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**A novel method for determining the small-strain shear modulus of soil  
using bender elements technique**

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22 ***Abstract***

23 Bender elements technique has become a popular tool for determining shear wave velocity  
24 ( $V_s$ ), hence the small-strain shear modulus of soils ( $G_{max}$ ), thanks to its simplicity and  
25 non-destructive character among other advantages. Several methods were proposed to  
26 determine the first arrival of  $V_s$ . However, none of them can be widely adopted as a standard  
27 and there is still an uncertainty on the detection of the first arrival.

28 In this study, bender elements tests were performed on lime-treated soil and both shear wave  
29 and compression wave velocities at various frequencies were measured. In-depth analysis  
30 showed that the S-wave received signal presents an identical travel time and opposite polarity  
31 compared with that of the S-wave components in P-wave received signal, especially at high  
32 frequency. From this observation, a novel interpretation method based on the comparison  
33 between the S-wave and P-wave received signals at high frequency is proposed. This method  
34 enables the determination of the arrival time of S-wave objectively, avoiding less reliable  
35 arrival pick-up point. Furthermore, the “ $\pi$ -point” method and cross correlation method were  
36 also employed and the obtained results agree well with those from the proposed method,  
37 indicating the accuracy and reliability of the latter. The effects of frequency on the shear  
38 wave velocity are also discussed.

39 *Keywords:* bender elements; signal interpretation; shear wave; compression wave; S+P interpretation  
40 method

## 41 **Introduction**

42 The small-strain shear modulus ( $G_{max}$ ) is a parameter of paramount importance in describing  
43 the elastic properties of soil. It is widely used in the analysis of dynamic problems in  
44 anti-seismic engineering (Zhou et al. 2005; Yang et al. 2009; Luke et al. 2013). It is also used  
45 to assess the soil stiffness in geo-environmental engineering (Tang et al. 2011; Gu et al. 2013;  
46 Hoyos et al. 2015). The value of  $G_{max}$  is usually determined from shear wave velocity ( $V_s$ )  
47 measurements either in the field or in the laboratory.

48 The measurement of  $V_s$  can be obtained from conventional laboratory experiments using  
49 resonant column (Hardin and Richart 1963; Anderson and Stokoe 1978; Fam et al. 2002),  
50 torsional shear tests (Iwasaki et al. 1978; Youn et al. 2008), flat transducers and  
51 accelerometers (Brignoli et al. 1996; Mulmi et al. 2008; Wicaksono et al. 2008) and  
52 piezoelectric bender elements technique (Dyvik and Madshus 1985; Viggiani and Atkinson  
53 1995; Brignoli et al. 1996; Jovičić et al. 1996; Pennington et al. 2001; Lings and Greening  
54 2001; Leong et al. 2005; Yamashita et al. 2009; Clayton 2011). The bender elements  
55 technique was first introduced by Shirley and Hampton (1978) and Shirley (1978) to soil  
56 testing, and due to its ability to measure the shear wave velocity of soil in a small-strain range,  
57 less than 0.001%, and in a wide range of stress conditions, it has taken a lot of interest of  
58 researchers. Nowadays, the bender elements transducers have been becoming a common  
59 laboratory tool, and have been incorporated in many geotechnical testing devices, such as  
60 oedometers (Dyvik and Olsen 1991; Zeng and Grolewski 2005; Sukolrat 2007), cubical and  
61 conventional triaxial apparatuses (Gajo et al. 1997; Jovičić and Coop 1998; Kuwano et al.  
62 1999; Pennington et al. 2001; Fioravante and Capoferri 2001; Sukolrat et al. 2006; Leong et

63 al. 2009; Finno and Cho 2011; Aris et al. 2012; Styler and Howie 2013), resonant column  
64 apparatuses (Ferreira et al. 2007; Youn et al. 2008; Gu et al. 2013).

65 Despite the popularity of its use, difficulties of signal interpretation of bender elements  
66 technique, mainly in the determination of the wave arrival time, are still remaining. Different  
67 methods and frameworks for the signal interpretation were reported (Lee and Santamarina  
68 2005; Viana da Fonseca et al. 2009; Leong et al. 2009; Arroyo et al. 2010), but none has been  
69 widely accepted as a standard. In fact, the complex phenomena of wave propagation in a soil  
70 sample have not been clearly understood yet, which represents an obstacle to the  
71 determination of the arrival point accurately and objectively (Camacho-Tauta et al. 2013).

72 In the field of geotechnical engineering, most tested materials with bender elements are soils  
73 with low stiffness, and generally, the chosen frequency of excitation voltage in most studies  
74 ranges from 1 to 30 kHz (Lee and Santamarina 2005; Sukolrat 2007; Youn et al. 2008; Viana  
75 da Fonseca et al. 2009; Leong et al. 2009). A few researchers used the bender elements  
76 technique for stiffer materials, including sandstones (Alvarado 2007), cement treated clay  
77 (Hird and Chan 2008), and argillaceous rocks (Arroyo et al. 2010). In these cases, the  
78 working frequency range is different, and higher input frequency should be chosen due to the  
79 higher resonant frequency of stiff material (Lee and Santamarina 2005).

80 This technical note puts forward an objective method for the signal interpretation of bender  
81 elements technique, with respect to the determination of the first arrival time of shear wave.

82 This method is based on the comparison of both S-wave and P-wave received signals  
83 obtained on the same sample with a single pair of bender elements to determine a unique  
84 arrival point. In this study, the bender elements tests were performed on compacted

85 lime-treated silt samples extending, thus, this technique to stiff soils. The results obtained by  
86 the proposed method are validated by comparing with those of other methods.

87

## 88 **Background**

### 89 *Working principal of Bender Elements*

90 The bender elements are piezo-electrical transducers, which consist of two thin piezo-ceramic  
91 bimorph sheets with external conducting surfaces, mounted together with a conductive metal  
92 shim at the centre. Details of the connection are given in Figure 1a. When an input waveform  
93 voltage is applied on the S-wave transmitter, one piezoceramic sheet extends and the other  
94 contracts, leading the transmitter to bend and generate a shear wave signal. The S-wave  
95 receiver bends when the shear wave arrives, propagating an electrical signal that can be  
96 visualised and measured by a digital oscilloscope. The operation of bender elements for  
97 S-wave transmission was well described by other researchers (Dyvik and Madshus 1985;  
98 Lings and Greening 2001; Lee and Santamarina 2005; Camacho Tauta et al. 2012).

99 Lings and Greening (2001) introduced a bender-extender element to transmit and receive  
100 both S-wave and P-wave with a single pair of transducers. The bender (or S-wave) receiver  
101 and transmitter in bender element testing also act as an extender (or P-wave) transmitter and  
102 receiver, respectively. Specifically, when an input voltage is applied on the extender  
103 transmitter, both two piezoceramic sheets with opposite polarisations extend or contract at the  
104 same time, causing the propagation of P-wave in a longitudinal direction. The P-wave wiring  
105 and transmitting are illustrated in Figure 1b. Note that the measurement of P-wave velocity is

106 mainly useful for unsaturated soils. Since in the saturated soils, P-wave usually travels much  
107 faster through water than through soil skeleton (Leong et al. 2009). Nevertheless, in this  
108 study, the degree of saturation of the tested material is lower than 95%. Bardet and Sayed  
109 (1993) who studied the Ottawa sand reported that the P-wave velocity became close to the  
110 value of dry sample when the degree of saturation decreased from 100% to 95%.

#### 111 *Determination of shear wave velocity*

112 During bender elements testing, both the transmitted and received signals are recorded to  
113 determine the travel time,  $t$ , of S-wave through a sample. The shear wave velocity,  $V_s$ , can  
114 then be calculated from the tip-to-tip travel length between the bender elements,  $L_{tt}$ , and  
115 travel time,  $t$ , as follows (Viggiani and Atkinson, 1995):

$$116 \quad V_s = \frac{L_{tt}}{t} \quad (1)$$

117 According to the theory of shear wave propagation in an elastic body, the small strain shear  
118 modulus,  $G_{\max}$ , can be determined by the following formula:

$$119 \quad G_{\max} = \rho V_s^2 \quad (2)$$

120 where  $\rho$  is the density of soil sample.

