometries, as long as there is enough X-ray absorption ${ }_{92}$ contrast between particles and the fluid.

The sample to image is placed on a rotating stage between an X-ray source and a 2D ${ }_{94}$ imaging device (Fig. 1), which monitors the transmission of X-rays through the sample. In 55 our case, the imaging device was a PaxScan2520V flat panel detector from Varian, made of 96 a CSI scintillator and a matrix of 1920x1536 pixels (among which 1840x1456 are effective), ${ }_{97}$ with a pixel width of $127 \mu \mathrm{~m}$. This provides a 2D radiograph, accounting for the total

$$
98
$$ 99 rotation angles of the sample (typically over $360^{\circ}$ in a lab X-ray CT device) then allows for 100 the reconstruction of the 3D map of absorption coefficients [21-23]. If particles and the fluid 01 do not have the same absorption coefficient, we can finally achieve to detect the particles. 2 This map is discretized over elementary volumes called voxels (i.e. volumic pixels). The з achievable minimum voxel size is given by the ratio of the size of the imaged sample over 204 the number of pixels in the detector width. The typical width of the CCD detectors used ${ }_{50}$ for X-ray CT is about 2000 pixels. Noting $D$, the diameter of the parallel plates geometry, 106 the minimum possible voxel size is $D / 2000$. To identify precisely the particles center, their ${ }_{107}$ diameter $d$ should be at least 10 voxels, i.e. $d \gtrsim D / 200$. The gap size $H$ between the plates 108 in the parallel plates geometry should contain at least 10 particles to avoid some finite size 109 effects, i.e. $H \gtrsim D / 20$, which is compatible with the study here but limits the possible 0 aspect ratios diameter/gap of the geometry to $D / H \lesssim 20$. We will show later that our 11 home-made algorithms allow us to refine particle centers positions at a sub-voxel resolution 2 of an order of $d / 100$, that ultimately allows for a fine characterization of the microstructure. ${ }_{3}$ Here (Fig. 2a), the diameter of the parallel plates geometry id $D=2 \mathrm{~cm}$, the gap of the 14 geometry is $H=2 \mathrm{~mm}$, the size of particles $d$ is between $100 \mu \mathrm{~m}$ and $200 \mu \mathrm{~m}$, the size of ${ }_{115}$ voxels is $12 \mu \mathrm{~m}$ and the final accuracy on the particles centers in most cases presented here 116 is $6 \mu \mathrm{~m}$, but it can be decreased down to $1 \mu \mathrm{~m}$ if necessary. ${ }_{25}$ particles do not move as long as stresses are smaller than the yield stress. Then we apply to 126 our suspension a given shear history, then stop the flow and image the material at rest with 127 a frozen structure, which is supposed to be close to the structure under flow. The underlying 128 hypothesis is that relaxation of the microstructure at rest is negligible. This flow-arresting ${ }_{129}$ technique was already used before for Brownian suspensions [18]. In a second time, we



FIG. 2. (a) Scheme of the parallel plates set-up: $D$ and $H$ are the plates diameter and gap, $d$ is the particles diameter, $\Omega$ and $\vartheta$ are the rotational and translational velocities of the top plate. (b) Schemes of the rotational and squeeze shear flows: the suspension velocity $u_{\theta}(r, z)$ and $u_{r}(r, z)$ are aligned with the azimuthal and radial directions respectively. (c) A picture processed from X-ray slices showing the rough surface with grooves of the set-up's plates.
succeed to use a Newtonian suspension with density-matched particles and fluid, to avoid 1 any buoyancy effects at rest. In principe, various types of suspensions may be imaged and 32 studied with our technique, as soon as particles or suspended objects do not move during the 133 time of a X-ray scan. For particles as small as a few $\mu m$, Brownian fluctuations may call for 134 even quicker X-ray micro-tomograph (e.g. Synchrotron source). In terms of concentration 135 of particles, we have successfully image in 3D suspensions at solid fractions as high as $50 \%$ 136 and it should be possible to go even higher.

TABLE I. Properties of particles and fluids used for the Newtonian and the visco-plastic suspensions: average diameter $d$ of particles, viscosity $\eta$ of the Newtonian fluid, elastic shear modulus $G^{\prime}$, yield stress $\tau_{y}$ and consistency $K$ of the visco-plastic fluid and density $\rho$ of the fluid and density-matched particles.

| PMMA | Newtonian | Density |
| :---: | :---: | :---: |
| $d(\mu \mathrm{~m})$ | $\eta($ Pa.s $)$ | $\rho$ |
| $170 \pm 12$ | 1.3 | 1.19 |


| PS | Yield stress |  | Density |  |
| :---: | :---: | :---: | :---: | :---: |
| $d(\mu \mathrm{~m})$ | $G^{\prime}(\mathrm{Pa})$ | $\tau_{y}(\mathrm{~Pa})$ | $K\left(\mathrm{~Pa} . \mathrm{s}^{0.5}\right)$ | $\rho$ |
| $138 \pm 8$ | 250 | 26 | 5.1 | 1.05 |

$$
\begin{equation*}
\tau=\tau_{y}+K \dot{\gamma}^{0.5}, \text { if } \tau \geq \tau_{y} \tag{1}
\end{equation*}
$$

160 valid in the steady flow regime, i.e. for stresses larger than the yield stress $\tau_{y}$. The elastic 161 contribution to its behaviour is accounted for by the elastic shear modulus $G^{\prime}=250 \mathrm{~Pa}$. 163 with and without particles, in [5] and [24] respectively.


FIG. 3. X-ray radiograph corresponding to the transmitted intensity of X-rays through the sample (the visco-plastic suspension) along one linear path (a), reconstructed horizontal slice encoding for X-ray absorption of voxels in the shear cell at a solid fraction $\phi \simeq 35 \%$ (b) and zoom of this slice (c) corresponding to the white rectangle drawn in (b). The scale bar drawn in (c) is 1 mm long. 180 slices of the reconstructed 3D image for a solid fraction $\phi \simeq 35 \%$. 204 conditions (no slip at the two plates) predict a simple shear of the fluid in the orthoradial ${ }_{205}$ plane $(\theta z)$, the single non-zero value of the shear rate tensor being:

$$
\begin{equation*}
\dot{\gamma}(r)=2 d_{z \theta}(r)=\Omega r / H \tag{2}
\end{equation*}
$$

6 with an azimuthal velocity:

$$
\begin{equation*}
u_{\theta}(r, z)=\Omega z r / H \tag{3}
\end{equation*}
$$

${ }_{207}(r, \theta, z)$ being the cylindrical coordinates [30]. The rotational shear is not homogeneous but 208 depends linearly on the radial direction $r$. This analysis holds not only for Newtonian fluids,

$$
\begin{equation*}
\dot{\gamma}=3 D \vartheta /\left(2 H^{2}\right) . \tag{4}
\end{equation*}
$$

${ }_{77}$ Note that by symmetry, the shear has a different sign in the upper and lower part of the ${ }_{8}$ suspension through the central surface at $z=0$.

In order to shear the material in a parallel plates geometry, it is first poured on the 20 bottom plate, then it is squeezed by the top plate to reach the required gap. So even if the interest is in a rotational shear, the material generally experiences first a squeeze flow during the loading of the fluid in the geometry, which might affect its initial state. That is why in the following, we will investigate the impact of both flows on the suspension microstructure.

## III. IMAGE ANALYSIS

An accurate identification of particle centers from the obtained 3D images is crucial for the 26 computation of pair vectors and pair distribution functions to characterize the microstructure ${ }^{27}$ of the suspension. Thus, we choose to develop home-made algorithms to be able to identify particles center at a sub-voxel resolution.

In all the text, the 3D images $I$ we work with are fields of absorption coefficients encoded 230 as intensity levels without unit measurement. In practice, 16 bits images have been used (grey levels between 0 and $2^{16}-1$ ).

Figure 4 shows the typical greylevel profile of a particle: grey levels of all voxels in the ${ }_{233}$ neighborhood of a particle are plotted against the distance $r$ to the voxel considered to be ${ }_{34}$ its center. Assuming the density of the particles to be uniform, one would have expected a Heaviside-like shape for such plots, with a uniform grey level for $r$ lower than the particle ${ }_{236}$ radius (of the order of 6 voxels), corresponding to the absorption coefficient of PMMA or ${ }_{237} \mathrm{PS}$, and another (larger) uniform grey level for larger $r$, corresponding to the absorption of


FIG. 4. Absorption levels (without unit measurement, included between 0 and $2^{16}-1$ ) of voxels over one particle and its close neighbourhood in the visco-plastic suspension at a solid fraction $\phi \simeq 40 \%$, as a function of the distance $r$ to its center, from the raw and filtered images $I$ (a) and $I_{f}(\mathrm{~b})$

238 the fluid.
Actual profiles first differ from ideal profiles because of natural random image noise, which ${ }_{241}$ indeed observed near $r=0$, up to a distance of about 2 to 3 voxels, where grey levels are ${ }_{242}$ essentially uniform on average but fluctuate randomly.

