MODEL FOR DELAMINATION PROPAGATION IN MULTILAYERED MATERIALS AT 0 /θ O INTERFACES: A COMPARISON BETWEEN EXPERIMENTAL AND FINITE ELEMENTS STRAIN ENERGY RELEASE RATES

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Abstract
Multilayered delaminated plates are analyzed here using an 2D plate model. Alternative to the existing three-dimensional finite element methods (3D-FEM), the proposed model, named LS1*, is a layerwise stress model proving significantly less computationally expensive while accurate and efficient [see 1, 2, 3, 4, 5]. In particular this paper uses experimental data from different test specimens (DCB, ELS, ADCB, MMF) in a finite element code, which is based on* LS1*, in order to calculate strain energy release rates (SERR) in different modes of delamination. The focus is on two types of delaminated interfaces 0/0 and 0/45. The obtained SERR results are in very good agreement with the experimental values and, in the case of mixed-mode delamination, they are as accurate as the SERR obtained by 3D-FE models. The other interesting property of the* LS1 *model is the very fast calculation speed as the SERR can be analytically deduced from interfacial stresses. This relation which only depends on the stacking sequence and the position of delamination is presented.*

1. Introduction

Various methods, viewpoints and criteria exist to take into account delamination, and stress concentrations. The present approach can be indexed in the Layerwise family (see a review in [6]), and the objective of this paper is then to present an efficient and accurate alternative to 3D methods for analyzing delaminated multilayered materials, here studied under classical loads in Mode I and/or II. The layerwise model proposed is a stress model [see 1, 2, 3, 4]. Referring to Carrera's nomenclature [6], the M4 model was renamed *LS1* model (Layerwise Stress approach with first-order membrane stress approximations per layer in the thickness direction [7]). In this model, each layer appears as a Reissner-Mindlin plate and the different layers are connected with interfacial stresses which are considered as generalized stresses of the model. Out-of-plane shear and normal stresses continuity is thus achieved at the interfaces. The principal dissimilarity between the *LS1* model and other existing layerwise models is that, in most cases, the layerwise models are either displacement or mixed stress-displacement approaches whereas

the LS1 model, directly inspired from Pagano's model [8], is a pure layerwise stress approach where there is no preliminary hypothesis on displacement fields. The method proposed in this study allows a full analysis of delaminated symmetric or asymmetric multilayered plates using the finite element *MPFEAP* based on the *LS1* model and implemented in the software GiD. This layer-wise model allows to access very directly and without any post-processing the values of the interfacial shears and normal stresses. It reduces drastically computation time compared to 3D finite elements and cohesive zone method (CZM) as it uses 2D plate finite elements. Results from *MPFEAP* code based on *LS1* model will be compared to experimental and 3D finite elements results from [9]. Test cases were performed on DCB, ELS, ADCB, MMF, and AMMF both UD and MD with 0/0 and 0/45 delaminated interfaces. The fracture toughness of these interfaces are calculated with the model by using the fact that this approach permits to deduce analytically mode separation from the interfacial stresses.

2. Description of the LS1 model

The model *LS1*, initially developed for calculating interfacial stresses, [1], is specifically devoted to the study of the interface phenomena, delamination initiation or sliding. The formulation of the *LS1* model (Layerwise Stress model with first-order membrane stress approximation per layer [7]), is presented in details for example in [7]. In the following formulation, x and y represent the in-plane directions and z is the thickness coordinate. h^i_- , h^i_+ and $\bar{h}^i$ are respectively the bottom, the top and the mid-plane z coordinate of layer i and $e^i = h^i_+ - h^i_-$ denotes the thickness of layer i. Greek alphabet subscripts (such as $\alpha, \beta, \gamma, \delta$) correspond to $\{x, y\}$ or $\{1, 2\}$.

