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## Can Uncertainty Justify Overlapping Policy Instruments to Mitigate Emissions?

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#### Abstract

This article constitutes a new contribution to the analysis of overlapping instruments to cover the same emission sources. Using both an analytical and a numerical model, we find that if there is a risk that the carbon price drops to zero and if the political unavailability of a  $CO_2$  tax (at least in the European Union) is taken into account, it can be socially beneficial to implement an additional instrument encouraging the reduction of emissions, for instance a renewable energy subsidy. Our analysis has both a practical and a theoretical purpose. It aims at giving economic insight to policymakers in a context of increased uncertainty concerning the future stringency of the European Emission Trading Scheme. It also gives another rationale for the use of several instruments to cover the same emission sources, and shows the importance of accounting for corner solutions in the definition of the optimal policy mix.

Keywords: Uncertainty, Policy overlapping, Mitigation policy, Energy policy, EU-ETS, Renewable energy, Corner solutions, Nil CO2 price, European Union.

#### 1. Introduction

All countries and regions having implemented climate policies seem to rely on several policy instruments, some of which covering the same emission sources, rather than a single one<sup>1</sup>. In the European Union, CO<sub>2</sub> emissions from the electricity sector are directly or indirectly covered by the EU Emission Trading System (ETS) (Ellerman et al., 2010), by energy-efficiency standards and energy-efficiency labels on electric motors and appliances (UE, 2009), by CO<sub>2</sub> or energy taxes (in some Member States), by energy-efficiency obligations<sup>2</sup> (in some Member States), and by renewable energy power (REP) subsidies, in the form of feed-in tariffs, feed-in premiums or REP portfolio obligations (in virtually all Member States).

This multiplicity of policy instruments is in sharp contrast to the so-called Tinbergen rule (Tinbergen, 1952) requiring in order to achieve a given number of targets that policymakers control an equal number of instruments. Unsurprisingly, this multiplicity has generated criticism by some economists who argue that the policy instruments complementing the EU ETS do not reduce CO<sub>2</sub> emissions (which are capped) but reduce the allowance price on the ETS market and generate costly economic distortions (Cf. for instance Böhringer and Keller (2011); Braathen (2007); Fischer and Preonas (2010) or Tol (2010)). Indeed, some abatement options, such as REP sources, are covered by several instruments and benefit from a higher implicit carbon price than others, such as coal-to-gas switch. The mix of instruments promoting the same abatement options is therefore suboptimal, at least in a simple economic model, as it disregards the equimarginal principle and leads to sometime antagonist interactions (Lecuyer and Bibas, 2011).

Yet, the multiplicity of policy instruments has been justified by some other economists, on several grounds. First, and most obviously, other policy targets such as air pollution reduction and se-

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<sup>&</sup>lt;sup>1</sup>The unconvinced reader is invited to look at the National Communications to the UNFCCC: http://unfccc.int/national\_reports/items/1408.php

<sup>&</sup>lt;sup>2</sup>Lees (2012) provides a recent survey of these systems in Europe, while Giraudet et al. (2012) discuss the costs and benefits of these systems.

curity of supply are differently impacted by the various CO<sub>2</sub> abatement options. Second, induced technical change may be higher for some options than for others. For instance, the deployment of photovoltaic panels is likely to induce more technical change than coal-to-gas switch (see Fischer and Newell (2008) for a review). Third, the slow diffusion of clean technology justifies implementing more costly but higher potential options, such as photovoltaic panels, before the cheaper but lower potential options, such as coal-to-gas switch (Vogt-Schilb and Hallegatte, 2011). Fourth, some market failures, regulatory failures or behavioral failures may reduce the economic efficiency of market-based instruments and justify additional policy instruments (Gillingham and Sweeney, 2010). For instance, the landlord-tenant dilemma reduces the efficiency of CO<sub>2</sub> pricing and can justify energy-efficiency standards in rented dwellings (de T'Serclaes and Jollands, 2007), while regulatory failures may lead to a too low carbon price, or prevent governments to commit to a high enough future carbon price (Hoel, 2012).

