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Dry granular flows: rheological measurements of the $\mu(I)$ -rheology

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Synopsis

Granular materials do not always flow homogeneously like fluids when submitted to external stress, but often form rigid regions that are separated by narrow shear bands where the material yields and flows. This shear localization impacts their apparent rheology, which makes it difficult to infer a constitutive behaviour from conventional rheometric measurements. Moreover, they present a dilatant behaviour, which makes their study in classical fixed-volume geometries difficult. These features led numerous groups to perform extensive studies with inclined plane flows, which were of crucial importance for the development and the validation of the $\mu(I)$ -rheology. Our aim is to develop a method to characterize granular materials with rheometrical tools. Using rheometry measurements in an annular shear cell, dense granular flows of 0.5 mm spherical and monodisperse beads are studied. A focus is placed on the comparison between the present results and the $\mu(I)$ -rheology. From steady state measurements of the torque and the gap under imposed shear rate $\dot{\gamma}$ and normal force F_N , we define an inertial number I . We show that, at low I (small $\dot{\gamma}$ and/or large F_N), the flow goes to a quasi-static limit, and the response in terms of dimensionless stress or internal friction coefficient $-\mu$ – and solid concentration $-\phi$ – profiles is independent of the inertial number. Upon increasing I (large $\dot{\gamma}$ and/or small F_N), dilation occurs and ϕ decreases while μ increases. The observed variations are in good agreement with previous observations of the literature (Jop et al. 2006; Hatano 2007). These results show that the constitutive equations $\mu(I)$ and $\phi(I)$ of granular materials can be measured with a rheometer.

32 **I. Introduction**

33 Granular matter shows both solid and fluid behavior. Of interest in many industrial processes
34 and in geophysics, granular flows are the focus of very active researches (Duran 2000). These
35 materials are very sensitive to various parameters: geometry of the flow, wall roughness, flow
36 rate, shape and size distribution of the grains, and coupling with the interstitial fluid
37 (Andreotti et al. 2013). Due to their macroscopic size, the interactions between the grains are
38 dissipative (friction and inelastic collisions); the energy lost is then transferred to internal
39 degrees of freedom. The lack of Brownian motion and the dissipative interactions, make the
40 granular material an intrinsically nonequilibrium system.

41 In the dry case – *without interstitial fluid* –, the rheology is solely governed by momentum
42 transfer and energy dissipation occurring in direct contacts between grains and with the walls.
43 Despite the seeming simplicity of the system, the behavior of dry granular material is very
44 rich and extends from solid to gaseous properties depending on the flow regime. In the
45 absence of a unified framework, granular flows are generally divided into three different
46 regimes. (i) At low shear, particles stay in contact and interact frictionally with their
47 neighbours over long periods of time. This “quasi-static” regime of granular flow has been
48 classically studied using modified plasticity models based on a Coulomb friction criterion
49 (Schofield & Wroth 1968; Becker and Lippmann 1977). The response in terms of velocity or
50 solid fraction profiles is independent of the shear rate (Roux & Combe 2002; GDR Midi
51 2004). Consequently, if the material remains homogeneous, this state only depends on
52 geometric data (shape and size distribution of the grains) and on the inter-granular friction
53 coefficient. (ii) Upon increasing the deformation rate, a viscous-like regime occurs and the
54 material flows more as a liquid (Forterre & Pouliquen 2008). In this intermediate regime, the
55 particles experience multi-contact interactions. (iii) At very high velocity, a transition occurs
56 towards a gaseous regime, in which the particles interact through binary collisions
57 (Goldhirsch 2003; Jenkins and Savage 1983).

58 For the modelling of dense granular flows, the concept of inertial number I has been widely
59 used and investigated with regard to its relationship with dynamic parameters, such as
60 velocity, stress, and friction coefficient which leads to constitutive relations for granular
61 flows. Thus, ‘*dynamic dilatancy*’ law and ‘*friction*’ law were deduced from discrete
62 simulation of two dimensional simple shear of a granular material without gravity (da Cruz et
63 al. 2005). Those results establish that the flow regime and rheological parameters depend on a
64 single dimensionless number that represents the relative strength of inertia forces with respect

65 to the confining pressure, or the combined effect of pressure and shear rate. Indeed, for a
66 three-dimensional granular medium made up of monodisperse spheres (d, ρ resp. the
67 particle's diameter and density) undergoing simple shear flow at a shear rate $\dot{\gamma}$ under an
68 applied normal confining stress σ , this dimensionless number – inertial number – is defined
69 as $I = \dot{\gamma}d / \sqrt{\sigma / \rho}$. Alternatively, one may use the Savage number which is the square of the
70 inertial number (Savage & Hutter 1989) or the Coulomb number (Ancy et al. 1999, see also
71 GDR Midi 2004; Baran & Kondic 2006; Hatano 2007; Luding 2008). Dimensionless number
72 I can also be seen as the ratio of $d / \sqrt{\sigma / \rho}$, the time scale for grains to rearrange due to the
73 confining stress σ , to the time scale $\dot{\gamma}^{-1}$ for deformation by the flow. Hence, it characterizes
74 the local ‘‘rapidity’’ of the flow. Thus it was observed that both dimensionless quantities: the
75 internal friction coefficient $\mu = \tau / \sigma$ and the solid fraction ϕ are functions of I (GDR Midi
76 2004; da Cruz et al. 2005; Hatano 2007). Thereby, the inertial number I opened a new path
77 unifying, in a single phenomenological law, many experimental and numerical data in a wide
78 variety of transient flows from the rotating drum to inclined plane flows, where large flowing
79 zones form. A local relation between an apparent friction coefficient and I then successfully
80 captures many aspects of these rapid granular flows (Savage & Hutter 1989; GDR Midi 2004;
81 Jop et al. 2006; Forterre & Pouliquen 2008).

