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1                    **Modeling soil desiccation cracking by analytical and numerical approaches**

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19 **Abstract**

20 An energy approach is proposed as a complement to the stress approach commonly considered for  
21 investigating soil desiccation cracking. The elastic strain energies before and after crack initiation are  
22 estimated by both numerical and analytical solutions. The energy released by cracking is then  
23 compared to the fracture energy to discuss crack initiation conditions. This leads to combined energy  
24 and stress conditions for crack initiation following Leguillon's theory. An approximate analytical  
25 solution is derived from a variational formulation of the porous elastic body equations. A cohesive  
26 zone model and Finite Element code are used to simulate crack propagation in an unsaturated porous  
27 body. This analysis shows that the energy criterion is reached before the stress criterion, and this can  
28 explain unstable crack propagation at the beginning. The approximate analytical solution allows  
29 predicting correctly the crack depth and opening in its initiation stage.

30

31 **Keywords:** desiccation cracks, crack initiation, crack propagation, energy criterion, cohesive zone  
32 model.

## 33        **1. Introduction**

34        Soil desiccation cracking is a common phenomenon in geotechnical engineering. The reduction of  
35        moisture content during desiccation causes soil shrinkage by the invasion of air into the soil pores. At  
36        the same time, matrix suction increases and leads to the development of tensile stress in restrained  
37        conditions (by a frictional boundary condition, suction gradient, concentrations of stress, heterogeneity  
38        of soil, *etc.*). This tensile stress can exceed the soil strength and create a desiccation crack network [1–  
39        4]. These cracks, created by tensile stresses, are different from localization shear bands that occur  
40        under other stress conditions. They have specific physical and geometrical properties and the  
41        theoretical and numerical approaches to determine their geometry is also different from the methods  
42        used for strain localization and shear band formation. The formation and propagation of desiccation  
43        cracks are hydro-mechanical processes that can affect the soil permeability and compressibility, as  
44        well as its mechanical strength. This could explain the effect of desiccation cracking on the instability  
45        of earth slopes [5–7].

46

47        The main methods for studying soil desiccation cracking are the following. The field and laboratory  
48        experiments [2,8–14] are used to study the morphology, geometry and development of the desiccation  
49        cracks and the effect of some parameters (*e.g.* specimen dimensions, boundary conditions, soil  
50        properties, and drying condition) on their formation. The theoretical methods [3,15–20] aim to predict  
51        the desiccation cracks geometry (depth and spacing between cracks). The numerical approaches are  
52        introduced to simulate the formation and propagation of cracks during desiccation or to characterize  
53        material properties that cannot be easily measured in laboratory because inaccessible. Different  
54        numerical methods have been used for this purpose: Finite Element Method (FEM), Discrete Element  
55        Method (DEM), JFEM (Joint Finite Element Method), and mesh-free method [21–30]. They have  
56        shown that the initiation and evolution of a crack network are influenced by various factors [1], which  
57        can be divided into two main groups: (*i*) the internal factors (soil characteristics, structure  
58        heterogeneity and sample dimensions); (*ii*) and the external factors (soil/mold interface,  
59        humidification/desiccation cycle, desiccation conditions). Quantitative studies on desiccation cracking

60 usually focus on crack density, volume change, crack geometry and spacing. Crack depth and spacing  
61 are key variables in the analysis of soil cracking. Various predictive models of soil cracking have been  
62 proposed, based on different approaches, to estimate these parameters.

63 Lachenbruch [15] developed an analytical approach to analyze the contraction cracks in basalt and  
64 permafrost by using Linear Elastic Fracture Mechanics (LEFM). The author introduced the fictitious  
65 stress notion and showed that: as a single crack propagated to a given depth, the stress field in the  
66 vicinity of the crack was perturbed; and that a distance away the stress asymptotically approached the  
67 initial value. An approach was proposed to predict the spacing between cracks from the theoretical  
68 stress relief field by assuming that the subsequent crack could propagate at approximately 5-10% of  
69 stress relief.

70 Konrad & Ayad [19] applied the concept introduced by Lachenbruch [15] to predict the crack depth at  
71 the onset of the formation of the primary cracks, as well as the average crack spacing. They calculated  
72 first the stress intensity factor  $K_I$  as a function of different crack depths. The depth of crack  
73 propagation at the time of initiation corresponded to that giving  $K_I$  equal to the soil's fracture  
74 toughness  $K_{IC}$ . The crack propagation was analyzed with a trapezoidal distribution of horizontal tensile  
75 stress. The crack spacing was determined from the horizontal stress field in the neighborhood of an  
76 existing crack calculated by the FEM. The prediction of crack spacing was similar to the work of  
77 Lachenbruch [15], in which the subsequent crack initiated in the vicinity of an existing crack when the  
78 horizontal stress reached 95% of the soil tensile strength. The analysis made by Konrad & Ayad [19]  
79 allowed predicting accurately the crack depth at the onset of the primary cracks by using the stress  
80 concept. However, the results depended on an arbitrary value of 95% of the tensile strength to initiate  
81 the subsequent crack. Moreover, this analysis did not give any information about the displacement  
82 fields of the soil and crack opening after crack initiation.

83 The LEFM, used to study the propagation of a single crack, was equally applied to predict the crack  
84 depth from the suction profile and soil properties [2,16]. However, the approaches developed in these  
85 works cannot be used to predict the crack spacing.

86 Penev & Kawamura [31] proposed a relationship to predict the crack spacing and the crack opening in  
87 a pavement layer composed of cement. The solution can be applied to a given thickness of material  
88 subjected to uniform shrinkage, and the crack depth was not considered. Chertkov [32,33] developed a  
89 physically based probabilistic model, using a multiple cracking and fragmentation model available for  
90 rocks, to predict the crack network geometry of swelling clayey soils. The model allowed estimating  
91 the crack opening, cross-sectional area and volume of cracks for a given water content profile and  
92 shrinkage curve of clay soil. However, two fundamental parameters of the model, the average spacing  
93 between cracks and the crack connectedness, were introduced in the model as a specific function of  
94 depth.

95 Moreover, using desiccation cracking test results and numerical simulation, Peron et al. [34] proposed  
96 a consistent explanation for the formation of desiccation crack patterns in soils. The authors gave an  
97 estimation of desiccation crack spacing by calculating the overall energy of the system. They assumed  
98 that the elastic strain energy was totally released during cracking and that all cracks penetrated to the  
99 full sample depth. This approach allowed estimating the number of blocks and thus the average crack  
100 spacing in a thin layer of soil, but it is not applicable for estimation of crack depth.

101 By using the LFEM, proposed by Lachenbruch [15] and developed by Konrad & Ayad [19], the crack  
102 propagation condition ( $K_I = K_{IC}$ ) can be satisfied by two values of crack depth. The larger value  
103 corresponds to the ultimate depth of the crack that propagates unstably, while the smaller value  
104 represents the initial crack depth necessary for the onset of unstable propagation. Sanchez et al. [9]  
105 performed laboratory experiments to investigate soil desiccation cracking. The experimental data  
106 showed that the crack reached the ultimate depth in a very short time and that the subsequent evolution  
107 of crack depth was more gradual, corresponding to a stable propagation phase. The instable crack  
108 propagation depth needs then specific analysis which is different from the gradual and stable  
109 propagation. The present paper focuses on the initial instable propagation crack. The depth reached by  
110 the crack during this phase is named "ultimate depth" although it can be followed by a further increase  
111 during a stable propagation phase. The existing approaches described above are based on stress or  
112 energy criterion. However, theoretical approaches, initially proposed by Leguillon [33], have shown

113 that a combined stress and energy criterion was needed to characterize crack initiation conditions at  
114 the surface of a homogeneous material [35,36]. A combined energy and stress approach was recently  
115 used [37] to analyze desiccation cracks spacing and depth. However, this work was based on very  
116 sharp simplifications such as a constant suction profile in a horizontal thin layer of soil with constant  
117 vertical stress and a shear stress linearly depending on the depth, etc.

118 In the present paper, a combined energy and stress approach is used as a complement to a moisture  
119 diffusion problem in unsaturated soil in order to model the desiccation cracking. Unsaturated soil  
120 models are usually extensions of saturated ones and this extension requires the definition of specific  
121 state variables. Various expressions for the effective stress in unsaturated materials have been  
122 proposed from early works of Roscoe et al. (1958) [38] and Bishop (1959) [39] to more recent works  
123 based on energetic approaches [40]. A comprehensive review of effective stress formulations can be  
124 found in [41]. Assuming the incompressibility of the solid matrix and the water phase, Houlsby (1997)  
125 [42] established a general expression for the effective stress in unsaturated material that is used, with  
126 some simplifications, in the present work. First, approximate solutions for the displacement fields of  
127 the soil before and after crack initiation are proposed. The elastic strain energies for these two  
128 moments are then calculated. The elastic energy released by cracking depends on both crack depth and  
129 spacing. Its estimation allows predicting the crack depth corresponding to a given crack spacing. This  
130 is supposed to be a first step towards a method predicting both the depth and spacing of desiccation  
131 cracking based on a combination of energy and stress approaches. The proposed displacement field  
132 allows calculating the crack opening and soil displacements after crack initiation. The approximate  
133 analytical method is compared to the results of numerical simulations using FEM. This latter includes  
134 a cohesive damage fracture law to model the moisture transfer, as well as the initiation and  
135 propagation of a single crack. These investigations provide approximate analytical solution for  
136 estimation of the crack depth due to soil desiccation in the initiation stage.

137

138 *Notation:* In the sequel, light-face (Greek or Latin) letters denote scalars; underlined letters ( $\underline{a}$ )  
 139 designate vectors and boldface letters ( $\mathbf{a}$ ) for second-order tensors; outline letters ( $\mathcal{L}$ ) are used for  
 140 fourth-order tensors.

## 141 2. Analytical approach

142 In this analysis, the soil is represented as a homogeneous porous material subjected to potential  
 143 cracking under desiccation. The study focuses on the initiation conditions and the geometry (*e.g.*  
 144 depth, opening and spacing) of cracks by combining stress and energy approaches. The simultaneous  
 145 apparition of cracks [34] is assumed to occur to create a crack network when the failure criterion is  
 146 reached. The study focuses on the desiccation crack initiation near the top surface of the soil where the  
 147 gravity-induced stresses are negligible. Thus, for the sake of simplicity, the gravity forces are  
 148 neglected in the sequel and the governing equations of the problem read:

$$div\boldsymbol{\sigma} = 0, \quad \boldsymbol{\sigma}' = \mathbf{C} : \boldsymbol{\varepsilon} \quad (1)$$

149 where  $\boldsymbol{\sigma}$  represents the total stress,  $\boldsymbol{\sigma}'$  the effective,  $\boldsymbol{\varepsilon}$  the strain and  $\mathbf{C}$  corresponds to elastic tensor  
 150 (linear isotropic with Lamé coefficients  $\lambda$  and  $\mu$ ).

151 The definition of the effective stress is a key question in porous materials. In the linear poroelasticity,  
 152 widely used to model fluid saturated porous materials, the following expression of the effective stress  
 153 is used:

$$\boldsymbol{\sigma}' = \boldsymbol{\sigma} + bp\boldsymbol{\delta} \quad (2)$$

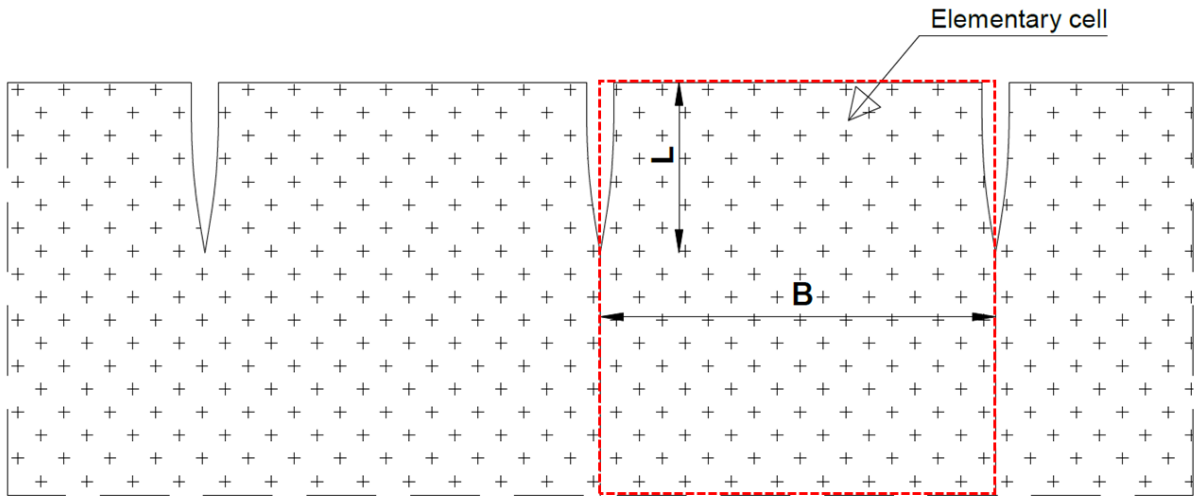
154 where  $b$  is the Biot coefficient,  $p$  is pore water pressure and  $\boldsymbol{\delta}$  the unit tensor. For unsaturated soils,  
 155 assuming the incompressibility of the solid matrix and the water phase, Houlsby (1997) [42]  
 156 established the following general expression for the effective stress:

$$\boldsymbol{\sigma}' = \boldsymbol{\sigma} - p_a\boldsymbol{\delta} + S(p_a - p_w)\boldsymbol{\delta} \quad (3)$$

157 where  $p_a$  designates the air pressure,  $p_w$  the water pressure and  $S$  the degree of saturation. This  
 158 expression of the effective stress was widely used for soil analyses and in particular for the non-linear,



159 elastoplastic or damage behavior of soils [43,44] . In this work, the air pressure is neglected. Several  
 160 works showed that crack initiation took place when the saturation degree  $S$  remained close to 1  
 161 ([10,11,45]). As a result, and in order to be able to apply the theoretical methods of LEFM and linear  
 162 poroelasticity in following analyses,  $S$  is assumed to be equal to 1 for the effective stress calculation in  
 163 this study. This does not exclude that the variation of  $S$  is considered in the moisture diffusion  
 164 problem. With this approximation, the same Eq. (2) can be formally used for the mathematical  
 165 treatments of unsaturated soil by taking  $p = -p_w$ . In the sequel for simplicity and unity of notations the  
 166 expression (2) is used in the theoretical relations but for the numerical models  $b$  is taken equal to 1.  
 167 The study focuses on the desiccation crack initiation near the top surface of the soil and for this  
 168 purpose, a regular geometry of cracks, characterized by a depth  $L$  and a spacing  $B$  (Figure 1), is  
 169 considered.

