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1 Multifractal characterisation of a simulated surface flow: a case study with
2 Multi-Hydro in Jouy-en-Josas, France

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9

10 Abstract

11 In this paper we suggest to innovatively use scaling laws and more specifically
12 Universal Multifractals (UM) to analyse simulated surface runoff and compare the retrieved
13 scaling features with the rainfall ones. The methodology is tested on a 3 km² semi-urbanised
14 with a steep slope study area located in the Paris area along the Bièvre River. First Multi-
15 Hydro, a fully distributed model is validated on this catchment for four rainfall events
16 measured with the help of a C-band radar. The uncertainty associated with small scale
17 unmeasured rainfall, i.e. occurring below the 1km x 1km x 5min observation scale, is
18 quantified with the help of stochastic downscaled rainfall fields. It is rather significant for
19 simulated flow and more limited on overland water depth for these rainfall events. Overland
20 depth is found to exhibit a scaling behaviour over small scales (10 m - 80 m) which can be
21 related to fractal features of the sewer network. No direct and obvious dependency between
22 the overland depth multifractal features (quality of the scaling and UM parameters) and the
23 rainfall ones was found.

24

25 1) Introduction

26

27 The combined effects of a growing urbanisation - approximately 80% of Europe's
28 population will live in cities by 2020 (EEA, 2014) - and potential increase of extreme events
29 as a consequence of climate change (IPCC, 2013) expose more and more people to surface
30 pluvial flooding. Pitt (2008) carried out a review on flood events in the United Kingdom and
31 showed that two thirds of the flood related damages were caused by surface water flooding.
32 Urban flooding has become a growing concern in Europe, hence a significant number of
33 European research projects address this issue, along with national counterparts. The purpose
34 of these projects is to increase the resilience of urban areas through improvement of both real
35 time management of extreme events and long term planning. We can cite FP7 SMARTesT
36 (<http://floodresilience.eu/>), CORFU (<http://www.corfu-fp7.eu/>), Climate KIC Blue Green
37 Dream (www.bgd.org.uk) or the INTERREG IV RainGain project (<http://www.raingain.eu>)
38 among others.

39 There is a need to improve the understanding of urban surface flow. Indeed, there is a
40 growing interest for 2D models in urban environment for both operational and research
41 applications (Bolle et al., 2006; Carr and Smith, 2006; Chen et al., 2007; Deltares, 2013; DHI,
42 2011; Giangola-Murzyn et al., 2014; Innovyze, 2012, 2103; Phillips et al., 2005; XP
43 Solutions, 2012). Such models aim at actually modelling processes in a physically based
44 manner, while the most commonly used semi-distributed models take them into account
45 through tailored lumped models. In case of overflow they simply consider a volume output
46 from the sewer system and deduce a local water depth, but the dynamical behaviour of the
47 water added on the ground is not addressed. Basically, urban surface flow is not commonly
48 perceived as a geophysical process and is therefore not addressed with geophysical tools

49 capable of grasping its intrinsic complexity visible across all scales. Indeed, it results from the
50 non-linear interactions between the highly spatially and temporally variable rainfall field, the
51 topography and the strongly inhomogeneous land use cover.

52 In this paper we suggest to use multifractal tools, which are commonly used in
53 geophysics to characterise and simulate fields extremely variable over a wide range of scales;
54 such as wind turbulence, rainfall, river flow or topography (see Schertzer and Lovejoy, 2011
55 for review). Such tools have seldom been used in an urban context. Gires et al. (2013, 2014b)
56 used them to downscale rainfall to quantify the uncertainty associated with small scale rainfall
57 variability, or to characterise the variability across scales of simulated flow in conduits
58 (sewer). To the knowledge of the authors it has never been used to study either surface runoff
59 flow (urban drainage) or surface flow in general including stream rivers. Investigating the
60 potential multifractal features of surface flow and notably whether it inherits rainfall features
61 is the main purpose of this paper and constitute its main novelty. In addition, this case study
62 will also be used to quantify the uncertainty associated with small scale rainfall variability,
63 not only on the simulated flow which has already been done on other catchments, but also on
64 the surface flow.

65 Given the lack of measurements of distributed data of surface runoff, outputs of a
66 numerical model are analysed. The model used is Multi-Hydro (El Tabach et al., 2009 for an
67 initial version and Giangola-Murzyn, 2014 for a recent one) developed at the Ecole des Ponts
68 ParisTech. It is implemented on a 3.017 km² peri-urban catchment in Jouy-en-Josas (South-
69 East of Paris), which exhibits steep slopes and both forest and urbanised areas. Achieving
70 such an analysis is relevant only if a distributed rainfall field is used as model input. Météo-
71 France radar mosaics with a resolution of 1 km in space and 5 min time (Tabary, 2007;
72 Tabary et al., 2007) for four events that occurred between 2009 and 2011 are used. When

73 needed, the rainfall field is downscaled both in space and time from the raw radar data, in
74 order to simulate the improvement that could be made with higher radar resolution.

75 The model and the study area data for its implementation are presented in details in
76 section 2. The multifractal framework and analysis methods are presented in section 3.
77 Results are discussed in section 4 and 5. More precisely, the validation of the model and
78 quantification of the uncertainty associated with small scale unmeasured rainfall variability on
79 both simulated sewer flow and maximum water depth is carried out in section 4. Multifractal
80 characterization of overland water depth is addressed in section 5. Main conclusions are
81 highlighted in section 6.

82

83

84 2) Model and catchment

85

86 2.1) The Multi-Hydro model

87

88 Multi-Hydro is a multi-module model whose goal is to model and predict the impacts
89 of rainfall events in urban and peri-urban areas. In this paper, there is an emphasis on heavy
90 rainfall events. Following the approach of various recent developments of hydrological
91 models (Djordjevic et al., 1999; Fletcher et al., 2013; Hsu et al., 2000; Jankowfsky, 2011;
92 Rodriguez et al., 2008); it makes different modules interact, each of them echoing a portion of
93 the water cycle in urban areas (surface runoff, infiltration, ground water flow, sewer flow).

94 Each of the modules integrated in Multi-Hydro relies on open-source software
95 packages that have already been widely used and validated by the scientific community. The
96 surface module is based on TREX (Two dimensional Runoff, Erosion and eXport model,
97 Velleux et al., 2011) which solves fluid mechanics equations for surface flow (diffusive wave

98 approximation of 2D Saint-Venant, see p. 6-7 of the TREX user manual) and infiltration
99 (simplification of Green and Ampt equation). The sewer or drainage module, which is based
100 on SWMM developed by the US Environmental Agency (Storm Water Management Model,
101 Rossman, 2010), is a 1D-model dealing with sewer flows through numerical solutions of
102 Saint-Venant 1D equations in pipes. The interactions between the surface and sewer flow is
103 handled through the gully pixels. These interactions (input or output of water) between the
104 surface and sewer flow are carried out every 3 min. When there is no overflow, gully pixels
105 are considered to have an infinite infiltration rate, and the water passing through them is
106 directly inputted into the corresponding node of the sewer model. This way of modelling
107 implies that a large transport capacity is assumed for gully, especially with 10 m pixel size as
108 in this paper (see below). Future developments of Multi-Hydro will enable to improve the
109 model with regards to this coarse assumption. They could notably rely on the experimental
110 and computational studies of gully inflow capacity, including 3D CFD studies, which analyse
111 phases in the flow, inlet capacity, reverse flow when the piezometric level in the sewer is
112 beyond the ground level (Despotovic et al., 2005; Djorjevic et al., 2005). In case of sewer
113 overflow through a node, the corresponding gully pixel is converted into a road pixel and the
114 water exiting the node is inputted on this pixel (considered as a source in TREX). There is
115 also a module handling ground water flow which was not included in this study to limit
116 computation time.

117 In order to run Multi-Hydro, data needs to be shaped in a standard format. Commonly
118 available Geographical Information System (GIS) data, such as land use and topography
119 provided in France by IGN (the French agency producing geographical information) are
120 inputted to MH-AssimTool (Richard et al., 2014). This software formats the inputs with the
121 desired resolution and makes Multi-Hydro a transportable model, rather easy to implement on
122 a new catchment. Once a resolution is chosen, one has to affect an elevation and a land use

123 class to each pixel. The elevation is obtained by an interpolation of the raw available data.
124 With regards to the land use, a priority order has been determined to assign a unique land use
125 class for a given pixel according to the hydrological importance of the given class instead of
126 the surface represented by this class: if a gully is located on a pixel, the entire pixel will be
127 considered as a gully. This process is repeated in the following order for this case study:
128 roads, houses, forest, grass, and water surface. See Ichiba et al. (2017) for a comparison with
129 other possible strategies.

130 In this paper, the model was implemented with pixels of size 10 m x 10 m. Given the
131 obtained results discussed below it was not found necessary to run it at higher resolution
132 which makes computation time too long. For an in-depth analysis of the relation between the
133 selected pixel size and simulated flow, which is not the purpose of this paper, refer to Ichiba
134 (2016). Multi-hydro was not calibrated, i.e. standard values for the parameters describing a
135 land use class are used (hydraulic conductivity, capillary suction, moisture deficit, Manning's
136 coefficient, depth of interception). Raw or downscaled radar data are used as input of the
137 model.

138

139 2.2) Presentation of the study area

140

141 The catchment studied in this article is located in Jouy-en-Josas (Yvelines County,
142 South-west of Paris). It occupies a 3.017 km² area, mainly on the left bank of the Bièvre
143 River. A small portion of the right bank near the river bed is also included. The remaining
144 portion of the right bank is drained to a small river that flows into the Bièvre River
145 downstream the outlet of the studied catchment. The Bièvre River is a tributary of the Seine
146 River which it meets in Paris. It flows through increasingly urbanised areas along its 33 km
147 path. This has led to strongly modify its natural bed, both in underground pipes which are

148 integrated in the storm water sewer system, or in a highly artificial open air bed. An effort is
149 currently undertaken to restore its “natural” aspect.

150 A striking feature of this catchment is that, unlike the previous ones studied with
151 Multi-Hydro (Giangola-Murzyn et al., 2014; Gires et al., 2014a), it exhibits steep slopes.
152 There is a difference of approximately 100 m between the plateau in the north of the
153 catchment, and the outlet of the catchment (Fig. 1). The downhill portion strengthens overland
154 runoff, and the combination of pluvial and fluvial processes on the river bank has led to
155 severe flooding in 1973 and 1982. Some details are available on the SIAVB (Syndicat
156 Intercommunal d’Assainissement de la Vallée de la Bièvre, the local authority in charge of
157 urban drainage of the area) website http://www.siavb.fr/gestion_des_crues.aspx. Urbanisation
158 and imperviousness are concentrated along the river bank, and on a housing estate along one
159 major North-South road. The remaining of this semi-urban catchment is mainly made of
160 forests. The sewer system is a separate one, and the storm water is routed into the Bièvre
161 River.

