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Urban Infrastructure Investment and Rent-Capture Potentials

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WORLD BANK GROUP

Climate Change Group

Office of the Chief Economist

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Abstract

In a context of rapid urbanization and energy transition, massive investments will be required to develop efficient public transport networks. Capturing the increase in land value caused by transport infrastructure (for example, through a betterment tax) appears a promising way to finance public transport. However, it is no trivial task, as it is difficult to anticipate the rent creation. This paper uses a simple city model based on urban economic theory to compute the rent created by improvements in public transport infrastructure in Paris, France. To apply in places where models or data are not available, a reduced form

of the model is shown to provide acceptable approximations of the rent creation. Simulations confirm that land value capture can finance a significant part of transport investments. The simulations also show that value capture potentials are influenced by what happens in the entire agglomeration. Simultaneous infrastructure investments in different parts of the city play a significant role, as they change overall accessibility patterns. Evolutions taking place in other cities also have a comparable influence. Non-local effects can change the total potential for land value capture and multiply this potential by as much as a factor of two.

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Urban Infrastructure Investment and Rent-Capture Potentials

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JEL: R14, R4, R51, H54, H71

1. Introduction

Efficient public transport systems in cities lie at the core of sustainable development ([Wright and Fulton, 2005](#); [Dodman, 2011](#); [Hoornweg et al., 2011](#); [UN-HABITAT, 2011](#)). However, financing both their development and maintenance at a global scale is challenging, because fares do not generally cover full costs. Capturing part of land value increase due to transport infrastructure construction, i.e. land value capture, has often been proposed as a promising alternative method of revenue generation and as a way to address this challenge (see for instance [Batt, 2001](#); [Martinez and Viegas, 2007](#); [Peterson, 2009](#)). It might be achieved through different policies such as land value taxes or land betterment taxes (see for instance [Plimmer and McGill, 2003](#); [Doherty, 2004](#); [Rybeck, 2004](#); [Medda, 2012](#)).

The concept of land value capture has a history which dates back to Henry George, in the late 19th century ([George, 1884](#); [Arnott and Stiglitz, 1979](#); [Arnott, 2004](#)). The basic idea is that successful public transport systems lead to higher land values, as a result of the increase in accessibility, which can be considered as an increase in rent for some landowners. Capturing some or all of this increment in land value (through a betterment tax) can enable the recovery of part of the capital cost of the transport investment. It has been used since then in different cities across the world. Recent examples include Bogotá ([Borrero et al., 2011](#); [Peterson, 2009](#)), London ([Medda, 2012](#)), Singapore, Hong Kong (Hong Kong SAR, China) ([Hui et al., 2004](#)), and various cities in Brazil, Argentina and India ([Walters, 2012](#)).

If such a betterment tax can be seen as an equitable, efficient ([Mattauch et al., 2013](#)) and easily understood levy, its implementation is however not a trivial task. It can lead to numerous difficulties, among which are political acceptability ([Booth, Philip A, 2012](#)), revenue volatility ([Peterson,](#)

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2009), administrative feasibility or inequality creation when for instance some inhabitants are asset-rich but cash-poor (Medda, 2012). Another main issue is revenue predictability, i.e. the fact that it is difficult to anticipate the rent creation. We focus this study on this last point.

Econometric studies have been extensively used to analyze past impact of transport developments on housing and land prices (see reviews in Smith and Gihring, 2006; Stefania Radopoulou et al., 2011; Salon and Shewmake, 2011). This enables to retrospectively quantify the benefits that land value capture could have brought, and serves as a basis for future estimations. However, it is not directly possible to fully rely on econometric-based coefficients to anticipate land value potential. Indeed, transport is not a “standard” public amenity: contrary to, say, the creation of an attractive park or the presence of a good school, accessibility is not an amenity per se (Mohring 1961). Accessibility is only valuable if it enables travel to interesting places. To adequately analyze benefits provided by increased accessibility, and therefore how much can be potentially capitalized in land prices, the whole city should be taken into account and whole-city models should be used.

Using first a theoretical urban economic model, and then an applied model, Nedum-2D, calibrated on Paris urban area, we propose here to show how urban models can capture important mechanisms that cannot be taken into account when using only constant elasticities (of land prices in response to transport investment) from the literature. To do that, we analyze and compare with the model rent creation and land value capture potential corresponding to different public transport investments in the Paris urban area. The results of the applied model that we use here are fully consistent with urban economic theory, but they provide numerical estimations, taking into account geographical specificity of a given city.

The same land price increase will have very different impacts on households when they are landowners or when they are renters. In one case, it is indeed an increase in wealth, whereas in the other case it is often translated into an increase in housing expenditures. The impact of transport improvement on household welfare therefore strongly depends on the hypothesis about who owns the land (Brueckner, 2005; Gusdorf and Hallegatte, 2007). In this study, we do not consider welfare impacts, and focus only on land price variations. We also consider that absentee landowners own the land, i.e. that land price variations do not directly impact household income, and that rents are not redistributed to the city population.

These simulations show that a significant fraction of infrastructure costs (almost all costs, under optimistic hypotheses) can be extracted from land value increase, and that one of the main factors influencing rent creation is population increase in the city due to the infrastructure construction. Simulations also show that land value capture potential is affected by other transport infrastructure developments in the city. Especially, in a sequence of several infrastructure developments, land value capture potential depends on the timing of each investment.

These results are derived on the case of the Paris agglomeration, but the qualitative findings are valid in any large city. Moreover, we are well aware that the data and models needed to replicate this analysis are not available in all locations, especially in developing countries. This is why we also propose a reduced form model that approximates the full model (within a 15% error margin in the case of Paris) and can be applied with limited resources and little data. This reduced form model can be used to assess the potential for rent capture and identify cases where more in-depth studies are necessary.

Section 2 recalls main urban economics results on rent creation, section 3 presents simulations on Paris, section 4 discusses our results and section 5 concludes.

2. A simple theoretical model

Let us first recall how the construction of a new public transport line can induce betterment, using a simple model derived from standard urban economic modeling (Alonso, 1964; Mills, 1967; Muth, 1969). Such a framework is particularly suited because it aims at explaining the link between accessibility and land price.

2.1. Urban economics framework

Let us use here a simple monocentric model (Fujita, 1989) and suppose that households choose their accommodation location and size by making a trade-off between the time and money they spend in transport (i.e. to commute to their jobs, which are all supposed to be in the center of the city) and the real estate price level (or, equivalently, between the proximity to the city center and the housing surface they can afford). Let us also suppose that households budget is divided between transport $T(x)$, housing $q(x).R(x)$ (dwelling size $q(x)$ multiplied by unitary rent $R(x)$) and composite good $Z(x)$ consumption:

$$\forall x, Y = Z + q.R + T$$

where x is the location in the city. We can model the household trade-off using the following utility function:

$$U(Z, q)$$

which is increasing with respect to Z and q . We suppose that, given rent level $R(x)$, and transport costs $T(x)$, households choose their housing $q(x)$ and composite good $Z(x)$ consumption to maximize their utility level. If several transport means are available, transport means choice can be included in the maximization, which will result in the choice of the cheapest transport mode.²

Finally, we suppose that absentee landowners own the land, and try to maximize the housing rent paid by inhabitants, under the condition that utility U should be constant across the city. This leads to a bid-rent function $R(T(x), \bar{U})$, where \bar{U} is a constant equal to the value of utility. For a given \bar{U} , the function $R(T(x), \bar{U})$ decreases as transport costs $T(x)$ increase (Fig. 1a).

The constant \bar{U} is given through a condition on the total population of the city. If $H(x)$ is total dwelling space available at each location x , population density is equal to $n(x) = \frac{H(x)}{q(x)}$ and total population in the city is equal to $P = \int n(x) = \int \frac{H(x)}{q(x)}$. $q(x)$ implicitly depends on rent level and transport costs, and for given transport costs $T(x)$ and a given total population P_0 , utility level \bar{U} can be obtained by solving the equation:

²Transport cost should rather be named a "generalized" transport cost, and includes transport monetary cost as well as costs associated to travel time and travel comfort.

$$P = \int \frac{H(x)}{q(R(T(x), \bar{U}), T(x))}$$

Any change in the city structure (through a variation in transport prices, or in income, for instance) will lead to a change in this equation. This can be either reflected by a change in \bar{U} , keeping P constant (“closed city case”) or by a change in P keeping \bar{U} constant (“open city case”).

It is often convenient to express all equations in terms of R_0 instead of \bar{U} , where R_0 is the rent level in the center of the city (i.e. where $T(x) = 0$). This can be done because there is a one to one relationship between the two values, and R_0 strictly decreases when \bar{U} increases. This is what we will do in the rest of this paper. Intuitively, increasing R_0 increases the total population in the city (it makes $q(x)$ decrease everywhere), whereas decreasing R_0 decreases total population.

A more detailed description of the model is given in [Appendix A](#), and more information about the model and its resolution can be found in the classic book by [Fujita \(1989\)](#).

2.2. Impact of new transport infrastructure

To see how the construction of a new public transport line can induce betterment, let us compare a city a to a city b exactly similar, except for the existence of better transport infrastructure.

[Fig. 1](#) illustrates this situation, in a simple case of a one-dimensional city where transport costs increase linearly in city a ([Fig. 1a](#)) and where one public transport station is constructed in city b ([Fig. 1b](#)).³

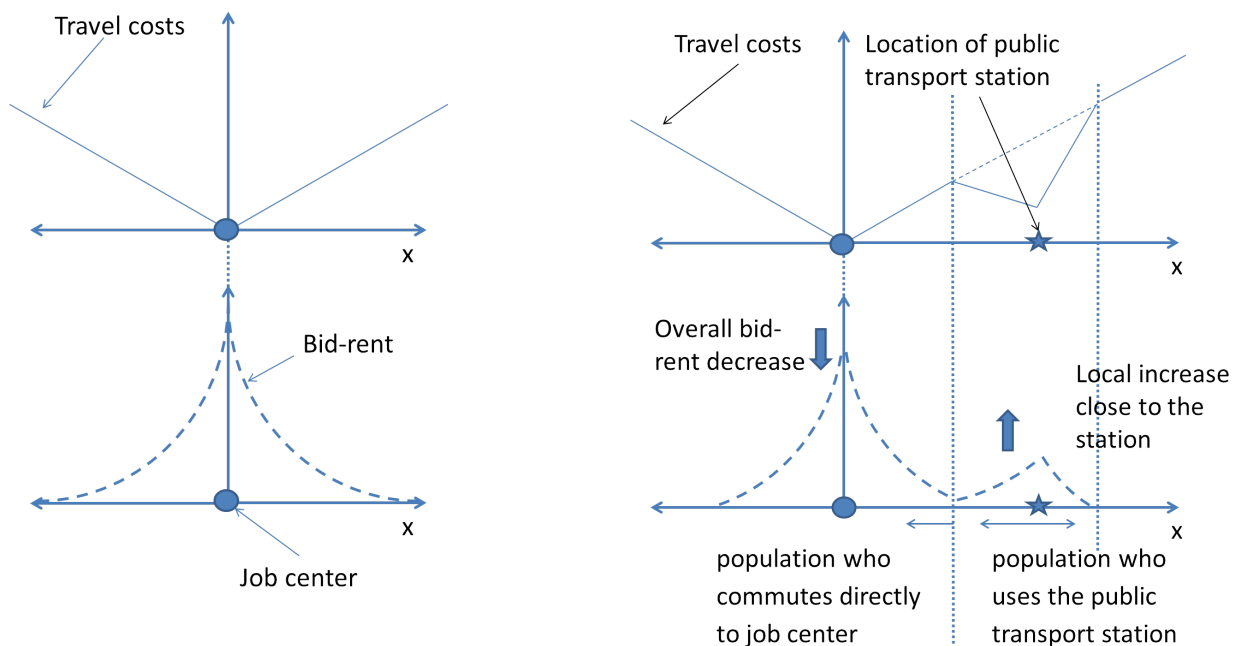
Intuitively, in places where the new infrastructure provides increased accessibility, $T(x)$ decreases which makes rents $R(R_0, T(x))$ increase. Therefore, in places where people use the new infrastructure, rents increase. This first phenomenon shows how increased accessibility can be capitalized in land rents, and therefore, land prices. However, another phenomenon can occur, which should not be forgotten : a variation of R_0 . This variation is determined by a hypothesis which should be made on the city population evolution.

2.2.1. Two different cases

Closed city case. Let us first suppose that the population of the city remains constant, or, equivalently, that city population evolution is exogenously given, and is similar in city a and city b . Because of increased rents, housing consumption $q(x)$ will decrease in places with improved accessibility, and population density will increase. To keep total population constant, rent level R_0^b in the center of the city b therefore has to be smaller than in city a . This makes rents decrease in city b in places where accessibility has not improved.⁴ This decrease in R_0^b corresponds to a gain in utility \bar{U}^b : because of transport improvement households utility has increased.

