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# COMPARISON OF HOMOGENIZATION SCHEMES TO PERIODIC AND RANDOM SIMULATIONS

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## ASSESSMENT OF THE EFFICIENCY OF THE INTERACTION DIRECT DERIVATIVE HOMOGENIZATION SCHEME BY COMPARISON TO FINITE ELEMENT SIMULATIONS.

### Average properties

At a macroscopic scale, for a porous medium with one pore family loaded with pressures  $p$ , the constitutive law writes [1]:

$$\begin{cases} \underline{\underline{\Sigma}} = \underline{\underline{C}}^{hom} : \underline{\underline{E}} - p\underline{\underline{B}} \\ \phi - f = \underline{\underline{B}} : \underline{\underline{E}} + pM \end{cases}$$

Where we call  $\underline{\underline{C}}^{hom}$  the homogenized stiffness tensor,  $\underline{\underline{B}}$  the Biot coefficient, and  $M$  the Biot modulus (inverse of the usual Biot modulus  $N$ ).

### Simulations using FreeFem++

We create a volume with pores in a 2d plane strain model, and apply:

- external loadings
- pressure in the pores

using periodic B.C., to determine the poroelastic constants by measurement of strain and stress averages.

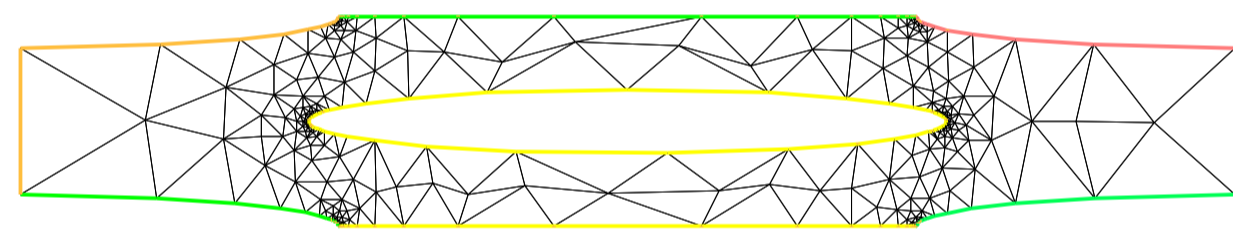


Fig. 1: Elliptical inclusions on an elliptical grid, periodic simulation

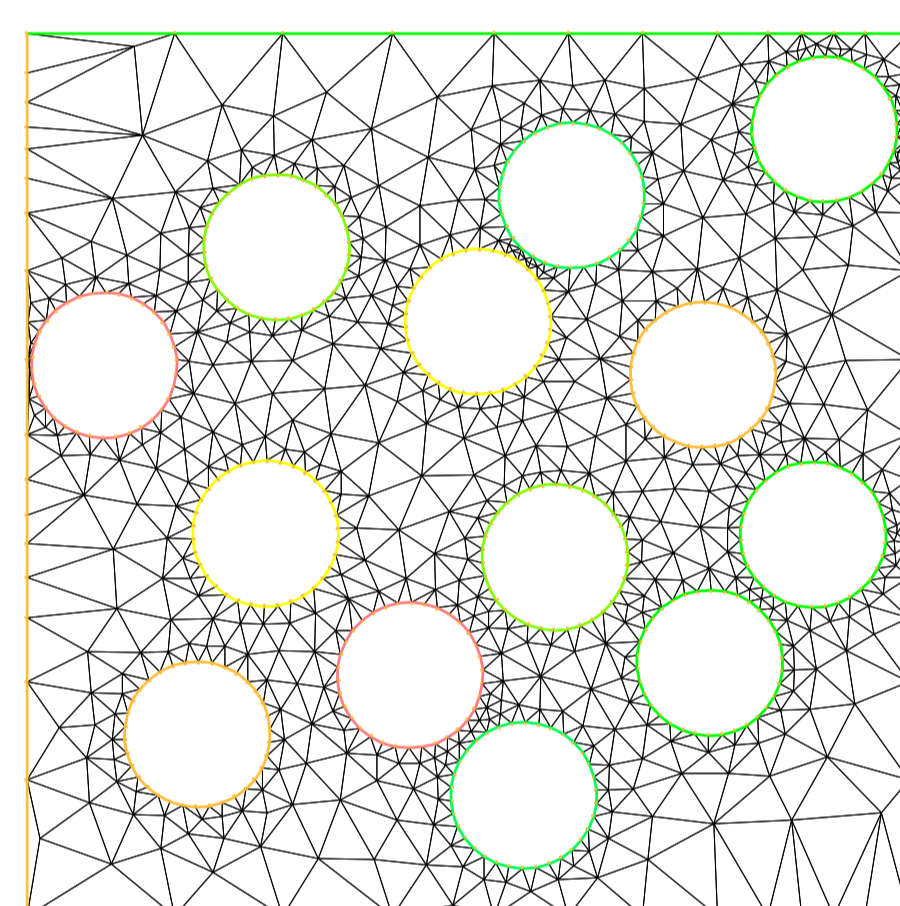
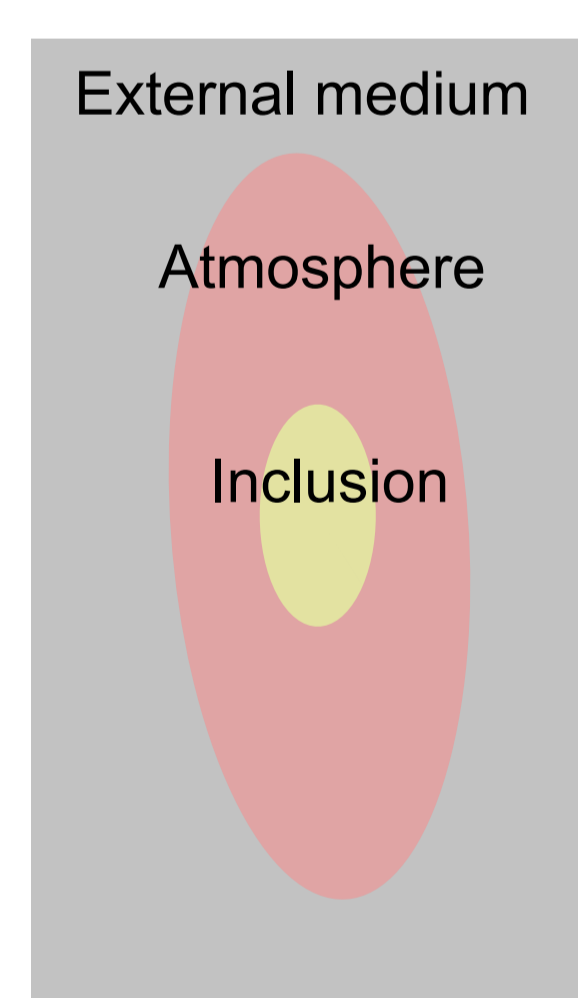