162 Following the severe flooding, the SIAVB has created 15 storage basins (integrated in
163 the landscape) along the Bièvre River to mitigate flooding risks. One, the Bassin des Bas Près,
164 is located just upstream the Jouy-en-Josas catchment. The outlet of this basin is equipped with
165 flow and height gauges operated in real time. There is a second measuring point of water
166 depth, few meters upstream the outlet of the catchment, at the “Pont de Pierre” (Fig. 1). This
167 gauge has been installed to monitor the river level and to protect a music school by triggering
168 a warning system in case of elevated height. Given the position of the two measuring points,
169 Multi-hydro will only be validated on the area drained by the sewer network represented in
170 green in Fig. 1. The forest corresponds approximately to 60% of the catchment (~ 2 km²).
171 Although it is only possible to validate the implementation of the model on a portion of the
172 catchment, the whole area is modelled to ensure the accuracy of flow over the areas actually

173 used for validation. The river is part of the storm water sewer system in Jouy-en-Josas and is
174 modelled as a pipe in Multi-Hydro drainage module. Indeed, through the city, the river bed is
175 highly artificial or even underground. The long and West – East oriented pipe located in the
176 South of the Basin (Fig. 1, left) is actually the Bièvre River.

177

178 2.3) Fractal dimensions of the impervious surfaces and of the sewer system

179

180 The studied catchment is located in a semi-urbanised area. The impervious surfaces
181 are highly relevant for hydrology since they basically correspond to areas where runoff is
182 quickly active during a storm event. Thanks to the determination of land use per pixel in MH-
183 AssimTool, the evaluation of the impervious areas can be done in an apparent simple way by
184 calculating the number of pixels of roads, buildings and gullies (since the water falling on
185 gully pixels is immediately routed to the sewer network, they are considered as impervious).

186 This impervious surface depends on the resolution at which it is computed. Indeed, an
187 imperviousness of 55%, 50%, 42%, 32% and 25% is obtained with pixels of size 20, 15, 10,
188 5, 2 m respectively. This is due to the priority order set in the data assimilation tool that
189 affects a land use for each pixel. This order prioritizes impervious areas (Fig. 2). Obviously
190 these values strongly depend on the approach implemented to affect a land use class to a
191 pixel. As previously mentioned, comparison with other approaches can be found in Ichiba et
192 al. (2017). Investigations on the possibility of having different pixel size according to the land
193 use should also be envisaged in the future, in order to for instance refine the pixels for roads
194 and gullies and coarser them for forests. Coming back to the imperviousness percentages
195 found in this paper, it is possible to use the notion of fractal dimension, which is scale
196 invariant, to explain these figures. The fractal dimension D_F of a geometrical set (here the
197 impervious pixels) is obtained with the help of the following equation:

198 $N_\lambda \approx \lambda^{D_F}$ Eq. 1

199 where N_λ is the number of impervious pixels, and λ is the resolution defined as the ratio
200 between the outer scale L of the phenomenon and the observation scale l ($\lambda = \frac{L}{l}$). It
201 characterizes the space occupied by a geometrical set in a scale invariant way. The symbol \approx
202 denotes an asymptotic convergence and absorbs slowly varying prefactors.

203 For the studied catchment, it appears that the impervious areas exhibit a fractal
204 dimension. Indeed Eq. 1 is plotted in log-log for the geometrical set consisting of the
205 impervious pixels at the 2-m resolution (imperviousness of 25 %), and a straight line is
206 retrieved on the whole range of scales, i.e. 2m-2048m (Fig. 3.a). This a basic feature of the
207 catchment. The fact that the points corresponding to the catchment representation at 20, 15,
208 10, 5, 2 m obtained with MH-AssimTool are along this straight line (circled cross on Fig. 3.a)
209 is simply a consequence of the priority order set for affecting a land use class to a pixel
210 (impervious classes are prioritised over pervious ones). This confirms the fact that even
211 though the represented imperviousness varies with scale, a feature (the fractal dimension) is
212 conserved and provides a quantification of the level of urbanisation. We find D_F equal to 1.73
213 for this catchment. In a previous study Gires et al. (2014a), found that for a highly urbanised
214 area in Seine-Saint-Denis (North-East of Paris), the fractal dimension was of 1.85 from on
215 scales ranging from 1 m to 1024 m. Given that this catchment is less urbanised, it was
216 expected to obtain a smaller fractal dimension.

217 The same study was performed on the sewer system (Fig. 3.b). In this case, the
218 geometrical set studied is the “rasterised” sewer system. If a pixel is crossed by a conduit
219 belonging to the storm water sewer network, then it is considered as part of the sewer system..
220 Two scaling regimes can be identified: from 10 m to 80 m the fractal dimension is 1.03 and
221 from 80 m to 1280 m it is 1.76. For small scales, the dimension is close to 1, and it simply
222 reflects the 1D intrinsic nature of the sewer system. For large scales, the structure of the

223 network becomes apparent, and exhibits a scaling behaviour. For large scales the value is
224 slightly smaller than the 1.85 found on the Seine-Saint-Denis catchment in Gires et al.
225 (2014a) which is consistent with the fact that this one is less urbanised. The similarity
226 between both fractal dimensions (imperviousness and large scale sewers) indicates that it is a
227 relevant way of quantifying a level of urbanisation for the area. See Gires et al. (2017) for an
228 extension of this approach to 10 areas in 5 European countries.

229

230 2.4) Rainfall data

231

232 Four rainfall events, which occurred between 2009 and 2011, are studied in this paper.
233 Simulations are performed using Météo-France radar mosaic, which provides a spatially
234 distributed data with a resolution of 1 km x 1 km x 5 min (the closest radar is the C-band one
235 of Trappes located 15 km West). For three events the data recorded with the help of a rain
236 gauge operated by the SIAVB located a few hundred meters south of the catchment at the
237 “Bassin des Bas Près” is also available. Because of (i) the standard 0.2 mm discretization
238 issue of the tipping bucket rain gauge (data is number of tips equal to 0.2 mm) which prevents
239 it from providing reliable intensity, (ii) the gap between the observation scales of the two
240 measuring devices (see Gires et al., 2014b, for an in-depth analysis of this issue) and (iii) the
241 fact that the rain gauge is furthermore outside of the catchment; it is not possible to use the
242 rain gauge data for other purpose than a rough check of the accuracy of radar data. It is done
243 by comparing the cumulative volumes of rainfall for each studied event which are displayed
244 in Table 1 along with their main features. Gires et al. (2014b) used data from dense network
245 of point measurement devices (rain gauges or disdrometers) distributed over 1 km² and
246 showed that the cumulative depth differences between devices could reach more than 40 %
247 for individual rainfall events (of the same order of magnitude as the one discussed here). They

248 showed with the help of numerical simulations that similar values were found simply taking
249 into account small scale rainfall variability. Here the maximum observed differences are 34%,
250 which suggests that the agreement between the two devices is acceptable, i.e. smaller than
251 expected uncertainty simply due to the scale gap between the two measuring devices. Authors
252 did not have access to longer time series of both radar and rain gauge to perform a more in-
253 depth evaluation of the radar versus rain gauge measurement for this specific point, which
254 would be the topic of another study. The temporal evolutions of the radar rain rate averaged
255 over the catchment are displayed in Fig. 4. These events were selected because they are heavy
256 ones. However they are not extreme ones, indeed over durations of 1 h and 4 h, only the 14
257 July 2010 event has a return period greater than 1 year (data from a rain gauge located in the
258 Paris area that was available to the authors was used to obtain these estimates). For the July
259 event, the return period is of about 1 year for a duration of 1 h and of about 2 years for a
260 duration of 4 h.

261

262 3) Methods

263 3.1) Multifractal framework

264 The Multifractal framework is used for several purposes throughout this paper to
265 characterize the variability across scales of fields, and is therefore presented here in a generic
266 way. Only basic properties are discussed here, and interested readers are referred to the recent
267 review by Schertzer and Lovejoy (2011) for more details. The general assumption of
268 multifractal fields is that they are generated by an underlying scale invariant multiplicative
269 cascade process. In such process, a structure at a given scale is divided into smaller structures
270 at smaller scale and the value of a child structure is equal to the value of the parent structure
271 multiplied of a random increment. The process is scale invariant in the sense that the way
272 structures are divided into sub-structures and the probability distribution of the random

273 multiplicative increments are the same at all scales. A consequence is that statistical
 274 properties of such fields are conserved across scales. More precisely let us denote ε_λ a field at
 275 resolution λ ($=L/l$, where l is the observation scale and L the outer scale of the phenomenon as
 276 for the fractal dimension definition). The probability of exceeding a given threshold (λ^γ),
 277 defined with the help of the scale invariant notion of singularity γ (the thresholds depend on
 278 the observation scale, but not the singularity),

279 $\Pr(\varepsilon_\lambda \geq \lambda^\gamma) \approx \lambda^{-c(\gamma)}$ Eq. 2,

280 and the moment of order q ,

281 $\langle \varepsilon_\lambda^q \rangle \approx \lambda^{K(q)}$ Eq. 3,

282 exhibit a power law relation with regards to the resolution at which they are computed. As for
 283 Eq. 1, the symbol \approx denotes an asymptotic convergence and absorbs slowly varying
 284 prefactors. Equations 2 and 3 define respectively the codimension function $c(\gamma)$ and the
 285 moment scaling function $K(q)$, which both fully characterize the variability across scales of
 286 the field. $c(\gamma)$ and $K(q)$ contain the same information and are related by a Legendre transform
 287 (Parisi and Frish, 1985). Eq. 2 can be understood from the simpler notion of fractal dimension
 288 (Eq. 1). Indeed, an intuitive interpretation of a multifractal field is that the geometrical sets
 289 made of each portion of the field greater than given thresholds are fractal and characterized by
 290 fractal dimensions. To be mathematically more rigorous the notion of threshold is replaced by
 291 the scale invariant one of singularity.

292 By generalizing the central limit theorem Schertzer and Lovejoy (1987) showed that
 293 any conservative scale-invariant multiplicative processes converge toward Universal
 294 Multifractals (in a similar way as re-normalized sum of identical and independent random
 295 variables converge toward normal distribution as long as their variance is defined). For
 296 Universal Multifractals (UM), i.e. this limit behaviour, $K(q)$ and $c(\gamma)$ functions are defined

297 with the help of only two relevant parameters with a physical interpretation. They are known
298 as UM parameters C_1 and α :

299 - C_1 is the mean intermittency which measures the average sparseness of the field. $C_1=0$ for a
300 homogeneous field.

301 - α is the multifractality index ($0 \leq \alpha \leq 2$) and measures how fast the intermittency evolves
302 when considering level of activity slightly different from the average one.

