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## Abstract

Soil air permeability ($k_a$) is an important factor that controls subsurface gas transport and exchange of gas across the soil-atmosphere interface. It is thus crucial to evaluate the spatial distribution of $k_a$ for both application and modeling purposes. However, relevant studies are still very limited, partly due to the fact that the dependence of $k_a$ on soil moisture levels cannot be directly included in the methods such as geostatistical techniques for analyzing the spatial distribution of $k_a$. To tackle this problem, the scaling scheme based on the similar media concept, which has been widely used in soil hydrology for characterizing spatial variability of soil hydraulic properties, was employed for scaling $k_a$ in this study. Four air permeability models, including Millington and Quirk (1960)-MQ, Hunt (2005)-HT, Brooks and Corey (1964)-BC, and Kawamoto et al. (2006)-KA, were selected to test this method using two independent datasets. For the first dataset that included $k_a$ measured for river sediments, all of the four models were able to delineate the spatial distribution of $k_a$ with a reference curve of $k_a$ and a set of scaling factors. Specifically, the MQ model gave the least satisfactory results due to the less flexibility of its form; whereas, there were no significant differences in the performances for the HT, BC, and KA models. For the second dataset that contained $k_a$ measured for agricultural soils, the overall performance of the four models for scaling $k_a$ deteriorated, largely due to the alterations in the microscopic structures of soil samples caused by repacking and compression of soil samples. Nonetheless, as the first attempt, this study shows the viability of using the similar media concept for scaling $k_a$. The merit of this method resides in the fact that the spatial variations of moisture conditions and soil properties can be simultaneously included for analyzing the spatial distribution of $k_a$. With a reference curve of $k_a$ and the distribution of scaling factors, this method would be particularly suitable for modeling subsurface gas transport.



***Keywords*:** Similar Media Concept, Scaling Factor, Soil Air Permeability, Air Permeability Model

## 1. Introduction

As one of the key factors that control subsurface gas transport and exchange of gas across the soil-atmosphere interface, soil air permeability ($k_a$) describes the ability of a soil to transmit gas. The rising interest in $k_a$ is manifested by its broad range of applications in various fields, including greenhouse gas emission (Ball et al., 1997; Conen et al., 1998), landfills (Jain et al., 2005; Wu et al., 2012), soil vapor extraction systems (Poulsen et la., 1998; Farhan et al., 2001), crop growth (Lipiec and Hatano, 2003; Barrios et al., 2005). Moreover, due to easy operation and cost effectiveness, $k_a$ measured near field capacity has been used to predict soil saturated hydraulic conductivity (Loll et al., 1999; Iversen et al., 2001).

Under natural conditions, $k_a$ is affected by a number of soil factors (e.g., air-filled porosity and pore size distribution) and moisture conditions, all of which show various degrees of spatial variations. As such, field measured $k_a$ exhibits significant spatial variability (Iversen et al., 2003). For application and modeling purposes, it is thus crucial to evaluate the spatial distribution of $k_a$ and its controlling factors. However, compared to relevant researches on soil hydraulic properties, only few studies are available on the spatial distribution of $k_a$, which nonetheless provided valuable insights into the understanding of the spatial pattern of $k_a$ (Poulsen et al., 2001; Iversen et al., 2003, 2004). Geostatistical techniques were mainly used in previous studies to analyze the spatial pattern of $k_a$. Based on the results of variograms, Iversen et al. (2003) found that the range of $k_a$ for sandy soils was larger than the one for a loamy soil, probably due to the difference in the depositional processes of those two types of soils. Although geostatistical techniques have been proven to be powerful tools for investigating naturally occurred phenomena, there are certain shortcomings for those techniques as pointed out by Henley (2001). Most notably, the underlying processes associated with studied targets cannot be explicitly considered in geostatistical techniques, which rather rely on statistical models for examining the spatial correlations of the targets.

With respect to $k_a$, the main issue of using geostatistical techniques stems from the impact of soil moisture on $k_a$. It has been well known that $k_a$ is highly dependent on soil moisture levels, but the spatial distribution of soil moisture cannot be directly included in the geostatistical analyses of $k_a$. To some degree, this is analogous to study the spatial distribution of soil unsaturated hydraulic conductivity without specifying moisture conditions. Therefore,

precautions were usually taken in previous field studies on the spatial distribution of $k_a$. As conjectured by Iversen et al. (2003), when soil moisture contents reach field capacity, the air flow takes place in the majority of soil pores; thus, the impact of moisture on $k_a$ can be neglected with moisture contents near and below field capacity. Although the assumption made by Iversen et al. (2003) is useful for studying the spatial distribution of $k_a$ under dry conditions, it may fail at regions with wet climates or shallow groundwater tables. For application purposes (e.g., landfills and soil vapor extraction systems), the inclusion of moisture in analyzing the spatial distribution of $k_a$ is also inevitable. Therefore, it is desirable to seek alternative methods to assess the spatial distribution of $k_a$ under the influence of soil moisture.

Along the line of above thinking, the scaling scheme based on the similar media concept may provide a promising approach to investigating the spatial distribution of $k_a$ with the consideration of moisture conditions. First introduced by Miller and Miller (1956), the similar media concept is based on the assumption that the internal geometry of similar media only differs by microscopic length scales that can be characterized by scaling factors. The purpose of this scaling approach is to coalesce a range of functional relationships into a single curve through scaling factors that depict the spatial distribution of those functional relationships. More specifically, scaling factors are used to relate soil properties at a given location to the mean properties at a reference location, which are invariant of moisture conditions. This scaling method has been widely used in soil hydrology to characterize spatial variability of soil hydraulic properties and soil hydraulic functions with associated model parameters (Warrick et al., 1977; Hopmans, 1987; Shouse and Mohanty, 1998; Zavattaro et al., 1999; Hendrayanto et al., 2000; Tuli et al., 2001). With a reference functional relationship and the distribution of scaling factors, this method is particularly suitable for modeling purposes (Peck et al., 1977; Kabat et al., 1997; Salvucci, 1998; Oliveira et al., 2006). Given the similarities between water flow and gas transport in soils, one can expect the viability of applying this scaling approach for investigating the spatial distribution of $k_a$ under the influence of soil moisture.

To our knowledge, this research was the first attempt to extend the use of the similar media concept for scaling $k_a$. The main objective of this study was to examine the feasibility of this approach using two datasets collected from USA and France. Four air permeability models were selected to delineate the functional relationship between $k_a$ and saturation degree of air. The

results of this study demonstrated the feasibility of using the similar media concept for scaling $k_a$, which also opened the door for utilizing this method for simulating gas transport in soils.

