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Auguste Gires, Ioulia Tchiguirinskaia, D
 Schertzer. Multifractal comparison of the outputs of two optical disdrometers. Hydrological Sciences Journal, 2016, 61 (9), pp.1641-1651. 10.1080/02626667.2015.1055270. hal-01707378

HAL Id: hal-01707378 https://enpc.hal.science/hal-01707378

Submitted on 12 Feb 2018

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7Abstract:

8 In this paper a Universal Multifractals comparison of the outputs of two types of 9collocated optical disdrometers installed on the roof of the Ecole des Ponts ParisTech is 10performed. A Campbell Scientific PWS100 which analyses the light scattered by the 11hydrometeors and an OTT Parsivel² which analyses the portion of occluded light are 12deployed. Both devices provide the binned distribution of drops according to their size and 13velocity. Various fields are studied across scales: rain rate (*R*), liquid water content (ρ), 14polarimetric weather radars quantities such the horizontal reflectivity (*Z*_h) and the specific 15differential phase (*K*_{dp}), and DSD parameters such as the total drop concentration (*N*_t) and the 16mass-weighted diameter (*D*_m).

17 For both devices a good scaling is retrieved on the whole range of available scales (2h 18–30s), except for the DSD parameters for which the scaling only holds down to few minutes. 19For *R*, the UM parameters are found equal to 1.5 and 0.2 for respectively α and C_1 . Results 20are interpreted with the help of the classical $Z_h - R$ and $R - K_{dp}$ radar relations.

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22Key words: multifractals, disdrometer, radar

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28 Rainfall measurement with the help of disdrometers is rapidly developing for point 29measurements. The first ones were impact disdrometers (Joss and Waldvogel 1967), and now 30optical ones (Loffler-Mang and Joss 2000, Ellis et al. 2006, Battaglia et al. 2010, Frasson et 31al. 2011) are more commonly used for operational and research purposes. The great advantage 32of these devices with regards to more conventional rain gauges is that they do not measure 33only a rain rate (or rainfall depth) but information about size and fall velocity for all the 34hydrometeors passing through the sampling area whose size is few tens of cm². The Drop Size 35Distribution (DSD) can then be computed from this raw data. From the DSD it is possible to 36estimate numerous rain related fields such as the rain rate (*R*) or the liquid water content (ρ) 37or even quantities measured directly by polarimetric weather radars such as the horizontal 38reflectivity (Z_h) and the specific differential phase (K_{dp}) (Jaffrain and Berne 2012, Leinonen et 39al. 2012, Verrier et al. 2013). This widens a lot the range of applications for disdrometer data. 40 Numerous experiments have been carried out to compare the rain rate output of 41various types of collocated disdrometers along with rain gauges (Miriovsky et al. 2004, 42Krajewski et al. 2006, Frasson et al., 2011, Thurai et al. 2011). The temporal evolution 43(Thurai et al 2011) or spatial distribution (Jaffrain and Berne 2011) of total drop concentration $44(N_t)$ and mass-weighted diameter (D_m), which are commonly used to fully characterize drop 45size distribution, have also been analysed. However these comparisons are usually done only 46at a single resolution, most commonly the maximum one available (i.e. using the time series 47at the recording time step). In this paper we suggest to carry out the comparison not only at a 48single scale but across scales. Considering various scales at once enables to obtained results 49robust over a wide range of scales. To achieve this, a theoretical framework relying on scale 50 invariant properties is implemented. Available data enables to carry out analysis on scales

51ranging from 30 s to approximately 2 hours for a punctual measure. Moreover not only R, ρ , 52or N_t and D_m , but also the radar quantities Z_h and K_{dp} will be investigated. The output data of 53two disdrometers, which have been deployed for few months on the roof of the Ecole des 54Ponts ParisTech building, will be used for this study. The two disdrometers are a Campbell 55Scientific PWS100 (Ellis et al. 2006, Campbell Scientific Ltd 2012), which has seldom been 56used in such study, and an OTT Parsivel² (Battaglia et al. 2010, OTT 2014).

The variability across scales will be quantified with the help of stochastic universal 58multifractals which have been extensively used to analyze, model, and simulate geophysical 59fields extremely variable over wide range of scales such as rainfall (Schertzer and Lovejoy 601987, 1997, Marsan et al. 1996, Olsson and Niemczynowicz 1996, Harris et al. 1997, de Lima 61and Grasman 1999, Lovejoy and Schertzer 2007, Nykanen 2008, Royer et al. 2008, de Lima 62and de Lima 2009, Mandapaka et al. 2009, de Montera et al. 2009, Verrier et al. 2010). This 63framework has seldom been applied to Z_h (see Tessier et al. 1993 for an example in space and 64not in time as here), and not to K_{dp} to the knowledge of the authors.

