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ABSTRACT

Multiframe image super-resolution is a technique to obtain a high-resolution image by fusing a sequence of low-resolution ones. This paper deals with a new approach to robust super resolution based on regularization framework. Since registration is an important step that ensures the success of super resolution algorithms, must choose the most suitable method. We suggest a new algorithm specified at low resolution images with small deformations using fourth-order partial differential equations (PDE) regularization in the last step of super resolution. The deformations are not parametric and differs from one image to another. We use a curvature registration specially because image are slightly deformed. Experimental results show the robustness of the proposed method compared to classical super resolution methods.

Keywords: robust, super resolution, curvature registration, PDE, MAP estimator, image restoration, regularization.

2000 Mathematics Subject Classification: 35G20, 35F20, 35J05, 49J40, 49M25.

1 Introduction

Multiframe super-resolution (SR) aim to improve the spatial resolution by combining detail of a set of low-resolution (LR) degraded images of the same scene to reconstruct a high resolution (HR) image of that scene. The degradation is modelled by different operators representing motion, blur, subsampling and additive noise. The SR process is possible if there exists a sub-pixel motion between the LR images. Therefore, each information about LR frame complete details of the original HR image. Robust Super-resolution (RSR) reconstruction is separate in three main steps: registration, interpolation and restoration. Registration is the process of estimating the deformation operator derived directly from the LR frame. In the interpolation step, the LR images are superimposed on to the HR image grid, while in the restoration step we remove noise and blur that is present in the degraded HR image.

The multiframe image super-resolution problem was first proposed by Tsai and Huang in E. Lee's 2003 (Tsai and Huang, 1984), many methods have been proposed to overcome the ill-posedness of this problem using a prior distribution on the image as a regularization (E. Lee, 2003; Milanfar, 2010; Sina Farsiu and Milanfar, OCTOBER 2004; Sanches and Marques, 2001). Regularization method is widely used to solve SR problem viewed as a maximum a posteriori approach. Popular Tikhonov regularizer was applied in SR problem in (Nguyen, October 3, 2006; E. Lee, 2003), Ng et al. used the total variation (TV) regularizer for video SR, and illustrate their efficient results using PSNR criteria (M. Ng, n.d.). Another interesting regularisation was proposed by Farsiu et al., they introduced the bilateral filter with the L_1 norm and showed that L_1 norm is more robust against noise and misregistration (Sina Farsiu, 2003; Sina Farsiu and Milanfar, OCTOBER 2004), hence in fact the name of the method RSR.

A key issue that guarantees the success of the SR algorithm is the image registration technique. The motion in LR images may include simple translations (Sina Farsiu and Milanfar, OCTOBER 2004; Term, 2007; Hiep Luong and Philips, 2006), affine (Elad, 2007; Tsai and Huang, 1984) as well as non parametric transformations (Thomas M. Lehmann and Spitzer, NOVEMBER 1999; Modersitzki, 2009; Modersitzki, 2007).

In this paper, we propose a curvature approach (Modersitzki, 2007) to register the LR frames estimating the deformations that differ between all LR images. The main of this technique is minimizing the distance between the estimation of the HR image and each upsampled LR image. In addition we propose a second PDE order regularisation in the debluring step (Lysaker, 2010; Bergounioux, 2010), since we know they success to recover smoother surfaces.
Numerical results indicate the superiority of the new SR algorithm in noise and misregistration removing compared with TV regularization. To evaluate the robustness of our algorithm we use the PSNR criteria.
This paper is organized as follows. Section 2 explains the main concepts of the SR. We justify the use of curvature registration and explain the use of second order PDE regularisation. Simulations and comparisons of our result with other classical methods are presented in Section 3.

2 Problem formulation

The observed images of a real scene are usually in low resolution due to some degradation operators. In practice, the acquired image is corrupted by noise, blur and decimation. In almost all cases the degradation is generated by inappropriate camera parameters or configuration.

In addition we have the effects of atmospheric turbulence. All these facts corrupt the resolution of the images, and improvement of resolution techniques is therefore required in those cases. We assume that the LR images are taken under the same environmental conditions using the same sensor.
The unknown and desired high-resolution image X is related to the captured low-resolution images Y_k (represented by a column vector of size M) by this formulation (Sina Farsiu and Milanfar, OCTOBER 2004):

$$Y_k = DF_kHX + E_k \qquad \forall k = 1, 2, ..., n \tag{2.1}$$

Where E_k represents the additive noise for each image, which is assumed to be zero-mean Gaussian distributed (with a standard deviation σ_k). H and D represent respectively the blurring operator of size $N \times N$ and decimation matrix of size $M \times N$. F_k is a geometric warp matrix representing a curvature transformation.