82 Following general results from simulations of planar shear (da Cruz et al. 2005; Iordanoff &
83 Khonsari 2004), and successful applications to inclined plane flows (Pouliquen & Forterre
84 2002; Silbert et al. 2003), the experiments of Jop and co-workers (Jop et al. 2006) were
85 carried out to quantify, for glass beads, the $\mu(I)$ –rheology from the quasi-static to the rapid
86 flow regime, corresponding to moderate inertial number (from 0.01 to 0.5) as:

$$87 \quad \mu = \mu_s + (\mu_2 - \mu_s) / (1 + I_0 / I) \quad (1)$$

88 in which μ_s, μ_2 and I_0 are three fitting parameters dependent on material properties.
89 According to this law, the internal friction coefficient μ goes from a minimum value μ_s for
90 very low I up to an asymptotic value μ_2 when I increases. The asymptotic value of μ at
91 high inertial number was not obtained by da Cruz and co-workers who observed an
92 approximately linear increase of the internal friction coefficient from the static internal
93 friction value:

$$94 \quad \mu = \mu_s + aI \quad (2)$$

95 where μ_s and a are two fitting parameters which depend on material properties. In 3D-
 96 simulation studies, Hatano did not either observe the asymptotic value of μ at high I : from
 97 $I = 10^{-4}$ to $I = 0.2$, he reported a law in which the friction coefficient increases as a power of
 98 the inertial number (Hatano 2007):

$$99 \quad \mu = \mu_s + aI^n \quad (3)$$

100 It should be pointed out however that this rheology agrees with earlier scaling relations
 101 stemming back to Bagnold (Bagnold 1954). Bagnold described a mechanism of momentum
 102 transfer between particles in adjacent layers that assumes instantaneous binary collisions
 103 between the particles during the flow. Under this assumption, the inverse strain rate is the
 104 only relevant time scale in the problem leading, for a constant solid fraction ϕ , to constitutive
 105 relations between the shear stress τ (resp. normal stress σ) and shear rate $\dot{\gamma}$ of the form
 106 $\tau = \rho d^2 f_1(\phi) \dot{\gamma}^2$ (resp. $\sigma = \rho d^2 f_2(\phi) \dot{\gamma}^2$) where f_1 and f_2 are functions of the solid fraction ϕ
 107 only. Bagnold's scaling has been verified for dry grains in both collisional and dense flow
 108 regimes (Jenkins & Savage 1983; Lois et al. 1987; Silbert et al. 2001; Lois et al. 2005; da
 109 Cruz et al. 2005). The main difference between the *Bagnold* and $\mu(I)$ approaches is that: in
 110 the first case, ϕ is kept constant for one given flow, whereas it varies freely and depends on
 111 the flow in the second case. Indeed, if the pressure is controlled, the solid fraction ϕ is free to
 112 adapt in the system with the evolution of other parameters. If ϕ is fixed however and the
 113 normal pressure measured, this will fix the value of I . Then when ϕ is fixed, the expression
 114 of I shows that the normal pressure should scale with the square of the shear rate as it was
 115 shown experimentally in annular parallel-plate shear cell (Savage & Sayed 1984).

116

117 The applicability of the $\mu(I)$ -rheology has been examined by various studies (Jop et al.
 118 2005, 2006; Hatano 2007; Forterre & Pouliquen 2008; Ruck et al. 2008; Aranson et al. 2008;
 119 Peyneau & Roux 2008; Staron et al. 2010; Gaume et al. 2011; Tripathi & Khakhar 2011;
 120 Chialvo et al. 2012; Azéma & Radjai, 2014; Gray & Edwards 2014; Edwards & Gray 2015)
 121 and many simulations and experiments have shown that the rheology is valid for various flow
 122 configurations for different choices of materials, although deviations from this rheology may
 123 take place for very slow (quasi-static) flows with small values of inertial number (Aranson et
 124 al. 2008; Staron et al. 2010; Gaume et al. 2011). Up to now, there is however in the literature,
 125 no experimental data from conventional rheology (*rheometer with conventional geometries*)
 126 describing the $\mu(I)$ -rheology of a dry granular material. Indeed, *Couette flows* of granular

127 materials are characterized by the formation of well-defined shear bands that resemble
128 qualitatively the behavior of a yield stress fluid, and make the interpretation of rheometric
129 data tricky. For slow flows, shear banding is generally very strong, with shear bands having a
130 typical thickness of five to ten grain diameters. In this regime of slow flow, the averaged
131 stresses and flow profiles become essentially independent of the flow rate, so that constitutive
132 relations based on relating stresses and strain rates are unlikely to capture the full physics
133 (Schofield & Wroth 1968; Nedderman 1992; Fenistein & van Hecke 2003; Fenistein et al.
134 2004). Moreover, due to the dilatant behaviour of granular materials, *constant solid fraction*
135 ϕ experiments classically made with a Couette shear-cell rheometer are much more difficult
136 to perform than *constant friction coefficient* μ experiments such as inclined plane flows.
137 When a granular matter is sheared, the spatial distribution of the shear rate is not always
138 homogeneous. Often, shear is localized near the system boundaries with a shear localization
139 width amounting to a few particle diameters. Nevertheless, depending on the boundary
140 conditions, confining pressure and shear velocity, the bulk of the granular system may exhibit
141 different behaviours. For high shear velocities and small confining pressures – *high I* –, the
142 granular matter flows homogeneously (Koval et al. 2009). However, at small inertial number,
143 it was indeed shown that in confined annular flow at small shear velocities and high confining
144 pressures – *small I* –, the shear may be not homogeneous and solid and fluid phases coexist
145 (Aharonov & Sparks 2002; Jalali et al. 2002).

146

147 In the present work, we show that it is not necessary to develop specific set-ups (such as the
148 inclined plane) to study dense granular flows. Indeed, we show that a simple annular shear
149 cell (Carr and Walker 1968, Savage and Sayed 1984) can be adapted to a standard rheometer
150 to study the rheology of granular materials under controlled confining pressure. It allows us in
151 particular to obtain the dilatancy law $\phi(I)$ and also to study very accurately the quasi-static
152 limit. Thus, from the steady state measurements of the torque and the gap during an imposed
153 shear flow under an applied normal confining stress σ , we report two laws in which the
154 internal friction coefficient and the solid fraction are functions of a single dimensionless
155 number: the inertial number I . An effort is then made to compare the present results to the
156 $\mu(I)$ –rheology described in the literature. Indeed, we show that at low inertial number I
157 (small $\dot{\gamma}$ and/or large σ), the flow goes to the quasi-static limit, and the response in terms of
158 internal friction coefficient μ and solid fraction ϕ profiles is independent of the inertial
159 number. Upon increasing I (large $\dot{\gamma}$ and/or small σ), ϕ decreases while μ increases. The

160 observed variations are in good agreement with previous observations of the literature.
161 Importantly, we also show that changing the initial gap size does not significantly affect these
162 results. This suggests that shear localization is mostly absent in the intermediate dense flow
163 regime, although it may still occur when the granular material is slowly sheared (in the quasi-
164 static regime).