³We speak here about "public transport stations" and about "infrastructure", but of course the transport improvement modeled here can be in practice any possible transport improvement, be it the construction of a new road, the increase of traffic speed, the increase in public transport comfort (leading to a reduction if transport generalized cost) etc.

⁴For instance, if utility is given by a Cobb-Douglas function of Z and q , rents are all divided by the same factor in city b in places where accessibility has not improved. See [Appendix C.1](#).



(a) Situation a with only private car transport.

(b) Example of a situation b with private car transport and a public transport station.

Figure 1: Transport costs and bid-rent with and without public transport station.

Adding both rents increase and decrease in city b (compared with city a) leads to a situation where rents decrease in all places where accessibility has not changed, and increase in places where accessibility has improved above a threshold. The technical proofs of these results can be found in [Appendix B](#).⁵

The extent of rent decrease in places with no increase in accessibility will depend on the magnitude of the change in transport cost, and it can be shown that the largest the change in transport costs, the largest the rent decrease in the rest of the city (cf. [Appendix B](#)). When transport improvement is localized (i.e. if transport accessibility is improved only in a small fraction of the city), then an increase in land prices close to the improved transport infrastructure will be counterbalanced by a general decrease in the rest of the city, i.e. in a much wider area. Land price decreases will therefore be of a smaller order of magnitude than land price increases.

The relative magnitude of the land price values decrease and increase are however directly dependent on the relative surfaces where they occur. An accessibility increase over a large fraction of the city (e.g. a general decrease in public transport prices) will lead to a much larger decrease in R_0 . It will therefore lead to a much larger decrease of rents in places where accessibility has not changed.

Finally, it should be noted that because of these positive and negative variations of rents, on average, rents in city b can be smaller or bigger than rents in city a depending on the shape of the

⁵These results have already been demonstrated in the seminal paper by [Arnott and Stiglitz \(1981\)](#). For the sake of exhaustivity, [Appendix B](#) presents proofs in a slightly more general case, but the ideas are the same.

initial transport network (Arnott and Stiglitz, 1981).⁶ A general land tax in the entire city may thus be inappropriate to finance the transport infrastructure.

Open city case. Let us now suppose that the population of the city does not remain constant. Because of the transport investment, it is for instance reasonable to anticipate that the city will become more attractive, and that its population will increase (we suppose here that all new inhabitants can get a job, i.e. the number of jobs in the city increases together with its population). If total population is larger in city b than in city a , it is no longer possible to use the former argument, and there is no reason that $R_0^b < R_0^a$.

A useful hypothesis is to suppose that the utility in the world outside of the city is constant, and that people are free to move in and out of the city. In this case, the population of the city will vary so that the utility of the inhabitants remains constant and equal to this utility in the “rest of the world” (the “open city” case).⁷

It is easy to prove that, in this case, $pop_a < pop_b$ and $R_0^b = R_0^a$ (see again Appendix B): the transport investment increases rents where transport accessibility increases, and does not impact them in the rest of the city. In this case, there is therefore a true capitalization of accessibility gains into land and housing rents. However, it is important to note that the change in total population in the city is the key factor, and determines whether capitalization will fully take place or not, and if rents will increase in all places where accessibility has increased or not.

2.2.2. Rent variation

To sum up, rent variation in the city will be the sum of the actual capitalization of accessibility change and of a global decrease of rents due to R_0 change, which will depend on the hypothesis made on city population evolution. It can actually be shown (see Eq. A.3 in Appendix A) that rent variation can be written the following way:⁸

$$dR(x) = \underbrace{\left(1 + \frac{\int_0^T \varepsilon_{R_0}^q \frac{1}{q} dT}{R_0}\right) . dR_0}_{\text{Global rents change (negative variation)}} - \underbrace{\frac{1}{q(x)} dT(x)}_{\text{Accessibility capitalization (positive variation)}} \quad (1)$$

Whereas the second term can lead to value capture, the first term prevents full capitalization (and thus capture) of accessibility gains. This first term depends on the scale of the transport investment (it increases with this scale) and decreases if city population is increased by the transport

⁶ R_0 is always lower in city b than in city a , but the average of rents across the city can be higher or lower in city b than in city a .

⁷Another interesting possibility, in a slightly different framework, is considered to derive Henri George Theorem (George, 1884; Arnott and Stiglitz, 1979; Arnott, 2004). It is the case in which the city population remains optimal with respect to households utility, when taking into account production and consumption of a pure local public good, whose cost is divided among all inhabitants, but whose benefit is shared by everybody (i.e. in a non-rivalrous way). In such a framework, aggregated land price increase due to transport infrastructure improvements is strictly equal to transport infrastructure improvements cost for the city (considered as an increase in pure local public good cost). This theorem can be extended to the macroeconomic scale (Edenhofer et al., 2013; Mattauch et al., 2013).

⁸where $\varepsilon_{R_0}^q = \frac{\partial q}{\partial R_0} \frac{R_0}{q}$ is dwelling size elasticity to rent in the city center.

investment. It is only in the limiting case of the open city that $dR(x) = -\frac{1}{q(x)}dT(x)$ and that all the added value can be captured.

This is an important idea when implementing land value capture because this means that, except if the transport investment has a significant impact on the city population evolution, land value capture has a smaller potential as a financing tool for large scale investment than for local investments.

Formula 1 can be greatly simplified when utility function is explicitly specified (Appendix C). A case which is especially interesting for its simplicity is the case in which utility function is given by a Cobb-Douglas function ($U(Z, q) = Z^\alpha q^\beta$ with $\alpha + \beta = 1$). In this case, we have (see Eq. C.2 in Appendix C) :

$$\frac{dR}{R(T, R_0)} = \frac{dR_0}{R_0} - \frac{dT}{\beta \cdot (Y - T)} \quad (2)$$

This formula enables to easily compute orders of magnitude of rents variation due to transport accessibility changes in the open city case. For instance, if $\beta = 0.3$ (this is approximately the case in Paris, for instance) and if transport represents, say, 15% of a household's budget⁹ and $\frac{dT}{T} = 10\%$ (i.e. $\frac{dT}{Y-T} = -1.76\%$), then $\frac{dR}{R} \approx 6\%$: a 10% reduction in transport (generalized) cost leads to a 6% increase in rents. This formula can also be used when variation in R_0 is negligible, i.e. when transport improvement is very localized in the city.

Computing an approximation of the expected variation in R_0 in a closed-city case (or, equivalently, population variation in the open-city case) can be done with a few approximations (see Appendix C.2.1). If we suppose that $Y \gg T$ (i.e. assume that most of the population lives in areas not too far from city center) and that $\frac{dT}{Y}$ is a constant τ for a fraction F of total population (we approximate that transport price variation has a certain negative value for a fraction of the city population, and is equal to zero for the rest of the population), then R_0 variation is approximately given by:

$$\frac{\delta R_0}{R_0} \approx \frac{\delta P}{P} + \frac{\alpha}{\beta} \cdot \tau \cdot F \quad (3)$$

where P is city population. Therefore, in the closed city case, $\frac{\delta R_0}{R_0} \approx \frac{\alpha}{\beta} \cdot \tau \cdot F$ and, in the open-city case, population variation is approximately given by $\frac{\delta P}{P} \approx -\frac{\alpha}{\beta} \cdot \tau \cdot F$. For instance, with previous values, if transport cost is decreased by 10% for 12% of the population, then $\frac{\delta R_0}{R_0} \approx -\frac{0.7}{0.3} \times 15\% \times 10\% \times 12\% = -0.42\%$. This decrease in R_0 is much smaller (more than 10 times) than the increase in R that we have computed with Eq. 2 for places with improved accessibility.

Combining Eq. 2 and Eq. 3, we get:

$$\frac{dR}{R} \approx \frac{\delta P}{P} + \frac{\alpha}{\beta} \cdot \tau \cdot F - \frac{1}{\beta} \cdot \tau = \frac{\delta P}{P} + \frac{\alpha \cdot F - 1}{\beta} \cdot \tau$$

⁹It should be noted that this share is defined here as transport generalized cost (including cost of time, for instance) divided by households budget. It can differ sensibly from actual transport cost share of households budget.

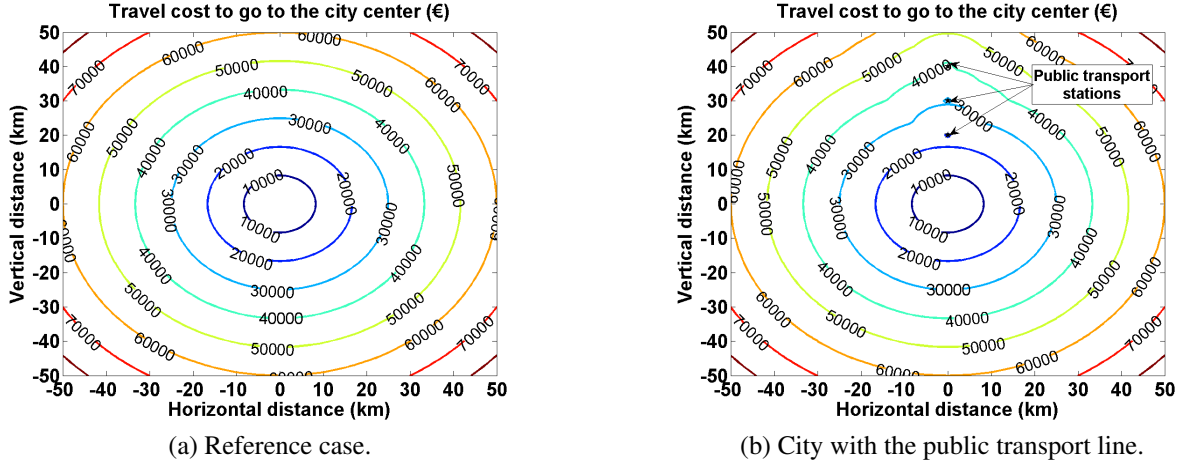


Figure 2: Transport costs in the city without the public transport line (Fig. 2a) and with the public transport line with three stations (Fig. 2b).

This formula approximates rent variation in places with significant improved accessibility (note that, if transport is improved, τ is negative). The decrease in R_0 becomes comparable to the increase in R due to transport improvement capitalization only if F becomes close to 100%, i.e. if most of the city benefits from transport improvements. In this case, if $F = 100\%$, then we have $\frac{\delta R_0}{R_0} \approx \frac{\delta P}{P} + \frac{\alpha}{\beta} \cdot \tau$ and $\frac{dR}{R} \approx \frac{\delta P}{P} - \tau$.

These approximations are computed supposing that buildings are exogenously given, and unaffected by transport infrastructure change (it can also be regarded as a description of short-term variations in rents). Endogenous building construction case is examined in Sec. 3.4.

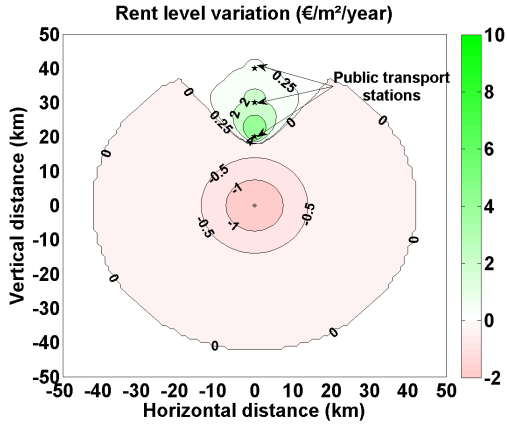
2.3. A simple two-dimensional simulation

For the sake of clarity, Fig. 2 and 3 show a simple numerical simulation, in a two-dimensional case, for a circular city. We simulate the impact of the development of a public transport line with three stations, in a city with initially no public transport.

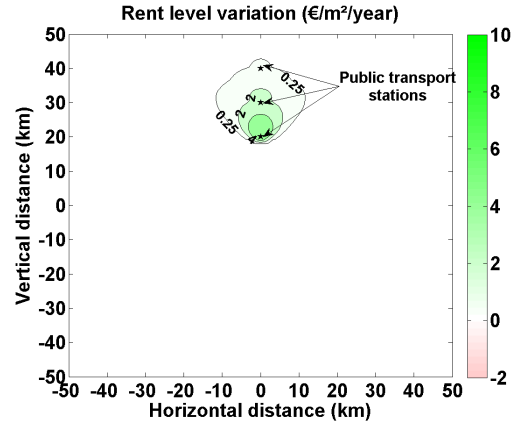
In this simulation, we suppose that the space is homogeneous for car travel: travel costs per distance to go to city center by car are the same everywhere in the city. We suppose also that, from a public transport station, travel cost to go to the center using public transport is 25% cheaper than the same journey using private car (Fig. 2). Numerical coefficients used are illustrative of Paris urban area, and are listed in Tab. 1.

