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1 **On the Use of the Similar Media Concept for Scaling Soil Air Permeability**

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16 **Abstract**

17 Soil air permeability (k_a) is an important factor that controls subsurface gas transport and
18 exchange of gas across the soil-atmosphere interface. It is thus crucial to evaluate the spatial
19 distribution of k_a for both application and modeling purposes. However, relevant studies are still
20 very limited, partly due to the fact that the dependence of k_a on soil moisture levels cannot be
21 directly included in the methods such as geostatistical techniques for analyzing the spatial
22 distribution of k_a . To tackle this problem, the scaling scheme based on the similar media concept,
23 which has been widely used in soil hydrology for characterizing spatial variability of soil
24 hydraulic properties, was employed for scaling k_a in this study. Four air permeability models,
25 including Millington and Quirk (1960)-MQ, Hunt (2005)-HT, Brooks and Corey (1964)-BC, and
26 Kawamoto et al. (2006)-KA, were selected to test this method using two independent datasets.
27 For the first dataset that included k_a measured for river sediments, all of the four models were
28 able to delineate the spatial distribution of k_a with a reference curve of k_a and a set of scaling
29 factors. Specifically, the MQ model gave the least satisfactory results due to the less flexibility of
30 its form; whereas, there were no significant differences in the performances for the HT, BC, and
31 KA models. For the second dataset that contained k_a measured for agricultural soils, the overall
32 performance of the four models for scaling k_a deteriorated, largely due to the alterations in the
33 microscopic structures of soil samples caused by repacking and compression of soil samples.
34 Nonetheless, as the first attempt, this study shows the viability of using the similar media
35 concept for scaling k_a . The merit of this method resides in the fact that the spatial variations of
36 moisture conditions and soil properties can be simultaneously included for analyzing the spatial
37 distribution of k_a . With a reference curve of k_a and the distribution of scaling factors, this method
38 would be particularly suitable for modeling subsurface gas transport.

39

40 **Keywords:** Similar Media Concept, Scaling Factor, Soil Air Permeability, Air Permeability
41 Model

42 1. Introduction

43 As one of the key factors that control subsurface gas transport and exchange of gas across the
44 soil-atmosphere interface, soil air permeability (k_a) describes the ability of a soil to transmit gas.
45 The rising interest in k_a is manifested by its broad range of applications in various fields,
46 including greenhouse gas emission (Ball et al., 1997; Conen et al., 1998), landfills (Jain et al.,
47 2005; Wu et al., 2012), soil vapor extraction systems (Poulsen et al., 1998; Farhan et al., 2001),
48 crop growth (Lipiec and Hatano, 2003; Barrios et al., 2005). Moreover, due to easy operation and
49 cost effectiveness, k_a measured near field capacity has been used to predict soil saturated
50 hydraulic conductivity (Loll et al., 1999; Iversen et al., 2001).

51 Under natural conditions, k_a is affected by a number of soil factors (e.g., air-filled porosity
52 and pore size distribution) and moisture conditions, all of which show various degrees of spatial
53 variations. As such, field measured k_a exhibits significant spatial variability (Iversen et al., 2003).
54 For application and modeling purposes, it is thus crucial to evaluate the spatial distribution of k_a
55 and its controlling factors. However, compared to relevant researches on soil hydraulic properties,
56 only few studies are available on the spatial distribution of k_a , which nonetheless provided
57 valuable insights into the understanding of the spatial pattern of k_a (Poulsen et al., 2001; Iversen
58 et al., 2003, 2004). Geostatistical techniques were mainly used in previous studies to analyze the
59 spatial pattern of k_a . Based on the results of variograms, Iversen et al. (2003) found that the range
60 of k_a for sandy soils was larger than the one for a loamy soil, probably due to the difference in
61 the depositional processes of those two types of soils. Although geostatistical techniques have
62 been proven to be powerful tools for investigating naturally occurred phenomena, there are
63 certain shortcomings for those techniques as pointed out by Henley (2001). Most notably, the
64 underlying processes associated with studied targets cannot be explicitly considered in
65 geostatistical techniques, which rather rely on statistical models for examining the spatial
66 correlations of the targets.

67 With respect to k_a , the main issue of using geostatistical techniques stems from the impact of
68 soil moisture on k_a . It has been well known that k_a is highly dependent on soil moisture levels,
69 but the spatial distribution of soil moisture cannot be directly included in the geostatistical
70 analyses of k_a . To some degree, this is analogous to study the spatial distribution of soil
71 unsaturated hydraulic conductivity without specifying moisture conditions. Therefore,

72 precautions were usually taken in previous field studies on the spatial distribution of k_a . As
73 conjectured by Iversen et al. (2003), when soil moisture contents reach field capacity, the air
74 flow takes place in the majority of soil pores; thus, the impact of moisture on k_a can be neglected
75 with moisture contents near and below field capacity. Although the assumption made by Iversen
76 et al. (2003) is useful for studying the spatial distribution of k_a under dry conditions, it may fail at
77 regions with wet climates or shallow groundwater tables. For application purposes (e.g., landfills
78 and soil vapor extraction systems), the inclusion of moisture in analyzing the spatial distribution
79 of k_a is also inevitable. Therefore, it is desirable to seek alternative methods to assess the spatial
80 distribution of k_a under the influence of soil moisture.

81 Along the line of above thinking, the scaling scheme based on the similar media concept may
82 provide a promising approach to investigating the spatial distribution of k_a with the consideration
83 of moisture conditions. First introduced by Miller and Miller (1956), the similar media concept is
84 based on the assumption that the internal geometry of similar media only differs by microscopic
85 length scales that can be characterized by scaling factors. The purpose of this scaling approach is
86 to coalesce a range of functional relationships into a single curve through scaling factors that
87 depict the spatial distribution of those functional relationships. More specifically, scaling factors
88 are used to relate soil properties at a given location to the mean properties at a reference location,
89 which are invariant of moisture conditions. This scaling method has been widely used in soil
90 hydrology to characterize spatial variability of soil hydraulic properties and soil hydraulic
91 functions with associated model parameters (Warrick et al., 1977; Hopmans, 1987; Shouse and
92 Mohanty, 1998; Zavattaro et al., 1999; Hendrayanto et al., 2000; Tuli et al., 2001). With a
93 reference functional relationship and the distribution of scaling factors, this method is
94 particularly suitable for modeling purposes (Peck et al., 1977; Kabat et al., 1997; Salvucci, 1998;
95 Oliveira et al., 2006). Given the similarities between water flow and gas transport in soils, one
96 can expect the viability of applying this scaling approach for investigating the spatial distribution
97 of k_a under the influence of soil moisture.

