Thick periodic plates homogenization, application to sandwich panels including chevron folded core

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Shear forces effects in heterogeneous plates?

- Raises many difficulties in laminated plates:
  - wrong shear deflection (transverse shear correction factors in laminates)
  - no accurate estimate of local stress generated (“free faces“ effect)
- Almost no simple method when the plate is periodic

$\Rightarrow$ apply a homogenization scheme derived from a new plate theory (Lebée and Sab, 2010) to a sandwich panel including a folded core

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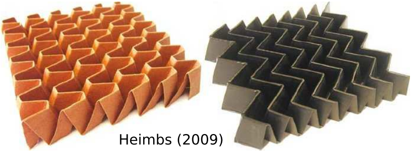
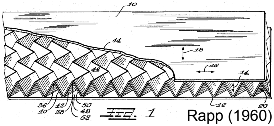
Folded Cores for sandwich panels


Heimbs (2009)


Rapp (1960)

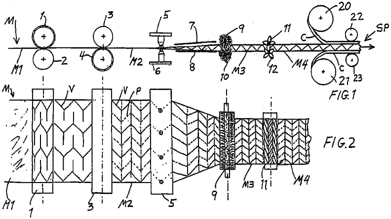
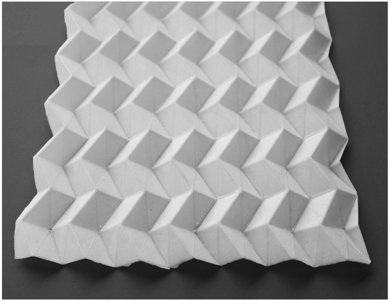


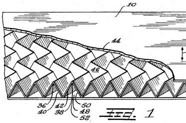
FIG.1
FIG.2

A cheaper substitute to honeycomb:

- Many possible materials (if foldable...: paper, metals...)
- Continuous process (Kehrle, 2004)

Folded Cores for sandwich panels





Rapp (1960)

A cheaper substitute to honeycomb:

- Many possible materials (if foldable...: paper, metals...)
- Continuous process (Kehrle, 2004)

$\Rightarrow$ the chevron pattern is investigated: assessment?

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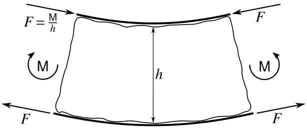
Sandwich panel simplified model

- Relevant loadings
  - Skins involved in bending:
    - traction/compression of the skins



Sandwich panel simplified model

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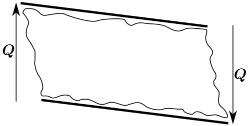
  - Core involved with shear forces:
    quantity of interest



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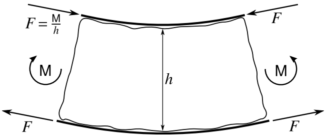
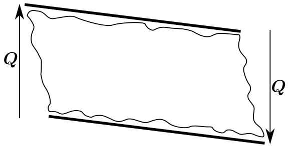
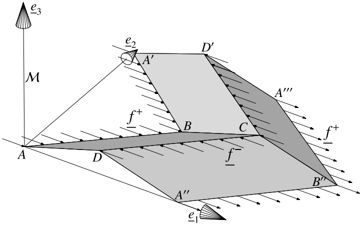


- Implicit contrast assumption:
  "skins are stiff compared to the core"



Sandwich panel simplified model

- Relevant loadings
  - Skins involved in bending:
    - traction/compression of the skins
  - Core involved with shear forces: quantity of interest
- Implicit contrast assumption:
  "skins are stiff compared to the core"
- Bounds from Kelsey et al. (1958) for shear forces stiffness
  - Apply uniform stress/displacement on a unit cell of the core, replacing skins.