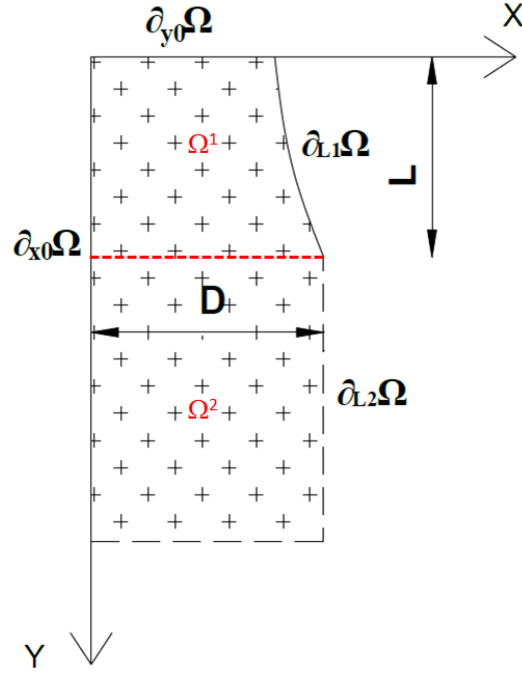


170

171

*Figure 1: Geometry of the problem*

172 The energy approach consists of the calculation of the elastic strain energies before and after crack  
 173 initiation with regular depth and spacing. The assumption of a regular set of cracks allows reducing  
 174 the model to an elementary cell of spacing  $B$  (Figure 1). The symmetry of this problem allows  
 175 reducing the model to a domain  $\Omega$  containing two subdomains  $\Omega^1$  and  $\Omega^2$  with four boundaries  
 176  $\partial_{x_0}\Omega$ ;  $\partial_{y_0}\Omega$ ;  $\partial_{L1}\Omega$ ;  $\partial_{L2}\Omega$  and a half spacing  $D$  ( $D = B/2$ ) presented in Figure 2 (the  $y$ -axis is oriented  
 177 downward).



178

179

Figure 2: Geometry of the model

180 The boundary conditions of the elementary cell considered in Figure 2 are:

$$\forall \underline{x} \in \partial_{x_0}\Omega \cup \partial_{L_2}\Omega; \quad u_x(\underline{x}) = 0 \quad (\text{I.1})$$

$$\forall \underline{x} \in \partial_{L_1}\Omega; \quad \sigma_{xx}(\underline{x}) = 0 \quad (\text{I.2})$$

181

$$\forall \underline{x} \in \partial_{y_0}\Omega; \quad \sigma_{yy}(\underline{x}) = 0 \quad (\text{I.3})$$

(I)

$$\forall \underline{x} \in \partial_{x_0}\Omega \cup \partial_{y_0}\Omega \cup \partial_L\Omega; \quad \sigma_{xy}(\underline{x}) = 0 \quad (\text{I.4})$$

$$\lim_{y \rightarrow \infty} \underline{u}(\underline{x}) = 0 \quad (\text{I.5})$$

182 The pore pressure field  $p$  taking negative values corresponding to suction, results from the moisture

183 transfer calculation in the numerical simulation. So, for mechanical calculation, it is supposed to be a

184 known function of  $y$  and:  $\lim_{y \rightarrow \infty} p(y) = 0$ . As the mechanism of desiccation cracking is related to

185 tensile stress due to increase of suction during desiccation, variation of the pressure in the positive

186 range has no effect on the desiccation cracking. The choice of this boundary condition simplifies the

187 solution while respecting these assumptions.

## 188 2.1. Variational formulation of the problem

189

190 A variational formulation of the local equations governing the deformation of solids is very commonly  
 191 used in order to build numerical approximate methods. The variational approach has been used  
 192 extensively in linear elasticity but also for non-linear problems. In particular for the analysis of crack  
 193 propagation it has been used in various works and a comprehensive review can be found in [46] . In  
 194 this section, a variational formulation of the system of equations (I) is established in order to build  
 195 approximate solutions. Then, Eq.(3) and the system (I) can be written under the effective stress form  
 196 as follows:

$$\forall \underline{x} \in \Omega; \operatorname{div} \boldsymbol{\sigma}' = b \nabla p \quad (\text{II.1})$$

$$\forall \underline{x} \in \partial_{x_0} \Omega \cup \partial_{L_2} \Omega; u_x(\underline{x}) = 0 \quad (\text{II.2})$$

$$\forall \underline{x} \in \partial_{L_1} \Omega; \sigma'_{xx}(\underline{x}) = bp(y) \quad (\text{II.3})$$

$$\forall \underline{x} \in \partial_{y_0} \Omega; \sigma'_{yy}(\underline{x}) = bp(0) \quad (\text{II.4}) \quad (\text{II})$$

$$\forall \underline{x} \in \partial_{x_0} \Omega \cup \partial_{y_0} \Omega \cup \partial_{L_1} \Omega; \sigma'_{xy}(\underline{x}) = 0 \quad (\text{II.5})$$

$$\lim_{y \rightarrow \infty} u(\underline{x}) = 0 \quad (\text{II.6})$$

197 This local formulation is equivalent to the variational problem explained below. A set of kinematically  
 198 admissible displacement fields  $U_A$ , satisfying the above displacement boundary conditions, is defined  
 199 as:  $U_A; \underline{u} / \forall \underline{x} \in \partial_{x_0} \Omega \cup \partial_{L_2} \Omega; u_x(\underline{x}) = 0$  and  $\lim_{y \rightarrow \infty} u(\underline{x}) = 0$

200 Thus, the solution of the system (II) is the displacement field that minimizes the following potential:

$$\Psi = \frac{1}{2} \int_{\Omega} \boldsymbol{\varepsilon} : \boldsymbol{\varepsilon} : \boldsymbol{\varepsilon} dw - \int_{\Omega} \underline{f} \cdot \underline{u} dw - \int_{\Gamma} \underline{T} \cdot \underline{u} ds \quad (4)$$

$$\text{Where: } \begin{cases} \underline{x} \in \Omega; \underline{f} = -b \nabla p & (\underline{f} \text{ is volumetric force}) \\ \partial_{\Gamma} \Omega = \partial_{L_1} \Omega \cup \partial_{y_0} \Omega \\ \underline{x} \in \partial_{L_1} \Omega; \underline{T} = bp(y) \underline{n}(\underline{x}) = bp(y) \underline{e}_x \\ \underline{x} \in \partial_{y_0} \Omega; \underline{T} = bp(0) \underline{n}(\underline{x}) = -bp(0) \underline{e}_y \end{cases} \quad (5)$$

201 By assuming that  $\boldsymbol{\varepsilon}^s$  and  $\underline{u}^s$  are the strain and displacement solutions of the system (II) for  $L \neq 0$ , the  
 202 following inequality can be written for every admissible displacement  $\underline{u} \in U_A$ :

$$\Psi(\underline{u}) \geq \Psi(\underline{u}^s) \quad (6)$$

203 Moreover, it can be established that, for the solutions  $\underline{u}^s$  and  $\varepsilon^s$  [47]:

$$\int_W \dot{\varepsilon}^s : \mathbb{f} : \varepsilon^s dw = \int_W \dot{f} \cdot \underline{u}^s dw + \int_{\Gamma_W} \dot{T} \cdot \underline{u}^s ds \quad (7)$$

204 From Eq.(4) and Eq.(7), the potential  $\Psi(\underline{u}^s)$  of the solutions can be deduced as follows:

$$\Psi(\underline{u}^s) = - \frac{1}{2} \int_W \dot{\varepsilon}^s : \mathbb{f} : \varepsilon^s dw \quad (8)$$

205 The elastic strain energy of the porous elastic material can be deduced from the following analysis. If

206  $\sigma$  designates the total stress, then a strain increment, produced by the external work  $dW$ , satisfies:  $dW$

207  $= \sigma : d\varepsilon$ . The porous-elastic constitutive law Eq.(2) can be written as:

$$\sigma = C : (\varepsilon - \varepsilon^L) \quad (9)$$

208 where  $\varepsilon^L = C^{-1} : bp\delta$ . Then the external work increment becomes:

$$dW = \sigma : d\varepsilon = (\varepsilon - \varepsilon^L) : C : d\varepsilon \quad (10)$$

209 In the present work, the pore pressure field is determined by a moisture transfer calculation, and the

210 effect of deformation on pore pressure is not considered. In addition, the crack propagation at the

211 initiation phase is very quick. Thus, the pore pressure fields before and after crack initiation are

212 supposed to be similar. Therefore,  $\varepsilon^L$  remains constant in the crack initiation phase. Eq. (10) is

213 integrated in:

$$dW = d \left[ \frac{1}{2} (\varepsilon - \varepsilon^L) : C : (\varepsilon - \varepsilon^L) \right] \quad (11)$$

214 The elastic strain energy released by the cracking process is then given by:

$$\Delta E = \frac{1}{2} \Delta \int_{\Omega} (\varepsilon - \varepsilon^L) : C : (\varepsilon - \varepsilon^L) d\Omega \quad (12)$$

215 The elastic strain energy before cracking,  $E^{before}$ , is analytically calculated from the displacements  
 216 field, which is the solutions of the system (II) with  $L = 0$ . The elastic strain energy after crack  
 217 initiation  $E^{after}$  is deduced from the displacement solution of the system (II) with  $L \neq 0$ . The variational  
 218 formulation allows establishing an upper estimation of the elastic strain energy after crack initiation:

$$E^{after} = \frac{1}{2} \int_{\Omega} (\boldsymbol{\varepsilon}^s - \boldsymbol{\varepsilon}^L) : \mathbf{C} : (\boldsymbol{\varepsilon}^s - \boldsymbol{\varepsilon}^L) d\Omega = \frac{1}{2} \int_{\Omega} \boldsymbol{\varepsilon}^s : \mathbf{C} : \boldsymbol{\varepsilon}^s d\Omega - \int_{\Omega} \boldsymbol{\varepsilon}^s : \mathbf{C} : \boldsymbol{\varepsilon}^L d\Omega + \frac{1}{2} \int_{\Omega} \boldsymbol{\varepsilon}^L : \mathbf{C} : \boldsymbol{\varepsilon}^L d\Omega \quad (13)$$

219 The second integral of this equation can be transformed as follows:

$$\dot{\mathbf{Q}}_{\mathbf{w}} \boldsymbol{\varepsilon}^L : \boldsymbol{\varepsilon} : \boldsymbol{\varepsilon}^s d\mathbf{W} = \dot{\mathbf{Q}}_{\mathbf{w}} b p \boldsymbol{\delta} : \boldsymbol{\varepsilon}^s d\mathbf{W} = \dot{\mathbf{Q}}_{\mathbf{w}} b p \boldsymbol{\nabla}_i u_i^s d\mathbf{W} \quad (14)$$

220 This Eq.(14) can be integrated by parts, yielding:

$$\dot{\mathbf{Q}}_{\mathbf{w}} b p \boldsymbol{\nabla}_i u_i^s d\mathbf{W} = \dot{\mathbf{Q}}_{\mathbf{w}} b p u_i^s n_i d\mathbf{W} - \dot{\mathbf{Q}}_{\mathbf{w}} b u_i^s \boldsymbol{\nabla}_i p d\mathbf{W} \quad (15)$$

221 Integration of  $\partial\Omega$  can be decomposed into integration of  $\partial_u\Omega$  (on which the displacement is  
 222 prescribed) and  $\partial_f\Omega$  (on which the surface traction is applied). Zero displacement is imposed on the  
 223 boundaries ( $\partial_{x_0}\Omega$  and  $\partial_{L_2}\Omega$ ). For these boundaries, the contribution of the first integral in Eq.(15)  
 224 vanishes because  $u_i^s = 0$ . The contribution to  $\partial_f\Omega$  corresponds to the prescribed forces  $\underline{T}$ . In addition,  
 225 by replacing the term  $b \boldsymbol{\nabla}_i p$  by  $-f_i$  (see Eq.(5)), the integral of Eq.(15) can be transformed as follows:

$$\dot{\mathbf{Q}}_{\mathbf{w}} b p \boldsymbol{\nabla}_i u_i^s d\mathbf{W} = \dot{\mathbf{Q}}_{\mathbf{w}} T_i u_i^s d\mathbf{W} + \dot{\mathbf{Q}}_{\mathbf{w}} f_i u_i^s d\mathbf{W} \quad (16)$$

226 From the equations (7), (14) and (16), it can be deduced that:

$$\dot{\mathbf{Q}}_{\mathbf{w}} \boldsymbol{\varepsilon}^L : \boldsymbol{\varepsilon} : \boldsymbol{\varepsilon}^s d\mathbf{W} = \dot{\mathbf{Q}}_{\mathbf{w}} \boldsymbol{\varepsilon}^s : \boldsymbol{\varepsilon} : \boldsymbol{\varepsilon}^s d\mathbf{W} \quad (17)$$