98 To our knowledge, this research was the first attempt to extend the use of the similar media
99 concept for scaling k_a . The main objective of this study was to examine the feasibility of this
100 approach using two datasets collected from USA and France. Four air permeability models were
101 selected to delineate the functional relationship between k_a and saturation degree of air. The

102 results of this study demonstrated the feasibility of using the similar media concept for scaling k_a ,
103 which also opened the door for utilizing this method for simulating gas transport in soils.

104

105 **2. Materials and Methods**

106 **2.1 Similar Media Concept in Soil Hydrology**

107 The use of the similar media concept for scaling soil water retention and hydraulic
108 conductivity curves is well documented in the literature (see the review by Vereecken et al.,
109 2007). So, only a brief overview is given here. Based on the similar media concept, it is assumed
110 that the microscopic structures (e.g., tortuosity, and relative particle size and pore size
111 distributions) of similar soils are identical and only differ by microscopic length scales that can
112 be characterized by scaling factors (Peck et al., 1977; Warrick et al., 1977). The scaling factor
113 (α) is thus defined by the ratio of the microscopic characteristic length of a soil (λ) to the
114 characteristic length of a reference soil (λ_m):

$$115 \quad \alpha_i = \frac{\lambda_i}{\lambda_m} \quad (1)$$

116 where $i=1, 2, \dots, L$ is the location of the soil and the subscript m denotes the reference soil. By its
117 definition, α is invariant of soil moisture conditions and only dependent upon the location of the
118 soil. As such, the soil water retention curve at any location can be scaled to the reference water
119 retention curve through α :

$$120 \quad h_i(\theta) = \frac{h_m(\theta)}{\alpha_{w,i}} \quad (2)$$

121 where h is the soil matric potential, θ is the volumetric moisture content, and the subscript w
122 denotes water. The scaling relationship of the soil hydraulic conductivity curve can be written as:

$$123 \quad K_{w,i}(\theta) = K_{w,m}(\theta) \alpha_{w,i}^2 \quad (3)$$

124 where K_w is the hydraulic conductivity. Due to the fact that soil porosity may vary across
125 locations, instead of θ , the saturation degree of moisture (S_w) is usually used (Warrick et al.,
126 1977):

127
$$S_w = \frac{\theta}{\phi} \quad (4)$$

128 where ϕ is the soil porosity.

129 To describe the reference curves of h_m and $K_{w,m}$, soil water retention and hydraulic
130 conductivity models (e.g., Brooks-Corey model and van Genuchten model) have been used,
131 although polynomial functions of $h_m(S_w)$ and $K_{w,m}(S_w)$ were also adopted (Warrick et al., 1977).
132 By optimizing the differences between measured and calculated $h(S_w)$ and $K_w(S_w)$ across all the
133 locations, the model parameters for the reference curves of h_m and $K_{w,m}$, and the scaling factor
134 $\alpha_{a,i}$ at each location can be obtained (Hopmans, 1987; Tuli et al., 2001).

135

136 **2.2 Extension of the Similar Media Concept for Scaling Soil Air Permeability**

137 Although the scaling theory based on the similar media concept has been widely used in soil
138 hydrology, there is a surprising lack of studies on its application for assessing the spatial
139 distribution of k_a . Strictly speaking, soil water permeability (k_w) should be used in the scaling
140 procedure (Eq. (3)), as regardless of fluid properties, soil permeability, whether it is k_w or k_a ,
141 represents the intrinsic properties of soils to transmit fluids and thus reflects the microscopic
142 structures of soils. However, given that the fluid of interest remains under the same conditions
143 (e.g., temperature), the fluid properties do not change across locations and conductivities (e.g.,
144 K_w) can be used in Eq. (3). Therefore, one can write a similar scaling relationship for k_a based on
145 Eq. (3):

146
$$k_{a,i}(S_a) = k_{a,m}(S_a) \alpha_{a,i}^2 \quad (5)$$

147 where $S_a=(1-S_w)$ is the saturation degree of air and the subscript a denotes air. For the same
148 reason for using S_w in scaling $h(S_w)$ and $K_w(S_w)$, S_a was used in this study instead of the
149 volumetric soil-air content. Note that α_a is also invariant of soil moisture conditions and only
150 dependent upon the location of the soil.

151 To delineate the reference relationship of $k_{a,m}$, four air permeability models were selected
152 (Ghanbarian-Alavijeh and Hunt, 2012). The first model was proposed by Millington and Quirk
153 (1960) (denoted as MQ hereafter):

154
$$k_a(S_a) = k_o \times S_a^2 \quad (6)$$

155 where k_o is the air permeability at the porosity ϕ .

156 The second model was developed by Hunt (2005) (HT), which is based on the continuum
157 percolation theory:

158
$$k_a(S_a) = k_o \left(\frac{S_a - S_{a,t}}{1 - S_{a,t}} \right)^2 \quad (7)$$

159 where $S_{a,t}$ is the percolation threshold used by Ghanbarian-Alavijeh and Hunt (2012). Note that
160 the HT model can be reduced to Eq. (6) by assuming $S_{a,t}=0$.

161 The third model was based on the hydraulic conductivity function of Brooks and Corey
162 (1964) (BC):

163
$$k_a(S_a) = k_o S_a^l \left[1 + (1 - S_a)^{1 + \frac{2}{\gamma}} \right] \quad (8)$$

164 where γ is the pore size distribution index, and l is the tortuosity-pore connectivity factor and
165 assumed to be 2 based on the percolation theory (Hunt and Ewing, 2009).

166 The last model was taken from Kawamoto et al. (2006) (KA):

167
$$k_a(S_a) = k_o \times (S_a)^{1+3/\eta} \quad (9)$$

168 where $(1+3/\eta)$ represents the combined effects of tortuosity and connectivity of air-filled pores.
169 In Kawamoto et al. (2006), the measured k_a at -100 cm H₂O was used instead of k_o . Essentially,
170 the MQ model is a simplified version of Eq. (9) with $\eta=3$.