121 Interpretation of bender element tests usually takes the advantage of the knowledge and  
122 experience gained into the development of in situ geophysical tests such as Down-Hole and  
123 Cross-Hole tests or even surface tests such as SASW tests (Stokoe et al. 2004; Viana da  
124 Fonseca et al. 2006).

#### 125 *Determination of travel time*

126 An accurate determination of the travel time is a crucial issue to get a reliable value of  $V_s$ ,

127 hence  $G_{\max}$ . The existing common methods for determining travel time can be classified into  
128 two categories: time domain and frequency domain methods.

129 The time domain methods determine the travel time directly from the time lag between the  
130 transmitted and received signals. Referring to different characteristic points, the time domain  
131 methods can be divided into “arrival-to-arrival” method, “peak-to-peak” method and “cross  
132 correlation method”.

133 The first method, based on the visual inspection of the received signal, is the most commonly  
134 used one; however, the determination of an accurate arrival point using this method is still  
135 controversial and quite subjective due to the complex received signal, wave’s reflexion and  
136 the near field effect. Many studies reported that the near field effect decreases with the  
137 increase of frequency or the ratio of the wave path length to wavelength,  $L_{tt}/\lambda$ . Arulnathan et  
138 al. (1998) reported that the near field effect disappears when this ratio is larger than 1.  
139 Pennington et al. (2001) pointed out that when the  $L_{tt}/\lambda$  values range from 2 to 10, a good  
140 signal can be obtained. Wang et al. (2007) advocated a ratio greater than or equal to 2 to  
141 avoid the near field effect. Similarly, a value of 3.33 was recommended by Leong et al. (2005)  
142 to improve the signal interpretation.

143 Figure 2 shows a typical single sinusoidal S-wave transmitted signal and its corresponding  
144 received signal. Generally, point “a” (the first deflection) is taken as the arrival of the near  
145 field component of received signal (Brignoli et al. 1996). Both point “b” (the first reversal)  
146 and point “c” (zero after first reversal) are chosen as the arrival points of S-wave by most  
147 researchers (Brignoli et al. 1996; Lee and Santamarina 2005; Youn et al. 2008; Yamashita et  
148 al. 2009; Arroyo et al. 2010).

149 The “peak-to-peak” method is also widely applied in the signal interpretation. In this method,  
150 time delay between the peak of transmitted signal and the first major peak of received signal  
151 (point “d” in Figure 2) is regarded as the travel time (Clayton et al. 2004; Ogino et al. 2015).  
152 Note that, since the frequency of received signal may be slightly different from that of  
153 transmitted signal, and that the nature of the soil and size of the sample often affect the shape  
154 of the signal which could present more than one peak, great attention should be paid to the  
155 calculation of travel time by the “peak-to-peak” method.

156 “Cross-correlation” method is another kind of time domain method. It assumes the travel time  
157 as the time shift corresponding to the peak value of cross-correlation function between the  
158 transmitted and received signals. The cross-correlation approach, first adopted by Viggiani  
159 and Atkinson (1995), is based on the following expression::

$$160 \quad CC_{xy}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T x^*(t)y(t + \tau)dt \quad (3)$$

161 where  $x(t)$  and  $y(t)$  are the received and transmitted signals respectively,  $T$  is the time record  
162 and  $\tau$  is the time shift between two signals. First, the transmitted and received signals are  
163 converted to their linear spectrums using Fast Fourier Transform. Then the cross-power  
164 spectrum can be built based on the linear spectrum of received signal and the complex  
165 conjugate of the linear spectrum of transmitted signal. Eventually, the maximum of the  
166 cross-correlation function gives the travel time of shear wave. However, the accuracy of the  
167 cross-correlation method is largely dependent on the quality of received signals. Many  
168 limitations due to complex characteristics of received signal or incompatible transformation  
169 have been reported by several researchers (Arulnathan et al. 1998; Viana da Fonseca et al.  
170 2009; Chan 2012).

171 Frequency domain methods estimate the travel time according to the relationship between the  
172 change in the phase angle, which corresponds to the phase shift between the transmitter and  
173 receiver signals, and input frequency (Greening and Nash 2004; Viana da Fonseca et al. 2009;  
174 Ogino et al. 2015). These methods can be applied using discrete method called “ $\pi$ -point”  
175 method (Greening and Nash 2004; Viana da Fonseca et al. 2009), or continuous method such  
176 as Frequency Spectral Analysis (Greening and Nash 2004; Viana da Fonseca et al. 2009; Kim  
177 et al. 2015). They were first performed on rock and mud samples by Kaarberg (1975) and  
178 later widely accepted by other researchers (Greening et al. 2003; Gutierrez 2007; Viana da  
179 Fonseca et al. 2009), thanks to their negligible effect of extraneous signals.

180 In the “ $\pi$ -point” method, a continuous sinusoidal wave at a single frequency is used as an  
181 input signal, the continuous sinuous wave transmitter and receiver are displayed in an X-Y  
182 plot on an oscilloscope, and the phase shift between these two signals is measured. The  
183 frequency of transmitter is increased very slightly from a low value, inducing a phase shift  
184 between the two signals. When these signals are in phase or out of phase, i.e. the phase  
185 differences are multiple  $N$  of  $\pi$  or  $(-\pi)$ , the corresponding frequency,  $f$ , and the number of  
186 wavelength,  $N$ , are recorded.

187 It is well known that velocity,  $V$ , can be determined from the wavelength,  $\lambda$ , and the  
188 frequency (Viana da Fonseca et al. 2009):

$$189 \quad V = \lambda f = f \frac{L}{N} \quad (4)$$

190 where the travel time,  $t$ , can be deduced from:

$$191 \quad t = \frac{N}{f} \quad (5)$$

192 This indicates that the slope of the  $N$ - $f$  plot represents the travel time.

193 Result given by this method is more objective than that obtained by time domain method.

194 However, it has the drawbacks of time consuming and limited interpretable points (Viana da  
195 Fonseca et al. 2009).

196 Compared with the time-consuming “ $\pi$ -point” method, continuous method (Frequency  
197 Spectral Analysis) provides more available information in a short period of time with less  
198 effort. Continuous method applies a sweep signal which has a wide frequency spectrum (for  
199 example: 0 – 20 kHz) as input wave and uses a spectrum analyzer to establish the relationship  
200 between the frequency and the phase change, as introduced by Greening et al. (2003),  
201 Greening and Nash (2004) and explained in details by Kim et al. (2015). Specifically, the  
202 spectrum analyzer computes the coherence function between the transmitted and received  
203 signals. Based on the coherence function and phase angle it provides, the travel time can be  
204 determined directly from the slope of the linear line representing the relationship between the  
205 frequency and phase angle. However, further analysis is necessary when applying this  
206 method if the result convergence cannot be reached quickly (Viana da Fonseca et al. 2009).

207 Viana da Fonseca et al. (2009) also proposed a practical framework which combines both  
208 time-domain and frequency-domain methods for an enhanced interpretation of the bender  
209 element testing results. Nevertheless, the differences of the travel time determined by the time  
210 domain method and frequency domain method are quite large, and the causes are still not  
211 well understood (Greening et al. 2003; Greening and Nash 2004; Viana da Fonseca et al.  
212 2009; Ogino et al. 2015). Therefore, there is still a strong need of searching a simple and  
213 objective approach for the bender element testing interpretation with a reliable determination

214 of the arrival time.

215 In this study, the “arrival-to-arrival” method, “peak-to-peak” method, and “ $\pi$ -point” method  
216 are evaluated through the interpretation of signals obtained on compacted lime treated soils.  
217 Furthermore, based on the observation that the S-wave received signal presents an identical  
218 travel time and opposite polarity compared with that of the S-wave components in P-wave  
219 received signal, especially at high frequency, a novel method namely S+P method is  
220 proposed and its accuracy is proved by the comparison with “ $\pi$ -point” method.