227 By replacing Eq. (17) in Eq. (13), the following is obtained:

$$E^{after} = - \frac{1}{2} \dot{\mathbf{Q}}_{\mathbf{w}} \boldsymbol{\varepsilon}^s : \boldsymbol{\varepsilon} : \boldsymbol{\varepsilon}^s d\mathbf{W} + \frac{1}{2} \dot{\mathbf{Q}}_{\mathbf{w}} \boldsymbol{\varepsilon}^L : \boldsymbol{\varepsilon} : \boldsymbol{\varepsilon}^L d\mathbf{W} \quad (18)$$

228 The same analysis can be done for the displacements and strains fields before cracking, denoted by  $\underline{u}^0$   
 229 and  $\boldsymbol{\varepsilon}^0$ , and it can be found that:

$$E^{before} = - \frac{1}{2} \dot{\mathbf{Q}}_{\mathbf{w}} \boldsymbol{\varepsilon}^0 : \boldsymbol{\varepsilon} : \boldsymbol{\varepsilon}^0 d\mathbf{W} + \frac{1}{2} \dot{\mathbf{Q}}_{\mathbf{w}} \boldsymbol{\varepsilon}^L : \boldsymbol{\varepsilon} : \boldsymbol{\varepsilon}^L d\mathbf{W} \quad (19)$$

230 The difference between elastic strain energies before and after crack initiation using the Eqs.(18), (19),  
 231 and (8) can be calculated as:

$$E^{before} - E^{after} = - \frac{1}{2} \dot{\mathbf{Q}}_w \boldsymbol{\varepsilon}^0 : \boldsymbol{\varepsilon} : \boldsymbol{\varepsilon}^0 dW + \frac{1}{2} \dot{\mathbf{Q}}_w \boldsymbol{\varepsilon}^s : \boldsymbol{\varepsilon} : \boldsymbol{\varepsilon}^s dW - \frac{1}{2} \dot{\mathbf{Q}}_w \boldsymbol{\varepsilon}^0 : \boldsymbol{\varepsilon} : \boldsymbol{\varepsilon}^0 dW - \Psi(\underline{u}^s) \quad (20)$$

232 Using Eq.(6) for an approximate solution  $\underline{u}$ , from Eq.(20), yields:

$$E^{before} - E^{after} \approx - \frac{1}{2} \dot{\mathbf{Q}}_w \boldsymbol{\varepsilon}^0 : \boldsymbol{\varepsilon} : \boldsymbol{\varepsilon}^0 dW - \Psi(\underline{u}) \quad (21)$$

233 The energy-based method for analyzing crack propagation consists of calculating the energy released  
 234 by cracking and comparing it with the surface energy needed to create a crack. If the fracture energy  
 235 dissipation rate is designated by  $G^c$ , with  $G^c = 2\gamma^s$  where  $\gamma^s$  is specific rupture energy per unit surface  
 236 (depth in 2D geometry), then the energy condition for crack propagation is expressed by:

$$E^{before} - E^{after} \geq G^c L \quad (22)$$

237 Thus, by using Eq.(21), a sufficient condition to satisfy the energy condition in (22) is:

$$- \Psi(\underline{u}) - \frac{1}{2} \dot{\mathbf{Q}}_w \boldsymbol{\varepsilon}^0 : \boldsymbol{\varepsilon} : \boldsymbol{\varepsilon}^0 dW \geq G^c L \quad (23)$$

238 A simple analysis shows that, for small perturbations close to the initial state, the left side of Eq.(23) is  
 239 positive. By designating:

$$\Delta \equiv - \Psi(\underline{u}) - \frac{1}{2} \dot{\mathbf{Q}}_w \boldsymbol{\varepsilon}^0 : \boldsymbol{\varepsilon} : \boldsymbol{\varepsilon}^0 dW \quad (24)$$

240 The energy criterion for crack propagation in Eq.(23) becomes:

$$\Delta/L \geq G^c \quad (25)$$

241 In the sequel, after calculating the energy before cracking, approximate solutions  $\underline{u}$  for the state after  
 242 crack initiation are fitted to  $\underline{u}^s$  to obtain close estimations of crack propagation conditions.

243 **2.2. Analytical solutions**

244 a) *Before cracking*

245 The displacement solution before cracking corresponds to  $L = 0$  in the system (II), and the condition  $u_x$   
 246  $= 0$  for every point with  $x = 0$  or  $x = D$ . The displacement solution is designated by  $\underline{u}^0$  as follows:

$$\begin{cases} u_x^0(x, y) = 0 \\ u_y^0(x, y) = -b \int_0^y P(x) dx \end{cases} \quad (26)$$

247 where  $P = \frac{p(y)}{\lambda + 2\mu}$  is dimensionless. The elastic strain energy before cracking can be calculated from

248 Eq.(19):

$$\frac{E^{before}}{\lambda + 2\mu} = b^2 D \frac{1 - \psi}{1 + 2\psi} \int_0^\infty P^2 dy \quad (27)$$

249 With  $\psi = \frac{\lambda}{\lambda + 2\mu}$ . The term  $\frac{1}{2} \int_{\Omega} \boldsymbol{\varepsilon}^0 : \boldsymbol{\varepsilon}^0 d\Omega$  in Eq.(24) can also be calculated as:

$$\frac{1}{2} \frac{\int_{\Omega} \boldsymbol{\varepsilon}^0 : \mathbf{C} : \boldsymbol{\varepsilon}^0 d\Omega}{\lambda + 2\mu} = \frac{1}{2} b^2 D \int_0^\infty P^2 dy \quad (28)$$

250 b) *After cracking*

251 Exact analytical solution doesn't exist for the stress and displacement fields of problem (II) with  $L \neq 0$ .

252 An approximate solution of the displacement field can be proposed by minimizing the potential  $\Psi(\underline{u})$

253 (Eq.(4)). This minimization is not applied in the space of all continuous functions in the domain  $\Omega$  but

254 only in a subdomain of polynomial functions of  $x$  and parameters  $a_n(y)$  that depend on depth. Every

255 function  $f(x, y)$  on the domain  $\Omega = [0, D] \times [0, \infty[$  can be decomposed as:

$$f(x, y) = \sum_{n=0}^{\infty} a_n(y) \frac{(x/D)^n}{n!} \quad (29)$$

256 An approximate solution is considered by considering only the first terms up to the second degree of  $x$   
 257 as follows:

$$\begin{cases} u_x(x, y) = a_0(y) + a_1(y)(x/D) + a_2(y)(x/D)^2/2 \\ u_y(x, y) = b_0(y) + b_1(y)(x/D) + b_2(y)(x/D)^2/2 \end{cases} \quad (30)$$

258 The analysis of the symmetries of the problem (*Figure 1*) shows that  $u_x$  must be an odd function of  $x$ ,  
 259 whereas  $u_y$  must be a pair function. This leads to  $a_0(y) = a_2(y) = b_1(y) = 0$ . The displacement field can  
 260 be written as:

$$\begin{cases} u_x(x, y) = a_1(y) x/D \\ u_y(x, y) = b_0(y) + b_2(y)(x/D)^2/2 \end{cases} \quad (31)$$

261 The functions  $a_1$ ,  $b_0$  and  $b_2$  are determined separately for  $y \in [0, L]$  and  $y \in [L, \infty]$ , supposing the  
 262 continuity of displacement in the entire domain and on the line  $y = L$ . Thus, two subdomains are  
 263 considered and denoted by  $\Omega^1 = [0, D] \times [0, L]$ ;  $\Omega^2 = [0, D] \times [L, \infty[$  (see *Figure 2*).

264 In  $\Omega^1$ , the displacement fields, which satisfy the boundary conditions (II.2) and (II.6), are determined  
 265 to find the elements of  $U_A$  defined in the previous section. The momentum balance equation (II.1) and  
 266 the force boundary conditions (II.4) should be satisfied in addition to the displacement boundary  
 267 conditions. Therefore, the solution globally satisfies the equations (II.1), (II.2), (II.4) and (II.5).  
 268 Additionally, the continuity of the displacement on the  $\partial_{12}\Omega$  interface line ( $y = L$ ) imposes  $u_x(D, L) = 0$ .  
 269 This leads to the following expressions for  $u_x$  and  $u_y$  in  $\Omega^1$  depending on only two constants,  $C_1$  and  
 270  $C_2$ :

$$\underline{x} \in \Omega^1; \begin{cases} u_x(x, y) = \frac{C_1}{D}(y-L)x \\ u_y(x, y) = u_y^0(x, y) + C_2 + \frac{\psi C_1}{D} \left( Ly - \frac{y^2}{2} \right) - \frac{C_1}{2D} x^2 \end{cases} \quad (32)$$

271 In  $\Omega^2$ , the continuity of the displacement on the  $\partial_{12}\Omega$  interface must be satisfied, in addition to the  
 272 displacement boundary conditions. It should be noted that  $u_y^0(x, y)$  (the solution before cracking)  
 273 satisfies the boundary conditions corresponding to  $\Omega^2$  but not the continuity of displacement for the



274 solution after cracking in the  $\partial_{12}\Omega$  interface. The correction term  $Du$  is defined to satisfy this  
 275 condition without changing other boundary conditions. The solution is proposed with:

$$Du = \frac{u_y(x, L) - u_y^0(x, L)}{P(L)} P(y) \quad (33)$$

276 Because  $\lim_{y \rightarrow \Psi} P(y) = 0$ , the condition (II.6) will be satisfied by this assumption. The general  
 277 displacement expression will be given in  $\Omega^2$  by:

$$\underline{x} \in \Omega^2; \begin{cases} u_x(x, y) = 0 \\ u_y(x, y) = u_y^0(x, y) + \left( C_2 + \frac{\psi C_1 L^2}{2D} - \frac{C_1}{2D} x^2 \right) \frac{P(y)}{P(L)} \end{cases} \quad (34)$$

278 The approximate displacement field given by Eqs. (32) and (34) satisfies all the conditions apart from  
 279 the boundary conditions (II.3) for the  $\Omega^1$  part and the momentum balance equation (II.1) for the  $\Omega^2$   
 280 part. In the sequel, the two constants  $C_1$  and  $C_2$  are determined by minimizing the potential  $\Psi(\underline{u})$  to  
 281 obtain the best approximation. Note that if  $C_1 = C_2 = 0$  in Eqs. (32) and (34), then the expression of  
 282 the displacement field before cracking can be obtained.

283 The expression of the potential  $\Psi(\underline{u})$ , depending on these two constants ( $C_1$  and  $C_2$ ) calculated by the  
 284 sum of the potential shown in Eq.(4) for the two subdomains  $\Omega^1$  and  $\Omega^2$ , is given as follows:

$$\Psi(\underline{u}) = (\lambda + 2\mu) \left[ -\frac{1}{2} b^2 D \int_0^\infty P^2 dy + C_1 b (\psi - 1) P_1 + \frac{(1 - \psi^2) C_1^2 L^3}{6D} + \frac{(1 - \psi) D C_1^2}{12 P_L^2} P_2 + \frac{1}{2 P_L^2} \left( \frac{1}{D} \chi^2 + \frac{C_1^2 D^3}{20} - \chi \frac{C_1 D}{3} \right) P_3 \right] \quad (35)$$

285 where  $\chi = \left( \frac{2DC_2 + \psi C_1 L^2}{2} \right)$  and the following constants depend on the pressure field:

$$286 \quad P_L = P(L), \quad P_1 = \int_0^L (y - L) P(y) dy, \quad P_2 = \int_L^\Psi P^2(y) dy, \quad P_3 = \int_L^\Psi (P'(y))^2 dy$$

287 It can be seen that the potential  $\Psi(\underline{u})$  is a quadratic function of ( $C_1, C_2$ ). Minimization with respect to  
 288 ( $C_1, C_2$ ), *i.e.*, the solution of the system of equations:

$$\frac{\int C_1}{\int C_2} \Psi(C_1, C_2) = \frac{\int C_1}{\int C_2} \Psi(C_1, C_2) = 0 \quad (36)$$

289 leads to the following result:

$$\begin{cases} C_1 = \frac{-6b(\psi-1)DP_L^2P_1}{2P_L^2(1-\psi^2)L^3 + (1-\psi)D^2P_2 + 2D^4P_3/15} \\ C_2 = \frac{-C_1}{2D} \left( \psi L^2 - \frac{D^2}{3} \right) \end{cases} \quad (37)$$

290 With this expression, the potential after crack initiation (Eq.(35)) is calculated as follows:

$$\begin{aligned} & \int \Psi(u) \\ & = (\lambda + 2\mu) \left[ -\frac{1}{2} b^2 D \int_0^\infty P^2 dy + C_1 b (\psi - 1) P_1 + \frac{(1-\psi^2)C_1^2 L^3}{6D} + \frac{(1-\psi)DC_1^2}{12P_L^2} P_2 + \frac{C_1^2 D^3}{90P_L^2} P_3 \right] \end{aligned} \quad (38)$$

291 From Eqs. (24), (28) and (38), it can be obtained that:

$$\frac{E}{\lambda + 2\mu} = -C_1 b (\psi - 1) P_1 - C_1^2 \left[ \frac{(1-\psi^2)L^3}{6D} + \frac{(1-\psi)D}{12P_L^2} P_2 + \frac{D^3}{90P_L^2} P_3 \right] \quad (39)$$

292 By replacing the  $C_1$  presented in Eq. (37), the normalized dissipated energy by cracking in the  
293 approximate energy approach becomes:

$$\frac{E}{L} = \frac{(\lambda + 2\mu)45DP_L^2b^2(\psi-1)^2P_1^2}{L \left[ 30P_L^2(1-\psi^2)L^3 + 2D^4P_3 + 15(1-\psi)D^2P_2 \right]} \quad (40)$$

294 It can be seen that this normalized dissipated energy by cracking in this approach is a function of the  
295 cracks depth  $L$ , the half spacing  $D$ , the soil properties though the term  $\psi$  and the given suction field  
296  $P(y)$ .