2.1. Generalized interface stresses, displacements and strains, constitutive and equilibrium equations

As explained, the LS1 model is a layerwise model with stress field approximations. Indeed, this model presents a stress approach based on Pagano's model [8], in which there is no hypothesis on displacement fields. In this model, the 3D stress components are considered as polynomial functions of z whose coefficients are expressed in terms of generalized stresses of the model. The in-plane stress components $\sigma_{\alpha\beta}$ are chosen as linear functions of z. According to the 3D equilibrium equations, the shear stresses $\sigma_{\alpha z}$ and the normal stress σ_{zz} are respectively quadratic and cubic polynomial functions of z.
Classically in-plane stress, moment and transverse shear resultants of layer i, are defined as generalized internal stresses (see [7]), and, more originally, interlaminar shear and normal stresses at interface $i, i+1$, respectively $\tau_\alpha^{i,i+1}$ and $\nu^{i,i+1}$ can also be defined as :

$$\tau_\alpha^{i,i+1}(x,y) = \sigma^i_{\alpha z}(x,y,h^i_+) = \sigma^{i+1}_{\alpha z}(x,y,h^{i+1}_-) \tag{1}$$

$$\nu^{i,i+1}(x,y) = \sigma^i_{zz}(x,y,h^i_+) = \sigma^{i+1}_{zz}(x,y,h^{i+1}_-) \tag{2}$$

By introducing the assumed stress fields into the Hellinger-Reissner functional and integrating with respect to z over the thickness of each layer, the expressions of generalized displacements may be deduced. These generalized displacements are in fact weighted-averages of the 3D displacements (see [1, 7]). In this way, five kinematic fields (three displacements $\mathbf{U}^i$ and two rotations ϕ^i) are introduced for each layer i (see [7]) and generalized strains which are deduced from the generalized displacements, are associated with these generalized stresses. In particular,

3 interface strains, $D_\alpha^{i,i+1}$ *and* $D_z^{i,i+1}$ are associated, respectively, with the generalized interface stresses defined above $\tau_\alpha^{i,i+1}$ and $\nu^{i,i+1}$.

$$D_\alpha^{i,i+1} = U_\alpha^{i+1} - U_\alpha^i - \left(\frac{e^i}{2}\Phi_\alpha^i + \frac{e^{i+1}}{2}\Phi_\alpha^{i+1}\right) \tag{3}$$

$$D_z^{i,i+1} = U_z^{i+1} - U_z^i \tag{4}$$

The derivation of the Hellinger-Reissner functional with respect to generalized stresses yields the constitutive equations of the model. Equilibrium and constitutive relations for layer i are on classical Reissner-Mindlin type (see [7]), and interface $i, i+1$ relations are exhibited as follows:

$$\begin{aligned} D_\alpha^{i,i+1} = &- \frac{1}{10}\left(S^i_{Q\alpha\beta}Q_\beta^i + S^{i+1}_{Q\alpha\beta}Q_\beta^{i+1}\right) - \frac{1}{30}\left(e^i S^i_{Q\alpha\beta}\tau_\beta^{i-1,i} + e^{i+1}S^{i+1}_{Q\alpha\beta}\tau_\beta^{i+1,i+2}\right) \\ &+ \frac{2}{15}\left(e^i S^i_{Q\alpha\beta} + e^{i+1}S^{i+1}_{Q\alpha\beta}\right)\tau_\beta^{i,i+1} \end{aligned} \tag{5}$$

$$D_z^{i,i+1} = \frac{9}{70}\left(e^i\, S^i_\nu\, \nu^{i-1,i} + e^{i+1}\, S^{i+1}_\nu\, \nu^{i+1,i+2}\right) + \frac{13}{35}\left(e^i\, S^i_\nu + e^{i+1}\, S^{i+1}_\nu\right)\nu^{i,i+1} \tag{6}$$

where Q_β^i are transverse shear resultant of layer i, $S^i_{\alpha\beta\gamma\delta}$, $S^i_{Q\alpha\beta}$ and S^i_ν are components of the compliance matrix of layer i as expressed for example in [5].

2.2. LS1 for delamination

In the original *LS1* model, the interfaces were considered as perfect and interface displacements were only due to the elastic generalized displacements in the neighboring layers. Now, if the role of physical interfaces has to be specifically taken into account (elastic or plastic sliding [10], thick elastic or plastic interface [11]), the elastic interface displacements described by equations (3) and (4) may highlight this new complexity and it was proposed to express elastic interface displacements as global displacements minus local anelastic ones. This later localized slidings $\gamma_x^{k,k+1}$, $\gamma_y^{k,k+1}$ and $\gamma_z^{k,k+1}$ due to the own interface behavior, can be, for instance, considered as elastic or plastic. Based on the previous model $LS\,1$, a C^0 finite element model, involving an eight-node isoparametric quadrilateral element with 5n d.o.f at each nodal point and four second-order Gaussian points was formulated. A program called MPFEAP (Multiparticle Finite Element Analysis Program) has been developed for the implementation of the proposed element and is used in this paper.