221

## 222 **Experimental methods**

### 223 *Tested Material*

224 The soil used in this study was a plastic silt, taken from an experimental embankment with  
225 the ANR project TerDOUEST (Terrassements Durables - Ouvrages en Sols Traités, 2008 -  
226 2012) at Héricourt, France. This soil was first air-dried, ground and sieved to 0.4 mm.  
227 Quicklime was used in the treatment. The soil powder was first mixed thoroughly with 2%  
228 lime, and then humidified to reach two target water contents (at dry or wet side of optimum).  
229 More details about the geotechnical properties of this silt and the preparation process of  
230 samples can be found in Wang et al. (2015). In this study, four compacted lime-treated  
231 samples (50 mm in diameter and 50 mm in height) with degree of saturation,  $S_r = 72\%$  at dry  
232 side and  $S_r = 93\%$  at wet side, were tested. The specific sample information is listed in Table  
233 1.

### 234 *Experimental Techniques*

235 The bender element system used in this study consists of two bender elements (one S-wave  
236 transmitter/P-wave receiver and one S-wave receiver/P-wave transmitter, as shown in Figure  
237 3), installed at the two extremities of the soil sample. Beforehand, a slot was carried out on  
238 the surface of each sample extremity with the same direction to facilitate the insertion of the  
239 protruded part of the bender elements, and a good alignment of the latter. Afterwards, the  
240 sample (50 mm in diameter and 50 mm in height) was placed on a home-made wooden  
241 sample holder (see Figure 4a) specially designed to forbid any wave transmission outside the  
242 soil sample, which may interfere with the signal arrival (Brignoli et al. 1996; Lee and  
243 Santamarina 2005). Additional force was provided by the holder to enhance a good contact  
244 between the benders and the sample. Special care was taken to avoid any cross-talk by  
245 improving the shielding and grounding of the system. The output signal ( $\pm 20V$  sine pulse)  
246 was generated by a function generator (*TTi* TG1010A) and amplified by a power amplifier.  
247 Both the transmitter and the receiver signals were captured with an oscilloscope (Agilent  
248 DSO-X 2004A). The set-up of the system used is presented in Figure 4a and details of the  
249 arrangement of devices are illustrated in Figure 4b. All the four different interpretation  
250 methods presented previously (“arrival-to-arrival”, “peak-to-peak”, “ $\pi$ -point” and S+P  
251 methods) were applied for each sample. As for the conventional “arrival-to-arrival” and  
252 “peak-to-peak” methods, a single pulse S-wave with various input frequency was used as  
253 transmitted signal. In S+P method, both S-wave and P-wave signals transmitted through the  
254 same sample by modifying the connection between the two benders, as illustrated in Figure  
255 4b. A continuous sweep signal (frequency increased slowly from 20 kHz to 50 kHz) was  
256 applied in the “ $\pi$ -point” method.

257 Prior to testing, calibration of bender elements (tip-to-tip calibration) was carried out by  
258 holding the two benders (transmitter and receiver) in contact with each other directly without  
259 sample. Both S-wave transmitter and P-wave transmitter were generated to measure the delay  
260 times for these two transmitters ( $t_{d\_S} = 5.5 \mu\text{s}$  and  $t_{d\_P} = 3.5 \mu\text{s}$ ). These delay times were  
261 accounted for in the calculation of travel time when applying the time domain method.

262

### 263 **Experimental Results**

264 Figure 5 presents the results of sample D1 with the time domain methods (conventional  
265 arrival-to-arrival method and peak-to-peak method). The S-wave transmitted signals with  
266 various frequencies are considered (dashed lines) and the S-wave received signals are  
267 presented in solid lines. In the arrival-to-arrival method, the first reversal in the received  
268 signal is chosen as the arrival point of S-wave. Besides, the first major peak is highlighted to  
269 calculate the time delay by the peak-to-peak method. It is observed that for the relatively low  
270 frequencies used here ( $f = 5, 10$  and  $15$  kHz), the interpretation of these received signals is  
271 ambiguous because of the evident near field effect. However, the signals at higher  
272 frequencies, from  $20$  kHz to  $50$  kHz, become quite clear and the near field less marked. These  
273 cases correspond to a ratio of wave path length to wavelength,  $L_{tt}/\lambda$ , larger than or equal to  $1.9$ ,  
274 and are in good agreement with those reported in the literature (Sanchez-Salinerio et al. 1986;  
275 Brignoli et al. 1996; Arulnathan et al. 1998; Pennington et al. 2001; Leong et al. 2005; Wang  
276 et al. 2007). The results obtained from the “ $\pi$ -point” method on the sample ( $w = 17\%$ , with a  
277 curing time,  $t_c = 25$  h) are shown in Figure 6. A good linear relationship between the number  
278 of wavelength and frequency is obtained. The travel time,  $t$ , can be determined as  $0.1441$  ms

279 directly from the slope of the matched line, according to Equation 5. Additionally, the good  
280 linear relationship observed also highlights that the frequency ranging from 25 to 50 kHz is  
281 reasonable and suitable for the determination of shear wave velocity by the time domain  
282 method.

283 In Figure 7, the S-wave and P-wave transmitted signals and the corresponding received  
284 signals are gathered. The arrival point of S-wave received signal can be determined by  
285 referring to P-wave received signal. It is well known that P-wave component travels the  
286 fastest, thus arrives first before the S-wave received signal. Specifically, the arrival point of  
287 S-wave received signal (point S) corresponds to the initial main excursion (point S<sub>p</sub>), which  
288 is in the opposite direction of movement compared to that of point S in the S-wave received  
289 signal. Apparently, the curvature of these points (S and S<sub>p</sub>) becomes much sharper when the  
290 input frequency increases up to 40 or 50 kHz, as shown in Figure 7c and 7d. Furthermore, the  
291 arrival point of P-wave received signal (point P) just corresponds to the arrival point of the  
292 near field components (point P<sub>s</sub>) in the S-wave received signal. This is in good agreement  
293 with the observation by Brignoli et al. 1996 - a “near field component” travelling at a similar  
294 velocity as P-wave was observed on a dry sample. Therefore, we propose this method,  
295 namely S+P method to determine the arrival point of S-wave: the arrival time is defined by  
296 point S (corresponding point S<sub>p</sub> in the P-wave received signal).

297 To verify the accuracy of this S+P method, Figure 8 collects all data of shear wave velocity  
298 obtained from different interpretation methods. The results obtained from the S+P method are  
299 compared with those from other methods. Note that in the arrival-to-arrival method, point b  
300 (first reversal as mentioned above) is chosen as the arrival point of S-wave. As for the result

301 by  $\pi$ -point method in Figure 8, the straight line represents a unique value of travel time which  
302 is determined as the input frequency slowly increasing from about 20 kHz to 50 kHz in each  
303 case. The differences between the shear wave velocities ( $V_s$ ) obtained from the conventional  
304 time domain methods (arrival-to-arrival method and peak-to-peak method) and that from the  
305 “ $\pi$ -point” method are about 30%. Similar ranges of difference are reported by other  
306 researchers (Viggiani and Atkinson 1995; Viana da Fonseca et al. 2009; Ogino et al. 2015).  
307 Nevertheless, the difference between the results obtained from S+P method and that taken  
308 from “ $\pi$ -point” method is only around 10% in all tests; while this difference shows a good  
309 agreement between the results obtained from S+P method and those from cross correlation  
310 method, especially in the high frequency range, as illustrated in Figure 8. Therefore, the  
311 accuracy of the proposed method (S+P method) can be confirmed.

312

### 313 **Discussions**

314 The results obtained in this study show that the S-wave received signals are unreadable at  
315 lower frequencies ( $f = 5, 10$  and  $15$  kHz), due to the influence of near field components. The  
316 effect of the latter is usually significant insofar as it obscures the shear wave arrival time  
317 point when the input frequency is low and the distance between the transmitter and receiver is  
318 short. Many researchers used the ratio of wave path length to wavelength  $L_{tt}/\lambda$  as an essential  
319 parameter to describe the near field effect. The near field effect can be reduced markedly with  
320 the increase of  $L_{tt}/\lambda$ , by improving the input frequency of transmitter or enlarging the distance  
321 between the transmitter and the receiver.