Our aim is not to discuss these justifications, but to introduce and discuss another rationale: the impact of uncertainty on abatement costs combined to the unavailability of the first-best instrument. It is well known since Weitzman (1974) that under uncertainty, the relative slope of the marginal abatement cost curve and marginal damage of emissions curve (labeled "marginal benefits" in Weitzman's framework) is key to choose between a price instrument (e.g. a CO<sub>2</sub> tax) and a quantity instrument (e.g. a cap-and-trade system, like the EU-ETS). More specifically, in the simplest form of Weitzman's (1974) model, the quantity instrument should be chosen if the marginal damage curve is steeper than the marginal abatement cost curve while the price instrument should be chosen if the marginal abatement cost curve is steeper. If the marginal damage curve is completely flat then a tax (set at the expected marginal damage) is the first-best instrument. In the case of climate change control, most researchers have concluded that on this ground, a tax should be preferred to a capand-trade system (e.g. Pizer (1999)). Indeed the marginal damage curve of CO<sub>2</sub> emissions over a few years period is relatively flat because CO<sub>2</sub> is a stock pollutant (Newell and Pizer, 2003). Actually, this argument is even stronger for policies covering only a small part of total emissions, such as the EU ETS; hence, with an uncertain marginal abatement cost curve, an ETS is less efficient than a tax, i.e. it brings a lower expected welfare.

Yet, in the EU, a meaningful CO<sub>2</sub> tax is out of reach because fiscal decisions are made under the unanimity rule, while a cap-and-trade system has been adopted thanks to the qualified majority rule which applies to environmental matters (Convery, 2009). Another main reason why cap-and-trade was chosen was for political economy reason in order to be able to alleviate opposition of e.g. electricity producers by means of free allocation of emission permits<sup>3</sup> (Boemare and Quirion, 2002).

The fact that the EU ETS is not optimal is illustrated by its history since its introduction in 2005, which shows how volatile the carbon price can be: it dropped to virtually zero in 2007 because allowance allocation in phase I was too generous (Ellerman and Buchner, 2008), recovered up to more than  $\in 30/tCO_2$  because allocation in phase II was tighter and dropped again sharply in 2009 following the economic crisis, down to €3/t CO<sub>2</sub> While economists disagree over in April 2013. the marginal damage of CO<sub>2</sub> emissions, commonly called the "social cost of carbon" (Perrissin Fabert et al., 2012), they would presumably agree that such a price evolution is inefficient: in some periods, the carbon price has prompted relatively expensive abatement options (up to  $\leq 30/t \text{ CO}_2$ ) while in other periods, cheaper abatement options have not been implemented. This potentially provides a rationale for correcting the ETS and/or for complementing it. Among the proposed corrections is the introduction of a price cap and a price floor. Since this proposal has been widely debated (e.g. Hourcade and Ghersi (2002)), we will not address it in this paper.

Conversely, to our knowledge only two papers have addressed the role of uncertainty on abatement costs on the effectiveness of multiple instruments. Mandell (2008) find that under some conditions, it is more efficient to regulate a part of emissions by a cap-and-trade program and the rest by an emission tax, than to use a single instrument. Admittedly, under such a mixed regulation, the marginal abatement cost differs across emission sources, which is inefficient, but the emission volume is generally

<sup>&</sup>lt;sup>3</sup> The ETS was also implemented as part of a long-term strategy aiming at setting clear targets for investors. As a market instrument, it also brings value as a coordination tool for investment efforts across a large range of sectors or parts of sectors.

closer to the ex post optimum than under a single instrument: following an increase in the marginal abatement cost, the tax yields too high an emission level while the cap-and-trade system yields a level which is too low, so these inefficiencies partly cancel out.

The other paper is by Hoel (2012, section 9) who studies the opportunity to subsidize REP in case of an uncertain future carbon tax. He studies the case of scientific uncertainty (damages caused by climate change are uncertain) and political uncertainty (the current government knows that there might be a different government in the future, and that this government may have a different valuation of emissions). He shows that scientific uncertainty justifies a subsidy to REP if REP producers are risk-averse. Under political uncertainty, results are more complex. If the current government expects the future government to have a lower valuation of emission reductions than itself, this tends to make the optimal subsidy positive. Hoel (2012) studies the impact of uncertainty, but only when the subsidy is combined to a tax, not when it is combined to an ETS — which is what the present article focuses on.

While we also address the role of uncertainty concerning abatement costs on the effectiveness of multiple instruments, our focus is on whether it makes sense to use several instruments to cover the same emission sources and not to cover different sources, as in Mandell's article (Mandell, 2008). More precisely, we assume that the EU cannot implement a CO<sub>2</sub> tax because of the above-mentioned unanimity rule but can implement an ETS. However some CO<sub>2</sub> abatement options (for illustration, REP) can be incentivised by a price instrument (in this case, a subsidy to REP, e.g. a feed-in tariff). In our model, without uncertainty on the energy demand level (and hence on abatement costs) or if uncertainty is low enough, using the REP subsidy in addition to the ETS is not cost-efficient because there is no reason to give a higher subsidy to REP than to other abatement options. However we find that this uncertainty provides a rationale for using the REP subsidy in addition to the ETS, if it is large enough to entail a risk of a nil carbon price<sup>4</sup>. Even though the first-best policy would be a  $CO_2$  tax, when the latter is unavailable, using both a REP subsidy and an ETS may provide a higher expected welfare than using an ETS alone.