## 2. Materials and Methods

### 2.1 Similar Media Concept in Soil Hydrology

The use of the similar media concept for scaling soil water retention and hydraulic conductivity curves is well documented in the literature (see the review by Vereecken et al., 2007). So, only a brief overview is given here. Based on the similar media concept, it is assumed that the microscopic structures (e.g., tortuosity, and relative particle size and pore size distributions) of similar soils are identical and only differ by microscopic length scales that can be characterized by scaling factors (Peck et al., 1977; Warrick et al., 1977). The scaling factor (α) is thus defined by the ratio of the microscopic characteristic length of a soil (λ) to the characteristic length of a reference soil ($\lambda_m$):

$$\alpha_i = \frac{\lambda_i}{\lambda_m} \tag{1}$$

where $i$=1, 2, …, $L$ is the location of the soil and the subscript $m$ denotes the reference soil. By its definition, α is invariant of soil moisture conditions and only dependent upon the location of the soil. As such, the soil water retention curve at any location can be scaled to the reference water retention curve through α:

$$h_i(\theta) = \frac{h_m(\theta)}{\alpha_{w,i}} \tag{2}$$

where $h$ is the soil matric potential, $\theta$ is the volumetric moisture content, and the subscript $w$ denotes water. The scaling relationship of the soil hydraulic conductivity curve can be written as:

$$K_{w,i}(\theta) = K_{w,m}(\theta)\ \alpha_{w,i}^2 \tag{3}$$

where $K_w$ is the hydraulic conductivity. Due to the fact that soil porosity may vary across locations, instead of $\theta$, the saturation degree of moisture ($S_w$) is usually used (Warrick et al., 1977):

$$S_w = \frac{\theta}{\phi} \quad (4)$$

where $\phi$ is the soil porosity.

To describe the reference curves of $h_m$ and $K_{w,m}$, soil water retention and hydraulic conductivity models (e.g., Brooks-Corey model and van Genuchten model) have been used, although polynomial functions of $h_m(S_w)$ and $K_{w,m}(S_w)$ were also adopted (Warrick et al., 1977). By optimizing the differences between measured and calculated $h(S_w)$ and $K_w(S_w)$ across all the locations, the model parameters for the reference curves of $h_m$ and $K_{w,m}$, and the scaling factor $\alpha_{a,i}$ at each location can be obtained (Hopmans, 1987; Tuli et al., 2001).

## 2.2 Extension of the Similar Media Concept for Scaling Soil Air Permeability

Although the scaling theory based on the similar media concept has been widely used in soil hydrology, there is a surprising lack of studies on its application for assessing the spatial distribution of $k_a$. Strictly speaking, soil water permeability ($k_w$) should be used in the scaling procedure (Eq. (3)), as regardless of fluid properties, soil permeability, whether it is $k_w$ or $k_a$, represents the intrinsic properties of soils to transmit fluids and thus reflects the microscopic structures of soils. However, given that the fluid of interest remains under the same conditions (e.g., temperature), the fluid properties do not change across locations and conductivities (e.g., $K_w$) can be used in Eq. (3). Therefore, one can write a similar scaling relationship for $k_a$ based on Eq. (3):

$$k_{a,i}(S_a) = k_{a,m}(S_a)\,\alpha_{a,i}^2 \quad (5)$$

where $S_a$=(1-$S_w$) is the saturation degree of air and the subscript *a* denotes air. For the same reason for using $S_w$ in scaling $h(S_w)$ and $K_w(S_w)$, $S_a$ was used in this study instead of the volumetric soil-air content. Note that $\alpha_a$ is also invariant of soil moisture conditions and only dependent upon the location of the soil.

To delineate the reference relationship of $k_{a,m}$, four air permeability models were selected (Ghanbarian-Alavijeh and Hunt, 2012). The first model was proposed by Millington and Quirk (1960) (denoted as MQ hereafter):

$$k_a(S_a) = k_o \times S_a^2 \quad (6)$$

where $k_o$ is the air permeability at the porosity $\phi$.

The second model was developed by Hunt (2005) (HT), which is based on the continuum percolation theory:

$$k_a(S_a) = k_o \left(\frac{S_a - S_{a,t}}{1 - S_{a,t}}\right)^2 \quad (7)$$

where $S_{a,t}$ is the percolation threshold used by Ghanbarian-Alavijeh and Hunt (2012). Note that the HT model can be reduced to Eq. (6) by assuming $S_{a,t}$=0.

The third model was based on the hydraulic conductivity function of Brooks and Corey (1964) (BC):

$$k_a(S_a) = k_o S_a^l \left[1 + (1 - S_a)^{1+\frac{2}{\gamma}}\right] \quad (8)$$

where γ is the pore size distribution index, and $l$ is the tortuosity-pore connectivity factor and assumed to be 2 based on the percolation theory (Hunt and Ewing, 2009).

The last model was taken from Kawamoto et al. (2006) (KA):

$$k_a(S_a) = k_o \times (S_a)^{1+3/\eta} \quad (9)$$

where (1+3/$\eta$) represents the combined effects of tortuosity and connectivity of air-filled pores. In Kawamoto et al. (2006), the measured $k_a$ at -100 cm $H_2O$ was used instead of $k_o$. Essentially, the MQ model is a simplified version of Eq. (9) with $\eta$=3.

In parallel to previous studies on scaling $h(S_w)$ and $K_w(S_w)$, the scaling factors $\alpha_{a,i}$ and the model parameters (given in Tables 2 and 3) for describing the reference curves of $k_{a,m}$ were obtained by optimizing the differences between measured and calculated $k_a$. The following objective function was used:

$$O = \sum_{i=1}^{L} \left\{ \sum_{j=1}^{M(i)} \left[ k_{a,i}^{obs}\left(S_{a,j}\right) - \alpha_{a,i}^2 \times k_{a,m}\left(S_{a,j}\right) \right]^2 \right\} \quad (10)$$

where $M(i)$ is the total number of observed $k_a$ at location $i$, $k_{a,i}^{obs}\left(S_{a,j}\right)$ is the $j$th observed $k_a$ corresponding to $S_{a,j}$ at location $i$, and $k_{a,m}(S_{a,j})$ is the calculated $k_a$ at $S_{a,j}$ from Eq. (6) to Eq.(9).