Fig. 2: Circular inclusions in a random simulation

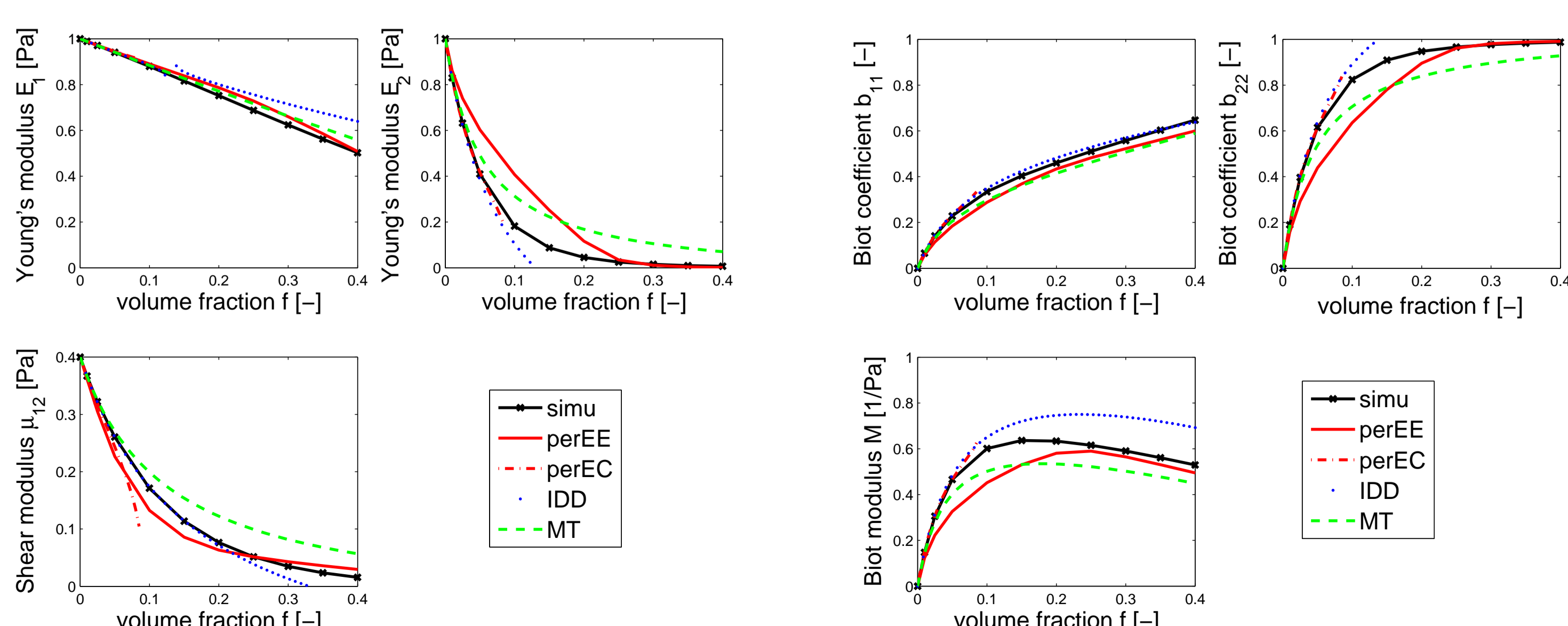
### Estimates

- Mori-Tanaka: **MT** (Inclusion embedded in the matrix, averages computed on a domain of the same shape as the inclusion, respecting volume fractions)
- Interaction-Direct-Derivative: **IDD** (Convenient simplification of the generalized self-consistent scheme, in which the inclusion is embedded in the matrix atmosphere, which is embedded in the average medium [2])



### Aligned elliptical pores, aspect ratio 0.1

We compare random simulations **simu**, periodic simulation with isotropic cell **perEC** and elliptic cell **perEE**, to the **IDD** scheme with circular atmosphere and the **MT** scheme. The **IDD** is failing because some coefficient reach their bound (0 or 1) far too early. **MT** does not show this problem. **IDD** needs to be used more carefully. **perEC** is accurate but cannot reach high volume fractions, **perEE** gives unsatisfactory results at intermediate volume fractions.



We modify the IDD scheme to improve the results according to a simple geometrical rule and an optimization procedure. The aspect ratio of the atmosphere  $\frac{b^d}{a^d}$  needs to change from 1 to that of the inclusion 0.1 when the volume fraction  $f$  increases.

