



**HAL**  
open science

## Multifractal characterisation of a simulated surface flow: A case study with Multi-Hydro in Jouy-en-Josas, France

Auguste Gires, Jean-Baptiste Abbes, Igor da Silva Rocha Paz, Ioulia  
Tchiguirinskaia, D Schertzer

### ► To cite this version:

Auguste Gires, Jean-Baptiste Abbes, Igor da Silva Rocha Paz, Ioulia Tchiguirinskaia, D Schertzer. Multifractal characterisation of a simulated surface flow: A case study with Multi-Hydro in Jouy-en-Josas, France. *Journal of Hydrology*, 2018, 558, pp.482 - 495. 10.1016/j.jhydrol.2018.01.062 . hal-01704091

**HAL Id: hal-01704091**

<https://enpc.hal.science/hal-01704091v1>

Submitted on 8 Feb 2018

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

1 Multifractal characterisation of a simulated surface flow: a case study with  
2 Multi-Hydro in Jouy-en-Josas, France

3 Auguste Gires\*<sup>1</sup>, Jean-Baptiste Abbes<sup>1</sup>, Igor da Silva Rocha Paz<sup>1,2</sup>, Ioulia Tchiguirinskaia<sup>1</sup>,  
4 Daniel Schertzer<sup>1</sup>

5 1. HMCO, *École des Ponts ParisTech, U. Paris-Est, Champs-sur-Marne, France,*

6 2. *Instituto Militar de Engenharia, Rio de Janeiro, Brazil*

7 \* Corresponding author: Auguste Gires, [auguste.gires@enpc.fr](mailto:auguste.gires@enpc.fr) (phone +33 1 64 15 36 48; fax  
8 +33 1 64 15 37 64)

9

10 Abstract

11 In this paper we suggest to innovatively use scaling laws and more specifically  
12 Universal Multifractals (UM) to analyse simulated surface runoff and compare the retrieved  
13 scaling features with the rainfall ones. The methodology is tested on a 3 km<sup>2</sup> semi-urbanised  
14 with a steep slope study area located in the Paris area along the Bièvre River. First Multi-  
15 Hydro, a fully distributed model is validated on this catchment for four rainfall events  
16 measured with the help of a C-band radar. The uncertainty associated with small scale  
17 unmeasured rainfall, i.e. occurring below the 1km x 1km x 5min observation scale, is  
18 quantified with the help of stochastic downscaled rainfall fields. It is rather significant for  
19 simulated flow and more limited on overland water depth for these rainfall events. Overland  
20 depth is found to exhibit a scaling behaviour over small scales (10 m - 80 m) which can be  
21 related to fractal features of the sewer network. No direct and obvious dependency between  
22 the overland depth multifractal features (quality of the scaling and UM parameters) and the  
23 rainfall ones was found.

24

25 1) Introduction

26

27         The combined effects of a growing urbanisation - approximately 80% of Europe's  
28 population will live in cities by 2020 (EEA, 2014) - and potential increase of extreme events  
29 as a consequence of climate change (IPCC, 2013) expose more and more people to surface  
30 pluvial flooding. Pitt (2008) carried out a review on flood events in the United Kingdom and  
31 showed that two thirds of the flood related damages were caused by surface water flooding.  
32 Urban flooding has become a growing concern in Europe, hence a significant number of  
33 European research projects address this issue, along with national counterparts. The purpose  
34 of these projects is to increase the resilience of urban areas through improvement of both real  
35 time management of extreme events and long term planning. We can cite FP7 SMARTesT  
36 (<http://floodresilience.eu/>), CORFU (<http://www.corfu-fp7.eu/>), Climate KIC Blue Green  
37 Dream ([www.bgd.org.uk](http://www.bgd.org.uk)) or the INTERREG IV RainGain project (<http://www.raingain.eu>)  
38 among others.

39         There is a need to improve the understanding of urban surface flow. Indeed, there is a  
40 growing interest for 2D models in urban environment for both operational and research  
41 applications (Bolle et al., 2006; Carr and Smith, 2006; Chen et al., 2007; Deltares, 2013; DHI,  
42 2011; Giangola-Murzyn et al., 2014; Innovyze, 2012, 2103; Phillips et al., 2005; XP  
43 Solutions, 2012). Such models aim at actually modelling processes in a physically based  
44 manner, while the most commonly used semi-distributed models take them into account  
45 through tailored lumped models. In case of overflow they simply consider a volume output  
46 from the sewer system and deduce a local water depth, but the dynamical behaviour of the  
47 water added on the ground is not addressed. Basically, urban surface flow is not commonly  
48 perceived as a geophysical process and is therefore not addressed with geophysical tools

49 capable of grasping its intrinsic complexity visible across all scales. Indeed, it results from the  
50 non-linear interactions between the highly spatially and temporally variable rainfall field, the  
51 topography and the strongly inhomogeneous land use cover.

52 In this paper we suggest to use multifractal tools, which are commonly used in  
53 geophysics to characterise and simulate fields extremely variable over a wide range of scales;  
54 such as wind turbulence, rainfall, river flow or topography (see Schertzer and Lovejoy, 2011  
55 for review). Such tools have seldom been used in an urban context. Gires et al. (2013, 2014b)  
56 used them to downscale rainfall to quantify the uncertainty associated with small scale rainfall  
57 variability, or to characterise the variability across scales of simulated flow in conduits  
58 (sewer). To the knowledge of the authors it has never been used to study either surface runoff  
59 flow (urban drainage) or surface flow in general including stream rivers. Investigating the  
60 potential multifractal features of surface flow and notably whether it inherits rainfall features  
61 is the main purpose of this paper and constitute its main novelty. In addition, this case study  
62 will also be used to quantify the uncertainty associated with small scale rainfall variability,  
63 not only on the simulated flow which has already been done on other catchments, but also on  
64 the surface flow.

65 Given the lack of measurements of distributed data of surface runoff, outputs of a  
66 numerical model are analysed. The model used is Multi-Hydro (El Tabach et al., 2009 for an  
67 initial version and Giangola-Murzyn, 2014 for a recent one) developed at the Ecole des Ponts  
68 ParisTech. It is implemented on a 3.017 km<sup>2</sup> peri-urban catchment in Jouy-en-Josas (South-  
69 East of Paris), which exhibits steep slopes and both forest and urbanised areas. Achieving  
70 such an analysis is relevant only if a distributed rainfall field is used as model input. Météo-  
71 France radar mosaics with a resolution of 1 km in space and 5 min time (Tabary, 2007;  
72 Tabary et al., 2007) for four events that occurred between 2009 and 2011 are used. When

73 needed, the rainfall field is downscaled both in space and time from the raw radar data, in  
74 order to simulate the improvement that could be made with higher radar resolution.

75 The model and the study area data for its implementation are presented in details in  
76 section 2. The multifractal framework and analysis methods are presented in section 3.  
77 Results are discussed in section 4 and 5. More precisely, the validation of the model and  
78 quantification of the uncertainty associated with small scale unmeasured rainfall variability on  
79 both simulated sewer flow and maximum water depth is carried out in section 4. Multifractal  
80 characterization of overland water depth is addressed in section 5. Main conclusions are  
81 highlighted in section 6.

82

83

84 2) Model and catchment

85

86 2.1) The Multi-Hydro model

87

88 Multi-Hydro is a multi-module model whose goal is to model and predict the impacts  
89 of rainfall events in urban and peri-urban areas. In this paper, there is an emphasis on heavy  
90 rainfall events. Following the approach of various recent developments of hydrological  
91 models (Djordjevic et al., 1999; Fletcher et al., 2013; Hsu et al., 2000; Jankowfsky, 2011;  
92 Rodriguez et al., 2008); it makes different modules interact, each of them echoing a portion of  
93 the water cycle in urban areas (surface runoff, infiltration, ground water flow, sewer flow).

94 Each of the modules integrated in Multi-Hydro relies on open-source software  
95 packages that have already been widely used and validated by the scientific community. The  
96 surface module is based on TREX (Two dimensional Runoff, Erosion and eXport model,  
97 Velleux et al., 2011) which solves fluid mechanics equations for surface flow (diffusive wave

98 approximation of 2D Saint-Venant, see p. 6-7 of the TREX user manual) and infiltration  
99 (simplification of Green and Ampt equation). The sewer or drainage module, which is based  
100 on SWMM developed by the US Environmental Agency (Storm Water Management Model,  
101 Rossman, 2010), is a 1D-model dealing with sewer flows through numerical solutions of  
102 Saint-Venant 1D equations in pipes. The interactions between the surface and sewer flow is  
103 handled through the gully pixels. These interactions (input or output of water) between the  
104 surface and sewer flow are carried out every 3 min. When there is no overflow, gully pixels  
105 are considered to have an infinite infiltration rate, and the water passing through them is  
106 directly inputted into the corresponding node of the sewer model. This way of modelling  
107 implies that a large transport capacity is assumed for gully, especially with 10 m pixel size as  
108 in this paper (see below). Future developments of Multi-Hydro will enable to improve the  
109 model with regards to this coarse assumption. They could notably rely on the experimental  
110 and computational studies of gully inflow capacity, including 3D CFD studies, which analyse  
111 phases in the flow, inlet capacity, reverse flow when the piezometric level in the sewer is  
112 beyond the ground level (Despotovic et al., 2005; Djorjevic et al., 2005). In case of sewer  
113 overflow through a node, the corresponding gully pixel is converted into a road pixel and the  
114 water exiting the node is inputted on this pixel (considered as a source in TREX). There is  
115 also a module handling ground water flow which was not included in this study to limit  
116 computation time.

117         In order to run Multi-Hydro, data needs to be shaped in a standard format. Commonly  
118 available Geographical Information System (GIS) data, such as land use and topography  
119 provided in France by IGN (the French agency producing geographical information) are  
120 inputted to MH-AssimTool (Richard et al., 2014). This software formats the inputs with the  
121 desired resolution and makes Multi-Hydro a transportable model, rather easy to implement on  
122 a new catchment. Once a resolution is chosen, one has to affect an elevation and a land use

123 class to each pixel. The elevation is obtained by an interpolation of the raw available data.  
124 With regards to the land use, a priority order has been determined to assign a unique land use  
125 class for a given pixel according to the hydrological importance of the given class instead of  
126 the surface represented by this class: if a gully is located on a pixel, the entire pixel will be  
127 considered as a gully. This process is repeated in the following order for this case study:  
128 roads, houses, forest, grass, and water surface. See Ichiba et al. (2017) for a comparison with  
129 other possible strategies.

130 In this paper, the model was implemented with pixels of size 10 m x 10 m. Given the  
131 obtained results discussed below it was not found necessary to run it at higher resolution  
132 which makes computation time too long. For an in-depth analysis of the relation between the  
133 selected pixel size and simulated flow, which is not the purpose of this paper, refer to Ichiba  
134 (2016). Multi-hydro was not calibrated, i.e. standard values for the parameters describing a  
135 land use class are used (hydraulic conductivity, capillary suction, moisture deficit, Manning's  
136 coefficient, depth of interception). Raw or downscaled radar data are used as input of the  
137 model.

138

## 139 2.2) Presentation of the study area

140

141 The catchment studied in this article is located in Jouy-en-Josas (Yvelines County,  
142 South-west of Paris). It occupies a 3.017 km<sup>2</sup> area, mainly on the left bank of the Bièvre  
143 River. A small portion of the right bank near the river bed is also included. The remaining  
144 portion of the right bank is drained to a small river that flows into the Bièvre River  
145 downstream the outlet of the studied catchment. The Bièvre River is a tributary of the Seine  
146 River which it meets in Paris. It flows through increasingly urbanised areas along its 33 km  
147 path. This has led to strongly modify its natural bed, both in underground pipes which are

148 integrated in the storm water sewer system, or in a highly artificial open air bed. An effort is  
149 currently undertaken to restore its “natural” aspect.

150 A striking feature of this catchment is that, unlike the previous ones studied with  
151 Multi-Hydro (Giangola-Murzyn et al., 2014; Gires et al., 2014a), it exhibits steep slopes.  
152 There is a difference of approximately 100 m between the plateau in the north of the  
153 catchment, and the outlet of the catchment (Fig. 1). The downhill portion strengthens overland  
154 runoff, and the combination of pluvial and fluvial processes on the river bank has led to  
155 severe flooding in 1973 and 1982. Some details are available on the SIAVB (Syndicat  
156 Intercommunal d’Assainissement de la Vallée de la Bièvre, the local authority in charge of  
157 urban drainage of the area) website [http://www.siavb.fr/gestion\\_des\\_crues.aspx](http://www.siavb.fr/gestion_des_crues.aspx). Urbanisation  
158 and imperviousness are concentrated along the river bank, and on a housing estate along one  
159 major North-South road. The remaining of this semi-urban catchment is mainly made of  
160 forests. The sewer system is a separate one, and the storm water is routed into the Bièvre  
161 River.