The second discrepancy with respect to the ideal situation takes the form of a gradual 244 increase of the average grey levels for $r$ varying from about 3 to 7 voxels. While there ${ }_{245}$ are still fluctuations associated to natural image noise, the smooth progressive increase of the average grey level is due to the limited spatial resolution of the CT imaging device. Indeed the grey level of a given voxel in the reconstructed image should be considered as ${ }_{248}$ a spatial convolution of the physical absorption coefficient of the material (relative to the 249 spectrum of the polychromatic X-ray beam) with some kernel of small finite size. This 50 kernel characterizes the resolving power of the imaging system. For CT imaging systems, 1 it is sensitive to various parameters, such as the finite spot size of the X-ray source (which ${ }_{252}$ can be evaluated to be about $8 \mu \mathrm{~m}$ for our imaging conditions), diffusion effects of the 53 scintillator, fill factor of the pixel matrix, blurring effects of electronic device,... A more ${ }_{254}$ detailed discussion can be found in [33]. In the present setup, and considering the grey level 5 profile presented in figure 4 , the radius of this kernel is likely to be of the order of 2 to 3

TABLE II. Values of the parameters $\ell_{\text {noise }}, r_{\text {neigh }}, r_{\text {min }}$ and $r_{\text {sym }}$ used for image analyses, with $d$, the diameter of particles

| $\ell_{\text {noise }}$ | $r_{\text {neigh }}$ | $r_{\text {min }}$ | $r_{\text {sym }}$ |
| :---: | :---: | :---: | :---: |
| 1 voxel | $\sim 7$ voxels | $\sim 6$ voxels | $\sim 7$ voxels |
| $\sim \mathrm{d} / 10$ | 0.6 d | 0.5 d | 0.6 d |
| $\sim 12 \mu \mathrm{~m}$ | $\sim 83 \mu \mathrm{~m}$ | $\sim 69 \mu \mathrm{~m}$ | $\sim 83 \mu \mathrm{~m}$ |

6 voxels, which explains that the sharp theoretical particle-matrix interface is degraded into ${ }_{7}$ a smooth interface with a thickness of about 5 voxels. 266 as the local minimum absorption and we get rid of false and multiple detections. Second, ${ }^{267}$ we refine particle center positions at a sub-voxel resolution by using the symmetry of the in268 creasing absorption around the center. Finally, we explain how we compute pair distribution 269 functions.

## A. Detection of particles

${ }_{271}$ We process the 3D images with 3D morphological operations within Matlab. We use as ${ }_{272}$ few as possible filtering operations and we introduce as few as possible tunable parameters, ${ }_{273}$ whose values are not severely chosen, so that our method depends only slightly on the value 274 of parameters and is efficient for various solid fractions without changing the value of any 275 parameter.

First, we smooth the raw 3D image $I$ by filtering high spatial frequency noise with a


FIG. 5. Two dimensional ( $\mathrm{a}, \mathrm{b}$ ) and three dimensional ( $\mathrm{c}, \mathrm{d}$ ) images of the visco-plastic suspension of length about $\sim 1 \mathrm{~cm}$ at a solid fraction $\phi \simeq 35 \%$ : raw images $I(\mathrm{a}, \mathrm{c})$ and filtered images $I_{f}(\mathrm{~b}$, d)
${ }_{277}$ Gaussian filter of characteristic size $\ell_{\text {noise }}=1$ voxel: $I_{f}=f * I$ with

$$
\begin{equation*}
f(x)=\exp \left(-\left(\frac{x}{2 \ell_{\text {noise }}}\right)^{2}\right) / \int_{-\infty}^{\infty} \exp \left(-\left(\frac{x}{2 \ell_{\text {noise }}}\right)^{2}\right) d x \tag{5}
\end{equation*}
$$

8 where the symbol $*$ refers to the convolution operator: $(f * I)\left(x_{0}\right)=\int_{-\infty}^{\infty} f(x) I\left(x-x_{0}\right) d x$, 9 and $I_{f}$ is the image we work with in the following steps (Fig. 4 and 5). This filtering operation has a long-range effect even in the case of a small value for the length $\ell_{\text {noise }}$, due to the extended range of integration in the convolution operation; in practice, the integration 282 is realized over a size of $21 \times 21 \times 21$ voxels, chosen after we checked that a larger size does not 283 change the value of $I_{f}$.

Second, we extract the positions of voxels showing a local minimum of absorption as a first estimation of the particle centers.

False detections corresponding to local extrema in the air, the plates or the fluid, as well 287 as multiple detections (several centers for the same particle) are possible. In practice, they ${ }_{288}$ represent less than $1 \%$ of all detections; however, we do get rid of these false and multiple 289 detections thanks to the two following steps.

- We first note that X-ray absorption coefficients of air $\mu_{\text {air }}$, particles $\mu_{\text {particles }}$ and suspending fluids $\mu_{\text {fluid }}$ are well separated and sorted according to:

$$
\begin{equation*}
\mu_{\text {air }}<\mu_{\text {particles }}<\mu_{\text {fluid }} \tag{6}
\end{equation*}
$$

To get rid of false detections, we build an histogram of the values of absorption of the detected extrema (Fig. 6). In practice, we do not use the value of the extrema, but the averaged value over a close neighbourhood around the extrema position, of


FIG. 6. Histogram (in fraction) of absorption values (without unit measurement) averaged over a close neighbourhood of size $r_{n e i g h}$ around the detected extrema, for the visco-plastic suspension at a solid fraction $\phi \simeq 35 \%$, before selection of extrema identified as real particles.
size $r_{\text {neigh }}$ taken equal to $60 \%$ of the mean diameter of particles (Tab. II), to avoid considering small dust (solid of tiny size). Also, this allows for a better separation in the histogram. On the histogram, we clearly distinguish the absorption values corresponding to particles, fluid and air. We kept only voxels of averaged absorption values between $\mu_{\text {min }}$ and $\mu_{\text {max }}$. In Figure 6, the width of the histogram results from the macroscopic inhomogeneities of absorption in space (due to a phenomenon called beam hardening) and likely bears the signature of particle distribution in space. False detections corresponding to local extrema in the plates have the same absorption value as particles, so a meticulous inspection of slices close to the plates and a manual exclusion may be needed if necessary.

- We compute distances between possible particle center positions to get rid of multiple detections. If two possible centers are closer than a threshold distance $r_{\text {min }}$, we keep only the one of minimal absorption. In the case of perfectly monodisperse particles and non-noisy images of infinite spatial resolution, $r_{\min }$ could be taken as $100 \%$ of the diameter of particles. Since particles are slightly polydisperse and at this step the accuracy of particle centers is only of $\pm 1$ voxel, we choose $r_{\text {min }}$ equal to $50 \%$ of the mean diameter of particles (Tab. II). Figure 7a shows the apparent volume fraction $\phi_{\infty}$ far from reference particles, computed from the number of neighbours detected by unit volume, for different values of the threshold distance $r_{\text {min }}$. The


FIG. 7. a) Volume fraction $\phi_{\infty}\left(r_{\text {min }}\right)$ far from reference particles, computed from the number of neighbours detected by unit volume, for different values of the threshold distance $r_{\text {min }}$ (minimal distance between two centers). b) Distribution of the interparticle distance for the identified first neighbours for all $r_{\text {min }} \leq 9$. The visco-plastic suspension ( $\mathrm{a}, \mathrm{b}$ ) has a solid fraction equal to $\phi \simeq 36 \%$.

$$
\begin{equation*}
\Delta I^{2}=\sum_{\Delta r \leq r_{s y m}}\left(I_{v x}\left(x_{0}+\Delta x, y_{0}+\Delta y, z_{0}+\Delta z\right)-I_{v x}\left(x_{0}-\Delta x, y_{0}-\Delta y, z_{0}-\Delta z\right)\right)^{2} \tag{7}
\end{equation*}
$$

${ }_{31}$ for values of $\Delta r=\sqrt{\Delta x^{2}+\Delta y^{2}+\Delta z^{2}}$ smaller than a threshold $r_{s y m}$, taken as $60 \%$ of the 12 mean diameter of particles (Tab. II).

A compromise between precision and computer time leads us to refine successively the 34 centers positions at 1 voxel first, then $1 / 2$ voxel, until $(1 / 2)^{n}$ voxel, with $n$, the integer such 5 that the chosen spatial resolution is reached.

Figure 8a shows the variations of $\Delta I$ with the displacement $\Delta x$ of the center in the $x$ direction in the range $\pm 2$ voxels, for different couple values $(\Delta y, \Delta z)$ of the displacement of the center in the $y$ - and $z$-directions: there is indeed a minimum $\Delta I_{\text {min }}$ allowing to identify the center. The same holds for $\Delta y$ and $\Delta z$ in the $y$ - and $z$-directions. Figure 8b shows the decrease of the minimum $\Delta I_{\text {min }}$ with $n$, the step of sub-voxel identification, what confirms 1 that the minimization and the sub-voxel refinement have still a physical sense till $n=5$, 32 allowing to reach a resolution on the particles center position of $(1 / 2)^{5}$ voxel $=0.375 \mu \mathrm{~m}$ ${ }_{353} \simeq \mathrm{~d} / 100$. Note that $n=-1$ (Fig. 8b) corresponds to the case of no interpolation of the ${ }_{354}$ images $I_{f}$, so that $\Delta I_{\min }(n=-1)$ quantifies the asymmetry of the signal around the minimal 55 absorption voxel. For the results presented in this paper, a spatial resolution of $6 \mu \mathrm{~m} \simeq \mathrm{~d} / 20$, i.e. $n=1$ is enough.