303 Great values of α and C_1 corresponds to strong extreme. A common tool to assess the
304 extremes of a field is the scale invariant notion of maximum probable singularity γ_s
305 observable (Hubert et al., 1993; Douglas and Barros, 2003; Royer et al., 2008; Gires et al.,
306 2014a). It is defined for a unique sample by

$$307 \quad c(\gamma_s) = d \quad \text{Eq. 4}$$

308 Where d is the dimension of the embedding space, i.e. $d = 1$ for time series and $d = 2$ for
309 maps.

310 The power spectrum (Fourrier transform of the auto-correlation function) of such
311 multifractal field exhibits a scaling relation with wave number k :

$$312 \quad E(k) \approx k^{-\beta} \quad \text{Eq. 5}$$

313 where β is the spectral slope.

314

315 3.2) Uncertainty associated with small scale rainfall

316 The purpose of this section is to explain the approach implemented to quantify the
317 uncertainty associated with small scales rainfall variability, i.e. which is occurring below the 1
318 km x 5 min scale currently provided by the C-band radar operating in this area. The same
319 methodology as in Gires et al. (2013, 2014a) is implemented, and only basic ideas are
320 explained here. Firstly, an ensemble of downscaled rainfall fields is generated, then each
321 realisation is inputted into the numerical model and finally the disparities within the ensemble

322 of outputs, which reflect the studied uncertainty, are analysed and quantified. 100 sample
 323 ensembles are used. The downscaling technique relies on the Universal Multifractal
 324 framework. It basically consists in stochastically continuing a space-time cascade process that
 325 has been validated on the available range of scales. The resolution of the downscaled rainfall
 326 field is 12 m in space and 20 s in time starting from the original 1 km and 5 min of the
 327 available radar data. The process has been validated down to such small scales (Gires et al.,
 328 2014b).

329 The disparities among the simulated ensembles are quantified with the help of quantile
 330 analysis. Let us first illustrate this with the flow output, but the same is done for maximum
 331 water depth at each pixel. For each time step the 5, 25, 75 and 95 % quantiles are computed,
 332 and give the envelop curves $Q_{0.05}$, $Q_{0.25}$, $Q_{0.75}$, and $Q_{0.95}$, respectively. The width between
 333 these curves characterizes the uncertainty interval on simulated flow. It is quantified with the
 334 help of two pseudo-coefficients of variation computed as:

$$335 \quad CV_{95}' = \frac{Q_{0.95}(t_{PF, radar}) - Q_{0.05}(t_{PF, radar})}{2 * PF_{radar}} \quad \text{Eq 6.a}$$

$$336 \quad CV_{75}' = \frac{Q_{0.75}(t_{PF, radar}) - Q_{0.25}(t_{PF, radar})}{2 * PF_{radar}} \quad \text{Eq 6.b}$$

337 where $t_{PF, radar}$ is the time of the peak flow simulated with the raw radar data
 338 (PF_{radar}).

339

340 3.3) Multifractal analysis of overland water depth maps

341 There is no distributed data available for overland water depth over large areas, but it
 342 is possible to study the fields obtained with the help of numerical simulations with spatially
 343 distributed rainfall as input. Maps of water depth during runoff at the end of each 3 min
 344 Multi-Hydro loop are studied.

345 Technically, in this paper an area of 128 x 128 pixels of size 10 m x 10 m is extracted
346 from the map of the catchment to carry out the analysis. Both ensemble analysis (i.e.
347 considering all successive maps as independent realisations of the same process and upscaling
348 them individually before taking the mean in Eq. 2 and 3) and individual time step analysis
349 (i.e. to obtain temporal evolutions of the various parameters) are performed. Finally, analyses
350 are done in 2D on the maps but also in 1D on the columns or the lines of pixels over the
351 catchment, in a North-South direction and in an East-West direction respectively (Fig. 5). The
352 purpose of this is to monitor a possible influence of the slope over the generated runoff
353 scaling properties.

354

355

356 4) Implementation of the Multi-Hydro model on the Jouy-en-Josas catchment

357

358 4.1) Validation with raw radar data

359

360 The validation of the model is achieved by comparing the water height measured at the
361 Pont-de-Pierre gauge with the simulated one. Before going on authors would like to highlight
362 that a proper validation on this case study is not possible given the available data, and will
363 therefore limit this section to checking that the model approximately behaves well. The main
364 reasons for this problem are:

365 - Only one measuring point is available for the whole catchment taking into account

366 approximately an area of 2 km².

367 - The uncertainty associated with this water level gauge is high. Indeed, it is not operated for
368 accurate hydraulic measurement but to trigger an alarm to evacuate a music school located
369 nearby. The main issue is that the shape of the river bed cross section at this point is not

370 available. The width was estimated at around 1.80 m, using aerial photography from IGN and
371 an approximate measure from few meters away. In order to correctly model the pipe, we used
372 Multi-Hydro and tested various types of conduits. Finally, we chose to model the Bièvre as a
373 circular pipe, with free surface of 2 m diameter, which is close to the approximate
374 measurement. This choice is only an approximation which does not take into account the
375 variations in time of this shape due the fact that the bottom of the river bed is not flat and
376 contains moving rocks and changing vegetation.

377 - There is a lack of available data on initial soil saturation which is one of main sources of
378 uncertainty and can biased runoff (see Shah et al., 1996; Zehe et al., 2005) especially at the
379 beginning of the event. In this paper, dry conditions were considered at the beginning of each
380 event. A sensitivity test was conducted by considering a saturated soil at the beginning. A
381 slight increase (few percent) of simulated flow was noted only during approximately the first
382 hour (not shown here). Having longer rainfall time series would enable to simulate the
383 catchment's behaviour some time before the event and limit the uncertainties associated with
384 this issue.

385 - The uncertainties on the water input in the Bièvre River at the outlet the Bas-Près storage
386 basin upstream the catchment are not quantified.

387 - Obviously there are some uncertainties on the radar rainfall measurement itself.

388 The simulation and measurement at the “Pont de Pierre” point for the selected rainfall
389 events are displayed in Fig. 6. For the 09-02-2009 event we observe a clear overestimation at
390 the beginning of the event. For the 14-07-2010 event Multi-Hydro with the radar rainfall data
391 reproduces well the two main peaks, but overestimates the first local maximum of rainfall
392 intensity and misses the second one. The 15-08-2010 event shows a greater variability in the
393 first half of the simulation (variations are more pronounced on the model than on the
394 measurements) but reproduces well the last peak. Finally, for the 15-12-2011 event, the Multi-

395 Hydro model reproduces well the first peak, but the flow decreases more rapidly than the
396 observations.

397 Given the available data on a limited number of events it is difficult to attribute the
398 observed discrepancies to one or several of the previously mentioned sources of uncertainty.
399 Proper validation would indeed require the analysis of much longer time series and more
400 accurate measurements with better position of sensors. Nevertheless, the obtained results do
401 not highlight strikingly wrong behaviour of simulated water heights in conduit, and enable to
402 partially reproduce observations. Finally, it seems that for some events the simulated flows
403 might be too noisy compared with observed water levels. This should not affect the UM
404 analysis that follows because the analyses carried out in this paper are spatial ones, i.e. maps
405 are studied and not time series so the potential effect should be limited. Keeping in mind the
406 previously mentioned limitations, results suggest that it remains relevant to use this
407 implementation of Multi-Hydro with a rather coarse 10 m resolution for testing its sensitivity
408 to small scale rainfall variability and analysing surface runoff with the help of multifractals.
409 The authors acknowledge that further investigations on other catchments with more
410 accurately validated models would be needed to fully confirm the findings discussed after.

411

412 4.2) Uncertainty associated with small scales rainfall variability

413 The envelop curves $Q_{0.05}$, $Q_{0.25}$, $Q_{0.75}$, and $Q_{0.95}$ are displayed in Fig. 7 for the 09-02-
414 2009 event for 5 conduits selected from upstream to downstream, which enables to analyse
415 the effect of the position of the conduit within the network. Link #4 corresponds to the Pont-
416 de-Pierre measurement, and #5 to the outlet of the catchment. As it can be seen in Fig. 7, link
417 #4 and #5 are located along the Bièvre River, and they take into account the significant base
418 flow in the river coming from upstream the Jouy-en-Josas catchment. It means that they are
419 obviously less sensitive to local rainfall variability. Similar curves were also generated for

420 water height (not shown) at the Pont-de-Pierre. The computed uncertainty is small and
421 certainly does not explain the discrepancies between simulations and measurements noticed in
422 Fig. 6, which are hence not simply due to effects of small scale rainfall variability.

423 CV'_{95} and CV'_{75} values computed for the selected conduits (Fig. 7) and the four events
424 are displayed in Table 2. As expected they decrease while considering more and more
425 downstream conduits. There is a sharp decrease in CV' when the Bièvre River is reached
426 because the base flow of the river dampens the effect of local small scale rainfall variability
427 occurring over the 3 km² catchment, but the uncertainty only associated with this effect
428 remains of roughly 10 % at the outlet whatever the event. The values for up-stream and mid-
429 stream pipes are great for all events, even for CV'_{75} which highlights a significant impact of
430 small scale rainfall variability on the simulated flow. The variability observed in the simulated
431 flow is basically due to the disparities in the simulated downscaled rainfall fields which are
432 transferred through the hydrological model (see Gires et al. 2012 for more detailed analysis of
433 this issue). Small scale rainfall data is needed to understand better, and plan better, some local
434 flooding due to sewer overflows which have been reported in some areas, notably the street
435 parallel to the Bièvre River bed in the city (just North of it), There does not seem to have a
436 straightforward relation between the computed uncertainty and the strength of the event (in
437 terms of maximum rainfall peak intensity over 5 min). Indeed, the tendency that could be
438 observed on the 09-02-2010, 15-08-2010 and 15-12-2011 (not a linear one as for example the
439 peak rainfalls are equal to approximately 7 and 24 mm.h⁻¹ for respectively the 15-08-2010 and
440 15-12-2011 event while the computed uncertainties are close) is not confirmed by the results
441 for the 14-07-2010 event (see Tab. 2). Finally, these values are comparable to the ones that
442 were obtained on a 1.5 km² highly urbanised catchment located 40 Km North-East on the
443 other side of the Paris area in Gires et al. (2014a). For this catchment, CV'_{95} values were
444 ranging from 21 to 56%, 26 to 94% and 22 to 50% from downstream to upstream for the same

445 09-02-2009, 15-08-2010 and 15-12-2011 events respectively (at a different location). The
446 values are slightly smaller for this catchment and this is likely to be due to lower level of
447 imperviousness resulting in a smaller portion of rainfall becoming immediately active.