The generalized reduced gradient method was used to optimize Eq. (10) with the geometric mean of the scaling factors assumed to be unity (Hopmans, 1987; Zavattaro et al., 1999; Hendrayanto et al., 2000):

$$(\prod_{i=1}^{L} \alpha_i)^{1/L} = 1 \quad (11)$$

To evaluate the performance of each air permeability model, the root mean square error (RMSE) was used:

$$RMSE = \left(\frac{1}{n}\sum_{k=1}^{n}(o_k - e_k)^2\right)^{\frac{1}{2}} \quad (12)$$

where $n$ is the total number of observed $k_a$ across all the locations, $o_k$ is either observed $k_{a,i}(S_{a,j})$ or scaled $k_{a,i}(S_{a,j})$ (i.e., observed $k_{a,i}(S_{a,j})\times\alpha_{a,i}^2$), and $e_k$ is the corresponding $k_{a,m}(S_{a,j})$ calculated from the reference curve of $k_{a,m}$.

## 2.3 Datasets

Two independent datasets collected from USA and France were used in this study to test the feasibility of applying the similar media concept for scaling $k_a$. The first dataset (denoted as Dataset I) included measured air permeability data using sediment samples taken from the channel of the Platte River near Kearney, NE (Figure 1). The streambed sediments at the site are moderately-sorted, and mainly consist of sands and gravels with very low organic matter contents (Table 1). A total of 25 samples with a length of 20 cm for each sample were collected using transparent polycarbonate tubes with an inside diameter of 5.1 cm. The tubes were vertically pressed to the depth of 20 cm into the sediments. The distance between neighboring sampling locations was 1 m. After taking out the sample from the river channel, the bottom of the tube was wrapped by a figberglass screening to prevent sediments from falling off. A falling head permeability test was carried out on site to determine the saturated hydraulic conductivity of the sediments inside the tube (Chen, 2005). Water temperatures were also recorded during the test to calculate water dynamic viscosity, which was used to calculate $k_w$.

After the measurements of saturated hydraulic conductivities, the sediment samples were brought back to the laboratory and air dried at room temperatures. On Day 2 and every three

205 days afterwards until Day 17, air permeability tests were performed using a portable air
206 permeameter (Figure 2), which resulted in 6 pairs of data points of $k_a$ and $S_a$ for each tested
207 sample. To measure $k_a$, the polycarbonate tubes were connected to the air permeameter, which
208 mainly consisted of an air tank, an air pressure regulator, a rotameter that measured the flow rate
209 of air through the sediments, and a manometer that recorded the pressure difference across the
210 length of the sediments. Darcy's Law was then used to calculate $k_a$ (Kirkham, 1947). Due to the
211 slippage of gas molecules near the pore wall, the flow of gas in porous media may exhibit non-
212 Darcian flow that leads to higher $k_a$ (Klinkenberg, 1941). Therefore, additional tests were
213 performed to examine the applicability of Darcy's Law by measuring the pressure difference
214 across the sample length under varying flow rates. The linearity of the results proved the
215 applicability of Darcy's Law and confirmed the air permeameter was operational for measuring
216 $k_a$. The merit of the air permeameter used in this study is that it allows to measure both saturated
217 hydraulic conductivity and $k_a$ of the same sample without the need to remove the sediments from
218 the permeameter for repacking. Thus, it minimizes the possible disturbance of the sample
219 structure. After each air permeability test, samples were immediately weighed to determine
220 moisture contents and corresponding volumetric soil-air contents. On Day 17, all of the samples
221 were dried in the oven for 24 h at the temperature of 105 °C to determine bulk density and
222 porosity. The results of the grain size analyses showed that the texture of the sediments was sand
223 according to the USDA classification (Table 1).

224 The second dataset (denoted as Dataset II) was taken from Tang et al. (2011) and contained
225 nine remolded soil samples from an experimental forest site at Le Breuil located in central
226 France. The reason for choosing Dataset II was that several air permeability models, including
227 the HT model, were successfully tested on those soil samples (Tang et al., 2011; Ghanbarian-
228 Alavijeh and Hunt, 2012). However, unlike the air permeameter used in Dataset I, a specially
229 designed oedometer was adopted by Tang et al. (2011) to measure $k_a(S_a)$ by vertically
230 compressing soil samples, which led to varying moisture contents under different compressional
231 stresses. Except for one sample with 6 data points, there were 8 pairs of $k_a$ and $S_a$ for each
232 sample. The texture of the soils was sandy loam and their physical properties are reported in
233 Table 1.

234

## 3. Results and Discussions

### 3.1 Dataset I

It has been well known that besides soil properties, $k_a$ is also affected by moisture conditions and thus $S_a$. With the drying process, the mean value of $S_a$ for Dataset I increased from 0.49 on Day 2 to 0.87 on Day 17. Consequently, the mean value of $k_a$ was almost quadrupled from 21.0 $\mu m^2$ on Day 2 to 83.6 $\mu m^2$ on Day 17, as more smaller pores opened up for transmitting air when the sediments became drier. The results from Dataset I again illustrate the importance of including moisture conditions in analyzing the spatial distribution of $k_a$. To further demonstrate the impact of $S_a$ on $k_a$, the results from four sediment samples in Dataset I (numbered as #1, #2, #3, and #4) are plotted in Figure 3. As expected, $k_a$ increased with increasing $S_a$ or with decreasing moisture levels; however, the $k_a(S_a)$ relationship also differed among different samples. More specifically, $k_a$ showed similar ranges for #1 (13.7 to 41.5 $\mu m^2$) and #2 (13.2 to 44.5 $\mu m^2$) with $S_a$ ranging from 0.40 to 0.81 and 0.56 to 0.86, respectively (Figures 3a and 3b). In comparison, $k_a$ for #3 (11.3 to 142.3 $\mu m^2$) and #4 (4.5 to 111.5 $\mu m^2$) varied over an order of magnitude with $S_a$ increasing from 0.46 to 0.90 and 0.42 to 0.89, respectively (Figures 3c and 3d). Clearly, a single curve of $k_a(S_a)$ is not sufficient to describe spatially distributed $k_a$, even at plot scales (e.g., a reach scale in this study). This presents a great challenge for simulating subsurface gas transport with spatially varied $k_a$ and associated model parameters. The same challenge is also faced by soil hydrologists to simulate water movement in vadose zone, where soil hydraulic properties may vary substantially across landscapes. One of the solutions to tackle this problem in soil hydrology relies on the use of scaling factors within the framework of the similar media concept (Kabat et al., 1997; Salvucci, 1998; Oliveira et al., 2006).