Fig. 3a shows the impact on rents of the development of such a transport infrastructure, in a closed city case. Rents increase along the public transport stations, and decrease everywhere else. The order of magnitude of the increase is almost 3 times larger than the order of magnitude of the decrease (Fig. 3b). However, the increase occurs in an area almost 6 times smaller than the decrease and overall land price variation is negative: total aggregated yearly land rent in the city decreases by 0.5% (almost 225 million euros).

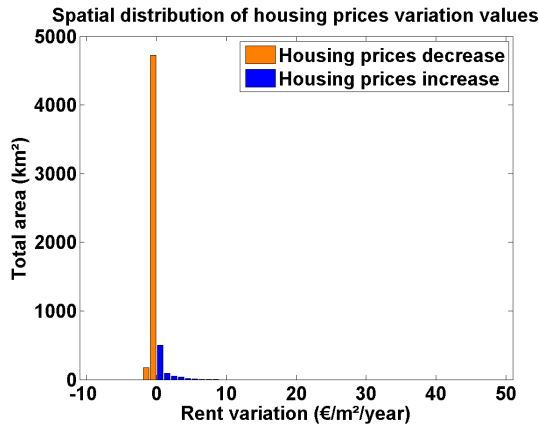
Fig. 3c shows a similar graph, but in the “open city” case. As in a closed city, rents increase along the public transport stations. However, they remain unchanged everywhere else. The his-



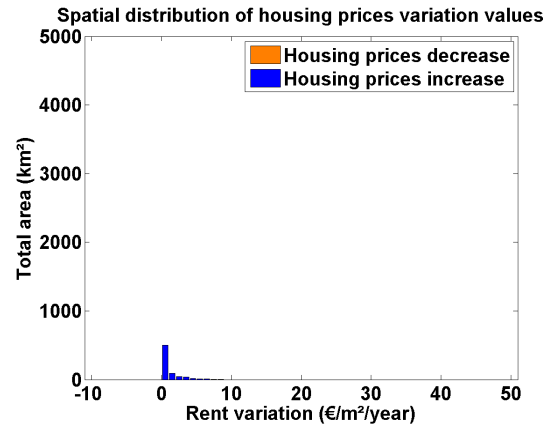
(a) Closed city case: rents decrease in all places where households do not use the public transport line, and increase in places where public transport is used.



(b) Open city case: rents do not change in all places where households do not use the public transport line, and increase in places where public transport is used.



(c) Spatial repartition of housing price variation values, closed city case. Taking into account population density, averaged housing price decrease and increase are respectively -0.56 and $+1.63$ €/m²/year; minimum and maximum values respectively -1.78 and $+8.15$ €/m²/year. Housing prices increase over 700 km² and decrease over 4400 km².



(d) Spatial repartition of housing price variation values, open city case. Averaged (taking into account population density) and maximum housing price increases are respectively $+1.69$ €/m²/year and $+8.53$ €/m²/year. Housing prices increase over 700 km². City population increases by 2% .

Figure 3: Example of a comparison of rent levels with, and without public transport line, in a simple circular city. Fig. 3a and 3c present the results in a closed city case, and Fig. 3b and 3d in an open city case.

coefficients	Value
α	0.3
β	0.7
Population	5 000 000 households
Y	50 000 €/year
$T(x)$	100 €/km/month
$H(x)$	constant, 1 m ² floor space/m ² ground space

Table 1: Coefficients of the numerical simulation. A Cobb-Douglas utility function is used: $U(Z, q) = Z^\alpha q^\beta$

togram of rent changes (Fig. 3d) is similar to the histogram in the closed city case (Fig. 3c), except that the negative part has disappeared, and that rents increase are slightly higher in each location. Overall land price variation is therefore positive: total aggregated yearly land rent in the city increases by 1.5% (i.e. almost 700 million euros). In this simulation, to maintain a constant utility level, the population has to increase by 2%.

3. Application to the Paris urban area

With standard urban economic modeling, when transport accessibility in a city is improved, rent variation is given by the competition between two opposing forces: accessibility capitalization and a global, negative, rent change. How the two effects compare, however, is unclear, and it depends on the city characteristics, i.e. on the calibration of this economic model.

To push the analysis further, and see what practical consequences these conclusions can have, we therefore need to focus on a case study, and calibrate a model derived from urban economic framework to compute rent creation and land value capture potential in a realistic case. We use in the following analysis a land-use transport interaction model (LUTI) called NEDUM-2D.

LUTI are applied models which aim at describing how transport investments can change a given city structure. Numerous models exist (for a review, see for instance [Hunt et al., 2005](#); [Iacono et al., 2008](#)), and NEDUM-2D has the particularity of being a simple model entirely based on urban economics framework. A few other LUTI models share these characteristics to some extent, and have been proposed and used in similar situations (cf. [Anderstig and Mattsson 1992](#); [Anas 1995](#); [Elhorst and Oosterhaven 2002](#)).

3.1. NEDUM-2D

NEDUM-2D ([Viguié, 2012](#); [Viguié and Hallegatte, 2012](#)) is a land-use transport interaction model which relies on the classical urban economics' framework. It aims at explaining the spatial distribution – across the city – of the costs of land and of real estate, housing surfaces, population densities and building heights and densities.

The equations of the model are slightly more complex than the equations presented in Sec. 2.1. One major difference is that housing construction $H(x)$ is endogenous and driven by changes

in rents.¹⁰ Another difference is that households can choose between different transport modes (a discrete choice model is used to compute modal shares). A third difference comes from the introduction of transaction costs and a rent border which have an influence on rent levels and city size.

We use this model calibrated on the Paris urban area, with a grid of 100km² with cells of 0.25 km². The model accounts for land-use constraints on the Paris agglomeration. Transportation costs include monetary costs such as the cost of gasoline and the cost of time. We assess them using the spatial structure of the Paris transportation networks (roads and public transport).

As described in [Appendix D](#), a validation of the model over the 1900–2010 period shows that the model reproduces the available data on the city’s evolution fairly faithfully and captures its main determinants. It also reproduces the spatial distribution of dwelling size, population density, and rents in the urban area fairly well. These results suggest that this tool can be used to inform policy decisions. This model, calibrated on Paris, has been used in several other studies ([Viguié and Hallegatte, 2012](#); [Viguié, 2012](#); [Avner et al., 2013](#); [Viguié et al., 2014](#)).

3.2. Increase in the speed of one local train

Let us analyze the effects of a theoretical increase by 25 % in the speed of one local train line (“RER B”) of Paris urban area. “RER B” is one of the main transport axis of Paris urban area, crossing Paris city, and linking this city with many of its suburbs. It has 47 stations, and an average 900,000 commuters daily traffic, which makes it the second most used public transport line in the Paris urban area.¹¹ Due to the planned important development of the places this line crosses, important modernizing works have been launched.¹²

The 25 % speed increase we are considering here is a theoretical example, that we will simply use as an illustration of potential improvement. Whereas we call it a “speed improvement”, it is important to note that, in the simulations, this improvement simply corresponds to a decrease in generalized transport costs, which could equivalently be interpreted, for instance, as improvements in comfort (AC, security etc.).

Closed city case. Let us first do an analysis in a closed city case, i.e. let us suppose that city population is exogenously determined and is not impacted by the transport investment. Fig. 4 shows a map of rents change due to the improvement in transport. As was observed in Sec. 2.3, rents increase in places with improved accessibility, and this is counterbalanced by a general decrease everywhere else. Rents decrease occurs over a much wider area (5800 km²) than rents increase (1000 km²), but the value of the decrease is extremely small and almost negligible compared to the increase : taking into account population density, averaged housing price decrease and increase

¹⁰To sum up, two main mechanisms drive the model. First, households choose where they will live and the size of their accommodation by assessing the trade-off between proximity to the city center and housing costs. Living close to the city center reduces transportation costs, but housing costs (per unit of area) are higher there. Second, we assume that landowners combine land with capital to produce housing: they choose to build more or less housing (that is, larger or smaller buildings) at a specific location depending on local real estate prices and construction costs.

¹¹Source: RATP (www.stif.org/IMG/pdf/Presentation_de_la_SNCF_et_de_la_RATP_au_comite_de_ligne_du_RER_B_du_3_novembre_2009.pdf).

¹²See for instance <http://www.modernisation-rerb.fr/>.

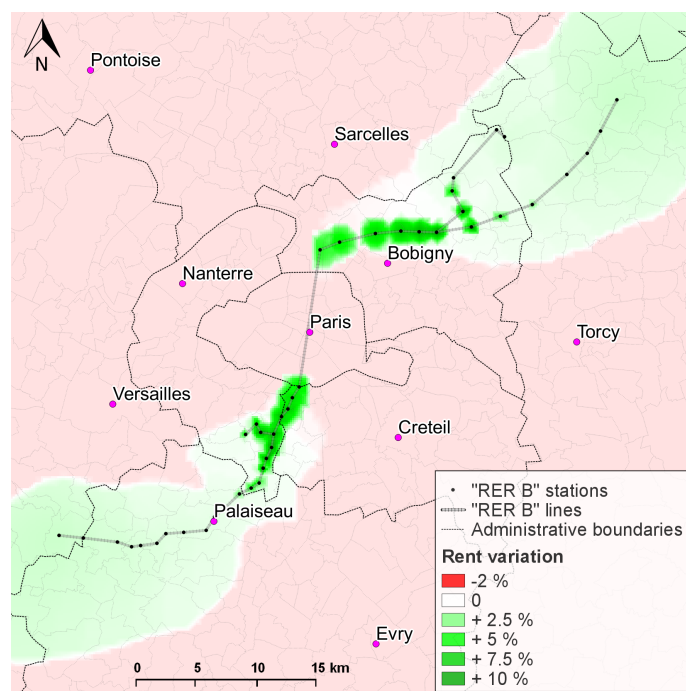


Figure 4: Simulated rents variation due to a 25% increase in the speed of one local train line (“RER B”).

are respectively -0.17 and +4.9 €/m²/year (global average is +0.60 €/m²/year); minimum and maximum values respectively -0.8 and +19 €/m²/year.

This last value corresponds to a maximum increase of housing prices by 10.7 %. This order of magnitude is coherent with the results of a number of econometric studies studying the impact on land prices of public transport lines construction or of the proximity to existing public transport stations (for a review, see [Ibeas et al., 2012](#)). For instance, [Rodríguez and Mojica \(2009\)](#) finds that the introduction of a bus rapid transit (BRT) system in Bogotá has led to real estate price increases up to 14%. [Munoz-Raskin \(2010\)](#) finds that, after the construction, real estate prices of properties close to the stations of this transport system are 8.7% higher than properties further away. [Kang and Cervero \(2009\)](#) find increase in property value up to 10% for properties close to stations of the new BRT system in Seoul.

Even if the transport improvement does not lead to a generalized increase in land prices, it is possible to capture part or all of price variation where it is positive. In the simulation, the aggregate increase in rents due to the transport improvement is equal to 250 million euros per year, when one restricts the sum to locations where rents increase (Fig. 5a).¹³

This figure can be compared with ex-ante evaluations made by the French government of the cost of future public transport improvements. For instance, as part of the “Grand Paris Express” transport project, 6 local train and subway lines (including “RER B” train line), 5 train stations and 10 streetcar lines should be improved between 2014 and 2017, as well as several bus lines

¹³With a 5% discount rate, this is equivalent to a net present value of 5 billions euros.

created, for an estimated 7 billion euros¹⁴. In a rough approximation, this means that the order of magnitude of the estimated cost of planned “RER B” train line improvements is around a billion euros. Rent value capture potential therefore seems comparable with this cost: a 1 billion euros sum could be covered in about 8 years, if, say, half of annual rent increase is captured.

What should be done in places where rents, and therefore land prices, decrease? There is in theory no easy answer, as a decrease in rent level is a gain for the inhabitants (they pay less for similar dwelling characteristics, including accessibility), but is a loss for landowners. In practice, however, the simulated rent decrease is so small that it is almost negligible, and this issue might pragmatically be discarded.

Comparison with the open city case. Let us now conduct a similar analysis in an open city case. As was explained in Sec. 2.3, in this case, rents increase in places with improved accessibility and do not change in the rest of the city.¹⁵ Rent increases are uniformly larger than in the closed city case. Quantitatively, we simulate that averaged (taking into account population density) and maximum housing price increases are respectively +4.6 €/m²/year and +20 €/m²/year. Housing prices increase over 1000 km².