Data and the retrieval of the various studied fields are presented in section 2 along Gowith a brief reminder of the Universal Multifractals (UM) framework and a presentation of retrieval of the Universal Multifractals (UM) framework and a presentation of R7the methodology implemented. Results are in section 4 where classical $Z_h - R$ of $R - K_{dp}$ Results are investigated with this data set, and scaling features and UM parameters estimates 69of the various fields are discussed.

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712) Data and methods

722.1) Description of the disdrometer data

The data used in this paper was collected between 27 September 2013 and 18 January view disdrometers installed on the roof of the Ecole des Ponts ParisTech building. They are both made of a transmitter that generates one or several laser sheet(s) and

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76receiver(s). The Campbell Scientific PWS100 computes size and fall velocity from light 77refracted by the hydrometeors (Ellis et al. 2006, Campbell Scientific Ltd 2012) whereas the 78OTT Parsivel² relies on occluded light (Loffler-Mang and Joss 2000, for an initial version; 79Battaglia et al. 2010, OTT 2014). The 50 recorded events of this period are used in this study. 80The criteria defining an event is a rainy period during which more than 1 mm is collected and 81that is separated by more than 15 min of dry conditions before and after. The main output of 82the disdrometers is a matrix with the number n_{ij} of drop recorded according to classes of 83equivolumic diameter (index *i*, and defined by a centre D_i and a width ΔD_i expressed in mm) 84and terminal fall velocity (index *j*, and defined by a centre v_i and a width Δv_j expressed in 85m.s⁻¹). This matrix is recorded for each 30s time step (Δt). Gires et al. (2014) which used the 86same data set noticed that the oblateness of drop was not properly taken into account in the 87PWS100 rationale and suggested a correction which is used here. Furthermore, as suggested 88by various authors (Jaffrain and Berne 2011, Kruger and Krajewski 2002, Thurai and Bringi, 892005) all the drops whose velocity was more than 60% different from what was expected by 90Beard's model (Beard 1977) according to its size were removed because considered as non 91meteorological measurements.

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93The rain rate for each time step is then computed as:

$$94 R_{PWS/Pars} = \frac{\pi}{6\Delta t} \sum_{i,j} \frac{n_{i,j} D_{PWS/Pars,i}}{S_{eff}(D_i)}$$
(1)

95 where $S_{eff}(D_i)$ is the sampling area of the device which is slightly modified 96according to the drop size to take into account potential edge effects for large drops. For the

97Parsivel² we have $S_{eff}(D_i) = L\left(W - \frac{D_i}{2}\right)$, where L = 180 mm and W = 30 mm are respectively 98the length and width of the sensing area (LW = 54 cm²) (OTT 2014). The PWS100 is not 99subject to this issue and S_{eff} is taken as constant equal to 40 cm² (Campbell Scientific Ltd 1002012). For the selected events the total rainfall depth collected is equal to 172 mm for the 101PWS100 and 154 mm for the Parsivel². A tipping bucket rain gauge is also located on the 102same roof and collected roughly 170 mm which is more in agreement with the PWS100, but it 103should not be "over-interpreted" given that its accuracy is not very high due to high rain rates 104and low level of maintenance at the beginning of the recording period. The normalized bias 105between the two disdrometers (computed for the time steps where R > 1mm/h) is equal to 1060.11; the correlation equals 0.96; the Nash-Sutcliffe efficiency coefficient equals 0.89 and the 107RMSE is 1.83. Although not negligible the differences between the two disdrometers are 108rather low compared to what is commonly observed (Miriovsky et al. 2004, Krajewski et al. 1092006, Frasson et al., 2011, Thurai et al. 2011).

110 A discrete drop size distribution (DSD) is computed from the available data as:

$$111 N(D_i) = \frac{1}{S_{eff}(D_i)\Delta D_i \Delta t} \sum_j \frac{n_{i,j}}{v_j} \quad (2)$$

112The number of drops with a diameter in the class *i* per unit volume (in m⁻³) is given by 113 $N(D_i) \Delta D_i$. As it is commonly done, we use the total drop concentration N_t (m⁻³) and mass-114weighted diameter D_m (mm) to characterize the DSD. In this framework the DSD is written as 115 $N(D) = N_t f(D_m)$; where it appears that D_m characterizes the shape of the DSD and N_t its 116total intensity. These two parameters are defined as (Jaffrain and Berne 2012a, Leinonen et al. 1172012):

$$118 N_t = \int_{D_{\min}}^{D_{\max}} N(D) dD \tag{3}$$

$$119 D_m = \frac{\int_{D_{\min}}^{D_{\max}} N(D) D^4 dD}{\int_{D_{\min}}^{D_{\max}} N(D) D^3 dD}$$
(4)