Navier

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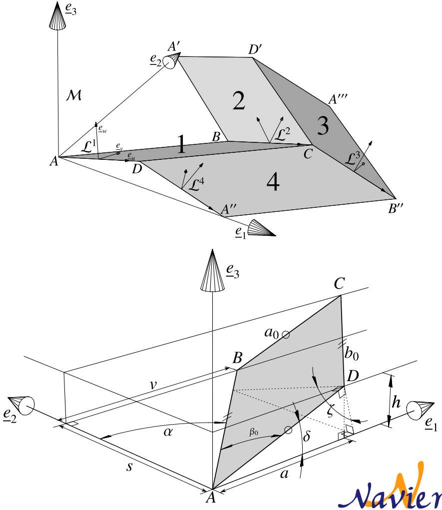
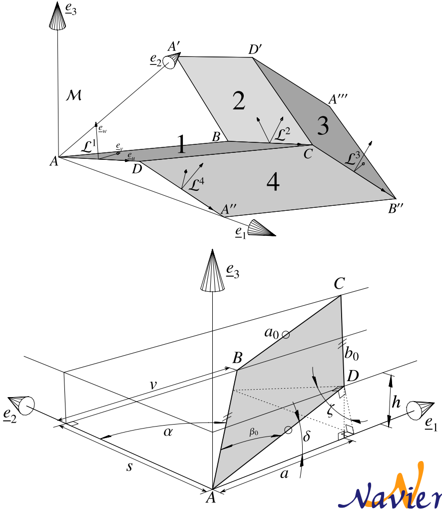
Closed-form bounds for shear forces stiffness

- Parameters (pattern reduced to a tilted parallelogram):
  - Facet thickness t_f
  - Shape ratio a_0/b_0
  - 2 orientation angles δ et ζ



Closed-form bounds for shear forces stiffness

- Parameters (pattern reduced to a tilted parallelogram):
  - Facet thickness t_f
  - Shape ratio a_0/b_0
  - 2 orientation angles δ et ζ
- Assumptions:
  - Plane-stress in facets
  - Piecewise uniform fields



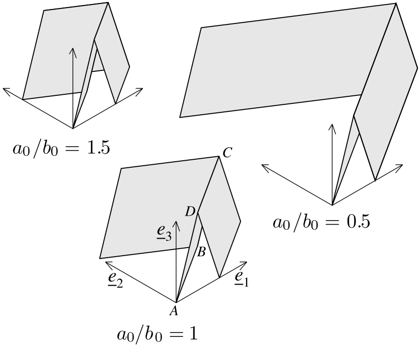
Closed-form bounds for shear forces stiffness

Results

- Manufactured cores (Nguyen et al., 2005):
  - $\delta = 72^\circ$ and $\zeta = 34^\circ$
  - Shape ratio: $0.5 < a_0/b_0 < 1.5$
- Normalized bounds $\mathcal{E} = F_{11}/\rho G_m h$:
  - $0.23 < \mathcal{E} < 0.71$: **very loose**

    $0 < \mathcal{E} < 1$

    $\rho G_m h$: Voigt upper bound



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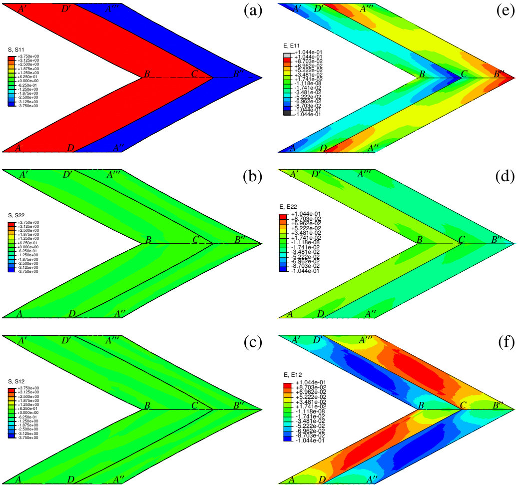
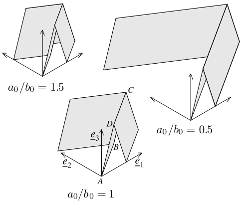
Finite Element bounds for shear forces stiffness