To get these results and maintain a constant utility level in the city, the total population has to increase by 3.2 %, i.e. almost 200,000 households. In this case, total rent creation induced by the transport improvement is equal to 290 million euros per year. The maximum potential land value increase which can be captured is therefore almost 16% higher than in the closed city case.

In practice, it is difficult to anticipate whether the transport infrastructure will lead to a significant city total population increase, or not. The link between transport network and a city attractiveness is far from direct and a number of different factors play a role. Important factors, among many others, are the evolutions taking place in other cities. Population increase due to transport improvement might indeed be zero if all other cities undertake similar transport improvements.

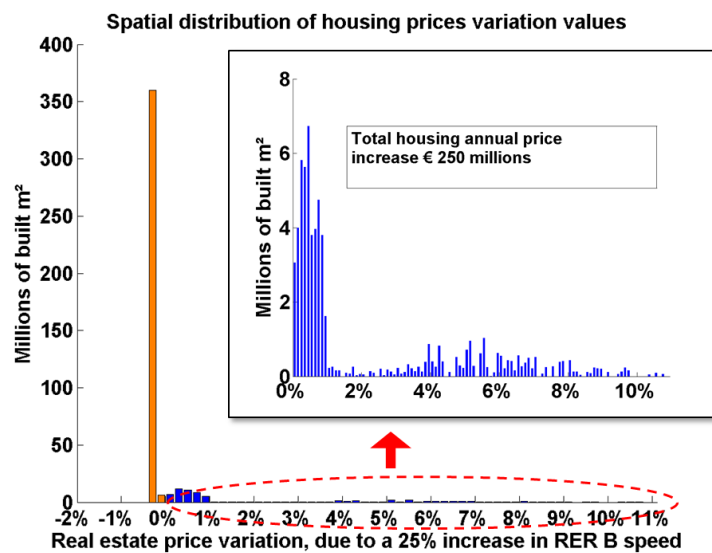
There are several ways to tax land value increase (Peterson, 2009). One way is to tax land value changes, in all places within a certain radius from newly built transport stations. Fig. 6 shows simulated total potential annual land value capture as a function of this radius. In the closed-city case, this value ceases to increase after about 1200m due to the inclusion within the radius of places where rent decrease.

In the open-city case, on the contrary, no such phenomenon occurs and the value keeps increasing as all places where rents increase tend to be included. However, the increase after 1200m is much slower than the increase before this threshold. This is for instance coherent with Ovenell (2007) (cited in Senior, 2009), who found a positive price effect on house prices of Metrolink Light Rail System construction in Greater Manchester, for houses located 0.5–1 km away from Metrolink stations.

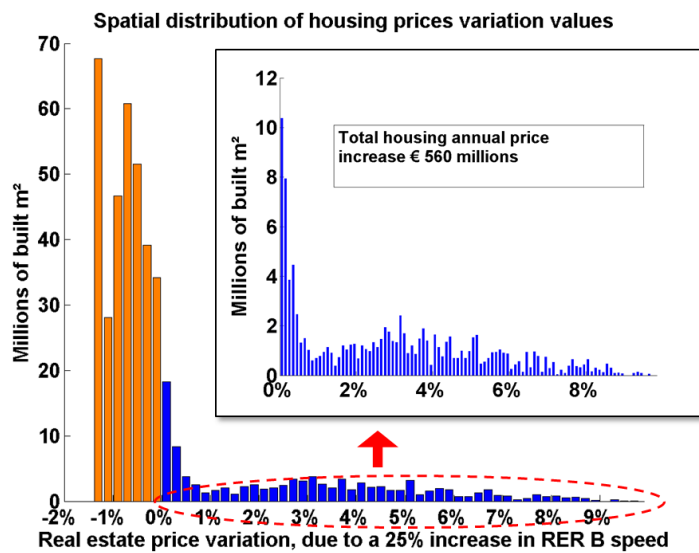
In practice, of course, a betterment tax could include a legal provision that it does not apply in places where rents or land price decrease because of the transport investment. However, it is

¹⁴See (Office of the French prime minister, 2013) or <http://www.societedugrandparis.fr/>.

¹⁵In this study, we do not consider congestion change due to increased population. City population increase due to the infrastructure construction could increase global congestion levels in the city, i.e. reduce accessibility in the rest of the city. We neglect this phenomenon here.



(a) 25% increase in the speed of one local train line (“RER B”) (see Sec. 3.2).



(b) Increase in speed affecting all local trains lines (see Sec. 3.3).

Figure 5: Repartition of land value gains and losses per year, for closed city simulations.

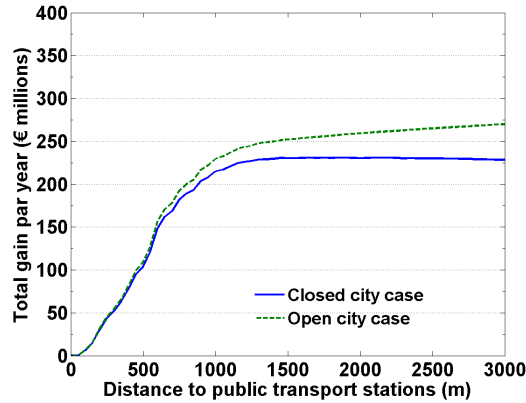


Figure 6: Land value capture potential when taxing all places within given distances from transport stations.

difficult to include such a provision because it is difficult to disentangle land price variation due to the transport investment from land price variation affecting the city and due to other factors. For instance, total population changes disconnected from the transport investment, macro-economic cycles or interest rates changes could globally impact land prices in the city. Similarly, city improvements such as parks, better schools, jobs etc. could change land prices locally.

3.3. Increase in the speed of all local trains

Let us now compare these simulations to what would happen in the case of a theoretical increase by 25 % in the speed outside Paris of all local trains lines at the same time. Fig. 7 shows a map of dwelling rents variations, which can be compared to Fig. 4.

Whereas in Fig. 4 accessibility increase is localized and occurs only in a fraction of the city (17% of the city surface, 16% of the population), in Fig. 7 it occurs in almost all the city (98% of the surface, 83% of the population). This leads to a much larger decrease in R_0 in this second case, and therefore to larger rent decreases (Fig. 5b).

Taking into account population density, averaged housing price decreases and increases are respectively -1.08 and + 5.7 €/m²/year (global average is + 0.96 €/m²/year); minimum and maximum values respectively -3.8 and +18 €/m²/year. The decreases are still lower than the increases, but are now of comparable magnitude.

Fig. 5b shows the distribution of land prices gains and losses. As in Sec. 3.2, it is possible to compute the aggregated increase in rents due to the transport improvement. Because of the large decrease in R_0 , there are many places where accessibility has improved, but where land prices decrease. This makes it impossible to capture accessibility gains through land value increase in many locations, and land value capture is less efficient as a financing tool than in previous case.

Simulated value of aggregated increase in rent is here equal to 560 million euros (Fig. 5b). This is slightly more than twice the value obtained for the improvement of “RER B” alone, whereas the investment required to improve all 9 local train lines is much higher than twice the investment to improve only one of the lines.

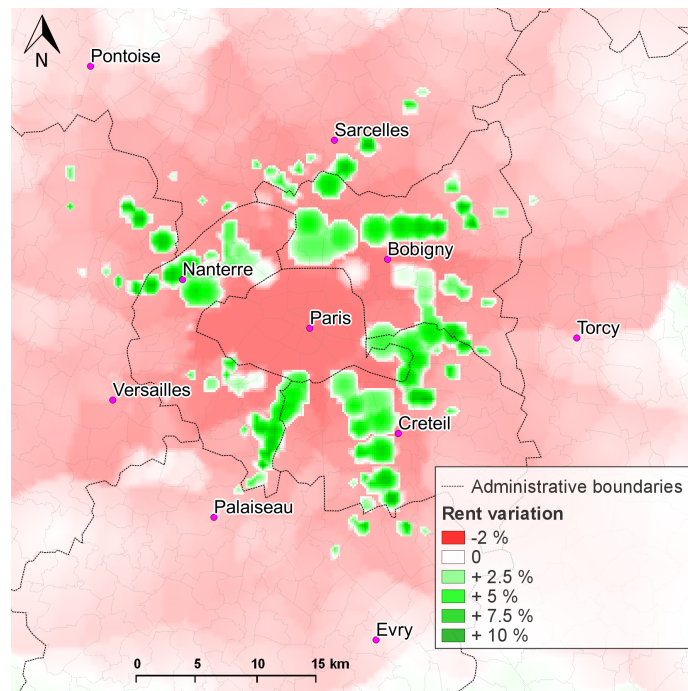


Figure 7: Simulated rent variation resulting from an increase in speed affecting all local trains lines, closed city case. Housing prices increase over 1600 km² and decrease over 5200 km². This map can be compared with Fig. 4.

Simulations in the open city case lead to a completely different picture. In this case, indeed, as was mentioned before, the decrease in R_0 is canceled, and accessibility gains are truly capitalized into land prices. Simulated value of aggregated increase in rent becomes equal to 1100 million euros. This almost twice the value obtained for the closed city gains: this illustrates how, when transport investment are widespread and increase accessibility in a large fraction of the city, the evolution of R_0 , and therefore the hypothesis on city population variation, becomes central.

It can be noted that, even in an open city case, there are decreasing returns in added value by transport investments. Maximum potential land value capture is only multiplied by a factor 4 when compared to Sec. 3.2, whereas the number of transport lines with improved speed is multiplied by 9. One of the reasons is the fact that there is some overlap between transport line accessibility zones. When some places benefit from improvements in several transport lines at the same time, total accessibility gain is simply the maximum of all gains, whereas total investment cost is the sum of all transport investments. This is strongly impacted by our monocentric hypothesis, which makes all transport lines provide accessibility to go to the exact same place, the center of Paris. This assumption overestimates the overlaps. In a more detailed, polycentric modelling, each transport line would enable to gain access to different places, reducing (but not canceling) this effect.

3.4. Comparison with reduced-form model

It is interesting to note that, because this model is based on urban economics equations, an approximation similar to the one presented in section 2.2.2 enables to approximate rent variations simulated by this model (see Appendix C and especially Appendix C.2.2) :

$$\frac{dR}{R} \approx \frac{dR_0}{R_0} - \frac{dT}{\beta \cdot (Y - T)} \quad (4)$$

and

$$\frac{\delta R_0}{R_0} \approx \frac{1}{\frac{b}{a} + 1} \frac{\delta P}{P} + \frac{\frac{b}{a \cdot \beta} + \frac{\alpha}{\beta}}{\frac{b}{a} + 1} \cdot \tau \cdot F \quad (5)$$

where α and β are the coefficient of the utility function, and a and b the coefficient of the housing production function (see Appendix D for a description of the meaning of these parameters). This reduced form is useful to get quickly orders of magnitude of rent value capture potential, without requiring complex numerical simulations. Let us compare the results of the reduced form to the simulations of the previous sections. For α , β , a and b , we use the same values as in NEDUM-2D (cf. Tab. D.3), and we assume that $\tau = -3.75\%$ and $R_0 = 27\text{€}/\text{m}^2/\text{month}$.

The value to give to F , the fraction of population with significant improved accessibility, is not straightforward. In Sec. 3.2 and 3.3, the fractions of population with reduced transport costs are respectively 16% and 83%. However, only part of these fractions actually experience the maximum decrease by 25% of transport costs. We suppose here that it is only for about 10% of the population with reduced transport costs that accessibility improves significantly.

Using these values, the results of the approximations are close to the results of the simulations (see Tab. 2) : less than 10% difference for variations in population P and rent in the center R_0 , and less than 15% difference for rent increase close to stations of improved public transport lines.

These approximations are computed supposing endogenous building construction driven by rents change. The approximations are different when supposing that buildings are exogenously given, and unaffected by transport infrastructure change (this case is examined in Sec. 2.2.2).

3.5. Land value capture and timing of investments

Let us finally have a closer look at the efficiency of land value capture as a financing tool, and assume that, as in Sec. 3.5, it is decided to capture land value changes within a certain radius of transport stations, to finance transport investment for the corresponding line. Fig. 8 shows potential value capture around “RER B” line stations when the speed of all local train lines is increased¹⁶, and compares it with the curve of Sec. 3.2.

In the closed city case (Fig. 8a), the value which can be captured appears to be 30% lower when all train lines are simultaneously improved: land value increase around one line is strongly dependent on what happens in the rest of the city, and, as noted in the previous section, land value capture is less efficient as a financing tool when the transport improvement increases in scale. The

¹⁶To prevent double-counting, this graph does not consider places which are within the given distance from the “RER B” stations, but which are closer to a station from another improved transport line. Indeed, it is supposed that land value capture in these places would be used to finance this other line.