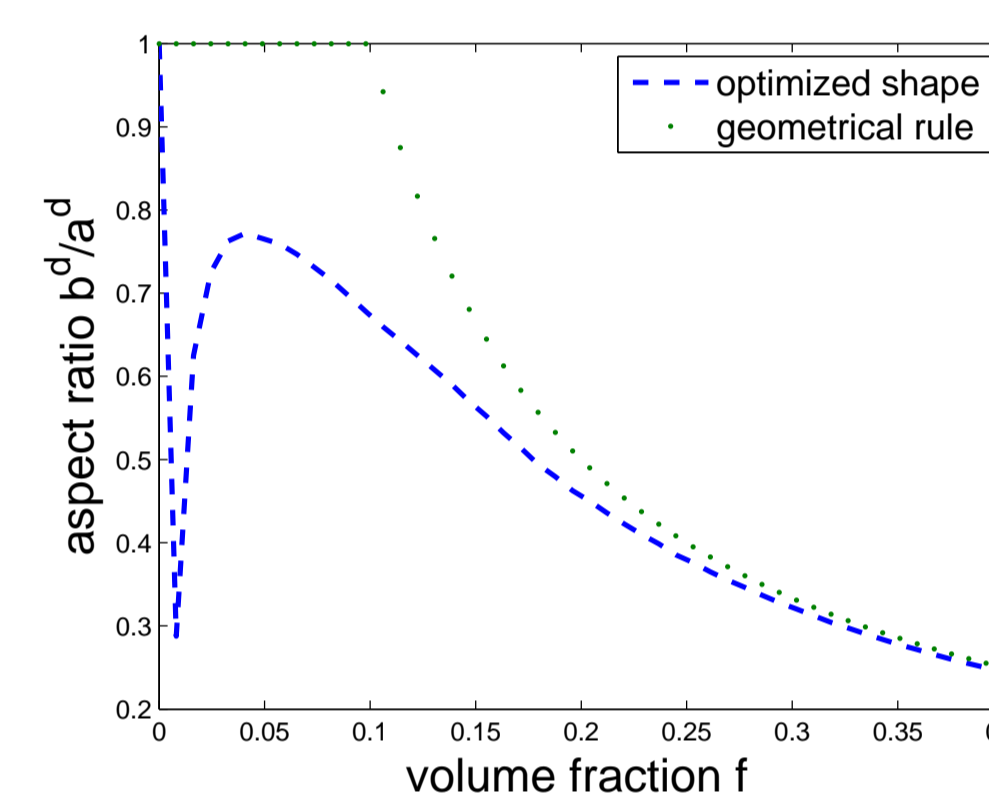
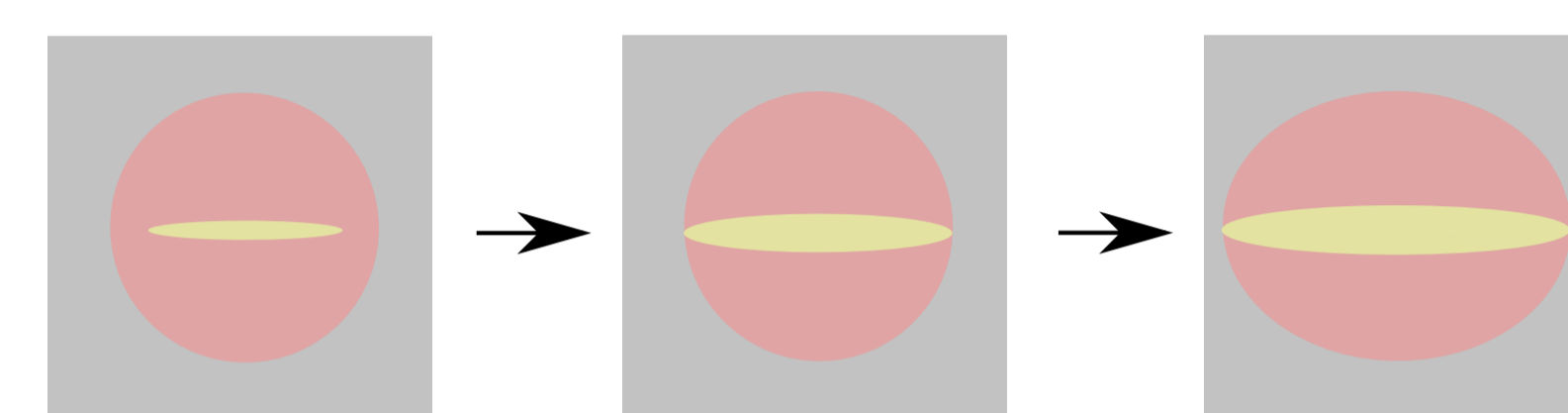


Fig. 3: Two possibilities for the evolution of the aspect ratio of the atmosphere