171 In parallel to previous studies on scaling $h(S_w)$ and $K_w(S_w)$, the scaling factors $\alpha_{a,i}$ and the
172 model parameters (given in Tables 2 and 3) for describing the reference curves of $k_{a,m}$ were
173 obtained by optimizing the differences between measured and calculated k_a . The following
174 objective function was used:

175
$$O = \sum_{i=1}^L \left\{ \sum_{j=1}^{M(i)} \left[k_{a,i}^{obs}(S_{a,j}) - \alpha_{a,i}^2 \times k_{a,m}(S_{a,j}) \right]^2 \right\} \quad (10)$$

176 where $M(i)$ is the total number of observed k_a at location i , $k_{a,i}^{obs}(S_{a,j})$ is the j th observed k_a
177 corresponding to $S_{a,j}$ at location i , and $k_{a,m}(S_{a,j})$ is the calculated k_a at $S_{a,j}$ from Eq. (6) to Eq.(9).

178 The generalized reduced gradient method was used to optimize Eq. (10) with the geometric mean
179 of the scaling factors assumed to be unity (Hopmans, 1987; Zavattaro et al., 1999; Hendrayanto
180 et al., 2000):

$$181 \quad (\prod_{i=1}^L \alpha_i)^{1/L} = 1 \quad (11)$$

182 To evaluate the performance of each air permeability model, the root mean square error
183 (RMSE) was used:

$$184 \quad RMSE = \left(\frac{1}{n} \sum_{k=1}^n (o_k - e_k)^2 \right)^{\frac{1}{2}} \quad (12)$$

185 where n is the total number of observed k_a across all the locations, o_k is either observed $k_{a,i}(S_{a,j})$ or
186 scaled $k_{a,i}(S_{a,j})$ (i.e., observed $k_{a,i}(S_{a,j}) \times \alpha_{a,i}^2$), and e_k is the corresponding $k_{a,m}(S_{a,j})$ calculated from
187 the reference curve of $k_{a,m}$.

188

189 2.3 Datasets

190 Two independent datasets collected from USA and France were used in this study to test the
191 feasibility of applying the similar media concept for scaling k_a . The first dataset (denoted as
192 Dataset I) included measured air permeability data using sediment samples taken from the
193 channel of the Platte River near Kearney, NE (Figure 1). The streambed sediments at the site are
194 moderately-sorted, and mainly consist of sands and gravels with very low organic matter
195 contents (Table 1). A total of 25 samples with a length of 20 cm for each sample were collected
196 using transparent polycarbonate tubes with an inside diameter of 5.1 cm. The tubes were
197 vertically pressed to the depth of 20 cm into the sediments. The distance between neighboring
198 sampling locations was 1 m. After taking out the sample from the river channel, the bottom of
199 the tube was wrapped by a fiberglass screening to prevent sediments from falling off. A falling
200 head permeability test was carried out on site to determine the saturated hydraulic conductivity
201 of the sediments inside the tube (Chen, 2005). Water temperatures were also recorded during the
202 test to calculate water dynamic viscosity, which was used to calculate k_w .

203 After the measurements of saturated hydraulic conductivities, the sediment samples were
204 brought back to the laboratory and air dried at room temperatures. On Day 2 and every three

205 days afterwards until Day 17, air permeability tests were performed using a portable air
206 permeameter (Figure 2), which resulted in 6 pairs of data points of k_a and S_a for each tested
207 sample. To measure k_a , the polycarbonate tubes were connected to the air permeameter, which
208 mainly consisted of an air tank, an air pressure regulator, a rotameter that measured the flow rate
209 of air through the sediments, and a manometer that recorded the pressure difference across the
210 length of the sediments. Darcy's Law was then used to calculate k_a (Kirkham, 1947). Due to the
211 slippage of gas molecules near the pore wall, the flow of gas in porous media may exhibit non-
212 Darcian flow that leads to higher k_a (Klinkenberg, 1941). Therefore, additional tests were
213 performed to examine the applicability of Darcy's Law by measuring the pressure difference
214 across the sample length under varying flow rates. The linearity of the results proved the
215 applicability of Darcy's Law and confirmed the air permeameter was operational for measuring
216 k_a . The merit of the air permeameter used in this study is that it allows to measure both saturated
217 hydraulic conductivity and k_a of the same sample without the need to remove the sediments from
218 the permeameter for repacking. Thus, it minimizes the possible disturbance of the sample
219 structure. After each air permeability test, samples were immediately weighed to determine
220 moisture contents and corresponding volumetric soil-air contents. On Day 17, all of the samples
221 were dried in the oven for 24 h at the temperature of 105 °C to determine bulk density and
222 porosity. The results of the grain size analyses showed that the texture of the sediments was sand
223 according to the USDA classification (Table 1).

224 The second dataset (denoted as Dataset II) was taken from Tang et al. (2011) and contained
225 nine remolded soil samples from an experimental forest site at Le Breuil located in central
226 France. The reason for choosing Dataset II was that several air permeability models, including
227 the HT model, were successfully tested on those soil samples (Tang et al., 2011; Ghanbarian-
228 Alavijeh and Hunt, 2012). However, unlike the air permeameter used in Dataset I, a specially
229 designed oedometer was adopted by Tang et al. (2011) to measure $k_a(S_a)$ by vertically
230 compressing soil samples, which led to varying moisture contents under different compressional
231 stresses. Except for one sample with 6 data points, there were 8 pairs of k_a and S_a for each
232 sample. The texture of the soils was sandy loam and their physical properties are reported in
233 Table 1.

234

235 3. Results and Discussions

236 3.1 Dataset I

237 It has been well known that besides soil properties, k_a is also affected by moisture conditions
238 and thus S_a . With the drying process, the mean value of S_a for Dataset I increased from 0.49 on
239 Day 2 to 0.87 on Day 17. Consequently, the mean value of k_a was almost quadrupled from 21.0
240 μm^2 on Day 2 to 83.6 μm^2 on Day 17, as more smaller pores opened up for transmitting air when
241 the sediments became drier. The results from Dataset I again illustrate the importance of
242 including moisture conditions in analyzing the spatial distribution of k_a . To further demonstrate
243 the impact of S_a on k_a , the results from four sediment samples in Dataset I (numbered as #1, #2,
244 #3, and #4) are plotted in Figure 3. As expected, k_a increased with increasing S_a or with
245 decreasing moisture levels; however, the $k_a(S_a)$ relationship also differed among different
246 samples. More specifically, k_a showed similar ranges for #1 (13.7 to 41.5 μm^2) and #2 (13.2 to
247 44.5 μm^2) with S_a ranging from 0.40 to 0.81 and 0.56 to 0.86, respectively (Figures 3a and 3b).
248 In comparison, k_a for #3 (11.3 to 142.3 μm^2) and #4 (4.5 to 111.5 μm^2) varied over an order of
249 magnitude with S_a increasing from 0.46 to 0.90 and 0.42 to 0.89, respectively (Figures 3c and
250 3d). Clearly, a single curve of $k_a(S_a)$ is not sufficient to describe spatially distributed k_a , even at
251 plot scales (e.g., a reach scale in this study). This presents a great challenge for simulating
252 subsurface gas transport with spatially varied k_a and associated model parameters. The same
253 challenge is also faced by soil hydrologists to simulate water movement in vadose zone, where
254 soil hydraulic properties may vary substantially across landscapes. One of the solutions to tackle
255 this problem in soil hydrology relies on the use of scaling factors within the framework of the
256 similar media concept (Kabat et al., 1997; Salvucci, 1998; Oliveira et al., 2006).

