Prediction of Asphalt Concrete Low-temperature Cracking Resistance on the Basis of Different Constitutive Models

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Abstract

The top-down cracking of asphalt concrete pavements caused by thermal factors are very common in Poland. Cracking can occur as a result of a single intensive event (severe temperature drop) or as a result of cyclic long-term less severe events (thermal fatigue). In both cases precise constitutive modeling of materials is a key issue for rational prediction of the pavement behavior. As a starting point the Thermal Stress Restrained Specimen Test (TSRST) in which the shrinkage proceeds due to temperature reduction is analyzed and compared with experiment results for chosen mix. The TSRST is modeled using the finite element method in a frame of thermo-mechanics with the so-called weak coupling between thermal and mechanical effects. Mechanical properties are taken into account by the constitutive relations of elasticity, visco-elasticity and continuum cracking models. Between continuum cracking models special place is devoted to cohesive zone model which is a new development in fracture mechanics. Cohesive zone model in many works is presented as the only solution for rational modeling of TSRST and this notion is also addressed herein.

Keywords: constitutive modeling; thermo-mechanics; finite element method; TSRST; asphalt concretes; cohesive zone model; cracking

1. Introduction

Among all effects that can affect pavement roads, temperature's impact is one of the hardest to predict. Each year in Poland, cracks occur on important roads because of the rapidly changing temperature, particularly in winter. Actually there can be two types of effects [1]. The first one is a fatigue phenomenon in which the material's failure is due to the repetition of many temperature cycles. The second one is a single event in which one significant temperature drop causes the cracking. In this article the latter case is addressed.

The first step in rational modeling of temperature effects in pavements is to model the experiment called TSRST (Thermal Stress Restrained Specimen Test), [2]. Obtained results allow to understand the influence of constitutive

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material modeling on proper prediction of material behavior for significant temperature changes. To achieve this goal, the thermo-elasticity (or thermo-visco-elasticity) theory and the finite element method, through the dedicated software ABAQUS is used [3]. The final cracking is taken into account through cohesive zone model (CZM), [4].

2. Formulation of initial-boundary value problem for TSRST test

2.1. Thermo-elasticity boundary value problem

From the momentum and angular momentum conservation principles, the thermodynamic laws, and from the geometrical relationship i.e. the relationship between the displacement vector $\mathbf{u}$ and the strain tensor $\boldsymbol{\varepsilon}$ result the following two equations for displacement formulation of thermo-elasticity problem with transient heat flow:

$$c\,\dot{\theta} + \mathrm{T}_0 \alpha \operatorname{div}\dot{\mathbf{u}} = \lambda_0 \nabla^2 \theta\,, \quad \mu(\theta)\nabla^2 \mathbf{u} + \left(\lambda(\theta) + \mu(\theta)\right) \operatorname{grad}\left(\operatorname{div}\mathbf{u}\right) + \mathbf{f} = \alpha(\theta) \operatorname{grad}\theta + \rho(\theta)\ddot{\mathbf{u}}\,. \tag{1}$$

In equation $(1)_1$ c is the specific heat capacity (per unit volume at a constant strain), T_0 reference temperature, α coefficient of linear expansion, and λ_0 thermal conductivity. In turn, in the equation $(1)_2$ $\mathbf{f}$ is a vector of volume forces, ρ the density of the material, while the "." indicates the time derivative. Other indications of differential operations are classic in the textbooks devoted to mechanics and thermodynamics [5]. The isotropy is assumed in both mechanical and thermal properties. The mechanical properties are described using the classical Hooke's relationship (or its generalization in the case of visco-elasticity relationships using the spectral decomposition and the Prony's series concept [6]) in the following form:

$$\boldsymbol{\sigma} = 2\mu(\theta)\boldsymbol{\varepsilon} + \left(\lambda(\theta)\operatorname{tr}\boldsymbol{\varepsilon} - \alpha\theta\right)\mathbf{I}\,, \tag{2}$$

where $\theta = T - T_o$. In (2) the isothermal Lame's elastic constants are present, which can be expressed by the technical material constants using the following relationships

$$\lambda = \frac{\nu E_o}{(1-2\nu)(1+\nu)}\,, \qquad \mu = \frac{E_o}{2(1+\nu)}\,, \tag{3}$$

where ν is the Poisson's ratio, and E_o is the initial Young's modulus at a given temperature. In the case of heat flow a classic Fourier constitutive relationship for isotropic materials is assumed:

$$\mathbf{q} = -\lambda_0 \operatorname{grad}\theta \tag{4}$$

in which $\mathbf{q}$ ([Wm^{-2}]) is the heat flux vector. In the analyzed initial-boundary value problem due to the temperature the Dirichlet and Neumann type boundary conditions can be assumed [5], while in the case of mechanical fields the stress and displacement type of boundary conditions can be assumed. It is also necessary to define the initial conditions which have to be compatible with boundary conditions.