120For the practical computation of these quantities (and the other ones), the integral is replaced 121by a summation over all the classes of diameter, i.e.:

$$122 \int_{D_{\min}}^{D_{\max}} f(D) N(D) dD \approx \sum_{i} f(D_{i}) N(D_{i}) \Delta D_{i}$$
(5)

Lastly the liquid water content (ρ in g.m⁻³) the horizontal reflectivity (Z_h in mm⁶.m⁻³) 124and the specific differential phase (K_{dp} in °.km⁻¹) are estimated with the help of the DSD from 125which they are basically various moments. These physical parameters are given by Jaffrain 126and Berne (2012a) and Leinonen et al. (2012):

$$127 \,\rho_{l} = \rho_{w} \frac{1}{10^{3}} \frac{4}{3} \pi \int_{D_{\min}}^{D_{\max}} N(D) \left(\frac{D}{2}\right)^{3} dD \tag{6}$$

128With ρ_{w} , the liquid water density in g.m⁻³,

$$129^{Z_{h,v}} = \frac{\lambda^4}{\pi^5 \left| \frac{m^2 - 1}{m^2 + 1} \right|} \int_{D_{\min}}^{D_{\max}} N(D) \sigma_{B;h}(D) dD$$
(7)

$$130 K_{dp} = \frac{10^{-3} 1800 \lambda}{\pi} \int_{D_{\min}}^{D_{\max}} \operatorname{Re}[S_{hh}(D) - S_{vv}(D)] N(D) dD$$
(8)

131Where $\sigma_{B;h}$ (in mm²) is the backscattering cross session at horizontal polarization, and 132Re(S_{hh/vv}) (in mm) is the real part of the forward scattering amplitude at horizontal/vertical 133polarization, λ is the radar wavelength (in mm) and *m* the complex refractive index of water. 134The scattering coefficients were computed with the help of the Python PyTMatrix library 135(Leinonen 2014) which relies on the T-Matrix code by Mishchenko et al. (1996). 136Computations were carried out for a radar wave length equal to 53.5 mm corresponding to C-137band radars (this wave length was chosen to facilitate comparison with other studies, because 138it is the most widely used and corresponds to the radar currently mostly used by Western 139European meteorological services), a temperature of 20°C (*m*=8.633 +1.289i) and an oblate 140spheroids model for drop shape with an axis ratio - equivolumic diameter relation 141corresponding to the one implemented in the Parsivel² rationale (Battaglia et al. 2010). The set 142up considered for drop orientation was the same as in Leinonen (2012); i.e. drops are partially 143aligned and a normal distribution (mean and standard deviation respectively equal to 0° and 1447°, in agreement with the findings of Bringi et al. 2008) characterizes the angle of the 145symmetry axis.

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1472.2) Methodology: UM framework

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149 The key elements of the theoretical framework of UM are presented here and the 150reader is invited to refer to Schetzer and Lovejoy (2011) for a recent review. In the following 151 ε_{λ} denotes a conservative field at resolution λ , defined as the ratio between the observation 152scale *l* and the outer scale *L* (λ =*L*/*l*). Practically ε_{λ} is obtained simply by up-scaling (averaging 153consecutive time steps) the measured field at the maximum resolution. If ε_{λ} is a multifractal 154field, then its statistical moment orders scale with resolution as:

 $155\left\langle \boldsymbol{\varepsilon}_{\lambda}^{q}\right\rangle \approx\boldsymbol{\lambda}^{K(q)} \quad (9)$

156Where K(q) is the moment scaling function that fully characterizes the variability across 157scales of the field ε_{λ} . The quality of the scaling is investigated with the help of the Trace 158Moment (TM) analysis which simply consists in plotting equation 9 in log-log, the slope of 159the obtained straight line being K(q).

160 Most multiplicative processes converge toward Universal Multifractal which are fully 161characterized with the help of only two scale invariant parameters C_1 and α (this a broad 162generalization of the central limit theorem, Schertzer and Lovejoy 1987, 1997). 163 C_1 is the mean intermittency co-dimension and measures the clustering of the (average) 164intensity at smaller and smaller scales (C_1 =0 for a homogeneous field). α is the multifractality

165index ($0 \le \alpha \le 2$) and measures the clustering variability with regards to intensity level. In 166this specific framework which is implemented here *K*(*q*) is given by:

167
$$K(q) = \frac{C_1}{\alpha - 1} (q^{\alpha} - q)$$
 (10)

168 The UM parameter are estimated in this paper with the help of the Double Trace 169Moment (DTM) technique (Lavallée et al. 1993).