3. Application of the LS1 to delaminated composite beams

In this part, the $LS\,1$ is applied for classical configurations of cracked composite beams. The case study described and studied in [9] allows for direct comparisons between 3D FEM, present approach and experimental results.

3.1. Case study [9]

The following case study is directly derived from Prombut & al. [9]. The main objective of this case study [9] was to develop a methodology for establishing crack propagation criteria of

unidirectional (UD) and multidirectional (MD) laminates at respectively 0/0 and 0/45 interfaces. In this work, the author focused on determining the critical strain energy release rate (SERR) in mode I, G_{Ic} under the conventional double cantilever beam (DCB) test method. The author used the asymmetric double cantilever beam (ADCB) and asymmetric mixed-mode flexure (AMMF) methods to obtain mixed-mode I + II loading with high mode I content in order to determine the G_{Ic} of the 0/45 interface.

3.2. Numerical data

All plies are made up of the same carbon-epoxy material (T700/M21) whose mechanical properties are as follows:

$$E_L = 98.62\ GPa\ ,\ E_T = E_N = 7.69\ GPa$$
$$G_{LT} = G_{LN} = 4.75\ GPa\ ,\ G_{TN} = 4.75\ GPa$$
$$\nu_{LT} = \nu_{LN} = 0.3\ ,\ \nu_{TN} = 0.3\ ,\ e = 0.26\ mm$$

To apply LS1 to this case study, elastic interface are chosen, a zero stiffness interface is put between the plies where delamination occurred, arm 1 and arm 2, while a quasi-perfect interface is considered (for numerical reason) in arm 3 by electing $k_x^{k,k+1} = k_y^{k,k+1} = k_z^{k,k+1} = 3500GPa$ to represent an infinite stiffness. Concerning the meshing, a 5x20 2D-mesh) with a refinement in the vicinity of the delamination front was enough to obtain the convergence of the interfacial stresses of the LS1 model at delamination front. In order to evaluate the model performance on delaminated composite beams for structural analysis, 12-layers unidirectional DCB and ADCB tests and 18-layers multidirectional ADCB test described in the previous paragraph were investigated and the displacements results of the *LS1* model for different crack lengths were compared to those of a 3D finite element model from Prombut &al. [9].

3.3. Displacements/crack length results

The results of each test configuration were obtained by averaging the experimental critical forces and displacements corresponding to different measured crack length in [9]. (Table 1) and (Table 2) results reports the average experimental load and displacement values obtained for each crack length, as well as the displacements obtained from the finite element models both from [9] and from MPFEAP named respectively $\delta - 3DFE$ and $\delta - MPFEAP$ for different test configurations. The correlation between the 3D finite elements and the *LS1* model is good. The average displacement difference between experimental and MPFEAP values is $< 7\%$ for the worst case with a really light and 2D mesh.

4. Strain energy release rate and mode ratio determination with a LS1 formulation

Using the *LS1* model and the VCCT method, the SERR and mode partitioning can be calculated using the interfacial stresses of the model at the delaminated interface and specific Ψ functions which depend only of the stacking sequence and position of delamination. This method is of great interest as it allows on simple cases pure analytical determination of the SERR mode ratio [10, 5]. For more complex structures interfacial stresses calculation will only need 2D-FE simulation while the Ψ functions expressions are analytical.
When VCCT method is applied using the LS1 interfacial stresses $\tau_x^{k,k+1}$, $\tau_y^{k,k+1}$ and $\nu^{k,k+1}$ defined

DCB	a(mm)/P(N)	45/51.19	50/49.63	55/47.61	60/46.27	65/45.11
	δ-EXP(mm)	5.76	7.41	9.35	11.16	13.31
	δ-3DFE(mm)	5.58	7.27	9.12	11.33	13.85
	δ-MPFEAP(mm)	5.32	6.93	8.72	10.84	13.31
ADCB	a(mm)/P(N)	45/41.98	50/40.75	55/40.73	60/39.72	65/37.20
	δ-EXP(mm)	8.06	10.35	13.01	16.02	19.01
	δ-3DFE(mm)	8.07	10.53	13.72	17.06	20.07
	δ-MPFEAP(mm)	7.98	10.11	12.82	15.78	18.99