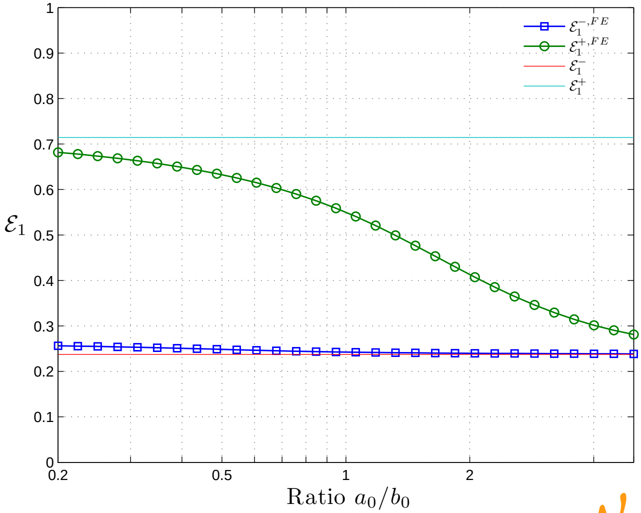
- Shell elements



Finite Element bounds for shear forces stiffness

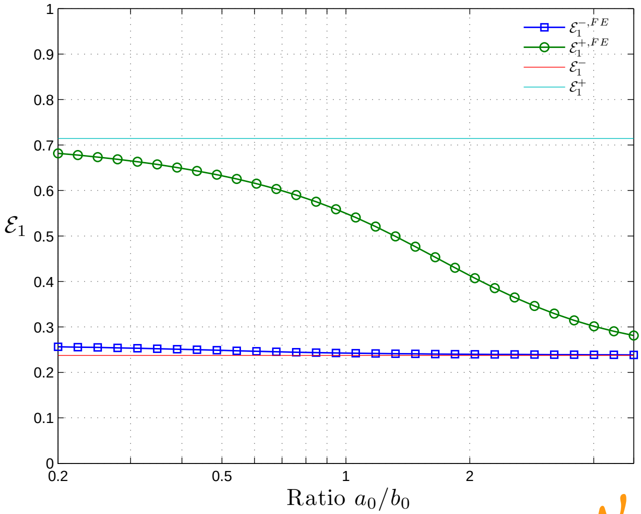
- Shell elements
- Consistent with analytical bounds

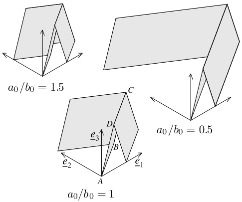




Finite Element bounds for shear forces stiffness

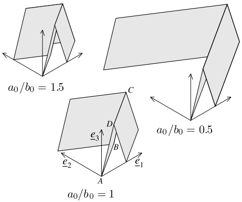
- Shell elements
- Consistent with analytical bounds
- **Still loose**: more discrepancy than for honeycombs

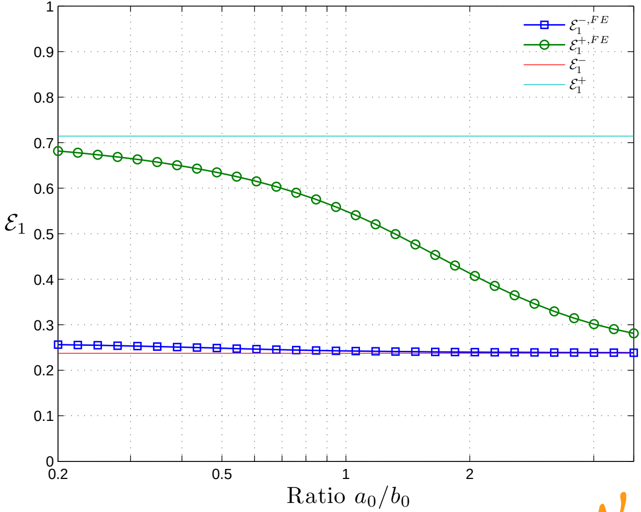




Finite Element bounds for shear forces stiffness

- Shell elements
- Consistent with analytical bounds
- **Still loose**: more discrepancy than for honeycombs
- Engineers often refer to the upper bound (rigid skins)





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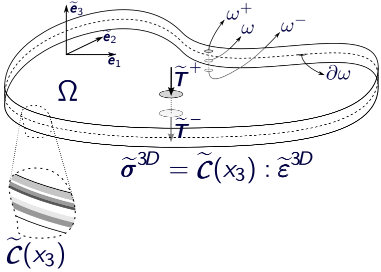
The 3D problem

$$\begin{cases} \sigma_{ij,j} = 0 \quad \text{on } \Omega. \\ \sigma_{ij} = C_{ijkl}(x_3)\varepsilon_{kl} \quad \text{on } \Omega. \\ \sigma_{i3} = T_i^{\pm} \quad \text{on } \omega^{\pm}. \\ \varepsilon_{ij} = \dfrac{1}{2}\left(u_{i,j} + u_{j,i}\right) \quad \text{on } \Omega. \\ u_i = 0 \quad \text{on } \partial\omega \times]-h/2, h/2[\end{cases}$$