162 Following the severe flooding, the SIAVB has created 15 storage basins (integrated in  
163 the landscape) along the Bièvre River to mitigate flooding risks. One, the Bassin des Bas Près,  
164 is located just upstream the Jouy-en-Josas catchment. The outlet of this basin is equipped with  
165 flow and height gauges operated in real time. There is a second measuring point of water  
166 depth, few meters upstream the outlet of the catchment, at the “Pont de Pierre” (Fig. 1). This  
167 gauge has been installed to monitor the river level and to protect a music school by triggering  
168 a warning system in case of elevated height. Given the position of the two measuring points,  
169 Multi-hydro will only be validated on the area drained by the sewer network represented in  
170 green in Fig. 1. The forest corresponds approximately to 60% of the catchment ( $\sim 2 \text{ km}^2$ ).  
171 Although it is only possible to validate the implementation of the model on a portion of the  
172 catchment, the whole area is modelled to ensure the accuracy of flow over the areas actually



173 used for validation. The river is part of the storm water sewer system in Jouy-en-Josas and is  
174 modelled as a pipe in Multi-Hydro drainage module. Indeed, through the city, the river bed is  
175 highly artificial or even underground. The long and West – East oriented pipe located in the  
176 South of the Basin (Fig. 1, left) is actually the Bièvre River.

177

178 2.3) Fractal dimensions of the impervious surfaces and of the sewer system

179

180 The studied catchment is located in a semi-urbanised area. The impervious surfaces  
181 are highly relevant for hydrology since they basically correspond to areas where runoff is  
182 quickly active during a storm event. Thanks to the determination of land use per pixel in MH-  
183 AssimTool, the evaluation of the impervious areas can be done in an apparent simple way by  
184 calculating the number of pixels of roads, buildings and gullies (since the water falling on  
185 gully pixels is immediately routed to the sewer network, they are considered as impervious).

186 This impervious surface depends on the resolution at which it is computed. Indeed, an  
187 imperviousness of 55%, 50%, 42%, 32% and 25% is obtained with pixels of size 20, 15, 10,  
188 5, 2 m respectively. This is due to the priority order set in the data assimilation tool that  
189 affects a land use for each pixel. This order prioritizes impervious areas (Fig. 2). Obviously  
190 these values strongly depend on the approach implemented to affect a land use class to a  
191 pixel. As previously mentioned, comparison with other approaches can be found in Ichiba et  
192 al. (2017). Investigations on the possibility of having different pixel size according to the land  
193 use should also be envisaged in the future, in order to for instance refine the pixels for roads  
194 and gullies and coarser them for forests. Coming back to the imperviousness percentages  
195 found in this paper, it is possible to use the notion of fractal dimension, which is scale  
196 invariant, to explain these figures. The fractal dimension  $D_F$  of a geometrical set (here the  
197 impervious pixels) is obtained with the help of the following equation:

198  $N_\lambda \approx \lambda^{D_F}$  Eq. 1

199 where  $N_\lambda$  is the number of impervious pixels, and  $\lambda$  is the resolution defined as the ratio  
200 between the outer scale  $L$  of the phenomenon and the observation scale  $l$  ( $\lambda = \frac{L}{l}$ ). It  
201 characterizes the space occupied by a geometrical set in a scale invariant way. The symbol  $\approx$   
202 denotes an asymptotic convergence and absorbs slowly varying prefactors.

203 For the studied catchment, it appears that the impervious areas exhibit a fractal  
204 dimension. Indeed Eq. 1 is plotted in log-log for the geometrical set consisting of the  
205 impervious pixels at the 2-m resolution (imperviousness of 25 %), and a straight line is  
206 retrieved on the whole range of scales, i.e. 2m-2048m (Fig. 3.a). This a basic feature of the  
207 catchment. The fact that the points corresponding to the catchment representation at 20, 15,  
208 10, 5, 2 m obtained with MH-AssimTool are along this straight line (circled cross on Fig. 3.a)  
209 is simply a consequence of the priority order set for affecting a land use class to a pixel  
210 (impervious classes are prioritised over pervious ones). This confirms the fact that even  
211 though the represented imperviousness varies with scale, a feature (the fractal dimension) is  
212 conserved and provides a quantification of the level of urbanisation. We find  $D_F$  equal to 1.73  
213 for this catchment. In a previous study Gires et al. (2014a), found that for a highly urbanised  
214 area in Seine-Saint-Denis (North-East of Paris), the fractal dimension was of 1.85 from on  
215 scales ranging from 1 m to 1024 m. Given that this catchment is less urbanised, it was  
216 expected to obtain a smaller fractal dimension.

217 The same study was performed on the sewer system (Fig. 3.b). In this case, the  
218 geometrical set studied is the “rasterised” sewer system. If a pixel is crossed by a conduit  
219 belonging to the storm water sewer network, then it is considered as part of the sewer system..  
220 Two scaling regimes can be identified: from 10 m to 80 m the fractal dimension is 1.03 and  
221 from 80 m to 1280 m it is 1.76. For small scales, the dimension is close to 1, and it simply  
222 reflects the 1D intrinsic nature of the sewer system. For large scales, the structure of the

223 network becomes apparent, and exhibits a scaling behaviour. For large scales the value is  
224 slightly smaller than the 1.85 found on the Seine-Saint-Denis catchment in Gires et al.  
225 (2014a) which is consistent with the fact that this one is less urbanised. The similarity  
226 between both fractal dimensions (imperviousness and large scale sewers) indicates that it is a  
227 relevant way of quantifying a level of urbanisation for the area. See Gires et al. (2017) for an  
228 extension of this approach to 10 areas in 5 European countries.

229

#### 230 2.4) Rainfall data

231

232 Four rainfall events, which occurred between 2009 and 2011, are studied in this paper.  
233 Simulations are performed using Météo-France radar mosaic, which provides a spatially  
234 distributed data with a resolution of 1 km x 1 km x 5 min (the closest radar is the C-band one  
235 of Trappes located 15 km West). For three events the data recorded with the help of a rain  
236 gauge operated by the SIAVB located a few hundred meters south of the catchment at the  
237 “Bassin des Bas Près” is also available. Because of (i) the standard 0.2 mm discretization  
238 issue of the tipping bucket rain gauge (data is number of tips equal to 0.2 mm) which prevents  
239 it from providing reliable intensity, (ii) the gap between the observation scales of the two  
240 measuring devices (see Gires et al., 2014b, for an in-depth analysis of this issue) and (iii) the  
241 fact that the rain gauge is furthermore outside of the catchment; it is not possible to use the  
242 rain gauge data for other purpose than a rough check of the accuracy of radar data. It is done  
243 by comparing the cumulative volumes of rainfall for each studied event which are displayed  
244 in Table 1 along with their main features. Gires et al. (2014b) used data from dense network  
245 of point measurement devices (rain gauges or disdrometers) distributed over 1 km<sup>2</sup> and  
246 showed that the cumulative depth differences between devices could reach more than 40 %  
247 for individual rainfall events (of the same order of magnitude as the one discussed here). They

248 showed with the help of numerical simulations that similar values were found simply taking  
249 into account small scale rainfall variability. Here the maximum observed differences are 34%,  
250 which suggests that the agreement between the two devices is acceptable, i.e. smaller than  
251 expected uncertainty simply due to the scale gap between the two measuring devices. Authors  
252 did not have access to longer time series of both radar and rain gauge to perform a more in-  
253 depth evaluation of the radar versus rain gauge measurement for this specific point, which  
254 would be the topic of another study. The temporal evolutions of the radar rain rate averaged  
255 over the catchment are displayed in Fig. 4. These events were selected because they are heavy  
256 ones. However they are not extreme ones, indeed over durations of 1 h and 4 h, only the 14  
257 July 2010 event has a return period greater than 1 year (data from a rain gauge located in the  
258 Paris area that was available to the authors was used to obtain these estimates). For the July  
259 event, the return period is of about 1 year for a duration of 1 h and of about 2 years for a  
260 duration of 4 h.

261

### 262 3) Methods

#### 263 3.1) Multifractal framework

264 The Multifractal framework is used for several purposes throughout this paper to  
265 characterize the variability across scales of fields, and is therefore presented here in a generic  
266 way. Only basic properties are discussed here, and interested readers are referred to the recent  
267 review by Schertzer and Lovejoy (2011) for more details. The general assumption of  
268 multifractal fields is that they are generated by an underlying scale invariant multiplicative  
269 cascade process. In such process, a structure at a given scale is divided into smaller structures  
270 at smaller scale and the value of a child structure is equal to the value of the parent structure  
271 multiplied of a random increment. The process is scale invariant in the sense that the way  
272 structures are divided into sub-structures and the probability distribution of the random

273 multiplicative increments are the same at all scales. A consequence is that statistical  
 274 properties of such fields are conserved across scales. More precisely let us denote  $\varepsilon_\lambda$  a field at  
 275 resolution  $\lambda$  ( $=L/l$ , where  $l$  is the observation scale and  $L$  the outer scale of the phenomenon as  
 276 for the fractal dimension definition). The probability of exceeding a given threshold ( $\lambda^\gamma$ ),  
 277 defined with the help of the scale invariant notion of singularity  $\gamma$  (the thresholds depend on  
 278 the observation scale, but not the singularity),

279  $\Pr(\varepsilon_\lambda \geq \lambda^\gamma) \approx \lambda^{-c(\gamma)}$  Eq. 2,

280 and the moment of order  $q$ ,

281  $\langle \varepsilon_\lambda^q \rangle \approx \lambda^{K(q)}$  Eq. 3,

282 exhibit a power law relation with regards to the resolution at which they are computed. As for  
 283 Eq. 1, the symbol  $\approx$  denotes an asymptotic convergence and absorbs slowly varying  
 284 prefactors. Equations 2 and 3 define respectively the codimension function  $c(\gamma)$  and the  
 285 moment scaling function  $K(q)$ , which both fully characterize the variability across scales of  
 286 the field.  $c(\gamma)$  and  $K(q)$  contain the same information and are related by a Legendre transform  
 287 (Parisi and Frish, 1985). Eq. 2 can be understood from the simpler notion of fractal dimension  
 288 (Eq. 1). Indeed, an intuitive interpretation of a multifractal field is that the geometrical sets  
 289 made of each portion of the field greater than given thresholds are fractal and characterized by  
 290 fractal dimensions. To be mathematically more rigorous the notion of threshold is replaced by  
 291 the scale invariant one of singularity.

292 By generalizing the central limit theorem Schertzer and Lovejoy (1987) showed that  
 293 any conservative scale-invariant multiplicative processes converge toward Universal  
 294 Multifractals (in a similar way as re-normalized sum of identical and independent random  
 295 variables converge toward normal distribution as long as their variance is defined). For  
 296 Universal Multifractals (UM), i.e. this limit behaviour,  $K(q)$  and  $c(\gamma)$  functions are defined

297 with the help of only two relevant parameters with a physical interpretation. They are known  
298 as UM parameters  $C_1$  and  $\alpha$ :

299 -  $C_1$  is the mean intermittency which measures the average sparseness of the field.  $C_1=0$  for a  
300 homogeneous field.

301 -  $\alpha$  is the multifractality index ( $0 \leq \alpha \leq 2$ ) and measures how fast the intermittency evolves  
302 when considering level of activity slightly different from the average one.

303 Great values of  $\alpha$  and  $C_1$  corresponds to strong extreme. A common tool to assess the  
304 extremes of a field is the scale invariant notion of maximum probable singularity  $\gamma_s$   
305 observable (Hubert et al., 1993; Douglas and Barros, 2003; Royer et al., 2008; Gires et al.,  
306 2014a). It is defined for a unique sample by

$$307 \quad c(\gamma_s) = d \quad \text{Eq. 4}$$

308 Where  $d$  is the dimension of the embedding space, i.e.  $d = 1$  for time series and  $d = 2$  for  
309 maps.

310 The power spectrum (Fourrier transform of the auto-correlation function) of such  
311 multifractal field exhibits a scaling relation with wave number  $k$ :

$$312 \quad E(k) \approx k^{-\beta} \quad \text{Eq. 5}$$

313 where  $\beta$  is the spectral slope.