Table 1. Simulated and experimental displacements for different crack lengths of unidirectional DCB and ADCB specimens

ADCB18	a(mm)/P(N)	85/26.21	90/25.17	95/24.82	100/24.28	105/22.83
	δ-EXP(mm)	15.01	16.93	19.32	22.37	23.87
	δ-3DFE(mm)	13.23	14.98	17.24	19.54	21.19
	δ-MPFEAP(mm)	12.63	14.31	16.51	18.75	20.33

Table 2. Simulated and experimental displacements for different crack lengths of multidirectional ADCB specimen

in (1) and (2) and slips $\gamma_x^{k,k+1}$, $\gamma_y^{k,k+1}$ and $\gamma_z^{k,k+1}$ introduced in paragraph 2.4. at the delaminated interface $k, k+1$, the following relations for the strain energy release rates in the different modes G_I, G_{II} and G_{III} are obtained and proposed firstly in [10]:

$$G_I = \frac{1}{2}\nu^{k,k+1}\gamma_z^{k,k+1} \tag{7}$$

$$G_{II} = \frac{1}{2}\tau_x^{k,k+1}\gamma_x^{k,k+1} \tag{8}$$

$$G_{III} = \frac{1}{2}\tau_y^{k,k+1}\gamma_y^{k,k+1} \tag{9}$$

Expressing $\gamma_x^{k,k+1}$, $\gamma_y^{k,k+1}$ and $\gamma_z^{k,k+1}$ as an analytically function of the $\tau_x^{k,k+1}$, $\tau_y^{k,k+1}$ and $\nu^{k,k+1}$ permits to obtain relations between strain energy release rates and interfacial stresses at crack front. These relations involve five ψ functions ψ^{ν}, ψ_{xx}^{τ}, ψ_{xy}^{τ}, ψ_{yy}^{τ} and ψ_{yx}^{τ} depending only of the stacking sequence and the position of the delamination [10]:

$$G_I = \psi^{\nu}(\nu^{k,k+1})^2 \tag{10}$$

$$G_{II} = \psi_{xx}^{\tau}(\tau_x^{k,k+1})^2 + \psi_{xy}^{\tau}\tau_y^{k,k+1}\tau_x^{k,k+1} \tag{11}$$

$$G_{III} = \psi_{yy}^{\tau}(\tau_y^{k,k+1})^2 + \psi_{yx}^{\tau}\tau_x^{k,k+1}\tau_y^{k,k+1} \tag{12}$$

These relations, (10) to (12), are general and for any cracked laminate.

	method	$G-EXP$	$G-3DFE$	$G-MPFEAP$
$DCB18U_{85}$	$G_I(J/m^2)$	392	421	409
$ELS\,18U_{85}$	$G_{II}(J/m^2)$	1211	1141	1176

Table 3. Average experimental and finite element SERR values for $DCB18U_{85}$ and $ELS\,18U_{85}$ test configurations

$MMF18U_{85}$	*method*	$G-EXP$	$G-3DFE$	$G-MPFEAP$
	$G_I(J/m^2)$	385	547	340
	$G_{II}(J/m^2)$	290	277	329

Table 4. Average experimental and finite element SERR values for $MMF18U_{85}$ test configuration, with same ψ^ν and ψ^τ_{xx} as used on $DCB18U_{85}$ and $ELS\,18U_{85}$

5. Application to case study

From Prombut & al. [9], 3D FE and MPFEAP calculations of SERR and mode partitioning were compared on DCB, ELS, MMF and ADCB unidirectional test specimens, and on ADCB18 multidirectional one. The same mesh and delaminated interface creation as in section 3.3 was used in GiD to obtain the values of the interfacial stresses $\tau_x^{k,k+1}$, $\tau_y^{k,k+1}$ and $\nu^{k,k+1}$ of the *LS1* and the critical strain energy release rates corresponding to experimental measures were determined using equations (10), (11) and (12).