448

449 In this paper, the uncertainty is computed not only on the simulated flow, but also on
450 the water depth in streets. As for the flow, for each realisation of downscaled rainfall field, the
451 maximum water depth over the whole simulation is retrieved for each pixel. A sample is
452 shown in Fig. 8.a for the 15-12-2011 event. The known hot spots are visible, although with
453 too high values. For example, the modelled maximum water depth reaches more than 15 cm
454 in the street along the Bièvre River bank in the city and the parallel street just north of it
455 (already mentioned in the previous paragraph). Although some flooding is regularly reported
456 by citizens to the SIAVB for these streets, such height was not reported for this event. In the
457 urbanized portion of the catchment the street network is visible on the maximum water depth
458 map, meaning the maximum values of water depth maps are reached on the corresponding
459 pixels. Lower values are found on the on roads/streets located on the steep portion of the
460 catchment because water moves faster in these areas. Same patterns and numerical values are
461 obtained for other realisations of the same event. Similar plots are obtained for the other
462 events with lower depths for the 09-02-2009 and 15-08-2010 (for which a lower cumulative
463 rainfall depth was recorded) and greater depths for the 14-07-2010 event. Then, as for the
464 flow analysis previously carried out, the uncertainty on this maximum water depth is
465 computed with the help of the 5 and 95% quantiles for each pixel and a pseudo-coefficient of
466 variation. Illustrations of the quantiles maps are shown in Fig. 8.b and 8.c for the 15-12-2011
467 event. Similar patterns are observed on the two maps, notably for the hotspots previously
468 mentioned which are visible on both maps. Maps of CV'_{95} for maximum depth are displayed
469 in Fig. 9 for the four rainfall events. It appears that the uncertainty is lower for the areas

470 where the greatest maximum depths are found (i.e. on roads) and is also lower for the heaviest
471 rainfall events. It reaches only few percents on the hottest points. The values (Fig. 9) are
472 anyway much smaller than those found for sewer flow (Table 2 and Fig. 7). This apparent
473 contradiction is likely to be due to the fact that most of the rain water is properly handled by
474 sewers and overflows are limited for these events. It means that for these events disparities in
475 local amounts will not be visible on ground levels, whereas they are indeed in sewer flows
476 and water depths. Further investigations with heavier rainfall events should be carried out to
477 confirm or not this interpretation. The areas with the greatest uncertainty are found in gardens
478 for the weakest event (09-02-2009), and correspond to places with a very small maximum
479 depth (smaller than 1mm), meaning that the hydrological relevance is not very high.

480

481

482 5) Multifractal characterization of overland water depth

483 Multifractal analyses of overland water depth during rainfall event are presented in
484 this paper for the 14-07-2010 and 15-12-2011 events which are the two heaviest ones in terms
485 of maximum rainfall intensity over 5 min (see Table 1).

486 Figure 10.a displays the spectral analysis of the water depth for the 14-07-2011 event.
487 Maps of water depth for each time steps during the event are used to carry out 2D ensemble
488 analyses. The quality of the scaling is low, with a coefficient of determination for the linear
489 regression equal to 0.42. The fact that the spectral slope is close to zero (β is found roughly
490 equal to 0.2) indicates that the field is conservative, i.e. its mean is conserved across scales. It
491 is therefore possible to implement directly on the field a Trace Moment (TM) analysis, which
492 consists in assessing the validity of Eq. 4 by plotting it in log-log. Perfect UM fields would
493 lead to straight lines. Figure 10.b shows the TM ensemble analysis performed over all the
494 time steps of the same 14-07-2011 event. Two scaling regimes can be identified: a small

495 scales regime from 10 m to 80 m (right part of Fig. 10.b) and a large scales regime from 80 m
496 to 1280 m (left part of Fig. 10.b). The coefficient of determination r^2 of the linear regression
497 for $q=1.5$ in Fig. 10.b is taken as an indication of the quality of the scaling. The scaling from
498 small scales (10 m - 80 m) is much more robust than for large scales (80 m to 1280 m), as
499 illustrated by the r^2 equal to respectively 0.99 and 0.91. Given the low quality of the scaling
500 for large scales, UM parameter estimates will not be reported and discussed for this regime
501 because they are not reliable. Furthermore, small scales are crucial for surface runoff because
502 it is at these scales that it is generated into the drainage system. The location of this break at
503 approximately 80 m indicates a possible physical interpretation. Indeed, it is the same location
504 as the break in the fractal analysis of the sewer network and corresponds roughly to the inter-
505 distance between roads. This would mean that this break is driven by the influence of the
506 collection of water by sewer network. The more robust scaling behaviour for surface flow is
507 found for the scales for which the sewer network does not behave yet as network but as
508 isolated linear pipes. Before going on, it should be mentioned that numerous pixels have very
509 small depth (see Fig. 8 for an illustration), for which the model uncertainties might be great.
510 These zeros values or spurious ones close to zero will affect the scaling analysis for small
511 moments (typically $q < 0.5$) through a multifractal phase transition (see Gires et al., 2012, for
512 a detailed analysis of this issue). Here the influence of this bias does not extend to moments
513 close to 1 around which the estimates of UM parameters are carried out, meaning that they are
514 not affected by this issue.

515

516 Although intrinsically less robust since scaling properties are statistical ones requiring
517 numerous data to be properly observed, TM analyses were also carried out independently on
518 each sampling time step of Multi-Hydro (3 min in this paper). The purpose is to see whether
519 there is an impact of the current rainfall rate on it. Figure 11 displays for the 14-07-2010 event

520 the temporal evolution of both the rainfall rate and the r^2 for $q=1.5$ in the TM analysis for the
521 two regimes identified in the ensemble analysis, i.e. small (10 m – 80 m) and large scales (80
522 m – 1280 m). For this event, two rainfall peaks are observed, and they both result in a sudden
523 loss of the scaling quality, more pronounced for large scales than small ones. For the first
524 peak (yellow bars on Fig. 11) the decrease of r^2 lasts approximately 20 min, while it lasts only
525 few minutes for the second peak (red bars on Fig 11). In both cases the quality of the scaling
526 behaviour improves again over few tens of minutes. The physical meaning of such loss is not
527 clear, but could be due to a bad representation of the surface flow process during intense
528 rainfall (it might take some time to retrieve a realistic surface flow simulation following a
529 sudden change in rainfall input), a bias in the geometrical repartition, or an intrinsic feature of
530 the process. For the latter, a possibility is that during intense rainfall period, the surface flow
531 exhibits more directly the rainfall features than its intrinsic ones which are retrieved once the
532 flow process has “adapted” to the new conditions. This would explain both the loss of scaling
533 quality and why scaling properties closer to rainfall ones are observed during these short
534 periods. Analysis with a higher resolution model would be needed to further investigate this
535 issue, which would also enable to have access to a wider range of small scales.

536 Similar features are retrieved for the other studied event (15-12-2011). Finally, it
537 should also be mentioned that similar results are also found when performing the analysis on
538 the North-South or West-East 1D-samples, which means that the preferential slope of the
539 catchment (North-South) does not seem to have an influence on the scaling features of the
540 simulated water depth. In terms of scaling quality, very similar results are also found with raw
541 radar data, or downscaled rainfall fields suggesting a limited impact of small scale rainfall
542 variability on these features. The same downscaling process as in section 2 is used.

543

544 UM parameters retrieved on the maximum water depth were computed for small
545 scales, and are displayed in Table 3 for the two events (14-07-2011 and 15-12-2012) and for
546 simulations with raw radar data and also a realisation of downscaled rainfall field with $\alpha=1.8$
547 and $C_1=0.1$ (other realisations yield very similar results). The temporal evolutions of α and C_1
548 for the 14-07-2011 event are shown in Figures 12.

549 It appears that the UM parameters are also affected by the “jumps” that were noticed
550 on r^2 in Fig. 11. Indeed after an intense period, sharp increase of α and decrease of C_1 are
551 noticed. These pronounced variations mean that the values obtained with ensemble analyses
552 should not be over-interpreted. Nevertheless few comments can be made. First the values of
553 C_1 are much greater than the ones reported for rainfall (typically 0.1-0.3 at small scale)
554 meaning that significant levels of water depth are much more concentrated than the rainfall
555 field, which reflects the influence of the physical processes associated with surface flow on
556 the transferred field, notably the flow concentration. The most relevant one is the topography
557 that routes water through specific paths and tends to concentrate it. Second the values of UM
558 parameters are quite different between the two events. These differences are much greater
559 than the ones observed on the rainfall fields (see Ichiba, 2016, for a detailed analysis of these
560 storms) at small scales. This suggests that the large scales rainfall pattern has a strong
561 influence on the retrieved parameters. Indeed, the topography and small scale rainfall features
562 are the same between the two simulations; the only difference is the large scale rainfall
563 features. Thirdly the values of γ_s are rather similar for both events (the differences between α
564 and C_1 tend to compensate themselves).

565 The temporal evolutions of the UM parameters obtained by inputting raw and
566 downscaled rainfall data are very similar. The differences are slightly more pronounced on the
567 values computed on ensemble analysis but as previously said this should not be over-
568 interpreted given the strong variations visible in the temporal analysis. This similarity

569 highlights the low influence of small scale rainfall variability on the retrieved parameters
570 which seems to be more dependent on features associated with surface flow process itself or
571 large scale rainfall.

572

573 In order to test the sensitivity of the results to small scales rainfall features, synthetic
574 rainfall fields with various sets of known parameters are used as input to Multi-Hydro
575 simulations. More precisely the pseudo-events tested last 30 min with an average intensity of
576 10 mm/h. Three pairs (α ; C_1) of parameters are tested: (1.8; 0.1), (1.8; 0.05), (1.4; 0.1). Figure
577 13 displays the temporal evolutions of the rain rates, r^2 , α and C_1 for water depth for the three
578 synthetic rainfall events.

579 The temporal evolution shows the same general tendency as the one observed with the
580 real events. A loss of scaling quality is observed during the event itself, and it improves
581 afterwards. α and C_1 have a constant behaviour during the rainfall, while they decrease and
582 increase respectively after the rainfall has stopped. The comparison of the UM parameters for
583 the overland maximum water depth shows that they do not seem to depend on the small scale
584 rainfall variability in this case. α is constant around 1.4 while C_1 is constant around 0.6 during
585 the rainfall. γ_s is again constant around 1.7 on average. The rainfall UM parameters do not
586 seem to modify the structure of the overland flow, and its geometrical distribution. Successive
587 simulations with the same parameters for synthetic rainfall yielded same results. A physical
588 explanation of the C_1 parameter could be that during the rainfall, the surface flow is more
589 homogenous due to a ubiquitous input of water. UM parameters on water depth are thus
590 closer to the rainfall ones (small C_1). However after the rain has stopped, the disparities of
591 simulated water depth are increased due to predominant pathways (roads) or topographic
592 depressions where the water can accumulate. The greater C_1 after the event could reflect this

593 fact. The smaller values of α mean that the disparities among the areas where water remains
594 tend to decrease after the rainfall event.