To have a thorough view of the effectiveness of using the similar media concept for scaling $k_a$, the results of unscaled and scaled $k_a$ from Dataset I are plotted in Figure 4 with the reference curves of $k_{a,m}$ for all of the air permeability models. For the purpose of clarity, the results of scaled $k_a$ for each model are also plotted separately in Figure 4. The scaled $k_a$ was calculated using the obtained scaling factors (e.g., Eq. (5)). The obtained fitting parameters and RMSE values for each model are given in Table 2. Note that the results of two sediment samples were removed from the analyses, as they exhibited irregular patterns between $k_a$ and $S_a$ (e.g., $k_a$ may decrease with increasing $S_a$). Those irregular patterns were suspected to be the consequence of

the collapse of root holes in the samples during the drying process. As expected, a positive trend existed between unscaled $k_a$ and $S_a$. When $S_a$ was lower than 0.7, unscaled $k_a$ largely remained below 60 μm$^2$, while unscaled $k_a$ varied about 8 times from approximately 20 to 160 μm$^2$ when $S_a$ reached above 0.8. Figure 5a further shows that a single curve of $k_a(S_a)$ is not sufficient to delineate the distribution of the unscaled $k_a$ as attested by the $k_{a,m}$ curves. By comparison, with the aid of scaling factors, the scaled $k_a$ coalesced along the $k_{a,m}$ curves with greatly reduced scatter for all of the four air permeability models. Note that scaled $k_a$ was systematically more scattered at the lower range of $S_a$ for all of the models. It was due to the fact that more weights were given to higher values of $k_a$ in Eq. (10) during the optimization process. If $k_a$ was replaced by log $k_a$ in Eq. (10), the scatter of scaled $k_a$ at the lower range of $S_a$ could be reduced; however, the overall performance of the models deteriorated. Compared to the other three $k_{a,m}$ curves, the one for the HT model slightly deviated; whereas, the resulting scaled $k_a$ was comparable for all of the four models (Figure 4b). Among all of the results, the RMSE value of the scaled $k_a$ for the MQ model was highest with $k_{a,m}$ overestimating scaled $k_a$ at the lower end of $S_a$ and underestimating scaled $k_a$ at the higher end of $S_a$ (Figure 4c). This was because without additional fitting parameters other than $k_o$, the form of the MQ model was the least flexible. By contrast, the performances of the other three $k_{a,m}$ curves improved at both ends of $S_a$ with the BC model showing the most satisfactory results (Figures 4d-4f); however, there appeared no significant differences in the performances of those three air permeability models for scaling $k_a$ (Table 2), indicating the applicability of those models for delineating the distribution of $k_a$ in the studied sediments.

The values of obtained $k_o$ varied among the four models (Table 2). The fitted $k_o$ in the KA model was highest with the value of 106.0 μm$^2$, while the one in the MQ model was only 92.5 μm$^2$. As a result, the scaled $k_a$ for the MQ model was lowest under the same level of $S_a$; however, the differences in the scaled $k_a$ were minimum among different models. The value of the obtained percolation threshold $S_{a,t}$ in the HT model was 0.168, which was comparable to the 0.1 threshold proposed by Ewing and Hunt (2006). In addition, the pore size distribution index γ (γ =3.067 in this study) in the BC model fell within the range for sandy soils (Ghanbarian-Alavijeh and Hunt, 2012). In summary, Figure 4 shows that for the studied sediments, the spatially distributed $k_a$ could be described by a reference curve of $k_{a,m}$ and a set of scaling factors, which attests the feasibility of using the similar media concept for scaling $k_a$ under the influence of

moisture. With a known reference curve of $k_{a,m}$ and the distribution of scaling factors, this method would be particularly suitable for modeling subsurface gas transport.

As previously explained, the spatially distributed $k_a$ is mostly controlled by moisture conditions and soil properties, both of which may vary at different spatiotemporal scales. Essentially, the impacts of moisture levels and $S_a$ on $k_a$ can be determined through the reference curve of $k_{a,m}$; whereas, the spatial variations of soil properties are embedded in the distribution of scaling factors, as scaling factors only depend on soil intrinsic properties (i.e., the microscopic characteristic length). The probability plots of the scaling factors (both $\alpha_{a,i}$ and $\ln\alpha_{a,i}$) are given in Figure 5 for the HT model. The resulting probability plots were similar for all of the four models, so only the ones for the HT model are presented here. The respective mean and standard deviation was 1.03 and 0.23 for $\alpha_{a,i}$, and 0 and 0.23 for $\ln\alpha_{a,i}$. The values of the standard deviations for $\alpha_{a,i}$ and $\ln\alpha_{a,i}$ were smaller than previously reported values derived from soil water retention and hydraulic conductivity data (Warrick et al., 1977; Hopmans, 1987; Hendrayanto et al., 2000), probably due to the smaller number of sediment samples used in this study. Although those previous studies showed a log normal distribution of scaling factors, the Kolmogorov-Smirnov Test did not reject the null hypothesis that $\alpha_{a,i}$ and $\ln\alpha_{a,i}$ were normally distributed at the significance level of 0.01. Thus, future studies are needed to include a larger number of samples, and to compare the distributions of scaling factors drawn separately from soil water retention and hydraulic conductivity data and from air permeability data.

To test the applicability of the derived reference curves of $k_{a,m}$, the values of $k_a$ on Day 17 were calculated using the obtained scaling factors and measured $S_a$ based on Eq. (5). As hypothesized by Iversen et al. (2003), when soil moisture contents reach field capacity, the air flow takes place in the majority of soil pores; thus, $k_a$ measured at or near field capacity can be used as a proxy of soil water permeability $k_w$ to predict saturated hydraulic conductivity. On Day 17, the average value of soil moisture contents for all of the sediment samples was 0.04, which was quite comparable to the observed field capacity of sandy soils (soil moisture data can be accessed from the High Plains Regional Climate Center at http://www.hprcc.unl.edu/awdn/soilm/). Based on the above reasoning, the calculated $k_a$ on Day 17 is plotted against $k_w$ derived from measured saturated hydraulic conductivities in Figure 6. Again, only the calculated $k_a$ for the HT model is shown here as a demonstration. It can be seen

from Figure 6 that the calculated $k_a$ was very close to $k_w$. On average, the calculated $k_a$ only underestimated $k_w$ by 11%. Figure 6 further corroborates the feasibility of using scaling factors within the similar media framework for describing spatially distributed $k_a$.

### 3.2 Dataset II

The previous section has demonstrated the feasibility of using the similar media concept for scaling $k_a$ in river sediments. To assess the viability of this method in agricultural soils, analyses were carried out for Dataset II. A mean value of 0.02 for $S_{a,t}$ in the HT model was obtained by Ghanbarian-Alavijeh and Hunt (2012) by fitting individual curves of $k_a$ to soil samples in Dataset II. Thus, to avoid the negative values of ($S_a$ -$S_{a,t}$) in Eq. (7), $S_{a,t}$ was fixed to be 0.02 for Dataset II. In addition, an unusually high value of $\gamma$ in the BC model was obtained from the optimization, which was caused by fitting the $k_{a,m}$ curve to comparatively low values of $k_a$ from one soil sample (the data points with $S_a$ between 0 and 0.2 shown in Figure 7a). Moreover, no physical constraints for the fitting parameters were considered in the optimization process and $k_a(S_a)$ in Eq. (8) became insensitive to $\gamma$ for large $\gamma$ values. Thus, instead of using this high value of $\gamma$, the class-averaged value of $\gamma$ for sandy loam was used, which was derived from soil water retention data (Rawls et al., 1982).