322 The results indicate also that the near field effect diminishes and a much clear received signal  
323 appears as the  $L_{tt}/\lambda$  value reaches 1.91 (in the arrival-to-arrival method). Similar conclusion  
324 can be made referring to the results of “ $\pi$ -point” method which shows a good linear  
325 relationship between the number of wavelength and frequency when the input frequency  
326 increases from 20 kHz (in case of  $L_{tt}/\lambda = 1.9$ ).

327 A higher frequency makes the received signal more readable in case of testing on a stiff  
328 material such as compacted lime-treated silt. However, the P-wave component can become  
329 significant when the excitation frequency is high (Brignoli et al. 1996). This is different from  
330 a near field component (Lee and Santamarina 2005). Bender element can also generate small  
331 compressive displacements even though the main displacements produced are of shear nature  
332 (Brignoli et al. 1996). More importantly, the small compressive components (P-wave  
333 components) can be prominent in case of high frequency being excited on stiff materials,  
334 resulting in slight interference with the shear components (S-wave components) which have  
335 relatively slower travel velocity. Conversely, some shear displacements would be generated  
336 simultaneously with the major excitation of compressive displacements. Note that the polarity  
337 of S-wave components is always contrary to that of P-wave components in the same signal. It  
338 is even important that the travel velocity of S-wave components in P-wave received signal is  
339 identical with that of the S-wave received signals.

340 According to the analyses above, the determination of arrival point of S-wave received  
341 signals becomes clear and objective using the S+P method. That is, based on the comparison  
342 between S-wave and P-wave received signals, a unique point, S (as illustrated in Figure 7),  
343 can be identified and it corresponds to the arrival point of S-wave. This point located in

344 S-wave received signal, corresponding exactly to the point S<sub>p</sub> in P-wave received signal,  
345 which shows opposite direction of movement in comparison with the point S in S-wave  
346 signal.

347 The results from S+P method show a small effect of input frequency (in a range from 20 to  
348 50 kHz) on the shear wave velocity. As Figure 8 shows, shear wave velocity increases  
349 slightly as the input frequency increases. Similar phenomena were reported by other authors.  
350 Youn et al. (2008) pointed out that the shear wave velocity obtained from time domain  
351 method presents a small increasing trend with the increase of excitation frequency. Yamashita  
352 et al. (2009) also noted that the shear modulus increases with the increase of excitation  
353 frequency. To a certain extent, it might be a possible reason to explain why a slight  
354 decreasing trend of arrival time is observed when the excitation frequency increases.  
355 Specifically, when the frequency is relatively low (as shown in Figure 7a and 7b), point S  
356 determined by S+P method as the arrival point is not very clear. By contrast, in the case of  
357 high frequency (as shown in Figure 7c and 7d), point S is easy to be distinguished due to a  
358 sharp curvature at point S<sub>p</sub> in the P-wave received signal. Brignoli et al. (1996) also  
359 recommended that high frequency should be used when measuring the P-wave components.

360

## 361 **Conclusions**

362 Due to the near field effect, reflected waves and soil properties, determining the shear wave  
363 arrival time accurately and reliably in bender elements testing is difficult and still  
364 controversial, and new methods need to be developed.

365 In this study, a novel method is proposed for a practical interpretation of the results from  
366 bender element tests, for unsaturated or nearly saturated soil specimen. This method, namely  
367 S+P method, is mainly based on the comparison between the S-wave and P-wave received  
368 signals, and enables the determination of the arrival point of S-wave signal in a more  
369 objective fashion. When a high frequency S-wave signal is excited, the P-wave components  
370 become evident, and are easy to be distinguished from the S-wave received signal. Note that  
371 the P-wave received signal also includes some S-wave components which arrive after the  
372 P-wave components. Based on the identical travel velocity of S-wave components in both  
373 S-wave received signal and P-wave received signal, and the opposite polarities between these  
374 two different S-wave components, a unique arrival point of S-wave can be determined.  
375 Comparisons between the results obtained by this method and those by the  $\pi$ -point method  
376 and cross correlation method were made, indicating the relevance of the proposed method. It  
377 is also worth noting that compared to the  $\pi$ -point method, the proposed method is less time  
378 consuming.

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## **References**

- Alvarado, G. 2007. Influence of Late Cementation on the Behaviour of Reservoir Sands. Diss. PhD thesis, Imperial College London.
- Anderson, D. G. and Stokoe, K. H. 1978. Shear Modulus: A Time-Dependent Soil Property. In: Dynamic Geotechnical Testing, ASTM International.
- Arroyo, M., Pineda, J. A. and Romero, E. 2010. Shear Wave Measurements Using Bender Elements in Argillaceous Rocks. *Geotech. Test. J.*, Vol. 33, pp. 1-11.
- Arulnathan, R., Boulanger, R. W. and Riemer, M. F. 1998. Analysis of Bender Element Tests. *Geotech. Test. J.*, Vol. 21, No. 2, pp. 120-131.
- Aris M., Benahmed N. and Bonelli S. 2012. A Laboratory Study on the Behaviour of Granular Material Using Bender Elements. *Euro. J. Environ. Civil. Engineer.*, Vol. 16, No. 1, pp. 97-110.
- Bardet, J. P. and Sayed, H. 1993. Velocity and Attenuation of Compressional Waves in Nearly Saturated Soils. *Soil Dyn. Earthq. Eng.*, Vol. 12, No. 7, pp. 391-401.
- Brignoli, E. G. M., Gotti, M. and Stokoe, K. H. 1996. Measurement of Shear Waves in Laboratory Specimens by Means of Piezoelectric Transducers. *Geotech. Test. J.*, Vol. 19, pp. 384-397.
- Camacho-Tauta, J. F., Reyes-Ortiz, O. J. and Jimenez Alvarez, J. D. 2013. Comparison between Resonant-Column and Bender Element Tests on Three Types of Soils. *Dyna*, Vol. 80, No. 182, pp. 163-172.
- Camacho-Tauta, J. F., Jimenez Alvarez, J. D. and Reyes-Ortiz, O. J. 2012 A Procedure to Calibrate and Perform Bender Element Test. *Dyna*, Vol. 79, No. 176, pp. 10-18,.
- Chan, C. M. 2012. Variations of Shear Wave Arrival Time in Unconfined Soil Specimens Measured with Bender Elements. *Geotech. Geo. Eng.*, Vol. 30, No. 2, pp. 461-468.
- Clayton, C. R. I., Theron, M. and Best, A. I. 2004. The Measurement of Vertical Shear-Wave Velocity Using Side-Mounted Bender Elements in the Triaxial Apparatus. *Géotechnique*, Vol. 54, No. 7, pp. 495-498.

- Clayton, C. R. I. 2011. Stiffness at Small Strain: Research and Practice. *Géotechnique*, Vol. 61, No. 1, pp. 5-37.
- Dyvik, R. and Madshus, C. 1985. Lab Measurements of  $G_{max}$  Using Bender Elements. Proceedings of the conference on the Advances in the Art of Testing Soil under Cyclic Conditions, New York: ASCE Geotechnical Engineering Division, pp. 186-196.
- Dyvik, R. and Olsen, T. S.. 1991.  $G_{max}$  Measured in Oedometer and DSS tests Using Bender Elements. Publikasjon-Norges Geotekniske Institutt, Vol. 181, pp. 1-4.
- Fam, M. A., Cascante, G., and Dusseault, M. B. 2002. Large and Small Strain Properties of Sands Subjected to Local Void Increase. *J. Geotech. Geoenviron., ASCE*, Vol. 128, No. 12, pp. 1018-1025.
- Ferreira, C., Viana da Fonseca, A. and Santos, J. A. 2007. Comparison of Simultaneous Bender Element Test and Resonant Column Tests on Porto Residual Soils. *Soil Stress-Strain Behavior: Measurement, Modeling and Analysis*, Springer Netherlands, pp. 523-535.
- Finno, R. and Cho, W. 2011. Recent Stress-History Effects on Compressible Chicago Glacial Clays. *J. Geotech. Geoenviron. Eng.*, Vol. 137, No. 3, pp. 197-207.
- Fioravante, V., and Capoferri, R. 2001. On the Use of Multi-Directional Piezoelectric Transducers in Triaxial Testing. *J. Geotech. Test.*, Vol. 24, No. 3, pp. 243-255.
- Gajo, A., Fedel, A. and Mongiovi, L. 1997. Experimental Analysis of the Effects of Fluid-Solid Coupling on the Velocity of Elastic Waves in Saturated Porous Media. *Géotechnique*, Vol. 47, No. 5, pp. 993-1008.
- Greening, P. D., Nash, D. F. T., Benahmed, N., Viana da Fonseca A. and Ferreira, C. 2003. Comparison of Shear Wave Velocity Measurements in Different Materials Using Time and Frequency Domain Techniques. Proceedings of Deformation Characteristics of Geomaterials, Lyon, France, pp. 381-386.
- Greening, P. D. and Nash, D. F. T. 2004. Frequency Domain Determination of  $G_0$  Using Bender Elements. *J. Geotech. Test.*, Vol. 27, No. 3, pp. 288-294.
- Gu, X., Yang, J. and Huang, M. 2013. Laboratory Measurements of Small Strain Properties of Dry Sands by Bender Element. *Soils Found.*, Vol. 53, No. 5, pp. 735-745.
- Gutierrez, G. A. 2007. Influence of Late Cimentation on the Behaviour of Reservoir Sands. Diss. PhD thesis, Imperial College London.
- Hardin, B. O. and Richart, Jr. F. E. 1963. Elastic Wave Velocities in Granular Soils. *J. Soil Mech. Found. Div.*, Vol. 89, No. SM1, pp. 33-65.
- Hird, C. and Chan, C. M. 2008. One-dimensional Compression Tests on Stabilized Clays