314

### 315 3.2) Uncertainty associated with small scale rainfall

316 The purpose of this section is to explain the approach implemented to quantify the  
317 uncertainty associated with small scales rainfall variability, i.e. which is occurring below the 1  
318 km x 5 min scale currently provided by the C-band radar operating in this area. The same  
319 methodology as in Gires et al. (2013, 2014a) is implemented, and only basic ideas are  
320 explained here. Firstly, an ensemble of downscaled rainfall fields is generated, then each  
321 realisation is inputted into the numerical model and finally the disparities within the ensemble

322 of outputs, which reflect the studied uncertainty, are analysed and quantified. 100 sample  
 323 ensembles are used. The downscaling technique relies on the Universal Multifractal  
 324 framework. It basically consists in stochastically continuing a space-time cascade process that  
 325 has been validated on the available range of scales. The resolution of the downscaled rainfall  
 326 field is 12 m in space and 20 s in time starting from the original 1 km and 5 min of the  
 327 available radar data. The process has been validated down to such small scales (Gires et al.,  
 328 2014b).

329 The disparities among the simulated ensembles are quantified with the help of quantile  
 330 analysis. Let us first illustrate this with the flow output, but the same is done for maximum  
 331 water depth at each pixel. For each time step the 5, 25, 75 and 95 % quantiles are computed,  
 332 and give the envelop curves  $Q_{0.05}$ ,  $Q_{0.25}$ ,  $Q_{0.75}$ , and  $Q_{0.95}$ , respectively. The width between  
 333 these curves characterizes the uncertainty interval on simulated flow. It is quantified with the  
 334 help of two pseudo-coefficients of variation computed as:

$$335 \quad CV_{95}' = \frac{Q_{0.95}(t_{PF, radar}) - Q_{0.05}(t_{PF, radar})}{2 * PF_{radar}} \quad \text{Eq 6.a}$$

$$336 \quad CV_{75}' = \frac{Q_{0.75}(t_{PF, radar}) - Q_{0.25}(t_{PF, radar})}{2 * PF_{radar}} \quad \text{Eq 6.b}$$

337 where  $t_{PF, radar}$  is the time of the peak flow simulated with the raw radar data  
 338 ( $PF_{radar}$ ).

339

### 340 3.3) Multifractal analysis of overland water depth maps

341 There is no distributed data available for overland water depth over large areas, but it  
 342 is possible to study the fields obtained with the help of numerical simulations with spatially  
 343 distributed rainfall as input. Maps of water depth during runoff at the end of each 3 min  
 344 Multi-Hydro loop are studied.

345           Technically, in this paper an area of 128 x 128 pixels of size 10 m x 10 m is extracted  
346 from the map of the catchment to carry out the analysis. Both ensemble analysis (i.e.  
347 considering all successive maps as independent realisations of the same process and upscaling  
348 them individually before taking the mean in Eq. 2 and 3) and individual time step analysis  
349 (i.e. to obtain temporal evolutions of the various parameters) are performed. Finally, analyses  
350 are done in 2D on the maps but also in 1D on the columns or the lines of pixels over the  
351 catchment, in a North-South direction and in an East-West direction respectively (Fig. 5). The  
352 purpose of this is to monitor a possible influence of the slope over the generated runoff  
353 scaling properties.

354

355

#### 356 4) Implementation of the Multi-Hydro model on the Jouy-en-Josas catchment

357

##### 358 4.1) Validation with raw radar data

359

360           The validation of the model is achieved by comparing the water height measured at the  
361 Pont-de-Pierre gauge with the simulated one. Before going on authors would like to highlight  
362 that a proper validation on this case study is not possible given the available data, and will  
363 therefore limit this section to checking that the model approximately behaves well. The main  
364 reasons for this problem are:

365 - Only one measuring point is available for the whole catchment taking into account

366 approximately an area of 2 km<sup>2</sup>.

367 - The uncertainty associated with this water level gauge is high. Indeed, it is not operated for  
368 accurate hydraulic measurement but to trigger an alarm to evacuate a music school located  
369 nearby. The main issue is that the shape of the river bed cross section at this point is not



370 available. The width was estimated at around 1.80 m, using aerial photography from IGN and  
371 an approximate measure from few meters away. In order to correctly model the pipe, we used  
372 Multi-Hydro and tested various types of conduits. Finally, we chose to model the Bièvre as a  
373 circular pipe, with free surface of 2 m diameter, which is close to the approximate  
374 measurement. This choice is only an approximation which does not take into account the  
375 variations in time of this shape due the fact that the bottom of the river bed is not flat and  
376 contains moving rocks and changing vegetation.

377 - There is a lack of available data on initial soil saturation which is one of main sources of  
378 uncertainty and can biased runoff (see Shah et al., 1996; Zehe et al., 2005) especially at the  
379 beginning of the event. In this paper, dry conditions were considered at the beginning of each  
380 event. A sensitivity test was conducted by considering a saturated soil at the beginning. A  
381 slight increase (few percent) of simulated flow was noted only during approximately the first  
382 hour (not shown here). Having longer rainfall time series would enable to simulate the  
383 catchment's behaviour some time before the event and limit the uncertainties associated with  
384 this issue.

385 - The uncertainties on the water input in the Bièvre River at the outlet the Bas-Près storage  
386 basin upstream the catchment are not quantified.

387 - Obviously there are some uncertainties on the radar rainfall measurement itself.

388         The simulation and measurement at the “Pont de Pierre” point for the selected rainfall  
389 events are displayed in Fig. 6. For the 09-02-2009 event we observe a clear overestimation at  
390 the beginning of the event. For the 14-07-2010 event Multi-Hydro with the radar rainfall data  
391 reproduces well the two main peaks, but overestimates the first local maximum of rainfall  
392 intensity and misses the second one. The 15-08-2010 event shows a greater variability in the  
393 first half of the simulation (variations are more pronounced on the model than on the  
394 measurements) but reproduces well the last peak. Finally, for the 15-12-2011 event, the Multi-

395 Hydro model reproduces well the first peak, but the flow decreases more rapidly than the  
396 observations.

397         Given the available data on a limited number of events it is difficult to attribute the  
398 observed discrepancies to one or several of the previously mentioned sources of uncertainty.  
399 Proper validation would indeed require the analysis of much longer time series and more  
400 accurate measurements with better position of sensors. Nevertheless, the obtained results do  
401 not highlight strikingly wrong behaviour of simulated water heights in conduit, and enable to  
402 partially reproduce observations. Finally, it seems that for some events the simulated flows  
403 might be too noisy compared with observed water levels. This should not affect the UM  
404 analysis that follows because the analyses carried out in this paper are spatial ones, i.e. maps  
405 are studied and not time series so the potential effect should be limited. Keeping in mind the  
406 previously mentioned limitations, results suggest that it remains relevant to use this  
407 implementation of Multi-Hydro with a rather coarse 10 m resolution for testing its sensitivity  
408 to small scale rainfall variability and analysing surface runoff with the help of multifractals.  
409 The authors acknowledge that further investigations on other catchments with more  
410 accurately validated models would be needed to fully confirm the findings discussed after.

411

#### 412 4.2) Uncertainty associated with small scales rainfall variability

413         The envelop curves  $Q_{0.05}$ ,  $Q_{0.25}$ ,  $Q_{0.75}$ , and  $Q_{0.95}$  are displayed in Fig. 7 for the 09-02-  
414 2009 event for 5 conduits selected from upstream to downstream, which enables to analyse  
415 the effect of the position of the conduit within the network. Link #4 corresponds to the Pont-  
416 de-Pierre measurement, and #5 to the outlet of the catchment. As it can be seen in Fig. 7, link  
417 #4 and #5 are located along the Bièvre River, and they take into account the significant base  
418 flow in the river coming from upstream the Jouy-en-Josas catchment. It means that they are  
419 obviously less sensitive to local rainfall variability. Similar curves were also generated for

420 water height (not shown) at the Pont-de-Pierre. The computed uncertainty is small and  
421 certainly does not explain the discrepancies between simulations and measurements noticed in  
422 Fig. 6, which are hence not simply due to effects of small scale rainfall variability.

423  $CV'_{95}$  and  $CV'_{75}$  values computed for the selected conduits (Fig. 7) and the four events  
424 are displayed in Table 2. As expected they decrease while considering more and more  
425 downstream conduits. There is a sharp decrease in  $CV'$  when the Bièvre River is reached  
426 because the base flow of the river dampens the effect of local small scale rainfall variability  
427 occurring over the 3 km<sup>2</sup> catchment, but the uncertainty only associated with this effect  
428 remains of roughly 10 % at the outlet whatever the event. The values for up-stream and mid-  
429 stream pipes are great for all events, even for  $CV'_{75}$  which highlights a significant impact of  
430 small scale rainfall variability on the simulated flow. The variability observed in the simulated  
431 flow is basically due to the disparities in the simulated downscaled rainfall fields which are  
432 transferred through the hydrological model (see Gires et al. 2012 for more detailed analysis of  
433 this issue). Small scale rainfall data is needed to understand better, and plan better, some local  
434 flooding due to sewer overflows which have been reported in some areas, notably the street  
435 parallel to the Bièvre River bed in the city (just North of it), There does not seem to have a  
436 straightforward relation between the computed uncertainty and the strength of the event (in  
437 terms of maximum rainfall peak intensity over 5 min). Indeed, the tendency that could be  
438 observed on the 09-02-2010, 15-08-2010 and 15-12-2011 (not a linear one as for example the  
439 peak rainfalls are equal to approximately 7 and 24 mm.h<sup>-1</sup> for respectively the 15-08-2010 and  
440 15-12-2011 event while the computed uncertainties are close) is not confirmed by the results  
441 for the 14-07-2010 event (see Tab. 2). Finally, these values are comparable to the ones that  
442 were obtained on a 1.5 km<sup>2</sup> highly urbanised catchment located 40 Km North-East on the  
443 other side of the Paris area in Gires et al. (2014a). For this catchment,  $CV'_{95}$  values were  
444 ranging from 21 to 56%, 26 to 94% and 22 to 50% from downstream to upstream for the same

445 09-02-2009, 15-08-2010 and 15-12-2011 events respectively (at a different location). The  
446 values are slightly smaller for this catchment and this is likely to be due to lower level of  
447 imperviousness resulting in a smaller portion of rainfall becoming immediately active.

448

449 In this paper, the uncertainty is computed not only on the simulated flow, but also on  
450 the water depth in streets. As for the flow, for each realisation of downscaled rainfall field, the  
451 maximum water depth over the whole simulation is retrieved for each pixel. A sample is  
452 shown in Fig. 8.a for the 15-12-2011 event. The known hot spots are visible, although with  
453 too high values. For example, the modelled maximum water depth reaches more than 15 cm  
454 in the street along the Bièvre River bank in the city and the parallel street just north of it  
455 (already mentioned in the previous paragraph). Although some flooding is regularly reported  
456 by citizens to the SIAVB for these streets, such height was not reported for this event. In the  
457 urbanized portion of the catchment the street network is visible on the maximum water depth  
458 map, meaning the maximum values of water depth maps are reached on the corresponding  
459 pixels. Lower values are found on the on roads/streets located on the steep portion of the  
460 catchment because water moves faster in these areas. Same patterns and numerical values are  
461 obtained for other realisations of the same event. Similar plots are obtained for the other  
462 events with lower depths for the 09-02-2009 and 15-08-2010 (for which a lower cumulative  
463 rainfall depth was recorded) and greater depths for the 14-07-2010 event. Then, as for the  
464 flow analysis previously carried out, the uncertainty on this maximum water depth is  
465 computed with the help of the 5 and 95% quantiles for each pixel and a pseudo-coefficient of  
466 variation. Illustrations of the quantiles maps are shown in Fig. 8.b and 8.c for the 15-12-2011  
467 event. Similar patterns are observed on the two maps, notably for the hotspots previously  
468 mentioned which are visible on both maps. Maps of  $CV'_{95}$  for maximum depth are displayed  
469 in Fig. 9 for the four rainfall events. It appears that the uncertainty is lower for the areas

470 where the greatest maximum depths are found (i.e. on roads) and is also lower for the heaviest  
471 rainfall events. It reaches only few percents on the hottest points. The values (Fig. 9) are  
472 anyway much smaller than those found for sewer flow (Table 2 and Fig. 7). This apparent  
473 contradiction is likely to be due to the fact that most of the rain water is properly handled by  
474 sewers and overflows are limited for these events. It means that for these events disparities in  
475 local amounts will not be visible on ground levels, whereas they are indeed in sewer flows  
476 and water depths. Further investigations with heavier rainfall events should be carried out to  
477 confirm or not this interpretation. The areas with the greatest uncertainty are found in gardens  
478 for the weakest event (09-02-2009), and correspond to places with a very small maximum  
479 depth (smaller than 1mm), meaning that the hydrological relevance is not very high.