Coefficients	Change in one local train line speed	Change in all local train lines speed
α, β, a and b	cf. Tab. D.3	
fraction of population with significant improved accessibility	2%	9%
R_0	27 €/m ² /month	
τ	-3.75%	
δR_0 as computed by Eq. 5, when $\delta P = 0$.	-0.8 €/m ² /year	-3.6 €/m ² /year
Simulated values in Sec. 3.2 and 3.3	-0.8 €/m ² /year	-3.8 €/m ² /year
$\frac{\delta P}{P}$ in the open city case, based on Eq. 5	3.5%	15.7%
Simulated values in Sec. 3.2 and 3.3	3.2%	14.6%
$\frac{dR}{R}$ as computed by Eq. 4	12.26%	11.40%
Maximum rent increase in NEDUM-2D simulations	10.76%	9.66%

Table 2: To compute the value of τ , we assume that transport generalized cost typically represents 15% of income and that transport time, which represents the biggest share of transport generalized cost, decreases by 25%, therefore $\tau = -15\% \times 25\% = -3.75\%$. We choose the same coefficients α, β, a and b as in NEDUM-2D simulations (see Tab. D.3). R_0 is taken as the maximum rent in NEDUM-2D simulation (see Fig. D.12a).

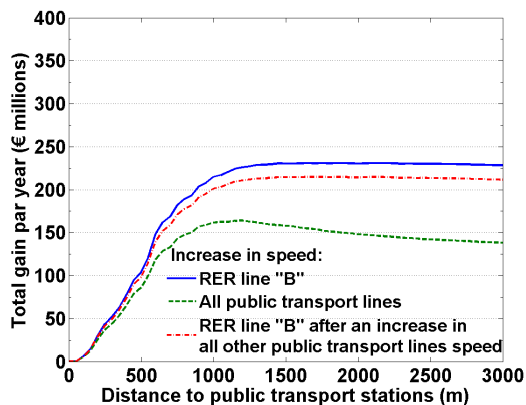
explanation is simple: as explained in Sec. 2.2, land value capture potential decreases with the scale of the investment, because of a greater impact on R_0 of large investments.

In the open city case (Fig. 8b), the value which can be captured in the “all lines improvement” scenario becomes almost equal to the value which can be captured in the “RER B improvement” scenario. Indeed, the open city hypothesis cancels out the variation of R_0 , and this makes rent variation truly reflect accessibility variation, which is the same in both scenarios. The slight difference (3%) comes from the overlap between different transport lines, which makes that, as noted in previous section, improving all public lines is less efficient in terms of accessibility change than improving one given line.

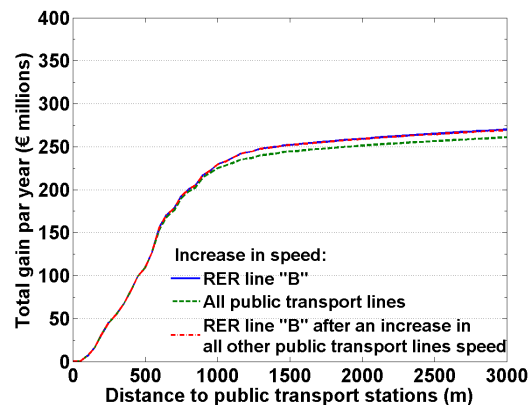
Timing of investments. Let us finally imagine a third, intermediate, case. It is a scenario in which the speed of “RER B” line is increased, such as in Sec. 3.2, but after all other local train line speeds have been similarly increased. This means that after the speed increase of line “RER B”, the situation will be exactly identical to the situation in Sec. 3.3.

This scenario is also represented on Fig. 8: both in open and closed city cases. It leads to a potential land value capture which is smaller than in the first scenario, but bigger than in the second. The reason is that, due to the overlap between transport lines, accessibility gained through the improvement of one line is bigger if the other lines have not been improved yet.

This shows that, when analyzing rent creation, the timing of transport investment is important : when improving/creating several transport lines, land value increase along a given line will be proportionally larger if this line is the first to be improved/created than if it is the last. Land value



(a) Closed city case. Compared to RER B improvement case, rent value capture potential is 29% smaller in all transport lines improvement case, and 7% smaller in the case of RER B improvement after all other lines.



(b) Open city case. Compared to RER B improvement case, rent value capture potential is 3.4% smaller in all transport lines improvement case, and 0.3% smaller in the case of RER B improvement after all other lines.

Figure 8: Land value capture potential if taxing all places withing given distances from transport stations.

capture potential will also be larger if each line is improved separately than if all lines are improved at the same time.¹⁷

This conclusion only holds if, in practice, there are no (or if there are imperfect) anticipations reflected in land prices. In case of perfect anticipations, if both transport lines improvements are anticipated together, then improving one or the other first, or improving both at the same time, leads to the same land price variation anyway. It therefore leads to the same value capture potential.

4. Discussion

In our simulations, land value capture potential appears comparable with price estimations of transport improvements works. This is coherent with literature estimations (see a review in [Smith and Gihring, 2006](#)). However, in practice, what is the relevance of our figures? In real life, many factors can influence rent creation and land value capture possibility. Let us review the limits of our conclusions.

Limitations due to model hypotheses. In this article, we only consider one household income class, and we therefore disregard transport improvement impact on income classes distribution. As empirically observed, transport accessibility change can play a strong role on the gentrification ([Lin, 2002](#)) or the pauperization ([Glaeser et al., 2008](#)) of neighborhoods. This can lead to strong

¹⁷When improving several lines at the same time, or when improving several lines successively, initial and final states are the same. The total difference in land prices is therefore also the same. Our conclusion is only true if value capture potential is computed by considering only *positive* land price variations and discarding negative ones.

increases or decreases in rents and land prices, possibly changing our conclusions. Taking into account this phenomenon, however, requires a much finer analysis than ours.

Similarly, our analysis did not consider amenities, i.e. places that households value per se. We therefore do not consider here how transport improvement could increase or decrease local quality of life (through increased or decreased sound or pollution nuisances, for instance). This effect is entangled with the former one, as amenities can be valued differently by different income classes (Brueckner et al., 1999).

Finally, as mentioned in Sec. 3.3, the fact that we only consider here one employment center has some implications on our assessment of potential land value capture, as it overestimates the overlaps between public transport lines accessibility. This will be addressed in an on-going study.

Impact on jobs. Our analysis only models household location choices : we do not consider firms choices and only investigate added value from housing. Transport improvement could have a strong impact on jobs and activities locations. This is roughly analyzed here by the comparison between "closed city case" and "open city case", but we did not do not explicitly model new economic activity generated by better infrastructure. In the open city case, we suppose that economic activity is a by-product of the increase in population, not the driving factor of population changes.

Economic activity change would impact rents and land prices either directly, through commercial demand for land, or indirectly, through an increased attractiveness for households. We therefore a priori underestimate land value changes.

Perfect land market. In our analysis, only economic processes drive land prices, and population density evolutions (through buildings construction or dwelling size changes). In practice, these evolutions are constrained by numerous regulations and market imperfections. NEDUM-2D only takes into account a few of these constraint (a limitation of maximum buildings density in Paris city center), but not all of them. This could have an impact on our conclusions, because this could limit land price increases in places where increased demand would exist, but where construction prospects or households movements are limited by such restrictions.

Inertia and dynamics. We use here a static equilibrium model, with no inertia on price evolution or population density change. In practice, this inertia could play an important role. For instance, land prices downward rigidity (Gao et al., 2009) could reduce the difference between the "closed" and the "open" city case, by preventing any significant price decrease. Similarly, any rigidity on households dwelling size evolution or building construction can cancel at least part of the land price decrease. Also, dynamic effects can create significant distributional effects, as explored in Gusdorf et al. (2008).

Exploring such a phenomenon would be an interesting complement to this work. Dynamic urban models such as the model defined in Vigié et al. (2014) could be used to do such an analysis.

5. Conclusion

In conclusion, using urban economics based models similar to NEDUM-2D provides estimations of land value capture potential, taking into account city systemic effect. The model that we

use is very simple, and disregards important phenomena that could strongly impact value capture potential, so caution is needed before using the exact figures we compute. However, a few important ideas emerge from our analysis.

First, the city systemic effect is particularly important to consider for large scale transport investments, and can dramatically change the magnitude of rent creation. It makes potential land value capture dependent on city population change, i.e. city attractiveness compared to other cities. This result warns against analysis realized at a lower scale than the urban area.

Another conclusion is the fact that value capture potential depends on the timing of investments. The value that is created is different when considering first an improvement in one given transport line, and, then, improvements in another transport line, than when considering improvement of both lines at the same time. And when considering land value capture potential for a given line, it can be different if this line is the first to be improved, or if the other line is improved first (Sec. 3.5).

In the theoretical case of the “open city”, i.e. with an adequate total population increase, transport accessibility is truly capitalized into land prices. However, if total population does not change (“closed city” case), capitalization does not fully occur, and the larger the scale of the transport project, the less rent is generated. In practice, if one considers that a reasonable bet is an intermediate evolution between these two limiting cases, it still means that land value capture has a smaller potential as a financing tool for large scale investments than for local investments.

Our results also indicate that, even without considering transport-related negative amenities such as air pollution or noise, there are by nature winners and losers when transport is improved. Except if the city population increases sufficiently (in which case, land price increases in all places), there are some places where land price increases, and some places where it decreases. This idea is actually not specific to transport, and valid for any amenity, such as the creation of a park, for instance. Indeed, in a not fully open city, an increase in land prices somewhere means that there has to be a decrease in land prices somewhere else.¹⁸

However, it should be noted that transport improvement is actually exactly capitalized into land prices, *when land prices are compared across the city*. Systemic effects can induce a general decrease of land prices in the city, but they do not change the fact that land prices close to the improved transport line will decrease less than land prices far from this line. Transport accessibility is capitalized in the difference in land prices across the city, not in land price evolution across time. The true question is whether it is politically or administratively feasible to capture this relative variation across the city, i.e. to tax a place because prices have decreased less than in other places in the city.

In practice, if a city follows a trajectory of sufficiently increasing land prices, due to external and exogenous factors (economic or demographic growth, interest rate changes etc.), then systemic effects could remain hidden and unnoticed. In such a case, the difference between small-scale and large-scale investment would disappear. This is part of a broader issue: one difficulty, when implementing land value capture, is the fact that land prices are not impacted only by the transport

¹⁸Of course, this is true only if dwelling size variation or housing construction are responsive enough to land price increase.

improvement, but also but many other evolutions which may take place at the same time. Some of these evolutions are local: increase in quality of life through city infrastructure improvement, better schools or socio-economic changes (such as gentrification). Some are city-wide, such as local economic development and population change. Others are national or even global, such as macroeconomic cycles, housing price bubbles and interest rate change. As indicated in Sec. 3.2, it could be difficult to disentangle land price variation due to the transport investment from land price variation due to other factors. The systemic effect we have analyzed in this study (the variation in R_0) acts exactly as these external factors, and prevents an exact capitalization of accessibility gains into land prices. Factors which lead to land price increase a priori favor the use of land value capture, whereas factors which lead to land price decrease prevent its full efficiency.

Our conclusions, here reached on the specific case of Paris, would remain valid in other urban areas, as long as an urban economics standard framework is supposed to be an adequate representation of reality. As indicated in Sec. 4, using a more complex model would enable getting finer conclusions, especially related to dynamic issues and firm choices. Both these questions are the subject of on going work. On the other hand, a full modeling analysis is not always possible, because of resource, time, or data availability issues. In this case, we propose a reduced form model that can help decision makers assess the orders of magnitude at stake, and determine whether more in-depth analyses are necessary.

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Appendix A. Urban economic framework

Notations. Let us recall briefly the main characteristics of classical monocentric city modelling (Fujita, 1989): we want to maximize

$$\begin{cases} U(x) = U(Z(x), q(x)) & \text{Household's utility function} \\ I = R(x) \cdot \sum_{\text{households living in } x} q(x) & \text{Landowners' income} \end{cases}$$

Under the constraints:

$$\begin{cases} Y = Z(x) + q(x) \cdot R(x) + T(x) & \text{Household's budget constraint} \\ \forall x, \sum_{\text{households living in } x} q(x) \leq H(x) & \text{Housing size constraint} \end{cases}$$

Landowners can choose $R(x)$, and households $q(x)$, $Z(x)$ and their location in the city. Variables $q(x)$, $Z(x)$ and $R(x)$ are always positive.

Mathematical hypotheses. We suppose that the utility function is twice differentiable, increasing at all $Z > 0$ and $q > 0$, and that all indifference curves are strictly convex. For simplicity, we also suppose that there is at least one location x in the city where $T(x) = 0$, i.e. one location where transport costs are equal to zero.