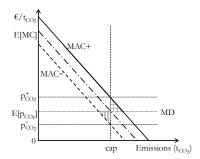
We demonstrate this result using three approaches. Section 2 presents the intuition in a graphical way. Section 3 develops an analytical model and presents some key analytical results based on the same intuition. Section 4 further completes the model and presents a numerical application on the European electricity sector. Section 5 concludes.

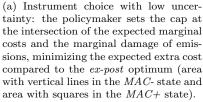
## 2. The possibility of a nil carbon price: justification and implications for instrument choice

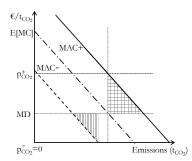
This section presents our main conclusion in an intuitive and graphical way. We study the possibility of a nil carbon price, unaccounted in Weitzman's seminal Prices vs. Quantities paper (Weitzman, 1974) or in the related literature, on optimal policy instrument choice. We show that using a REP subsidy in addition to the ETS improves expected welfare in so far as uncertainty on the demand level is large enough to entail a possibility of a nil carbon price, i.e. if there is a possibility that demand for GHG quotas turns out to be so low, compared to its expected value, that the ETS cap becomes non-binding.

Before introducing the intuition, let us give some elements justifying the possibility of a nil carbon price, in the light of the experience with cap-andtrade systems. An allowance price dropping to zero in an ETS is not unrealistic at all, and happened in some of the most well-known ETS worldwide. In the EU ETS, the carbon price dropped to zero at the end of the first period (in 2007). It would have done so in the second period (2008-2012) again without the possibility to bank allowances for the next period (2013-2020) and the likelihood of a political intervention to sustain the price. In the Regional Greenhouse Gas Initiative (RGGI), which covers power plant CO<sub>2</sub> emissions from North-Eastern US states, phase one carbon emissions fell 33% below cap (Point carbon, 2012). Consequently, the price remained at the auction reserve price, below \$2/tCO<sub>2</sub>. The cap also turned out to be higher than emissions in the tradable permit program to control air pollution in Santiago, Chile (Coria and Sterner, 2010) and in the UK greenhouse gas ETS (Smith and Swierzbinski, 2007). Even in the US SO<sub>2</sub> ETS, the price is now below \$1/tSO<sub>2</sub> (Schmalensee and Stavins, 2012),

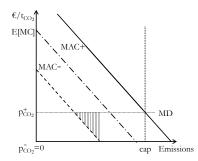
<sup>&</sup>lt;sup>4</sup>Since we use an expected welfare maximization model with a subjective probability distribution, we do not distinguish between risk and uncertainty.







(b) Instrument choice with high uncertainty: here setting a cap at the intersection of the expected marginal costs and the marginal damage of emissions does not minimize the total costs. The carbon price is nil in the MAC-state.



(c) Instrument choice with high uncertainty: setting the cap at the intersection of the MAC+ marginal costs and the marginal damage of emissions minimizes the costs in the MAC+ state with no additional costs in the MAC- state.

Figure 1: The implications of the possibility of a nil carbon price on optimal policy instrument choice.

vs. more than  $$150/{\rm tSO_2}$  ten years before, because new regulations and the decrease in high-sulfur fuels consumption have reduced emissions below the cap.

Figure 1 present graphically the implications of the possibility of a nil carbon price on optimal policy instrument choice. For our purpose, it is more convenient to draw the marginal cost and marginal damage as a function of emissions rather than as a function of abatement (as in Weitzman's paper), because we are interested in the uncertainty of unabated emissions. Let's assume that the Marginal Damage MD is known with certainty and is perfectly flat. We do not model the uncertainty on the marginal damage side since it is well known that this uncertainty matters only when correlated with abatement cost (Weitzman, 1974; Stavins, 1996). In our model, as in these two papers, adding (uncorrelated) uncertainty on marginal damages from emissions would not influence the ranking of instruments. Let's further assume than the marginal abatement cost curve is uncertain and can take with an equal probability two values, MAC+ and MAC-<sup>5</sup>, representing for instance the two extreme cases of a probability distribution. This uncertainty on the MACs captures economic uncertainty, as well as uncertainty on the technological costs (Quirion, 2005). In Figure 1a, uncertainty is lower (MAC- (decreasing dashed line) and MAC+(decreasing solid line) are closer) than in Figure 1b and 1c.