480

481

## 482 5) Multifractal characterization of overland water depth

483         Multifractal analyses of overland water depth during rainfall event are presented in  
484 this paper for the 14-07-2010 and 15-12-2011 events which are the two heaviest ones in terms  
485 of maximum rainfall intensity over 5 min (see Table 1).

486         Figure 10.a displays the spectral analysis of the water depth for the 14-07-2011 event.  
487 Maps of water depth for each time steps during the event are used to carry out 2D ensemble  
488 analyses. The quality of the scaling is low, with a coefficient of determination for the linear  
489 regression equal to 0.42. The fact that the spectral slope is close to zero ( $\beta$  is found roughly  
490 equal to 0.2) indicates that the field is conservative, i.e. its mean is conserved across scales. It  
491 is therefore possible to implement directly on the field a Trace Moment (TM) analysis, which  
492 consists in assessing the validity of Eq. 4 by plotting it in log-log. Perfect UM fields would  
493 lead to straight lines. Figure 10.b shows the TM ensemble analysis performed over all the  
494 time steps of the same 14-07-2011 event. Two scaling regimes can be identified: a small

495 scales regime from 10 m to 80 m (right part of Fig. 10.b) and a large scales regime from 80 m  
496 to 1280 m (left part of Fig. 10.b). The coefficient of determination  $r^2$  of the linear regression  
497 for  $q=1.5$  in Fig. 10.b is taken as an indication of the quality of the scaling. The scaling from  
498 small scales (10 m - 80 m) is much more robust than for large scales (80 m to 1280 m), as  
499 illustrated by the  $r^2$  equal to respectively 0.99 and 0.91. Given the low quality of the scaling  
500 for large scales, UM parameter estimates will not be reported and discussed for this regime  
501 because they are not reliable. Furthermore, small scales are crucial for surface runoff because  
502 it is at these scales that it is generated into the drainage system. The location of this break at  
503 approximately 80 m indicates a possible physical interpretation. Indeed, it is the same location  
504 as the break in the fractal analysis of the sewer network and corresponds roughly to the inter-  
505 distance between roads. This would mean that this break is driven by the influence of the  
506 collection of water by sewer network. The more robust scaling behaviour for surface flow is  
507 found for the scales for which the sewer network does not behave yet as network but as  
508 isolated linear pipes. Before going on, it should be mentioned that numerous pixels have very  
509 small depth (see Fig. 8 for an illustration), for which the model uncertainties might be great.  
510 These zeros values or spurious ones close to zero will affect the scaling analysis for small  
511 moments (typically  $q < 0.5$ ) through a multifractal phase transition (see Gires et al., 2012, for  
512 a detailed analysis of this issue). Here the influence of this bias does not extend to moments  
513 close to 1 around which the estimates of UM parameters are carried out, meaning that they are  
514 not affected by this issue.

515

516         Although intrinsically less robust since scaling properties are statistical ones requiring  
517 numerous data to be properly observed, TM analyses were also carried out independently on  
518 each sampling time step of Multi-Hydro (3 min in this paper). The purpose is to see whether  
519 there is an impact of the current rainfall rate on it. Figure 11 displays for the 14-07-2010 event

520 the temporal evolution of both the rainfall rate and the  $r^2$  for  $q=1.5$  in the TM analysis for the  
521 two regimes identified in the ensemble analysis, i.e. small (10 m – 80 m) and large scales (80  
522 m – 1280 m). For this event, two rainfall peaks are observed, and they both result in a sudden  
523 loss of the scaling quality, more pronounced for large scales than small ones. For the first  
524 peak (yellow bars on Fig. 11) the decrease of  $r^2$  lasts approximately 20 min, while it lasts only  
525 few minutes for the second peak (red bars on Fig 11). In both cases the quality of the scaling  
526 behaviour improves again over few tens of minutes. The physical meaning of such loss is not  
527 clear, but could be due to a bad representation of the surface flow process during intense  
528 rainfall (it might take some time to retrieve a realistic surface flow simulation following a  
529 sudden change in rainfall input), a bias in the geometrical repartition, or an intrinsic feature of  
530 the process. For the latter, a possibility is that during intense rainfall period, the surface flow  
531 exhibits more directly the rainfall features than its intrinsic ones which are retrieved once the  
532 flow process has “adapted” to the new conditions. This would explain both the loss of scaling  
533 quality and why scaling properties closer to rainfall ones are observed during these short  
534 periods. Analysis with a higher resolution model would be needed to further investigate this  
535 issue, which would also enable to have access to a wider range of small scales.

536         Similar features are retrieved for the other studied event (15-12-2011). Finally, it  
537 should also be mentioned that similar results are also found when performing the analysis on  
538 the North-South or West-East 1D-samples, which means that the preferential slope of the  
539 catchment (North-South) does not seem to have an influence on the scaling features of the  
540 simulated water depth. In terms of scaling quality, very similar results are also found with raw  
541 radar data, or downscaled rainfall fields suggesting a limited impact of small scale rainfall  
542 variability on these features. The same downscaling process as in section 2 is used.

543

544 UM parameters retrieved on the maximum water depth were computed for small  
545 scales, and are displayed in Table 3 for the two events (14-07-2011 and 15-12-2012) and for  
546 simulations with raw radar data and also a realisation of downscaled rainfall field with  $\alpha=1.8$   
547 and  $C_1=0.1$  (other realisations yield very similar results). The temporal evolutions of  $\alpha$  and  $C_1$   
548 for the 14-07-2011 event are shown in Figures 12.

549 It appears that the UM parameters are also affected by the “jumps” that were noticed  
550 on  $r^2$  in Fig. 11. Indeed after an intense period, sharp increase of  $\alpha$  and decrease of  $C_1$  are  
551 noticed. These pronounced variations mean that the values obtained with ensemble analyses  
552 should not be over-interpreted. Nevertheless few comments can be made. First the values of  
553  $C_1$  are much greater than the ones reported for rainfall (typically 0.1-0.3 at small scale)  
554 meaning that significant levels of water depth are much more concentrated than the rainfall  
555 field, which reflects the influence of the physical processes associated with surface flow on  
556 the transferred field, notably the flow concentration. The most relevant one is the topography  
557 that routes water through specific paths and tends to concentrate it. Second the values of UM  
558 parameters are quite different between the two events. These differences are much greater  
559 than the ones observed on the rainfall fields (see Ichiba, 2016, for a detailed analysis of these  
560 storms) at small scales. This suggests that the large scales rainfall pattern has a strong  
561 influence on the retrieved parameters. Indeed, the topography and small scale rainfall features  
562 are the same between the two simulations; the only difference is the large scale rainfall  
563 features. Thirdly the values of  $\gamma_s$  are rather similar for both events (the differences between  $\alpha$   
564 and  $C_1$  tend to compensate themselves).

565 The temporal evolutions of the UM parameters obtained by inputting raw and  
566 downscaled rainfall data are very similar. The differences are slightly more pronounced on the  
567 values computed on ensemble analysis but as previously said this should not be over-  
568 interpreted given the strong variations visible in the temporal analysis. This similarity



569 highlights the low influence of small scale rainfall variability on the retrieved parameters  
570 which seems to be more dependent on features associated with surface flow process itself or  
571 large scale rainfall.

572

573 In order to test the sensitivity of the results to small scales rainfall features, synthetic  
574 rainfall fields with various sets of known parameters are used as input to Multi-Hydro  
575 simulations. More precisely the pseudo-events tested last 30 min with an average intensity of  
576 10 mm/h. Three pairs ( $\alpha$ ;  $C_1$ ) of parameters are tested: (1.8; 0.1), (1.8; 0.05), (1.4; 0.1). Figure  
577 13 displays the temporal evolutions of the rain rates,  $r^2$ ,  $\alpha$  and  $C_1$  for water depth for the three  
578 synthetic rainfall events.

579 The temporal evolution shows the same general tendency as the one observed with the  
580 real events. A loss of scaling quality is observed during the event itself, and it improves  
581 afterwards.  $\alpha$  and  $C_1$  have a constant behaviour during the rainfall, while they decrease and  
582 increase respectively after the rainfall has stopped. The comparison of the UM parameters for  
583 the overland maximum water depth shows that they do not seem to depend on the small scale  
584 rainfall variability in this case.  $\alpha$  is constant around 1.4 while  $C_1$  is constant around 0.6 during  
585 the rainfall.  $\gamma_s$  is again constant around 1.7 on average. The rainfall UM parameters do not  
586 seem to modify the structure of the overland flow, and its geometrical distribution. Successive  
587 simulations with the same parameters for synthetic rainfall yielded same results. A physical  
588 explanation of the  $C_1$  parameter could be that during the rainfall, the surface flow is more  
589 homogenous due to a ubiquitous input of water. UM parameters on water depth are thus  
590 closer to the rainfall ones (small  $C_1$ ). However after the rain has stopped, the disparities of  
591 simulated water depth are increased due to predominant pathways (roads) or topographic  
592 depressions where the water can accumulate. The greater  $C_1$  after the event could reflect this

593 fact. The smaller values of  $\alpha$  mean that the disparities among the areas where water remains  
594 tend to decrease after the rainfall event.

595         This study seems to highlight the fact that UM parameters  $\alpha$  and  $C_I$  for water depth are  
596 rather relying on the large scale structure of the rainfall and on the catchment features, while  
597 the maximum observable singularity  $\gamma_s$  is conserved for all events. Further studies could  
598 infirm or confirm the fact that  $\gamma_s$  depends on the studied catchment. The temporal evolutions  
599 of the UM parameters also deeply rely on the rainfall rate. Synthetic events with block  
600 structures enabled to stand out rather simple general tendencies. They become more complex  
601 with real rainfall, when the intensity has a higher temporal variability.

602         The temporal evolutions of the UM parameters also enable to quantify a catchment  
603 response time. Due to the sampling time step of the simulations, the uncertainty associated  
604 with the response is of 3 min. Still, it can be noted that in urban catchments (or semi-urban  
605 here), the response time of water depth UM parameters to the beginning of a rainfall or to an  
606 important peak of intensity is almost non-existent. This is due to the presence of impervious  
607 area over which rainfall directly transfer into surface runoff.

608

609

## 610 6) Conclusions

611

612         The Multi-Hydro model was implemented on the Jouy-en-Josas catchment in the Paris  
613 area. This 3 km<sup>2</sup> semi-urbanised catchment exhibits sharp slopes, and a dense area along the  
614 Bièvre River bed. It has often been damaged by major pluvial and fluvial flooding, before the  
615 construction of storage basins along the river path. The model was validated on this new  
616 catchment on four rainfall events with the help of the data from a height gauge near the outlet.  
617 Rainfall radar data with a resolution of 1km x 1km x 5min was used.

618           Then ensembles of downscaled rainfall fields were used to quantify the sensitivity of  
619 the model outputs to small scales unmeasured rainfall variability, i.e. occurring below the  
620 resolution of the available raw radar data. It appears that it is rather significant on flow  
621 simulated in conduits with pseudo coefficients of variations ranging from 90 % upstream to  
622 10% downstream. This confirms previous results obtained on a 1.5 km<sup>2</sup> flat highly urbanised  
623 catchment also in the Paris area. The methodology was extended here to simulated water  
624 depth, and it was found that the sensitivity was much lower than for conduits' flow. This is  
625 likely to be due to the fact that the sewer system is mainly able to cope with the storm water  
626 for these events limiting the amount of surface runoff.