Figure 9 shows the original (non interpolated, i.e. at 1 voxel resolution, $n=-1$ ) absorp${ }_{88}$ tion signal $I_{f}$ (without unit measurement) over one particle as a function of distance from


FIG. 8. a) Variations of $\Delta I$ (Eq. 7) with the displacement $\Delta x$ of the center in the $x$-direction for the first step of sub-voxel identification $n=0$. Each line corresponds to different couple values of the displacement of the center ( $\Delta y, \Delta z$ ) in the $y$ - and $z$-directions. b) Variations of $\Delta I_{\text {min }}$ with the number of steps of sub-voxel identification $n . \Delta I(n=-1)$ quantifies the asymmetry of the signal around the minimal absorption voxel, i.e. without any interpolation of the image $I_{f}$. Note that here $\Delta I$ was normalized by the number of voxels for which it has been computed to allow for a comparison at different sub-voxel resolutions
the center, for the initial center (determined from the local extrema) (Fig. 9a) and for the centers refined successively with $n=0$, 1 , and 2 in Figure 9b, c and d respectively. We see ${ }_{1}$ that for $\Delta r \leq r_{s y m}$ the scattering of data (or asymmetry of the signal around its center) 32 decreases at each step of sub-voxel identification, that is quantified by $\Delta I$. Even at 1 voxel resolution (for $n=0$ ), we already note that the center of symmetry (which is used to plot Figure 9b) is not always the minimal voxel of absorption (which is used to plot Figure 9a).

## C. Pair distribution functions

From particle positions within the parallel plates geometry, we are able to study how 7 particles are spatially distributed in the suspending fluid. In addition to information on the 568 structure of the suspension at the macroscopic scale from global positions of particles within the geometry, we do get information on the microstructure of the suspension at the particle ${ }_{370}$ scale from relative positions of particles (relative to each other) through the computation of ${ }_{371}$ pair distribution functions [7].

The pair distribution function $g(\vec{r})$ is the probability of finding a particle pair separated


FIG. 9. Absorption levels (without unit measurement) of voxels over one particle and its close neighbourhood in the visco-plastic suspension at a solid fraction $\phi \simeq 40 \%$ as a function of the distance $\Delta r$ to its center, through the different steps ( $n=-1,0,1,2$ in a, b, c, d) of sub-voxel identification based on symmetrisation
${ }^{373}$ by the vector $\vec{r}$ normalized by the mean particle density, so that the asymptotic value of $g$ ${ }_{374}$ for large values of $\|\vec{r}\|$ is:

$$
\begin{equation*}
g(\|\vec{r}\| \rightarrow \infty)=1 \tag{8}
\end{equation*}
$$

${ }_{375}$ in the absence of macroscopic structure or long-range correlations. Equivalently, $g(\vec{r})$ is a 376 normalized probability of finding a (test) particle located at $\vec{r}_{0}+\vec{r}$ with another (reference) ${ }^{377}$ particle located at $\vec{r}_{0}$. This last definition suggests naturally how to compute $g(\vec{r}): N_{\text {ref }}$ ${ }_{378}$ particles are subsequently chosen to be the reference particles; we identify the $N^{\text {pair }}$ test ${ }^{379}$ particles separated by $\vec{r}$ from each reference particle, corresponding to the number of pairs 380 characterized by $\vec{r}$ in the region of interest.

In the following, $\overrightarrow{r_{0}}=\left(r_{0}, \theta_{0}, z_{0}\right)$ and $\overrightarrow{r_{0}}+\vec{r}=(r, \theta, z)$ are the cylindrical coordinates 382 (attached to the axis of the rheometer circular plates) of a reference and of a test particle


FIG. 10. Histogram (in fraction) of the interparticle 3D (solid lines) and 2D (dashed lines with crosses) distances for the first neighbours identified in slices of different thicknesses $\Delta=d / 8, d$ and $2 * d$, with $d$, the particles' diameter. The visco-plastic suspension has a solid fraction equal to $\phi \simeq 38 \%$.
located in $M_{0}$ and $M$. ${ }_{385}$ for example underestimating the statistics in the direction of a boundary - test particles ${ }_{386}$ shall be selected around their reference particle within an isotropic region (e.g. that does ${ }_{387}$ not cross any boundary). To be able to study the microstructure even close to boundaries ${ }_{388}$ (solid plates or free interfaces), we use adaptative spheres of variable radius, equal to the 9 minimal distance between the reference particle and the closest boundary, as the area where 390 searching test particles around a reference particle. In this case, we have to update the 391 number of reference particles $N_{\text {ref }}$ that is no more constant for all values of $\vec{r}$, especially 32 for large values of $\|\vec{r}\|$. This choice is an alternative to exclusion of regions of investigation ${ }_{393}$ as in [14] and to a geometry-dependent correction introduced in the computation of $g(\vec{r})$ as 394 in [11].

The 3D pair distribution function $g(\vec{r})$ is a function of 3 scalar variables of size $\sim N^{3}$ in 6 the discretized space, with $N$, the number of voxels in a given direction of space. However, ${ }_{397}$ for visualization, we generally plot the values of $g(\vec{r})$ taken in three orthogonal slices (data ${ }_{398}$ of size $\sim N^{2}$ ). As relevant slices, we choose the cylindrical 'planes' attached to the axis of 99 the rheometer circular plates, as naturally suggested by the cylindrical symmetry of the set400 up and the flow. So, to reduce computation time, we generally compute only three 2D pair

401 distribution functions: $g_{r}$ in the orthoradial 'plane' $(\theta z)$ normal to $r$ (being a toroidal ring or 402 a cylinder), $g_{\theta}$ in the radial plane ( $r z$ ) normal to $\theta$ and $g_{z}$ in the horizontal plane $(\theta r)$ normal ${ }_{403}$ to $z$. So, the region where searching for test particles can be reduced to the intersection of 404 a sphere and the elementary cylindrical 'plane' of interest attached to the reference particle. ${ }_{405}$ In practice, defining the elementary planes as strictly 2D slices of cylindrical coordinates ${ }_{406} r=r_{0}, \theta=\theta_{0}$ or $z=z_{0}$ for a reference particle in $M_{0}\left(r_{0}, \theta_{0}, z_{0}\right)$ does not allow to sample ${ }_{407}$ enough test particles and get enough statistics, so we affect elementary thicknesses to the 8 cylindrical 'planes'. Whereas the definition of the thicknesses $\Delta z$ of horizontal planes ( $\theta r$ ) 409 and $\Delta r$ of orthoradial 'planes' $(\theta z)$ is obvious, the thickness of radial planes $(r z)$ offers ${ }_{410}$ several choices (see Appendix A). We choose the definition of:

$$
\begin{equation*}
\Delta h=r \sin \left(\theta-\theta_{0}\right) \tag{9}
\end{equation*}
$$

${ }_{411}$ corresponding to a constant euclidean thickness (or minimal distance of $M$ from the ra${ }_{412}$ dial direction $O M_{0}$ ), because it prevents a non symmetric pattern on the pair distribution ${ }_{413}$ function $g_{\theta}$ (see Appendix A). We checked the influence of the value $\Delta$ of the thicknesses ${ }_{414}$ of the planes $\Delta=\Delta z=\Delta r=\Delta h$ on our results. Figure 10 compares two distributions: ${ }_{415}$ the 3D distances (real distance) and the 2D distances (projected in the slice) between first ${ }_{416}$ neighbours of particles in an elementary cylindrical plane for different thicknesses $\Delta$ be${ }_{417}$ tween $d / 8$ and $2 * d$. The 3D distances are always equal or larger than $d$ with a maximum ${ }_{418}$ probability for $d$. But the 2D distances are not, because they do not represent well real 3D ${ }_{419}$ distances in slices of thicknesses larger than $d$, as demonstrated by the distributions that do ${ }_{420}$ not super-impose for $\Delta>d$. This leads us to choose for the thickness of elementary cylin${ }_{421}$ drical 'planes' $\Delta=d$, with $d$ the particle diameter, as a compromise between accuracy and ${ }_{422}$ statistics. Moreover we checked that this choice allows to preserve the quality and quantity ${ }_{423}$ of the pdfs.
${ }^{424}$ Different choices for the characterization of the pair vector $\vec{r} \equiv \overrightarrow{M_{0} M}$ (relative coordinates ${ }_{425}$ between test and reference particles located in $M$ and $M_{0}$ ) are possible (see Appendix A). ${ }_{426}$ Euclidean coordinates can in principle be chosen but we believe that non euclidean coor${ }_{427}$ dinates, curvilinear projected along the cylindrical 'planes' aligned with the circular flow ${ }_{428}$ streamlines, are more relevant to characterize the microstructure induced by a simple shear ${ }_{429}$ flow. We choose for coordinates of the pair vector $\vec{r}=(\rho, \ell, \xi)$ :

$$
\begin{equation*}
\rho=r-r_{0}, \ell=r\left(\theta-\theta_{0}\right), \xi=z-z_{0} \tag{10}
\end{equation*}
$$