Since the marginal damage of emissions MD is known with certainty and perfectly flat, a price instrument (like a  $\mathrm{CO}_2$  tax) is optimal, both ex-ante and ex-post. On the opposite, a quantity instrument (like an emission cap or the EU-ETS) is generally not optimal ex-post because the cap does not follow the (ex-post) optimal emission level. Let's analyze how a risk-neutral policy maker minimizing expected cost (or maximizing expected welfare) would set the cap.

In Figure 1a, with a low uncertainty, the policy maker would set the optimal cap at the intersection between the marginal damage of emissions and the expected marginal abatement cost curve (the dotted-dashed line). This is also the expected emission level under a price instrument. The expected carbon price would then equal the marginal damage of emissions  $^6$ , although ex post, the carbon price would be either higher  $(p_{\rm CO_2}^+)$  or lower  $(p_{\rm CO_2}^-)$  than the expected carbon price  $(E[p_{\rm CO_2}])$ . The cost of the quantity instrument compared to

<sup>&</sup>lt;sup>5</sup>Noted MC in Weitzman (1974).

<sup>&</sup>lt;sup>6</sup> This equality (in expectation) between the price instrument and the quantity instrument regarding price and quantity is dubbed "certainty equivalence" by Hoel and Karp (2001). They find that while the equivalence prevails with additive uncertainty (a shift of the marginal abatement cost curve as in Weitzman's original paper), it does not under multiplicative uncertainty (a change in the slope of the marginal abatement cost curve). In this paper, we find that even with additive uncertainty on abatement costs, this principle does not prevail if there is a possibility that the price drops to zero.

the price instrument (or to the optimum) is given by the area with squares (in case of a higher than expected cost) or by the area with vertical lines (in case of a lower than expected cost). All this is consistent with Weitzman's standard model.

Conversely, in Figure 1b which features a large uncertainty, setting the optimal cap at the intersection between the marginal damage and the expected marginal abatement cost curve (vertical dotted line) does not minimize the expected cost: such a cap would not be binding in the MAC- state, but it would entail a significant cost, both in the MAC-state (the area with vertical lines) and in the MAC+ state (the area with squares).

A better solution (Figure 1c) is to set a more lenient cap which equalizes the marginal abatement cost and marginal damages of emissions only in the MAC+ state: the extra cost compared to the price instrument would then be nil in the MAC+ state while it would still equal the area with vertical lines in the MAC- state. In other words, the policy-maker now neglects the MAC- state, knowing that in such an eventuality, the cap is non-binding anyway; rather he sets the cap which is optimal is the high-cost state.

Notice in Figure 1c that in the MAC+ state, the marginal abatement cost equals the marginal damage; hence the welfare loss from a marginal additional effort would only be of the second order. Conversely, in the low-cost state, the marginal abatement cost is below the marginal damage; hence the welfare gain from a marginal additional effort would be of the first order. Consequently, an additional policy instrument might improve welfare even if it entails additional abatement in both states of nature, and even if it is imperfect — for example, because it targets only a subset of abatement options, like a REP subsidy.

Having explained the intuition of our main results, we now turn to the presentation of the analytical model.

## 3. Key analytical results in a stylized electricity market

To discuss the implications of a possible nil carbon price on the electricity sector, we model in this section a stylized European electricity market with an uncertain demand. This uncertainty on the electricity demand results in an uncertain abatement effort for any given emission cap, and hence in an uncertain MAC, as in the previous section.

We first present the equations and the programs of the producers and the social planner. The setting presented here corresponds to a mix with an ETS and a REP subsidy. Appendix C and following present the other settings used in our analytical results.

#### 3.1. Analytical framework and equations

We represent three types of agents: a social planner, representative electricity producers and representative consumers. The social planner maximizes an expected welfare function by choosing the optimal level of various instruments depending on the available instrument set: a carbon tax, an emission cap for the electricity sector or a REP subsidy. For demonstration purposes we focus in the model presentation on a setting with an emission cap and a renewable subsidy.

The emission cap can be interpreted as a stylized representation of the EU-ETS. The future level of electricity demand is uncertain, with a risk that the carbon price drops to zero in case of low demand. The electricity market is assumed to be perfectly competitive and we assume a 100% pass-through of the emission allowance.

The model is a two-stage framework. In the first stage, the social planner chooses the level of the various policy instruments, facing an uncertainty about the level of future electricity demand. In the second stage, the electricity producers maximize their profit given the policy instrument levels and the demand function.