627           After using them to downscale the radar data, Universal Multifractals are used in an  
628 innovative way to characterize the surface flow process -through simulated water depth for  
629 each 10 m x 10 m pixel over 3 min time steps- during rainfall events. UM parameters  $\alpha$  and  
630  $C_1$ , and the composite parameter  $\gamma_s$  are evaluated on the outputs of Multi-Hydro. Two scaling  
631 regimes are identified for this field and estimates are only reliable for small scales, i.e. 10m -  
632 80m, and related to the fractal feature of the sewer system which exhibits a scale break at the  
633 same scale. There is a loss of the quality of the scaling during intense rainfall periods and UM  
634 parameters get closer to rainfall ones. A possible interpretation is that during this short period,  
635 a mixture of the scaling behaviour of both surface flow and rainfall is observed. After the  
636 event scaling is improved and features more specific to surface flow processes are retrieved  
637 with a field strongly concentrated and variability among the wet areas dampened ( $C_1$  greater  
638 than 1 and  $\alpha$  smaller than 1). Small scale rainfall features do not seem to strongly influence  
639 the results which depend more on large scales rainfall spatio-temporal patterns for these  
640 events which do not trigger much sewer overflow.

641           The conclusions found with the help of this innovative methodology are not as  
642 straightforward as the authors would have hoped. Further investigations with other rainfall

643 events, other catchments, notably with denser monitoring network including in-sewer  
644 measurements, should be carried out to strengthen the results. Higher resolution models  
645 should also be tested to extend the range of available scales for the small scales regime to  
646 obtain more reliable estimates of scaling features. Such new analysis would enable to  
647 generalize the behaviour of the scaling and of the UM parameters which describes the surface  
648 flows, and eventually to link them to other geometrical features of the catchment, such as the  
649 fractal dimension of its impervious surface, of the roads (which are the preferential path for  
650 surface flows) or of the sewer system. This paper should be seen as a promising first step that  
651 hints at innovative techniques relying on scale invariance properties to analyse how the  
652 rainfall extremes are either dampened or enhanced by hydrological models and also to  
653 quantify the extremes at very high spatial resolution (typically 1 m) without having to run the  
654 model at these resolutions which would require too much time especially for real time  
655 applications.

656

657

#### 658 Acknowledgments

659 Authors acknowledge partial financial support from the European Interreg IV RainGain  
660 project ([www.raingain.eu](http://www.raingain.eu)), the Chair “Hydrology for resilient cities” endowed by Veolia, and  
661 the Ile-de-France regional R2DS RadX@IdF project. Authors would also like to thank Hervé  
662 Cardinal from the SIAVB for providing data and fruitful discussions.

663

664

665

#### 666 6) References

667

668 (EEA), E.E.A., 2014. <http://www.eea.europa.eu/articles/analysing-and-managing-urban->  
669 [growth](#) (consulted on 20-10-2014).

670 Bolle, A. et al., 2006. Hydraulic modelling of the two-directional interaction between sewer  
671 and river systems, Urban Drainage Modelling and Water Sensitive Urban Design,  
672 Merlbourne.

673 Carr, R.S. and Smith, G.P., 2006. Linking of 2D and Pipe hydraulic models at fine spatial  
674 scales, Urban Drainage Modelling and Water Sensitive Urban Design, Melbourne,  
675 Australia.

676 Chen, A.S., Djordjević, S., Leandro, J. and Savić, D.A., 2007. The urban inundation model  
677 with bidirectional flow interaction between 2D overland surface and 1D sewer  
678 networks, Proceedings of NOVATECH, Lyon, France.

679 Deltares, 2013. Sobek Suite V2.13, Deltares, Delft, NL.

680 Despotovic, J., Plavsic, J., Stefanovic, N., Pavlovic D., 2005. Inefficiency of storm water  
681 inlets as a source of urban floods. *Water Sci Technol*, 15(2):139–145

682 Djordjevic, S., Prodanovic, D. and Maksimovic, C., 1999. An approach to simulation of dual  
683 drainage. *Water Science and Technology*, 39(9): 95-103.

684

685 Djordjević, S., Prodanović, D., Maksimović, Č., Ivetić, M., Savić D., 2005. SIPSON –  
686 Simulation of Interaction between Pipe flow and Surface Overland flow in Networks.  
687 *Water Science and Technology*, 52 (5): 275-283.

688 Douglas, E.M. and Barros, A.P., 2003. Probable maximum precipitation estimation using  
689 multifractals: Application in the eastern United States. *Journal of Hydrometeorology*,  
690 4(6): 1012-1024.

691 El Tabach, E., Tchiguirinskaia, I., Mahmood, O. and Schertzer, D., 2009. Multi-Hydro: a  
692 spatially distributed numerical model to assess and manage runoff processes in peri-

693 urban watersheds, Final conference of the COST Action C22 Urban Flood  
694 Management, Paris 26/27.11.2009, France.

695 Fletcher, T.D., Andrieu, H. and Hamel, P., 2013. Understanding, management and modelling  
696 of urban hydrology and its consequences for receiving waters: A state of the art.  
697 *Advances in Water Resources*, 51(0): 261-279.

698 Giangola-Murzyn, A. et al., 2014. Multi-Hydro : an open access coupling model for urban  
699 hydrology (submitted). *Journal of Hydro-Informatics*.

700 Gires, A., Schertzer, D., Tchiguirinskaia, I., Lovejoy, S., Maksimovic, C., Onof, C., Simoes,  
701 N., 2011b. Impact of unmeasured rainfall variability on urban discharge: a case study  
702 in a multifractal framework. *Houille Blanche - Revue Internationale de l'Eau*(4): 37-  
703 42.

704 Gires, A., Tchiguirinskaia, I., Schertzer, D. and Lovejoy, S., 2012. Influence of the zero-  
705 rainfall on the assessment of the multifractal parameters. *Advances in Water*  
706 *Resources*, 45, 13-25.

707 Gires, A., Tchiguirinskaia, I., Schertzer, D. and Lovejoy, S., 2013. Multifractal analysis of a  
708 semi-distributed urban hydrological model. *Urban Water Journal*, 10(3): 195-208.

709 Gires, A. et al., 2014a. Impacts of small scale rainfall variability in urban areas: a case study  
710 with 1D and 1D/2D hydrological models in a multifractal framework. *Urban Water*  
711 *Journal*: 1-11.

712 Gires, A. et al., 2014b. Influence of small scale rainfall variability on standard comparison  
713 tools between radar and rain gauge data. *Atmospheric Research*, 138(0): 125-138.

714 Gires, A., Tchiguirinskaia, I., Schertzer, D., Ochoa-Rodriguez, S., Willems, P., Ichiba, A.,  
715 Wang, L.-P., Pina, R., Van Assel, J., Bruni, G., Murla Tuyls, D., and ten Veldhuis,  
716 M.-C., 2017. Fractal analysis of urban catchments and their representation in semi-

717 distributed models: imperviousness and sewer system, *Hydrol. Earth Syst. Sci.*, 21,  
718 2361-2375, <https://doi.org/10.5194/hess-21-2361-2017>.

719 Hsu, M.H., Chen, S.H. and Chang, T.J., 2000. Inundation simulation for urban drainage basin  
720 with storm sewer system. *Journal of Hydrology*, 234(1-2): 21-37.

721 Hubert, P. et al., 1993. Multifractals and extreme rainfall events. *Geophys. Res. Lett.*, 20:  
722 931-934.

723 Ichiba, A. (2016). X-band radar data and predictive management in urban hydrology, Ph.D.  
724 thesis, Université Paris-Est.

725 Ichiba, A., Gires, A., Tchiguirinskaia, I., Schertzer, D., Bompard, P., and Ten Veldhuis, M.-  
726 C., 2017. Scale effect challenges in urban hydrology highlighted with a distributed  
727 hydrological model, *Hydrol. Earth Syst. Sci.*, accepted.

728 Innovyze, 2012. InfoWorks CS v13.0.6.

729 Innovyze, 2013. Infoworks ICM v.3.5.

730 Institute, D.H., 2011. Mike Urban.

731 IPCC, 2013. Working Group 1 contribution to the IPCC fifth report climate change 2013: the  
732 physical science basis, summary of policy makers. Technical report.

733 Jankowfsky, S., 2011. Understanding and modelling of hydrological processes in small peri-  
734 urban catchments using an object-oriented and modular distributed approach.  
735 Application to the Chaudanne and Mercier sub-catchments (Yzeron catchment,  
736 France). PhD Thesis., Lyon, France.

737 Parisi, G. and Frish, U., 1985. A multifractal model of intermittency. In: M. Ghill, R. Benzi  
738 and G. Parisi (Editors), *Turbulence and predictability in geophysical fluid dynamics*.  
739 Elsevier North Holland, New-York, pp. 111-114.

740 Phillips, B., Yu, S., Thompson, G. and Silva, N., 2005. 1D and 2D Modelling of Urban  
741 Drainage Systems using XP-SWMM and TUFLOW, 10th International Conference on  
742 Urban Drainage, Copenhagen, Denmark.

743 Pitt, M., 2008. The Pitt Review : Learning lessons from the 2007 floods,  
744 [http://webarchive.nationalarchives.gov.uk/20100807034701/http://archive.cabinetoffice](http://webarchive.nationalarchives.gov.uk/20100807034701/http://archive.cabinetoffice.gov.uk/pittreview/thepittreview/final_report.html)  
745 [e.gov.uk/pittreview/thepittreview/final\\_report.html](http://webarchive.nationalarchives.gov.uk/20100807034701/http://archive.cabinetoffice.gov.uk/pittreview/thepittreview/final_report.html) (accessed 20-10-2014).

746 Richard, J., Giangola-Murzyn, A., Gires, A., Tchiguirinskaia, I. and Schertzer, D., 2014. Gis  
747 data Assimilation interface for distributed hydrological models. Environmental  
748 Modelling and Software (submitted).

749 Rodriguez, F., Andrieu, H. and Morena, F., 2008. A distributed hydrological model for  
750 urbanized areas - model development and application to case studies. Journal of  
751 Hydrology, 351: 268-287.

752 Rossman, L.A., 2010. Storm Water Management Model, User's Manual. Version 5.0. U.S.  
753 Environmental Protection Agency, EPA/600/R-05/040.

754 Royer, J.-F., Biauou, A., Chauvin, F., Schertzer, D. and Lovejoy, S., 2008. Multifractal  
755 analysis of the evolution of simulated precipitation over France in a climate scenario.  
756 C.R Geoscience, 340: 431-440.

757 Schertzer, D. and Lovejoy, S., 1987. Physical modelling and analysis of rain and clouds by  
758 anisotropic scaling and multiplicative processes. J. Geophys. Res., 92(D8): 9693-  
759 9714.

760 Schertzer, D. and Lovejoy, S., 2011. Multifractals, generalized scale invariance and  
761 complexity in geophysics. International Journal of Bifurcation and Chaos, 21(12):  
762 3417-3456.



763 Shah, S.M.S., O'Connell, P.E. and Hosking, J.R.M., 1996. Modelling the effects of spatial  
764 variability in rainfall on catchment response. 1. Formulation and calibration of a  
765 stochastic rainfall field model. *Journal of Hydrology*, 175(1-4): 67-88.

766 Solution, X., 2012. XPSWMM V.12.

767 Tabary, P., 2007. The new French operational radar rainfall product. Part I: Methodology.  
768 *Weather and Forecasting*, 22(3): 393-408.

769 Tabary, P. et al., 2007. The new French operational radar rainfall product. Part II: Validation.  
770 *Weather and Forecasting*, 22(3): 409-427.

771 Velleux, M.L., England, J.F. and Julien, P.Y., 2011. TREX Watershed Modelling Framework  
772 User's Manual: Model Theory and Description. Department of civil engineering, Colorado  
773 State University, Fort Collins: 106p.

774

775

776 **Tables:**

777

	Radar rain depth (mm)	Rain gauge depth (mm)	Duration (min)	Peak intensity over 5 min (mm/h)
09-02-2009	9.4	<i>Unavailable</i>	725	5.12
14-07-2010	43.2	35.2	1020	52.06
15-08-2010	27.8	20.8	1745	7.56
15-12-2011	26.2	29.6	785	24.26

778 Table 1: Main features for the four studied rainfall events. Cumulative depth are computed  
779 over the whole event. For the radar data averages over the catchment are displayed.

780

781

Event / Link	#1	#2	#3	#4	#5
09-02-2009	63 / 16	35 / 15	10 / 7.2	4.0 / 1.7	4.8 / 2.1
14-07-2010	76 / 22	27 / 13	7.1 / 3.6	7.5 / 3.2	7 / 3.1
15-08-2010	70 / 20	38 / 16	26 / 12	9.3 / 3.9	8.5 / 3.8
15-12-2011	60 / 23	50 / 22	28 / 12	11 / 4.1	8.7 / 3.9

782 Table 2:  $CV'_{95}$  and  $CV'_{75}$  in % (first and second figure respectively) for the five selected

783 conduits and four rainfall events.