### 3. Numerical approach

297 In this work, approximate analytical solutions are compared with the results obtained by the numerical  
298 method. Simulations are performed with the Finite Element code *Porofis* [48] which is a research code  
299 derived from the commercial code DISROC built for coupled hydro-mechanical processes in porous  
300 fissured materials. The equations used in this paper, for governing hydraulic flow and mechanical

301 deformation in cracked and unsaturated porous materials, have been presented in detail in previous  
 302 papers [49–51]. They are here briefly summarized.

### 303 **3.1. Hydraulic behavior**

304 The moisture transfer process in a homogeneous porous and unsaturated material representing the soil  
 305 is simulated. The flow in the soil around the cracks is governed by Darcy's law, while the flow in the  
 306 cracks is governed by the cubic law [52], and they satisfy the mass conservation condition. During  
 307 desiccation, the suction evolution is related to the degree of saturation by the Van Genuchten model  
 308 [53] expressed by the following equation:

$$\frac{S - S_{res}}{1 - S_{res}} = \frac{1}{(1 + (\theta p)^n)^m} \quad (41)$$

309 where  $S_{res}$  is residual degree of saturation and  $\theta$ ,  $n$ ,  $m$  are constants.

310 The equation that allows determining the flow in the soil matrix with an assumption of incompressible  
 311 fluid can be written as follows [54]:

$$div\left(\frac{\mathbf{k}}{\rho g} \nabla p\right) = C_M \frac{\partial p}{\partial t} \quad (42)$$

312 where  $C_M = S \left( \frac{1}{N} + \frac{\phi S'}{S} \right)$ ,  $\mathbf{k}$  is the soil hydraulic conductivity,  $g$  is the gravitational acceleration,  $N$  is  
 313 the Biot modulus,  $\phi$  represents the porosity, and  $S'$  is the derivative  $dS/dp$  calculated from the water  
 314 retention curve.

315 For the cracks, the transversal conductivity between the two crack walls is infinitely high. This implies  
 316 that the pressure is continuous between the two opposite faces of the crack and that the pressure takes  
 317 the same value  $p$  for a given point along the crack. The equation, which allows calculating the pressure  
 318 for every location  $\underline{s}$  along the crack, can be written as below:

$$\nabla_s \cdot (c \partial_s p) = r^{mf} + r^f \quad (43)$$

319 where  $r^{mf} = \|\underline{v}\| \cdot \underline{n}$ ;  $r^f = \frac{\partial e}{\partial t}$ ,

320 In this equation,  $\nabla_s \cdot ()$  designates the divergence in the crack. The fluid-crack mass exchanges are  
321 taken into account by the jump of fluid velocity across the crack  $\|\underline{v}\|$  by presenting the discontinuity  
322 with velocity values  $\underline{v}^+$  and  $\underline{v}^-$  for two faces of the crack.  $\underline{n}$  is the normal unit vector to the crack  
323 surface.  $c$  is the tangential conductivity of the crack (parallel with the two crack walls) that can be  
324 evaluated with the crack opening  $e$  by the cubic law.

### 325 **3.2. Mechanical behavior – Cohesive zone model for the crack**

326 For the mechanical problem, the crack initiation and propagation are modeled by placing joint  
327 elements with Cohesive Zone Model (CZM) behavior. The CZM has been proposed as an alternative  
328 to Linear Elastic Fracture Mechanics to model crack propagation and takes its origin from the works  
329 of Dugdale [55], Barenblatt [56]. It is based on two key parameters: tensile strength and work of  
330 separation (or fracture energy) [57,58]. Thus, the CZM incorporates both energy and strength criteria  
331 [59]. Numerous developments have been introduced since the original Dugdale-Barenblatt's model to  
332 consider compression, shear and mixed loadings [60–65]. The CZM has been widely used because it  
333 avoids stress singularity at the fracture tip and can be easily implemented in a numerical code [66,67].  
334 The CZM has been used more recently to study fracture propagation or hydric cracking in soil and  
335 rock type materials [68] and for microstructural approach of porous fractured solids [69,70].

336 An advanced CZM model was proposed by Pouya and Bemani [63], which covers both normal and  
337 shear loads. This model was successfully applied to the analysis and modeling of a set of tensile and  
338 shear, monotonic and cyclic fracture experiments on different rocks. This model is available in the  
339 numerical code Porofis and was used for the present study.

340 The following equation is used to simulate the cohesive damage crack behavior:

$$\underline{\sigma} = (1 - d)\underline{R}\underline{u} \quad (44)$$

341 where  $\underline{\sigma}$  is the stress vector on the matrix/crack interface surface,  $\underline{n}$  is the normal unit vector on this  
 342 surface,  $\mathbf{R}$  is the joint stiffness tensor and  $d$  a scalar damage variable.  
 343 The strength criterion  $F(\underline{\sigma}, d)$  for this model depends on three parameters  $\sigma_R$ ,  $C$  and  $\varphi$  (which represent  
 344 respectively the tensile strength, cohesion and friction angle of the interface) and a function  $g(d)$   
 345 controlling the strength degradation with damage variable  $D$ . It has the following expression (See  
 346 Figure 3a):

$$F(\underline{\sigma}, d) = \tau^2 - \sigma_n^2 \tan^2 \varphi + 2g(d)\sigma_c \sigma_n - g^2(d)C^2 \quad (45)$$

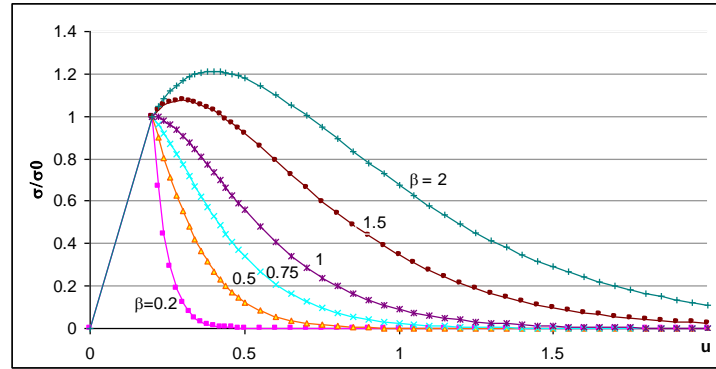
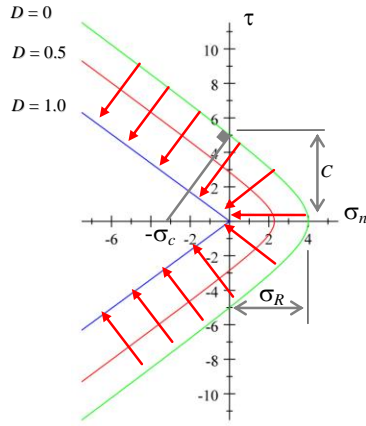
347 In mode I propagation, the damage evolution law induces a relation between  $d$  and opening  $u_n$  for a  
 348 monotonic traction loading given by the following relation:

$$\begin{cases} d = 0 & \text{for } u_n < u_0 \\ d = 1 - e^{-\frac{(u_n - u_0)}{\beta u_0}} & \text{for } u_n \geq u_0 \end{cases} \quad (46)$$

349 where  $u_n$  is the normal component of the displacement discontinuity  $\underline{u}$  through the two crack walls,  
 350  $u_0 = \frac{\sigma_R}{R_m}$  is the elastic displacement limit, with  $R_m$  the normal component of the joint stiffness, and  $\beta$  a  
 351 parameter related to the ductility of the damage process (see Figure 3b). The function  $g$  has the  
 352 following expression:

$$g(D) = (1-d)(1 - \beta \ln(1-d)) \quad (47)$$

353 For the intact state,  $d = 0$ ,  $g = 1$  and a hyperbolic shape criterion can be obtained, which is similar to  
 354 those proposed by Ekelin [71] for soils and Carol al. [62] for quasi-brittle materials. At the final  
 355 failure state,  $d \rightarrow 1$  and  $g \rightarrow 0$  and purely frictional interface with zero cohesion can be found.



356

357

(a)

(b)

358 Figure 3. (a) Evolution of the criterion from the intact condition (green) to a frictional law (blue)  
 359 during damage process (red arrows)), and (b) Traction-Separation law of the model for mode I  
 360 propagation [63]

361

362 In the context of desiccation cracking, the crack propagation occurs mainly under the mode I and it is  
 363 mainly controlled by the parameter  $\sigma_R$  of the model related to the tensile strength of the material. The  
 364 parameter  $C$  has no significant effect on the propagation process.

365 The fracture energy dissipation rate  $G^c$  can be related to the stress intensity factor  $K_{IC}$ , the elasticity  
 366 modulus and the Poisson ratio in the framework of the LEFM [72]. Moreover, the equivalence of the  
 367 LEFM and the CFM has been established in terms of crack propagation. The fracture energy  
 368 dissipation rate  $G^c$  is obtained by the integral of the surface under the traction-separation law and its  
 369 expression as a function of cohesive crack parameters is proposed as follows [63]:

$$G^c = \left[1/2 + \beta(\beta + 1)\right] \frac{\sigma_R^2}{R_m} \quad (48)$$

370 The effective stress is formulated to describe the mechanical behavior of the soil matrix and the failure  
 371 criterion of the cohesive cracks. The suction is calculated in the matrix from the governing equations  
 372 given in the previous section, whereas the crack is supposed to be empty and therefore no fluid

373 pressure exists in the crack. The effective stress takes different values in the crack (joint elements) and  
374 in the matrix whereas the continuity of total stress is ensured at the interface between the matrix and  
375 the joint elements.

376 The crack opening  $e$  changes with the deformation from the initial value  $e_0$  to:

$$e = e_0 + u_n \quad (49)$$

377 This crack opening change is considered to modify its hydraulic conductivity. The soil matrix is  
378 assumed to be an isotropic elastic linear material obeying Eq.(2).

### 379 ***3.3. Hydro-mechanical coupling***

380 The coupling between mechanical and hydraulic problems is performed by a sequential resolution of  
381 the two problems and the interactions between them. For each time increment, the hydraulic problem  
382 is calculated in the beginning by Eqs. (42) and (43). The outputs corresponding to soil suction, degree  
383 of saturation and hydraulic conductivity of soil are then used as inputs for the mechanical problem.  
384 For the soil matrix, the hydro-mechanical coupling (Eq.(2)) allows updating the effective stress and  
385 calculating the soil strain. The effect of soil strain on the pore pressure field is ignored. The  
386 mechanical problem results are then input into the hydraulic problem to change the crack opening  $e$   
387 (Eq.(43)) and update the hydraulic conductivity of the crack  $c$  following the cubic law [52].

388

## **4. Comparison between analytical and numerical results**

389

### ***4.1. Model parameters and boundary conditions for numerical simulation***

390 The numerical simulation presented here focuses on a single crack propagation supposing that the  
391 crack spacing is known. A rectangular sample of 1 m in width, supposed to represent the crack  
392 spacing, and 4 m in height, supposed to be greater than the ultimate crack depth, is simulated (see  
393 *Figure 4*). Joint elements are inserted by the code *Porofis* [48] in the FEM model on the crack  
394 propagation path which is considered to be known in the present work. The initial stiffness of the

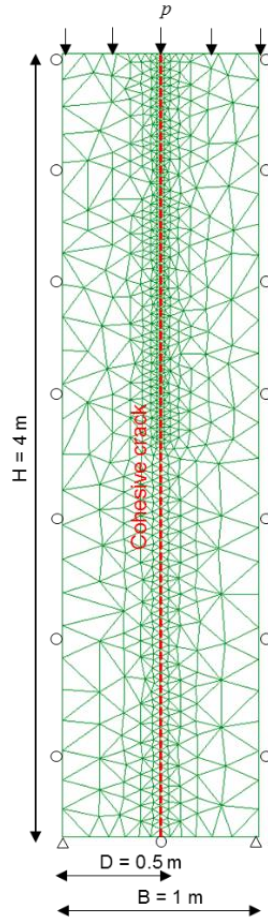
395 crack, before damage, is high and so the presence of the joint elements in the model does not affect the  
396 deformability of the model before cracking.

397 For the mechanical boundary conditions: the horizontal displacement is fixed on the two lateral sides;  
398 the vertical displacement is prevented at the bottom of the sample; and the top surface is free to move  
399 to simulate the settlement. For the hydraulic boundary condition, the desiccation is applied on the top  
400 surface of the sample by a pore pressure that decreases with time. The variation of the applied pore  
401 pressure with time can be expressed by the following function:

$$p = p_0(1 - e^{-\alpha t}) \quad (50)$$

402 where  $p_0$  is the final pore pressure on the top surface and  $t$  is time. In this equation,  $\alpha$  represents the  
403 desiccation rate: a higher  $\alpha$  corresponds to a faster desiccation. The geometry and the boundary  
404 condition of the simulation are presented in *Figure 4*. The mesh is refined around the cohesive crack  
405 line, almost on the first half from the surface in order to better capture the crack propagation.





406

407

*Figure 4: Geometry and boundary conditions of the model*

408 As explained above, the cohesive crack was used to model the desiccation cracks initiation and  
 409 propagation. Several studies indicated that the desiccation cracking occurs mainly in opening mode  
 410 (mode I) [15,16,19,73] and this mode is also assumed in the present work. Thus, the main parameters

411 of the cohesive crack are: normal joint stiffness  $R_m$ , tensile strength  $\sigma_R$ , and initial crack opening  $e_0$ .