## 3.1.1. Step 1: the producer profit maximization problem

We consider two types of electricity generation: fossil fuels (f) and REP (r). The electricity producers can also make abatement investments (a) to comply with the emission cap. Those abatements are assumed for simplicity to be independent from the level of fossil-based production. They refer for instance to investments making coal-fueled power plants able to cope with some share of biomass, CCS investments or allowance purchases on the Clean Development Mechanism (CDM) market. p is the electricity wholesale price.

Producers face an aggregate emission cap  $\Omega$  and benefit from a REP subsidy  $\rho$ .  $\phi$  is the carbon price emerging from the allowance market, equal to the shadow value of the emission cap constraint. We assume a 100% pass-through from allowance costs

to whole sale price. In our framework,  $\rho$  can be seen as a feed-in premium for instance. The producer maximizes its profit  $\Pi$  (Table 1 describes all the variables and parameters).

$$\max_{f,r,a} \Pi(p, f, r, a, \phi, \rho) = p \cdot f + (p + \rho) \cdot r \qquad (1)$$
$$- C_f(f) - C_r(r)$$
$$- AC(a) - PC(f, a, \phi)$$

where  $C_f(f)$  and  $C_r(r)$  are the production costs from fossil fuel and REP respectively. We assume decreasing returns for REP and constant returns for emitting power plants  $(C_f'(f)>0,C_r'(r)>0,C_f''(f)=0$  and  $C_r''(r)>0).$  The decreasing returns assumption is justified as the best production sites are used first and further REP development implies investing in less and less productive sites. On the contrary, emitting technologies such as combined cycles power plants or advanced coal power plants are easily scalable and thus do not generate a scarcity rent (Jonghe et al., 2009; Fischer, 2010; Fischer and Preonas, 2010). AC(a) is the Abatement Cost function of the electricity producers, independent of fossil or REP production and  $PC(f, a, \phi)$  is the allowance Purchasing Cost. The cost functions have a classical linear-quadratic form:

$$C_f(f) = \iota_f \cdot f$$

$$C_r(r) = \iota_r \cdot r + \frac{r^2}{2\sigma_r}$$

$$AC(a) = \frac{\sigma_a}{2}a^2$$

$$PC(f, a, \phi) = \phi \cdot (\tau \cdot f - a)$$

With  $\iota_f$  and  $\iota_r$  the intercepts (iota like intercept) of the fossil fuel and the REP marginal supply function respectively and  $\sigma_r$  the slope (sigma like slope) of the REP marginal supply function<sup>7</sup>.  $\sigma_a$  is the slope of the marginal abatement cost curve for the electricity producer and  $\tau$  is the average unabated carbon intensity of fossil fuel-based electricity production. We define a linear downward sloping electricity demand function  $d(\cdot)$  (with  $d'(\cdot) < 0$ ) whose intercept depends on the state of the world. We consider two different states s occurring with a probability  $\mathcal{P}_s$ , one with a high demand  $(d_+(p))$  and one with a low demand  $(d_-(p))$ . The demand function is defined as:

$$d(p) = \iota_d \pm \Delta - \sigma_d \cdot p$$

with the intercept being  $\iota_d + \Delta$  in the high-demand state of the world and  $\iota_d - \Delta$  in the low-demand state. The equilibrium conditions on the electricity and the emission markets thus depend on the state of the world.

$$f + r = d(p) \tag{2}$$

is the demand constraint. In each state of the world, the electricity supply has to meet the demand on the electricity market.

$$\begin{cases} \tau \cdot f_{-} - a_{-} < \Omega \\ \phi_{-} = 0 \end{cases} \quad \text{or} \quad \begin{cases} \tau \cdot f_{+} - a_{+} = \Omega \\ \phi_{+} > 0 \end{cases}$$
 (3)

expresses the joint constraint on emissions and carbon price. In the high-demand state of the world, total emissions cannot be higher than the cap  $\Omega$  and the carbon price is therefore strictly positive. In the low-demand state, we assume that the emission cap constraint is non-binding, hence the carbon price is nil.

The first order conditions of the producer maximization problem are the following:

$$p = \iota_f + \tau \phi \tag{4}$$

Fossil fuel producers will equalize marginal production costs with the wholesale market price, net from the price of emissions.

$$\rho + p = \iota_r + \frac{r}{\sigma_r} \tag{5}$$

REP producers will equalize marginal production costs with the wholesale market price, net from the subsidy.

$$\sigma_a a = \phi \tag{6}$$

Fossil fuel producers will equalize the marginal abatement cost with the carbon price.