Equilibrium. Equilibrium leads to a uniform utility level across the city (see the proof in Fujita, 1989) :

$$\forall x, U(Z(x), q(x)) = U_0$$

The limit of the city is given by the condition: $Y > T(x)$.

In the city, rent levels are given by the bid-rent function:

$$R(x) = \Psi(R_0(U_0), T(x)) = \max_{Z, q} \left\{ \frac{Y - T(x) - Z}{q} \mid U(Z(x), q(x)) = U_0 \right\} \quad (\text{A.1})$$

Where $\Psi(R_0(U_0), T(x))$ is strictly decreasing with T and increasing with $R_0(U_0)$, and where $\Psi(R_0(U_0), 0) = R_0(U_0)$. $R_0(U_0)$ is a bijective function, strictly decreasing with U_0 . It represents rent level in the middle of the city, where transport price is zero.

Dwelling size $q(x)$ is given by:

$$q(x) = s(R(x), T(x)) = \arg \max_q U(Y - T(x) - q \cdot R(x), q)$$

Using eq. A.1, we can write $q(x)$ as a function of $R_0(U_0)$ and $T(x)$, or as a function of $R_0(U_0)$ and $R(x)$, due to the fact that there is a one-to-one correspondence between $T(x)$ and $R(x)$, if U_0 is given.

$$q(x) = s(R(x), T(x)) = S(R_0(U_0), T(x))$$

An important property of the function S , which will be useful in the following demonstrations, is the fact that it is strictly increasing with T , when U_0 is given. Indeed, keeping U_0 constant, an

increase in T will induce a non-zero decrease in R . As U_0 is constant, q and Z will move along an indifference curve, and as these curves are strictly convex, a decrease in R will result in an increase in q and a decrease in Z .

Remark. Using envelope theorem (here again, see [Fujita, 1989](#)) it is possible to show that $\frac{\partial R}{\partial T} = -\frac{1}{q} \cdot \frac{dT}{dx}$, i.e.

$$\frac{\partial R}{\partial T} = -\frac{1}{q} \quad (\text{A.2})$$

and therefore:

$$\begin{aligned} R(R_0, T) &= R(R_0, 0) + \int_0^T \frac{\partial R}{\partial T} dT = R_0 - \int_0^T \frac{1}{q(R_0, T)} dT \\ \Rightarrow dR &= dR_0 - \frac{1}{q} dT - \left(\int_0^T \frac{\partial(\frac{1}{q})}{\partial R_0} dT \right) \cdot dR_0 \\ \Leftrightarrow dR &= dR_0 - \frac{1}{q} dT + \left(\int_0^T \frac{\partial q}{\partial R_0} \frac{1}{q^2} dT \right) \cdot dR_0 \\ \Leftrightarrow dR &= dR_0 - \frac{1}{q} dT + \left(\int_0^T \varepsilon_{R_0}^q \frac{1}{q} dT \right) \cdot \frac{dR_0}{R_0} \\ \Leftrightarrow dR &= \left(1 + \frac{\int_0^T \varepsilon_{R_0}^q \frac{1}{q} dT}{R_0} \right) \cdot dR_0 - \frac{1}{q} dT \end{aligned} \quad (\text{A.3})$$

where $\varepsilon_{R_0}^q = \frac{\partial q}{\partial R_0} \frac{R_0}{q}$ is dwelling size elasticity to rent in the city center.

Appendix B. Propositions and proofs

Appendix B.1. Main hypothesis

We consider here two cities a and b which are strictly identical, except for transport prices T_a and T_b :

$$\begin{cases} U_a(Z, q) = U_b(Z, q) = U(Z, q) & \text{Same utility function} \\ Y_a = Y_b & \text{Same income} \\ \forall x, H_a(x) = H_b(x) = H(x) & \text{Same buildings} \\ \forall x, T_a(x) \geq T_b(x) & \text{Transport prices are cheaper in city } b \end{cases}$$

In all these demonstrations, we will use the same set of joined hypotheses:

Definition 1. Hypothesis \mathcal{H} : we suppose that there is a non measure-zero set A of x where we have simultaneously $T_a(x) > T_b(x)$, $H(x) > 0$, and $Y > T_b(x)$.

\mathcal{H} simply means that there is at least one location x in the city with some housing space and where transport is strictly cheaper in city b than in city a .

Appendix B.2. Close city case

In the close city case, the sum of inhabitants in the city has to be equal to the total population pop , given exogenously:

$$\int_x n(x)dx = \int_x \frac{H(x)}{q(x)}dx = pop \quad \text{total population constraint}$$

Definition 2. Hypothesis \mathcal{P} : We suppose that total population in cities a and b is identical : $pop_a = pop_b$.

Lemma 3. If hypothesis \mathcal{H} and \mathcal{P} are true, then $R_a^0 > R_b^0$ (i.e. if there is a place in city a with some housing space and where transport is strictly more expensive than in city b , then the rent in the city center is higher in city a than in city b).

Proof. Let us do a proof by contradiction and suppose that $R_a^0 \leq R_b^0$. In this case, in each location x , we have:

$$R_a(x) = \Psi(R_a^0, T_a(x)) \leq \Psi(R_b^0, T_a(x)) \leq \Psi(R_b^0, T_b(x)) = R_b(x)$$

Therefore:

$$q_a(x) = s(R_a(x), T_a(x)) \geq s(R_b(x), T_b(x)) = q_b(x) \quad (\text{B.1})$$

However, for any x in A , this inequality is strict, as $R_a(x) \geq R_b(x)$ and $T_a(x) > T_b(x)$ (cf. [Appendix A](#)). Indeed:

$$q_a(x) = s(R_a(x), T_a(x)) = s(\Psi(R_a^0, T_a(x)), T_a(x)) \geq s(\Psi(R_b^0, T_a(x)), T_a(x))$$

and

$$\forall x \in A, s(\Psi(R_b^0, T_a(x)), T_a(x)) = S(R_b^0, T_a(x)) > S(R_b^0, T_b(x)) = q_b(x)$$

therefore:

$$\forall x \in A, q_a(x) > q_b(x)$$

Since, we also have, from Eq. [B.1](#):

$$\forall x \notin A, q_a(x) \geq q_b(x)$$

This means that:

$$POP_a = \int_{x/Y > T_a(x)} \frac{H(x)}{q^a(x)} dx < \int_{x/Y > T_b(x)} \frac{H(x)}{q^b(x)} dx = POP_b$$

whereas, by hypothesis, POP_a and POP_b should be equal. \square

Proposition 4. *If hypothesis \mathcal{H} and \mathcal{P} are true, then in places where $T_a(x) = T_b(x)$, $R_a(x) \geq R_b(x)$. More specifically, in these places, either $R_a(x) > R_b(x)$ or $R_a(x) = R_b(x) = 0$ (i.e., x is outside of the city).*

Proof. Let us call Ψ the bid-rent function expressed as a function of R^0 and T . It is a classical result that Ψ is an increasing function of T and a strictly increasing function of R^0 (cf. Fujita (1989)) inside the city, i.e. where $Y > T(x)$. When $Y \leq T$, $\Psi(R^0, T) = 0$ for any R^0 . Following lemma 3, we have:

$$R_a(x) = \Psi(R_a^0, T_a) \geq \Psi(R_b^0, T_b) = R_b(x)$$

$$\text{If } R_a(x) > 0, \text{ then } R_a(x) = \Psi(R_a^0, T_a) > \Psi(R_b^0, T_a) = R_b(x).$$

$$\text{If } R_a(x) = 0, \text{ then } R_a(x) = \Psi(R_a^0, T_a) = \Psi(R_b^0, T_a) = R_b(x). \quad \square$$

Proposition 5. *If hypothesis \mathcal{H} and \mathcal{P} are true, then the larger the area where $T_a(x) > T_b(x)$, or/and the larger the decrease in transport prices, the bigger the decrease in rents in areas where transport prices do not change.*

Proof. Let us imagine a third city c where $T_a(x) \geq T_b(x) \geq T_c(x)$, with a non measure-zero set of x where we have simultaneously $T_b(x) > T_c(x)$, $H(x) > 0$, and $Y > T_a(x)$ (and therefore $Y > T_a(x) \geq T_b(x)$). Applying proposition 4 to cities b and c leads to:

$$\begin{cases} R_a(x) < R_b(x) < R_c(x) & \text{if } R_a(x) > 0 \\ R_a(x) = R_b(x) = R_c(x) & \text{if } R_a(x) = 0 \end{cases}$$

$$\text{in places where } T_a(x) = T_b(x) = T_c(x). \quad \square$$

Appendix B.3. Open city case

In the open city case, utility level U_0 in the city is given exogenously. The sum of inhabitants in the city varies to maintain this constant utility level.

Definition 6. Hypothesis O : We suppose that utilities in cities a and b are identical, therefore : $U_a = U_b$.

In an open city, rents are unaffected by the new transport infrastructure in places where people do not use it, and rents are increased in places where people use it:

Proposition 7. *If hypothesis \mathcal{H} and O are true, then in places where $T_a(x) = T_b(x)$, $R_a(x) = R_b(x)$. In places where $T_a(x) > T_b(x)$ and $Y > T_a(x)$, $R_a(x) < R_b(x)$.*

Proof. Let us consider a location x . If x is outside the city (i.e. $Y < T_a(x)$), we have $R_a(x) = R_b(x) = 0$.

If x is inside the city, let us suppose that $T_a(x) = T_b(x)$. In this case, we have:

$$\begin{cases} Y - T_a(x) = Z_a(x) + q_a(x) \cdot R_a(x) = Z_b(x) + q_b(x) \cdot R_b(x) \\ U(Z_a(x), q_a(x)) = U(Z_b(x), q_b(x)) \end{cases}$$

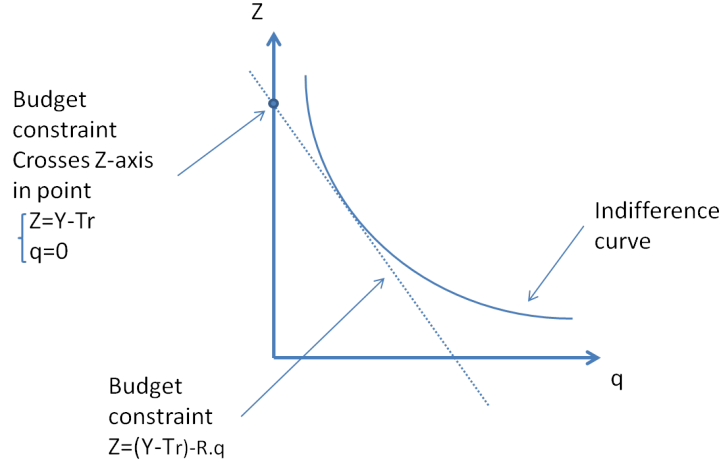


Figure B.9: In both cities a and b , the indifference curve is the same ($U_a = U_b$) and the budget constraint line crosses Y-axis in the point $Z = Y - T$. From this point, there is only one budget constraint line which is tangent to the indifference curve, because this curve is strictly convex.

As indifference curves are strictly convex there is only one value for R , Z and q such that these equalities are satisfied and such that q and Z maximize the utility. This can be seen on Fig. B.9: In both city a and b , the indifference curve is the same ($U_a = U_b$) and the budget constraint line crosses Z-axis in the point $Z = Y - T_a(x)$. From this point, there is only one budget constraint line which is tangent to the indifference curve, because this curve is strictly convex. The slope of this line gives R , which is therefore identical in cities a and b . Similarly, the contact point between this line and the indifference curve gives Z and q , which are therefore also identical in both cities.

In particular, in the center of the city, where $T = 0$, we have $R_0^a = R_0^b$.

Let us now consider a location x inside the city, where $T_a(x) < T_b(x)$. We have:

$$R_a(x) = \Psi(R_a^0, T_a) < \Psi(R_a^0, T_b) = \Psi(R_b^0, T_b) = R_b(x)$$

because $\Psi(R^0, T)$ is strictly decreasing with T .

□

Proposition 8. *If hypothesis \mathcal{H} and O are true, in any location x inside the city, the larger the decrease in transport prices, then the bigger the increase in rent*

Proof. The proof is straightforward : Let us imagine a location x inside the city and a third city c where $T_a(x) > T_b(x) > T_c(x)$, Applying proposition 7 to cities b and c leads to:

$$R_a(x) < R_b(x) < R_c(x)$$

□

Appendix C. Approximations and simplifications

Without any complex city modelling, it is possible to compute easily an approximation of the change in R_0 and population in the closed and open city cases with a few hypotheses. If households utility function is a Cobb-Douglas function, it is possible to simplify the formulas even further.