2.2. TSRST modelling

Based on analysis of the geometry, material properties and boundary conditions at the analyzed initial-boundary value problem corresponding to TSRST test it can be concluded that it is possible to use as many as three planes of symmetry Oxy, Oxz, Oyz. Finally, it can be assumed that in this case it is possible to model 1/8 of the sample marked in Fig. 1 with dimensions 15x15x100mm (the original beam has the dimensions 30x30x200mm). As a consequence of the symmetry for the nodes located on the plane ABOE displacement boundary condition u_x = 0, for plane BCFO u_y = 0, and for plane OFGE u_z = 0 were assumed. In addition, on the planes ADGE and CDGF zero stress boundary conditions were adopted. On the plane ABCD all components of the displacement vector were assumed to be zero.

In addition to the above boundary conditions of a mechanical nature, it is also necessary to formulate appropriate boundary conditions for transient heat flow. So on planes ADGE and CDGF the Dirichlet type boundary conditions in the form of $T(t) = T_0 + v_T t$ were assumed (in analysed case $T_0 = 10^oC$ and $v_T = -5^oC/3600s$), and on the planes ABOE, BCFO, OFGE and ABCD Neumann type conditions on the heat flux in the form $\mathbf{q} = \mathbf{0}$.
The region is modelled with 10x10x50 *C3D8RHT* elements for the beam, and 10x10x1 *COH3D8* elements for the cohesive zone. In cases when CZM is used the cohesive elements section with the initial thickness equal to 0.5mm at the end of the beam is applied.

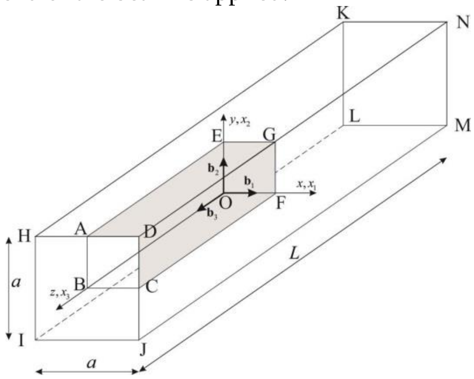

Fig 1. Modeled region and symmetry conditions for TSRST test

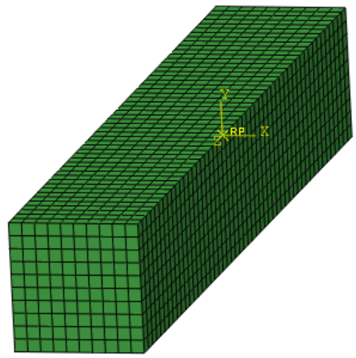

Fig 2. FEM mesh of 1/8 part of the sample

In the analyzed problem as a result of adopted boundary conditions, the decreasing temperature at the bounding planes, and consequently the inside temperature because of the heat flow causes shrinkage and the stress in the direction of the z-axis increase until breakage of the sample. In this paper four variants of the problem are analyzed: a) the sample material was taken as elastic (excluding CZM) and the temperature was varied in the whole region of the sample as homogenous, b) the sample material was taken as visco-elastic (without CZM) and the temperature was varied in the whole region of the sample as homogenous, c) the sample material was taken as visco-elastic, the temperature was varied in the whole region of the sample and CZM was created, d) the sample material was taken as visco-elastic, also heat flow and CZM were assumed, e) in that variant every assumption is the same as in (d), but the Poisson’s coefficient was changed from 0.3 to 0.4.

3. Estimation of parameters for chosen constitutive models

The material parameters for the elasticity and visco-elasticity models were determined on the basis of the experimental results presented in [2]. In paper [2] the results of the master curve determination leading to the Prony's model parameters were presented, but here they are not applied, because authors have reasonable doubts about the correctness of presented parameters estimation.

3.1. Elasticity

In case of elasticity constitutive relationships, the Poisson’s coefficient ν is assumed to be constant and equal to $\nu = 0.3$, which is a common assumption in analysis of asphalt concretes [7]. The influence of Poisson’s coefficient is also addressed in section 4.

Table 1. Value of initial Young modulus as a function of temperature

Temperature (°C)	-20	-15	-10	-5	0	5	10
Young Modulus value E_o (MPa)	19764	18426	16806	13797	10466	6976	4263

The thermal parameters were assumed on the basis of the following works [8], [9] and [10]. It was assumed that α =21.5·10^{-6} , λ_0 =0.74 $\left(W/\left(m\,^oC\right)\right)$ and c =880 $\left(J/\left(kg\,^oC\right)\right)$. The initial Young modulus like in (2) depends on the temperature and the experimental values are presented on Tab. 1.