5.1. ψ functions calculation

As previously mentioned with LS1 model, it is possible to calculate directly the strain energy release rates for each mode as in (10), (11) and (12), using the same ψ functions no matter the crack length or loading conditions as they depend only on the stacking sequence and the z-position of the delaminated interface.
Here, ψ^ν and ψ^τ_{xx} (the only functions involved in the following study as $\tau_y^{k,k+1} = 0$) were calculated for an 18-layers unidirectional specimen and were used to calculate strain energy release rates in respectively double cantilever beam ($DCB18U_{85}$) and end load split ($ELS\,18U_{85}$) test configuration with a delamination length of $85mm$. These elementary calculations were made using equations (10) and (11) with the interfacial stresses $\tau_x^{k,k+1}$ and $\nu^{k,k+1}$ found numerically with MPFEAP. The strain energy release rates G_I and G_{II} were then compared with the ones found experimentally and calculated with finite elements by Prombut & al. in [9] as reported in (Table 3). The good correlation between the experimental SERR and the *LS1* model despite the light mesh and direct calculation of the SERR shows the efficiency of this approach.

To validate the fact that these functions are independent of loading cases but depend only of material and stacking sequence, the previously calculated values of ψ^ν and ψ^τ_{xx} were then used to calculate the strain energy release rates in mode I and mode II of another case loading of the same 18-layers unidirectional sequence, a mixed-mode flexure ($MMF18U_{85}$) specimen with same geometry as $DCB18U_{85}$ and $ELS\,18U_{85}$. Table 4) shows a very good correlation between experimental SERR and the *LS1* model (a bit better than with 3D finite elements).

DCB	a(mm)/P(N)	45/51.19	50/49.63	55/47.61	60/46.27	65/45.11
	$G_I - EXP(J/m^2)$	426	479	527	560	602
	$G_I - 3DFE(J/m^2)$	451	515	567	629	694
	$G_I - MPFEAP(J/m^2)$	436	494	539	593	645

Table 5. Average experimental and finite element SERR values for DCB test configuration

ADCB	a(mm)/P(N)	85/34.17	90/31.93	95/31.31	100/29.56	105/28.73
	$G_I - EXP(J/m^2)$	321	314	337	332	346
	$G_I - 3DFE(J/m^2)$	327	319	340	335	347
	$G_I - MPFEAP(J/m^2)$	315	304	325	322	335
	$G_{II} - EXP(J/m^2)$	59	58	62	61.5	64
	$G_{II} - 3DFE(J/m^2)$	46	45	48	47	49
	$G_{II} - MPFEAP(J/m^2)$	70	68	73	72	75

Table 6. Average experimental and finite element SERR values for ADCB18 unidirectional test configuration

5.2. Calculation of SERR for different delamination length in DCB and ADCB

To enforce the validation of the method, other stacking sequences are studied, a 12 layers DCB specimen and a ADCB unidirectional and multidirectional ones.
The associated ψ functions are calculated and the strain energy release rates for an initial delamination length $a_0 = 45mm$ (for DCB) and $a_0 = 85mm$ (for ADCB). Then, other values of SERR for different delamination lengths of same specimens were calculated using the same ψ functions, calculated for the initial delamination length, and the different interfacial stresses results corresponding to different crack length given by MPFEAP. (Table 5), (Table 6) and (Table 7) results report the average experimental SERR values and mode partitioning, as well as those obtained from the finite element models both from [9] and from MPFEAP named respectively $G - 3DFE$ and $G - MPFEAP$. The correlation between the 3D finite elements and the *LS1* model is quite good.

6. Conclusion

In order to test the numerical formulation based on the *LS1* for delamination propagation analyzes, strain energy release rates comparisons were made between the *LS1* model and a 3D-FEM from [9] for different test specimens such as DCB for pure mode I, ELS for pure mode II and

ADCB	a(mm)/P(N)	85/26.21	90/25.17	95/24.82	100/24.28	105/22.83
	$G_I - 3DFE(J/m^2)$	272	280	301	318	309
	$G_I - MPFEAP(J/m^2)$	273	280	303	320	311
	$G_{II} - 3DFE(J/m^2)$	16	16	17	18	17
	$G_{II} - MPFEAP(J/m^2)$	15	15	16	18	16

Table 7. Average finite element SERR values for ADCB18 multidirectional test configuration

ADCB for mixed-mode I/II and two different delaminated interfaces 0/0 and 0/45. The delaminated and non-delaminated interfaces were represented numerically by respectively a zero and a quasi-infinite stiffness. It has been shown that the strain energy release rate values determined analytically using interfacial stresses from the *LS1* model are in really good agreement with experimentation and 3D-FEM which is remarkable given the very direct and light method used in this paper. The main advantage of using this model is that it only needs a 2D-FE calculation with low mesh refinement, reducing drastically computational time.

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