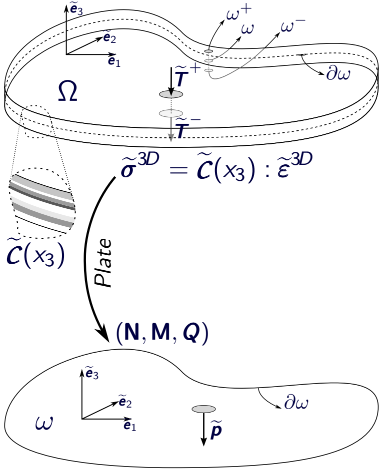
- Laminated plate
- Clamped plate
- Only out-of-plane loading, per unit surface

$$\boldsymbol{T}^{\pm} = \frac{p_3}{2}\boldsymbol{e}_3$$

Plate stress and equilibrium equations

- Plate stresses

$$\begin{cases} \mathrm{M}_{\alpha\beta}\left(x_1, x_2\right) = \int_{-\frac{h}{2}}^{\frac{h}{2}} x_3 \sigma_{\alpha\beta} dx_3 \\ Q_\alpha\left(x_1, x_2\right) = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{\alpha 3} dx_3 \end{cases}$$



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- Equilibrium equations:

$$\begin{cases} \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{\alpha 3,3} dx_3 \\ \int_{-\frac{h}{2}}^{\frac{h}{2}} x_3 \sigma_{\alpha\beta,\beta} dx_3 \end{cases} \Rightarrow \begin{cases} Q_{\alpha,\alpha} + p_3 = 0 \\ \mathrm{M}_{\alpha\beta,\beta} - Q_\alpha = 0 \end{cases}$$

Boussinesq (1871); Mindlin (1951)...

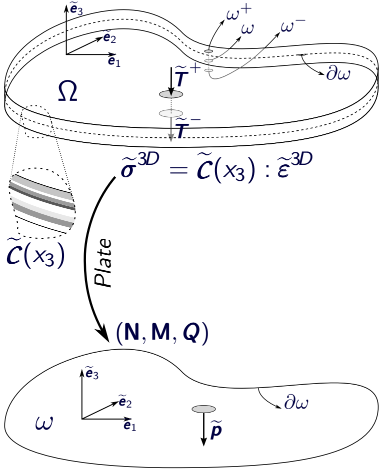

Plate stress and equilibrium equations

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- Equilibrium equations:

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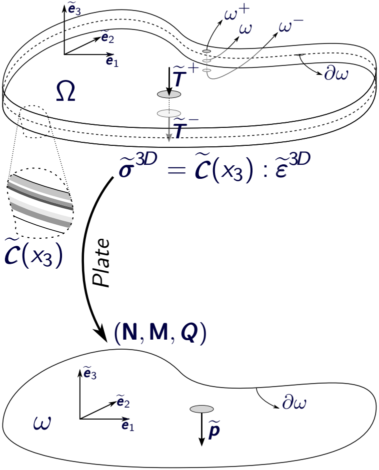
- $\boldsymbol{\sigma}^{(\mathrm{M})}$: asymptotic expansion
  $\boldsymbol{\sigma}^{(Q)}$: ??



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Reissner's approach for a homogeneous plate

- Bending stress fields (asymptotic expansion: $\varepsilon_{\alpha\beta} = x_3 \kappa_{\alpha\beta}$):

$$\boldsymbol{\sigma}^{(\mathrm{M})} = \frac{12x_3}{h^3} \begin{pmatrix} \mathrm{M}_{11} & \mathrm{M}_{12} & 0 \\ \mathrm{M}_{12} & \mathrm{M}_{22} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

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- Volume force related to shear forces:

$$\boldsymbol{\sigma}^{(\mathrm{M})}_{ij,j} = \frac{12x_3}{h^3} \begin{pmatrix} \mathrm{M}_{1\alpha,\alpha} \\ \mathrm{M}_{2\alpha,\alpha} \\ 0 \end{pmatrix} = \frac{12x_3}{h^3} \begin{pmatrix} Q_1 \\ Q_2 \\ 0 \end{pmatrix} = f_i^{(Q)}$$

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- Transverse shear unit-load problem (Reissner, 1945):

$$\begin{cases} \sigma^{(Q)}_{ij,j} + f^{(Q)}_i = 0 \\ \sigma^{(Q)}_{i3} = 0 \text{ on } x_3 = \pm h/2 \end{cases} \Rightarrow \sigma^{(Q)}_{ij} = \frac{3}{2h}\left(1 - \frac{4x_3^2}{h^2}\right)\begin{pmatrix} 0 & 0 & Q_1 \\ 0 & 0 & Q_2 \\ Q_1 & Q_2 & 0 \end{pmatrix}$$