170 A common framework to deal with a non-conservative field ϕ_{λ} (i.e. we have $\langle \phi_{\lambda} \rangle \neq 1$) 171 is to assume it can written as

$$172\,\phi_{\lambda} = \varepsilon_{\lambda}\,\,\lambda^{-H} \tag{11}$$

173where *H* is the non-conservation parameter (*H*=0 for conservative fields), and ε_{λ} a

174conservative field characterized by a moment scaling function $K_c(q)$ depending only on UM 175parameters C_1 and α . More physically, this non-conservativeness means that the studied field 176 ϕ_{λ} (the observations here) exhibits stronger correlations than the ones obtained with the help 177of a simple multifractal cascade and an additional fractional integration is needed to represent 178it. The moment scaling function K(q) of ϕ_{λ} is given by:

$$179 K(q) = K_c(q) - Hq$$
 (12)

180 *H* can be estimated with the help of (Tessier et al. 1993):

$$181\,\beta = 1 + 2H - K_c(2) \quad (13)$$

182where β is the spectral slope. It is the exponent of the power law that characterizes over large 183range of wave numbers the power spectrum of a scaling field:

$$184 E(k) \propto k^{-\beta} \qquad (14)$$

185 Before going on let us clarify the relations between the notions of stationarity and 186conservation. For stochastic processes, stationarity refers to the fact that given statistics are

187 invariant with respect to time translations. When no given statistics are mentioned, it usually 188corresponds to the strongest case of stationarity, i.e. the probability itself is time translation 189 invariant and therefore all the statistical moments are stationnary The classical case of 190"second order stationarity" is presumably the most cited case and is particularly important for 191additive processes. It corresponds to time translation invariance of the second order moments 1920f the increments. For multiplicative processes, conservation means that a given statistic is 193strictly independent from scale, in general this statistic is the mean of the field (H=0 in Eq. 19411). The latter implies a given form of stationarity because at all scales the fluctuations remain 195around this mean. However, these fluctuations are not only easily wilder than for an additive 196process, but their amplitude generally increases with smaller and smaller scales. Conversely, 197non-conservation implies a strong non stationarity, starting with the fact that the mean has a 198scaling behaviour (Eq. 11). For instance, a spectral analysis will detect the non conservation ($199 H \neq 0$), due to the departure of the spectral slope, which is a second order statistic, from that 2000f "a pink noise" (k^{-1}), where the exponent one corresponds here to the dimension of the 201embedding space (time series are studied here). With multifractal fields there is a further 202correction with the $K_c(2)$ (Eq. 13) which is not negligible (in the range 0.1-0.4 for the 203retrieved UM parameters).

204 The TM and DTM techniques are designed for analysing conservative fields (*H*=0) 205and remain reliable as long as *H*<0.5. In case of greater *H*, they should be implemented not on 206 ϕ_{λ} , but on the underlying conservative field ε_{λ} . A fractional integration of order *H* (equivalent 207to a multiplication by k^{H} in the Fourier space) is theoretically required to estimate ε_{λ} from ϕ_{λ} . 208However a common approximation (Lavallée et al. 1993) which provides reliable results 209consists in taking ε_{Λ} at the maximum resolution simply equal to the renormalized absolute 210value of the fluctuations of the field i.e.:

$$211 \varepsilon_{\Lambda} = \frac{\left|\phi_{\Lambda}(i+1) - \phi_{\Lambda}(i)\right|}{\left\langle \left|\phi_{\Lambda}(i+1) - \phi_{\Lambda}(i)\right|\right\rangle}$$
(15)

212and then upscaling this field at other resolutions λ .

Spectral and multifractal analyses are implemented on ensemble average over various 214samples, i.e. each sample is considered as a realization of the process. For example it means 215that each sample is up-scaled and taken to the power *q* independently before taking the 216average in equation 9. The studied samples are extracted in the following way from the 217selected events: for each event (i) a sample size is chosen (necessarily a power of two); (ii) the 218maximum number of samples for this event is computed; (iii) the portion of the event of 219length equal to the sample size multiplied by the number of samples found in (ii) with the 220greatest cumulative depth is extracted; (iv) the extracted series is cut into various samples.

Given that the sample size is a power of two there is obviously some data lost during 222the selection process. Here the percentage of available data actually used is equal to 36, 65, 82 223and 91% for samples of size 512, 256, 128 and 64 times steps of 30s respectively. The chosen 224sample size should result from a trade off between the width of the available range of scales 225which should be as great as possible, and the amount of wasted data which reduces the 226reliability of the estimates. Here we chose to analyse the scaling properties with samples of 227size 256 (approximately 2h, 46 samples available), and given that no scaling break was 228identified on the main studied fields, UM parameters were estimated with samples of size of 22964 (approximately 30 min, 259 samples available) to benefit from the use of more data. 230Finally it should be mentioned that since D_m is not defined when there are no drops, only the 231samples containing drops at all time steps are used for this field.