FIG. 11. Two particles $M_{0}\left(r_{0}, \theta_{0}, z_{0}\right)$ and $M(r, \theta, z)$ in the global cylindrical framework and definition of the pair vector $\vec{r} \equiv \overrightarrow{M_{0} M}$ characterizing their separation in curvilinear coordinates (along circular flow lines): $\vec{r}=(\rho, \ell, \xi)=\left(r-r_{0}, r\left(\theta-\theta_{0}\right), z-z_{0}\right)$
${ }_{430}$ as shown in Figure 11. This choice does not induce any bias in the symmetry of the pair ${ }_{431}$ distribution functions $g_{r}, g_{\theta}$ or $g_{z}$ (see Appendix A). Note that in the following, when showing 432 a 2D pdf $g\left(g_{r}, g_{\theta}\right.$ and $\left.g_{z}\right)$, we will use alternatively cartesian coordinates (couples among $\rho$, ${ }_{433} \ell$ or $\left.\xi\right)$ or polar coordinates $\left(\rho_{2 d}, \phi_{2 d}\right)$ for a pair of particles in the plane of interest. In the ${ }^{434}(\theta z),(r z)$ and $(\theta r)$ planes, cartesian and polar coordinates are respectively related through ${ }_{436}\left(\rho_{2 d} \cos \phi_{2 d}, \rho_{2 d} \sin \phi_{2 d}\right)=(\ell, \xi),(\rho, \xi)$ and $(\rho, \ell)$.
${ }_{437}^{43}$ Altogether, with the mean particle density (or number of particles per unit volume):

$$
\begin{equation*}
n_{0 v}=N / V=\phi /\left(\pi d^{3} / 6\right), \tag{11}
\end{equation*}
$$

${ }_{438}$ with the total number of particles $N$, the total volume of the suspension $V$, the particle ${ }_{439}$ concentration (or solid fraction) $\phi$, the particle diameter $d$, the formula for the 2D pair ${ }_{440}$ distribution functions $g_{r}, g_{\theta}$ and $g_{z}$ in the $(\theta z),(r z)$ and $(\theta r)$ planes respectively (for spatial ${ }_{441}$ resolutions $\delta \rho$ and $\left.\delta \ell, \delta \xi\right)$ are:
${ }_{442} \bullet g_{r}(\ell, \xi)=P_{r}(\ell, \xi) / n_{0 v}$, with $P_{r}(\ell, \xi) \delta V_{r}=N_{r}^{\text {pair }}(\ell \pm \delta \ell / 2, \xi \pm \delta \xi / 2) / N_{r e f}$,

443

- $g_{\theta}(\rho, \xi)=P_{\theta}(\rho, \xi) / n_{0 v}$, with $P_{\theta}(\rho, \xi) \delta V_{\theta}=N_{\theta}^{p a i r}(\rho \pm \delta \rho / 2, \xi \pm \delta \xi / 2) / N_{r e f}$,

444

- $g_{z}(\rho, \ell)=P_{z}(\rho, \ell) / n_{0 v}$, with $P_{z}(\rho, \ell) \delta V_{z}=N_{z}^{p a i r}(\rho \pm \delta \rho / 2, \ell \pm \delta \ell / 2) / N_{r e f}$,

45 with $N_{r e f}$, the number of reference particles, and $(\rho, \ell, \xi)$, the coordinates of a pair vector ${ }_{466}$ defined in Eq. (10). While $g$ refers to a pair distribution function, $P$ refers to a probability ${ }_{447}$ of finding a pair of particles and $N^{\text {pair }}$ to a number of pairs of particles. More precisely, ${ }_{448} N_{z}^{\text {pair }}(\rho \pm \delta \rho / 2, \ell \pm \delta \ell / 2)$ is the number of pairs of coordinates $(\rho \pm \delta \rho / 2, \ell \pm \delta \ell / 2)$ in cylindrical ${ }_{453}$ sampling volumes (defining the spatial resolution for $g$ in 2D) are equal to:

$$
\begin{equation*}
\delta V_{r}=\Delta r \delta \ell \delta \xi, \delta V_{\theta}=\delta \rho \Delta h \delta \xi, \text { and } \delta V_{z}=\delta \rho \delta \ell \Delta z . \tag{12}
\end{equation*}
$$

${ }_{54}$ Here, the thickness of the 'planes' are chosen as $\Delta z=\Delta r=\Delta h=d$ and usually the spatial 55 resolution for pair distribution functions $g$ is chosen as $\delta \rho=\delta \ell=\delta \xi=1 / 2$ voxel. Note 456 that these spatial averages may lead to some underestimate of peak values of $g$ whether 7 there are large variations of $g$, that may lead to some discrepancies between theoretical and ${ }_{58}$ experimental values. Finally, in three dimensions, the expression for the $\operatorname{pdf} g$ is:

$$
\begin{equation*}
g(\vec{r} \pm \delta \vec{r})=N^{\text {pair }}(\vec{r} \pm \delta \vec{r}) / N_{\infty}^{\text {pair }} \text { with } N_{\infty}^{\text {pair }}=N_{r e f} n_{0 v} \delta V, \tag{13}
\end{equation*}
$$



FIG. 12. a) X-ray radiograph of a non sheared drop of a visco-plastic suspension at a solid fraction $\phi \simeq 35 \%$. b) 3D plot of centers of all detected particle (in blue) and reference particles (in green) for the characterization of the microstructure

## IV. MICROSTRUCTURE

## A. Bulk microstructure of a non sheared visco-plastic suspension

First, we characterize the initial microstructure of a visco-plastic suspension poured on the bottom plate of the rheometer, but before any loading and shear history. A suspension 88 is prepared by simply mixing 'by hand' the particles and the fluid in a cup with a spatula, with the goal of achieving a mixing close to chaotic mixing. With this procedure, we aim to prepare a material that is homogeneous and isotropic. It is then degased to remove possible air bubbles (in a vacuum or in a centrifuge). Upon pouring, the visco-plastic suspension has the shape of an irregular drop due to its yield stress (Fig. 12). Here, we analyze the microstructure in a toroidal region $\left(R_{0}=0.36 D / 2 \pm 0.30 D / 2\right.$ and $\left.Z_{0}=2.4 H \pm 1.2 H\right)$ inside the drop far from solid surfaces and free interfaces (Fig. 12b).

In its initial configuration, we observe that the suspension has at first order an isotropic microstructure: the pair distribution function $g(\vec{r})$ does not depend much on the direction $\vec{r} /\|\vec{r}\|$ (Fig. 13 and 19). Figure 13 shows $g_{r}(\ell, \xi), g_{\theta}(\rho, \xi)$ and $g_{z}(\rho, \ell)$ : they are roughly the same in the three orthogonal 2D 'planes', with a circular symmetry in each 2 D 'plane'. $g(\vec{r})$ mostly depends only on the distance $\|\vec{r}\|$. Figure 14b shows the 1D scalar pdf $g(\|\vec{r}\|)$ : o $g(\|\vec{r}\|)=0$ for (and only for) distances $\|\vec{r}\| \lesssim d$; it has a maximal value for $\|\vec{r}\| \simeq d$ and a second local maxima for $\|\vec{r}\| \simeq 2 d$; then $g$ tends to 1 for larger values of $\|\vec{r}\|$.

Note that a closer inspection of the 2D pdfs (Fig. 13b) shows that they are not exactly the same and not exactly isotropic: the averages of $g_{r}$ and $g_{\theta}$ for distances $\rho_{2 d}$ close to its ${ }^{4} 4$ maxima ( $\rho_{2 d}=d \pm d / 6$ ) as a function of $\phi_{2 d}$ (represented in a polar plot) are not perfectly
a)



FIG. 13. a) Three dimensional microstructure of a visco-plastic suspension in a non sheared configuration (drop) at a solid fraction $\phi \simeq 35 \%$ : 2D pair distribution functions $g_{r}(\ell, \xi), g_{\theta}(\rho, \xi)$ and $g_{z}(\rho, \ell)$. b) Polar plots of $<g\left(\phi_{2 d}\right)>_{\rho_{2 d}=d \pm d / 6}$ in the three planes of interest $(\theta z),(r z)$ and $(\theta r)$

b)

$$
g_{z}(\rho, \ell) \text {. b) Polar plots of }\left\langle g\left(\phi_{2 d}\right)>_{\rho_{2 d}=d \pm d / 6} \mathrm{in} \text { the three planes of interest }(\theta z),(r z) \text { and }(\theta r)\right.
$$

FIG. 14. Numerical microstructure from a simulation of finite-size particles following a naive rule of exclusion and redistribution for a solid fraction $\phi \simeq 36 \%$ : a) 2D pair distribution function $g(\ell, \xi)$ and b) 1D scalar pair distribution function $g\left(\rho_{2 d}\right)$ super-imposed with the experimental 1D pdf for a non sheared suspension
${ }_{495}$ circular but bear a signature of a slight over-population of pairs of particles roughly aligned 496 with the gravity. This slight micro-structuration is not visible in the ( $\theta r$ ) plane (on $g_{z}$ ). ${ }_{497}$ This may be attributed to vertical flows of the suspension when it is poured on the plate.

This almost isotropic experimental pdf is now compared to the numerical pdf obtained when simulating a random distribution of finite-size particles, with a naive rule of exclusion 500 and redistribution (Fig. 14). The finite-size effect introduced here, arbitrarily governed 501 (i.e. without introducing physical forces), has to be seen as a minimal steric constraint. A

b)


FIG. 15. Three dimensional microstructure of a visco-plastic suspension after loading and squeezing in a parallel plates geometry at a solid fraction $\phi \simeq 35 \%$ : 2D pair distribution functions $g_{r}(\ell, \xi), g_{\theta}(\rho, \xi)$ and $g_{z}(\rho, \ell)$ for $z<0$ (a) and for $z>0$ (b)

502 I 510 particles, to the arbitrary rule of redistribution of over-lapping particles and to a specific ${ }_{511}$ spatial distribution of particles in a non sheared suspension. Finally, the fluid does not seem 512 to have any significant impact on the microstructure during the mixing. 515 bottom plate. The top plate is then translated downwards, thus squeezing the visco-plastic ${ }_{516}$ suspension between the two plates. The flow induced by this squeeze flow is an inhomoge${ }_{517}$ neous simple shear flow, which has been thoroughly described in the literature [30-32] and 518 is briefly discussed in part IID.