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Revisiting Reissner's approach

- Building the body force imposes the introduction of the full bending gradient:

  $f^{(Q)}$ becomes $f^{(\mathtt{R})}$, where: $\mathtt{R}_{\alpha\beta\gamma} = \mathsf{M}_{\alpha\beta,\gamma}$

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- We define the full bending gradient local stress field:

  $\boldsymbol{\sigma}^{BG} = \boldsymbol{\sigma}^{(\mathsf{N})} + \boldsymbol{\sigma}^{(\mathsf{M})} + \boldsymbol{\sigma}^{(\mathtt{R})}$

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  $\boldsymbol{\sigma}^{BG} = \boldsymbol{\sigma}^{(\mathsf{N})} + \boldsymbol{\sigma}^{(\mathsf{M})} + \boldsymbol{\sigma}^{(\mathtt{R})}$

- Mechanical meaning of $\mathbf{R}$

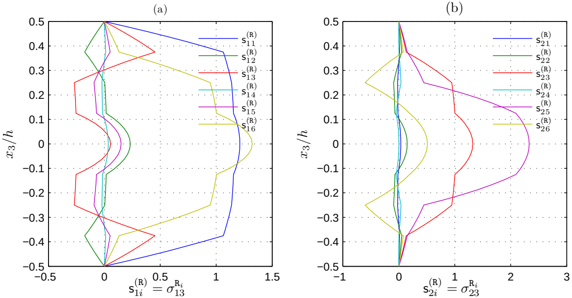
$$\begin{cases} Q_1 = \mathtt{R}_{111} + \mathtt{R}_{122} = \mathsf{M}_{11,1} + \mathsf{M}_{12,2} \\ Q_2 = \mathtt{R}_{121} + \mathtt{R}_{222} = \mathsf{M}_{21,1} + \mathsf{M}_{22,2} \end{cases}$$

Thus:

$\mathtt{R}_{111} = \mathsf{M}_{11,1}$: Cylindrical Bending part of Q_1
$\mathtt{R}_{221} = \mathsf{M}_{22,1}$: Pure warping
$\mathtt{R}_{121} = \mathsf{M}_{12,1}$: Torsion part of Q_2
$\mathtt{R}_{112} = \mathsf{M}_{11,2}$: Pure warping
$\mathtt{R}_{222} = \mathsf{M}_{22,2}$: Cylindrical Bending part of Q_2
$\mathtt{R}_{122} = \mathsf{M}_{12,2}$: Torsion part of Q_1

Major features of the Bending-Gradient theory

- Enables the distinction between each component of $\mathtt{R}_{\alpha\beta\gamma}$:



- The exact extension of RM model to laminated plates:
  - If the plate is homogeneous, BG is turned into RM model
  - The restriction of the BG to a RM model is not unique (except if homogeneous)
- A successful application to highly anisotropic laminated plates
  - Excellent estimate of transverse shear fields and deflection
  - Local fields converge with slenderness (St Venant Solution)

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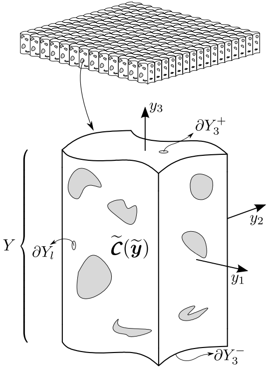
Extension to periodic plates

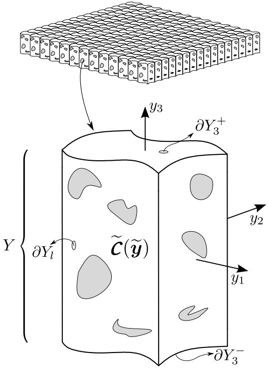
- Unit-cell and average estimates



Extension to periodic plates

- Unit-cell and average estimates
- Bending auxiliary problem (Caillerie, 1984)