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2333) Results and discussion

2343.1) Standard radar relationships

Given that it was possible to retrieve both rain rates and radar parameters, it is possible analyse the validity, for this data set, of the standard power law relations that are commonly between these quantities:

$$238 Z_h = a R^b \quad (16)$$

$$239\,R = cK_{dp}^{\ d} \qquad (17)$$

240These relations will be used in the following sections to help in the interpretation of the 241observed scaling features exhibited by each quantity. Various authors (Campos and Zawadzki 2422000, Jaffrain and Berne 2012b, Verrier et al. 2013) noticed a strong sensitivity of the 243 estimates of *a* and *b* on the method implemented to compute them. Here we performed an 244orthogonal linear regression, which does not assume any dependent variable, on the logs of 245 Z_h , *R* and K_{dp} . The regressions are performed only for the time steps for which $K_{dp} > 10^{-2}$ 246because we noticed that small values in which we are not interested had a strong influence on 247the retrieved parameters. Figures 1.a and 1.b display the regressions for the two relations and 248both devices, and the retrieved values for *a*, *b*, *c* and *d* are shown in Table 1. It appears that the 249quality of the fitting is comparable for the two relationships and slightly better for the 250PWS100 than for the Parsivel² ($r^2 \sim 0.8$ vs. 0.75). The estimates are similar for both devices. 251More precisely for the $Z_h - R$ relation *a* is slightly greater and *b* slightly smaller for the 252PWS100 that for the Parsivel², meaning that the effects of each other are compensating. For 253the $R - K_{dp}$ relationship *c* and *d* are both slightly greater for the PWS100, meaning a given 254 value of K_{dp} will systematically yield greater estimates of *R* with the PWS100 values. The 255estimates of the exponents are in the range of those commonly observed (Jaffrain and Berne 2562012b, Figuras I ventura et al. 2013, Verrier et al. 2013). With regards to *a* values they are in 257 agreement with those found by Verrier et al. (2013), and in the upper range of those reported 258by Jaffrain and Berne (2012b). The *c* values are similar to those found by Jaffrain and Berne 259(2012b). Relations 16 and 17 are studied only at the maximum resolution (30 s) and the strong 260scale dependency of the parameters *a*, *b*, *c* and *d* is not investigated here (see Verrier et al. 2612013 for an analysis of this issue for $Z_h - R$ relation). Parameters are computed here taking 262into account all the events at once, meaning that "climatic" (keeping in mind only 4 months of 263data are used) estimates are studied. An event based analysis will be discussed in future 264works. It is indeed not needed for the purpose of this paper, which is to quantify the scaling 265variability observed by the two disdrometers on various fields.

Finally the influence of using the retrieved "climatic" parameters for computing rain 267rates from radar parameters is assessed. With this purpose, the rain rates computed either 268directly from raw data (*R*) or through the radar relations 16 and 17 (respectively R_{Z-R} and 269 $R_{R-\kappa_{dp}}$) once Z_h and K_{dp} have been estimated with raw data are compared. Figure 1.c and 1.d 270display, for the PWS100 data, a scatter plot for the $Z_h - R$ and $R - K_{dp}$ relations 271respectively. Similar curves are obtained for the Parsivel² data and not shown here. The 272scattering around the bisector is not negligible with a 30 s time step, and is more pronounced 273for the $Z_h - R$ than for the $R - K_{dp}$ relation which means that the use of "climatic" values is 274less acceptable for the former. It should also be noted that there is a tendency of 275underestimating large rain rate with the $R - K_{dp}$ relation, indeed $R_{R-\kappa_{dp}}$ is systematically 276smaller than *R* for *R*>50 mm/h. The total rainfall depths between the three techniques are very

277similar with roughly 3% or less differences.

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2793.2) Scaling behaviour

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The scaling features of the various fields are studied in this section. Figure 2.a displays 282the spectral analysis (i.e. equation 14 in log-log plot) of the rain rate for the PWS100. A very 283good scaling (i.e. a straight line) is observed on the whole range of available scales (30s – 2h).

284The spectral slope is greater than the embedding dimension of the field (1 for time series) 285suggesting that *R* is non-conservative (this will be confirmed in the next session with the 286estimation of *H*). A practical consequence is that the TM analysis which assumes a 287conservative field should not be implemented on the field itself but only on its conservative 288part which can the approximated by the absolute value of its fluctuations (equation 15). The 289results are displayed on Fig. 2.b, where the good scaling with a unique regime is confirmed. 290The r^2 for q = 1.5 which is taken as an indication of the quality of the scaling is greater than 2910.99. The same analysis carried out directly on the field yields non aligned points with a 292flattening for small scales ($r^2 = 0.94$ for q = 1.5). Same curves for spectral and TM analyses 293for the Parsivel² data are shown in Fig. 2.c and 2.d respectively. It appears that very similar 294results are found for the Parsivel² data. Similar curves for both devices are also obtained for 295the other studied quantity (ρ , K_{dp} , Z_h , N_b , D_m), therefore in the following only the curves for 296PWS100 will be showed and discussed in this sub-section.