We Also suppose that the noise is independent and identically distributed (i.i.d), following a Gaussian distribution with a standard deviation σ:

$$p(E_k) = \frac{1}{(2\pi)^{\frac{N}{2}}\sigma^N} exp\{\frac{-E_k^{\intercal}E_k}{2\sigma^2}\} \tag{2.2}$$

Due to the complexity of the problem, we separate it in four steps. This is the same approach used in the RSR algorithm (Sina Farsiu and Milanfar, OCTOBER 2004).

1. Fusing the low-resolution images Y_k into a blurred HR version $B = HX$.
2. Finding the estimation of the HR image X from the blurring one B.
3. Computing the warp matrix F_k for each image using curvature registration.
4. Resolving the magnification problem.

We will detail these steps in the next subsections.

2.1 The fusing step

The first part of our algorithm is computing the blurred HR version $B = HX$. Then we use the maximum likelihood estimator (ML) which suggests the choice of $\widehat{B}$ that maximizes the likelihood function (2.6).

$$\begin{aligned} \widehat{B} &= \underset{B}{\text{argmax}}\{p(Y_k/B)\} && (2.3) \\ &= \underset{B}{\text{argmin}}\{-log(p(Y_k/B))\} \\ &= \underset{B}{\text{argmin}} \sum_{k=1}^{n} \|Y_k - DF_kB\|^2_{\mathbb{L}^2} && (2.4) \end{aligned}$$

In this case, the matrix $(DF_k)^{\intercal}(DF_k)$ is positive definite, the problem is well posed, and has a unique solution:

$$\widehat{B} = \sum_{k=1}^{n}((DF_k)^{\intercal}(DF_k))^{-1}(DF_k)^{\intercal}Y_k \tag{2.5}$$

If the problem is ill posed we use TV as a robust regularization, and the minimization problem become:

$$\widehat{B} \quad = \quad \underset{B}{\text{argmin}} \sum_{k=1}^{n} \|Y_k - DF_k B\|_{\mathbb{L}^2}^2 + \|\nabla B\|_1 \tag{2.6}$$

2.2 Deconvolution and denoising Step

Since X has been known in the presence of white noise, the vector measured Y_k is also a Gaussian one. Via Bayes rule, finding the HR image $\widehat{X}$ is equivalent to solve the minimization problem (2.7) using the Maximum a posteriori (MAP).

$$\begin{aligned}
\widehat{X}_{MAP} &= \underset{X}{\text{argmax}}\{p(X/\widehat{B})\} \\
&= \underset{X}{\text{argmax}}\{\frac{p(\widehat{B}/X).p(X)}{p(\widehat{B})}\} \\
&= \underset{X}{argmin}\{-log(p(\widehat{B}/X)) - log(p(X))\} \qquad (2.7)
\end{aligned}$$

where $p(\widehat{B}/X)$ represents the likelihood term and $p(X)$ denotes the prior knowledge in the high-resolution image. To solve this problem we need to describe the prior Gibbs function (PDF): p.

2.3 The prior Gibbs function

To describe the PDF function we us a second order TV regularisation as we know they robustness to remove noise and misregistration in smoother surfaces.

$$p(X(x)) = c.\exp\left\{-\alpha||f(|\nabla^2 X(x)|)||_1\right\} \tag{2.8}$$

Where f is a linear growth increasing function defined: $\mathbb{R} \longrightarrow \mathbb{R}^+$.
And c is a normalizing constant, guaranteeing that the integral over all x is 1.
$|.|$: is the euclidean norm of $\mathbb{R}^4$.
α: is the regularisation parameter verifying $0 < \alpha < 1$

2.4 The construction of warp matrix F_k

We obtain the warp matrix F_k for each frame trough a curvature registration algorithm, after a transition from discrete to continuous images using 2-linear interpolation. Let us denote by $Y_k(x)$ the intensity of the kth image of coordinate $x \in \Omega \subset \mathbb{R}^2$ where Ω is defined as the domain of the image. So the expression of the new continuous image $Y_k(x)$ (Modersitzki, 2007) is given by this formulation :

$$\begin{aligned}
Y_k(x) &= \sum_{k\in\{0,1\}^2} Y_k\left(\frac{E(n_1x_1)+k_1}{n_1}, \frac{E(n_2x_2)+k_2}{n_2}\right) \\
&\times \prod_{j=1}^{2}\left(\frac{(-1)^{k_j}}{n_j}(E(x_jn_j)+1-k_j-x_jn_j)\right) \qquad (2.9)
\end{aligned}$$