165

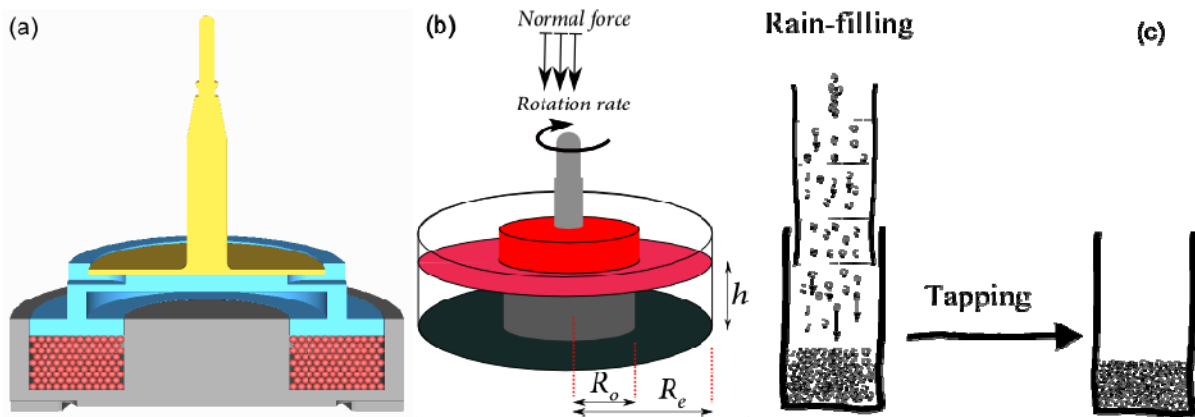
166 **II. Materials and methods**

167 To investigate the steady flows of dry granular materials and determine the $\mu(I)$ -rheology,
168 three main features are required: (i) to avoid shear banding, (ii) to apply a confining stress in
169 the velocity gradient direction, and (iii) to allow volume fraction variations. If one wants to
170 use a rheometer, (i) implies that the use of a Couette cell should be avoided since it is
171 characterized by a shear stress inhomogeneity that naturally leads to shear-banding. Both the
172 cone-and-plate and the parallel-plate geometries allow a normal force to be imposed in the
173 velocity gradient direction; however, the analysis of the cone-and-plate flow can be performed
174 only at a single gap value, i.e. it cannot be used to characterize a material whose the volume
175 varies under shear. On the other hand, any gap variation in parallel-plate geometry can be
176 accounted for in the determination of the shear rate value. Moreover, shear banding should in
177 principle be avoided in this last geometry since the shear stress is independent of the vertical
178 position in the gap (Macosko 1993). Nevertheless, the shear rate varies along the radial
179 position and is equal to zero in the center. One should thus try to avoid the use of the central
180 zone of the gap. Finally, the material should be confined by lateral walls to make sure that any
181 gap variation actually leads to a material volume fraction variation. These requirements led us
182 to develop a home made annular shear cell, inspired by Boyer et al. 2011, in which pressure-
183 imposed measurements can be performed as shown in Figure 1. Annular shear cells have been
184 extensively used to characterize the flow of pharmaceutical powders and dry granular
185 materials (Carr and Walker 1968, Savage and Sayed 1984, Schulze 1996; Schwedes 2003).

186 We use a granular material made of rigid polystyrene beads (from Dynoseeds) of density
187 $\rho = 1050 \text{ kg/m}^3$, of diameter $d = 0.5 \text{ mm}$ (with a standard deviation of 5%). Spherical beads
188 fill the annular box between two static concentric cylinders with respectively an inner and
189 outer radii of $R_i = 21 \text{ mm}$ and $R_o = 45 \text{ mm}$. The width of the annular trough is about $48d$
190 leading to a ratio of inner to outer wall radii of 0.46. We also used another annular channel
191 with the same width but with a larger ratio of inner to outer cylinder radii equal to 0.61
192 ($R_i = 38 \text{ mm}$ and $R_o = 62 \text{ mm}$). We have verified that changing the ratio of inner to outer

193 cylinder radii of the annular shear cell does not significantly affect the results. The filling
 194 height (initial gap, h_0) of the annular box is adjustable from a few grain diameters (typically
 195 $5d$) to $30d$. The cylinders were finished as smoothly as possible to permit the granular
 196 material to slip there as readily as possible. For that, they are made of polyoxymethylene
 197 (POM) resin which exhibits a low friction coefficient due to the flexibility of the linear
 198 molecular chains. We have measured the friction coefficient at the wall μ_w between
 199 polystyrene beads and a plane made of POM by measuring with our rheometer the sliding
 200 stress under different normal stresses such as in Jenike's shear tester (Jenike 1964; Schwedes
 201 2003); it is found to be very small: $\mu_w \approx 0.05$.

202



203

204

205 Figure 1: (a-b) Cross section of the annular plane shear flow. The shear and pressure are
 206 provided by a ring which is assembled on a *Kinexus* rheometer by *Malvern* that is free to
 207 move vertically while maintaining a constant rotation rate or shear rate and imposed pressure.
 208 (c) Rain-filling coupled to tapping to get the denser piling sample ('D').

209

210 The experiments were performed initially on a very dense piling $\phi_0 \approx 0.625$, close to the so
 211 called random close packing of 0.637 (Torquato et al. 2000; Camenen et al. 2012) obtained by
 212 combining a rain-filling and tapping the box (Ovarlez et al. 2003) to get a reasonably uniform
 213 packing (Figure 1b). However, we will show below that the steady state obtained when the
 214 material is sheared is the same for an initially looser piling (Figure 2c). Granular beads are
 215 then driven by the ring-shaped upper boundary made of PMMA, which is assembled on a
 216 *Kinexus Pro* rheometer by *Malvern*. To avoid wall slip, both the moving upper boundary and
 217 the static lower boundary are serrated, with 0.5 mm ridges which correspond to the size of
 218 grains (Shojaee et al. 2012).