- Incorporating Shear Wave Velocity Measurements. *J. Geotech. Test.*, Vol. 31, No. 2, pp. 495.
- Hoyos, L. R., Suescún-Florez, E. A. and Puppala, A. J. 2015. Stiffness of Intermediate Unsaturated Soil from Simultaneous Suction-Controlled Resonant Column and Bender Element Testing. *Eng. Geol.*, Vol. 188, pp. 10-28.
- Iwasaki, T., Tatsuoka, F. and Takagi, Y. 1978. Shear Moduli of Sands under Cyclic Torsional Shear Loading. *Soils Found.*, Vol. 18, No. 1, pp. 39-56.
- Jovičić, V., Coop, M. R. and Simić, M. 1996. Objective Criteria for Determining  $G_{max}$  from Bender Element Tests. *Geotechnique*, Vol. 46, No. 2, pp. 357-362.
- Jovičić, V. and Coop, M. R. 1998. The Measurement of Stiffness Anisotropy in Clays with Bender Element Tests in the Triaxial Apparatus. *J. Geotech. Test.*, Vol. 21, No. 1, pp. 3-10.
- Kaarsberg, E. A. 1975. Elastic-wave Velocity Measurements in Rocks and Other Materials by Phase-delay Methods. *Geophysics*, Vol. 40, pp. 955-960.
- Kim, T., Zapata-Medina, D. G. and Vega-Posada, C. A. 2015. Analysis of Bender Element Signals during Triaxial Testing. *Revista Facultad de Ingeniería Universidad de Antioquia*, No. 76, pp. 107-113.
- Kuwano, R., Connolly T. M. and Jardine, R. J. 1999. Anisotropic Stiffness Measurements in a Stress-Path Triaxial Cell. *J. Geotech. Test.*, Vol. 23, No. 2, pp. 141-57.
- Lee, J. S. and Santamarina, J. C. 2005. Bender Elements: Performance and Signal Interpretation. *J. Geotech. Geoenvironmental Eng.*, Vol. 131, pp. 1063-1070.
- Leong, E. C., Yeo, S. H. and Rahardjo, H. 2005. Measuring Shear Wave Velocity Using Bender Element. *Geotech. Test. J.*, Vol. 28, pp. 488-498.
- Leong, E. C., Cahyadi, J. and Rahardjo, H. 2009. Measuring Shear and Compression Wave Velocities of Soil Using Bender–Extender Elements. *Can. Geotech. J.*, Vol. 46, pp. 792-812.
- Ling, M. L. and Greening, P. D. 2001. A Novel Bender/Extender Element for Soil Testing. *Géotechnique*, Vol. 51, No. 8, pp. 713-7.
- Luke, B., Werkema, D., and Andersen, S. 2013. Laboratory Investigation of Seismic Effects of Nanoparticle Dispersions in Saturated Granular Media. *Proceeding of the 18<sup>th</sup> International Conference on Soil Mechanics and Geotechnical Engineering*, Paris.
- Mulmi, S., Sato, T. and Kuwano, R. 2008. Performance of Plate Type Piezo-Ceramic Transducers for Elastic Wave Measurements in Laboratory Soil Specimens. *Seisan Kenkyu*, Vol. 60, No. 6, pp. 565-569.

- Ogino, T., Kawaguchi, T., Yamashita, S. and Kawajiri, S. 2015. Measurement Deviations for Shear Wave Velocity of Bender Element Test Using Time Domain, Cross-Correlation, and Frequency Domain Approaches. *Soils Found.*, Vol. 55, pp. 329-342.
- Pennington, D. S., Nash, D. F. T. and Lings, M. L. 2001. Horizontally Mounted Bender Elements for Measuring Anisotropic Shear Moduli in Triaxial Clay Specimens. *Geotech. Test. J.*, Vol. 24, No. 2, pp. 133-144.
- Sanchez-salintero, I., Roesset, J. M. and Stokoe, K. H. 1986 Analytical Studies of Body Wave Propagation and Attenuation. Geotechnical Engineering Report No. GR86-15, Texas Univ at Austin Geotechnical Engineering Center.
- Shirley, D. J. 1978. An improved shear wave transducer. *J. Acoust. Soc. Am.*, Vol. 63, No. 5, pp. 1643-1645.
- Shirley, D. J. and Hampton, L. D. 1978. Shear Wave Measurements in Laboratory Sediments. *J. Acoust. Soc. Am.*, Vol. 63, No. 2, pp. 607-613.
- Stokoe, K. H. Joh, S. H. and Woods, R. D. 2004. Some Contributions of in situ Geophysical Measurements to Solving Geotechnical Engineering Problems. Proceedings of ISC-2: Geotechnical and Geophysical Site Characterization. Viana da Fonseca and Mayne, Eds., Porto, Portugal, 19-22 September, Millpress Science Publishers, Rotterdam, Netherlands. Vol. 1, pp. 97-132.
- Styler M. A. and Howie, J. A. 2013 Continuous Monitoring of Bender Element Shear Wave Velocities During Triaxial Testing. *Geotech. Test. J.*, Vol. 36, No. 5, pp. 649-659.
- Sukolrat J., Nash D.F.T., Ling, M.L. and Benahmed N. 2006 The Assessment of Destructuration of Bothkennar Clay Using Bender Elements. In *Soft Soil Engineering: Proceedings of the Fourth International Conference on Soft Soil Engineering*, Vancouver, Canada, pp.471.
- Sukolrat J. 2007. Structure and Destructuration of Bothkennar Clay. Diss. PhD thesis, University of Bristol.
- Tang, A., Vu, M. and Cui, Y. 2011. Effects of the Maximum Grain Size and Cyclic Wetting/Drying on the Stiffness of a Lime-Treated Clayey Soil. *Géotechnique*, Vol. 61, No. 5, pp. 421-429.
- Viana da Fonseca, A. V., Carvalho, J., Ferreira, C., Santos, J. A., Almeida, F., Pereira, E., Feliciano, J., Grade, J. and Oliveira, A. 2006. Characterization of a Profile of Residual Soil from Granite Combining Geological, Geophysical and Mechanical Testing Techniques. *Geotech. Geologic. Eng.*, Vol. 24, No. 5, pp.1307–1348.
- Viana da Fonseca, A. V., Ferreira, C. and Fahey, M. 2009. A Framework Interpreting Bender Element Tests, Combining Time-Domain and Frequency-Domain Methods. *Geotech.*