784

785

786

Event	Rainfall input	$\alpha$	$C_1$	$\gamma_s$
14-07-2010	Raw radar data	1.55	0.62	1.52
	Downscaled rainfall	1.25	0.90	1.68
15-12-2011	Raw radar data	0.95	1.42	1.74
	Downscaled rainfall	0.99	1.22	1.65

787 Table 3: UM parameters for small scales (10 m – 80 m) computed with the help of a 2D

788 analysis with either raw radar data or a realisation of downscaled rainfall field (with  $\alpha=1.8$

789 and  $C_1=0.1$ ) as rainfall input for the 14-07-2010 and 15-12-2011 events.

790

791

792

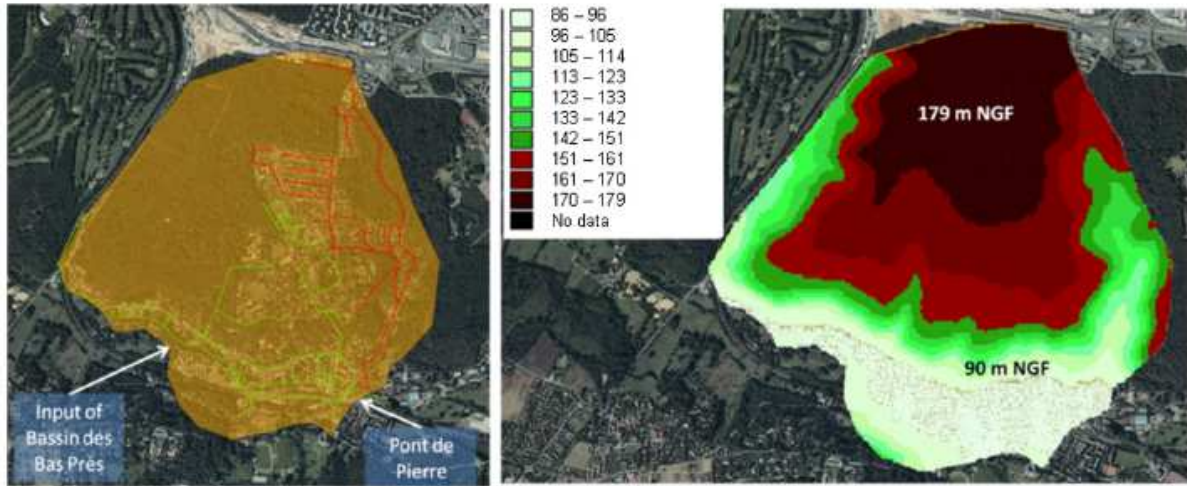
793

794

795 Figure captions:

796

797

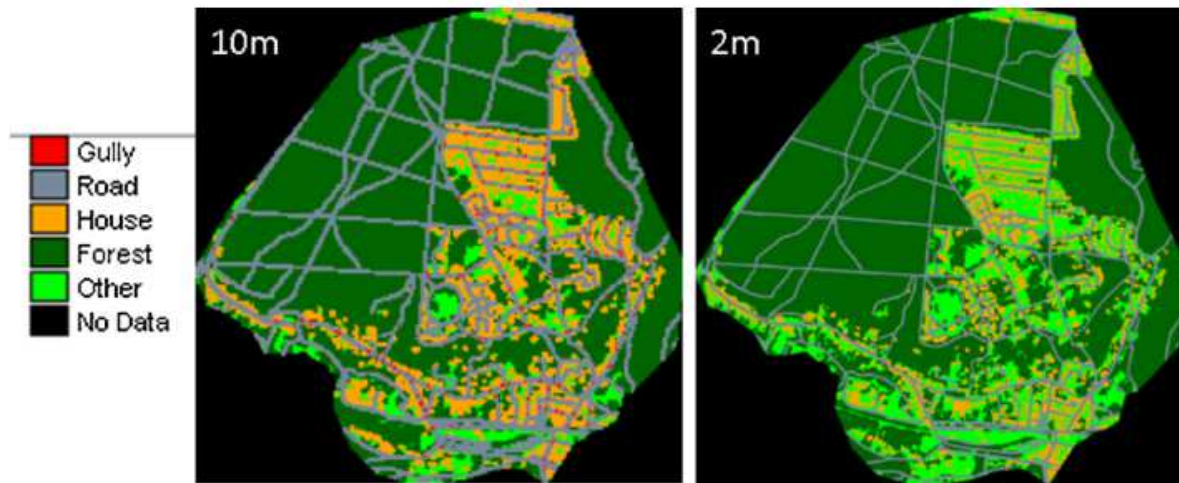


798

799 Figure 1: Maps of the Jouy-en-Josas catchment: (left) aerial photography and sewer system

800 (The green portion of the sewer network corresponds to the portion over which validation is

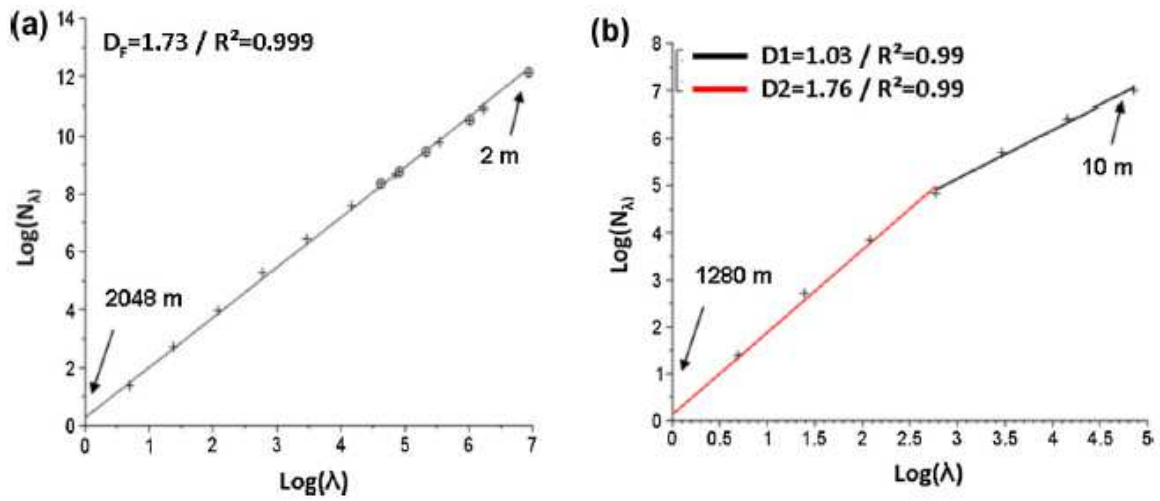
801 possible), (right) elevation in m.



802

803 Figure 2: Map of the land use obtained with the help of MH-AssimTool over the Jouy-en-

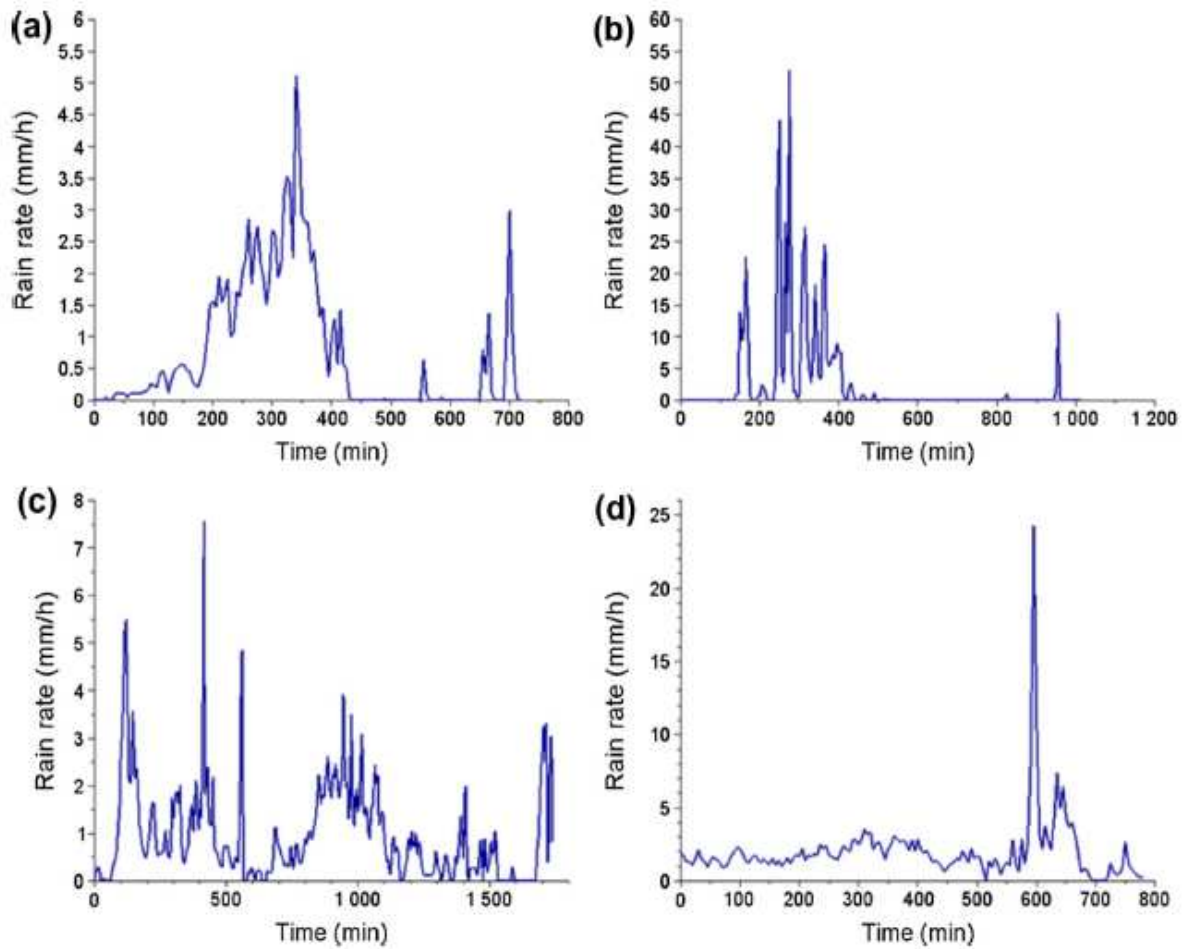
804 Josas catchment for two different resolutions.



805

806 Figure 3: (a) Evaluation of the fractal dimension of the impervious area for the studied  
 807 catchment (Eq. 1 in log-log plot). The circle points correspond to the figures obtained from  
 808 the map generated with the help of MH-AssimTool at various resolutions. (b) Evaluation of  
 809 the fractal dimension of the sewer system.