The technique developed here for the 3D characterization of microstructure allows us


FIG. 16. Pair distribution function $g_{\phi}(\rho, \xi)$ after a squeeze flow in the shear plane for $z<0$ (a) and for $z>0$ (b)


FIG. 17. Pair distribution functions $g_{r}, g_{\theta}$ and $g_{z}$ averaged for distances $\rho_{2 d}=d \pm d / 6$ plotted as a function of the angle $\phi_{2 d}$ in polar coordinates after a squeeze flow for $z<0$ (a) and for $z>0$ (b) for a visco-plastic suspension at a solid fraction $\phi \simeq 35 \%$

520 to investigate if and how the simple shear in the $(r z)$ plane induced by this squeeze flow ${ }_{521}$ changes the microstructure of the suspension, initially nearly isotropic as seen above. The ${ }_{522}$ plane in the middle of the gap, defined by $z=0$, is an axis of symmetry of the flow. To take ${ }_{523}$ into account the different sign of the shear rate (velocity gradient) in the upper and lower ${ }_{524}$ part of the suspension, we analyse here the microstructure separately in both semi-planes ${ }_{525} z<0$ and $z>0$.

In Figures 15 and 16, we observe that an anisotropic microstructure develops in the shear ${ }_{527}$ plane $(r z)$ (velocity-velocity gradient plane), while it remains approximately isotropic in the

528 529 530 531
two other cylindrical planes $(\theta z)$ and $(\theta r)$ where the suspension does not experience any shear. Contrary to the nearly isotropic microstructure observed previously in the initial configuration, the pdf $g(\vec{r})$ is no more simply a scalar function of $\|\vec{r}\|$ but depends both on the distance $\|\vec{r}\|$ and the direction $\vec{r} /\|\vec{r}\|$. As in shear-induced structuration observed in the literature in the case of the rotational shear of a Newtonian suspension [7, 9-14], we observe here a depletion of particles in close contact ( $\left.\rho_{2 d} \simeq d\right)$ located in the extensional area close to the direction of the velocity (near the angles $+15^{\circ}$ and $+195^{\circ}$ for $z<0$ and $+165^{\circ}$ and $+345^{\circ}$ for $\left.z>0\right)$. Moreover, Figure 19 shows the 1D scalar plot of $g(\|\vec{r}\|)$ : its maxima is larger and tightens (spreads less) in comparison with the case of the non sheared suspension.

As previously seen, a closer inspection of the 2D pdfs (Fig. 17 and 19) by looking at the averages of $g_{r}, g_{z}$ and $g_{\theta}$ for distances $\rho_{2 d}$ close to its maxima ( $\rho_{2 d}=d \pm d / 6$ ) as a function of $\phi_{2 d}$ shows a secondary structuration in addition to the depletion of particles in the $(r z)$ plane: $g_{r}$ and $g_{z}$ exhibit an 'hexagonal' shape, that may be the signature of a more complex flow occurring in the parallel plates than a simple shear flow as approximated in the framework of lubrification when $D / H$ is large (the flow has also a velocity component aligned with the direction of translation of the bottom plate, which might not be negligible).

As a consequence, a suspension initially characterized by a nearly isotropic microstructure in 3D, develops at first an anisotropic microstructure when loaded in the parallel plates geometry, even before any imposed shear history. Such impact of loading on the microstructure of suspension should be observed in most rheometrical devices (cone-and-plate, Couette, Poiseuille, ...). If the characterization of an isotropic structure is needed, a possible rheometrical tool is the vane in cup geometry, classically used to study gels: isotropy can be achieved by chaotic mixing in the cup, and then the insertion of the vane into the cup should not affect the material's structure.

## C. Bulk microstructure of a visco-plastic suspension after a rotational shear flow

After the loading of the visco-plastic suspension in the parallel plates geometry, a simple shear flow in the $(\theta z)$ plane is imposed thanks to the rotation of the top plate until a stationary state is reached in terms of shear stress.

As already observed for a Newtonian suspension (numerically and experimentally) [7, 914] and as reported in Ovarlez et al. [5] for a visco-plastic suspension, the microstructure


FIG. 18. a) Three dimensional microstructure: pair distribution functions $g_{r}(\ell, \xi), g_{\theta}(\rho, \xi), g_{z}(\rho, \ell)$ after a steady rotational shear at a shear rate $\dot{\gamma}=10^{-2} \mathrm{~s}^{-1}$ of a visco-plastic suspension at a solid fraction $\phi \simeq 35 \%$. b) Pair distribution function $g_{r}, g_{\theta}$ and $g_{z}$ averaged for $\rho_{2 d}=d \pm d / 6$ plotted as a function of the angle $\phi_{2 d}$ in polar coordinates.

558 becomes anisotropic in the $(\theta z)$ shear plane (velocity-velocity gradient plane), while it is ap${ }_{559}$ proximately isotropic in the two other 'planes', as shown in Figure 18 and 19: this anisotropy 560 is referred to in the literature as a fore-aft asymmetry. In 3D, it means that the positions 561 of particle pairs previously correlated in the $(r z)$ plane due to the squeeze flow, decorrelate ${ }_{562}$ thanks to the rotational shear and the particles reorganize themselves relatively to each 563 other, leading to some new correlations in the $(\theta z)$ plane.

These correlations between particles develop at the scale of particle pairs in close contact, ${ }_{565}$ leading to a depletion of particles in the extensional area, in contrast with the compressional ${ }_{566}$ area, as well as a primary over-population roughly aligned with the direction of the flow ${ }_{567}\left(+170^{\circ}\right)$. This anisotropy can be quantified thanks to the plots of $g\left(\phi_{2 d}\right)$ (the average of ${ }_{568} g\left(\rho_{2 d}, \phi_{2 d}\right)$ for $\left.\rho_{2 d}=d \pm d / 6\right)$ plotted in polar coordinates in Figure 18b: $g\left(\phi_{2 d}\right)$ has the 569 shape of a 'butterfly'.

More precisely, Figure 18 shows that for distances $\rho_{2 d}$ close to the value of the particles 571 diameter, $g_{r}$ has minimal values for the angles $+30^{\circ}$ and $+210^{\circ}$, corresponding to a decrease 572 of the number of pairs in the extensional stress domain; and it has maximal values for the ${ }_{573}$ angles $+170^{\circ}$ and $+350^{\circ}$, corresponding to an increase of the number of particle pairs aligned 574 roughly with the flow. These two principal extrema can be referred to as an extensional575 depletion of particle pairs (for the minima of $g$ ) and a flow-alignment of particle pairs (for 576 the maxima of $g$ ). Whereas the extensional-depletion of particle pairs was already reported


FIG. 19. Scalar pair distribution functions $<g\left(\rho_{2 d}\right)>_{\phi_{2 d}}$ averaged for all values of $\phi_{2 d}$ and $<g\left(\phi_{2 d}\right)>_{\rho_{2 d}=d \pm d / 6}$ averaged for distances $\rho_{2 d}=d \pm d / 6$ plotted as a function of the distance $\rho_{2 d}$ and the angle $\phi_{2 d}$ respectively in cartesian coordinates for a visco-plastic suspension at a solid fraction $\phi \simeq 35 \%$. In the first, second and third lines, $<g\left(\rho_{2 d}\right)>_{\phi_{2 d}}$ and $<g\left(\phi_{2 d}\right)>_{\rho_{2 d}=d \pm d / 6}$ are computed from $g_{z}, g_{\theta}$ and $g_{r}$ in the $(\theta r),(r z)$ and $(\theta z)$ planes respectively. Light blue lines correspond to a non sheared suspension, dark blue ones to a suspension sheared by a squeeze flow (in the two bottom and top half-planes $z<0$ and $z>0$ ) and red lines correspond to a suspension sheared by rotation.


FIG. 20. a-b) Pdf in the shear plane ( $\theta z$ ) for a steady rotational shear of a visco-plastic suspension at a solid fraction $\phi \simeq 35 \%$ for a low ( $\dot{\gamma}=10^{-2} \mathrm{~s}^{-1}$, a) and a high ( $\dot{\gamma}=10 \mathrm{~s}^{-1}$, b) shear rate. c) Pdf averaged for $\rho_{2 d}=d \pm d / 6$ as a function of the angle $\phi_{2 d}$ in polar coordinates for the two shear rates. d) Some profiles of the pdf at different positions: $g_{r}(\ell)$ for $\xi=-d$ and $\xi=d$ for the two shear rates
${ }_{607}$ history [5]. This was attributed to the difference in the microstructures jammed at rest after ${ }_{608}$ the preshear. As a conclusion, the shear-thinning rheology of the suspending visco-plastic ${ }_{609}$ fluid, the presence of a yield stress or its elasticity may be responsible of the influence of the ${ }_{610}$ shear rate on the microstructure and the related rheology.