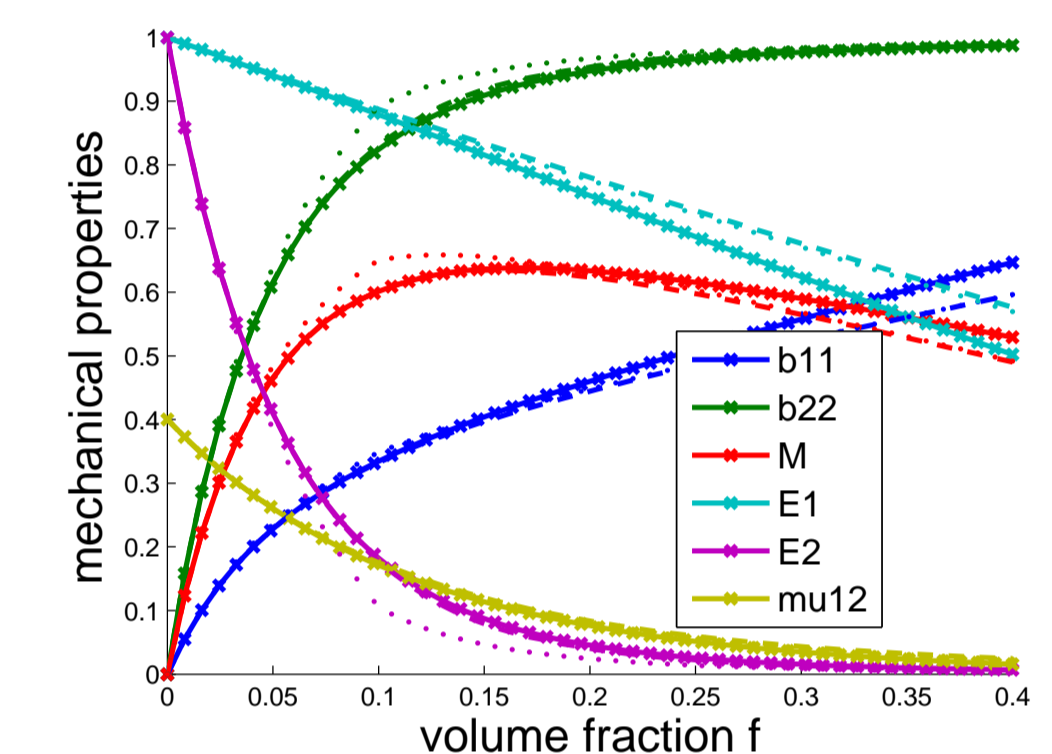
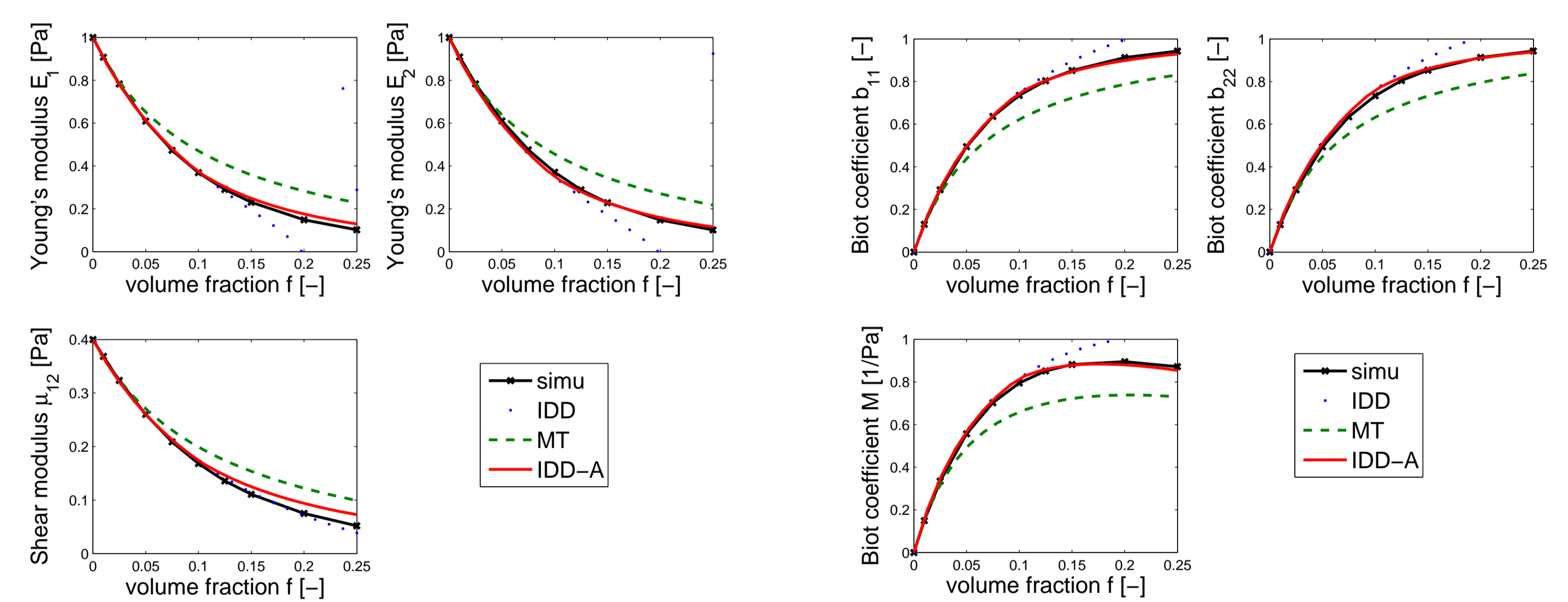


Fig. 4: Efficiency of this modification. (×): simulations, (●): geometrical rule, (---): optimized shape

The geometrical rule is less satisfactory than the optimized shape, but is simpler. We call the **IDD** estimate built by this modification **IDD-A**. It is not new [3].

### Isotropically oriented elliptical pores, aspect ratio 0.1

Finally we compare three estimates to simulations in the case of randomly oriented pores. The results obtained with **IDD-A** are very satisfactory.



### Conclusion

The **IDD** scheme, when used with adapted shapes for the atmospheres, gives good results to predict the homogenized properties of crack-like pores, whether aligned or isotropically oriented.

### Acknowledgements

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### References

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