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For elastic-perfectly plastic structures under prescribed loading histories, the celebrated Melan's theorem [1,2] gives a sufficient condition for the evolution to become elastic in the large-time limit. That situation, classically referred to as shakedown, is associated with the intuitive idea that the plastic strain tends to a limit as time tends to infinity. The Melan's theorem has the distinctive property of being path-independent, i.e. independent on the initial state of the structure. Regarding fatigue design, shakedown corresponds to the most beneficial regime of high-cycle fatigue, as opposed to the regime of low-cycle fatigue which typically occurs if the plastic strain does not converge towards a stabilized value.

This communication addresses the extension of Melan's theorem to situations in which the elastic moduli are fluctuating in time. Such time fluctuations may result from significant variations of the temperature. In a lot of practical situations, structural elements are indeed submitted to thermomechanical loading histories in which variations of the temperature are large enough for the temperature dependence of the material not to be negligible. The case of the temperature-dependent yield limits has been considered in [3]. In contrast, the case of temperature-dependent elastic moduli remains a long standing issue, even though the practical importance of that problem has been recognized early [4]. It has been conjectured that Melan's theorem could be extended to temperature-dependent (or time-dependent) elastic moduli [5], but no theoretical result is available. This communication aims at providing results in that direction, with a special emphasis on time-periodic variations [6].

If Melan's condition is satisfied, we show that shakedown indeed occurs provided the time fluctuations of the elastic moduli satisfy a certain condition (which in particular is fulfilled if the time fluctuations are not too large). We provide a counterexample which shows that setting such a constraint on the elastic moduli is necessary to reach path-independent theorems as proposed. A simple mechanical system is studied as an illustrative example.

References

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