## E. Bulk microstructure of a Newtonian suspension after a rotational shear flow

${ }_{612}$ We now analyse the 3D microstructure of a Newtonian suspension in our parallel plates ${ }_{613}$ geometry.
${ }_{614}$ It is already known from experiments and numerical simulations that an anisotropic ${ }_{615}$ microstructure develops in sheared Newtonian suspensions [7, 10, 14]. However, in the
a)


FIG. 21. 3D pair distribution functions in the shear plane in a parallel plates geometry for a Newtonian suspension of particles: for $\phi \simeq 36 \%$ (a) and $\phi \simeq 44 \%$ (b)

616 literature, most pdfs are 2 D , in the shear plane. The more reliable data were obtained ${ }_{617}$ in a Couette geometry, which presents a stress inhomogeneity in the shear plane, whereas ${ }_{618}$ the stress inhomogeneity in the parallel plates geometry we use is in the velocity gradient${ }_{619}$ vorticity $(z \theta)$ plane. Another difference between these two geometries is that shear stresses ${ }_{620}$ and shear rates are imposed in a Couette geometry and in a parallel plates one respectively. 621 Endly, whereas the microstructure in $[10,14]$ is imaged under a flow, we are using a flow${ }_{622}$ arresting technique[18], so there may be some relaxation of the microstructure. In the ${ }_{623}$ following, we compare our 3D pdf obtained in our parallel plates set-up with 2D data ${ }_{624}$ obtained in a Couette geometry by Blan et al. [14] for the same Newtonian suspension.

In the experiments presented here, we focus on steady simple shear flows controlled ${ }_{626}$ by the rotation of the top plate of the parallel plates rheometer. Figure 21 shows the ${ }_{627}$ microstructure through pair distribution functions in the three cylindrical 'planes' $g_{r}(\ell, \xi)$, ${ }_{628} g_{\theta}(\rho, \xi)$ and $g_{z}(\rho, \ell)$ : it is not the same in the three planes. $g_{r}(\ell, \xi)$ becomes anisotropic in ${ }_{629}$ the shear plane, while it is still nearly isotropic in the two other 'planes'. Figure 21 shows ${ }_{630}$ the microstructure of two Newtonian suspensions for two different solid fractions $\phi \simeq 36 \%$ ${ }_{631}$ and $\phi \simeq 44 \%$ : they are the same.
${ }_{632}$ We can show an extensional-depletion of particle pairs and a flow-alignment of particle ${ }_{633}$ pairs. Secondary maxima and minima are observed too here and (experimental and numeri$\left.{ }_{634} \mathrm{cal}\right)$ results from Blanc $[14,24]$ show that these two local maxima and minima are visible for ${ }_{635}$ solid fractions larger than $\phi \simeq 30 \%$. For comparison, Figure 22 reports two 2D pdfs mea${ }_{636}$ sured by Blanc et al for $\phi \simeq 35 \%$ and $\phi \simeq 45 \%$ for the same suspension but in a different


FIG. 22. Pair distribution functions in the shear plane from the literature (Blanc et al [14]) in a Couette geometry (co-centric cylinders) for a Newtonian suspension of particles: in the bulk for $\phi \simeq 35 \%$ (a) and $\phi \simeq 45 \%$ (b). Here $a$ is the particles radius, whereas $d$ is the diameter. Note the different color scales in a and b.
 655 nian and visco-plastic suspensions is the dependence on the shear rate for visco-plastic and 66 the independence on the shear rate for Newtonian. However, in all cases (visco-plastic or ${ }_{57}$ Newtonian, parallel plates or Couette geometry), the microstructure becomes anisotropic 8 in the shear plane characterized by both an extensional-depletion of particle pairs and a 9 flow-alignment of particle pairs at similar angles. Finally, there may be a difference of quan60 titative values of the pdf (and of the microstructure) of a suspension in a Couette geometry 61 and in a parallel plates one, where shear stresses and shear rates are imposed respectively 62 and where a stress inhomogeneity is in the shear plane and in the velocity gradient-vorticity ${ }_{3}(z \theta)$ plane respectively. Endly, whereas the microstructure in $[14]$ is imaged under a flow, 64 here we are using a flow-arresting technique[18], so there may be some relaxation of the 5 microstructure.

## F. Microstructure near solid walls: macroscopic layering

In our parallel plates geometry, the suspension is confined between two plates and the gap 668 is not so large compared to the size of particles $(H / d \simeq 14$ for the visco-plastic suspension 669 and 12 for the Newtonian one) - this aspect ratio being common for such set-ups. These 670 solid boundaries induce some layering of particles, as shown by the solid fraction profiles 671 as a function of $z$ for smooth and rough surface plates (Fig. 24). Even when the surfaces


FIG. 24. Vertical concentration profiles close to a solid boundary for smooth (a) versus rough (b) surfaces after a rotational shear of $\approx 5 s^{-1}$ for a visco-plastic suspension at a solid fraction $\phi \simeq 40 \%$.


FIG. 25. Pair distribution function $g_{r}$ in the shear plane $(\theta z)$ computed near a smooth solid wall located in the bottom (a) and in the bulk far from any solid wall (b).

672 have been roughened (Fig. 2c and 24b), we may observe a few alternate layers of high and ${ }_{673}$ low solid fraction values, suggesting about 1 layer of aligned particles laying parallel to the 674 walls. In comparison with the case of smooth surfaces (Fig. 24a), the layering is reduced for ${ }_{675}$ rough surfaces (Fig. 24b), and is localized close to the solid boundaries, whereas layering 676 comes deep inside the gap for smooth surfaces.

677 This macroscopic layering close to solid boundaries is also visible on pair distribution 678 functions in the shear plane, when comparing pdfs computed close to solid boundaries with ${ }_{679}$ pdfs computed in the bulk (far from any solid surface) for smooth solid plates (Fig. 25). A ${ }_{680}$ periodic pattern of strips is superimposed on the bulk microstructure. This effect was already ${ }_{681}$ reported by $[24,35]$ for Newtonian suspensions, so this layering does exist for suspensions 682 of different rheology.
${ }_{683}$ As a conclusion, studying the bulk microstructure imposes to be far enough from solid


FIG. 26. Pair distribution function $g_{r}$ in the orthoradial plane $(\theta z)$ computed in the bulk (i.e. $R_{0}<0.60 \mathrm{D} / 2,1.2 H<Z_{0}<2.2 H$ ) (a) and near the orthoradial free surface (i.e. $0.60 \mathrm{D} / 2<R_{0}$, $1.2 H<Z_{0}<2.2 H$ ) (b) in the non sheared configuration (drop). The later pdf (b) is not observed anywhere else in the suspension than in the upper part of the suspension drop.


FIG. 27. a-b) Pair distribution function $g_{r}$ in the orthoradial plane $(\theta z)$ computed in the bulk (i.e. $R_{0}<0.60 D / 2,0.5 H / 2<Z_{0}$ ) (a) and near the orthoradial free surface (i.e. $0.60 D / 2<R_{0}$, $0.5 H / 2<Z_{0}$ ) (b) in the loaded and squeezed configuration. The later pdf (b) is not observed anywhere else in the suspension than in the quarter cylindrical part of the suspension near the top plate.

684 boundaries, even if they are rough.


FIG. 28. Pair distribution function $g_{r}$ in the shear plane $(\theta z)$ far from (a, c) and close to (b, d) the orthoradial free interface for a Newtonian (a, b) and a visco-plastic suspension (c, d)
G. Microstructure near free interfaces: alignment of particle doublets with free

A more subtle structuration is observed here near the orthoradial free interface, probably ${ }_{688}$ enhanced when coupled with a shear flow (or with a flow in general). This structuration is ${ }^{689}$ observed both in non sheared (drop) and sheared (by squeeze and rotation flows) suspensions 690 (Fig. 26, 27 and 28): this is an alignment of particle doublets (two particles in close contact) ${ }_{691}$ with the free interface along $\phi_{2 d} \simeq 0^{\circ}$ and $\phi_{2 d} \simeq 180^{\circ}$. However, whereas this alignment ${ }_{692}$ is systematically observed for rotational shear experiments, it is less pronounced and more ${ }_{693}$ localized in experiments of squeeze shear and in non loaded drop. This suggests that the 694 occurrence of secondary flows in the squeeze flow (see part IV B) and when pouring the 695 non loaded drop (see part IV A) may play a role in this structuration, due to a coupling 96 between free interfaces and flows. The pair distribution function exhibits a larger maxima ${ }_{997}$ in the orthoradial plane $g_{r}(\ell, \xi)$ for values of $\ell \simeq d$ and $\xi \simeq 0$ when computed near the free


FIG. 29. Radial profiles of density (number per unit volume) of particle pairs aligned with the local flow $\left(\vec{e}_{\theta}\right) \phi_{\text {pair }}\left(R_{0}\right)$ for a Newtonian $(\phi \simeq 45 \%$, a) and a visco-plastic suspension ( $\phi \simeq 38 \%$, b), for uniform radial profiles of solid fractions $\phi\left(R_{0}\right)$.
${ }_{698}$ interface (Fig. 26, 27 and 28), that is not visible on pdf computed in the bulk (far from the 699 orthoradial free interface). This maxima of the pdf corresponds to an over-population of 700 particle doublets in close contact in the azimuthal (flow) direction; it is counter-balanced 701 by the apparition of a minima (drop, squeeze) or by a decrease of the secondary minima ${ }_{702}$ (rotation), resulting in a modification of the shape of the pdf. This feature may recall the ${ }_{703}$ flow-induced structural transition (from disorder to order to disorder as the Peclet number 704 is increased) observed in numerical simulations of Brownian suspensions [8, 15-18].