The unscaled and scaled $k_a$ from Dataset II are presented in Figure 7, and the obtained fitting parameters and RMSE values are given in Table 3. Overall, the unscaled $k_a$ from Dataset II exhibited a much wider distribution from 0.5 to 1980.8 $\mu m^2$. Some of the remolded samples (e.g., the one with the maximum $k_a$ of 1980.8 $\mu m^2$) also showed different ranges of $k_a$ (Figure 7a), which was probably caused by the repacking processes. As expected, the improvement of the scaled $k_a$ from the MQ model was least satisfactory (Figure 7c). By comparison, the performances of the HT and BC models were considerably improved; however, the obtained $k_{a,m}$ tended to overestimate most of the scaled $k_a$ (Figures 7d and 7e), owing to the fact that the fitting parameter (or shape factor) $S_{a,t}$ in the HT model and $\gamma$ in the BC model were fixed during the optimizations. Without the need to fix the fitting parameter $\eta$ in the KA model, the $k_{a,m}$ curve seemed to more reasonably delineate the distribution of the scaled $k_a$ over the entire range of $S_a$ (Figures 7f), although it should be noted that a very high value of $k_o$ was obtained for the KA

355 model in order to match the data points from the sample with the maximum $k_a$ of 1980.8 μm$^2$.
356 Using the same remolded samples from Dataset II, Ghanbarian-Alavijeh and Hunt (2012)
357 showed that the HT model was better than the other three models (e.g., MQ, BC, and KA) to
358 describe the $k_a(S_a)$ relationship for individual samples; however, no significant improvement of
359 the HT model was found in this study for scaling $k_a$. One of the possible reasons for the different
360 conclusions about the model performances was due to the differences in the optimization
361 procedures. In Ghanbarian-Alavijeh and Hunt (2012), the $k_a(S_a)$ curves were optimized for
362 individual samples and some of the fitting parameters (e.g., γ in the BC model) were obtained
363 through optimizing soil retention data rather than soil permeability data.

364 In comparison to the results from Dataset I, the overall performance of the four selected air
365 permeability models for scaling $k_a$ was less satisfactory for Dataset II, which can be attributed to
366 two reasons. First, for the soil samples in Dataset II, soil structures might have been altered
367 during the repacking processes. Secondly, in order to achieve different moisture levels, soil
368 samples were compressed using an oedometer (Tang et al., 2011), which unavoidably changed
369 the microscopic structures of soils. The repacking processes and the compression of soil samples
370 might have led to variations in the microscopic structures among different soil samples and thus
371 the less satisfactory performance of using the similar media concept for scaling $k_a$. Nonetheless,
372 Figure 7 shows that the use of scaling factors was able to narrow down the scatter of $k_a$ around
373 the $k_{a,m}$ curves for the HT, BC, and KA models. In terms of the distribution of scaling factors
374 from Dataset II, the respective mean and standard deviation for the HT model was 1.01 and 0.14
375 for $\alpha_{a,i}$, and 0 and 0.13 for $\ln\alpha_{a,i}$. Again, the values of the standard deviations for $\alpha_{a,i}$ and $\ln\alpha_{a,i}$
376 were smaller than the previously reported values derived from soil water retention and hydraulic
377 conductivity data (Warrick et al., 1977; Hopmans, 1987; Hendrayanto et al., 2000).

378

379 **4. Conclusions**

380 The use of the scaling factors based on the similar media concept was tested for scaling air
381 permeability $k_a$ using two independent datasets in this study. With a reference curve of $k_{a,m}$ and a
382 set of scaling factors, this method was shown to be able to delineate the spatial distribution of $k_a$
383 for the first dataset, which included $k_a$ measured under different moisture conditions by drying
384 river sediment samples. For the second dataset that contained $k_a$ measured for agricultural soils,

however, the use of the similar media concept for scaling $k_a$ was less successful. It was most likely due to the alterations in the microscopic structures of soil samples caused by repacking and compression of soil samples. The merit of this method resides in the fact that the spatial variations of moisture conditions and soil properties can be simultaneously included for analyzing the spatial distribution of $k_a$. At any given location, the impact of $S_a$ and thus moisture levels on $k_a$ can be quantified by the reference curve of $k_{a,m}$; whereas, soil properties are embedded in the scaling factors. Since this study was the first attempt to apply the similar media concept for scaling $k_a$, more studies are needed to test this method on soils with different textures and to examine the impacting factors that control the distribution of scaling factors. It would be also useful to compare scaling factors derived independently from air permeability data, and from soil water retention and hydraulic conductivity data, which may further elucidate the physical meanings of those scaling factors. Future studies are also needed to incorporate this method in modeling subsurface gas transport.

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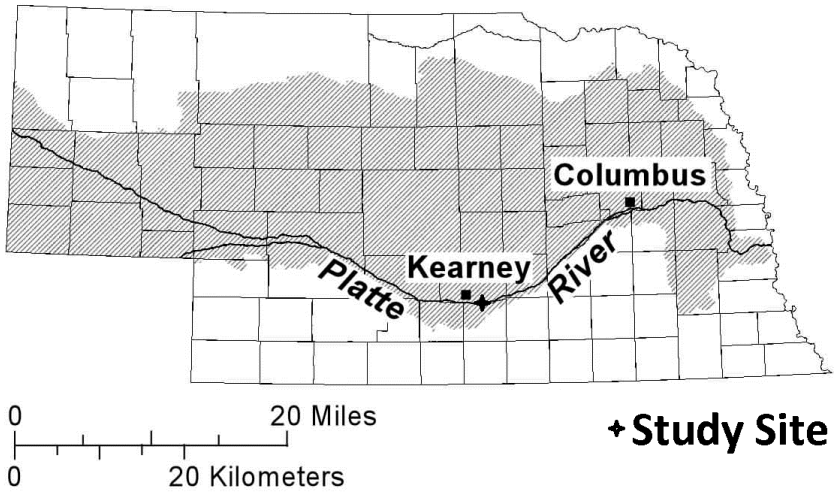
495 dependence of near-saturated hydraulic conductivity. *Soil Science Society of America*
496 *Journal* **63**: 486–492.