412 The crack tensile strength  $\sigma_R$  was taken to be equal to the soil tensile strength. The initial value of the  
 413 normal stiffness  $R_m$  is taken to be sufficiently high, and that of the hydraulic conductivity (related to

414 the initial opening  $e_0$ ) is sufficiently small so as to have negligible effects on the global elasticity and  
 415 permeability of the model before cracking. The parameter  $\beta$  in the damage model corresponds to the

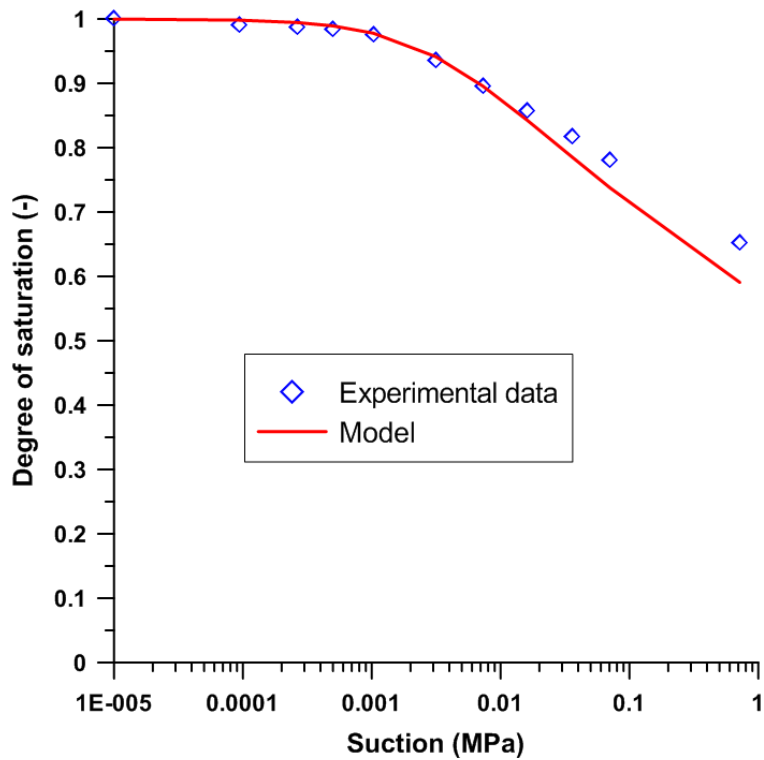
416 ductility of the material and can be obtained from the experimental curves [74]. In this work,  $\beta$  was  
 417 taken to be equal to 1, which implies that the tensile stress of the fracture starts to decrease at the onset

418 of damage (see Figure 3). The parameters  $C_{coh}$  and  $\varphi$  do not affect the mode-I crack propagation  
 419 considered in the present work. However, these parameters must satisfy the inequality

420  $C_{coh} / \tan \varphi > \sigma_R$  for the hyperbolic strength surface. For this reason,  $C_{coh}$  was chosen as

421  $C_{coh} = 1.5 \sigma_R \tan \varphi$ .

422 The soil parameters are identical to those used by Konrad & Ayad [19]. The water retention curve is  
 423 fitted from the experimental curve [19] with two constants  $\theta$ ,  $n$  ( $m = 1-1/n$ ) of the Van Genuchten  
 424 model (Eq.(41)). The *Figure 5* shows the water retention curves given by the experimental data [19]  
 425 and the model.



426

427 *Figure 5 : Water retention curve (Experimental data from [19])*

428 *Table 1* presents the main parameters for the soil studied [19], for the cohesive crack and for the  
 429 desiccation loading function used in numerical simulation. In this table,  $E$  represents the soil elasticity  
 430 modulus,  $\nu$  is the Poisson ratio and  $k_s$  is the hydraulic conductivity of soil at saturated state.

431 *Table 1: Parameters of the numerical simulation*

Soil	$E$ (MPa)	$\nu$ (-)	$k_s$ (m/s)	Water retention curve (Van Genuchten model)			
				$\theta$ (MPa <sup>-1</sup> )	$n$ (-)	$m$ (-)	$S_{res}$ (-)

	10	0.3	$5 \times 10^{-6}$	310	1.1	0.09	0.02
--	----	-----	--------------------	-----	-----	------	------

432

Crack	$R_{tt}$ (MPa/m)	$R_{mm}$ (MPa/m)	$R_m = R_{nt}$ (MPa/m)	$\sigma_R$ (MPa)	$C_{coh}$ (MPa)	$\varphi$ (°)	$\beta$ (-)	$e_0$ (m)
	1	10000	0	0.01	0.006	20	1	$10^{-5}$

433

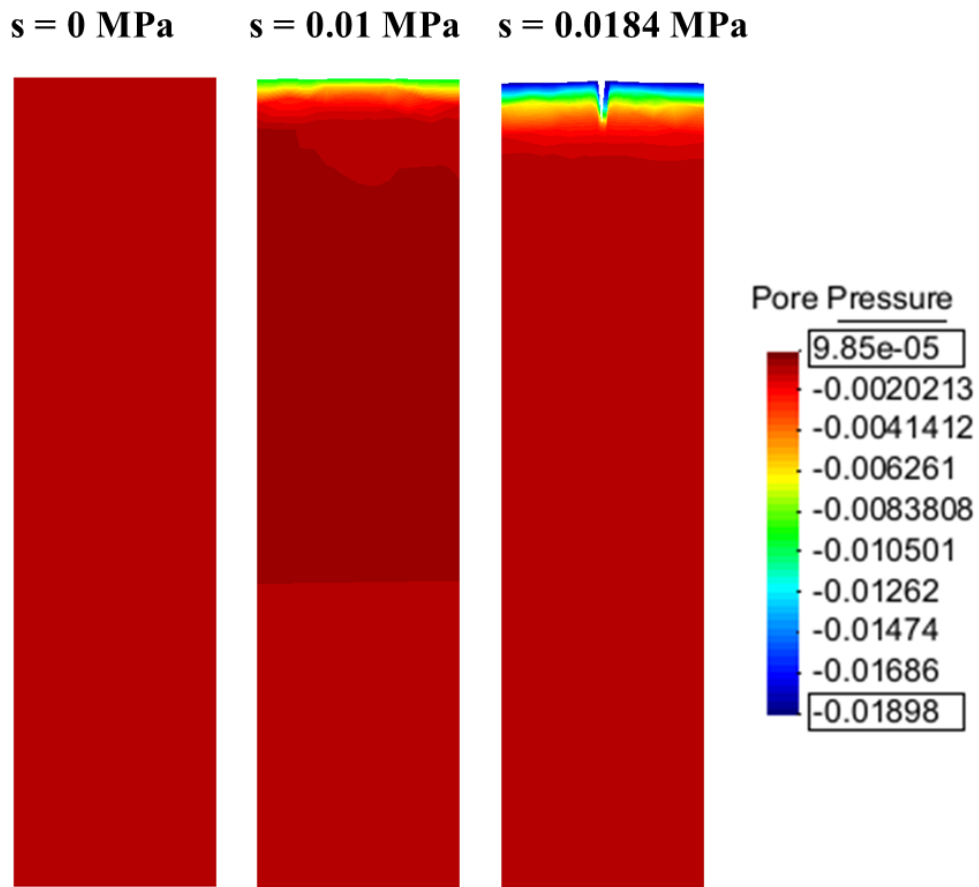
Desiccation rate	$p_0$ (MPa)	$\alpha$ (-)
	-0.03	50

434

435

4.2. Suction evolution and two phases of the desiccation process following the numerical approach

436



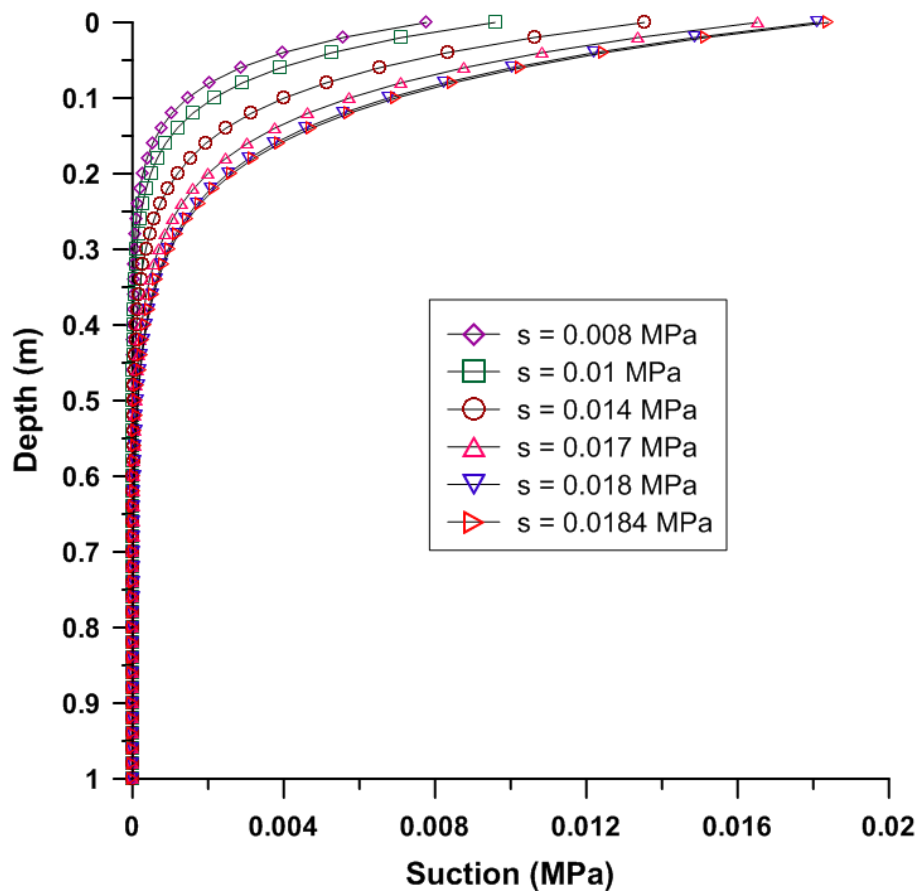
437

438

439

(a)

440



441

(b)

442

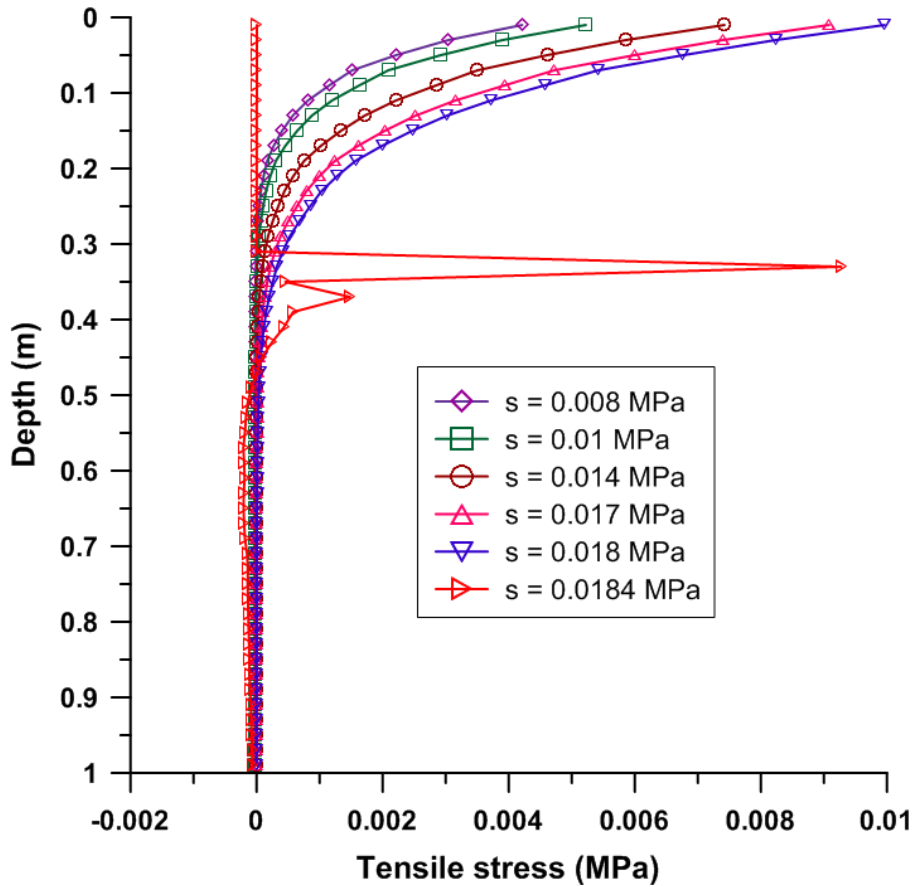
443 *Figure 6: Suction profile with different suctions at the top surface (following the numerical approach)*

444 The *Figure 6a* shows the pore pressure distribution, including the deformed shape of the sample for  
445 three suctions at the top surface calculated by the numerical approach. Before cracking, the pore  
446 pressure is almost homogenous in each horizontal section. The suction profiles are presented in the  
447 *Figure 6b* for various suctions at the top surface. It can be seen that the suction increases (or pore  
448 pressure decreases) gradually on the top surface due to the applied boundary condition. This suction  
449 penetrates from the top to the deeper parts of the sample due to moisture transfer. The highest suction  
450 is always on the top surface, and it decreases gradually with the depth. Two phases can be identified  
451 from *Figure 6a*. In the first phase ( $s = 0.01$  MPa, for example), the sample presents only settlement  
452 without cracking, and the presence of the cohesive crack does not influence the pore pressure

453 distribution. In the second phase ( $s = 0.0184$  MPa), the crack initiated partially by an opening, and the  
454 crack hydraulic conductivity increases by representing higher suction values around its location  
455 (*Figure 6a*). The suction at the initiation of the crack was about 0.018 MPa. This suction is close to the  
456 air-entry value (0.02 – 0.03 MPa) observed on the water retention curve plotted in the *Figure 5*. This  
457 result is in agreement with previous experimental observations [10,11,45].