257 To have a thorough view of the effectiveness of using the similar media concept for scaling
258 k_a , the results of unscaled and scaled k_a from Dataset I are plotted in Figure 4 with the reference
259 curves of $k_{a,m}$ for all of the air permeability models. For the purpose of clarity, the results of
260 scaled k_a for each model are also plotted separately in Figure 4. The scaled k_a was calculated
261 using the obtained scaling factors (e.g., Eq. (5)). The obtained fitting parameters and RMSE
262 values for each model are given in Table 2. Note that the results of two sediment samples were
263 removed from the analyses, as they exhibited irregular patterns between k_a and S_a (e.g., k_a may
264 decrease with increasing S_a). Those irregular patterns were suspected to be the consequence of

265 the collapse of root holes in the samples during the drying process. As expected, a positive trend
266 existed between unscaled k_a and S_a . When S_a was lower than 0.7, unscaled k_a largely remained
267 below $60 \mu\text{m}^2$, while unscaled k_a varied about 8 times from approximately 20 to $160 \mu\text{m}^2$ when
268 S_a reached above 0.8. Figure 5a further shows that a single curve of $k_a(S_a)$ is not sufficient to
269 delineate the distribution of the unscaled k_a as attested by the $k_{a,m}$ curves. By comparison, with
270 the aid of scaling factors, the scaled k_a coalesced along the $k_{a,m}$ curves with greatly reduced
271 scatter for all of the four air permeability models. Note that scaled k_a was systematically more
272 scattered at the lower range of S_a for all of the models. It was due to the fact that more weights
273 were given to higher values of k_a in Eq. (10) during the optimization process. If k_a was replaced
274 by $\log k_a$ in Eq. (10), the scatter of scaled k_a at the lower range of S_a could be reduced; however,
275 the overall performance of the models deteriorated. Compared to the other three $k_{a,m}$ curves, the
276 one for the HT model slightly deviated; whereas, the resulting scaled k_a was comparable for all
277 of the four models (Figure 4b). Among all of the results, the RMSE value of the scaled k_a for the
278 MQ model was highest with $k_{a,m}$ overestimating scaled k_a at the lower end of S_a and
279 underestimating scaled k_a at the higher end of S_a (Figure 4c). This was because without
280 additional fitting parameters other than k_o , the form of the MQ model was the least flexible. By
281 contrast, the performances of the other three $k_{a,m}$ curves improved at both ends of S_a with the BC
282 model showing the most satisfactory results (Figures 4d-4f); however, there appeared no
283 significant differences in the performances of those three air permeability models for scaling k_a
284 (Table 2), indicating the applicability of those models for delineating the distribution of k_a in the
285 studied sediments.

286 The values of obtained k_o varied among the four models (Table 2). The fitted k_o in the KA
287 model was highest with the value of $106.0 \mu\text{m}^2$, while the one in the MQ model was only 92.5
288 μm^2 . As a result, the scaled k_a for the MQ model was lowest under the same level of S_a ; however,
289 the differences in the scaled k_a were minimum among different models. The value of the
290 obtained percolation threshold $S_{a,t}$ in the HT model was 0.168, which was comparable to the 0.1
291 threshold proposed by Ewing and Hunt (2006). In addition, the pore size distribution index γ (γ
292 =3.067 in this study) in the BC model fell within the range for sandy soils (Ghanbarian-Alavijeh
293 and Hunt, 2012). In summary, Figure 4 shows that for the studied sediments, the spatially
294 distributed k_a could be described by a reference curve of $k_{a,m}$ and a set of scaling factors, which
295 attests the feasibility of using the similar media concept for scaling k_a under the influence of

296 moisture. With a known reference curve of $k_{a,m}$ and the distribution of scaling factors, this
297 method would be particularly suitable for modeling subsurface gas transport.

298 As previously explained, the spatially distributed k_a is mostly controlled by moisture
299 conditions and soil properties, both of which may vary at different spatiotemporal scales.
300 Essentially, the impacts of moisture levels and S_a on k_a can be determined through the reference
301 curve of $k_{a,m}$; whereas, the spatial variations of soil properties are embedded in the distribution of
302 scaling factors, as scaling factors only depend on soil intrinsic properties (i.e., the microscopic
303 characteristic length). The probability plots of the scaling factors (both $\alpha_{a,i}$ and $\ln\alpha_{a,i}$) are given in
304 Figure 5 for the HT model. The resulting probability plots were similar for all of the four models,
305 so only the ones for the HT model are presented here. The respective mean and standard
306 deviation was 1.03 and 0.23 for $\alpha_{a,i}$, and 0 and 0.23 for $\ln\alpha_{a,i}$. The values of the standard
307 deviations for $\alpha_{a,i}$ and $\ln\alpha_{a,i}$ were smaller than previously reported values derived from soil water
308 retention and hydraulic conductivity data (Warrick et al., 1977; Hopmans, 1987; Hendrayanto et
309 al., 2000), probably due to the smaller number of sediment samples used in this study. Although
310 those previous studies showed a log normal distribution of scaling factors, the Kolmogorov-
311 Smirnov Test did not reject the null hypothesis that $\alpha_{a,i}$ and $\ln\alpha_{a,i}$ were normally distributed at the
312 significance level of 0.01. Thus, future studies are needed to include a larger number of samples,
313 and to compare the distributions of scaling factors drawn separately from soil water retention and
314 hydraulic conductivity data and from air permeability data.