## List of Figures

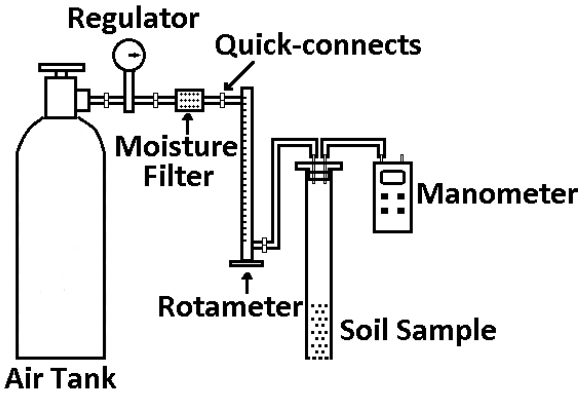



517

518 Figure 1 The sampling location for Dataset I near Kearney, NE

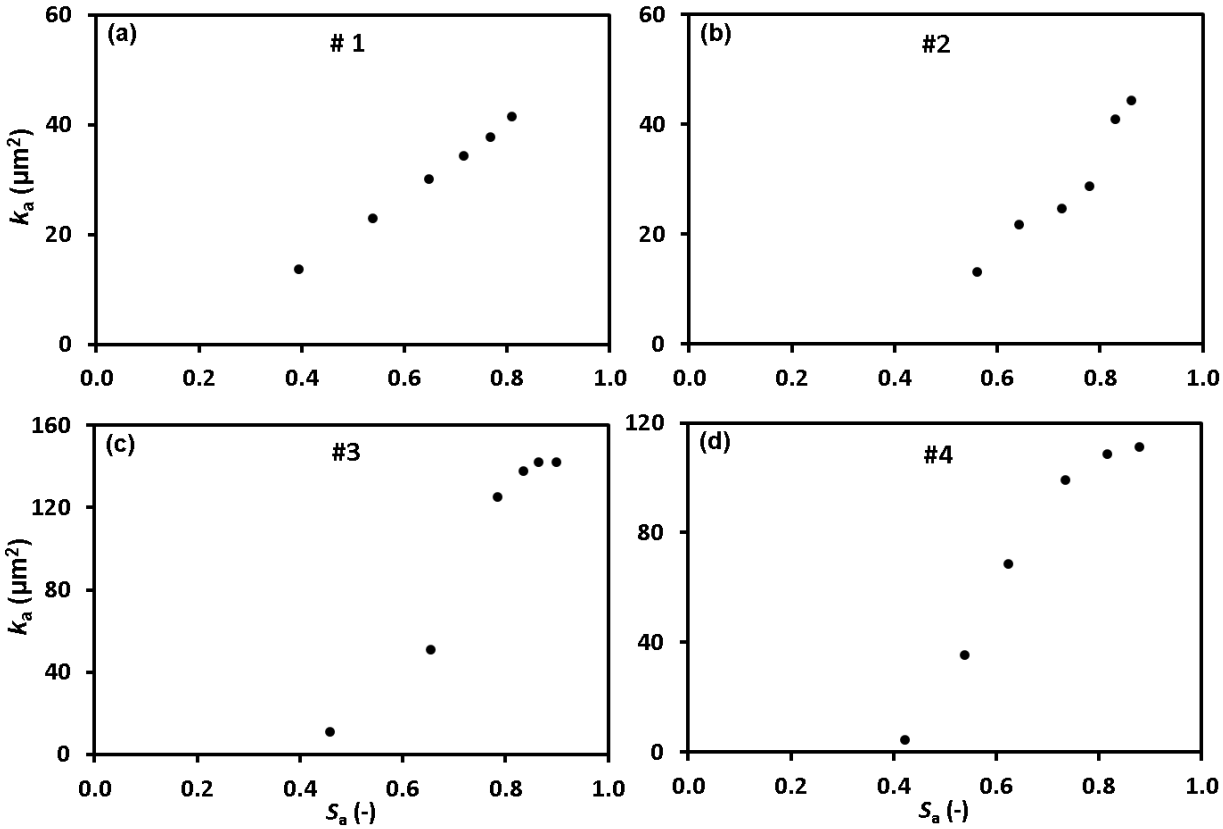
519

520




521

522 Figure 2 Schematic of the air permeameter used for measuring air permeability in Dataset I




523

524 Figure 3 Examples of the relationship between saturation degree of air ($S_a$) and air permeability
525 ($k_a$) from Dataset I