458 The mechanism of crack initiation can be further studied in *Figure 7* by the stress criterion. *Figure 7*  
459 shows the tensile stress evolution of all joint elements along the line of the crack from the top surface  
460 to 1 m depth. It can be observed that, from  $s = 0.008$  MPa to  $s = 0.017$  MPa, the tensile stress develops  
461 gradually in the upper part of the model (from  $y = 0$  to  $y = 0.5$  m), and the highest stress is on the top  
462 surface, which corresponds to the desiccation condition (*Figure 6*). In this period, the tensile stress  
463 increases but remains smaller than the tensile strength (0.01 MPa). Therefore, all joint elements  
464 remain in the elastic phase in which no damage occurs and no crack can be observed. It is the first  
465 phase explained above where the sample presents only settlement without cracking (*Figure 6a*). When  
466  $s = 0.018$  MPa, the tensile stress at the top surface ( $y = 0$ ) reaches the tensile strength (0.01 MPa), the  
467 damage phase begins, and the second phase with the initiation of the crack starts. The propagation of  
468 the crack in the crack initiation phase occurs suddenly. When  $s = 0.0184$  MPa, all joint elements from  
469  $y = 0$  to  $y = 0.31$  m are damaged. This damage is represented by the stress relaxation. This can be  
470 considered as the critical moment that distinguishes the two phases of the analysis: before and after  
471 crack initiation.

472



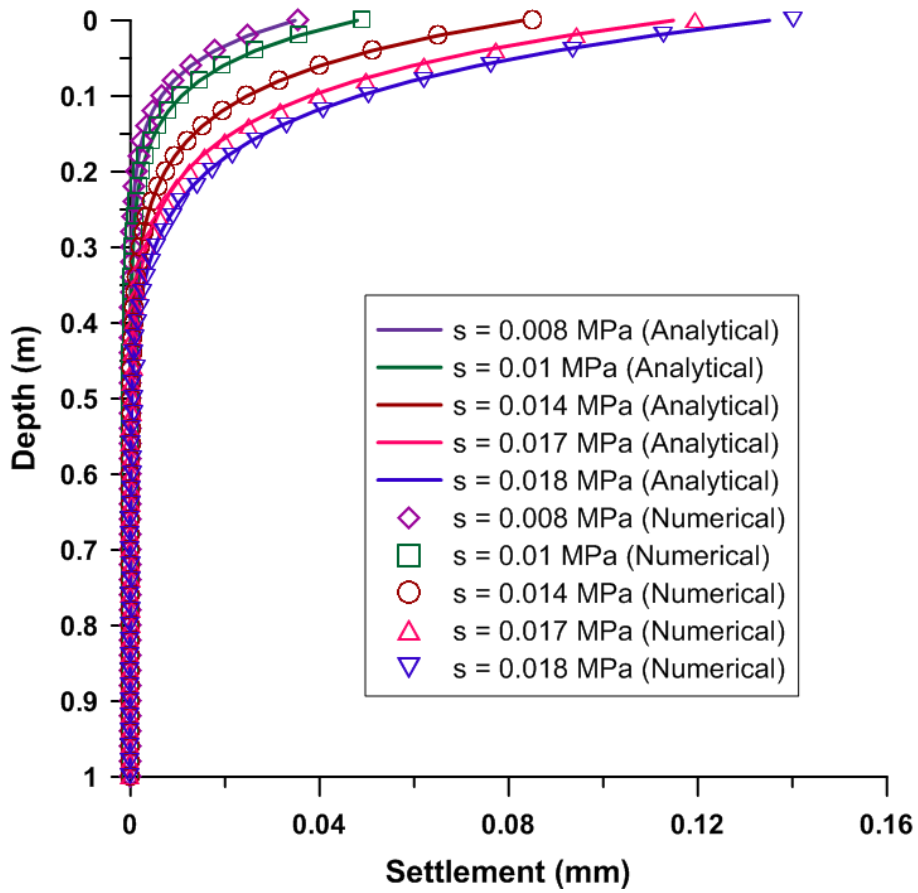
473

474 *Figure 7: Tensile stress profile at various suctions on top surface following the numerical approach*

475

#### 4.3. Displacements and energy before cracking

476 As explained above, the horizontal displacement  $u_x$  is assumed to be zero and the settlement  $u_y$  varied  
 477 only vertically before cracking. This settlement can be calculated analytically with the soil parameters  
 478 and the suction profiles (Eq. (26)). From the given pore pressure distribution presented in *Figure 6b*,  
 479 the settlement with the depth of the model is analytically calculated to compare with the numerical  
 480 simulation results (see *Figure 8*). That shows good agreement between the settlements calculated by  
 481 analytical solution (lines) and the ones calculated by numerical simulation (points). The settlement  
 482 increases during the desiccation, and the maximum settlement is on the top of the sample.



483

484

Figure 8: Settlement field before cracking

485

486

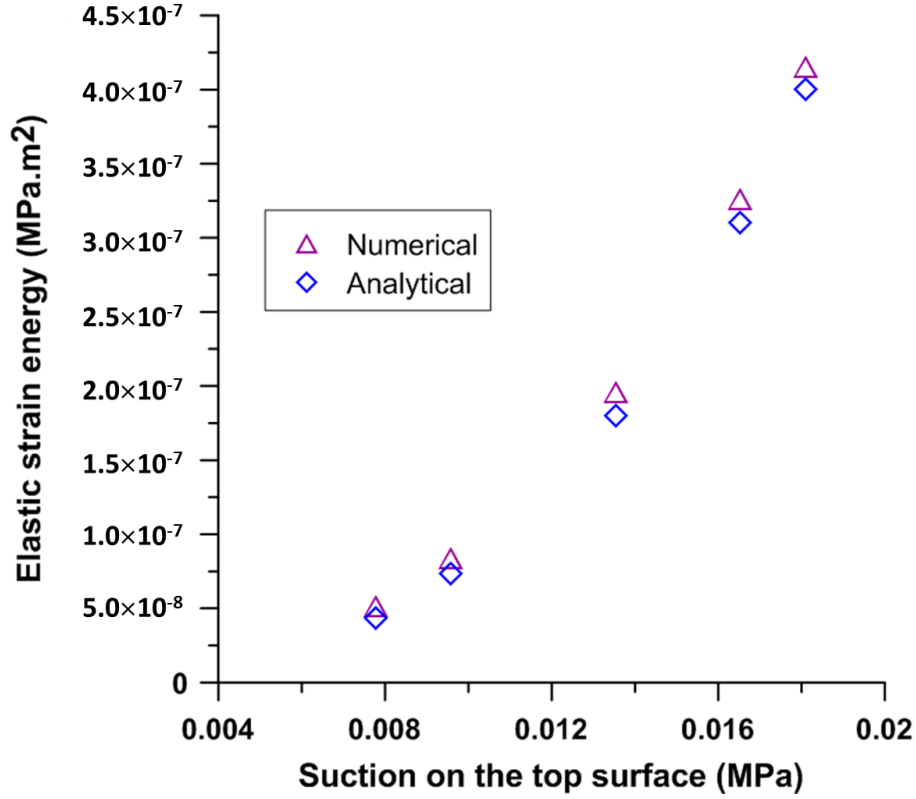


Figure 9: Elastic strain energy versus suction on the top surface

487

488

489

490 The elastic strain energy before cracking can be analytically calculated as a function of the soil  
 491 parameters and the suction evolution (Eq. (27)). In the numerical simulation, the elastic strain energy

492  $E^{num}$  of the sample is calculated by the following equation:

$$E^{num} = \left[ \frac{1}{2} \int_{\Omega} (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^L) : \mathbf{C} : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^L) + \frac{1}{2} \int_{\Omega} \underline{\mathbf{u}} : \mathbf{R} : \underline{\mathbf{u}} \right] dv \quad (51)$$

493 where  $\underline{\mathbf{u}}$  and  $\mathbf{R}$  are the opening and normal stiffness of the joint elements.

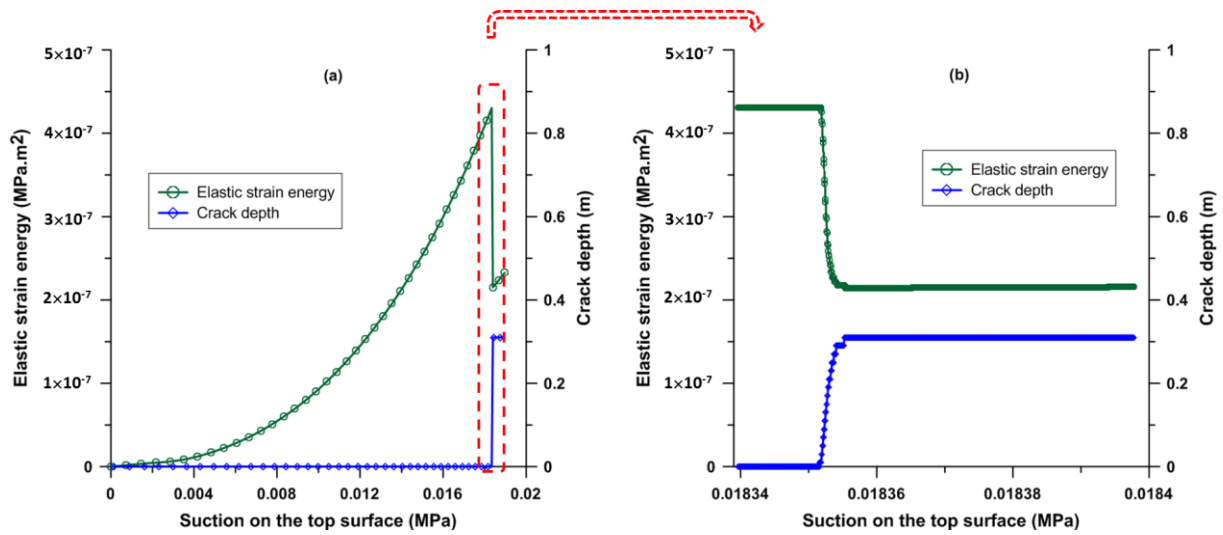
494 The Figure 9 shows good agreement between the elastic strain energy evolution for the numerical  
 495 simulation and the analytical results.

#### 496 4.4. Crack depth in the initiation phase

497 As mentioned above, in this simulation, the crack is initiated when  $s = 0.0184$  MPa. The pore pressure  
 498 profile at this moment can be applied in the analytical approach to calculate the energy released by  
 499 cracking and to predict the crack depth in its initiation stage for a known half-spacing  $D$ .



500 The energy evolution of the model during desiccation and the evolution of the crack depth in this  
501 period are initially determined by the numerical simulation. In the numerical simulation, the crack  
502 depth is considered as the distance from the top surface to the last damaged joint element in which the  
503 damage variable is equal to 1. During the desiccation, the tensile stress increases due to the increase of  
504 suction, and the damage process begins when the tensile stress reaches the tensile strength (*Figure 7*).  
505 The elastic strain energy of the sample is calculated by Eq.(48). *Figure 10* shows the elastic strain  
506 energy and crack depth evolution calculated by the numerical simulation. At the beginning, from  $s = 0$   
507 to  $s = 0.01835$  MPa, the elastic strain energy of the sample increases due to the increase of suction  
508 (*Figure 10a*). During this period, the crack remains closed. At  $s = 0.01835$  MPa, the elastic strain  
509 energy decreases markedly, and the energy is dissipated due to the initiation of a crack with a depth of  
510 0.31 m. It can be seen that this crack is propagated with two main phases: in the first phase, the crack  
511 propagates suddenly in a very short time (instable propagation phase) to reach an ultimate length (see  
512 *Figure 10b*), and in the second one, a stable and gradual propagation of the crack occurs.  
513



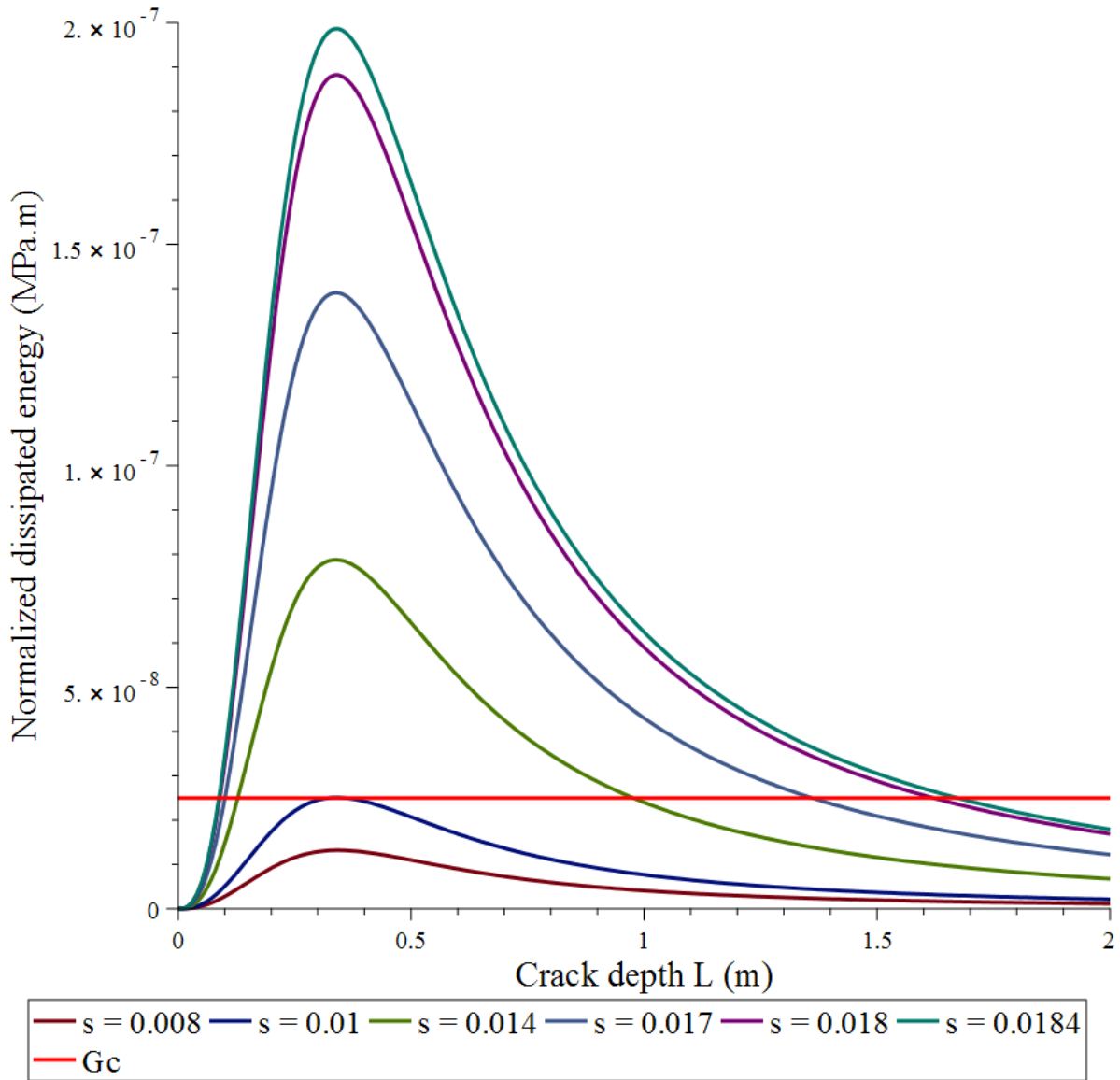
514  
515 *Figure 10: Elastic strain energy and crack depth versus suction on the top surface by the numerical*  
516 *approach*

517 In the analytical approach, the pore pressure profiles numerically calculated (*Figure 6b*) are applied  
518 (Eq.(40)) to calculate the normalized dissipated energy  $\downarrow/L$  for different crack depths. In the energy  
519 approach, the crack initiates when the energy criterion is satisfied, *i.e.*,  $\downarrow/L \geq G^c$  (Eq.(25)).