Appendix C.1. Rent variation

Let us suppose that $\varepsilon_{R_0}^q$ is constant over the urban area (this is for instance what we get if households utility function is a Cobb-Douglas function, see below). Eq. A.3 can be greatly simplified:

$$\begin{aligned}
 dR &= \left(1 + \varepsilon_{R_0}^q \cdot \frac{\int_0^T \frac{1}{q} dT}{R_0} \right) \cdot dR_0 - \frac{1}{q} dT \\
 &= \left(1 - \varepsilon_{R_0}^q \cdot \frac{\int_0^T \frac{\partial R}{\partial T} dT}{R_0} \right) \cdot dR_0 - \frac{1}{q} dT \\
 &= \left(1 - \varepsilon_{R_0}^q \cdot \frac{R(T, R_0) - R_0}{R_0} \right) \cdot dR_0 - \frac{1}{q} dT \\
 &= \underbrace{\left(-\varepsilon_{R_0}^q \cdot \frac{R(T, R_0)}{R_0} + 1 + \varepsilon_{R_0}^q \right)}_{\text{negative variation due to change in } R_0} \cdot dR_0 + \underbrace{\left(-\frac{1}{q} \right)}_{\text{optional positive local variation in } R} dT
 \end{aligned} \tag{C.1}$$

Let us now suppose further that households utility function is a Cobb-Douglas function with constant returns ($U(Z, q) = Z^\alpha \cdot q^\beta$ with $\alpha + \beta = 1$). In this case, we have:

$$q = \frac{\beta \cdot (Y - T)}{R(T, R_0)} = \frac{\beta \cdot Y^{\frac{1}{\beta}}}{R_0} \cdot (Y - T)^{-\frac{\alpha}{\beta}}$$

this enables to compute that, as noted above, $\varepsilon_{R_0}^q$ is constant over the urban area, and equal to -1 . Eq. C.1 then becomes:

$$\frac{dR}{R(T, R_0)} = \frac{dR_0}{R_0} - \frac{dT}{\beta \cdot (Y - T)} \tag{C.2}$$

Open city case. If R_0 does not change, but T changes, then we have $\frac{dR}{R} = -\frac{dT}{\beta \cdot (Y - T)}$. This formula enables to easily compute orders of magnitude of rents variation due to transport accessibility changes in the open city case (see section 2.2.2).

Closed city case. In the closed-city case, we have to add to the last formula the effect of the variation in R_0 . We can note that, in places where T remains the same, Eq. C.2 shows that rents are all multiplied by the same factor (i.e. $\frac{dR}{R} = \frac{dR_0}{R_0}$). Next section shows how to compute an order of magnitude of the variation in R_0 .

Appendix C.2. Variation in R_0

Appendix C.2.1. Exogenous building construction

Let us first consider, as in Sec. 2, that buildings are exogenously given, and unaffected by transport infrastructure change (it can also be regarded as a description of short-term variations in rents). Population is given by: $P = \int \frac{H}{q}$, therefore

$$\begin{aligned} dP &= - \int \frac{H}{q} \frac{dq}{q} = - \int \frac{H}{q} \frac{\partial q}{\partial R_0} \frac{dR_0}{q} - \int \frac{H}{q} \frac{\partial q}{\partial T} \frac{dT}{q} \\ &= - \int n \cdot \varepsilon_{R_0}^q \cdot \frac{dR_0}{R_0} + \int n \cdot \varepsilon_{\bar{Y}}^q \cdot \frac{dT}{\bar{Y} - T} \end{aligned}$$

where $n = \frac{H}{q}$ is population density, and $\varepsilon_{R_0}^q = \frac{\partial q}{\partial R_0} \cdot \frac{R_0}{q}$ and $\varepsilon_{\bar{Y}}^q = \frac{\partial q}{\partial (Y-T)} \cdot \frac{Y-T}{q}$ dwelling size elasticity to respectively rent in the center of the urban area and to available income \bar{Y} (income minus transport cost: $\bar{Y} = Y - T$).¹⁹ If $\varepsilon_{R_0}^q$ and $\varepsilon_{\bar{Y}}^q$ are constant over the urban area (this is also what we get if households utility function is a Cobb-Douglas function), then, noting that $P = \int n$, we have:

$$\begin{aligned} \frac{dP}{P} &= - \varepsilon_{R_0}^q \cdot \frac{dR_0}{R_0} + \varepsilon_{\bar{Y}}^q \cdot \frac{\int n \cdot \frac{dT}{\bar{Y} - T}}{P} \\ \Leftrightarrow \frac{dR_0}{R_0} &= - \frac{1}{\varepsilon_{R_0}^q} \frac{dP}{P} + \frac{\varepsilon_{\bar{Y}}^q}{\varepsilon_{R_0}^q} \cdot \frac{\int n \cdot \frac{dT}{\bar{Y} - T}}{P} \end{aligned}$$

To approximate the variation in R_0 , we have to do two other approximations. First, we suppose that $Y \gg T$, i.e. we assume that most of the population lives in areas not too far from city center. Second, we assume that $\frac{dT}{\bar{Y}}$ is a constant τ for a fraction F of total population (we approximate that transport price variation has a certain negative value for a fraction of the city population, and is equal to zero for the rest of the population). In this case, we have:

$$\delta R_0 \approx - \frac{R_0}{\varepsilon_{R_0}^q} \frac{\delta P}{P} + \frac{\varepsilon_{\bar{Y}}^q}{\varepsilon_{R_0}^q} \cdot R_0 \cdot \tau \cdot F \quad (\text{C.3})$$

This equation can also be used to compute $\frac{\delta P}{P}$ in the open city case: $\frac{\delta P}{P} = \varepsilon_{\bar{Y}}^q \cdot \tau \cdot F$. There is another way to write this equation: if δR_0 is R_0 variation in the closed-city case, then population variation in the open-city case is given by

$$\frac{\delta P}{P} = \varepsilon_{R_0}^q \frac{\delta R_0}{R_0}$$

¹⁹In this computation, we assume that Y does not change, so $d\bar{Y} = d(Y - T) = -dT$. If Y can vary, too, then other terms have to be added to the equation.

Case of a Cobb-Douglas utility function. If households utility function is a Cobb-Douglas function with constant returns, then we have $\varepsilon_{R_0}^q = -1$ and $\varepsilon_Y^q = -\frac{\alpha}{\beta}$, and Eq. C.3 becomes:

$$\delta R_0 \approx R_0 \frac{\delta P}{P} + \frac{\alpha}{\beta} \cdot R_0 \cdot \tau \cdot F \quad (\text{C.4})$$

so, in the closed city case $\delta R_0 \approx \frac{\alpha}{\beta} \cdot R_0 \cdot \tau \cdot F$ and, in the open city case, $\frac{\delta P}{P} = -\frac{\alpha}{\beta} \cdot \tau \cdot F$.

Appendix C.2.2. NEDUM-2D model approximation: Endogenous building construction

Equations C.3 or C.4 cannot be directly used to approximate results of NEDUM-2D model because of endogenous building construction driven by rents change (Muth, 1969; Fujita, 1989). However, a similar result can be derived, noting that in this case:

$$dP = \int \frac{dH}{q} - \int \frac{H}{q} \frac{dq}{q}$$

And

$$\begin{aligned} \int \frac{dH}{q} &= \int \frac{1}{q} \cdot \left(\frac{\partial H}{\partial R_0} dR_0 + \frac{\partial H}{\partial T} dT \right) \\ &= \int \frac{\partial H}{\partial R_0} \frac{dR_0}{q} + \frac{\partial H}{\partial T} \frac{dT}{T} \\ &= \int \frac{H}{q} \cdot \varepsilon_{R_0}^H \cdot \frac{dR_0}{R_0} - \int \frac{H}{q} \cdot \varepsilon_Y^H \cdot \frac{dT}{Y-T} \end{aligned}$$

where $\varepsilon_{R_0}^H = \frac{\partial H}{\partial R_0} \cdot \frac{R_0}{H}$ and $\varepsilon_Y^H = \frac{\partial H}{\partial(Y-T)} \cdot \frac{Y-T}{H}$. If, here again, we suppose that $\varepsilon_{R_0}^H$ and ε_Y^H are constant over the urban area, which is for instance what we get if housing production function and utility function are both Cobb-Douglas functions, we have:

$$\begin{aligned} \frac{dP}{P} &= \left(\varepsilon_{R_0}^H - \varepsilon_{R_0}^q \right) \cdot \frac{dR_0}{R_0} - \left(\varepsilon_Y^H - \varepsilon_Y^q \right) \cdot \frac{\int n \cdot \frac{dT}{Y-T}}{P} \\ \Rightarrow \frac{\delta R_0}{R_0} &\approx \frac{1}{\varepsilon_{R_0}^H - \varepsilon_{R_0}^q} \frac{\delta P}{P} + \frac{\varepsilon_Y^H - \varepsilon_Y^q}{\varepsilon_{R_0}^H - \varepsilon_{R_0}^q} \cdot \tau \cdot F \end{aligned} \quad (\text{C.5})$$

As with Eq. C.3, this equation can be used to compute $\frac{\delta P}{P}$ in the open city case: if δR_0 is R_0 variation in the closed-city case, then population variation in the open-city case is given by:

$$\frac{\delta P}{P} = \left(\varepsilon_{R_0}^q - \varepsilon_{R_0}^H \right) \cdot \frac{\delta R_0}{R_0} \quad (\text{C.6})$$

Case of Cobb-Douglas functions. If households utility and housing production functions are both Cobb-Douglas functions with constant returns (use the same notations as in [Appendix D](#): utility function $U = Z^\alpha q^\beta$ and housing production function $\tilde{H} = AL^a \tilde{K}^b$ and $H = \frac{\tilde{H}}{L}$ and $K = \frac{\tilde{K}}{L}$), then we have $\varepsilon_{R_0}^q = -1$ and $\varepsilon_Y^q = -\frac{\alpha}{\beta}$, as above, and $\varepsilon_{R_0}^H = \frac{b}{a}$ and $\varepsilon_Y^H = \frac{b}{a\beta}$, because

$$H = A \cdot \left(\frac{\rho}{b.R.A} \right)^{-\frac{b}{a}} = A \cdot \left(\frac{bA}{\rho} \right)^{\frac{b}{a}} \cdot R_0^{\frac{b}{a}} \cdot \left(\frac{Y-T}{Y} \right)^{\frac{b}{a\beta}}$$

Eq. [C.5](#) then becomes:

$$\frac{\delta R_0}{R_0} \approx \frac{1}{\frac{b}{a} + 1} \frac{\delta P}{P} + \frac{\frac{b}{a\beta} + \frac{\alpha}{\beta}}{\frac{b}{a} + 1} \cdot \tau \cdot F \quad (\text{C.7})$$

so, in the closed city case $\delta R_0 \approx \frac{\frac{b}{a\beta} + \frac{\alpha}{\beta}}{\frac{b}{a} + 1} \cdot R_0 \cdot \tau \cdot F$ and, in the open city case, $\frac{\delta P}{P} = - \left(\frac{b}{a\beta} + \frac{\alpha}{\beta} \right) \cdot \tau \cdot F$.

Appendix D. Description of NEDUM-2D model

For more information on NEDUM-2D model, see [Viguié \(2012\)](#); [Viguié and Hallegatte \(2012\)](#).

Appendix D.1. Equations

We model the household trade-off using the following utility function:

$$U = Z^\alpha q^\beta$$

where α and β are coefficients ($\alpha + \beta = 1$), q the surface of the households' dwelling and Z the money remaining after the household has paid its rent and a commuting round-trip per day to the center of Paris. The cost of transportation includes the monetary cost of transportation and the cost associated with the trip duration, which we consider as an actual loss of income. Such a functional form is consistent with the fact that the share of household income devoted to housing expenditures is relatively constant over time and space ([Muth, 1969](#); [Thorsnes, 1997](#)). Household income constraint reads:

$$Y = Z + qR + t_r$$

where Y is the average household income, R is the rent per square meter, and t_r the transportation costs (monetary cost added with time cost).