219

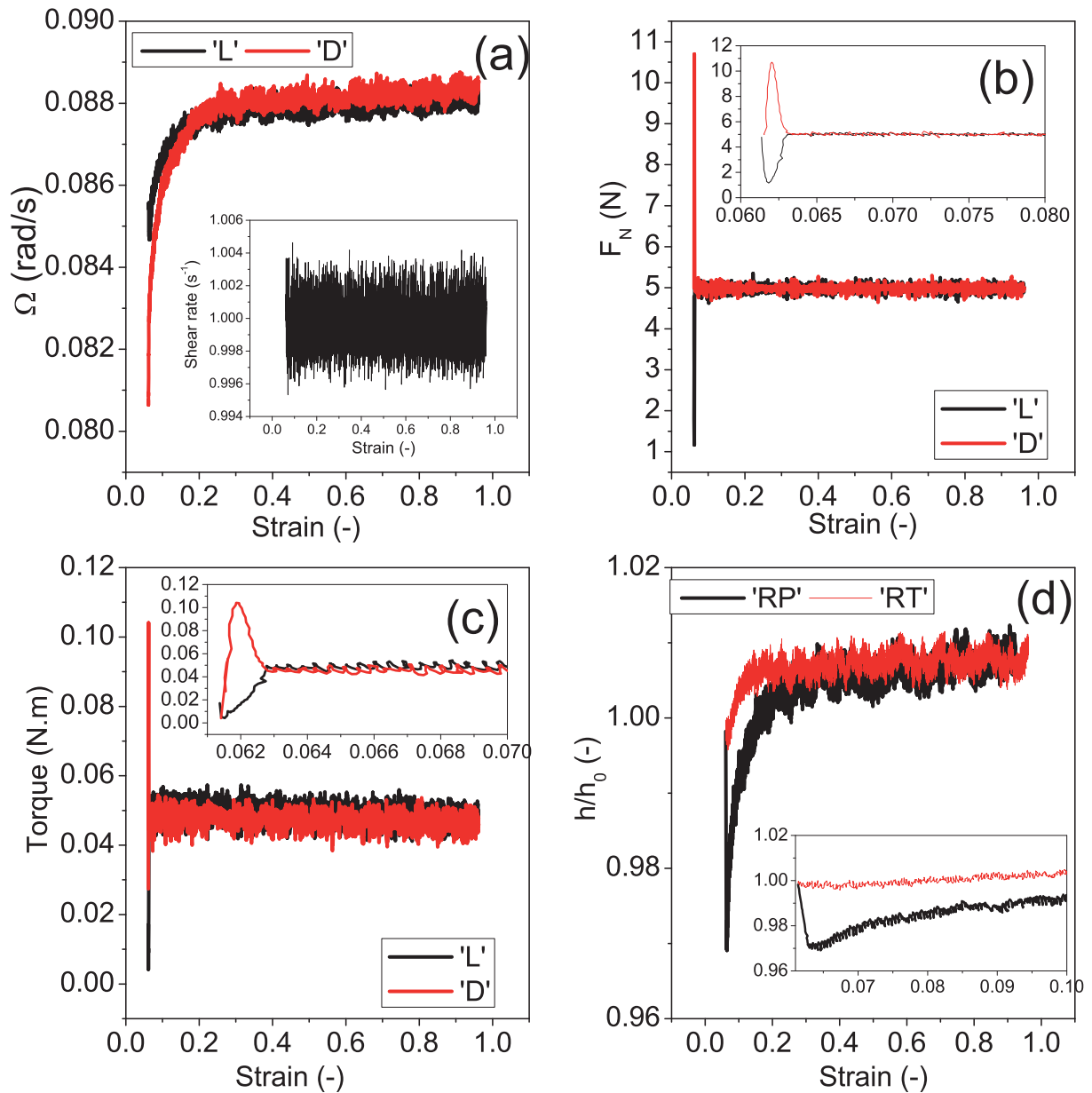
220 In our rheometer, instead of setting the value of the gap size for a given experiment, as in
221 previous studies (Schulze 1996; Schwedes 2003) and generally in rheometric measurements,
222 we impose the normal force (i.e. the confining normal stress) and then, under shear, we let the
223 gap size vary in order to maintain the desired value of the normal force. We then have access
224 to instantaneous measurements of the driving torque T and the gap h for imposed normal
225 force F_N and shear rate $\dot{\gamma}$: in this case, the solid fraction ϕ is not fixed, but adjusts to the
226 imposed shear. However, it remains important to notice that, in order to keep the imposed
227 shear rate constant, the rheometer adjusts the rotation velocity Ω since the gap varies as will
228 be discussed below.

229

230 A typical measurement is shown in Figure 2, where we start out with a given gap $h_0 \approx 6d$,
231 impose a constant shear rate $\dot{\gamma}$ and normal force F_N and measure the torque T and the gap
232 h as a function of strain (or time). The system reaches a steady state after a certain amount of
233 shear strain but we carefully compare the transient dynamics of these quantities, beginning
234 with freshly poured grains for two preparations, ‘rain piling’ and ‘rain coupled to tapping
235 piling’ (resp.) which allow us to get the looser (‘L’) piling and the denser (‘D’) piling (resp.)
236 samples with an initial solid fraction of 0.612 and 0.625 (resp.).

237 Let us start with the imposed quantities. In both cases, a small variation of the rotation
238 velocity is seen (Figure 2a). As discussed above, this variation is a signature of the shear-
239 induced gap variations; it ensures that the shear rate is constant at any time or strain (see
240 inset). We also see that, in both cases, the normal force reaches quickly the stationary targeted
241 value and remains steady during the whole experiment (Figure 2b). Nevertheless, the short
242 transient behavior is different: in the looser sample (‘L’), a significant decrease of the normal
243 force is initially observed while a very sharp increase is observed in the densest piling (‘D’).
244 The reason is that the densest system initially wants to dilate, and the rheometer response is
245 not fast enough to allow for this fast dilation at constant normal force. By contrast, the loose
246 sample initially wants to compact, and again the rheometer response is not fast enough to
247 allow for this fast compaction at constant normal force.

248



249

250 Figure 2: Comparison between samples from the two different pilings: rain piling (which
 251 allows us to get the looser piling sample 'L') and rain piling coupled to tapping (that lets us to
 252 get the denser piling sample 'D') of the (a) rotational rate against strain; Inset: the imposed
 253 shear rate vs. strain, (b) imposed normal force as a function of strain, (c) driving torque and
 254 (d) gap size rescaled by the initial gap against strain. The insets in (b-c-d) are the same data
 255 showing the sharp increase/decrease of the normal force, the overshoot of the torque and the
 256 gap change at the beginning of shearing.

257

258 The transient dynamics of the measured quantities (torque and gap size) also shows a clear
 259 difference between the two samples. In Figure 2c, we see that, in the loosest piling ('L'), the
 260 torque increases before reaching the steady state regime. By contrast, there is an overshoot

261 within the densest sample ('D'), followed by a fast decrease of the torque: the peak strongly
262 depends on the imposed normal force and/or on the shear rate; the decrease corresponds to the
263 continuous dilation of the material under shear. Indeed, in the same time, the gap is not fixed,
264 but adjusts to the imposed shear (Figure 2d). Shear dilation is observed: the measured gap
265 increases up to a constant value since the average of the initial solid fraction is very close to
266 the random close packing. In this regime, shearing necessarily implies immediate dilatancy of
267 the granular media. In contrast, an initially looser sample first compact instead of dilate under
268 shear: when the system is driven under a fixed normal pressure, the granular packing
269 undergoes compaction before dilation into the final state once sufficiently compacted (Wroth
270 1958).