- Test. J., Vol. 32, pp. 91-107.
- Viggiani, G. and Atkinson, J. H. 1995. Interpretation of Bender Element Tests. *Géotechnique*, Vol. 45, No. 1, pp. 149-154.
- Wang, Y., Cui, Y. J., Tang, A. M., Tang, C. S. and Benahmed, N. 2015. Effects of Aggregate Size on Water Retention Capacity and Microstructure of Lime-Treated Silty Soil. *Geotech. Lett.*, Vol. 5, No. 4, pp. 269-274.
- Wang, Y. H., Lo K. F., Yan, W. M. and Dong, X. B. 2007. Measurement Biases in the Bender Element Test. *J. Geotech. Geoenviron. Eng.*, Vol. 133, No. 5, pp. 564-574.
- Wicaksono, R. I., Tsutsumi, Y., Sato, T., Koseki, J. and Kuwano, R.. 2008. Stiffness Measurements by Cyclic Loading, Trigger Accelerometer, and Bender Element on Sand & Gravel. *Deformational Characteristics of Geomaterials*, Atlanta, GA, Vol. 2, pp. 733-39.
- Yamashita, S., Kawaguchi, T., Nakata, Y., Mikami, T., Fujiwara, T. and Shibuya, S. 2009. Interpretation of International Parallel Test on the Measurement of  $G_{max}$  Using Bender Elements. *Soils Found.*, Vol. 49, pp. 631-650.
- Yang, J. and Yan, X. R. 2009. Site Response to Multi-Directional Earthquake Loading: A Practical Procedure. *Soil. Dyn. Earthq. Eng.*, Vol. 29, No. 4, pp. 710-721.
- Youn, J. U., Choo, Y. W. and Kim, D. S. 2008. Measurement of Small-Strain Shear Modulus  $G_{max}$  of Dry and Saturated Sands by Bender Element, Resonant Column, and Torsional Shear Tests. *Can. Geotech. J.*, Vol. 45, pp. 1426-1438.
- Zeng, X. W., and Grolewski, B. 2005. Measurement of  $G_{max}$  and Estimation of  $K_0$  of Saturated Clay Using Bender Elements in an Oedometer. *Geotech. Test. J.*, Vol. 28, No. 3, pp. 264-274.
- Zhou, Y. G., Chen, Y. M and Huang, B. 2005. Experimental Study of Seismic Cyclic Loading Effects on Small Strain Shear Modulus of Saturated Sands. *J. Zhejiang Univ. Sci. A.*, Vol. 6, No. 3, pp. 229-236.

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**Table 1 Sample characteristics**

Sample	Dry density (Mg/m <sup>3</sup> )	Water content (%)	Degree of saturation (%)	Curing time (Hours)
D1	1.65	17	72	25
D2	1.65	17	72	528
W1	1.65	22	93	27
W2	1.65	22	93	504

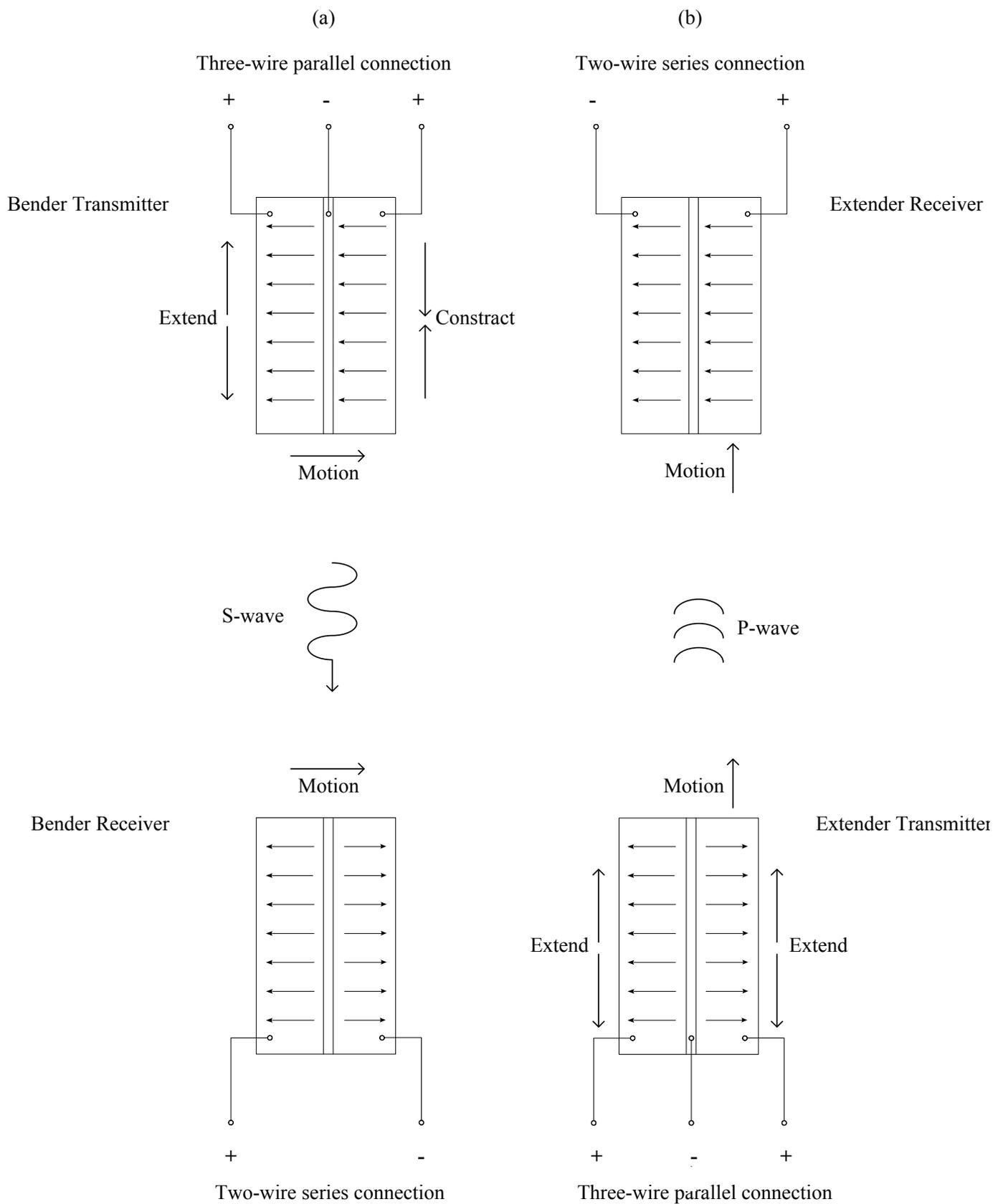


Figure 1. Sketch of bender and extender connection: (a) transmitting S-wave; (b) transmitting P-wave (after Lings and Greening 2001)