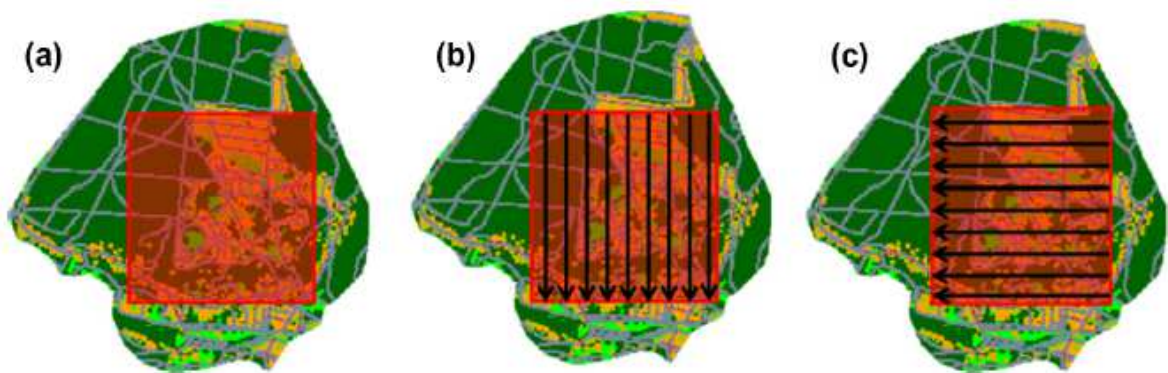
810



811

812 Figure 4: Average over the catchment of the rainfall radar intensity in mm/h over 5 min time

813 steps for the four events: (a) 09-02-2009, (b) 14-07-2010, (c) 15-08-2010, (d) 15-12-2011

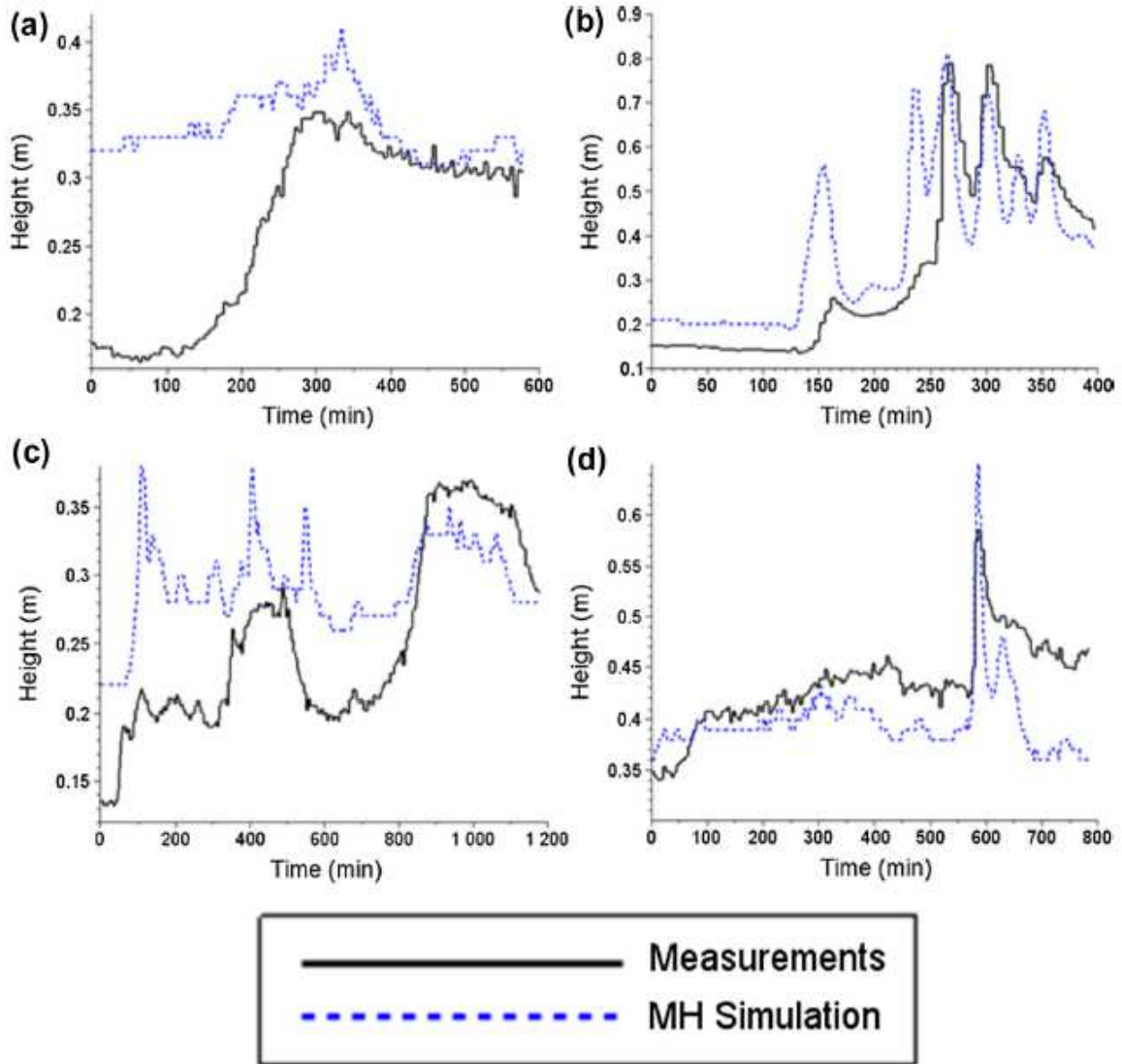


814

815 Figure 5: Illustration of the samples studied in the multifractal analysis of overland water

816 depth at the end of each 3-min Multi-Hydro loop: (a) 2D maps, (b) 1D vertical columns (N-S

817 direction), (c) 1D horizontal rows (W-E direction).

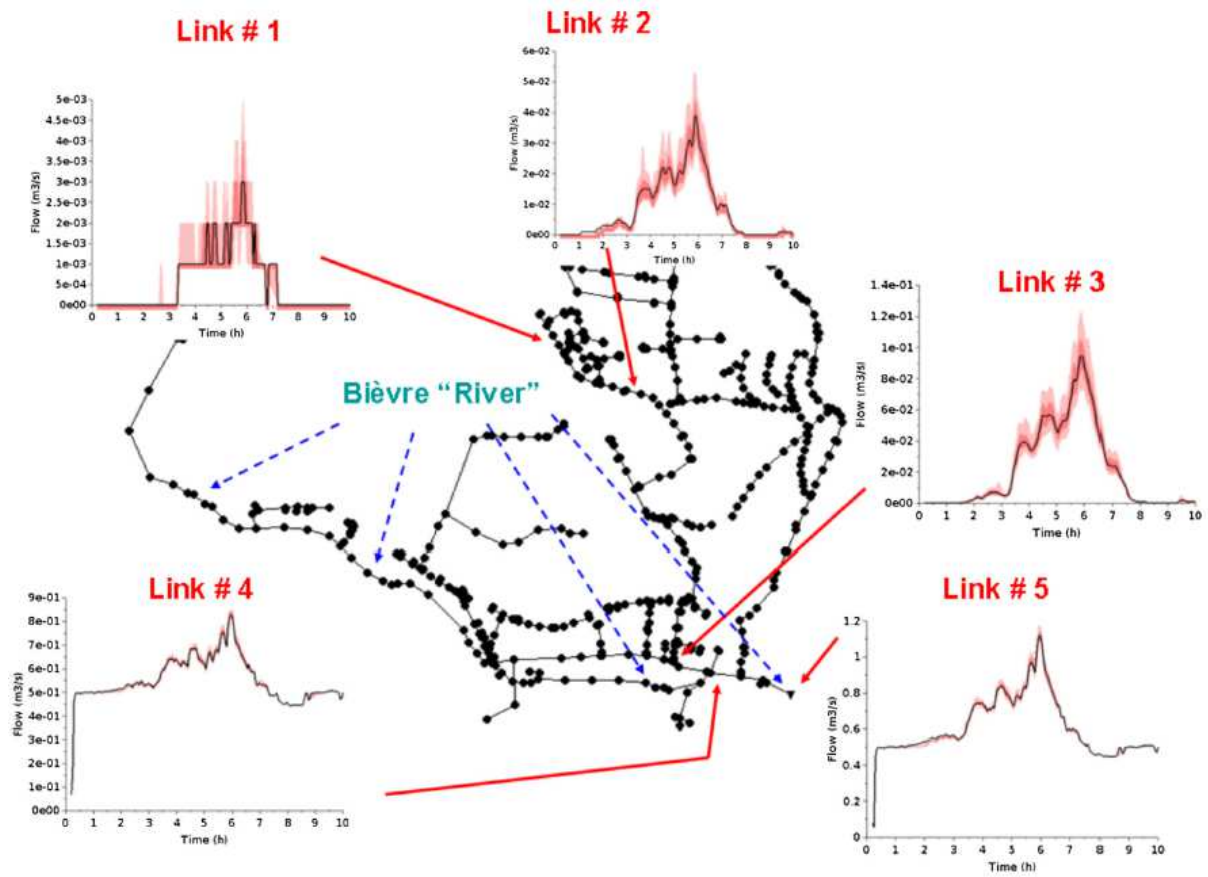


818

819 Figure 6: Water height simulated with the help of Multi-Hydro using raw radar data as rainfall

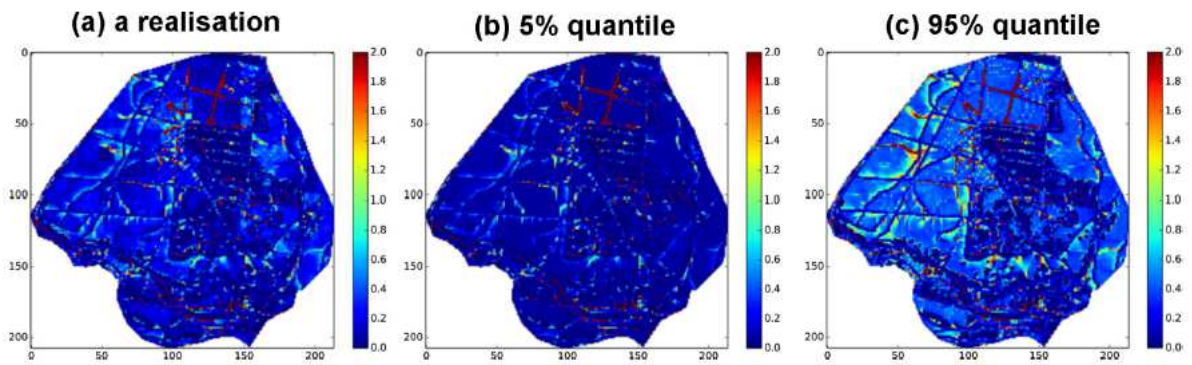
820 input, and measurements at the Pont-de-Pierre for the four events: (a) 09-02-2009, (b) 14-07-

821 2010, (c) 15-08-2010, (d) 15-12-2011



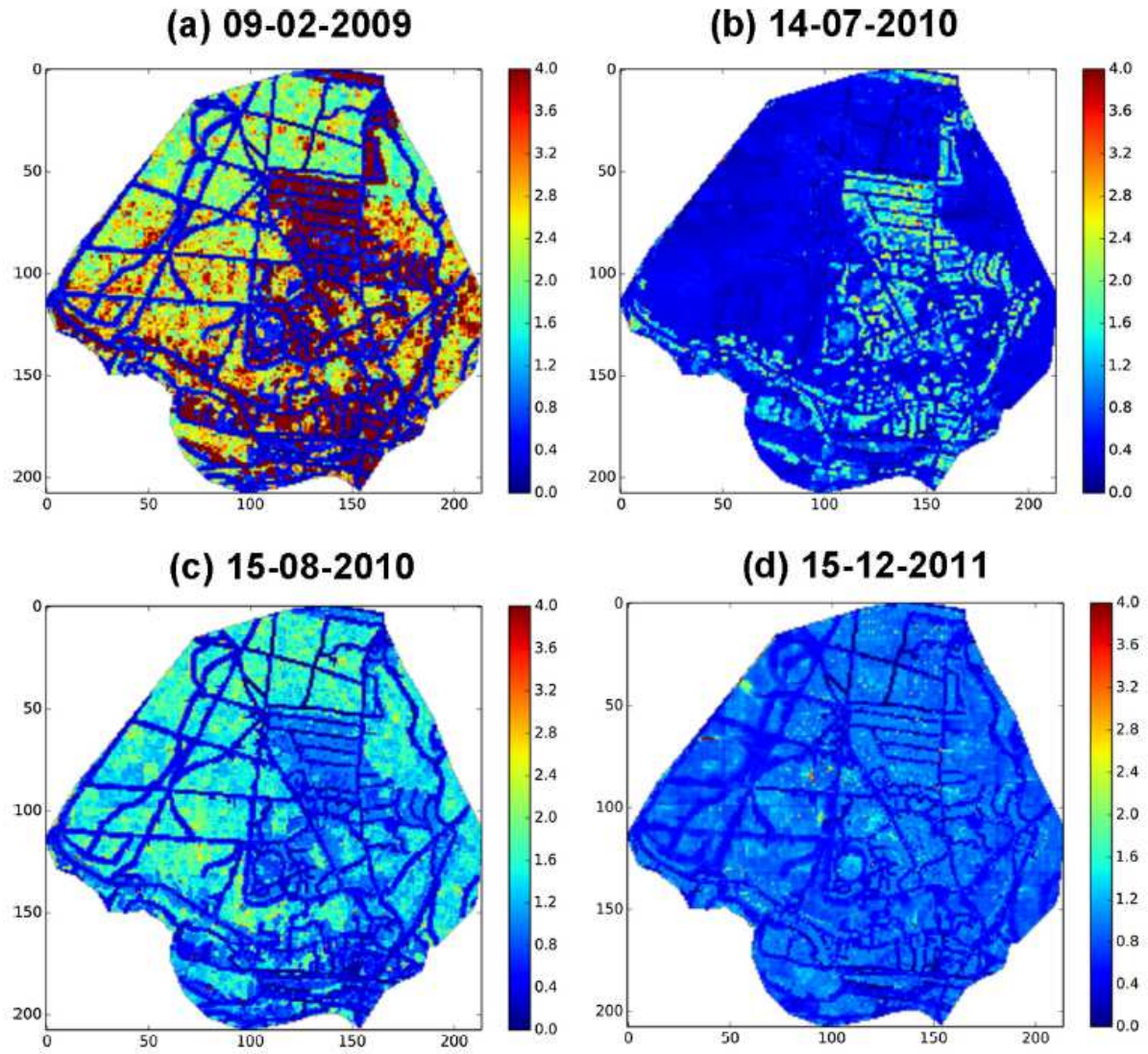
822  
823  
824  
825

Figure 7: Simulated flow with the raw radar data (black),  $Q_{0.25}$  and  $Q_{0.75}$  (dark pink colour),  $Q_{0.05}$  and  $Q_{0.95}$  (light pink colour) for 5 conduits of the studied catchment for the 09-02-2009 event.