Very similar results are found for ρ and K_{dp} (Fig. 3) with a very good scaling on the 298whole range of available scales on both the spectra and the TM analysis, which also has to be 299conducted on the fluctuations of the field. The good scaling behaviour retrieved for K_{dp} is not 300surprising and was actually expected if relation 17 is correct. Indeed a power (K_{dp} here) of a 301multifractal field (R here) should also behave as a multifractal field (this is the basic concept 302behind the DTM technique). To the knowledge of the authors, the multifractal behaviour of 303 K_{dp} has not yet been studied in time with disdrometer data this way, and it opens new 304perspectives. A potential one would be to compare these outputs with similar analysis 305performed in space with data provided by weather data. This would enable to study scaling 306relations in a spatio-temporal framework with a quantity directly measured by the radar (so 307far the only device providing "rather" high resolution space-time data of rain related fields) 308without having to rely on tailored relations that may introduce biases in the scaling behaviour 309as it is the case for the rain rate.

310 The situation for Z_h is more complex. Indeed the energy spectrum (Fig. 4.a) is not 311linear and could be interpreted as exhibiting two breaks, one at roughly 6 min⁻¹ and the other 312one (the minimum on the curve) at roughly 2 min⁻¹. Authors do not have explanation for this 313behaviour which is not retrieved on *R* and K_{dp} . It was not expected and suggests that the 314" climatic" relation 16 does not hold very well. Indeed if it was true, a good scaling behaviour 315would be observed on the spectra. To confirm this, the same analysis were carried out on the 316 quantity aR^b , where *R* is the rain rate studied before and *a* and *b* are the "climatic" values 317estimated in section 3.1. As expected for this analysis a good scaling behaviour is indeed 318retrieved on both the spectra (Fig. 4.c) and the TM analysis (Fig. 4.d). It should be mentioned 319that the greater scattering of the points for the $Z_h - R$ than the $R - K_{dp}$ relation observed on 320Fig. 1 is consistent with the fact that we found a good scaling behaviour on R and K_{dp} and not 321*Z*_h. Quite surprisingly the scaling breaks observed on the spectra are not visible on the TM 322analysis (Fig. 4.b) where a unique regime is observed. It was implemented on the fluctuations 323of the field (Eq. 15) for which there are a slightly better scaling than for the field itself. 324However given the lack of scaling observed in the spectral analysis, the TM analysis might be 325not very reliable and should not be over-interpreted.

Finally we analysed the scaling features of N_t and D_m which are used to characterize 327the shape of the DSD. For N_t we find a good spectral behaviour but limited to the range 2 min 328– 2 h, as a flattening of the spectra is observed for small scales (Fig. 5.a). Similarly to the 329previously studied fields, a spectral slope greater that one is found meaning that the TM 330analysis should not be performed on the field directly but on its fluctuations (equation 15). It 331is displayed in Fig. 5.b and it appears that a good scaling behaviour is retrieved on a range of 332scales (4 min – 2 h) similar to the one observed on the spectra (Fig. 5.a). Considering the

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333whole range of scales would lead to a r^2 for q = 1.5 equal to 0.96 whereas is of 0.99 on the 334limited range of scales. Similar scaling regimes are observed on D_m 's spectra (Fig. 6.a), but 335the slope is much lower meaning that the TM analysis should be conducted directly on the 336field. It yields a good scaling behaviour with a r^2 for q = 1.5 equal to 0.99 (it is of 0.87 if the 337TM analysis is conducted on the fluctuations) (Fig. 6.b). The flattening of the spectra (almost 338a horizontal slope) for small scales corresponds to what would be observed for a white noise. 339This would suggest that N_t and D_m exhibit a scaling structure down to few minutes and behave 340as a random homogeneous variable for smaller scales. It is not possible to confirm this 341interpretation with the help of this data. More data at higher resolution, which would extend 342the range of available small scales, would be needed to achieve this. A possible interpretation 343of this could simply be that the sampling uncertainty is more visible at smaller scales and for 344these quantities than the other ones.

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3463.3) UM parameters

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Estimates of UM parameters for the various studied fields and the two devices are 349reported in Table 2. It is timely to mention that the proportion of zeros is quite low (12 %, 350with a fractal dimension of 0.96), which means that estimates are not biased by the 351multifractal phase transition for small moment orders which is associated with them (see 352Gires et al., 2012 for a in-depth analysis of this effect). Given the low quality of the scaling in 353the spectral analysis for Z_h , the values of β and H are not shown for this field since they are 354not reliable.