$$\mathcal{P}^{(\kappa)} \begin{cases} \widetilde{\boldsymbol{\sigma}}^{(\kappa)} \cdot \widetilde{\boldsymbol{\nabla}} = 0 \\ \widetilde{\boldsymbol{\sigma}}^{(\kappa)} = \widetilde{\boldsymbol{\mathcal{C}}}\,(\widetilde{\boldsymbol{y}}) : \widetilde{\boldsymbol{\varepsilon}}^{(\kappa)} \\ \widetilde{\boldsymbol{\varepsilon}}^{(\kappa)} = y_3 \widetilde{\boldsymbol{\kappa}} + \widetilde{\boldsymbol{\nabla}} \otimes^s \widetilde{\boldsymbol{u}}^{per} \\ \widetilde{\boldsymbol{\sigma}}^{(\kappa)} \cdot \widetilde{\boldsymbol{e}}_3 = 0 \text{ on free faces } \partial Y_3^{\pm} \\ \widetilde{\boldsymbol{\sigma}}^{(\kappa)} \cdot \widetilde{\boldsymbol{n}} \text{ skew-periodic on lateral edge } \partial Y_l \\ \widetilde{\boldsymbol{u}}^{per}(\widetilde{\boldsymbol{y}})\ (y_1, y_2)\text{-periodic on lateral edge } \partial Y_l \end{cases}$$



$\rightarrow$ gives:

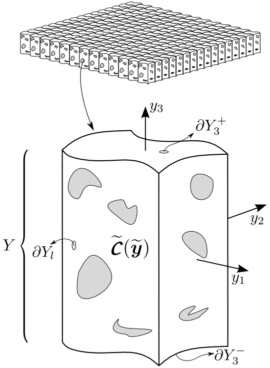
Localization related to curvature $\boldsymbol{\kappa}$
Bending compliance tensor: $\boldsymbol{d}$

$\rightarrow$ enable the derivation of $\boldsymbol{f}^{(\mathtt{R})}$

Extension to periodic plates

- Unit-cell and average estimates
- Bending auxiliary problem (Caillerie, 1984)
- Shear auxiliary problem

$$
\mathcal{P}^{(\mathrm{R})}
\begin{cases}
\widetilde{\boldsymbol{\sigma}}^{(\mathrm{R})} \cdot \widetilde{\boldsymbol{\nabla}} + \widetilde{\boldsymbol{f}}^{(\mathrm{R})}(\widetilde{\boldsymbol{y}}) = 0 \\
\widetilde{\boldsymbol{\sigma}}^{(\mathrm{R})} = \widetilde{\boldsymbol{\mathcal{C}}}(\widetilde{\boldsymbol{y}}) : \left(\widetilde{\boldsymbol{\nabla}} \otimes^s \widetilde{\boldsymbol{u}}^{(\mathrm{R})}\right) \\
\widetilde{\boldsymbol{\sigma}}^{(\mathrm{R})} \cdot \widetilde{\boldsymbol{e}}_3 = 0 \text{ on free faces } \partial Y_3^{\pm} \\
\widetilde{\boldsymbol{\sigma}}^{(\mathrm{R})} \cdot \widetilde{\boldsymbol{n}} \text{ skew-periodic on lateral edge } \partial Y_l \\
\widetilde{\boldsymbol{u}}^{(\mathrm{R})}(\widetilde{\boldsymbol{y}}) \; (y_1, y_2)\text{-periodic on lateral edge } \partial Y_l
\end{cases}
$$



$\rightarrow$ gives:

Localization related to $\mathbf{R}$
Shear compliance tensor: $\mathbb{f}$

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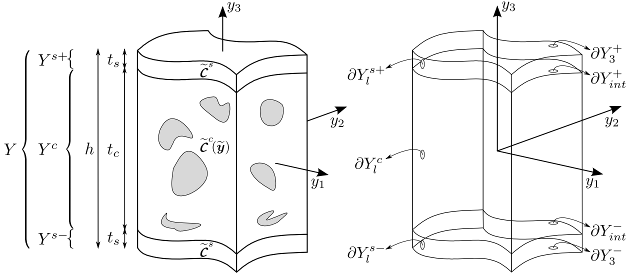
Justification of the classical approach (sandwich theory)

- Divide in 3 layers
  (homogeneous skins and heterogeneous core)

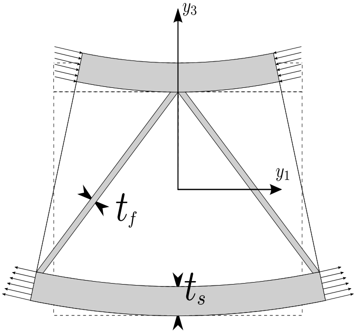


Justification of the classical approach (sandwich theory)

- Divide in 3 layers (homogeneous skins and heterogeneous core)
- Bending auxiliary problem
  - Contrast assumption $\Leftrightarrow t_f \ll t_s$:
    $\rightarrow t_s/t_f$ Contrast ratio