With:
$n = (n_1, n_2)$ is the size of Y_k and $E(x)$ is the integer part of x.
In the first step we choose arbitrarily one image Y_i from Y_k as an image of reference, and we try to find the deformations u_k between Y_i and the other images, such that:

$$Y_i(x) = Y_k(u_k(x)) \qquad \text{for } k \neq i \quad \text{and} \quad \forall x \in \Omega$$

An intuitive way to find the deformation u_k between the images is to minimize the so-called distance measure $\mathcal{D}$. Since the problem is ill-posed, we have to choose an appropriate regularization $\mathcal{R}$.
The image registration problem is now to find a minimizer u_k of the variational problem (2.10) defined by the functional $\mathcal{J}$:

$$\mathcal{J}(u_k) = \mathcal{D}(Y_i, Y_k, u_k) + \beta\mathcal{R}(u_k) \quad \text{for} \quad u_k \in \mathcal{T} \tag{2.10}$$

Where $\mathcal{T}$ denotes the set of admissible transformations, and β is the regularisation parameter.
A typical choice for the distance $\mathcal{D}$ is the mass-preserving MP measure defined as:

$$\mathcal{D}_{MP}(Y_i, Y_k, u_k) = \int_\Omega \left(Y_k(u_k(x))det(\nabla u_k(x)) - Y_i(x)\right)^2 dx \tag{2.11}$$

In this paper we use a curvature regularisation based on strain tensor. This tensor is generally defined via the displacement v_k, and we suppose that $u_k(x) = x + v_k(x)$, so $\bigtriangledown u_k(x) = I_2 + \bigtriangledown v_k(x)$.

$$S_{curv}(u_k) = \sum_{i=1}^{2} \int_\Omega (\triangle u_{k_i})^2 dx \tag{2.12}$$

The registration problem is now well defined in (2.13).

$$\min_{u_k} \mathcal{J}_{curv}(u_k) \tag{2.13}$$

With:

$$\mathcal{J}_{curv}(u_k) = \mathcal{D}_{MP}(Y_i, Y_k, u_k) + \beta S_{curv}(u_k) \tag{2.14}$$

To solve this problem, numerical schemes are required after a discretization of the domain and the objective function $\mathcal{J}_{curv}$. Since we have a curvature registration, we choose a cell-centred grid in the discretization step (Modersitzki, 2007). The objective function depends on the discretization h and defined in (2.15).

$$\mathcal{J}^h_{curv}(u^h_k) = \mathcal{D}^h_{MP}(Y^h_i, Y^h_k, u^h_k) + \beta S^h_{curv}(u^h_k) \tag{2.15}$$

To solve the minimisation problem (2.14) we use the Newton Gauss method (Jorge Nocedal, 2006), since this problem is nonlinear. We finally find the deformation $\hat{u_k}$ between the k low resolution images Y_k. We can now define easily the matrix of deformation F_k easily using $\hat{u_k}$.

2.5 Resolution of the MAP estimator problem

Since we have defined the prior function p and the operators F_k, we rewrite the equation of the MAP estimator:

$$\begin{aligned}\widehat{X}_{MAP} &= \underset{X}{argmin}\{\sum_{x\in\Omega}\|HX(x)-\widehat{B}(x)\|_1 \\ &+ \alpha||f(|\nabla^2 X(x)|)||_1\} \end{aligned} \tag{2.16}$$

where Ω contains all the pixels on the HR grid X. The norm $\|HX-\widehat{B}\|_1$ is used because it is very robust against outliers (Sina Farsiu and Milanfar, OCTOBER 2004).
To minimise the problem (2.16) we use the classical steepest descent algorithm. We finally have the HR image $\widehat{X}$ as follows.

$$\begin{aligned}\widehat{X}_{n+1}(x) &= \widehat{X}_n(x)-\alpha^{'}\{H^{\intercal}\mathsf{sing}(H\widehat{X}_n(x)-\widehat{B}(x)) \\ &+ \alpha\, div^2(g(|\nabla^2 X(x)|)\nabla^2 X(x))\} \end{aligned} \tag{2.17}$$

Where the second order divergence operator $div^2 : (\mathbb{R}^{n\times m})^4 \longrightarrow \mathbb{R}^{n\times m}$ with the adjointness property:

$$div^2 X.Y = X.\nabla^2 Y,\, \forall X \in (\mathbb{R}^{n\times m})^4,\, Y \in \mathbb{R}^{n\times m}$$

And:

$$g(s) = \frac{f^{'}(s)}{s}$$

3 RESULTS

In this section we evaluate the performance of the proposed algorithm specified at slightly deformed low resolution images. We construct a synthetic LR image to test our algorithm, and compare it with the SR using TV regularization and RSR algorithm with bilateral regularization (Sina Farsiu and Milanfar, OCTOBER 2004). The peak-signal-to-noise ratio (PSNR) is used to measure the quality of our approach. The SR algorithm is used after an curvature registration, while we use the same data for the RSR resolution and our proposed method. we choose the Lisa (the Simpson's) (1) as the original image with size 256×256.