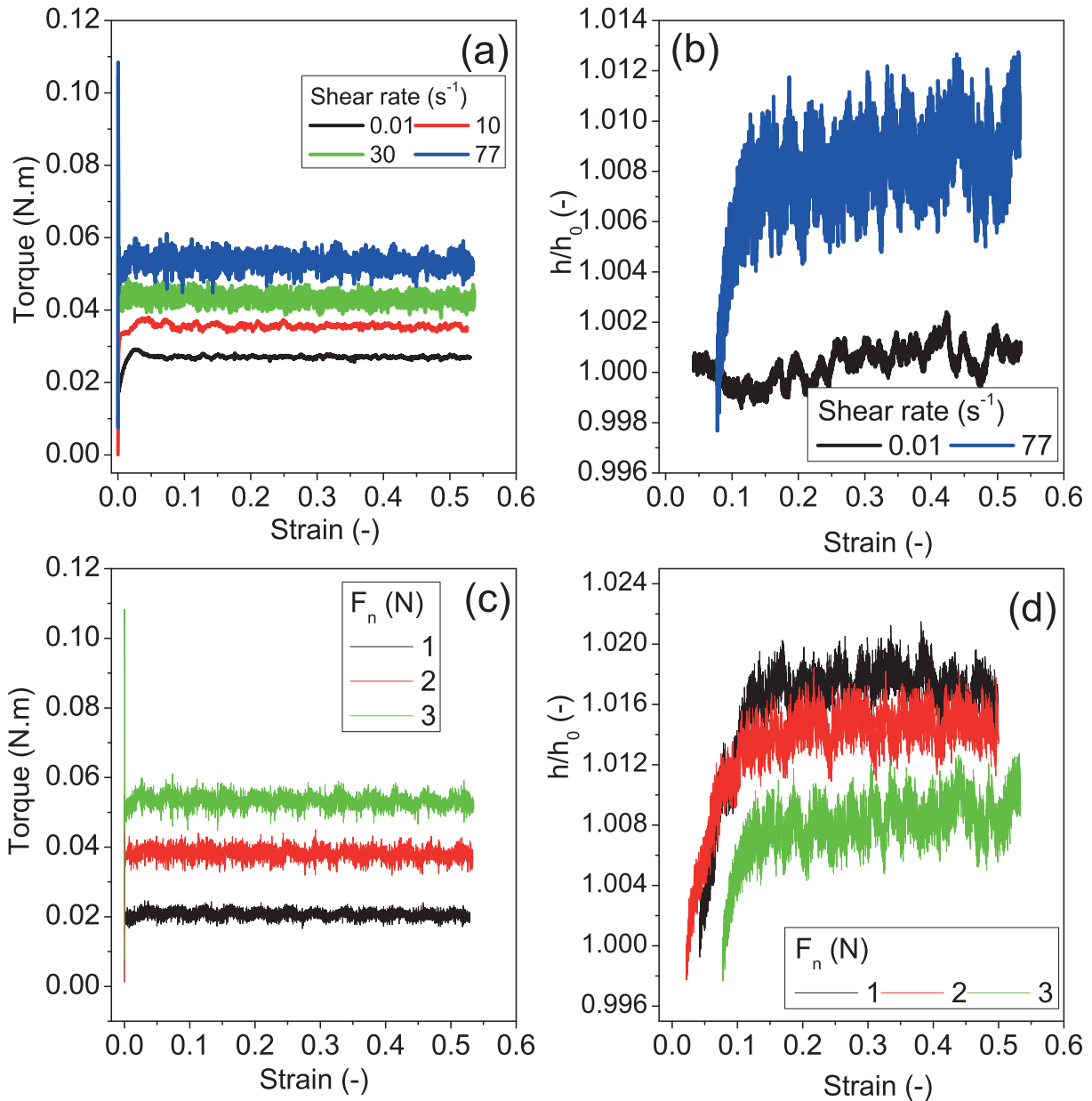
271
272 The most important observation is that the same steady state is reached (same torque and
273 same solid fraction) for the two initial states. This indicates that the material history has been
274 erased, and that we are actually studying the material steady response as a function of $\dot{\gamma}$ and
275 F_N only. As a consequence, there is no need for the sample preparation for the grains we
276 study, namely monodisperse spheres (note that this might not be a general result). All the
277 annular shear cell requires is that the sample is sheared at the required normal pressure until
278 the critical state is reached. However it is necessary to adopt a reference packing state and
279 ensure that the piling method used allows us to achieve sufficient repeatability for a given set
280 materials and system parameters. Hence, we prepare all samples in the same way: a very
281 dense material, from a rain piling coupled to tapping, is used in all the experiments presented
282 below (the initial average volume fraction is 0.625).

283

284 **III. Experimental results and analyses**

285 We have measured the driving torque and the gap as a function of shear strain for various
286 imposed normal force F_N (between 1 and 5N) and applied shear rate $\dot{\gamma}$ (between 0.01
287 and 77 s⁻¹) for a given gap $h_0 \approx 6d$.

288 In Figure 3 (a, b), we show those measurements for $F_N = 3N$ and various $\dot{\gamma}$. At low shear
289 rate, the driving torque increases slowly before reaching a steady plateau within strain of
290 order of unity. Meanwhile, the gap fluctuates around its initial value (we show the gap size
291 rescaled by its initial value before shearing h/h_0 called the rescaled gap in the following.).



292

293 Figure 3: Evolution as a function of the strain at 3N imposed normal force under different
 294 applied shear rates of: (a) the driving torque and (b) the rescaled gap size (only 2 curves are
 295 shown for clarity). Evolution as a function of the strain at 77 s⁻¹ imposed shear rate under
 296 different applied normal forces of: (c) the driving torque and (d) the gap size rescaled by its
 297 initial value before shearing.

298

299 Upon increasing the imposed shear rate, an overshoot occurs: its amplitude increases with
 300 increasing the shear rate. In steady state, a rate dependence of the torque is observed (Figure
 301 3a). Moreover, increasing the imposed shear rate causes an increase of the gap size (Figure
 302 3b) allowing to quantify the dynamic dilatancy of the granular material. Notice that, with

303 increasing the applied shear rate, large fluctuations of the driving torque and also of the gap
304 size evolution occur in steady state flows.

305 Similarly, in experiments in which different normal forces are imposed at a given shear rate
306 (Figure 3c, d), the steady torque is observed to increase while the steady solid fraction (steady
307 gap) decreases when the normal force is increased.

308 Once the above described experiments are combined, we can obtain the constitutive laws of
309 the dry granular material i.e. the dependence of the steady solid fraction ϕ and the ratio
310 between shear and normal stresses τ/σ variation on shear rate. Indeed, from macroscopic
311 quantities T , Ω , F_N and h , the shear stress τ , the normal stress σ , shear rate $\dot{\gamma}$, and the
312 solid fraction ϕ can be computed.

313

314 In the annular plate-cup shear geometry, the driving torque is converted into shear stress using
315 the equation:

$$316 \quad T = \int_{R_i}^{R_o} 2\pi \tau r^2 dr \quad (4)$$

317 where τ is the shear stress, and R_i and R_o are inner and outer radii of the annular trough.

318 If one neglects the radial velocity gradient (Clever et al. 2000; Coste 2004) into the annular
319 trough, the shear stress is quasi-independent of the radial position r and thus, integrating Eq.
320 (4) yields the shear stress as:

$$321 \quad \tau = 3T / 2\pi(R_o^3 - R_i^3) \quad (5)$$

322 Eq. (5) holds because the lateral contribution of wall frictions on the stress distribution within
323 the granular media can be neglected. Indeed, for $h = 30d$, this relative contribution can be
324 estimated to be of the order of μ_w / μ and we will see below that $\mu_w / \mu \ll 1$.