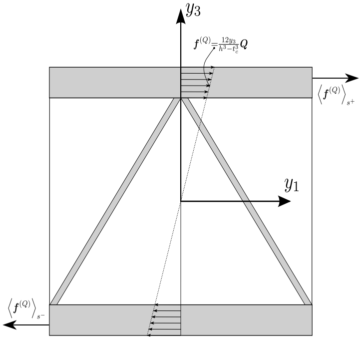
$\Rightarrow$ Skins under traction/compression

$\Rightarrow$ Core not involved in Bending stiffness



Justification of the classical approach (sandwich theory)

- Divide in 3 layers
  (homogeneous skins and heterogeneous core)
- Bending auxiliary problem
- Shear auxiliary problem
  - $\widetilde{\boldsymbol{f}}^{(\mathrm{R})}$ becomes $\widetilde{\boldsymbol{f}}^{(Q)}$
  - The BG is degenerated into RM model
  - $\widetilde{\boldsymbol{f}}^{(Q)}$ confirms the classical intuition
  - Proof of the bounds from Kelsey et al. (1958)



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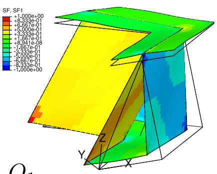
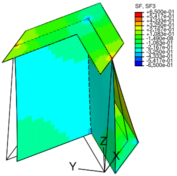
Application to the chevron pattern

Shear forces localization $\boldsymbol{\sigma}^{(Q)}$

- Overall shearing of the core



Q_1

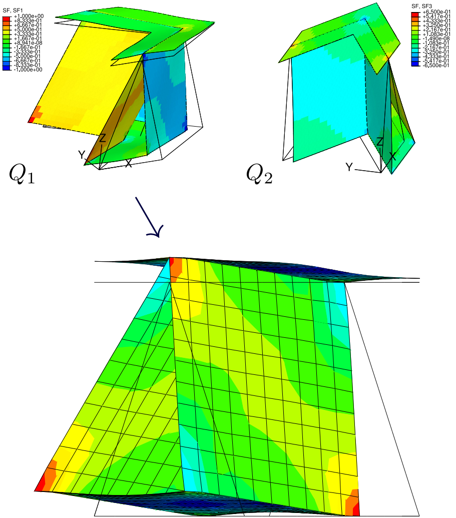


Q_2

Application to the chevron pattern

Shear forces localization $\boldsymbol{\sigma}^{(Q)}$

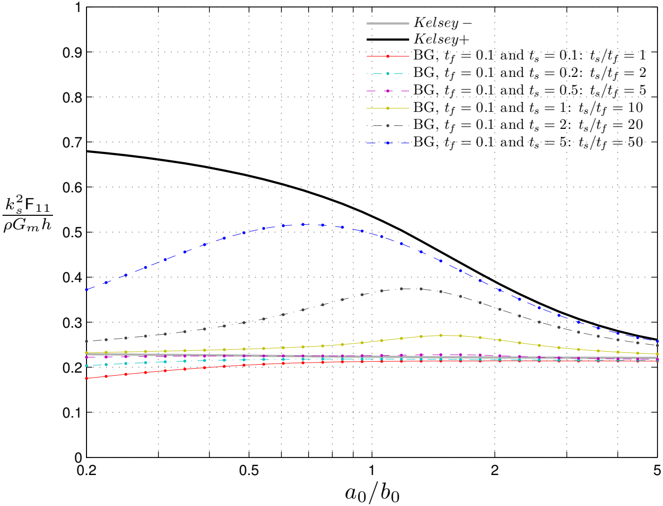
- Overall shearing of the core
- Out-of-plane skins distorsion