705 This can be observed at the macroscopic scale by computing $\phi_{\text {pair }}$, the density (number ${ }_{706}$ per unit volume, normalized by the solid fraction) of particle doublets in close contact aligned ${ }_{707}$ with $\vec{e}_{\theta}$ in toroidal regions at different radial positions $R_{0}$ (Fig. 29). This density is found 708 to increase close to the free interface in a region of width about $D / 8$ (Fig. 29), despite 709 some spatial fluctuations of $\phi_{\text {pair }}$ We may wonder if this local alignment is due to a steric 710 constraint of the free interface or to capillary stresses due to the contact lines between air, 711 particles and the fluid, and a coupling with solid walls or with some flows.

712 We observe this specific microstructure near the free interface both for Newtonian and ${ }_{713}$ for visco-plastic suspensions in a parallel plates geometry (Fig. 28, Fig. 29). Interestingly, 714 this allows us to provide an explanation for the observation of a peak in the flow direction 715 for the first experimental pdfs of Newtonian suspensions reported in the literature [10] 716 and reprinted here (Fig. 30), which has long been a puzzle. Indeed, pdfs from Parsi et ${ }_{717}$ al. [10] were computed from optical observations of the first layer of particles on top of


FIG. 30. Pair distribution functions from the literature in a Couette geometry (co-centric cylinders) for a Newtonian suspension of particles: a) close to the top free surface for $\phi=40 \%$ from [10], b) far from the top free surface for $\phi \simeq 35 \%$ from [14]; the top free surface being also the shear plane.

718 a Couette geometry (co-centric cylinders), that is from the observation of particles close ${ }_{719}$ to the horizontal free interface, which is also the shear plane. Their results contrast with 720 the pdfs reported by [14], computed from the particle positions in the bulk of a Couette ${ }_{721}$ flow and reprinted here (Fig. 30), in which such a peak is absent. The same conclusion ${ }_{722}$ can thus be drawn from these studies (near and far from the free interface) as from our ${ }^{723}$ observations: some peak appears in the pdf in the shear plane corresponding to an over${ }^{724}$ population of particle doublets in close contact in the flow direction near the free surface ${ }_{725}$ (Fig. 28, Fig. 30). Note that in both cases (parallel plates and Couette geometry) the free 726 surface is orthogonal to vorticity; no prediction can yet be made from our observations for ${ }^{727}$ a free surface orthogonal to the velocity gradient, as in a flow along an inclined plane.

728 As a conclusion, studying the bulk microstructure imposes to be far enough from both ${ }_{729}$ free interfaces and solid boundaries; this precludes optical studies on non-index-matched 730 suspensions.

## 731 V. CONCLUSION

In this paper, we present a technique we developed to image in 3D hard particle sus${ }_{733}$ pensions with X-ray tomography. It allows for the three dimensional characterization of ${ }_{734}$ the microstructure in space - in different planes and at different positions in the set-up

735 ${ }_{737}$ is possible, even for particle pairs in close contact. an example). characterized in 3D, both in the bulk and near the interfaces. X-ray tomograph but may demand a quicker X-ray source. changes the anisotropy of the microstructure.

The dedicated set-ups used here for imaging have been built so that they can also be used in a rheometer to perform rheological measurements. It is thus possible to correlate the 3D-microstructures and the rheological properties for a same shear history (see [5] for

To illustrate the technique presented here, we apply it to two materials: a visco-plastic suspension and a Newtonian suspension, meaning that our non-Brownian particles are suspended in a visco-plastic and a Newtonian fluid respectively. The Newtonian suspension is the same as used by Blanc et al. [14], but it is loaded here in a different geometry and

In principle, all the details of our technique are given so that it should be possible to reproduce them. Our method should work for many suspensions (with a contrat of X-ray between particles and the fluid), as soon as particles do not move during the time of a X-ray scan. This may prevent us from imaging Brownian particles $(d \lesssim 1 \mu m)$ with a conventional

After a rotational shear, we find approximately the same shear-induced anisotropic pair distribution function (in the shear plane) in the bulk of the Newtonian suspension as in Blanc et al. [14]. Moreover, we find that it is nearly isotropic in the two other planes.

The visco-plastic suspension is characterized in different configurations. Its microstructure is found to be nearly isotropic in 3D after preparation, showing that it is possible to design such a nearly isotropic suspension. It becomes anisotropic in the shear plane whatever the shear is (rotational shear or continuous squeeze); loading in any geometry (except the vane-in-cup) thus leads to an anisotropic initial state. The shear-induced pair distribution function for the visco-plastic suspension is found to be similar to that for a Newtonian suspension, except at high shear rates. Indeed, as a consequence of the non-linear nature of the visco-plastic fluid (either the yield-stress or the shear-thinning viscosity), the shear rate

For the two hard non-Brownian particles suspensions used here (Newtonian and viscoplastic suspensions), we find some layering close to solid walls, especially if they are smooth, 66 and a more surprising layering close to the free interface. This latter is not as obvious as


FIG. 31. Two particles $M$ and $M_{0}$ in the global cylindrical framework and four examples for the definition of the pair vector $\vec{r} \equiv \overrightarrow{M_{0} M}$ characterizing their separation: euclidean ( $\vec{r}=\overrightarrow{M_{0} M}$ ) or curvilinear - along circular flow lines - coordinates, with orthogonal or not orthogonal axis 768 scale of some particle pairs only: it induces a flow-alignment of particle doublets at close 769 contact, in regions of quite large extent close to free interfaces. This effect is likely to be a 770 consequence of a coupling between free interfaces and shear; it allows to explain previous 771 data of the literature [10].


FIG. 32. Influence on the pair distribution function $g_{z}$ computed in $(\theta r)$ planes (in 2D (a), and in 1D (b)) of the definition of the pair vector $\vec{r}$ (for the four definitions written in the text). Our choice, Eq. (10) does not induce any bias in the symmetry of the pair distribution functions between negative and positive values of $\rho$

787 comparison in Figure 32 of $g_{z}$ computed in $(\theta r)$ planes for reference particles in a toroidal ${ }_{788}$ ring at a small radius for the four previous different definitions of $\vec{r}$.

789 In order to get enough statistics, we need to define a thickness for the elementary planes 790 where we compute 2D pair distribution functions (see Part III C). Whereas the definition of 791 the thicknesses $\Delta z$ of horizontal planes $(\theta r)$ and $\Delta r$ of orthoradial 'planes' $(\theta z)$ is obvious, 792 the thickness of radial planes $(r z)$ offers several choices. Different possible choices for the 793 thickness of radial planes $(r z)$ are: constant euclidean thickness $r \sin \left(\theta-\theta_{0}\right)$, constant wedge 794 angle $\theta-\theta_{0}$, or constant arc length $r\left(\theta-\theta_{0}\right)$. Figure 33 shows the pair distribution function 795 $g_{\theta}$ computed in ( $r z$ ) planes (in 2D and in 1D) for the three previous different definitions 796 of the thickness. Whereas the definitions of a constant wedge angle and of a constant arc 797 length show a bias (there is an asymmetry between the left and right sides), the definition of 798 a constant euclidean thickness does not. That is why we choose to work with radial planes ${ }_{799}(r z)$ of thickness $\Delta h=r \sin \left(\theta-\theta_{0}\right)$.


FIG. 33. Influence on the pair distribution function $g_{\theta}$ computed in radial ( $r z$ ) planes (in 2D (a) and in 1D (b)) of the definition of their thicknesses (for the three different definitions written in the text). The choice of a constant euclidean thickness $\Delta h=r \sin \left(\theta-\theta_{0}\right)$, Eq. 9, does not induce any bias in the symmetry of the pair distribution functions between negative and positive values of $\rho$

## Appendix B: Statistical precision of pair distribution functions

Here we discuss the statistics in terms of number of particles required for fine sampling and 802 good precision of the pdf $g$ from its following expression (see Part III C), where $N^{\text {pair }}(\vec{r} \pm \delta \vec{r})$ ${ }_{803}$ is the number of pairs of particles separated by $\vec{r} \pm \delta \vec{r}$ and $N_{r e f}$ is the total number of the 4 reference particles:

$$
\begin{equation*}
g(\vec{r} \pm \delta \vec{r})=N^{\text {pair }}(\vec{r} \pm \delta \vec{r}) / N_{\infty} \tag{B1}
\end{equation*}
$$

with:

$$
\begin{equation*}
N_{\infty}=N_{r e f} n_{0 v} \delta V, \tag{B2}
\end{equation*}
$$

806 the number of pairs separated by a vector $\vec{r}$ such that $\|\vec{r}\| \rightarrow \infty$ in the sampling volume $\delta V$ ${ }_{807}$ in the space of pair vectors delimited by $\vec{r} \pm \delta \vec{r}$. Note that $n_{0 v} \delta V$ is the number of particles ${ }^{808}$ in the sampling volume $\delta V$. The precision $\Delta g$ on $g(\vec{r})$, or the minimal detected variation, corresponding to the variation of the integer $\Delta N^{\text {pair }}=1$, is:

$$
\begin{equation*}
\Delta g=1 / N_{\infty}=\left(N_{\text {ref }} n_{0 v} \delta V\right)^{-1} \tag{B3}
\end{equation*}
$$





FIG. 34. Scalar pair distribution function $g(\|\vec{r}\|)$ for different numbers of reference particles $N_{\text {ref }}$ increasing from arbitrary small values to larger values (but for constant numbers of reference particles with $\|\vec{r}\|$, i.e. non adaptative spheres).