325

326 The normal stress can be also calculated from the normal force as follow:

$$327 \quad \sigma = F_N / \pi(R_o^2 - R_i^2) \quad (6)$$

328 Notice that for $h = 10d$, the imposed normal stress is larger than the hydrostatic pressure
329 ρgh once F_N is larger than $0.25N$, meaning that gravity may be neglected for the range of
330 imposed normal forces.

331

332 Besides, assuming that the velocity gradient is approximately uniform over the depth and
 333 width of the annular trough and a no-slip condition exists at the rough upper and lower
 334 shearing walls, one can estimate the mean shear rate as:

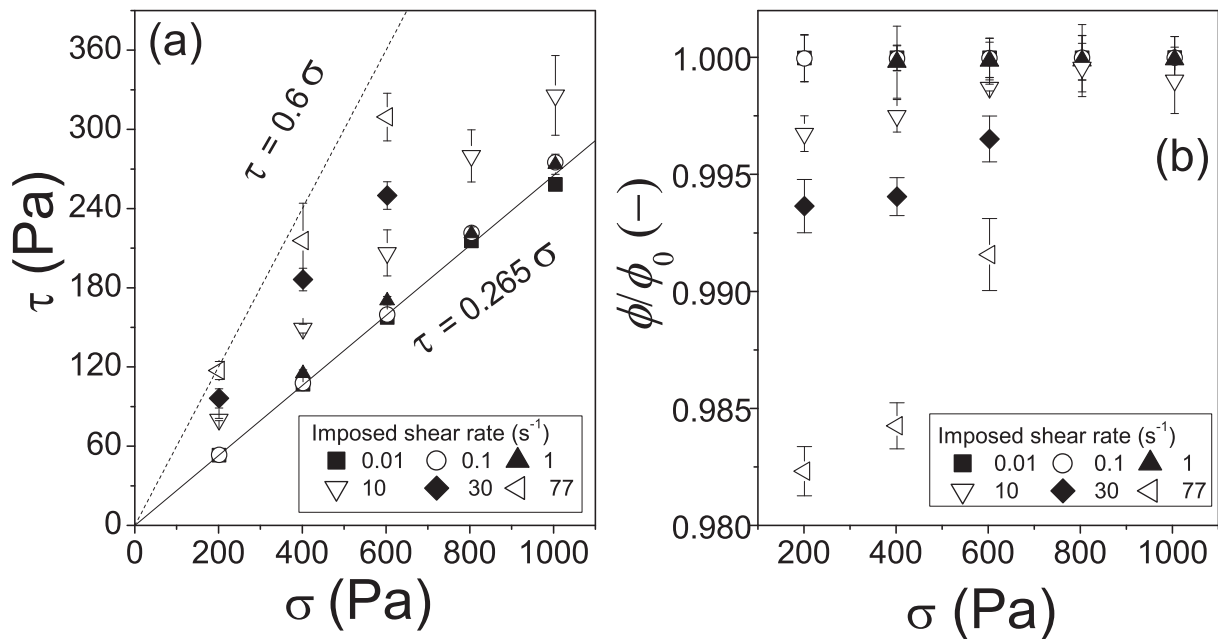
$$335 \quad \dot{\gamma} = \Omega(R_o + R_i)/2h \quad (7)$$

336 And the mean shear strain is given by:

$$337 \quad \gamma = \theta(R_o + R_i)/2h \quad (8)$$

338 where θ is the angular displacement.

339 With this analysis, one can plot the shear stress in the steady state as a function of the normal
 340 stress as in Figure 4a. The first observation is that a linear relationship between the shear and
 341 normal stresses is seen for all imposed shear rates, with a slope that increases from 0.265 to
 342 0.6 with increasing the shear rate. If an internal friction coefficient μ is defined as the ratio
 343 between shear and normal stresses, we evidence here that μ is rate dependent: it increases
 344 with $\dot{\gamma}$.



345
 346 Figure 4: Plot of the shear stress (a) and of the solid fraction rescaled by its initial value
 347 before shearing (b) as a function of the normal stress. The stress and the gap are measured in
 348 the steady state, for different imposed shear rate. The error bars come from three experiment
 349 runs.

350
 351 The second observation is that, for all imposed shear rates, the steady value of the solid
 352 fraction decreases when one decreases the normal stress (Figure 4b). Indeed, since the grains

353 cannot escape from the cell, one can measure unambiguously the solid fraction from the gap
 354 variation as:

$$355 \quad \phi = m / \pi \rho h (R_o^2 - R_i^2) \quad (9)$$

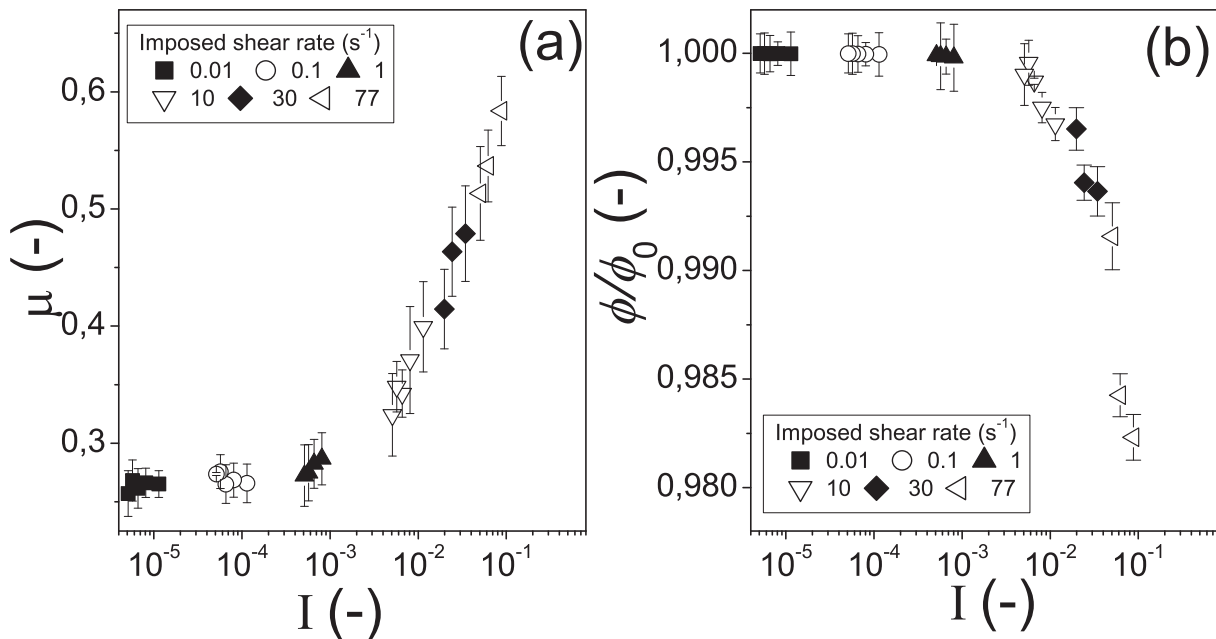
356 in which m represents the mass of grains. Thus $h_o/h = \phi/\phi_0$ will simply reflect the impact of
 357 shear and confinement on dilatancy.