The values of the market variables  $(p, f, r, a, \phi)$  as a function of policy instruments are found by solving the system of equations (2) to (6). They represent the reaction functions of the electricity producer.

<sup>&</sup>lt;sup>7</sup>The supply functions are the expression of the quantity produced as a function of price. This corresponds to the inverse of the marginal cost function, and the slope of the supply function  $(\sigma_r)$  is the inverse of the slope of the marginal cost function  $(\frac{1}{\sigma_r})$ . We constructed the Renewable cost function this way in order to keep the dimension of  $\sigma_r$  consistent with the slope of the demand function  $\sigma_d$ , allowing for some simplifications in the equations.

	Dimension	Description
f	(MWh)	Electricity from fossil fuels
r	(MWh)	Electricity from REP sources
p	(€/MWh)	Wholesale power price
a	$(tCO_2)$	Abatements from power sector
$\phi$	$(\in/tCO_2)$	Carbon price
$\rho$	(€/MWh)	REP subsidy
Ω	$(tCO_2)$	Emission cap
$\sigma_a$	$(\in/ tCO_2^2)$	Slope of power sector MACC
$\sigma_d$	(MWh <sup>2</sup> /€)	Slope of demand function
$\sigma_r$	(MWh <sup>2</sup> /€)	Slope of RE supply function
$\delta$	$(\in / tCO_2)$	Marginal environmental damage
$\lambda$	-	Probability of the high-demand state
$\Delta$	(MWh)	Variance of demand
$\tau$ (	$(tCO_2/MWh)$	Average carbon intensity (fossil fuels)
$\iota_f$	(€/ MWh)	Intercept of fossil fuel supply function
$\iota_r$	(€/ MWh)	Intercept of RE supply function
$\iota_d$	$({\in}/~\mathrm{MWh})$	Intercept of demand function

Table 1: Variables and parameters used in the models

## 3.1.2. Step 2: the social planner's expected welfare maximization problem

The social planner, assumed risk-neutral and giving the same weight to consumers and producers, faces an uncertain future demand and has a limited number of possible policy instruments (i.e. an emission cap and a REP subsidy) to maximize the expected welfare. We assume no social externality on the public funding, as this would imply that all public goods become more expensive, including the environment. We would have to add a deadweight loss on the revenues from the emission cap allowances transfers, and distinguish several cases with and without auction. We keep therefore our welfare function as simple as possible:

$$\max_{\Omega,\rho} EW(\Omega,\rho) = \sum_{s \in \text{states}} \mathcal{P}_s(CS(p))$$

$$+ \Pi(p, f, r, a, \phi) - dam(f, a)$$

$$- \rho \cdot r + PC(f, a, \phi))$$
(7)

 $\mathcal{P}_s$  is the probability of the two states of the world:  $\mathcal{P}_+ = \lambda$  and  $\mathcal{P}_- = (1 - \lambda)$ ,  $\lambda \in [0, 1]$ . CS(p) is the consumer surplus and dam(f, a) is the environmental damage function from the GHG emissions. The last two terms of the expected welfare cancel pure transfers between agents included in the profit functions. The consumer surplus CS and the damage function are taken as simple as possible for clarity. In particular, consumer are assumed risk-neutral:

$$CS(p) = \int_0^{d(p)} d^{-1}(q)dq - p \cdot d(p)$$
$$dam(f, a) = \delta \cdot (\tau f - a)$$

With  $\delta$  the constant environmental damage coefficient (Newell and Pizer, 2003). After having substituted the market variables in the expected welfare function (7) with the reaction functions coming from the producer problem we maximize the expected welfare. The first-order conditions give the optimal levels of the policy instruments across all states ( $\rho^*$  and  $\Omega^*$ ).

## 3.2. Social optimum when the carbon price is nil in the low-demand state

**Proposition 1.** When the carbon price is nil in the low-demand state of the world, the optimal renewable subsidy is strictly positive.