315 To test the applicability of the derived reference curves of $k_{a,m}$, the values of k_a on Day 17
316 were calculated using the obtained scaling factors and measured S_a based on Eq. (5). As
317 hypothesized by Iversen et al. (2003), when soil moisture contents reach field capacity, the air
318 flow takes place in the majority of soil pores; thus, k_a measured at or near field capacity can be
319 used as a proxy of soil water permeability k_w to predict saturated hydraulic conductivity. On Day
320 17, the average value of soil moisture contents for all of the sediment samples was 0.04, which
321 was quite comparable to the observed field capacity of sandy soils (soil moisture data can be
322 accessed from the High Plains Regional Climate Center at
323 <http://www.hprcc.unl.edu/awdn/soilm/>). Based on the above reasoning, the calculated k_a on Day
324 17 is plotted against k_w derived from measured saturated hydraulic conductivities in Figure 6.
325 Again, only the calculated k_a for the HT model is shown here as a demonstration. It can be seen

326 from Figure 6 that the calculated k_a was very close to k_w . On average, the calculated k_a only
327 underestimated k_w by 11%. Figure 6 further corroborates the feasibility of using scaling factors
328 within the similar media framework for describing spatially distributed k_a .

329

330 **3.2 Dataset II**

331 The previous section has demonstrated the feasibility of using the similar media concept for
332 scaling k_a in river sediments. To assess the viability of this method in agricultural soils, analyses
333 were carried out for Dataset II. A mean value of 0.02 for $S_{a,t}$ in the HT model was obtained by
334 Ghanbarian-Alavijeh and Hunt (2012) by fitting individual curves of k_a to soil samples in Dataset
335 II. Thus, to avoid the negative values of $(S_a - S_{a,t})$ in Eq. (7), $S_{a,t}$ was fixed to be 0.02 for Dataset
336 II. In addition, an unusually high value of γ in the BC model was obtained from the optimization,
337 which was caused by fitting the $k_{a,m}$ curve to comparatively low values of k_a from one soil
338 sample (the data points with S_a between 0 and 0.2 shown in Figure 7a). Moreover, no physical
339 constraints for the fitting parameters were considered in the optimization process and $k_a(S_a)$ in
340 Eq. (8) became insensitive to γ for large γ values. Thus, instead of using this high value of γ , the
341 class-averaged value of γ for sandy loam was used, which was derived from soil water retention
342 data (Rawls et al., 1982).

343 The unscaled and scaled k_a from Dataset II are presented in Figure 7, and the obtained fitting
344 parameters and RMSE values are given in Table 3. Overall, the unscaled k_a from Dataset II
345 exhibited a much wider distribution from 0.5 to 1980.8 μm^2 . Some of the remolded samples
346 (e.g., the one with the maximum k_a of 1980.8 μm^2) also showed different ranges of k_a (Figure
347 7a), which was probably caused by the repacking processes. As expected, the improvement of
348 the scaled k_a from the MQ model was least satisfactory (Figure 7c). By comparison, the
349 performances of the HT and BC models were considerably improved; however, the obtained $k_{a,m}$
350 tended to overestimate most of the scaled k_a (Figures 7d and 7e), owing to the fact that the fitting
351 parameter (or shape factor) $S_{a,t}$ in the HT model and γ in the BC model were fixed during the
352 optimizations. Without the need to fix the fitting parameter η in the KA model, the $k_{a,m}$ curve
353 seemed to more reasonably delineate the distribution of the scaled k_a over the entire range of S_a
354 (Figures 7f), although it should be noted that a very high value of k_o was obtained for the KA

355 model in order to match the data points from the sample with the maximum k_a of $1980.8 \mu\text{m}^2$.
356 Using the same remolded samples from Dataset II, Ghanbarian-Alavijeh and Hunt (2012)
357 showed that the HT model was better than the other three models (e.g., MQ, BC, and KA) to
358 describe the $k_a(S_a)$ relationship for individual samples; however, no significant improvement of
359 the HT model was found in this study for scaling k_a . One of the possible reasons for the different
360 conclusions about the model performances was due to the differences in the optimization
361 procedures. In Ghanbarian-Alavijeh and Hunt (2012), the $k_a(S_a)$ curves were optimized for
362 individual samples and some of the fitting parameters (e.g., γ in the BC model) were obtained
363 through optimizing soil retention data rather than soil permeability data.

364 In comparison to the results from Dataset I, the overall performance of the four selected air
365 permeability models for scaling k_a was less satisfactory for Dataset II, which can be attributed to
366 two reasons. First, for the soil samples in Dataset II, soil structures might have been altered
367 during the repacking processes. Secondly, in order to achieve different moisture levels, soil
368 samples were compressed using an oedometer (Tang et al., 2011), which unavoidably changed
369 the microscopic structures of soils. The repacking processes and the compression of soil samples
370 might have led to variations in the microscopic structures among different soil samples and thus
371 the less satisfactory performance of using the similar media concept for scaling k_a . Nonetheless,
372 Figure 7 shows that the use of scaling factors was able to narrow down the scatter of k_a around
373 the $k_{a,m}$ curves for the HT, BC, and KA models. In terms of the distribution of scaling factors
374 from Dataset II, the respective mean and standard deviation for the HT model was 1.01 and 0.14
375 for $\alpha_{a,i}$, and 0 and 0.13 for $\ln\alpha_{a,i}$. Again, the values of the standard deviations for $\alpha_{a,i}$ and $\ln\alpha_{a,i}$
376 were smaller than the previously reported values derived from soil water retention and hydraulic
377 conductivity data (Warrick et al., 1977; Hopmans, 1987; Hendrayanto et al., 2000).

378

379 **4. Conclusions**

380 The use of the scaling factors based on the similar media concept was tested for scaling air
381 permeability k_a using two independent datasets in this study. With a reference curve of $k_{a,m}$ and a
382 set of scaling factors, this method was shown to be able to delineate the spatial distribution of k_a
383 for the first dataset, which included k_a measured under different moisture conditions by drying
384 river sediment samples. For the second dataset that contained k_a measured for agricultural soils,

385 however, the use of the similar media concept for scaling k_a was less successful. It was most
386 likely due to the alterations in the microscopic structures of soil samples caused by repacking
387 and compression of soil samples. The merit of this method resides in the fact that the spatial
388 variations of moisture conditions and soil properties can be simultaneously included for
389 analyzing the spatial distribution of k_a . At any given location, the impact of S_a and thus moisture
390 levels on k_a can be quantified by the reference curve of $k_{a,m}$; whereas, soil properties are
391 embedded in the scaling factors. Since this study was the first attempt to apply the similar media
392 concept for scaling k_a , more studies are needed to test this method on soils with different textures
393 and to examine the impacting factors that control the distribution of scaling factors. It would be
394 also useful to compare scaling factors derived independently from air permeability data, and
395 from soil water retention and hydraulic conductivity data, which may further elucidate the
396 physical meanings of those scaling factors. Future studies are also needed to incorporate this
397 method in modeling subsurface gas transport.