358 In order to analyse the results in term of $\mu(I)$ -rheology, one has to define an inertial number
 359 such as:

$$360 \quad I = \dot{\gamma} d / \sqrt{\sigma / \rho} \quad (10)$$

361 in which σ is the normal stress in the steady state regime. It varies between 10^{-7} and 0.1 in
 362 the range of applied normal force and shear rate. This corresponds to the usual range of quasi-
 363 static to dense flow regimes. It should be noted that our annular shear geometry does not
 364 allow higher values of I to be studied.

365



366

367 Figure 5: Constitutive law for different sets of mean shear rates and imposed normal forces
 368 (a) ‘friction law’ i.e. effective internal friction coefficient as a function of inertial number; (b)
 369 ‘dynamic dilatancy law’ i.e. solid fraction as a function of inertial number. The error bars
 370 come from three experiment runs.

371

372 Figure 5a shows how μ vary throughout the flow regimes since I characterizes the local
 373 ‘‘rapidity’’ of the flow. All the data obtained for different sets of shear rate and normal force
 374 collapse on a single curve $\mu = \tau/\sigma$ vs. I . For low inertial number, the internal friction

375 coefficient tends to a finite value $\mu_s \approx 0.265$ and increases with increasing I . At the same
376 time, the solid fraction variation ϕ/ϕ_0 with the inertial number I is shown in Figure 5b. Once
377 again, all the data collapse on a single curve. At low I , ϕ/ϕ_0 is quasi-constant: this is the
378 quasi-static regime. When I increases, the inertia starts influencing the flow and the system
379 becomes rate dependent: the ratio ϕ/ϕ_0 decreases; this regime corresponds to the dense flow
380 in which the granular material dilates.

381
382 Moreover, in order to check the robustness of our results, we have varied the initial size of the
383 gap. Here, with our annular shear geometry, the same experiments discussed above are made
384 with different gap sizes from $6d$ to $22d$ and different sets of imposed shear rate and normal
385 force (Table 1).

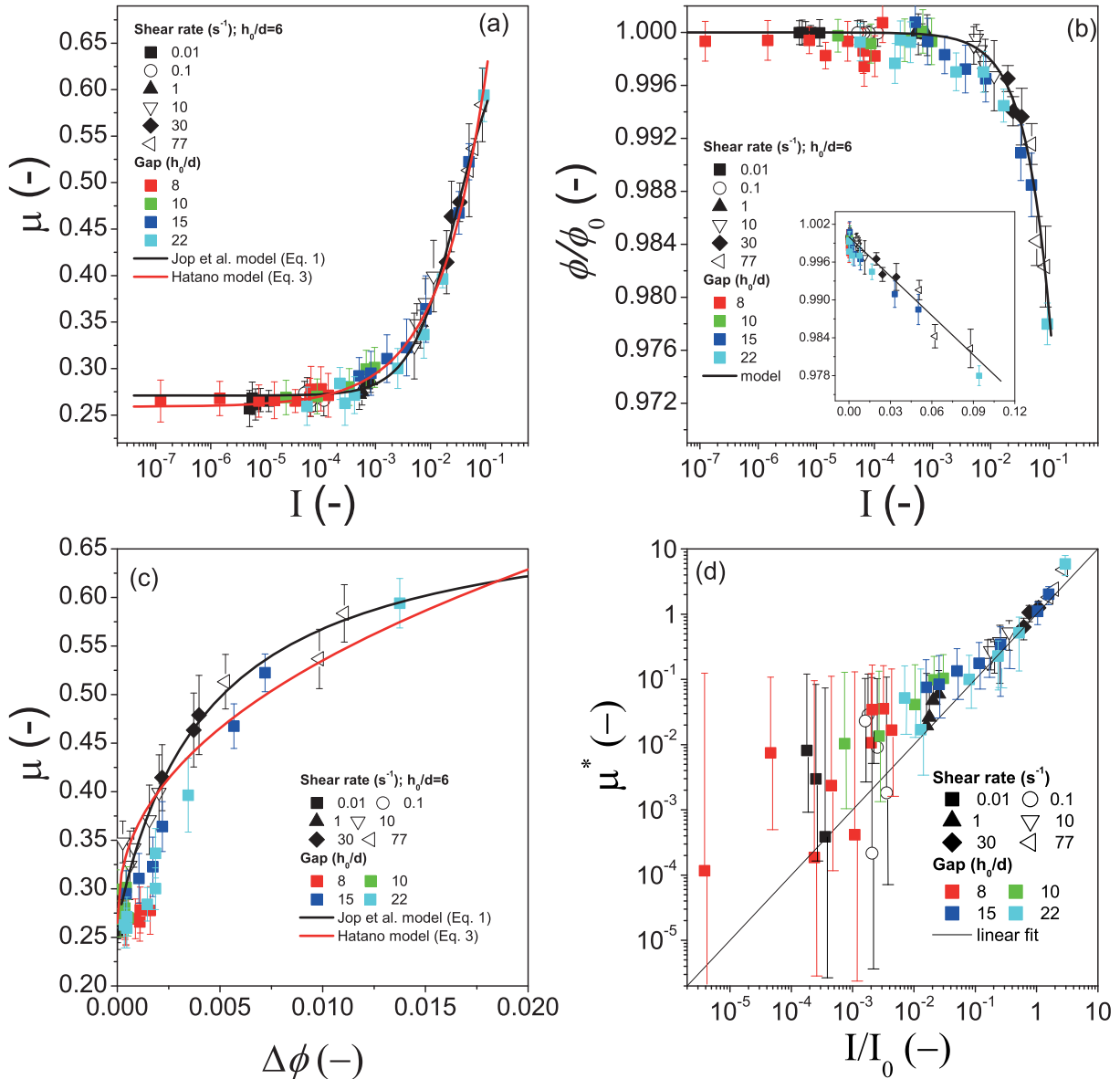
Shear rate (s^{-1})	Normal force F_N (N)	Gap (h_0/d)
0.01	1-2-3-4-5	6-8-15
0.1	1-2-3-4-5	6-8-10
1	1-2-3-4-5	6-10-15
10	1-2-3	6-15
30	1-2-3	6-15-22
77	1-2-3	6-15-22

386 Table 1: Values of the imposed shear rate and normal force for each gap size.

387
388 It is shown in Figure 6 that changing the gap does not significantly affect these results. This
389 suggests a total absence of shear localization at high inertial number. However, at small
390 inertial number, the resolution of our measurements is not sufficient to dismiss the possibility
391 that shear localization arises. It was indeed shown that in confined annular flow at small shear
392 velocities and high confining pressures – small I –, the shear may be not homogeneous and
393 solid and fluid phases coexist (Aharonov & Sparks 2002; Jalali et al. 2002).