826  
827  
828  
829  
830

Figure 8: For the 15-12-2011 event. (a) Map of the computed maximum water depth for a realisation of the downscaled rainfall field. (b) 5% quantile map of the maximum water depth over 100 realisations. (c) 95% quantile map of the maximum water depth over 100 realisations. Unit is m.



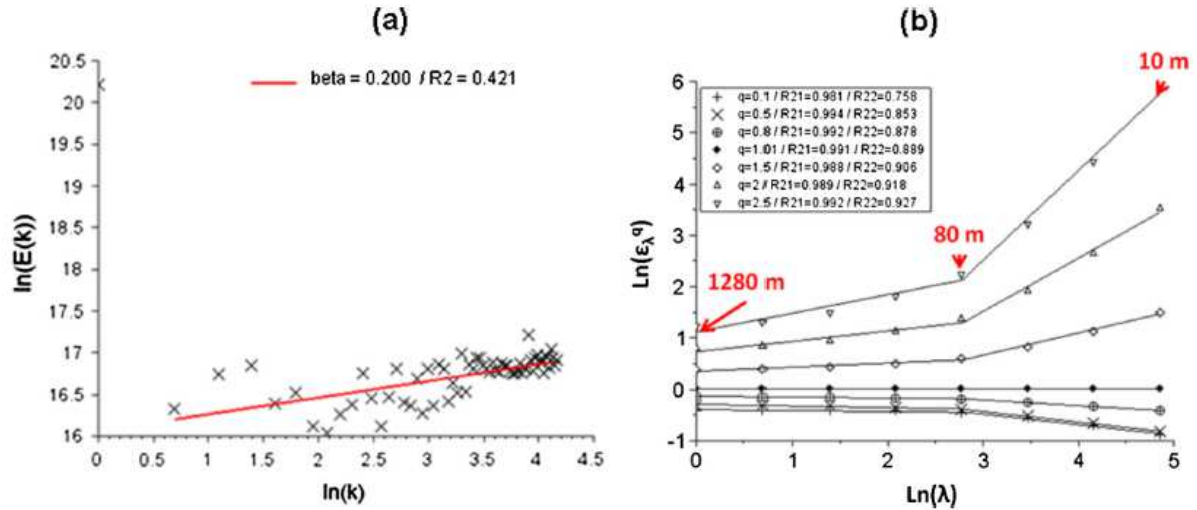
831

832 Figure 9: Map of  $CV'_{95}$  (in %) for the maximum water depth for the 09-02-2009 (a), 15-08-

833 2010 (b) and 15-12-2011 (c) events.

834

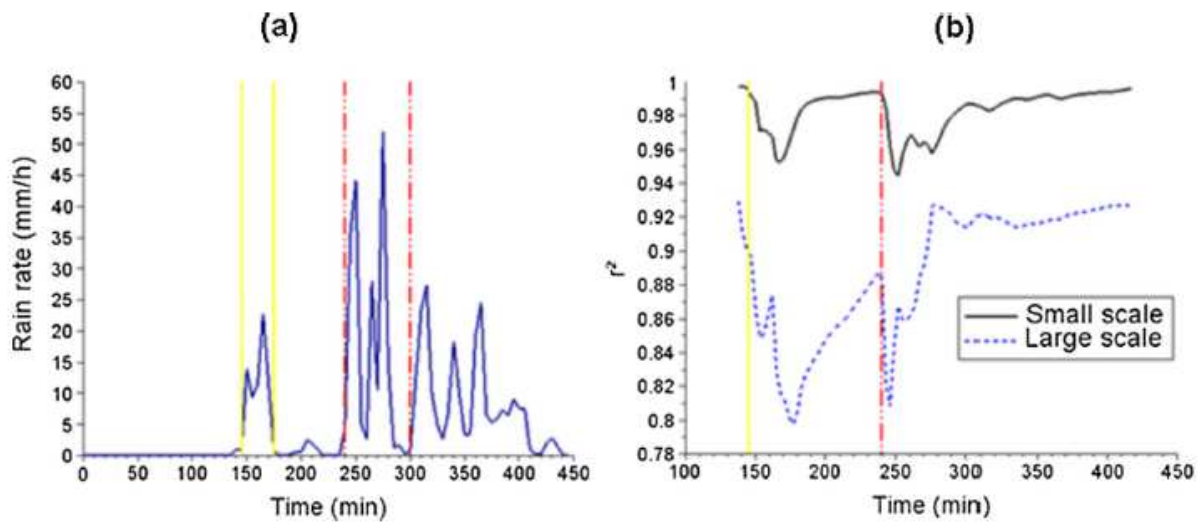




835

836 Figure 10: For the 14-07-2010 event and 2D ensemble analysis over all the time steps: (a)

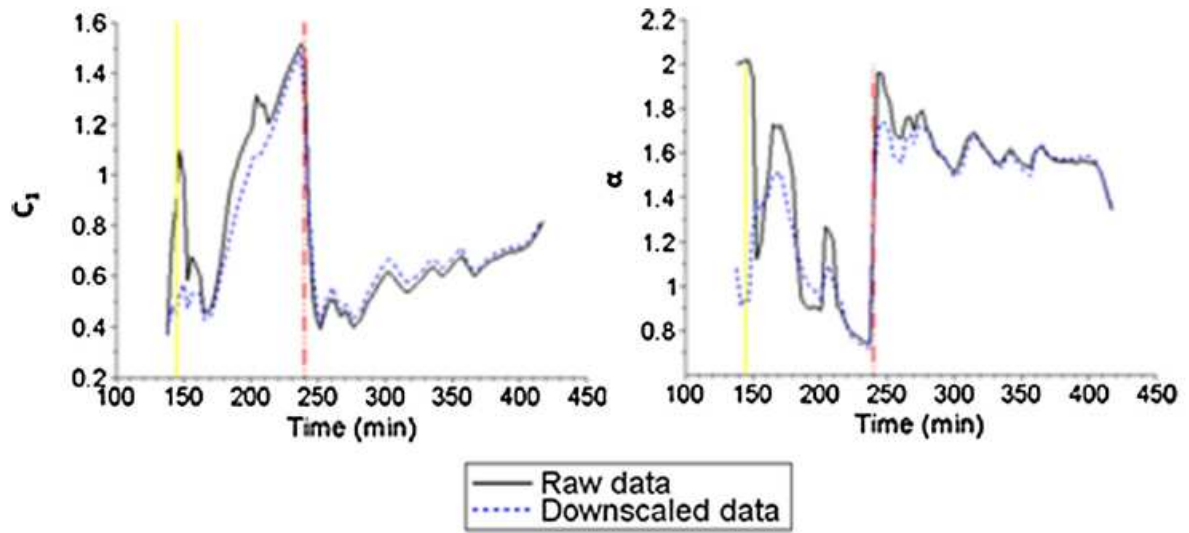
837 Spectral analysis, i.e. Eq. 5 in log-log plot; (b) TM analysis, i.e. Eq. 3 in log-log plot.



838

839 Figure 11: For the 14-07-2010 event: (a) Temporal evolution of the rain rate; (b) Temporal

840 evolution of the  $r^2$  for  $q=1.5$  in the TM analysis for the two regimes identified in Fig. 10.

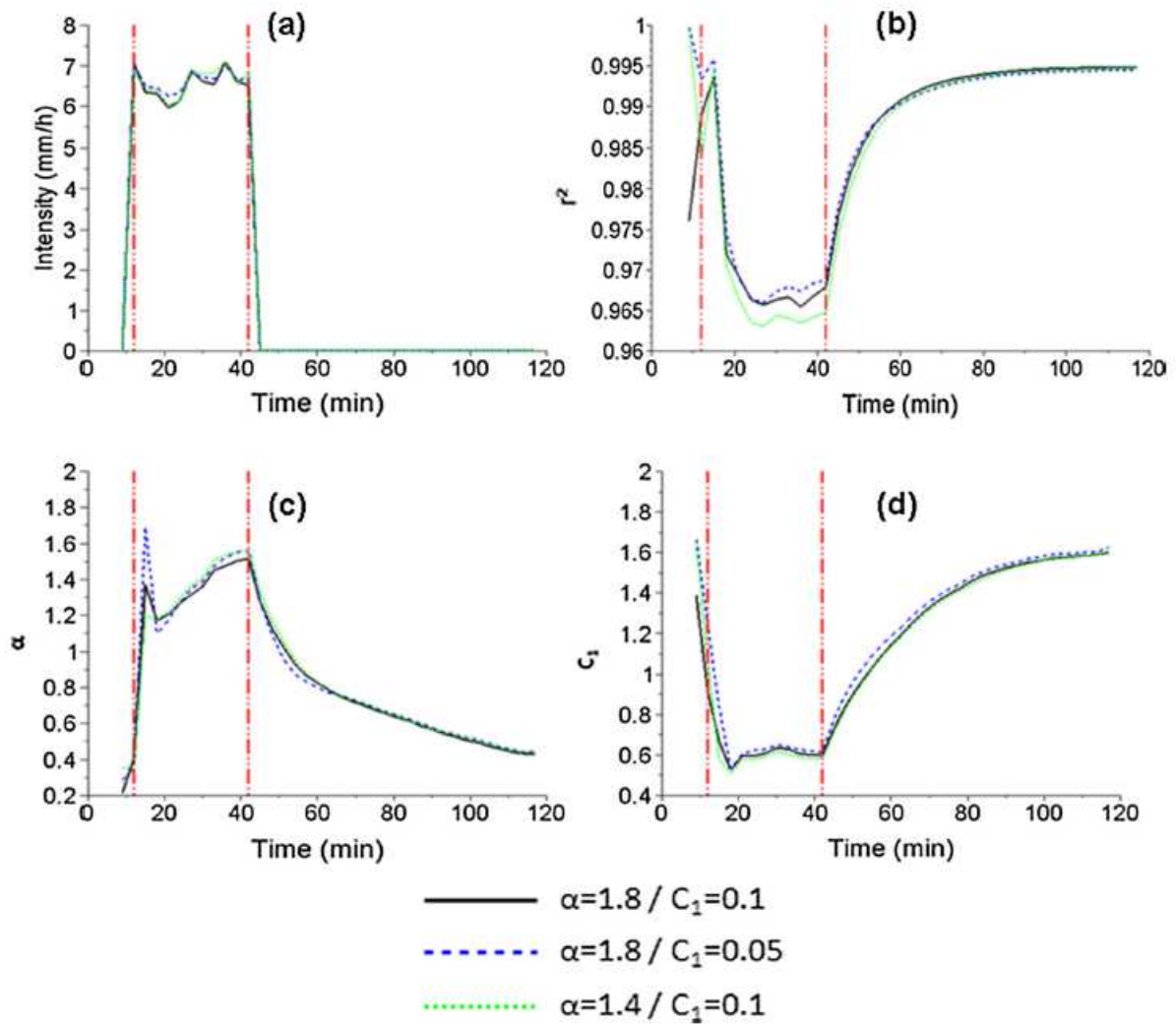


841

842 Figure 12: Temporal evolution of the UM parameters  $\alpha$  and  $C_1$  of the maximum water depth

843 field over 3 min for small scales (10 - 80 m) for the 14-07-2010 rainfall event.

844



845

846 Figure 13: For three synthetic rainfall events with different sets of UM parameters; temporal

847 evolution of the average rain rate over the catchment (a), and for the simulated overland

848 maximum water depth,  $r^2$  (b),  $\alpha$  (c) and  $C_1$  (d) for small scales (10 m-80 m).

849