398

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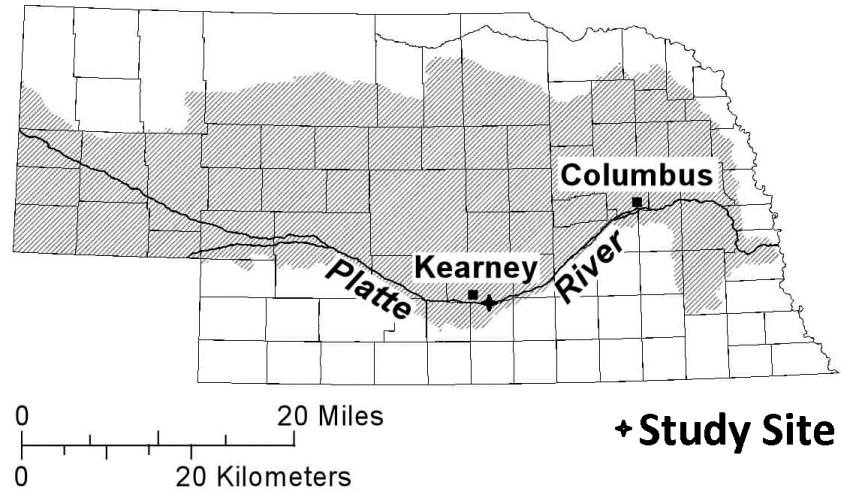
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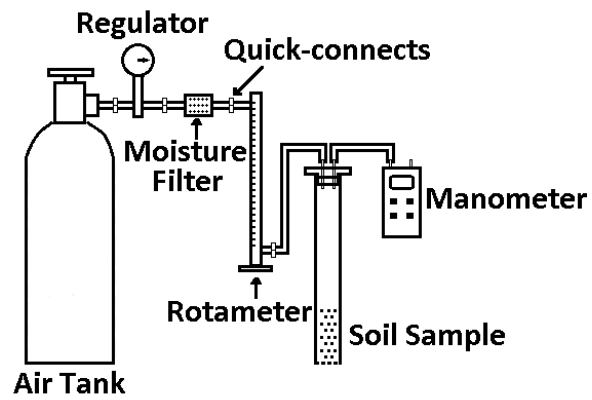
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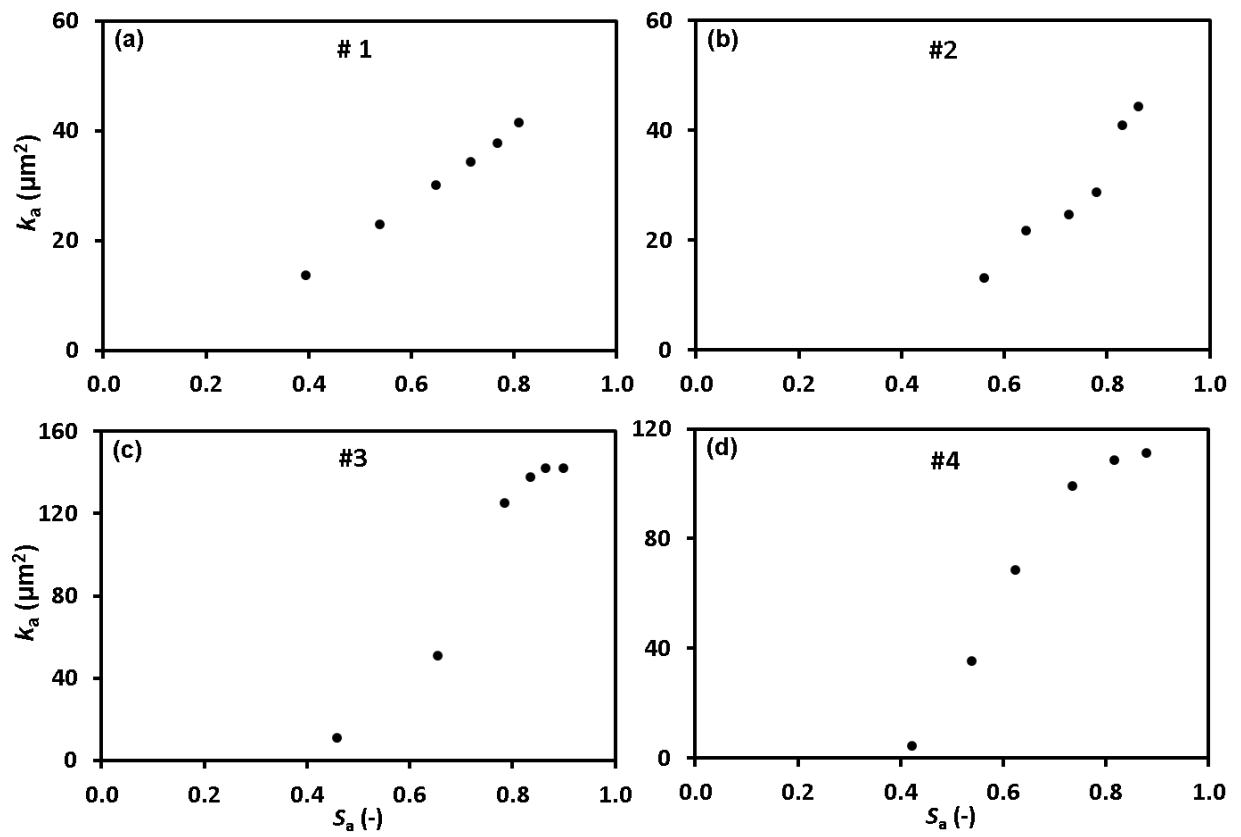
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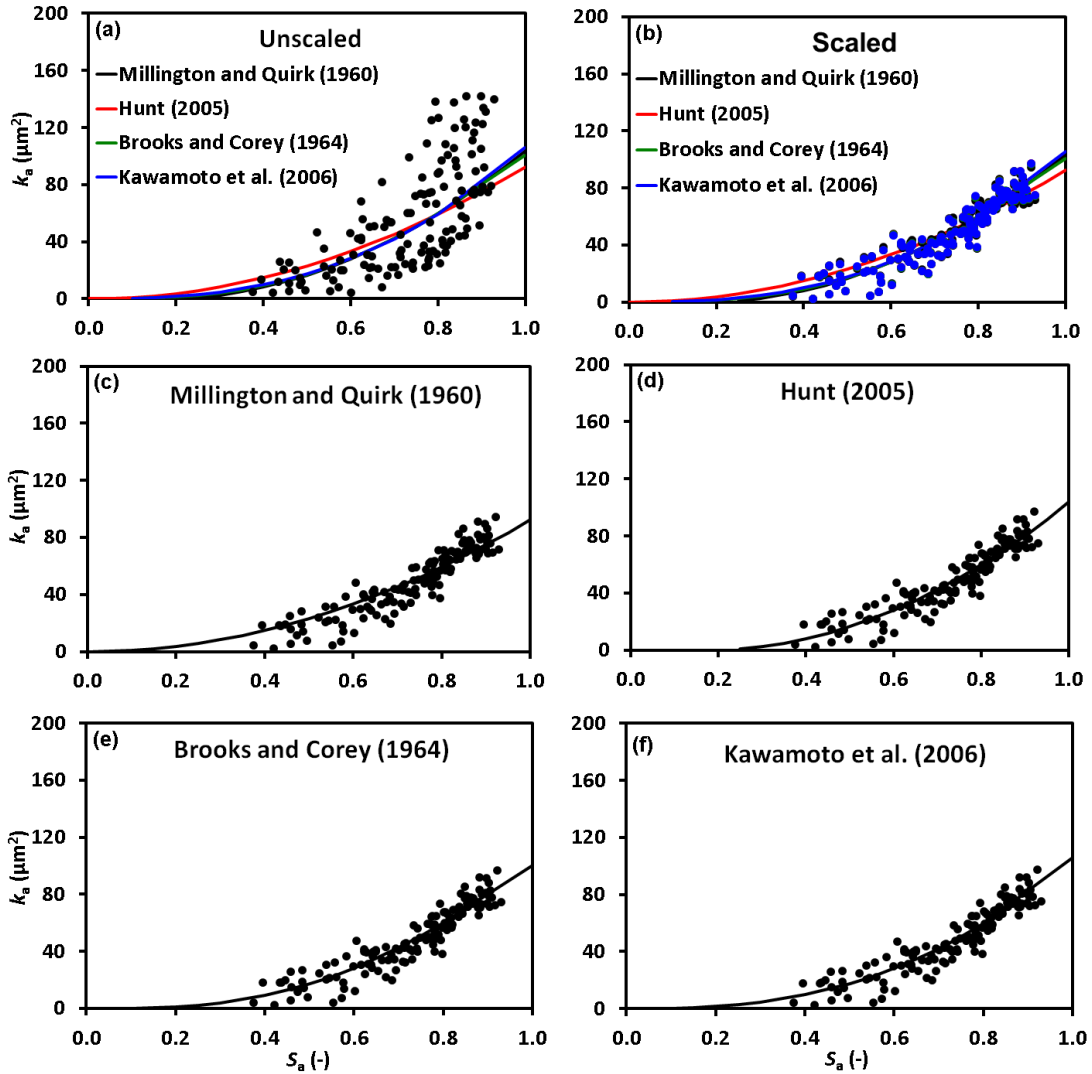