394
395 We now compare these experimental results with existing models such as those of Jop et al.
396 Eq. (1) and Hatano Eq. (3) on dry granular flows in terms of quasi-static and dense flow
397 behaviours. Concerning the ‘friction law’, our experimental data include points in range of I
398 from 10^{-7} to 0.1 which covers quasi-static and dense flow regimes. Our measurements then
399 show that, in this range of I , both models can describe our data. Using a classical least
400 squares method, the fit of the data gives: $I_0 \approx 0.032 \pm 0.003$, $\mu_s \approx 0.271 \pm 0.002$ and

401 $\mu_2 \approx 0.68 \pm 0.02$ for the ‘Jop *et al.* model’; $\mu_s \approx 0.259 \pm 0.002$, $a \approx 1.12 \pm 0.05$ and
 402 $n \approx 0.49 \pm 0.02$ for the ‘Hatano model’. In Figure 6b, we show the ‘dynamic dilatancy law’,
 403 i.e., the variation of the solid fraction ϕ as a function of the inertial number I . We observe
 404 that ϕ decreases linearly with I , starting from a maximum value ϕ_c in the quasi-static regime
 405 where the granular material is very dense, close to the maximum solid fraction. The ‘dynamic
 406 dilatancy law’ is thus given as: $\phi/\phi_c = 1 - (1 - \phi_m/\phi_c)I$ with $\phi_c \approx \phi_0 \approx 0.625$ and
 407 $\phi_m \approx 0.495 \pm 0.002$ in agreement with (Pouliquen *et al.* 2006; Jop *et al.* 2005).



408
 409 Figure 6: Constitutive law for different sets of mean shear rates and imposed normal forces,
 410 and different gap sizes (Table 1): (a) ‘friction law’ i.e. internal friction coefficient as a
 411 function of inertial number; (b) ‘dynamic dilatancy law’ i.e. solid fraction as a function of
 412 inertial number; inset: the same data in linear scale. The solid line is $\phi/\phi_c = 1 - (1 - \phi_m/\phi_c)I$

413 with $\phi_c \approx \phi_0 \approx 0.625$ and $\phi_m \approx 0.495$; (c) Internal friction coefficient vs. the reduced solid
 414 fraction ($\Delta\phi = \phi_c - \phi$); (d) Reduced internal friction coefficient as a function of the reduced
 415 inertial number. The line is $\mu^* = I/I_0$. For each gap, the imposed shear rate varies from 0.01
 416 to 77 s^{-1} depending of the imposed normal force in order to obtain either a low I (small $\dot{\gamma}$
 417 and/or large F_N) or a high I (large $\dot{\gamma}$ and/or small F_N). The error bars come from three
 418 experiment runs.

419

420 Combining the ‘*dynamic dilatancy*’ and the ‘*friction*’ laws, these data show that the internal
 421 friction coefficient μ strongly depends on the solid fraction: it decreases towards μ_s when ϕ
 422 tends to the maximum solid fraction (as shown on Figure 6c in which $\Delta\phi = \phi_0 - \phi$ is the
 423 reduced solid fraction). Moreover, following Staron and co-workers (Staron et al. 2010), we
 424 defined a reduced internal friction coefficient μ^* as:

$$425 \quad \mu^* = (\mu - \mu_s)/(\mu_2 - \mu) \quad (11)$$

426 We plot in the main panel of Figure 6d the resulting μ^* vs. I/I_0 data points. It holds
 427 remarkably well with a prefactor of unity. Satisfying the ‘*Jop et al. model*’ implies indeed that
 428 $\mu^* = I/I_0$ which is almost the case for our data except for small values of I ($I < I_0 = 0.03$)
 429 wherein deviations seem to take place. The relationship between μ^* and the reduced inertial
 430 number is not clear. These deviations, observed in the quasi-static regime, were also
 431 mentioned in recent studies which indicate that this rheology (the ‘*Jop et al. model*’) may be
 432 not sufficient to describe the complex phenomena occurring at the flow threshold such as
 433 intermittent flows (Mills et al. 2008), and is maybe strictly valid only for relatively large
 434 inertial numbers (e.g., $I > 0.02$ as noted by Staron et al. 2010 and $I > 0.005$ by Gaume et al.
 435 2011). The reason for that is not clear yet but one possible explanation is shear localization
 436 that arises most often in confined annular flow at small imposed shear and high pressure
 437 (small inertial number I) (Aharonov & Sparks 2002; Jalali et al. 2002; Koval 2009). Such
 438 behavior of granular materials has not yet been fully understood and no consistent and general
 439 formalism can predict it successfully (Kamrin 2012). In contrast to discrete numerical
 440 simulations (Wang et al. 2012) and theoretical studies (Jenkins & Richman 1985; Richman &
 441 Chou 1988; Jenkins 1992), the study of shear localization structure with experimental
 442 methods is rather difficult. The visualization of the granular interface is usually limited to the
 443 free surface or bottom layers (Fenistein and van Hecke 2003). Recently, MRI has been used

444 to study the granular rheology (velocity and solid fraction profiles) inside the granular system
445 (Moucheront et al. 2010); this tool may thus be a great help in understanding the behaviour at
446 low I . A change in the roughness of a boundary bottom might be used to modify the flow
447 properties such as the wall slip velocity (Shojaee et al. 2012).

448

449 **IV. Conclusion**

450 We have developed a rheometrical method in order to study dense granular flows with a
451 rheometer under imposed confining normal stress and applied shear rate. From the steady
452 state measurement of the torque and the gap, the internal friction coefficient, the solid fraction
453 and the inertial number I are measured. For low I , the flow goes to the quasi-static limit and
454 the internal friction coefficient and the solid fraction profiles are independent of I . Upon
455 increasing I , dilation occurs and the solid fraction decreases linearly when I increases while
456 the friction coefficient increases. The observed variations are in good agreement with
457 previous observations of the literature. As a consequence, we bring evidence that rheometric
458 measurements can be relevant to describe dry granular flows. However, additional
459 experimental work should be carried out in order to measure the dependence of the boundary
460 layer constitutive law on the state of the bulk material, so as to be able to describe properly
461 the rheology when approaching the quasi-static limit.

462

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466

467 **References**

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