*Proof.* The optimal levels of the policy instruments across all states are given by solving the first-order conditions of the welfare maximization problem (7) (see Appendix E).

$$\Omega^{\star} = \tau \Delta + \tau \iota_d + \tau \iota_r \sigma_r$$

$$- \tau (\sigma_d + \sigma_r)(\iota_f + \delta \tau) - \frac{\delta}{\sigma_a}$$

$$\rho^{\star} = (1 - \lambda)\delta \tau \frac{1 + \sigma_a \sigma_d \tau^2}{1 + \sigma_a (\sigma_d + \sigma_r - \lambda \sigma_r) \tau^2}$$
(9)

knowing that all parameters are positive, and using the reaction functions from the profit maximization problem (1), we can write:

$$0 < \rho^{\star} < \delta \tau \tag{10}$$

Results follow directly.  $\hfill\Box$ 

If we considered only one certain state, we would fall back on the first-best optimum characterized by a REP subsidy equal to zero and the emission cap set so as to equalize the carbon price with the marginal damage  $\delta$ . The cap is set to be optimal in the high-demand state only, and does not depend on the probability distribution. We see here in (10) that the optimal subsidy is a portion of the marginal environmental damage (see also (12) below), and is weighted by the probability of the low-demand-state  $(1 - \lambda)$ .

By substituting the optimal levels of policy instruments in the reaction functions, we obtain the socially optimal level of all market variables for both states of demand (see Appendix E).

While in a first-best world the carbon price would equal the marginal environmental damage, in this second-best setting, the optimal carbon price in the high-demand state is lower because the REP subsidy also reduces emissions. The expected carbon price  $\mathcal{E}_{\phi} = \sum_{s \in \text{states}} \mathcal{P}_s \cdot \phi_s$  can be rearranged into:

$$\sum_{s \in \text{states}} \mathcal{P}_s \cdot \phi_s = \delta \cdot \frac{\lambda (1 + \sigma_a \sigma_d(\tau)^2)}{1 + \sigma_a (\sigma_d + \sigma_r - \lambda \sigma_r) \tau^2}$$
(11)

The term in the denominator expresses the substitutions taking place when the abatement through carbon pricing only is no longer optimal.

**Proposition 2.** When the carbon price is nil in the low-demand state of the world, the renewable subsidy equivalent in  $\in$ / $tCO_2$  is equal to the marginal damage of emissions minus the expected carbon price.

*Proof.* Combining (9) and (11) gives:

$$\frac{\rho^{\star}}{\tau} = \delta - \mathcal{E}_{\phi} \tag{12}$$

The proof follows directly.

In (12),  $\frac{\rho^{\star}}{\tau}$  is the marginal abatement effort through REP promotion and  $\mathcal{E}_{\phi}$  is the expected marginal abatement effort through carbon pricing. The simple intuition behind this result is that since the expected carbon price is below the marginal damages, the additional instrument, e.g. the REP subsidy, is also used to reduce emissions.

Since the carbon price is nil in the low-demand state, the expected carbon price decreases with the probability of the high-demand state (everything else being equal). Equation (12) reveals that the optimal subsidy moves accordingly to keep the global expected mitigation effort constant and equal to the marginal damage.

### 3.3. Expected emissions with various instrument

As mentioned in section 2, in Weitzman's model (Weitzman, 1974) with an additive uncertainty on the marginal abatement cost curve, the expected emissions are the same with a price or a quantity instrument. This is no longer the case in our model.

Par.	Meaning of an increase in the parameter	Sign of partial der.
$\sigma_a$	Higher abatement cost	+
$\sigma_d$	More elastic power demand	=
$\sigma_r$	Cheaper REP	_
δ	Higher marginal damage	-
Δ	Higher demand variance	+
λ	Higher probability of the high-demand state	_

Table 2: Signs of partial derivatives of the difference between the cap minus the emissions in the low-demand state for a nil carbon price; – indicates a negative partial derivative, + indicates a positive partial derivative and ? indicates an ambiguous sign.

**Proposition 3.** If there is a risk that the carbon price equals zero in the low-demand state of the world, expected emissions vary with the instrument mix.

Proof. See Appendix A. 
$$\Box$$

The expected emissions are lower in the secondbest setting (with an ETS and a REP subsidy) than in the third (with an ETS alone) and even lower with a first-best carbon tax.

The expected carbon price changes also. It is lowest in the second-best setting when it is optimal to implement a REP subsidy along with the emission cap.

The drop between first-best and second-best is mostly due to the nil carbon price in the low-demand state of the world. When comparing third-best and second-best, the carbon price is lower because another instrument, the REP subsidy, is now also used to reduce emissions.

## 3.4. Boundary condition for having a nil carbon price in the low-demand state of the world

As discussed in the graphical anlaysis in Section 2, the carbon price drops to zero when the optimal cap no longer crosses the low-demand MAC curve. In this section we investigate the effect of a change in the main parameters on the boundary between the positive-carbon price and the nil-carbon price spaces.