520 *Figure 11* shows the normalized dissipated energy  $\downarrow\uparrow/L$  (analytically calculated) versus crack depth for  
521 various suctions at the top surface. The fracture energy dissipation rate  $G^c$  calculated by Eq.(48) is also  
522 plotted in this figure. It can be seen that the energy criterion is satisfied with  $s = 0.01$  MPa when the  
523 peak of the normalized dissipated energy curve  $\downarrow\uparrow/L$  reaches  $G^c$ . If the energy criterion alone was  
524 sufficient for crack propagation, this state could be the onset of crack propagation. However, *Figure 7*  
525 shows that, at this moment, the tensile stress is still smaller than the tensile strength. According to the  
526 Leguillon's theory the two criteria must be satisfied for the crack initiation takes place. This is well  
527 confirmed by the numerical simulation results since, even though the energy criterion is satisfied at  
528 this moment, this crack does not initiate. Then, the energy strain of the sample continues to  
529 accumulate with increasing suction. When, the tensile stress reaches the soil tensile strength ( $s =$   
530  $0.01835$  MPa), *Figure 7*, the criteria of both energy and stress are satisfied, and, at this moment  
531 precisely the crack starts to initiate and propagate (instantaneous energy drop and jump in the crack  
532 depth), *Figure 10*.

533 It is interesting to note that the energy criterion provides also an information on the crack depth.  
534 According to the energy criterion, the crack depth must satisfy the condition  $\downarrow\uparrow/L \geq G^c$ . *Figure 11*  
535 shows that, at the crack initiation moment,  $s \approx 0.01835$  MPa, this condition is satisfied for  $L = 0.08$  to  
536  $1.65$  m. The crack depth  $L$  corresponding to the peak value of  $\downarrow\uparrow/L$  is approximately equal to  $0.33$  m  
537 for all the curves. This value of the crack depth is close to the one obtained by the numerical approach  
538 (see *Figure 10*). Note that Konrad &Ayad [19] used LEFM to study the propagation of desiccation  
539 cracks in the same soil and found an ultimate crack depth of approximately  $0.30$  m with average  
540 spacing of  $1-1.2$  m (half-spacing is close to  $0.5$  m, as in the case of the present study). These results  
541 were in agreement with in situ observation [14,16].

542



543

544 *Figure 11: Normalized dissipated energy versus crack depth for various suctions on the top surface*

545 *and the fracture energy dissipation rate  $G^c$  by the analytical approach*

546

#### **4.5. Displacements and energy after crack initiation**

547 In this section, the proposed approximate analytical solution of the displacement field after crack

548 initiation is compared with the numerical results. For this purpose, the suction distribution when  $s =$

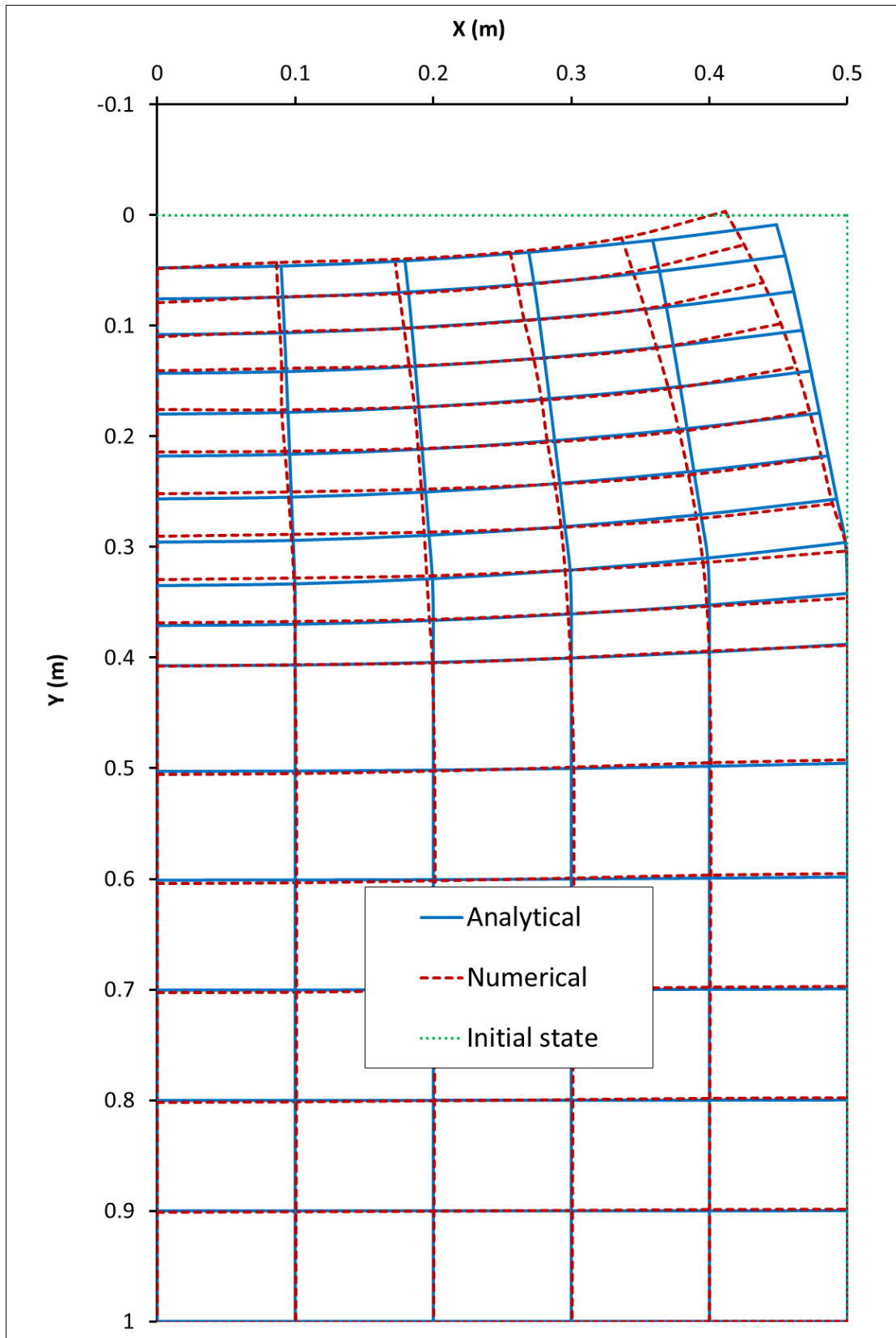
549  $0.01834$  MPa and a crack depth of  $L = 0.31$  m are considered in the analytical solution (Eqs. (32), (34)

550 and (37)) to calculate the displacement field. The left side of the crack (from  $x = 0 - 0.5$  m) is studied

551 in this part. As mentioned above, the subdomain  $1 \Omega^1$  contains the crack with  $y \leq L = 0.31$  m, and the

552 displacements can be calculated by Eq. (32). For the subdomain  $2 \Omega^2$  below the crack with  $y > L =$   
553 0.31 m, the displacements can be calculated by Eq. (34), and two constants  $C_1$  and  $C_2$  can be calculated  
554 by Eq. (37).

555 *Figure 12* shows the deformed mesh calculated by the numerical simulation (dashed lines) and the  
556 analytical solution (continuous lines). The displacement presented here is amplified 300 times for a  
557 clear visualization. The horizontal displacement in the crack line ( $x = 0.5$  m) shows the opening of the  
558 crack. In this analytical solution, the horizontal displacement is approximated as a linear function of  $y$ .  
559 It can be observed that the horizontal displacement is maximal on the crack line and decreases from  
560 this line to the left side. In addition, the settlement after crack initiation is not uniform in each  
561 horizontal section, but the maximal settlement is obtained on the left side (center of the unit cell  
562 because of the symmetry conditions), and the minimal settlement is obtained on the crack line. A  
563 reasonable agreement between the numerical simulation results and the approximate analytical  
564 solution can be observed.



565

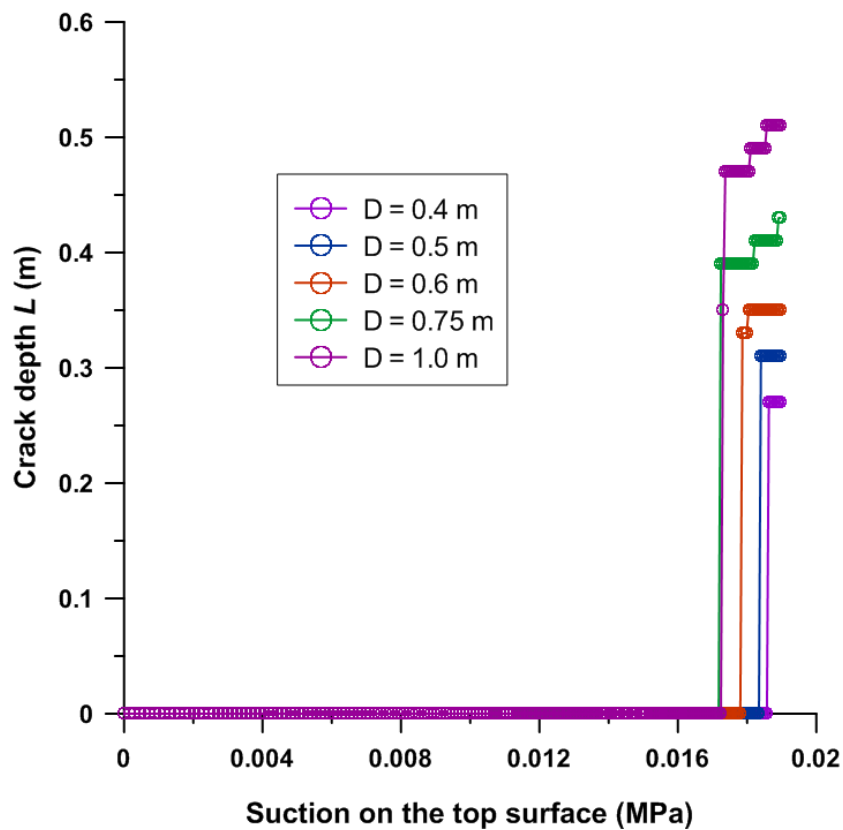
Figure 12: Deformed mesh according to the analytical and numerical results

566

567 **4.6. Crack depth versus cracks spacing**

568 To investigate the effect of the half-spacing  $D$  on the numerical results, the following values are  
569 considered: 0.4 m, 0.5 m, 0.6 m, 0.75 m and 1.0 m by using different meshes. In these tests, the same  
570 soil parameters and the same boundary conditions of the previous test are used. The numerical results  
571 show that the crack initiation occurs with a similar suction profile for all five tests ( $s = 0.016 - 0.019$   
572 MPa). This suction profile is applied to calculate the normalized dissipated energy  $\downarrow/L$  using Eq. 39.  
573 *Figure 13* presents the evolution of the crack depth numerically calculated for the five tests with  
574 different half-spacing  $D$  values. At the beginning, the crack is not initiated yet, which corresponds to  
575 the elastic phase of joint elements. When the damage criterion is reached, the crack propagates  
576 markedly to reach the ultimate depth. After this moment, the crack propagates slowly. From this  
577 figure, the ultimate crack depth in the crack initiation phase for each test can be determined by the  
578 numerical simulation.