We illustrate in figure (2) four of the $N=30$ input low-resolution frames chosen arbitrary and blurring with 5×5 Gaussian blur kernel with standard deviation equal to 1.5, and sub-sampling by a factor of 3. In addition we add a noise E_k arbitrary in each frame. The parameters chosen for our algorithm are $\alpha^{'}=0.1$, $\alpha=0.6$ and $maxiter=100$ iteration for the steepest descent, finally we choose $\beta=0.1$ the curvature registration regularization.

In figure (4d) We show the image obtained using different algorithm compared with our method. To measure the robustness of the proposed algorithm we use the PSNR in the table (3), we can clearly show the efficiency of our algorithm.

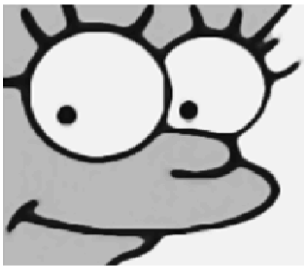


Figure 1: The original image of peppers

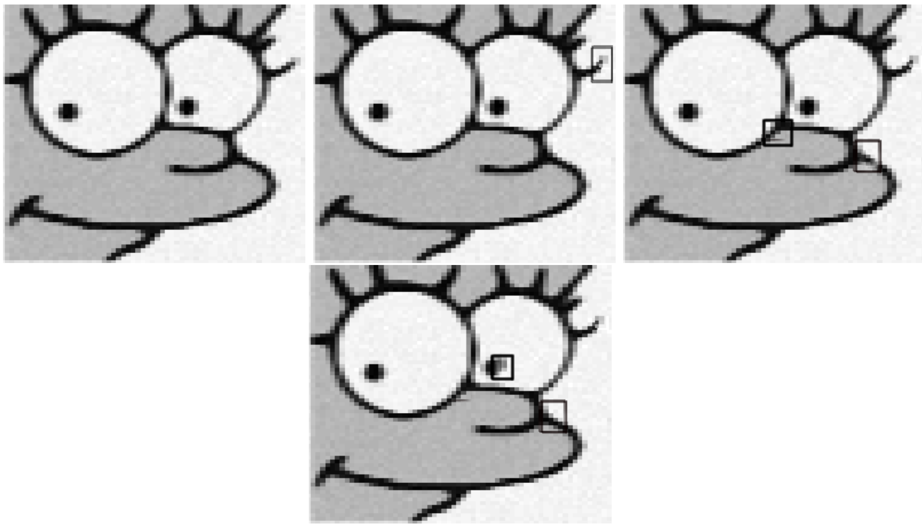


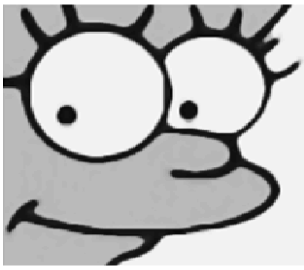
Figure 2: Four of low resolution images

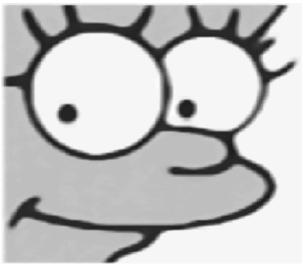
Image obtained with \ σ noise	$\sigma = 5$	$\sigma = 10$
TV Regularization	24.1	23.22
RS resolution	26.12	24.22
proposed method	27.3	26.44

Figure 3: PSNR results obtained by each method with Gaussian noise

4 CONCLUSION

In this paper, we present a new approach to the Robust SR image reconstruction problem based on curvature registration ans new PDE regularization in the restoration step. The proposed algorithm differs from the others in the registration and restoration step, here we use a non parametric curvature one. Using different images and small deformations, we can see that our algorithm has better results compared with other methods in the literature.


(a) Original image


(b) TV regularization


(c) RSR algorithm

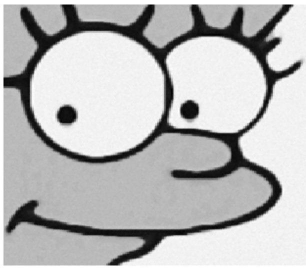

(d) proposed regularization

Figure 4: Our result compared with the classical approaches

Acknowledgment

We are grateful to the anonymous referee for the corrections and useful suggestions that have improved this article.

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