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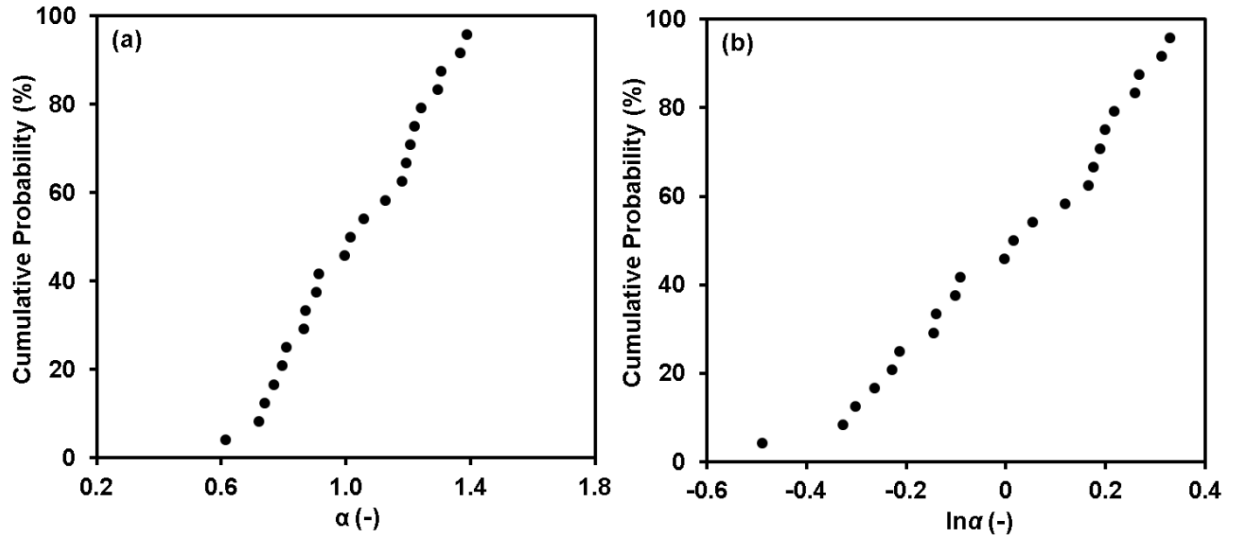
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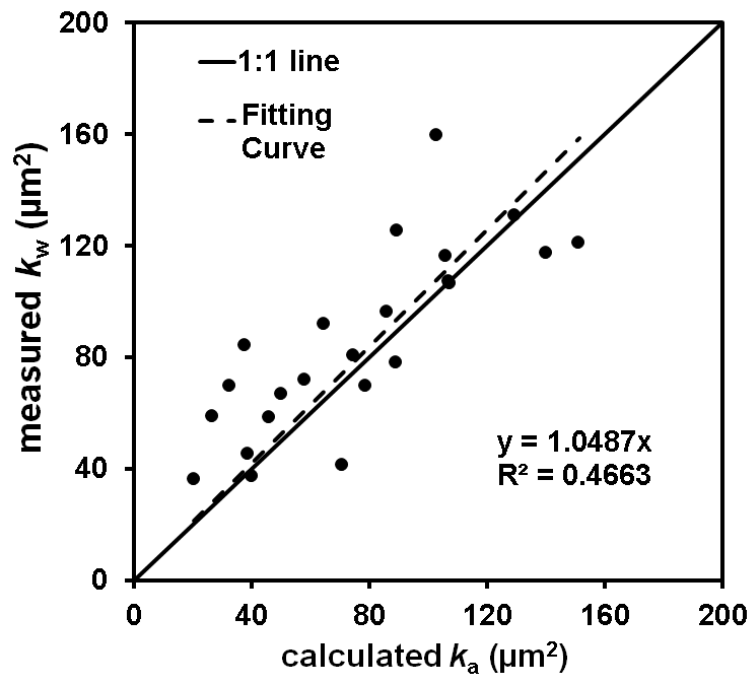
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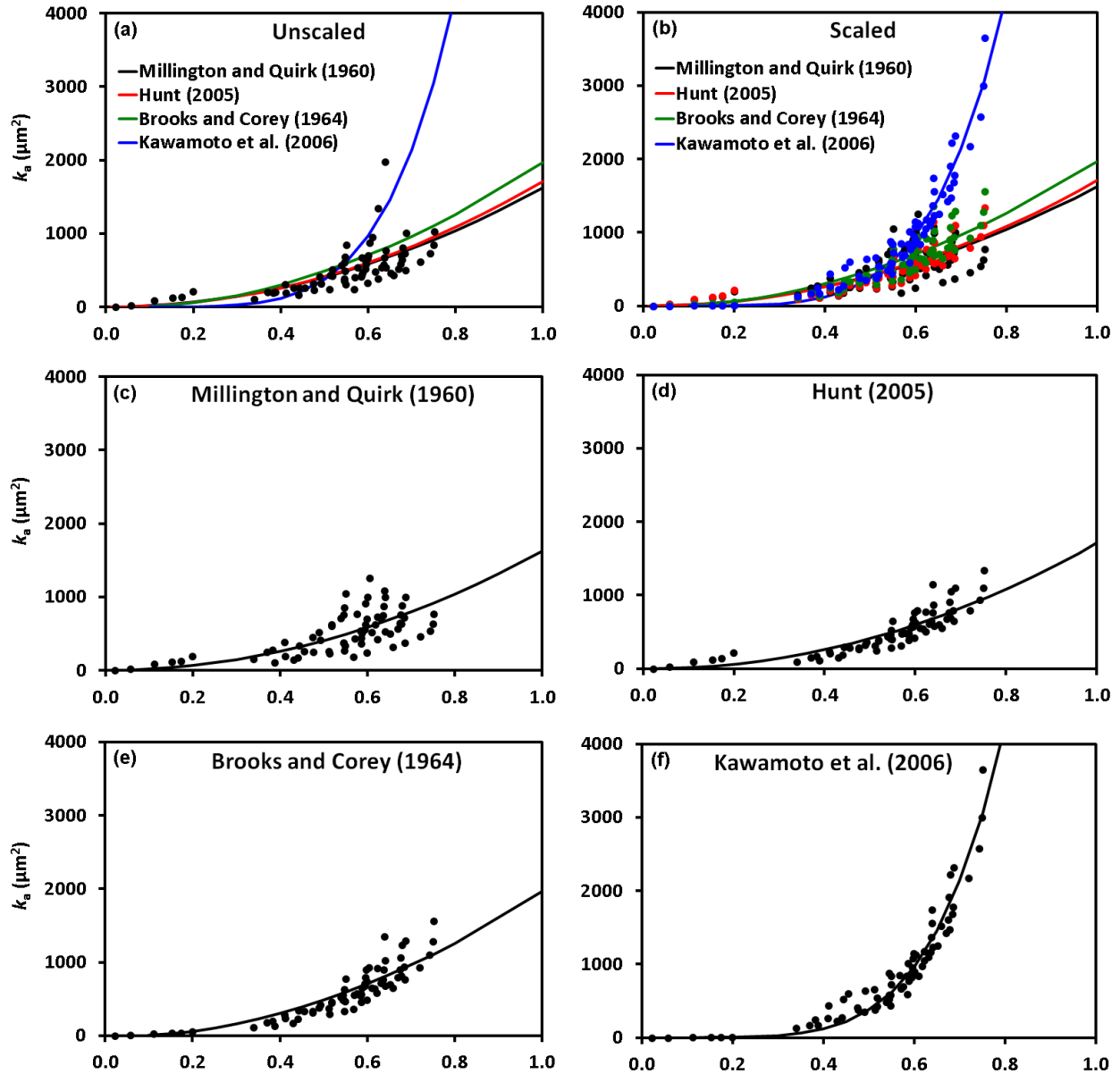
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551 Table 2 Fitting parameters for the reference curves of $k_{a,m}$ with the corresponding RMSE values
552 based on Dataset II