595 This study seems to highlight the fact that UM parameters α and C_I for water depth are
596 rather relying on the large scale structure of the rainfall and on the catchment features, while
597 the maximum observable singularity γ_s is conserved for all events. Further studies could
598 infirm or confirm the fact that γ_s depends on the studied catchment. The temporal evolutions
599 of the UM parameters also deeply rely on the rainfall rate. Synthetic events with block
600 structures enabled to stand out rather simple general tendencies. They become more complex
601 with real rainfall, when the intensity has a higher temporal variability.

602 The temporal evolutions of the UM parameters also enable to quantify a catchment
603 response time. Due to the sampling time step of the simulations, the uncertainty associated
604 with the response is of 3 min. Still, it can be noted that in urban catchments (or semi-urban
605 here), the response time of water depth UM parameters to the beginning of a rainfall or to an
606 important peak of intensity is almost non-existent. This is due to the presence of impervious
607 area over which rainfall directly transfer into surface runoff.

608

609

610 6) Conclusions

611

612 The Multi-Hydro model was implemented on the Jouy-en-Josas catchment in the Paris
613 area. This 3 km² semi-urbanised catchment exhibits sharp slopes, and a dense area along the
614 Bièvre River bed. It has often been damaged by major pluvial and fluvial flooding, before the
615 construction of storage basins along the river path. The model was validated on this new
616 catchment on four rainfall events with the help of the data from a height gauge near the outlet.
617 Rainfall radar data with a resolution of 1km x 1km x 5min was used.

618 Then ensembles of downscaled rainfall fields were used to quantify the sensitivity of
619 the model outputs to small scales unmeasured rainfall variability, i.e. occurring below the
620 resolution of the available raw radar data. It appears that it is rather significant on flow
621 simulated in conduits with pseudo coefficients of variations ranging from 90 % upstream to
622 10% downstream. This confirms previous results obtained on a 1.5 km² flat highly urbanised
623 catchment also in the Paris area. The methodology was extended here to simulated water
624 depth, and it was found that the sensitivity was much lower than for conduits' flow. This is
625 likely to be due to the fact that the sewer system is mainly able to cope with the storm water
626 for these events limiting the amount of surface runoff.

627 After using them to downscale the radar data, Universal Multifractals are used in an
628 innovative way to characterize the surface flow process -through simulated water depth for
629 each 10 m x 10 m pixel over 3 min time steps- during rainfall events. UM parameters α and
630 C_1 , and the composite parameter γ_s are evaluated on the outputs of Multi-Hydro. Two scaling
631 regimes are identified for this field and estimates are only reliable for small scales, i.e. 10m -
632 80m, and related to the fractal feature of the sewer system which exhibits a scale break at the
633 same scale. There is a loss of the quality of the scaling during intense rainfall periods and UM
634 parameters get closer to rainfall ones. A possible interpretation is that during this short period,
635 a mixture of the scaling behaviour of both surface flow and rainfall is observed. After the
636 event scaling is improved and features more specific to surface flow processes are retrieved
637 with a field strongly concentrated and variability among the wet areas dampened (C_1 greater
638 than 1 and α smaller than 1). Small scale rainfall features do not seem to strongly influence
639 the results which depend more on large scales rainfall spatio-temporal patterns for these
640 events which do not trigger much sewer overflow.

641 The conclusions found with the help of this innovative methodology are not as
642 straightforward as the authors would have hoped. Further investigations with other rainfall

643 events, other catchments, notably with denser monitoring network including in-sewer
644 measurements, should be carried out to strengthen the results. Higher resolution models
645 should also be tested to extend the range of available scales for the small scales regime to
646 obtain more reliable estimates of scaling features. Such new analysis would enable to
647 generalize the behaviour of the scaling and of the UM parameters which describes the surface
648 flows, and eventually to link them to other geometrical features of the catchment, such as the
649 fractal dimension of its impervious surface, of the roads (which are the preferential path for
650 surface flows) or of the sewer system. This paper should be seen as a promising first step that
651 hints at innovative techniques relying on scale invariance properties to analyse how the
652 rainfall extremes are either dampened or enhanced by hydrological models and also to
653 quantify the extremes at very high spatial resolution (typically 1 m) without having to run the
654 model at these resolutions which would require too much time especially for real time
655 applications.

656

657

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666 6) References

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771 Velleux, M.L., England, J.F. and Julien, P.Y., 2011. TREX Watershed Modelling Framework
772 User's Manual: Model Theory and Description. Department of civil engineering, Colorado
773 State University, Fort Collins: 106p.

774

775

776 **Tables:**

777

	Radar rain depth (mm)	Rain gauge depth (mm)	Duration (min)	Peak intensity over 5 min (mm/h)
09-02-2009	9.4	<i>Unavailable</i>	725	5.12
14-07-2010	43.2	35.2	1020	52.06
15-08-2010	27.8	20.8	1745	7.56
15-12-2011	26.2	29.6	785	24.26

778 Table 1: Main features for the four studied rainfall events. Cumulative depth are computed
779 over the whole event. For the radar data averages over the catchment are displayed.

780

781

Event / Link	#1	#2	#3	#4	#5
09-02-2009	63 / 16	35 / 15	10 / 7.2	4.0 / 1.7	4.8 / 2.1
14-07-2010	76 / 22	27 / 13	7.1 / 3.6	7.5 / 3.2	7 / 3.1
15-08-2010	70 / 20	38 / 16	26 / 12	9.3 / 3.9	8.5 / 3.8
15-12-2011	60 / 23	50 / 22	28 / 12	11 / 4.1	8.7 / 3.9

782 Table 2: CV'_{95} and CV'_{75} in % (first and second figure respectively) for the five selected

783 conduits and four rainfall events.

784

785

786

Event	Rainfall input	α	C_1	γ_s
14-07-2010	Raw radar data	1.55	0.62	1.52
	Downscaled rainfall	1.25	0.90	1.68
15-12-2011	Raw radar data	0.95	1.42	1.74
	Downscaled rainfall	0.99	1.22	1.65

787 Table 3: UM parameters for small scales (10 m – 80 m) computed with the help of a 2D

788 analysis with either raw radar data or a realisation of downscaled rainfall field (with $\alpha=1.8$

789 and $C_1=0.1$) as rainfall input for the 14-07-2010 and 15-12-2011 events.

790

791

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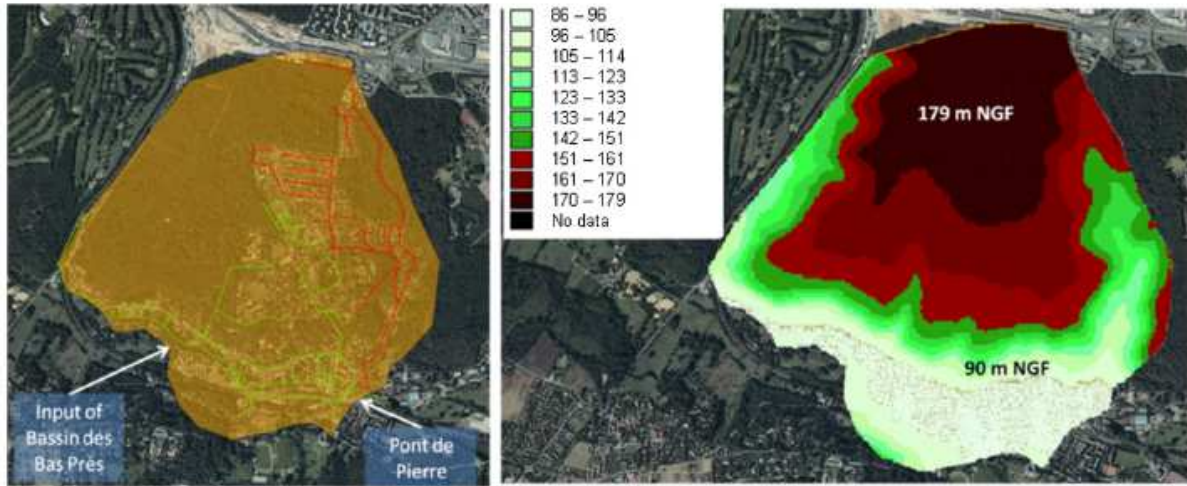
793

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795 Figure captions:

796

797

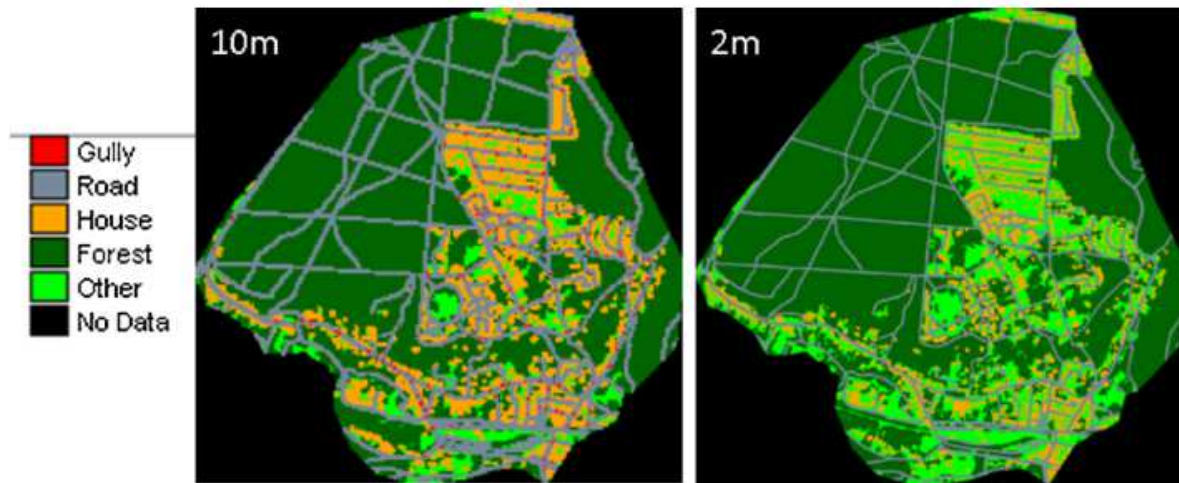


798

799 Figure 1: Maps of the Jouy-en-Josas catchment: (left) aerial photography and sewer system

800 (The green portion of the sewer network corresponds to the portion over which validation is

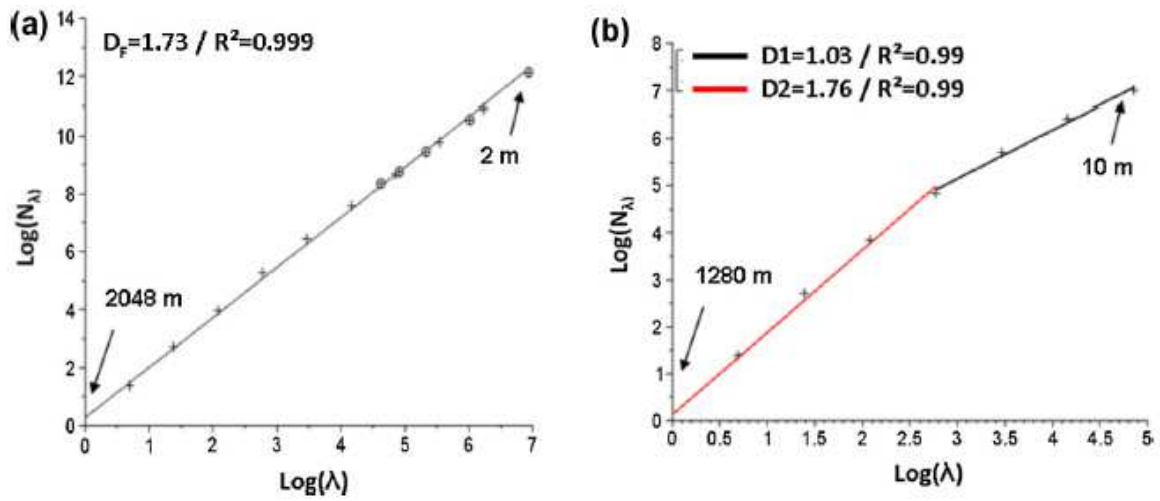
801 possible), (right) elevation in m.