355 For all the fields we find *H* values greater than 0.5, except for D_m , which confirms that 356the studied fields are non-conservative, and that the UM parameters α and C_1 should indeed 357be estimated on their fluctuations as it was done. The estimates of the characteristic scaling

358parameters are very similar for the two devices. It means that despites a roughly 10% 359difference in terms of rain rate; they both record the same variability across scales. The 360estimates for the rain rate, which is the field that has been mostly studied, are comparable 361 with the ones usually found for this range of scales by authors focusing the analysis on the 362rainy portions (de Montera et al. 2009, Mandapaka et al. 2009, Verrier et al. 2010, Gires et al. 3632013), although with slightly smaller values of α which were commonly reported to 1.8. With 364regards to $D_{\rm m}$ and $Z_{\rm h}$ the differences between the two devices are more pronounced but it is 365harder to interpret this fact given that the lower quality of the scaling for these fields implies 366less reliable estimates.

367 It is possible to give an insight into radar relations 16 and 17 through the UM 368parameters estimates. Indeed if a field is multifracal, then as previously mentioned a power of 369it is also multiractal, and there is furthermore a relation between the α and C_1 (Tessier et al.

3701993, Lovejoy et al. 2008) More precisely, if $R = cK_{dp}^{d}$, we have:

(18.b)

$$371 \alpha_{R} = \alpha_{K_{dp}}$$
 (18.a)
 $372 C_{1,R} = d^{\alpha} C_{1,K_{dp}}$ (18.

373Here for both devices we find a roughly 0.2 difference between α computed for the two fields. 374With regards to C_1 , for the PWS100 data $C_{1,R} / d_{PWS}^{\alpha_R} = 0.29$ (the value is slightly smaller 375 when considering $\alpha_{K_{dp}}$ in the relation) which is in rather good agreement with the 0.23 376 retrieved on the K_{dp} field. For the Parsivel² data $C_{1,R} / d_{Pars}^{\alpha} = 0.35$ which yields a value 377 greater than the 0.25 retrieved on the K_{dp} field. The same computations can be carried out for 378the $Z_h - R$ relation. Indeed if $Z_h = aR^b$, one expects:

$$379\,\alpha_{Z_h} = \alpha_R \qquad (19.a)$$

 $380 C_{1,Z_h} = b^{\alpha} C_{1,R} \qquad (19.b)$

381The difficulty here is that the agreement between the estimates of α is worse than for the 382 $R - K_{dp}$ relation and the scaling on Z_h is not very good and reliable. With regards to C_1 we

383have $b_{PWS}^{\alpha_{PWS,R}}C_{1,R_{PWS}} = 0.32$ (a slightly greater value is found using α_{Z_h}) which is quite

384close to 0.34 found on Z_h , and $b_{Pars}^{\alpha_{Pars}} C_{1,R_{Pars}}^{\alpha_{Pars}} = 0.39$ (a slightly greater value is found 385using α_{Z_h}) which is quite different from the 0.51 found on Z_h . Finally it should be mentioned 386that the estimates of α and C_1 on the field $a_{PWS} R_{PWS}^{b_{PWS}}$ (see Fig. 4.c and 4.d for scaling 387curves) are respectively 1.55 and 0.28 which is roughly in agreement with equation 19 (388 $b_{PWS}^{\alpha_{PWS}} C_{1,R_{PWS}} = 0.32$). The differences noticed with regards to the underlying theoretical 389framework highlight the limitation of the "climatic" relations 16 and 17. The better agreement 390in equations 18 and 19 for the PWS100 data is consistent with the greater quality of the fitting 391of relations 16 and 17 for the PWS100 than for the Parsivel² (Fig. 1). The fact the α are equal 392for R, ρ and N_t also suggests that a power-law relation between these quantities could be 393investigated, which is not surprising since these quantities correspond to various moments of 394the DSD (Sempere-Torres et al. 2000, Lee et al. 2004, Lovejoy and Schertzer 2008, Verrier et 395al. 2013), even more directly than Z_h and K_{dp} , but this is beyond the scope of this paper.

396

3974) Conclusions

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In this paper the output data provided by two optical disdrometers is analysed; the 400Campbell Scientific PWS100 based on the analysis of the light refracted by drops and the

401OTT Parsivel² based on the analysis of light occluded by drops. Not only the rainfall rate, but 402also the DSD parameters N_t and D_m , and the radar parameters Z_h and K_{dp} are studied. 403Furthermore the analysis is not performed only at the maximum resolution as it is commonly 404done but across scales with the help of the theoretical framework of Universal Multifractals.