553 Table 1 Summaries of the physical properties of sediment and soil samples from Dataset I and
 554 Dataset II

	Dataset I	Dataset II
Texture *	Sand	Sandy Loam
Porosity	0.30	0.52
Gravel (>2mm)	10.9	-
Coarse Sand (2-0.5 mm)	53.0	-
Medium and Fine Sand (0.5-0.05 mm)	36.0	-
Silt and Clay (<0.05 mm)	0.1	-
Sand (2-0.05mm)	-	19.0
Silt (0.002-0.05mm)	-	23.0
Clay (<0.002mm)	-	58.0

555 * USDA classification

556

557 Table 2 Fitting parameters for the reference curves of $k_{a,m}$ with the corresponding RMSE values
 558 based on Dataset I

Air Permeability Model	k_o (μm^2)	$S_{a,t}$ (-)	γ (-)	η (-)	RMSE	
					Unscaled k_a	Scaled k_a
Millington and Quirk (1960)	92.5	-	-	-	28.4	8.9
Hunt (2005)	103.6	0.168	-	-	27.4	8.0
Brooks and Corey (1964)	100.6	-	3.067	-	33.0	7.8
Kawamoto et al. (2006)	106.0	-	-	1.893	27.4	7.9

559

560

561 Table 3 Fitting parameters for the reference curves of $k_{a,m}$ with the corresponding RMSE values
 562 based on Dataset II

Air Permeability Model	k_o (μm^2)	$S_{a,t}$ (-)	γ (-)	η (-)	RMSE	
					Unscaled k_a	Scaled k_a
Millington and Quirk (1960)	1625.3	-	-	-	228.7	217.3
Hunt (2005)	1709.4	0.02	-	-	229.5	135.0
Brooks and Corey (1964)	1966.2	-	0.378*	-	257.3	156.7
Kawamoto et al. (2006)	13390.6	-	-	0.724	719.7	188.7

563 * Taken from Rawls et al. (1982)