802

803 Figure 2: Map of the land use obtained with the help of MH-AssimTool over the Jouy-en-

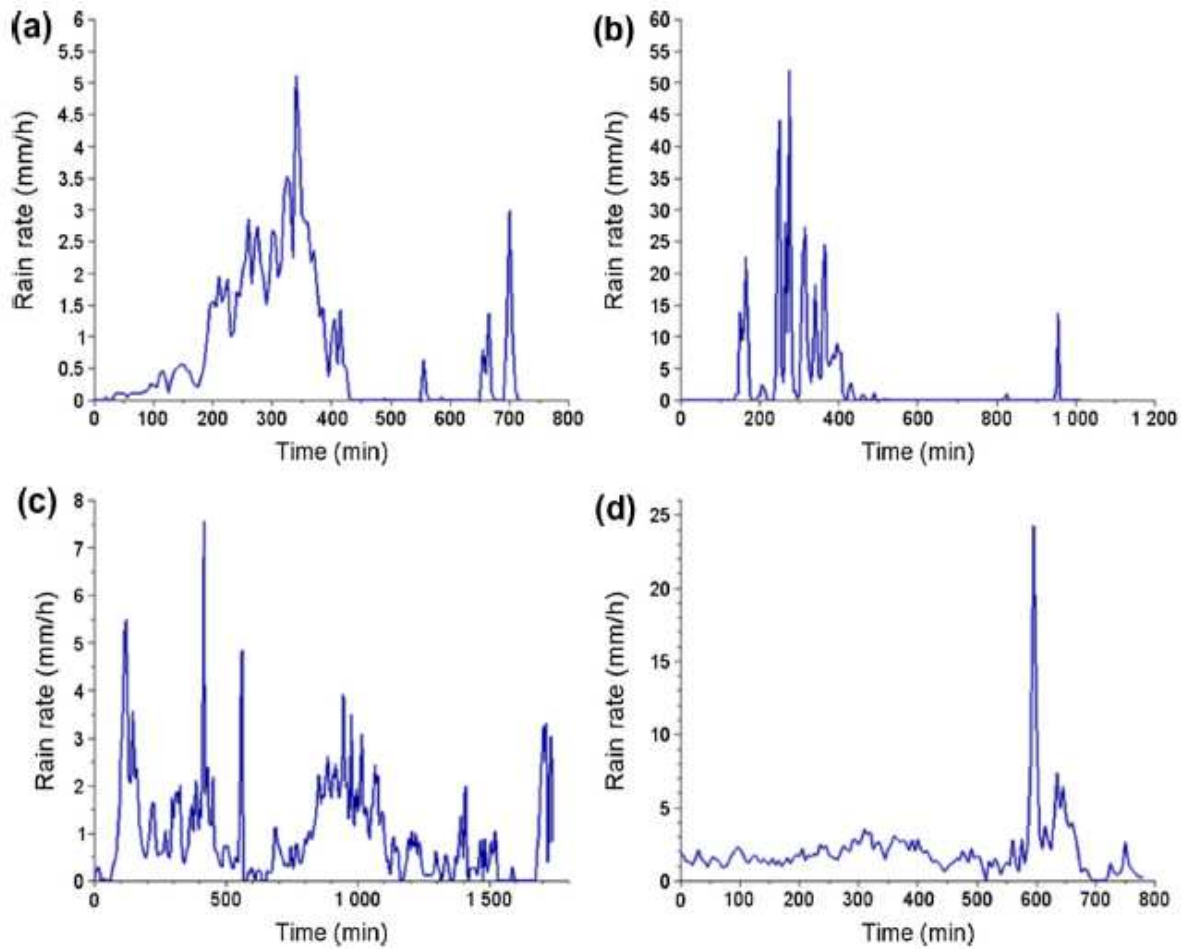
804 Josas catchment for two different resolutions.



805

806 Figure 3: (a) Evaluation of the fractal dimension of the impervious area for the studied
 807 catchment (Eq. 1 in log-log plot). The circle points correspond to the figures obtained from
 808 the map generated with the help of MH-AssimTool at various resolutions. (b) Evaluation of
 809 the fractal dimension of the sewer system.

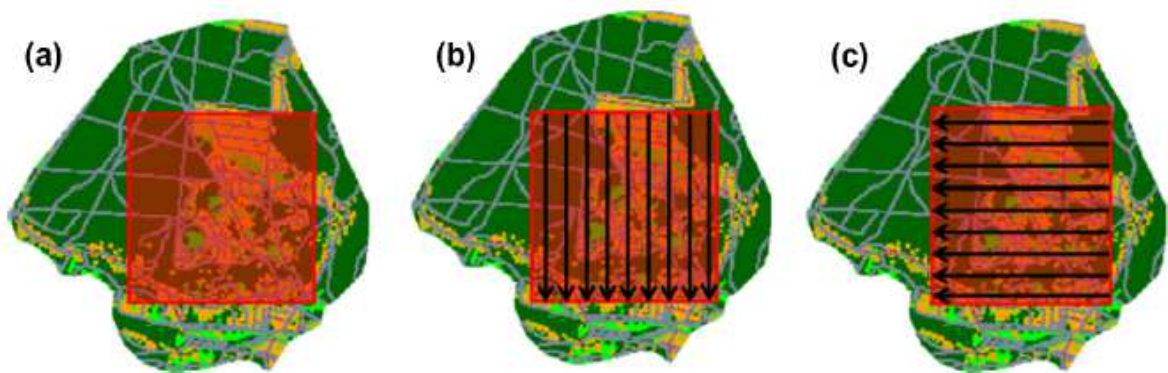
810



811

812 Figure 4: Average over the catchment of the rainfall radar intensity in mm/h over 5 min time

813 steps for the four events: (a) 09-02-2009, (b) 14-07-2010, (c) 15-08-2010, (d) 15-12-2011

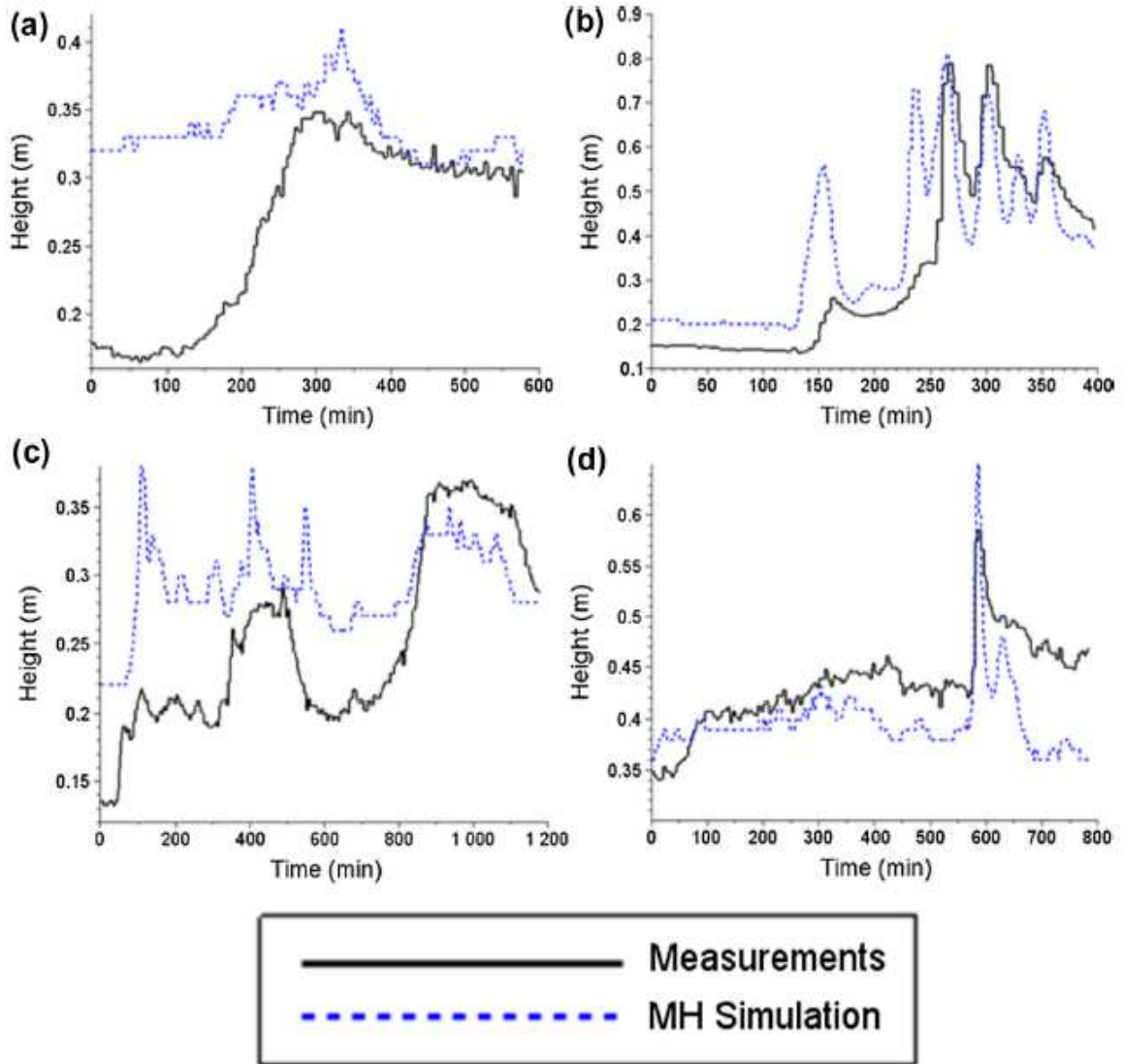


814

815 Figure 5: Illustration of the samples studied in the multifractal analysis of overland water

816 depth at the end of each 3-min Multi-Hydro loop: (a) 2D maps, (b) 1D vertical columns (N-S

817 direction), (c) 1D horizontal rows (W-E direction).

