



Corrigendum to “Combining Galerkin approximation techniques with the principle of Hashin and Shtrikman to derive a new FFT-based numerical method for the homogenization of composites” [Comput. Methods Appl. Mech. Engrg. 217–220 (2012) 197–212]

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Corrigendum to “Combining Galerkin approximation techniques with the principle of Hashin and Shtrikman to derive a new FFT-based numerical method for the homogenization of composites” [Comput. Methods Appl. Mech. Engrg. 217-220 (2012) 197–212]

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Assumption 1 in our original paper can be replaced with the following, less stringent assumption.

Assumption 1. *There exists $\lambda > 0$ such that at any point $\mathbf{x} \in \Omega$, the eigenvalues of $[\mathbf{C}(\mathbf{x}) - \mathbf{C}_0]$ are greater than λ in absolute value.*

Proof of Theorem 4 requires that the local stiffness be bounded from below and above. Therefore, Assumption 2 must be altered as follows

Assumption 2. *There exists $\kappa_{\max} > \kappa_{\min} > 0$ and $\mu_{\max} > \mu_{\min} > 0$ such that at any point $\mathbf{x} \in \Omega$*

$$\kappa_{\min} \leq \kappa(\mathbf{x}) \leq \kappa_{\max}, \quad \mu_{\min} \leq \mu(\mathbf{x}) \leq \mu_{\max}.$$

Then, the end of the proof of Theorem 4 (starting from “Taking advantage of the isotropy”) must be modified as follows.

Proof of Theorem 4. [...] Taking advantage of the isotropy of both local and reference materials, the above volume averages can be expanded

$$\begin{aligned} |\Omega| \overline{\boldsymbol{\tau}^+ : (\mathbf{C} - \mathbf{C}_0)^{-1} : \boldsymbol{\tau}^+} &= \int_{\kappa(\mathbf{x}) > \kappa_0} \frac{\|\boldsymbol{\tau}^{\text{hyd}}(\mathbf{x})\|^2}{d[\kappa(\mathbf{x}) - \kappa_0]} d\Omega \\ &+ \int_{\mu(\mathbf{x}) > \mu_0} \frac{\|\boldsymbol{\tau}^{\text{dev}}(\mathbf{x})\|^2}{2[\mu(\mathbf{x}) - \mu_0]} d\Omega, \end{aligned}$$

and

$$\begin{aligned} |\Omega| \overline{\boldsymbol{\tau}^- : \mathbf{S}_0 : (\mathbf{S} - \mathbf{S}_0)^{-1} : \mathbf{S}_0 : \boldsymbol{\tau}^-} &= \int_{\kappa(\mathbf{x}) < \kappa_0} \frac{\kappa(\mathbf{x}) \|\boldsymbol{\tau}^{\text{hyd}}(\mathbf{x})\|^2}{d\kappa_0 [\kappa_0 - \kappa(\mathbf{x})]} d\Omega \\ &+ \int_{\mu(\mathbf{x}) < \mu_0} \frac{\mu(\mathbf{x}) \|\boldsymbol{\tau}^{\text{dev}}(\mathbf{x})\|^2}{2\mu_0 [\mu_0 - \mu(\mathbf{x})]} d\Omega. \end{aligned}$$

From Assumption 2, we first have

$$\begin{aligned} |\Omega| \overline{\boldsymbol{\tau}^+ : (\mathbf{C} - \mathbf{C}_0)^{-1} : \boldsymbol{\tau}^+} &\geq \frac{\int_{\kappa(\mathbf{x}) > \kappa_0} \|\boldsymbol{\tau}^{\text{hyd}}(\mathbf{x})\|^2 d\Omega}{d(\kappa_{\max} - \kappa_0)} \\ &+ \frac{\int_{\mu(\mathbf{x}) > \mu_0} \|\boldsymbol{\tau}^{\text{dev}}(\mathbf{x})\|^2 d\Omega}{2(\mu_{\max} - \mu_0)}, \end{aligned}$$

then

$$|\Omega| \overline{\boldsymbol{\tau}^- : \mathbf{S}_0 : (\mathbf{S} - \mathbf{S}_0)^{-1} : \mathbf{S}_0 : \boldsymbol{\tau}^-} \geq \frac{\kappa_{\min} \int_{\kappa(\mathbf{x}) < \kappa_0} \|\boldsymbol{\tau}^{\text{hyd}}(\mathbf{x})\|^2 d\Omega}{d\kappa_0 (\kappa_0 - \kappa_{\min})} + \frac{\mu_{\min} \int_{\mu(\mathbf{x}) < \mu_0} \|\boldsymbol{\tau}^{\text{dev}}(\mathbf{x})\|^2 d\Omega}{2\mu_0 (\mu_0 - \mu_{\min})}.$$

from which the following bound results

$$a(\boldsymbol{\tau}, \boldsymbol{\varpi}) \geq \frac{\alpha}{|\Omega|} \int_{\Omega} [\|\boldsymbol{\tau}^{\text{hyd}}(\mathbf{x})\|^2 + \|\boldsymbol{\tau}^{\text{dev}}(\mathbf{x})\|^2] d\Omega = \alpha \|\boldsymbol{\tau}\|_{\mathbb{V}}^2, \quad (49)$$

with¹

$$\alpha = \min \left\{ [d(\kappa_{\max} - \kappa_0)]^{-1}, [2(\mu_{\max} - \mu_0)]^{-1}, \kappa_{\min} [d\kappa_0 (\kappa_0 - \kappa_{\min})]^{-1}, \mu_{\min} [2\mu_0 (\mu_0 - \mu_{\min})]^{-1} \right\}.$$

The proof of the first statement is complete, since $\|\boldsymbol{\varpi}\|_{\mathbb{V}} = \|\boldsymbol{\tau}\|_{\mathbb{V}}$, and $\alpha > 0$. \square

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¹In this definition of α , it is assumed that $\kappa_{\min} < \kappa_0 < \kappa_{\max}$ and $\mu_{\min} < \mu_0 < \mu_{\max}$. The proof remains valid if any of these inequalities are not verified. For example, if $\mu_{\min} \geq \mu_0$, then, from Assumption 2, $\mu(\mathbf{x}) \geq \mu_0$ at any point $\mathbf{x} \in \Omega$. In other words, the integral over the set of points $\mathbf{x} \in \Omega$ such that $\mu(\mathbf{x}) < \mu_0$ is null. Then Eq. (49) still holds, provided that α is defined as follows

$$\alpha = \min \left\{ [d(\kappa_{\max} - \kappa_0)]^{-1}, [2(\mu_{\max} - \mu_0)]^{-1}, \kappa_{\min} [d\kappa_0 (\kappa_0 - \kappa_{\min})]^{-1} \right\}.$$