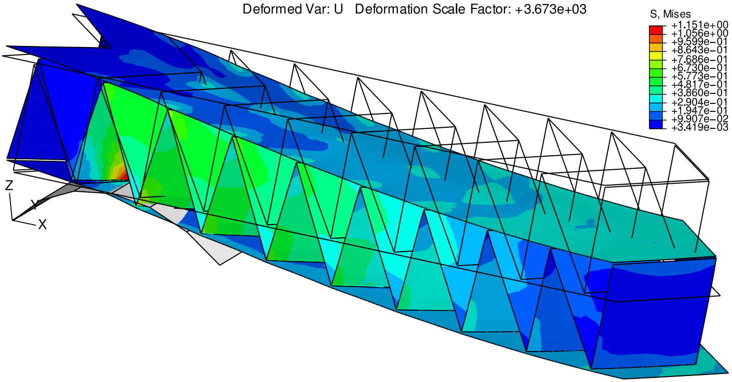
Application to the chevron pattern

Shear forces localization $\boldsymbol{\sigma}^{(Q)}$

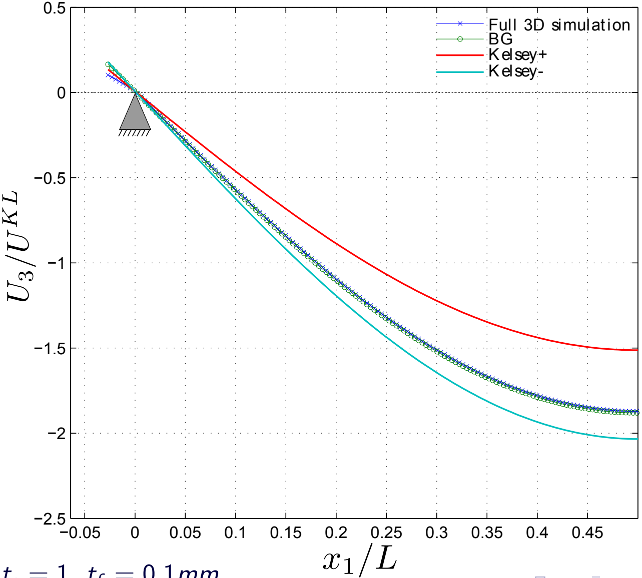
- Overall shearing of the core
- Out-of-plane skins distorsion
- Critically influence shear force stiffness



Comparaison with full 3D simulation



Comparaison with full 3D simulation



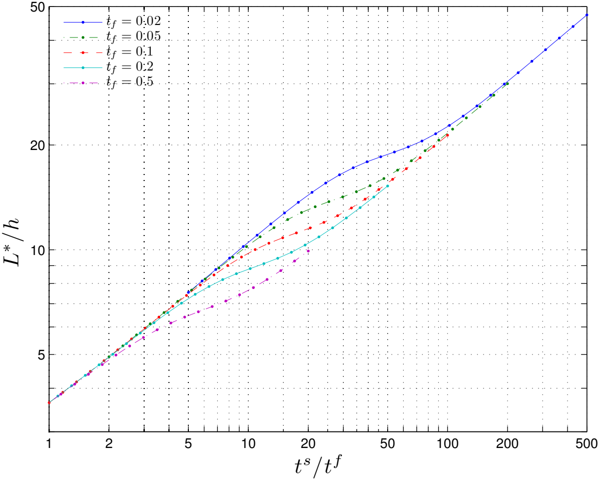
$h = 30mm,\ t_s = 1,\ t_f = 0.1mm$

Is skin distorsion really critical?

- mid-span deflection:

$$U_3 = U_3^{KL}\left(1+\left(\frac{L^*}{L}\right)^2\right)$$

where $L^* = \pi\sqrt{\frac{\mathcal{D}_{1111}}{\mathsf{F}_{11}}}$



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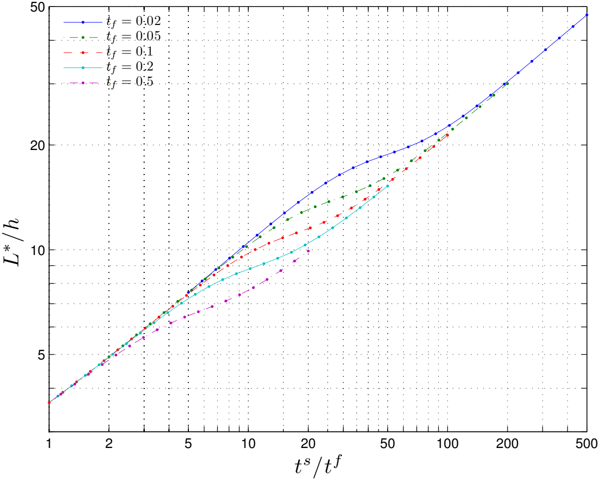
- The shift "stiff/compliant skins" occurs for usual contrast ratios:

$$\frac{t_s}{t_f} = 20 \text{ and } \frac{L}{h} = 20$$

$$\Rightarrow 25\% < \left(\frac{L^*}{L}\right)^2 < 60\%$$



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- Outlooks
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