We will usually compute the pdf on toroidal rings of thicknesses $\Delta R$ and $\Delta Z$ centered 811 on $R_{0}$ and $Z_{0}$ (of volume $\vartheta=2 \pi R_{0} \Delta R \Delta Z$ ), so that the number of reference particles is 812 approximately:

$$
\begin{equation*}
N_{r e f} \simeq n_{0 v} \vartheta \tag{B4}
\end{equation*}
$$

${ }_{813}$ For a given density $n_{0 v}$ and chosen precisions on the spatial resolution $\delta V$ and on the pdf ${ }_{814} \Delta g$, this prescribes a minimum number of reference particles from Eq. (B3):

$$
\begin{equation*}
N_{r e f}=\left(n_{0 v} \delta V \Delta g\right)^{-1} \tag{B5}
\end{equation*}
$$

815 or equivalently a minimum volume $\vartheta$ for the toroidal ring from Eq. (B4).
$8_{816}$ As a numerical example, if we want a precision $\Delta g=10^{-2}$ on $g(\vec{r})$ and a spatial resolution ${ }_{817}$ for the $2 \mathrm{D} \operatorname{pdf} \delta V=\Delta \delta^{2}$ with $\Delta=\mathrm{d}=140 \mu \mathrm{~m}=12 \mathrm{vx}$ and $\delta=1 \mathrm{vx}=12 \mu \mathrm{~m}$, for a solid ${ }_{818}$ fraction $\phi=35 \%$ and $\mathrm{d}=140 \mu \mathrm{~m}\left(n_{0 v}=244 \mathrm{~mm}^{-3}\right)$, we need at least $2.10^{4}$ reference ${ }_{819}$ particles in the region of interest. Otherwise, we may see the discrete nature of variations ${ }_{820}$ of $g(\|\vec{r}\|)$ (Fig. 34). This effect of discretization may appear anyway for large inter-particle 821 distances, because of the use of adaptative spheres. As a compromise, these spatial averages ${ }_{822}$ may lead to some underestimate of peak values of $g$, whether there are large variations of $g$.

## ACKNOWLEDGMENTS

This work was supported by a grant from Agence National de la Recherche (ANR 2010 ${ }_{825}$ JCJC 0905 01-SUSPASEUIL). The Laboratoire Navier microtomograph has been acquired
[1] Thrane L. N., L. F. Nielsen, M. Brandl, and M. R. Geiker. On the effect of coarse aggregate fraction and shape on the rheological properties of self-compacting concrete. Cem. Concr. Aggregates, 24(1):3-6, 2002.
[2] Mahaut F., X. Chateau, P. Coussot, and G. Ovarlez. Yield stress and elastic modulus of suspensions of noncolloidal particles in yield stress fluids. J. Rheol., 52(1):287-313, 2008.
[3] Denn M. M. and J. F. Morris. Rheology of non-brownian suspensions. Annual Review of Chemical and Biomolecular Engineering, 5:203-228, 2014.
[4] Chateau X., G. Ovarlez, and K. L. Trung. Homogenization approach to the behavior of suspensions of noncolloidal particles in yield stress fluids. J. Rheol., 52(2):489-506, 2008.
[5] Ovarlez G., F. Mahaut, S. Deboeuf, N. Lenoir, S. Hormozi, and X. Chateau. Flows of suspensions of particles in yield stress fluids. J. Rheol., 59:1449-1486, 2015.
[6] Hafid H., G. Ovarlez, F. Toussaint, P.H. Jezequel, and N. Roussel. Effect of particle morphological parameters on sand grains packing properties and rheology of model mortars. Cem. Concr. Res., 80:44-51, 2016.
[7] Morris J. A review of microstructure in concentrated suspensions and its implications for rheology and bulk flow. Rheol. Acta, 48:909923, 2009.
[8] Nazockdast E. and J. F. Morris. Microstructural theory and the rheology of concentrated colloidal suspensions. J. Fluid Mech., 713:420-452, 2012.
[9] Gadala-Maria F. and A. Acrivos. Shear-induced structure in a concentrated suspension of solid spheres. J. Rheol., 24(6):799-814, 1980.
[10] Parsi F. and F. Gadala-Maria. Fore-and-aft asymmetry in a concentrated suspension of solid spheres. J. Rheol., 31(8):725-732, 1987.
[11] Rampall I., J. R. Smart, and D. T. Leighton. The influence of surface roughness on the particlepair distribution function of dilute suspensions of non-colloidal spheres in simple shear flow. J. Fluid Mech., 339:1-24, 1997.
[12] Blanc F., F. Peters, and E. Lemaire. Experimental signature of the pair-trajectories of rough spheres in the shear-induced microstructure in non-colloidal suspensions. Phys. Rev. Lett.,

107(208302), 2011.
[13] Blanc F., F. Peters, and E. Lemaire. Local transient rheological behavior of concentrated suspensions. J. Rheol., 55(4):835-854, 2011.
[14] Blanc F., E. Lemaire, A. Meunier, and F. Peters. Microstructure in sheared non-brownian concentrated suspensions. J. Rheol., 57:273-292, 2013.
[15] Phung T. N., J. F. Brady, and G. Bossis. Stokesian dynamics simulation of brownian suspensions. J. Fluid Mech., 313:181-207, 1996.
[16] Foss D. R. and J. F. Brady. Structure, diffusion and rheology of brownian suspensions by stokesian dynamics simulation. J. Fluid Mech., 407:167200, 2000.
[17] Morris J. F.and B. Katyal. Microstructure from simulated brownian suspension flows at large shear rate. Phys. Fluids, 14(1920), 2002.
[18] Gao C., S. D. Kulkarni, J. F. Morris, and J. F. Gilchrist. Direct investigation of anisotropic suspension structure in pressure-driven flow. Phys. Rev. E, 81:041403, 2010.
[19] Cheng X., J. H. McCoy, J. N. Israelachvili, and I. Cohen. Imaging the microscopic structure of shear thinning and thickening colloidal suspensions. Science, 333(6047):1276-1279, 2011.
[20] Goyon J., A. Colin, G. Ovarlez, A. Ajdari, and L. Bocquet. Spatial cooperativity in soft glassy flows. Nature, 454(7200):84-87, 2008.
[21] Maire E., J. Y. Buffiere, L. Salvo, J. J. Blandin, W. Ludwig, and J. M. Letang. On the application of x-ray microtomography in the field of materials science. Adv. Eng. Mater., 3:539546, 2001.
[22] Hsieh J. Computed tomography: principles, design, artifacts, and recent advances. Wiley Interscience, Bellingham Washington USA, 2009.
[23] Heindel T. J. A review of x-ray flow visualization with applications to multiphase flows. Journal of Fluids Engineering, 133(7):074001, 2011.
[24] Blanc F. Rhéologie et microstructure des suspensions concentrées non browniennes. PhD thesis, Université Nice Sophia Antipolis, 2011.
[25] Feldkamp L.A., L.C. Davis, and J.W. Kress. Practical cone-beam algorithm. JOSA A, 1(6):612-619, 1984.
[26] Amon A., P. Born, K. E. Daniels, J. A. Dijksma, K. Huang, D. Parker, M. Schroter, R. Stannarius, and A. Wierschem. Preface: Focus on imaging methods in granular physics. Review of Scientific Instruments, 88(051701), 2017.
${ }_{866}$ [27] Ni W.J. and H. Capart. Crosssectional imaging of refractiveindexmatched liquidgranular flows.

888 [28] Dijksman J. A., F. Rietz, K. A. Lorincz, M. van Hecke, and W. Losert. Invited article: Refrac-

895 [31] Ovarlez G., Q. Barral, and P. Coussot. Three-dimensional jamming and flows of soft glassy 896 materials. Nature Materials, 9(2):115-119, 2010.
${ }_{897}$ [32] Ramachandran A. and D. T Jr Leighton. Particle migration in concentrated suspensions 898 undergoing squeeze flow. J. Rheol., 54(3):563-589, 2010.
${ }_{899}$ [33] Rueckel J., M. Stockmar, F. Pfeiffer, and J. Herzen. Spatial resolution characterization of a

901 [34] More and more iterations are needed for convergence at higher solid fractions, but convergence Exp. Fluids, 56:163, 2015. tive index matched scanning of dense granular materials. Review of Scientific Instruments, 83(011301), 2012.

29] Gao Y. and Kilfoil M. L. Accurate detection and complete tracking of large populations of features in three dimensions. Optics Express, 17(4685), 2009.

30] Macosko C. W. Rheology: principles, measurements, and applications. Wiley-VCH, New York, 1994. x-ray microct system. Applied Radiation and Isotopes, 94:230-234, 2014. is still obtained for solid fraction as high as $36 \%$. If needed, it should be checked that it still holds for larger solid fractions.
[35] Yeo K. and M. R. Maxey. Ordering transition of non-brownian suspensions in confined steady shear flow. Phys. Rev. E, 81:051502, 2010.


[^0]:    * sdeboeuf@dalembert.upmc.fr; http://www.ida.upmc.fr/ / deboeuf/