3.2. Viscoelasticity

In the case of visco-elastic material properties as mentioned above the constitutive model described in detail in [6] was used. In order to use it, it is necessary to prepare experimental data in special way (creation of the master curve on the base of the experimental data as in [7]) and to determine the parameters for the Prony's model using nonlinear optimization methods. For this purpose, suitable programs were developed in Excel, whose results are presented below by comparing the experimental results with the results of Prony's model predictions, see Fig. 3.

For master curve creation the time-temperature equivalence principle and the WLF (Williams-Landel-Ferry) relation were applied. Here, we have assumed $-20°C$ as the reference temperature T_0. The constant value a_T is determined by the WLF law:

$$log(a_T) = -\frac{C_1(T-T_0)}{C_2+T-T_0}, \qquad (5)$$

where C_1 and C_2 are positive values which are intrinsic to the material. The next step will be to model the master curve with the Prony's series model. The general form of Young modulus expansion into the Prony's series can be written in the following form

$$E(t) = E_0 - \sum_{i=1}^{N} E_i \left(1 - \exp\left(-\frac{t}{\tau_i}\right)\right). \qquad (6)$$

For optimization it was assumed that number of series elements is $N = 8$ and relaxation times are assumed as $\tau_i = 2{\cdot}10^{i-1}$. In order to establish E_i parameters many specific methods can be applied, see. [7]. In this paper, the original approach in which the optimization process will be determined not only for E_i but also for C_1 and C_2 was used. The optimization problem is formulated as minimization of the quadratic error function f between the experimental data (denoted as $\tilde{E}(t,T)$) and the Prony's series expectations written in the following form:

$$f(E_0,..,E_N,C_1,C_2) = \sum\left(\frac{\tilde{E}(t,T) - E(\alpha_T t)}{\tilde{E}(t,T)}\right)^2 \qquad (7)$$

with summation over all experimental data points. To find optimal solution quasi-Newton method called BFGS method with Wolfe's linear searching procedure programmed in Excel was used [11]. The parameters which allow plotting prediction of the master curve by Prony's series model as shown in Fig 3 are presented in Tab. 2. In this case, normalised error for $f\left(E_0,..,E_N,C_1,C_2\right)$ was close to 0.85.

3.3. Cohesive Zone Model

Material in the cohesive zone in this study is modeled as an elastic-brittle, see [3,4]. Until the critical state of the interface is not met cohesive elements have elastic properties described by the following relation:

$$\mathbf{t} = \mathbf{K}\hat{\boldsymbol{\varepsilon}} \qquad (8)$$

where $\mathbf{t}$ is a stress vector in the interface ($\mathbf{t} = \boldsymbol{\sigma}\mathbf{n}$), $\mathbf{K}$ is the second order tensor characterizing the stiffness of the interface, and $\hat{\boldsymbol{\varepsilon}}$ is a strain vector in the interface described as follows:

$$\hat{\boldsymbol{\varepsilon}} = \varepsilon_n \mathbf{e}_n + \varepsilon_s \mathbf{e}_s + \varepsilon_t \mathbf{e}_t = \frac{\delta_n}{l_0}\mathbf{e}_n + \frac{\delta_s}{l_0}\mathbf{e}_s + \frac{\delta_t}{l_0}\mathbf{e}_t \qquad (9)$$

In (9) where $i = n, s, t$, the $\{\mathbf{e}_i\}$, is an orthonormal vector basis in the interface. Index n stands for normal direction, while s and t describe two orthogonal tangent directions. In addition δ_i are the displacements in the interface, and l_0 is the initial thickness of the cohesive elements layer. In the analyzed problem, the following stiffness values were assumed: K_{ii} =6.5MPa and K_{ij} =0.0, see [2]. The criterion for damage is assumed as the following function of the three stress vector components:

$$max\left(\frac{< t_n >}{t_n^{\max}}, \frac{t_s}{t_s^{\max}}, \frac{t_t}{t_t^{\max}}\right) = 1\,. \tag{10}$$

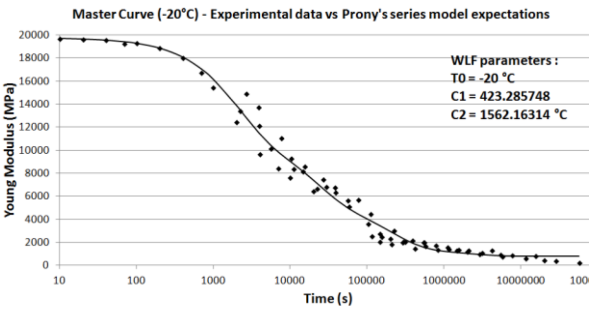

Fig 3. Master curve and experimental data shifted with WLF

Table 2. Prony's series model parameters

Index i	τ_i (s)	E_i (MPa)	$g_i = k_i = \frac{E_i}{E_0}$
1	0.2	13.9748847	0.00070708
2	2	14.1906212	0.000718
3	20	33.4424683	0.00169208
4	200	40.9668277	0.00207278
5	2 000	8234.41315	0.41663345
6	20 000	5725.45435	0.28968862
7	200 000	4243.24886	0.21469404
8	2 000 000	666.612764	0.03372835