We assume that absentee landowners own the land, and that they combine land with capital to produce housing. The housing production function reads, in a classical way:

$$\tilde{H} = AL^a \tilde{K}^b$$

where A , a and b are coefficients ($a + b = 1$), \tilde{H} the housing surface built, L the land surface occupied by the buildings and \tilde{K} the financial capital used for construction. If we call $H = \frac{\tilde{H}}{L}$ and $K = \frac{\tilde{K}}{L}$ housing surface and capital per land unit, the benefit of land owners reads therefore :

$$\Pi = R.H - (\delta + \rho)K$$

Main Data	
Urban area population	5,101,300 households
Fraction of ground surface devoted to housing	0.45
Households average income	56,098 €
Transport times and costs in Paris urban area	cf. Appendix D.2
Interest rate	$\delta = 5\%$
Built capital depreciation time	$\rho = 0.5\%$
Calibrated parameters	
Households utility function parameter	$\beta = 0.3$
Coefficients of construction cost function	$A = 1.57 * 10^{-3}$ and $a = 0.07$
Maximum floor-area ratio	cf. Appendix D.2
Cost associated with travel time	cf. Appendix D.2
Rent determining city border	$R_0 = 12\text{€}/m^2$

Table D.3: Summary of main data and calibration parameters

Π is the profit per land unit, ρ represents the joined effect of real estate capital depreciation and annual taxes paid by land owners on the real estate capital, and δ the interest rate. The metropolitan area boundary is defined by a rent R_0 , below which it is not profitable to build housing building (this value corresponds both to other use of the land like agriculture and to transaction costs in the building and renting process). Developers build to maximize their profit: at each point of the metropolitan area they construct, i.e. choose K , to maximize Π under the constraint that $\frac{\tilde{H}}{L}$ ratio is limited by a land-use constraint (see detail below).

Appendix D.2. Calibration

Tab. [D.3](#) presents the numerical data we used in our simulations. In absence of adequate data for some parameters, for instance the cost of time and construction costs, these parameters have been calibrated on the Paris structure in 2006. A detailed comparison of model results with available data is provided below, and shows a good agreement on the model with observed urban evolutions.

Maximum floor-area ratio in the center of Paris. We include in the model a constraint on the maximum density allowed. This constraint reflects actual land-planning constraints which prevents building which are too high. Calibration process leads to 1.5 as the value of this maximum floor-area ratio. In the simulations, this constraint is only binding in the center of Paris.

This value may seem low as most buildings in Paris have approximately 6 floors, which would induce a ratio of about 6 at the center of Paris. However, our ratio is only taking into account housing surface, and not the total built surface, and the discrepancy is simply caused by built surface intended for purposes other than housing (it includes, on the one hand, corridors and lobbies in buildings dedicated to housing and, on the other hand, all buildings not dedicated to housing: offices, shops, museums, train stations, office buildings, schools, universities, etc.).

Generalized transport prices. To compute generalized transport prices, we used data about walking times, actual transport times and prices in public transport (underground, regional trains and suburban trains) and private transport (during rush hours, for an average car). At each location, the generalized transport cost is computed for each transport mode, and a logit weighting is used to compute the modal shares²⁰. In the simulations, changes in public and private transport prices lead therefore to modal shifts.

Construction costs. The calibration process provides construction costs between 3000 €/m² for a housing-surface/land-surface ratio of 1 and 3300 €/m² for a ratio of 5. We compare in Sup. Fig. D.10 the calibrated costs to construction cost estimates from the Centre Scientifique et Technique du Bâtiment (CSTB), a French public institution providing analysis and research on construction and housing issues. These data are partial, since they are prices announced by developers in several public procurement documents and in various estimates of building construction costs, as well as technical documents.

What emerges from CSTB data is an average cost of construction of 1300 €/m² before tax, or approximately 1500 €/m² including all taxes, which increases slightly as the building becomes higher. However, these estimates are quite uncertain: because of the diversity of types of buildings that it is possible to build, it is difficult to obtain a cost that can be used as a reference cost. The order of magnitude of the calibrated cost seems to agree with the order of magnitude of the data. These data present however a less convex profile than calibrated data.

An explanation of the discrepancy may be that the so-called “actual” costs in CSTB data are direct construction costs, while in reality developers consider also additional costs when the height of buildings increases. These additional costs include administrative costs (building permits etc.) and financial costs (the risk associated with a larger investment cost), and technical costs (duration and technical difficulty of the works). (Castel, 2005, 2007)

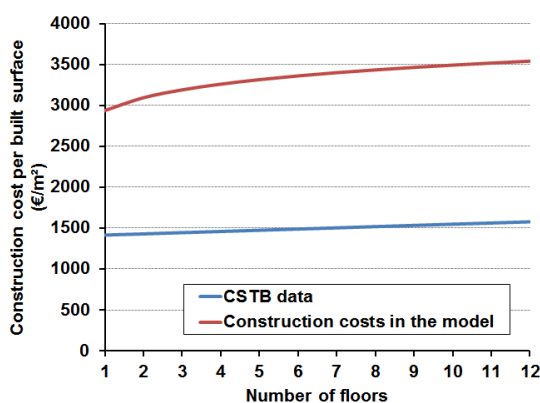


Figure D.10: Construction costs.

²⁰More rigorously, at each location, the logit weighting is computed on each price divided by the minimum price at this location.

Cost of time. In the model, rents (per surface unit) decrease when moving away from the center of Paris because households have to pay a generalized transportation cost, which is the sum of a perceived monetary cost (interpreted here as the cost of fuel) and of the cost associated with transport time, assuming that households do a round-trip per day towards the center of Paris. In the simulation, cost associated with transport time represents generally the bigger part of generalized cost, and the way we assess this cost has an important role in our results.

Numerous studies have dealt with this issue, but no conclusive result exists on this complex subject. In Ile-de-France, French Government's Strategic Analysis Center proposed to use net hourly wage as an estimate for commuting time cost, but explained that the value of actual commuting time cost depends greatly on several factors such as households characteristics or modal choice. (Boiteux and Baumstark, 2001)

Due to the importance of time cost choice in the simulation, we calibrated time cost instead of using an a priori fixed value. We computed this cost using our data on rent spatial distribution: out of these data, assuming our model perfectly exact, it is indeed possible to estimate a theoretical generalized transportation cost. Assuming that this generalized cost reflects the sum of the direct cost of transport and of the cost associated with transport time, and assuming that households do a round-trip per day towards the center of Paris, the transport time cost was estimated as a function of journey time.

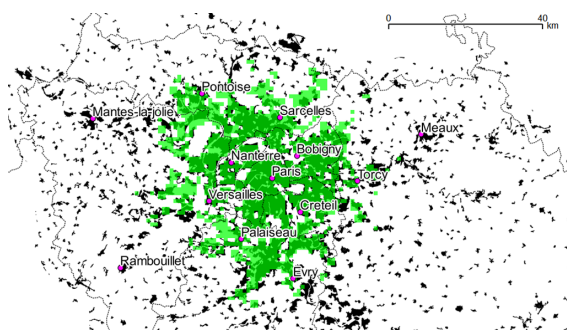
Marginal time cost seems to decrease with travel time, and we chose to model simply this decrease using a piecewise affine function. This representation leads us to use a cost time worth 105% of the net hourly wage when the travel time is less than 25 min (or, equivalently, when the distance to the center of Paris is less than 15 km), then a very low cost (6.6% of the net hourly wage) for portions of journey in excess of this limit. The value of time for journeys during less than 25 min is therefore very close to commuting time cost in Ile de France according to French Government's Strategic Analysis Center.

This observed decrease in marginal time cost can be attributed to the limits of our approach, in particular to the monocentric framework and to the hypothesis that households do a round-trip per day towards the center of Paris. In the real world, in places where travel time exceeds 25 minutes, a large fraction of households do not commute to the center of Paris. This leads to a shorter average trip length than in the mono-centric case, and using actual average trip length would enable to use more realistic time cost values and smaller total fuel costs for locations far from Paris city center. In absence of needed data, we did not take into account explicitly this variation in trip length, and modeled it with a non-linear time cost.

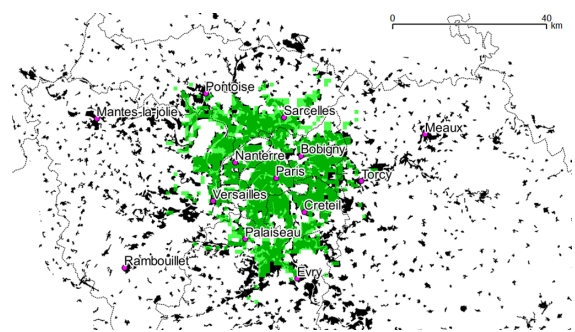
Appendix D.3. Validation: urbanized surface evolution

As can be seen on Fig. D.11a, the model reproduces well the current Paris urban area. The main mismatch is in the west (Mantes la Jolie) and in the south of the urban area (Melun), where the model does not capture observed urbanized areas. These two zones correspond to cities which were built long before being included in Paris urban area, and are important employment centers on their own, whereas the model only represents built areas due to Paris urban area sprawl.

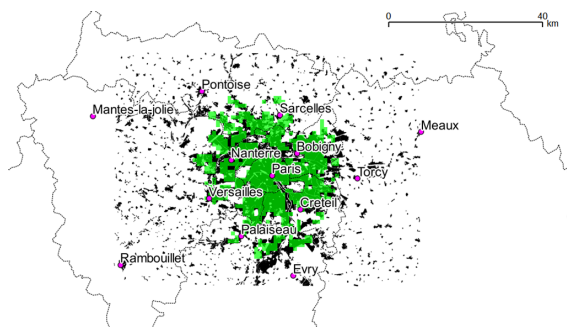
It is possible to use this model to simulate city evolution from 1900 to 2006. For instance, Fig. D.11d, Fig. D.11c and Fig. D.11b compare simulated urban area with actual urbanized area, in 1900, 1960 and 1982, respectively. Because of the lack of data, we used the same transport



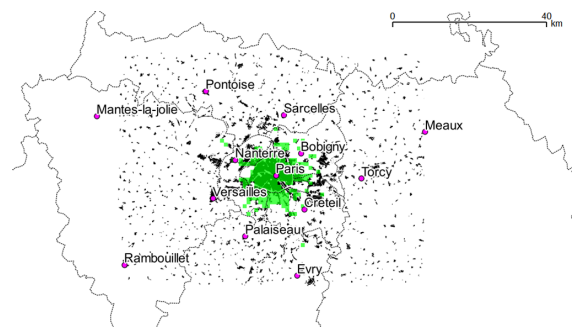
(a) 2006



(b) 1990

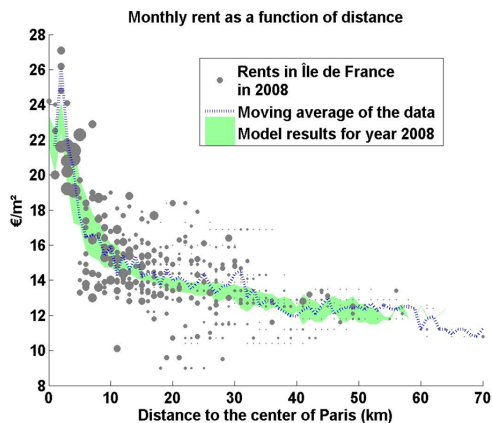


(c) 1960

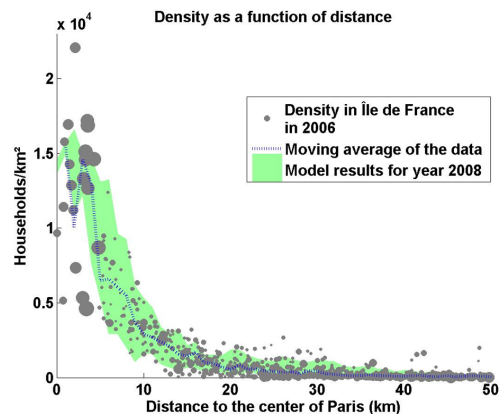


(d) 1900

Figure D.11: Simulated urbanized area compared to actual urbanized area. Actual urban area appears in black (Source: IAU, MOS database), whereas model simulation appears in transparent green.



(a) Rents (Data source: CLAMEUR). The model explains 55.1 % of the two-dimensional variance of the data.



(b) Population density (Data source: INSEE). The model explains 74.7% of the two-dimensional variance of the data.

Figure D.12: Rents and population density computed by the model (green area) and from data. Dots represent data for individual localities. The dotted line represents the average value of data at a given distance from Paris center.

network as in 2006 to do these three simulations, and the description of the city in 1900 is not as good as for the following years²¹. However, large-scale trends between 1900 and 2006 are well described, suggesting that the model captures the main determinants of city shape evolution.

Appendix D.4. Validation: city structure

As shown in Fig. D.12a the model describes the distribution of rents across the city in 2008 satisfactorily. In two dimensions, the model explains 51.8 % of the variance in rents. When all areas at a given distance from the center are averaged, the model explains 89.5 % of the uni-dimensional variance. Figure D.12b shows that there is also a good agreement between the model and data in terms of population density. The model explains 77.2 % of the two-dimensional variance in population density, and 95.9 % of the uni-dimensional variance.

Model and data seem to match well on the urban area scale, even if local differences can be large, due to the lack of several locally-important characteristics (e.g., public services supply and local amenities).

²¹For instance, bus and tramway networks are not modeled, whereas they were of great importance at the beginning of the 20th century.