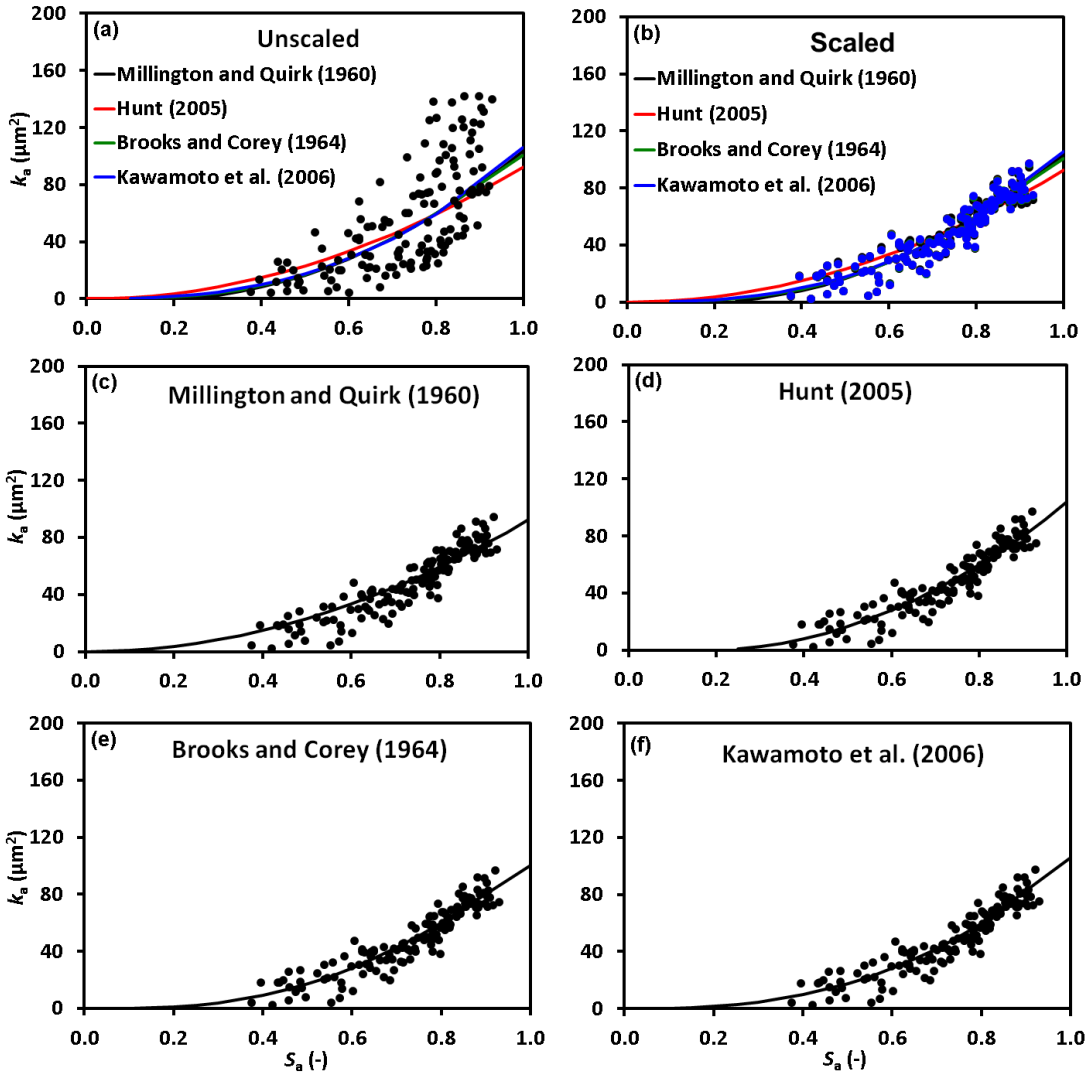



Figure 4 Unscaled and scaled air permeability ($k_a$) with the reference curves of $k_{a,m}$ for the selected air permeability models from Dataset I: (a) unscaled $k_a$, (b) scaled $k_a$ for all of the four air permeability models, (c) scaled $k_a$ for the Millington and Quirk (1960) model, (d) scaled $k_a$ for the Hunt (2005) model, (e) scaled $k_a$ for the Brooks and Corey (1964) model, and (f) scaled $k_a$ for the Kawamoto et al. (2006) model

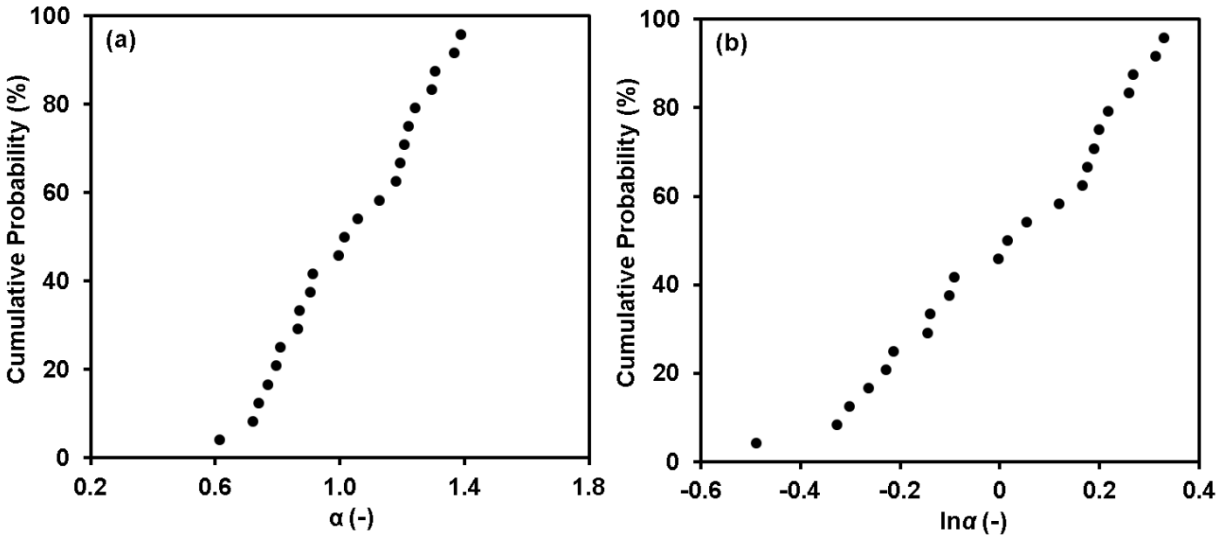



Figure 5 Probability plots for the scaling factors ($\alpha_a$) obtained from the Hunt (2005) model based on Dataset I

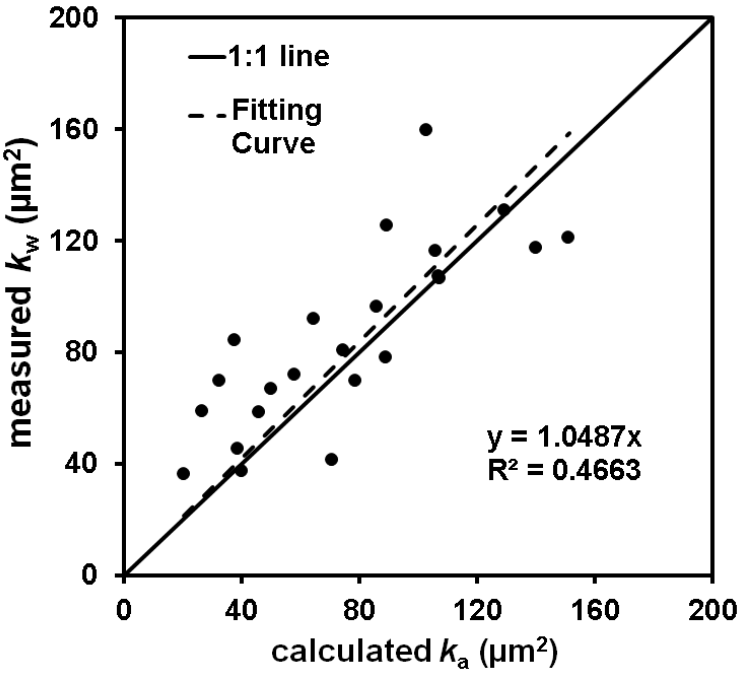



Figure 6 Comparison of calculated air permeability ($k_a$) from the Hunt (2005) model and measured water permeability ($k_w$) based on Dataset I. The measured saturation degree of air on Day 17 was used to calculate $k_a$.