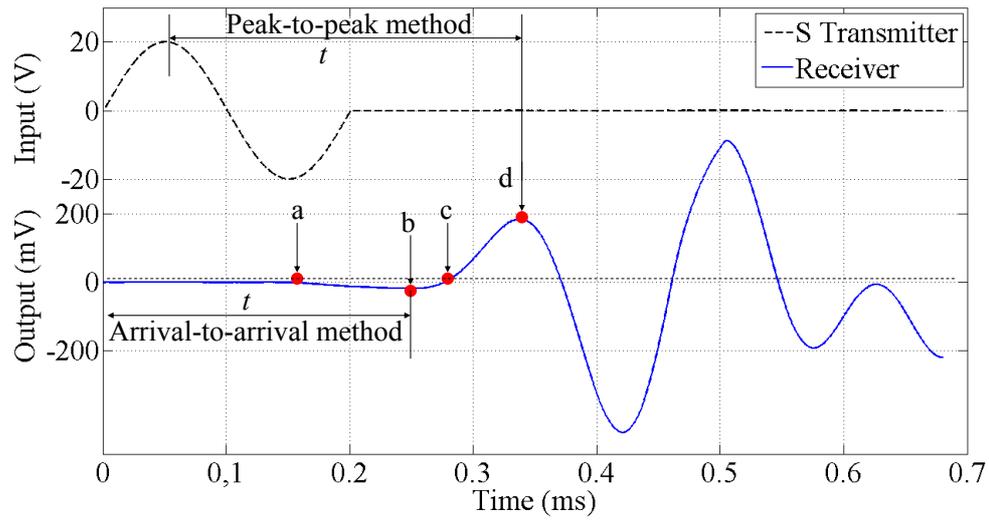
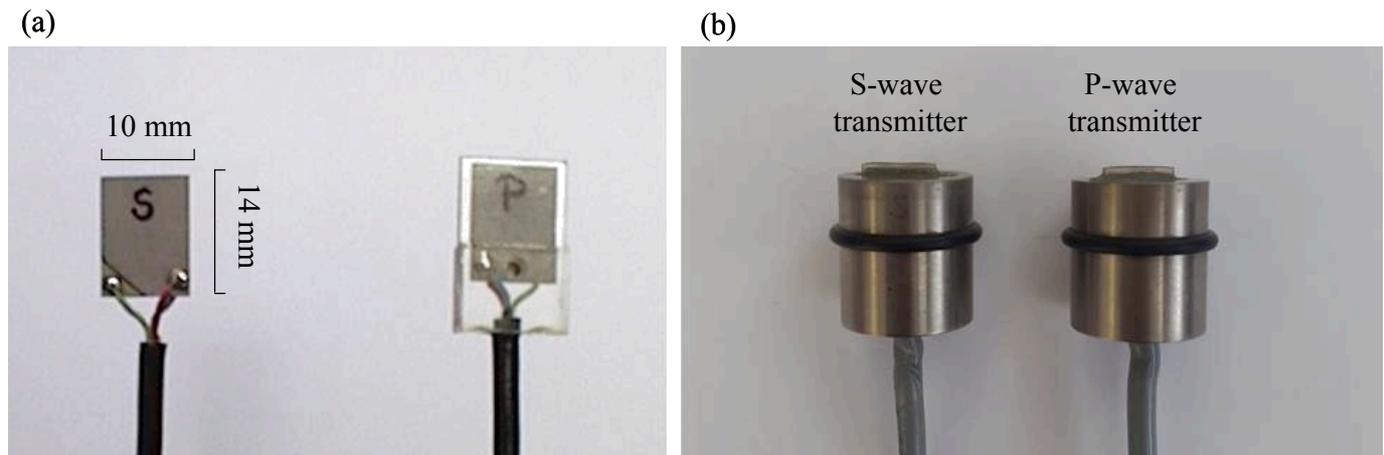


Figure 2. Typical S-wave transmitted and received signals



**Figure 3. Bender elements transducers: (a) wiring of two bender elements; (b) GDS bender elements waterproofed and encapsulated in pot**

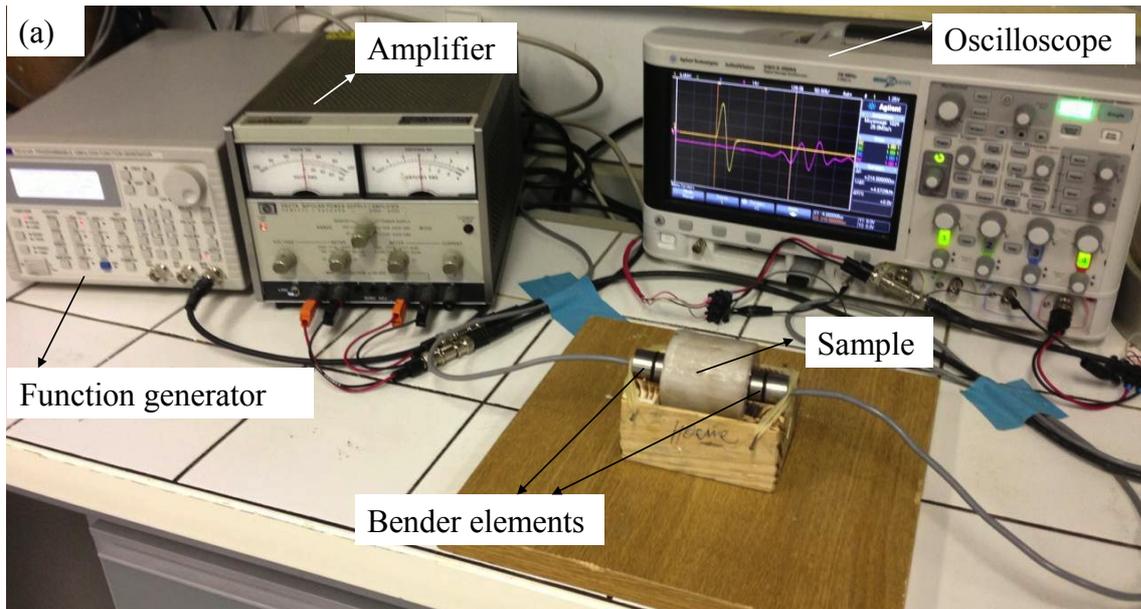
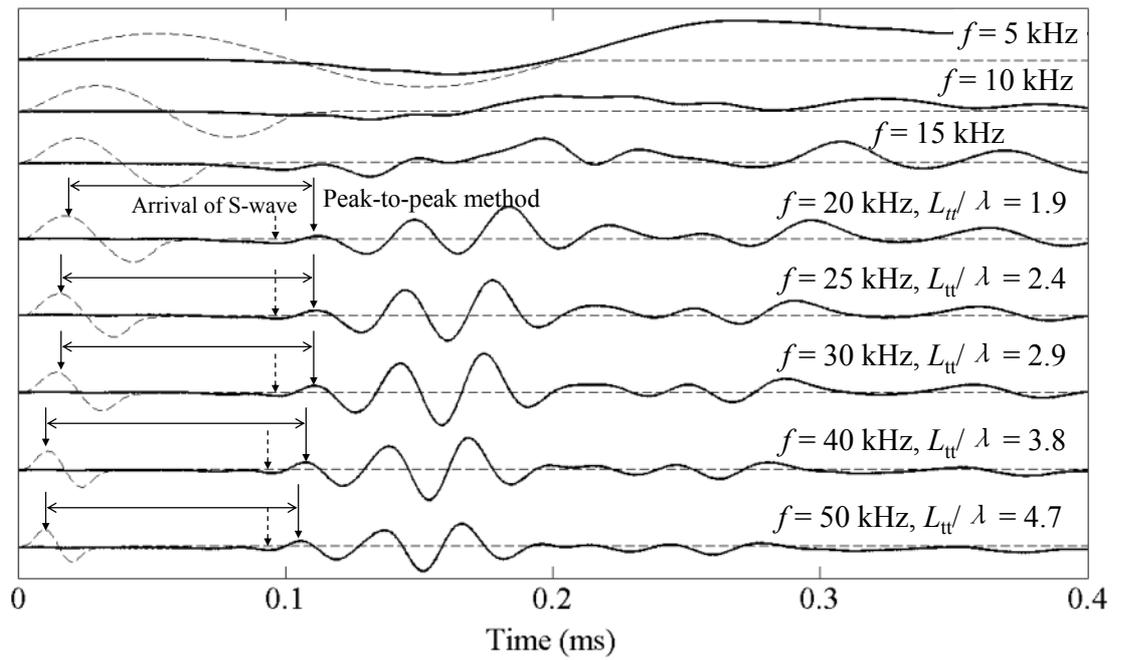
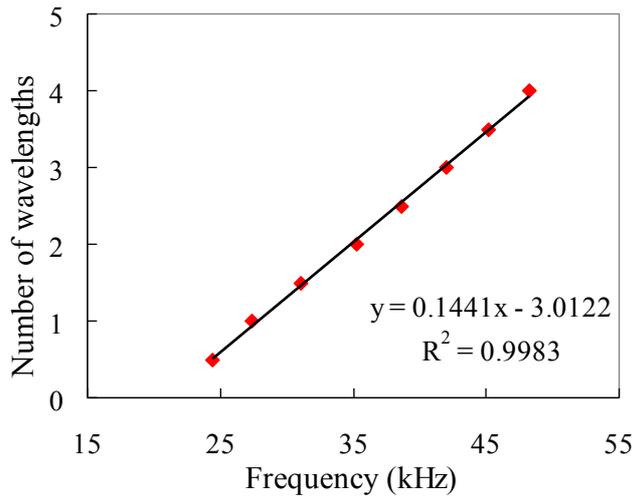