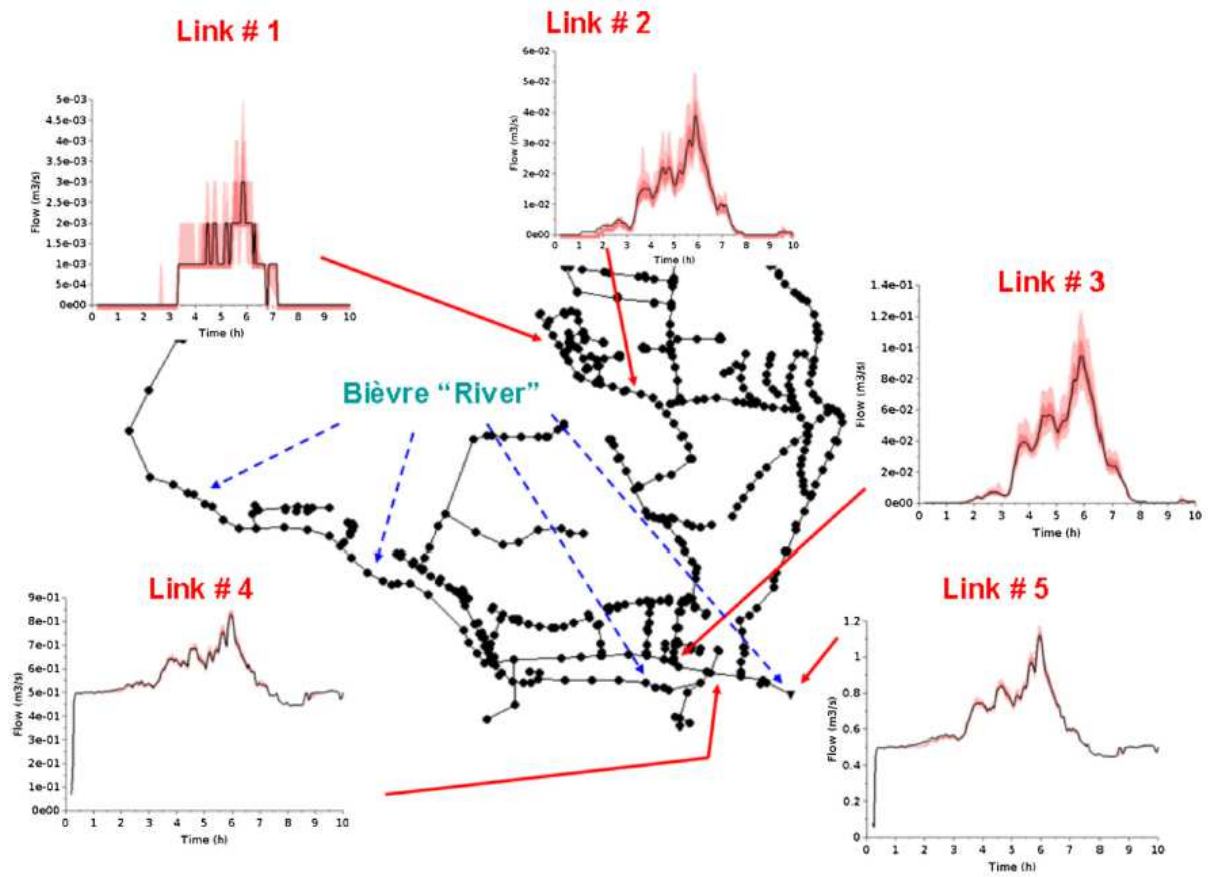


818

819 Figure 6: Water height simulated with the help of Multi-Hydro using raw radar data as rainfall

820 input, and measurements at the Pont-de-Pierre for the four events: (a) 09-02-2009, (b) 14-07-

821 2010, (c) 15-08-2010, (d) 15-12-2011

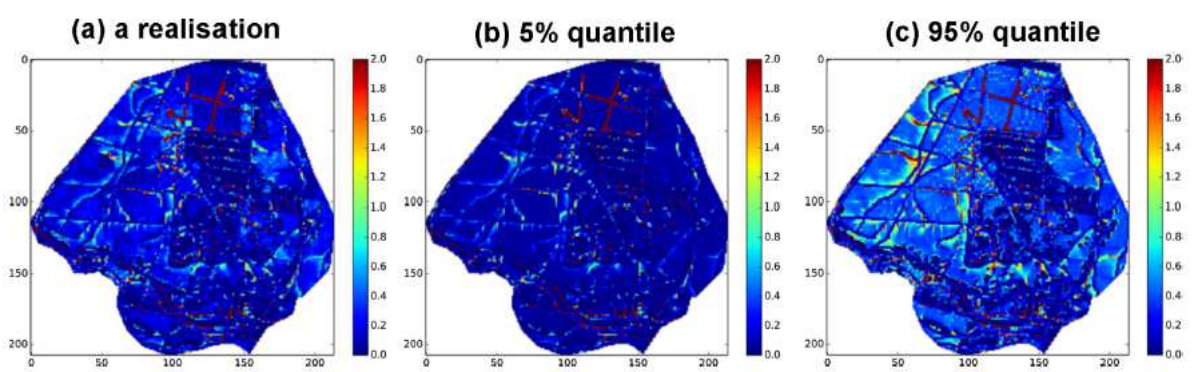


822

823 Figure 7: Simulated flow with the raw radar data (black), $Q_{0.25}$ and $Q_{0.75}$ (dark pink colour),

824 $Q_{0.05}$ and $Q_{0.95}$ (light pink colour) for 5 conduits of the studied catchment for the 09-02-2009

825 event.



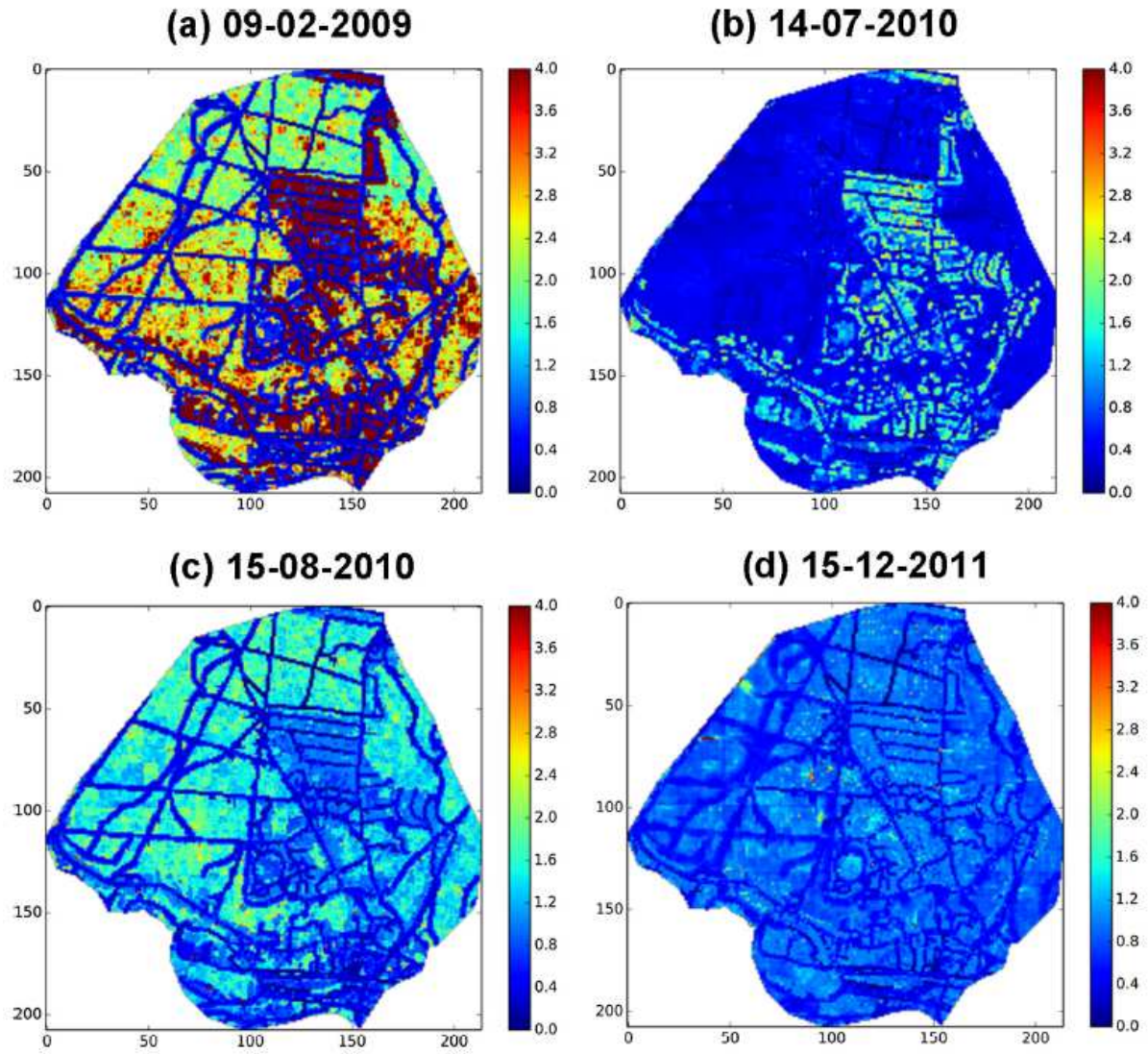
826

827 Figure 8: For the 15-12-2011 event. (a) Map of the computed maximum water depth for a

828 realisation of the downscaled rainfall field. (b) 5% quantile map of the maximum water depth

829 over 100 realisations. (c) 95% quantile map of the maximum water depth over 100

830 realisations. Unit is m.