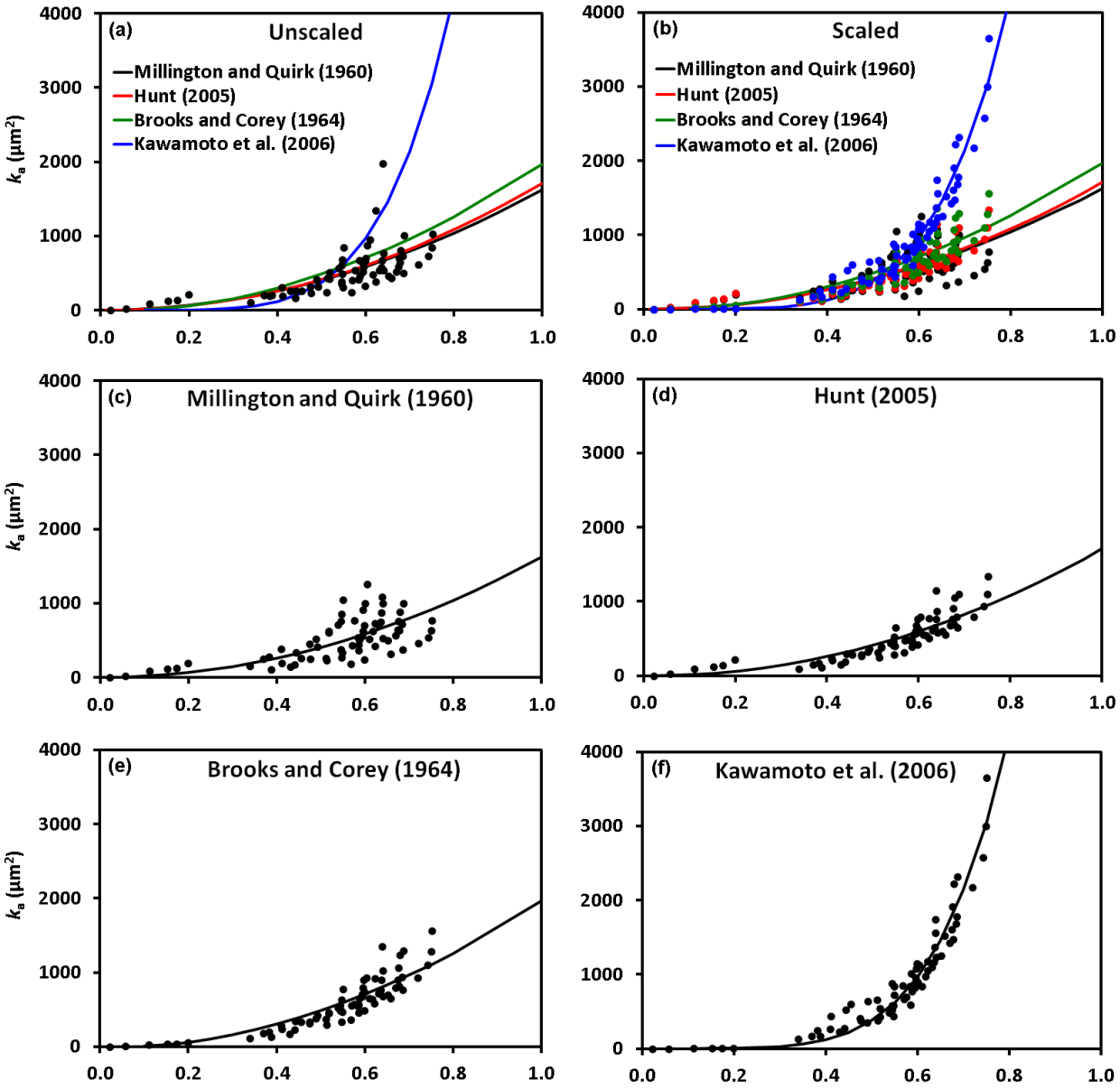



Figure 7 Unscaled and scaled air permeability ($k_a$) with the reference curves of $k_{a,m}$ for the selected air permeability models from Dataset II: (a) unscaled $k_a$, (b) scaled $k_a$ for all of the four air permeability models, (c) scaled $k_a$ for the Millington and Quirk (1960) model, (d) scaled $k_a$ for the Hunt (2005) model, (e) scaled $k_a$ for the Brooks and Corey (1964) model, and (f) scaled $k_a$ for the Kawamoto et al. (2006) model

## List of Tables

553 Table 1 Summaries of the physical properties of sediment and soil samples from Dataset I and
554 Dataset II

| | Dataset I | Dataset II |
|---|---|---|
| Texture* | Sand | Sandy Loam |
| Porosity | 0.30 | 0.52 |
| Gravel (>2mm) | 10.9 | - |
| Coarse Sand (2-0.5 mm) | 53.0 | - |
| Medium and Fine Sand (0.5-0.05 mm) | 36.0 | - |
| Silt and Clay (<0.05 mm) | 0.1 | - |
| Sand (2-0.05mm) | - | 19.0 |
| Silt (0.002-0.05mm) | - | 23.0 |
| Clay (<0.002mm) | - | 58.0 |

555 * USDA classification

556

557 Table 2 Fitting parameters for the reference curves of $k_{a,m}$ with the corresponding RMSE values
558 based on Dataset I

| Air Permeability Model | $k_o$ (μm$^2$) | $S_{a,t}$ (-) | $\gamma$ (-) | $\eta$ (-) | RMSE | |
|---|---|---|---|---|---|---|
| | | | | | Unscaled ka | Scaled ka |
| Millington and Quirk (1960) | 92.5 | - | - | - | 28.4 | 8.9 |
| Hunt (2005) | 103.6 | 0.168 | - | - | 27.4 | 8.0 |
| Brooks and Corey (1964) | 100.6 | - | 3.067 | - | 33.0 | 7.8 |
| Kawamoto et al. (2006) | 106.0 | - | - | 1.893 | 27.4 | 7.9 |

559

560

561 Table 3 Fitting parameters for the reference curves of $k_{a,m}$ with the corresponding RMSE values
562 based on Dataset II

| Air Permeability Model | $k_o$ (μm$^2$) | $S_{a,t}$ (-) | $\gamma$ (-) | $\eta$ (-) | RMSE | |
|---|---|---|---|---|---|---|
| | | | | | Unscaled ka | Scaled ka |
| Millington and Quirk (1960) | 1625.3 | - | - | - | 228.7 | 217.3 |
| Hunt (2005) | 1709.4 | 0.02 | - | - | 229.5 | 135.0 |
| Brooks and Corey (1964) | 1966.2 | - | 0.378* | - | 257.3 | 156.7 |
| Kawamoto et al. (2006) | 13390.6 | - | - | 0.724 | 719.7 | 188.7 |

563 * Taken from Rawls et al. (1982)