In condition (10), $t_i^{\max}$ are the maximum value of the stress vector components, respectively determined in the tensile test in n direction and the shearing test respectively in s and t directions (here all maximum values were assumed as equal to 5.4MPa). After damage initiation it is assumed that the elastic properties of the interface defined by the scalar parameter D are degraded according to the following formulas:

$$\mathbf{t} = (1 - D)\,\bar{\mathbf{t}}\,,\; D = \frac{\delta_m^f\left(\delta_m^{\max} - \delta_m^0\right)}{\delta_m^{\max}\left(\delta_m^f - \delta_m^0\right)} \tag{11}$$

where $\bar{\mathbf{t}}$ is a stress vector evaluated from the relation (8) with the assumption that there is no damage. All appearing in (11) displacements have interpretation of so called effective displacements and their interpretation can be found in [3]. The exact parameters for the damage evolution (altogether with cracking energy) were assumed exactly like in [2].

4. Results and final remarks

The five variants of task modelling TSRST with different constitutive models presented in section 2 were solved using FEM and ABAQUS. Obtained results are presented in the form of graphs of the stress component σ_{zz} as a function of temperature, cf. Fig. 4. On the basis of obtained results the following conclusions can be formulated:

- Neglecting the visco-elastic properties of mineral-asphalt mixtures in low temperatures leads to multiple overestimations (depending on the process development) of global stiffness of material.
- Consideration of the TSRST test simulation as a transient heat flow problem in relation to the task in which temperature is changed in the whole region does not seem to be justified. However, this conclusion should not be generalized on the problem in which the layered structures subjected to thermal boundary conditions are analyzed.

- Taking into account the cohesive model with degradation of its elastic properties after crack initiation allows for rational prediction of the sample cracking moment. Its disadvantage is the fact that after the occurrence of crack it is not possible to model sharp drop of stiffness to zero.
- In the solved task with visco-elastic properties, CZM and transient heat flow the influence of the Poisson’s ratio (which in the analyzed model is assumed to be constant) is negligible.

This work is the first step on the way to the rational modeling of layered pavement structures subjected to severe thermal boundary conditions (vary low temperatures or very high gradients resulting from very rapid temperature changes). Conclusions presented above can be helpful for rational constitutive models for asphalt mixtures choice, also reducing the possibility of over-simplification. The study shows how detailed material data is needed for correct modeling of asphalt mixtures. The compatibility of the best constitutive model with experiment is satisfactory, but can be improved. Among the parameters that have a significant impact on the results and are not normally determined in the standard road laboratories are: specific heat, thermal conductivity, linear expansion coefficient, which in this work were taken as independent of temperature based on the literature. The correct determination of this parameters as a function of temperature for the analyzed material will improve the accuracy of the experiment prediction, compare also considerations contained in [8].

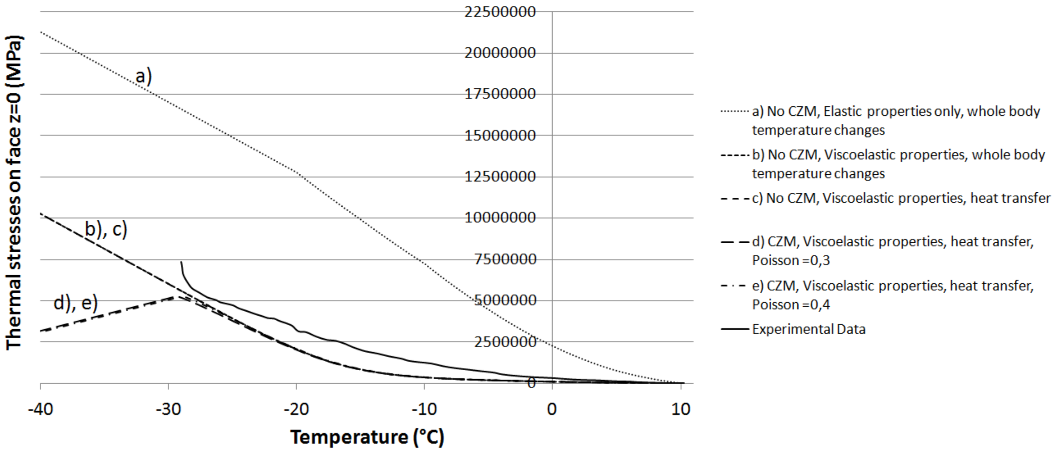

Fig 4. The stress tensor component σ_{zz} as a function of temperature for different constitutive models

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