It appears that despite a roughly 10 % differences in terms of rain rate, the multifractal 406analysis yield very similar results for the two measuring devices, with a slightly worse scaling 407observed on the Parsivel² data. *R* exhibits a very good scaling on the whole range of available 408scales (30 s – 2h) with *H*=0.7, *C*₁=0.2 and α =1.5 which confirms findings of previous studies. 409A very good scaling is also retrieve on *K*_{dp} and UM parameters estimates are in rather good 410agreement with what would be expected if the standard relation *R* = *cK*_{dp}^{*d*} is implemented 411with "climatic" parameters computed for this data set. The scaling of *Z*_h is worse, especially 412on the spectra, which highlights some limitations of the power law relation *Z*_h = *aR*^b. Finally 413the scaling behaviour only holds on the range few min – 2 h for *N*_t and *D*_m, and there are some 414hints at a possible random uniform behaviour for smaller scales possibly associated with 415sampling uncertainty.

These results suggest new ways of comparing the outputs of disdrometers by using 417other fields than the rain rate, and also scaling analysis. The results are particularly promising 418for K_{dp} which is also directly measured by polarimetric weather radars, contrarily to the rain 419rate for which non-trivial transformations potentially biasing the observed scaling are 420implemented. Multifractal investigations in a spatio-temporal framework on K_{dp} radar data 421should be carried out to improve knowledge about rainfall as a space-time process. More data, 422including spatial ones, should be analysed to confirm the possibility of characterizing DSD 423parameters with the help of UM, and hence the possibility of developing coupled multifractal 424cascades to actually simulate DSD fields.

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426Acknowledgments

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428Authors thanks Serge Botton (from the "Département Positionnement Terrestre et Spatial" of 429the ENSG) for facilitating access to the roof where the disdrometers are installed. Authors 430greatly acknowledge partial financial support form the Chair "Hydrology for Resilient Cities" 431(sponsored by Veolia) of Ecole des Ponts ParisTech, EU NEW INTERREG IV RainGain 432Project (www.raingain.eu), and EU Climate KIC Blue Green Dream (www.bgd.org.uk) 433

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539

540**Table:**

541

		PWS100	Parsivel ²
$Z_h - R$	a	349	313
	b	1.49	1.63
$R-K_{dp}$	С	17.3	15.3
A	d	0.72	0.66

542Table 1: Parameters computed for the $Z_h - R$ relation (Equation 16) and the $R - K_{dp}$

543 relation (Equation 17)

		R	ρ	$K_{ m dp}$	$Z_{ m h}$	$N_{ m t}$	$D_{ m m}$
PWS100	β	2.05	2.08	1.88	-	2.07	1.41
	α	1.51	1.53	1.35	1.72	1.58	1.66
	C_1	0.18	0.16	0.23	0.34	0.11	0.005
	Н	0.67	0.67	0.62	-	0.63	0.21
Parsivel ²	β	1.96	2.08	1.71	-	2.11	1.33
	α	1.57	1.54	1.33	2.00	1.60	2.10
	C_1	0.18	0.16	0.25	0.51	0.11	0.006
	H	0.63	0.67	0.55	-	0.64	0.17

545Table 2: Scaling parameters of the various studied fields for the PWS100 and Parsivel² data

Figure caption list:



552Figure 1: Computation of the parameters of the $Z_h - R$ (a) and $R - K_{dp}$ (b) relations 553(equations 16 and 17 respectively in log-log plot). Scatter plot of R_{Z-R} (c) and $R_{R-K_{dp}}$ (d) 554versus *R* for the PWS100 data

555



557Figure 2: Scaling analysis for *R* measured by the PWS100: (a) Spectral analysis, i.e. equation 55814 in log-log plot; (b) Trace Moment (TM) analysis, i.e. equation 9 in log-log plot. (c) and (d) 559Same as (a) and (b) for *R* measured by the Parsivel².



562Figure 3: Scaling analysis for K_{dp} measured by the PWS100: (a) Spectral analysis, i.e. 563equation 14 in log-log plot; (b) Trace Moment (TM) analysis, i.e. equation 9 in log-log plot 564



566Figure 4: (a) – (b) Same as in Fig. 3 but for Z_h measured by the PWS100, in (a) a break is 567considered for k = 20-30 (~ 6 min⁻¹) and k = 60 (~ 2 min⁻¹); (c) – (d) Same as in Fig. 3 but for 568 aR^b measured by the PWS100 and "climatic" parameters a and b found in section 2.





571Figure 5: Same as in Fig. 3 but for N_t measured by the PWS100, in (a) a break is considered 572for k=70 (~ 2 min⁻¹), in (b) a break is considered for λ =32 (4 min)



575Figure 6: Same as in Fig. 3 but for D_m measured by the PWS100, in (a) a break is considered 576for k=70 (~ 2 min⁻¹), in (b) a break is considered for λ =32 (4 min)