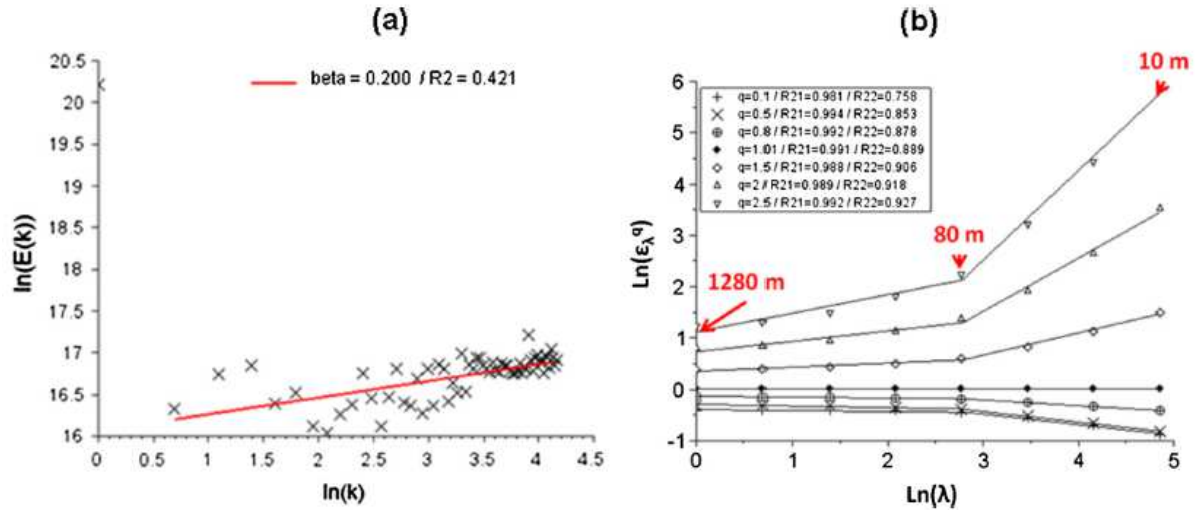


831

832 Figure 9: Map of CV'_{95} (in %) for the maximum water depth for the 09-02-2009 (a), 15-08-

833 2010 (b) and 15-12-2011 (c) events.

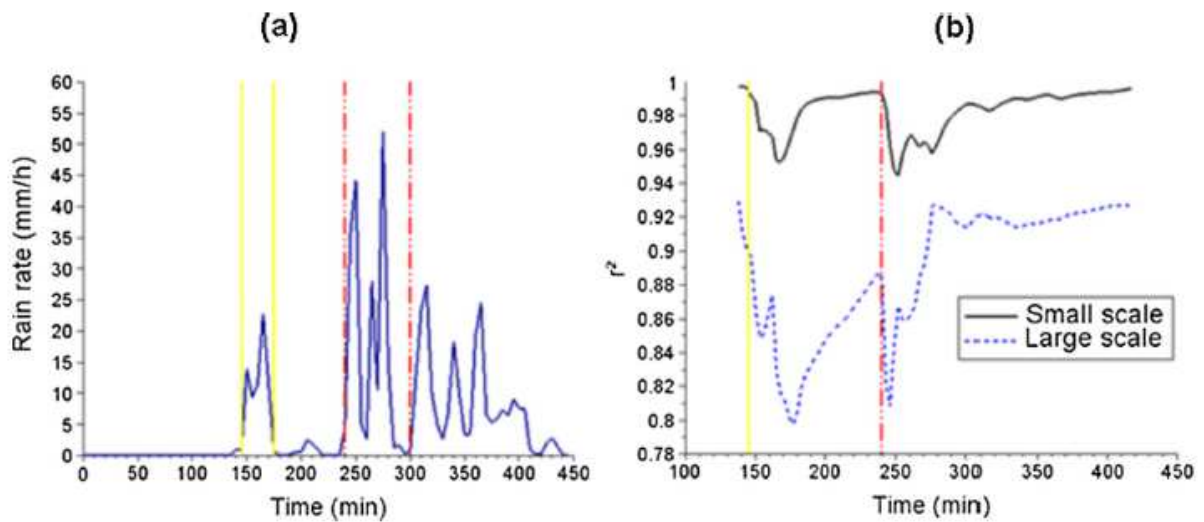
834



835

836 Figure 10: For the 14-07-2010 event and 2D ensemble analysis over all the time steps: (a)

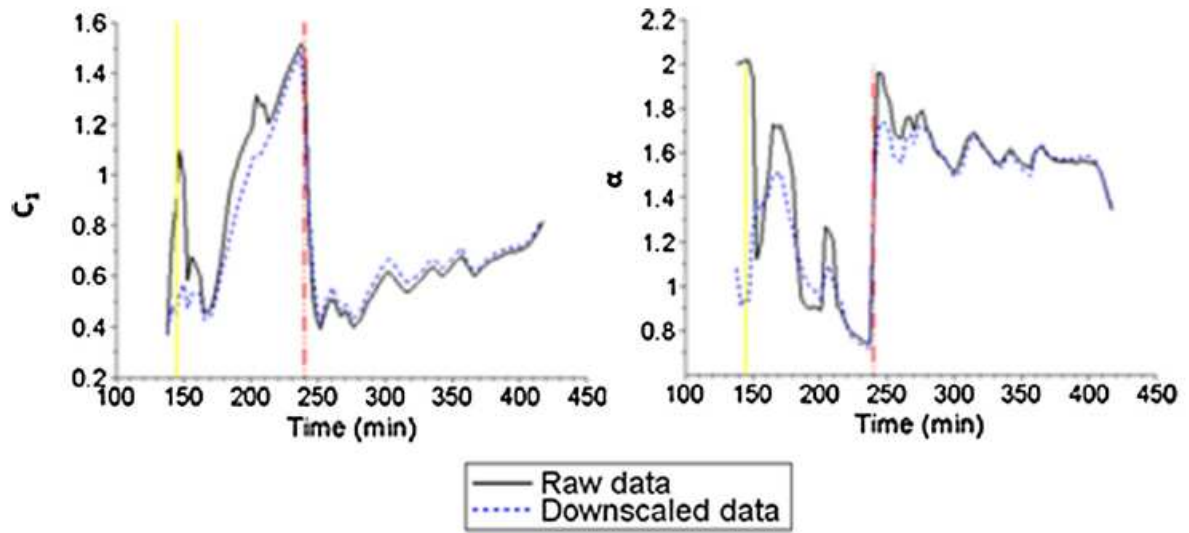
837 Spectral analysis, i.e. Eq. 5 in log-log plot; (b) TM analysis, i.e. Eq. 3 in log-log plot.



838

839 Figure 11: For the 14-07-2010 event: (a) Temporal evolution of the rain rate; (b) Temporal

840 evolution of the r^2 for $q=1.5$ in the TM analysis for the two regimes identified in Fig. 10.

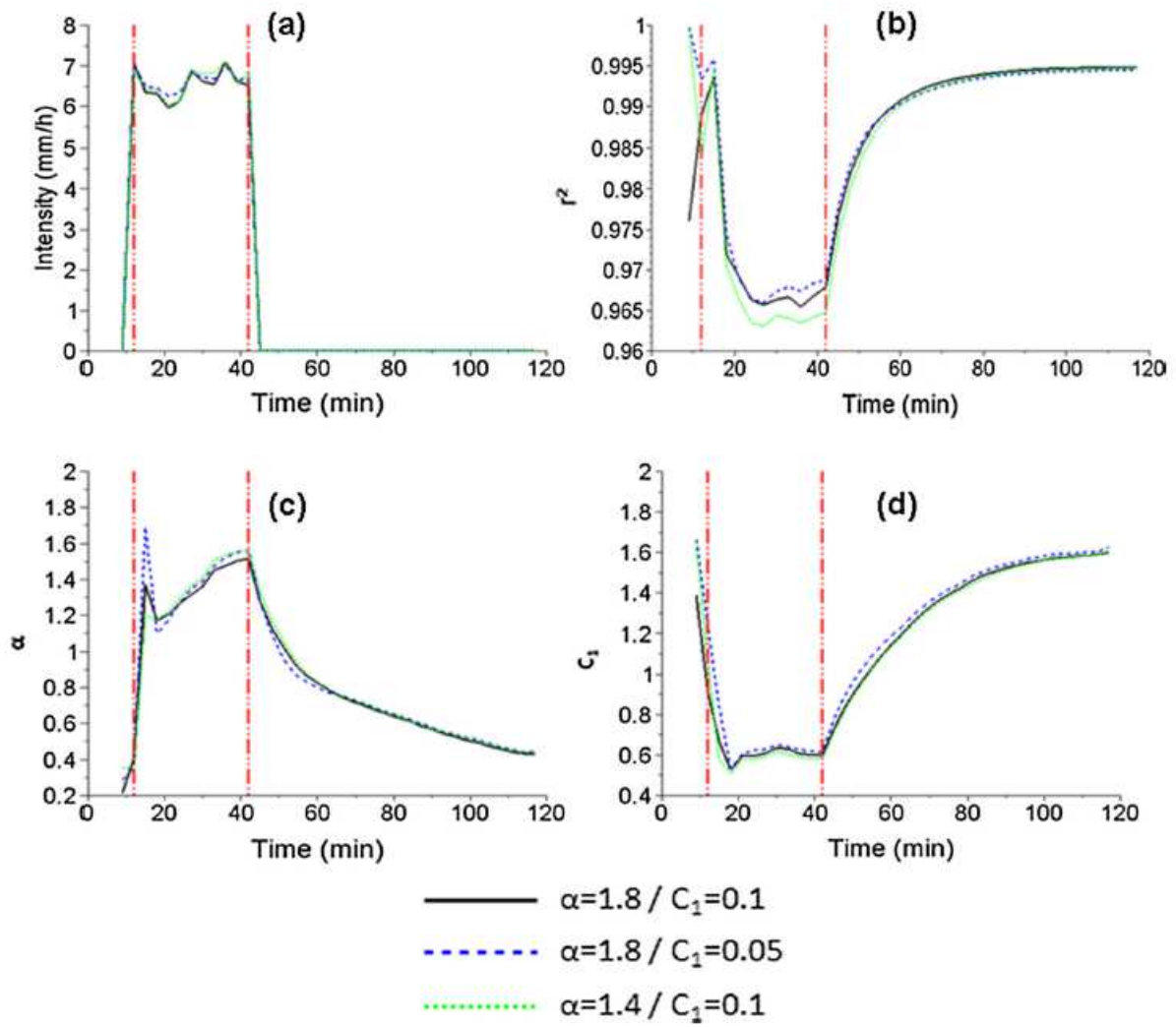


841

842 Figure 12: Temporal evolution of the UM parameters α and C_1 of the maximum water depth

843 field over 3 min for small scales (10 - 80 m) for the 14-07-2010 rainfall event.

844



845

846 Figure 13: For three synthetic rainfall events with different sets of UM parameters; temporal

847 evolution of the average rain rate over the catchment (a), and for the simulated overland

848 maximum water depth, r^2 (b), α (c) and C_1 (d) for small scales (10 m-80 m).

849