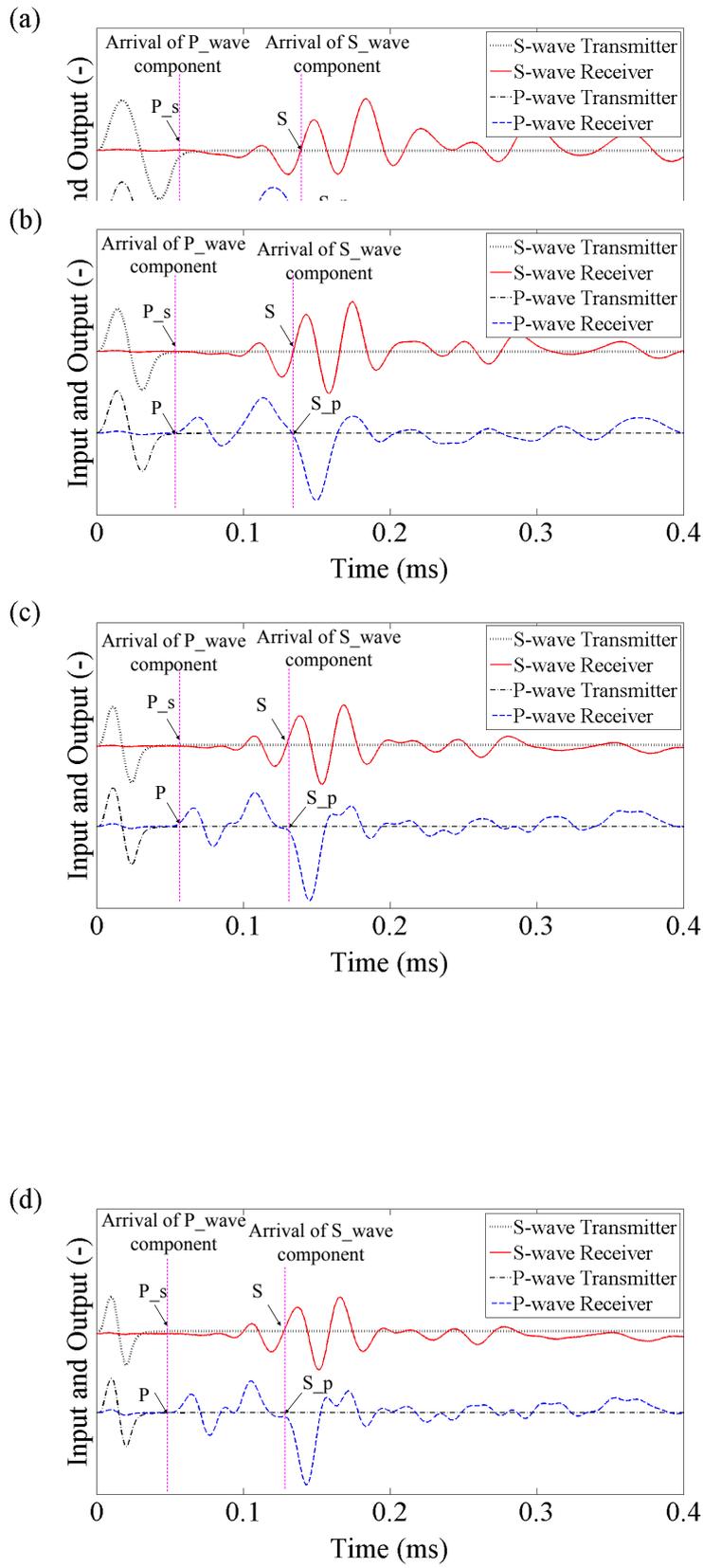
Figure 4. Setup of the bender element testing: (a) photo of the setup; (b) schematic diagram of the setup



**Figure 5. Measurements by traditional time domain methods (arrival-to-arrival method and peak-to-peak method) at different frequencies on a lime-treated sample (D1)**



**Figure 6. Measurements by “ $\pi$ -point” method on a lime-treated soil (D1)**



**Figure 7. Determination of the arrival point by S+P method on a lime-treated soil (D1): (a)  $f = 20$  kHz; (b)  $f = 30$  kHz; (c)  $f = 40$  kHz; (d)  $f = 50$  kHz**

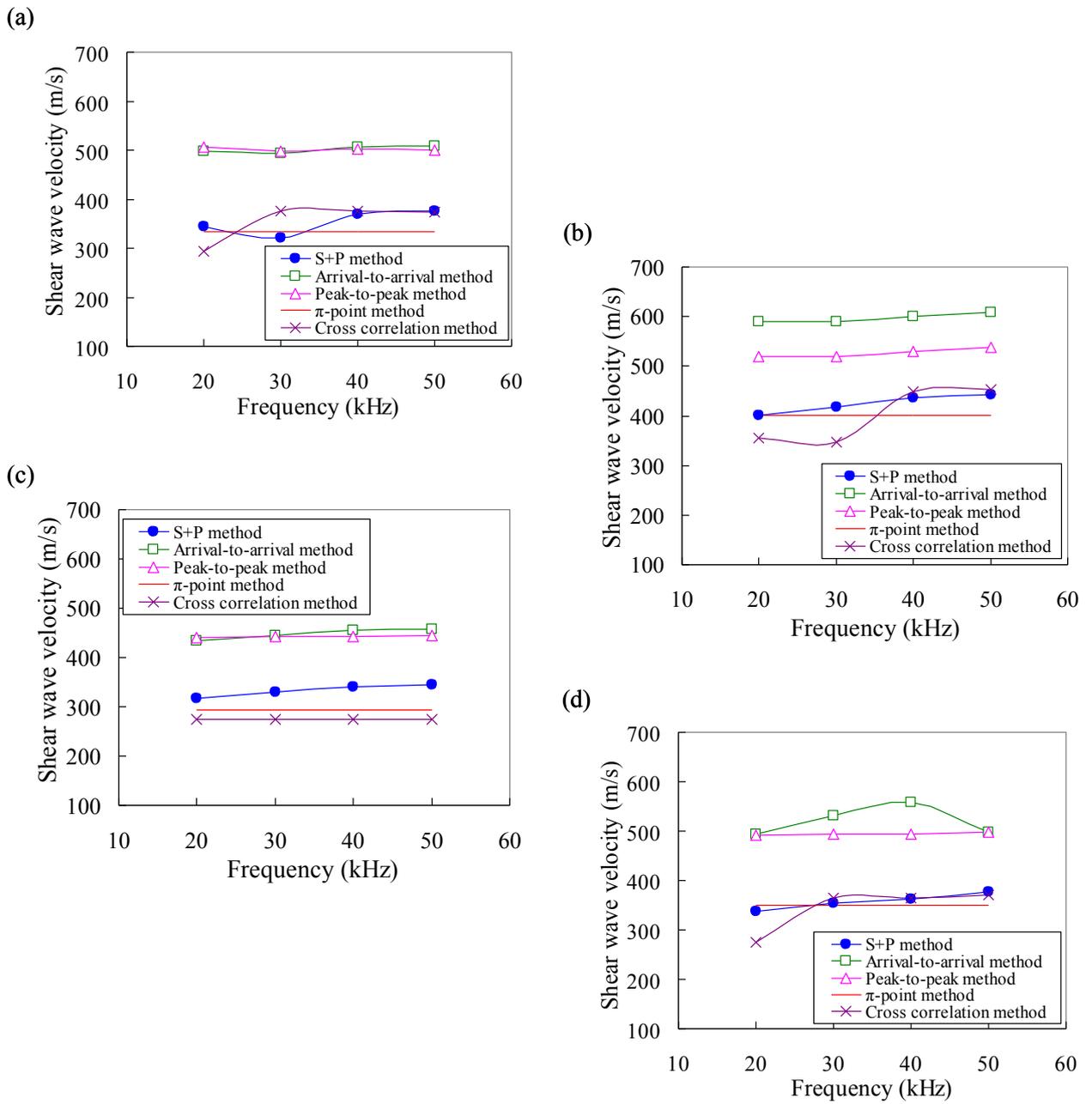


Figure 8. Comparisons of the measurements of shear wave velocity by different methods: (a) D1; (b) D2; (c) W1; (d) W2