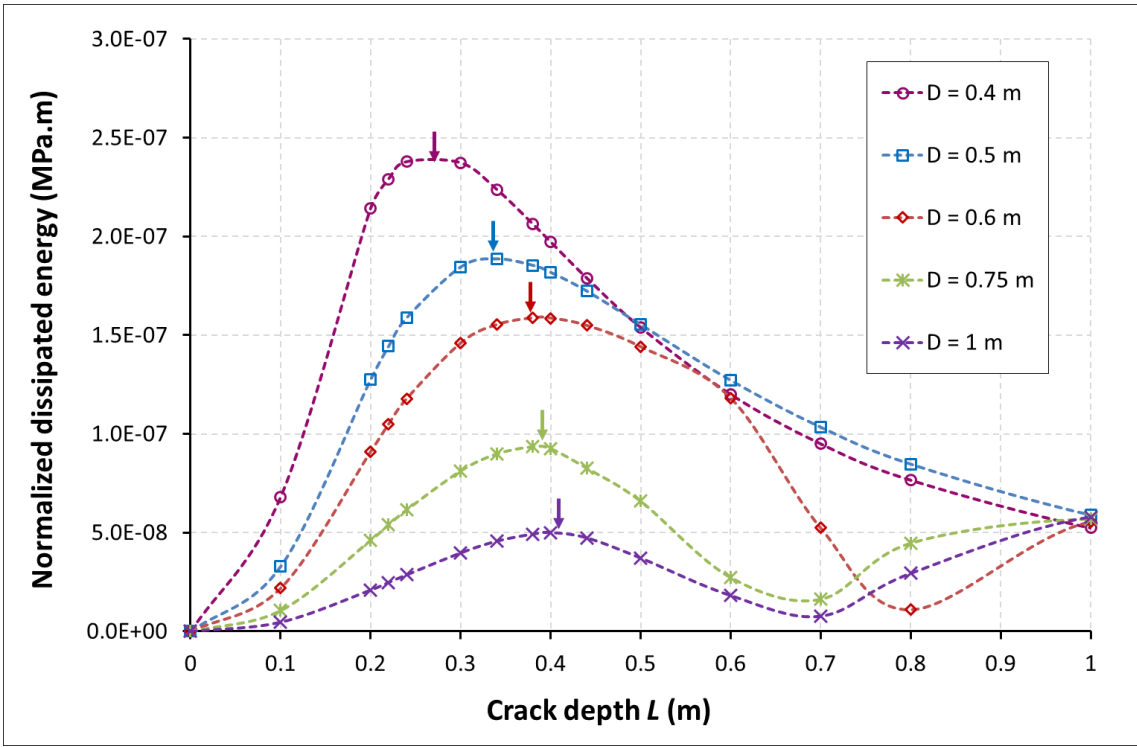
579



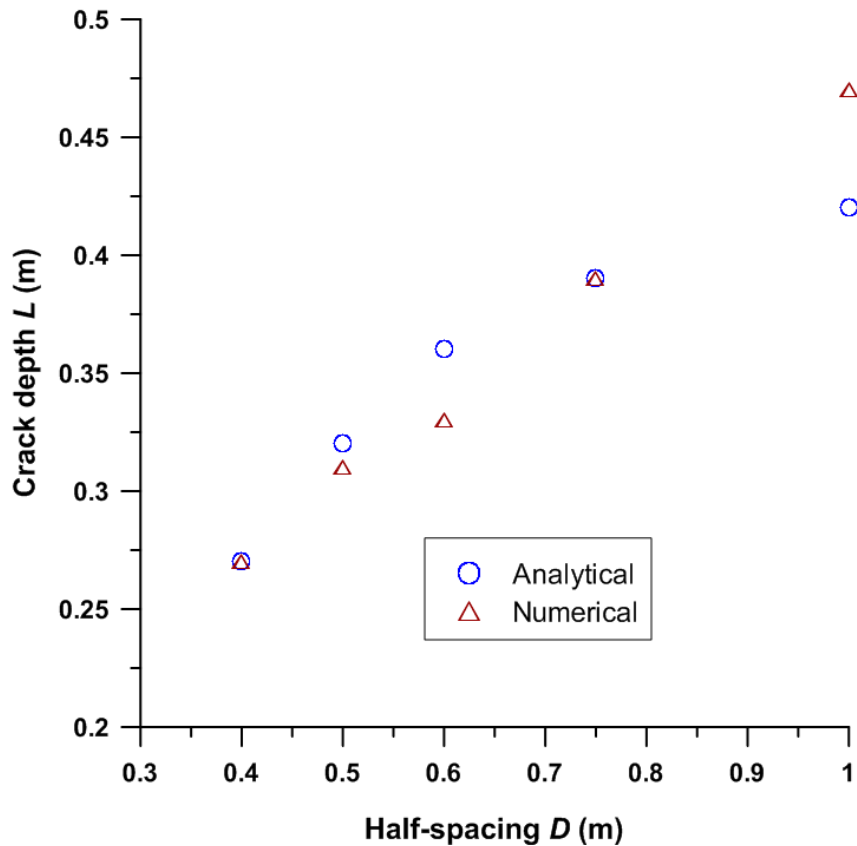
580

581 *Figure 13: Crack depth versus suction on the top surface for various half-spacing values calculated by*  
582 *the numerical simulation*

583 *Figure 14* shows the normalized dissipated energy  $\downarrow/L$  versus the crack depth  $L$  for various  $D$  values  
584 calculated by the analytical approach (Eq.(40)). These results allow determining the crack depth  
585 obtained by the energy approach if the crack depth corresponding to the peak of the  $\downarrow/L$  curve is  
586 accepted. *Figure 15* presents the crack depth for different half-spacing values calculated by the  
587 numerical simulation and the analytical (energy) approach. A good agreement between the two  
588 approaches confirms that the crack depth analytically calculated is equal to the depth corresponding to  
589 the maximal normalized dissipated energy. From this conclusion, the proposed energy approach can  
590 be used to predict the crack depth in its initiation phase with a given spacing.  
591



592  
593 *Figure 14: Normalized dissipated energy versus crack depth at crack initiation for different values of*  
594 *the half-spacing  $D$  by the analytical approach*



595

596 *Figure 15: Crack depth versus half-spacing calculated by the numerical and analytical approaches*

597

## 5. Discussions

598 The analytical study in this work is based on the approximation of the displacement field after crack  
 599 initiation by simple functions of  $x$  and  $y$ . A second-degree polynomial function of  $x$  with coefficients  
 600 depending on  $y$  is considered. The dependency on  $y$  is also restricted to a second-degree polynomial  
 601 function along the crack depth. This approximation could be improved to obtain more accurate results.  
 602 However, this simple solution seems to give reasonable values of displacement (*Figure 12*) and a good  
 603 prediction of crack depth compared to the numerical results (*Figure 15*).

604 The proposed analytical solutions allow calculating the elastic energy released by cracking which  
 605 depends on both crack depth and spacing. They allow predicting a crack depth corresponding to a  
 606 given crack spacing and vice versa. The crack spacing can be considered as the distance between two  
 607 neighboring opened cracks. Numerically, this spacing can be obtained by placing a set of cohesive  
 608 cracks in the model. Some cracks will be opened and the spacing will be detected automatically. But



609 the simulation of the multi-crack case is very challenging. In order to simplify the numerical  
610 simulation for a given crack spacing, a case of a single crack is investigated in this work. The cracking  
611 process is then limited to the initiation and propagation of a single cohesive crack located in the  
612 middle of the sample. Besides, in the present work the crack depth calculated analytically from  
613 fracture energy analysis is compared with that obtained by numerical simulation; this latter combines  
614 both the stress and energy criteria. This result is also in good agreement with the analysis of Konrad &  
615 Ayad based on LEFM [19] and in situ observation [14,16]. As mentioned above, the soil parameters  
616 and the water retention curve used in this study are identical to those used by Konrad & Ayad [19].  
617 The numerical results show that the crack initiates at a suction on the surface of 18 kPa, which is close  
618 to that observed by Konrad & Ayad [14] on the field (20 kPa). In addition, the value for the crack  
619 depth experimentally observed was 0.3 m and the theoretical value obtained in the present work is also  
620 similar (0.31 m for 1 m spacing introduced in the model).

621 The proposed energy approach was established by assuming that a set of cracks with the same depth  
622 and same spacing appears instantaneously. This simplifies the problem and allows studying only one  
623 half of a periodic cell containing a crack. In reality, the cracking pattern evolves to form sequentially  
624 different families of cracks ([8,75]). First, primary cracks are developed dividing the soil surface into  
625 cells and then subsequent drying tends to subdivide these cells in the form of secondary and tertiary  
626 crack families. In addition, the crack patterns observed experimentally by Peron et al. [34] showed that  
627 the crack network formation can result from the combination of two processes, “sequential infilling”  
628 and “simultaneous growing,” since the cracks tend to appear either successively or simultaneously.  
629 The sequential infilling occurs to create different families of cracks, but the simultaneous growing can  
630 occur within each family. The “sequential infilling” concept for desiccation cracking should be  
631 invoked only when cells of an intact material with a reduced, well-defined size can be individualized  
632 [34]. The initiation of crack is influenced by the two major factors : stress distribution and presence of  
633 flaws [28]. Therefore, the simultaneous propagation can occur in the long specimens with few flaws  
634 [34]. Crack “simultaneous growing” in large mud slabs, forming a regularly spaced crack pattern

635 within a short amount of time, is also a common observation [14,76].The present approach represents  
636 the “simultaneous growing” process which can occur and create the cracks in one family.

637 The existence of an unstable or instantaneous crack propagation, corresponding to the crack initiation  
638 phase considered in the present work, has already been observed experimentally in [9] where the crack  
639 reached the ultimate depth in a short time. This ultimate depth corresponding to the unstable  
640 propagation was also calculated by theoretical investigations [15,19]. It is interesting to note that, by  
641 using cohesive joints elements, this phenomenon is found automatically as a result of numerical  
642 simulation of the process. It is also interesting to note that the energy criterion is satisfied before the  
643 stress criterion, and thus, at the initiation moment, some energy excess is available to make the  
644 fracture propagate up to the ultimate depth. This vision conforms to Leguillon’s theory [35,77] for  
645 crack initiation conditioned by a double stress and energy criteria. However, there is a difference in the  
646 present study concerning the stress condition. In Leguillon’s theory, the stress on the entire crack  
647 length has to be greater than or equal to the tensile strength of the material. In this study, based on  
648 cohesive zone models, the damage in the joint elements, representing the crack propagation process, is  
649 triggered when the stress in the first element on the top surface of the sample reaches the stress limit.  
650 As a result, the stress condition would be satisfied when the stress reaches the limit just on the soil  
651 surface. The opening of the first element on the top surfaces modifies then the tensile stress profile  
652 inducing the crack propagation (Fig. 6). Besides, according to Amarasiri & Kodikara [24], when the  
653 available strain energy is larger than the required fracture energy for a full crack to develop, the crack  
654 can propagate to the full depth uncontrollably. They also used the cohesive crack method with  
655 softening law to model the desiccation cracking and indicated that in some cases there is more energy  
656 than needed for fracture propagation. In these cases (where the crack opening at which the stress drops  
657 to zero is small), the extra energy is likely to be converted mostly to kinetic energy, and cracks will  
658 propagate instantaneously. However, the kinetic energy is not considered explicitly in the present  
659 work.

660

## 6. Conclusions

661 In the present work, a complementary energy approach is proposed to analyze soil desiccation  
662 cracking, which is commonly investigated by a stress approach. It allows predicting the ultimate crack  
663 depth for a given crack spacing in the initiation phase by calculating the elastic strain energies before  
664 and after crack initiation.

665 The results show that the energy criterion is reached before the stress criterion. However, the crack  
666 initiates when both criteria are satisfied. The energy dissipation depends on the crack spacing and  
667 depth. When the stress condition is satisfied, the energy condition is already satisfied for a set of  
668 possible pair values (depth, spacing). The numerical analysis shows that the crack depth and its related  
669 spacing can be predicted by the maximum dissipated energy per unit crack depth. This assumption is  
670 based on numerical simulation results and requires further investigation. However, the results obtained  
671 by this assumption show good agreement for the crack depth and the displacement field after crack  
672 initiation between the analytical and numerical results.

673 Despite some limitations, the approximate analytical solution established in this study allows  
674 estimating of the crack depth for given soil parameters, as well as the crack opening and soil  
675 settlement, which are in good agreement with the numerical results. In the future work, the multi-crack  
676 numerical simulation would be considered to confirm the findings of the present work.

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680

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848 **List of symbol:**

Symbol	Definition
$\varphi$	Potential
$B$	Cracks spacing
$b$	Biot coefficient
$c$	Tangential hydraulic conductivity of crack
$C_{\text{coh}}$	Cohesion of joint intact
$\mathfrak{f}$	Linear isotropic elastic tensor
$d$	Damage variable
$D$	Half-spacing
$e$	Crack opening
$e_0$	Initial crack opening
$E$	Soil elasticity modulus
$E^{\text{after}}$	Elastic strain energy after crack initiation
$E^{\text{before}}$	Elastic strain energy before crack initiation
$E^{\text{num}}$	Numerical elastic strain energy
$\underline{f}$	Volumetric force
$g$	gravitational acceleration
$G^c$	Fracture energy dissipation rate
$k$	Soil hydraulic conductivity
$k_s$	Hydraulic conductivity of soil at saturated state
$K_I$	Stress intensity factor
$K_{IC}$	Soil's fracture toughness
$L$	Ultimate depth of crack
$\underline{n}$	Normal unit vector to the fracture surface
$N$	Biot modulus
$p$	Pore pressure



$p_0$	Final applied suction on the top surface
$S$	Degree of saturation
$S_{res}$	Residual degree of saturation
$\underline{u}$	Displacement vector
$u_n$	Normal component of displacement
$u_0$	Elastic displacement limit
$t$	Time
$\underline{T}$	Prescribed forces
$\mathbf{R}$	Joint stiffness tensor
$R_{nn}$	Normal component of joint stiffness
$\ \underline{v}\ $	Discontinuity velocity through two crack faces
$\alpha$	Constant representing desiccation rate
$\beta$	Parameter related to the material ductility
$\gamma^s$	Specific rupture energy per unit surface
$\varphi$	Friction angle
$\delta$	Kronecker or Unit tensor
$\varepsilon$	Strain tensor
$\varepsilon^0 ; \underline{u}^0$	Strain and displacement solutions (before crack initiation)
$\varepsilon^s ; \underline{u}^s$	Strain and displacement solutions (after crack initiation)
$\Downarrow$	Dissipated energy by cracking
$\lambda ; \mu$	Lamé coefficients
$\nu$	Poisson ratio
$\phi$	Porosity
$\theta, n, m$	Constants of Van Genuchten model
$\rho$	Fluid density
$\sigma$	Total stress tensor

$\sigma'$  Effective stress tensor

$\sigma_R$  Tensile strength