Par.	Meaning of an increase in the parameter	$ \rho $ : REP subsidy	$\Omega$ : Emission cap
$\sigma_a$	Higher abatement cost	]0;1[	+
$\sigma_d$	More elastic power demand	]-1;0[	-
$\sigma_r$	Cheaper REP	]0;1[	?
δ	Higher marginal damage	1	_

Table 4: Elasticity of instrument variables with respect to various parameters. ]-1;0[ indicates an elasticity between 0 and -1; ]0;1[ indicates an elasticity between 0 and 1; + or - indicate respectively a positive or negative elasticity; ? indicates an indeterminate sign of the elasticity.

Proposition 4. On the boundary, the carbon price in the low-demand state drops to zero as mitigation options (abatements and REP) become more expensive, uncertainty on the level of the electricity demand grows, the demand gets more inelastic, the environmental damage gets lower and the low-demand state gets more probable.

*Proof.* We compute the equilibrium conditions of the model without making any assumption on the emission or the carbon price levels in the low-demand state (see Appendix B). The expression for emissions, being a decreasing function of the carbon price, give the expression of the MAC curve in the low-demand state.

The difference between the emission cap and the low-demand state MAC curve for  $\phi_{-}=0$  give then a test of the positivity of the carbon price in the low-demand state. When emissions at  $\phi_{-}=0$  are below the cap, the carbon price is nil, and when emissions are above the cap, the carbon price is positive.

Table 2 gives the sign of the partial derivative of the difference between the cap minus the emissions in the low-demand state for a nil carbon price. On the boundary, if this difference increases, the carbon price drops to zero; if it decrases, the carbon price rises above zero.

#### 3.5. Variables' elasticity with respect to parameters

As a preliminary step to the numerical sensitivity analysis presented in Section 4, Table 3 and Table 4 show the sign of the elasticity of all variables with

respect to various parameters in the  $2^{nd}$  Best setting (instrument mix  $M_2$ , see Appendix C), and indicate whether they are above or below 1.

**Proposition 5.** the optimal subsidy  $\rho^*$  rises as abatement is more expensive, production from REP sources is cheaper, electricity demand is less elastic to electricity price and the marginal environmental damage from GHG emissions rises.

Proof. Table 4 shows the sign of variation of the optimal levels of policy instruments when various parameters change<sup>8</sup>. A positive elasticity indicates a positive variation when a parameter increases, and an absolute elasticity smaller than one indicates that a 1% change in that parameter will cause a less than 1% change in the variable. We see that the elasticity of  $\rho$  with respect to  $\sigma_a$  and  $\sigma_r$  is positive but smaller than 1, with respect to  $\sigma_d$  it is negative but smaller than one and the elasticity with respect to  $\delta$  is 1. The proof follows directly.

The explanation of this result is straightforward: more REP should be installed when the environmental damage is higher, when REP are cheaper and when the other ways to reduce emissions, i.e. abatement and energy savings become more expensive. Similarly, a higher abatement cost naturally leads to a less stringent emission cap  $\Omega$ , while a higher marginal damage and a more elastic electricity demand (which means higher energy savings for a given change in electricity price) lead to a more stringent cap. The impact of cheaper REP on the optimal cap is ambiguous: on the one hand, it reduces the overall cost of cutting emissions, leading to a more stringent cap, but on the other hand it pushes to an increased use of the other policy instrument, the subsidy, which minors the importance of the emission cap.

Table 3 shows that in state –, there is no abatement, the carbon price is nil and the electricity price is solely determined by the supply curve, so the parameters considered in Table 3 have no effect on these variables. However, they have an indirect effect on  $f_-$  and  $r_-$  since they impact  $\rho$ . Hence, the considered parameters increase the amount of REP  $r_-$  and they decrease the amount of fossil-fuel electricity  $f_-$  when they increase the REP subsidy  $\rho$ .

 $<sup>^8{\</sup>rm Elasticities}$  have been calculated in Mathematica. The Mathematica notebook is available upon request from the contact author

Param.	Meaning of an increase in the parameter	Level of demand (state)	f fossil fuel	r REP	p electricity price	a abatements	$\phi$ CO <sub>2</sub> price
$\sigma_a$	Higher abatement cost	High (+)	+	_	] - 1;0[	< -1	] - 1;0[
		Low(-)	_	+	0	0	0
$\sigma_d$	More elastic power demand	High (+)	?	+	]0;1[	]0;1[	]0;1[
		Low(-)	?	_	0	0	0
$\sigma_r$	Cheaper REP	High (+)	?	?	] - 1;0[	] - 1; 0[	] - 1;0[
		Low(-)	?	?	0	0	0
δ	Higher marginal damage	High (+)	_	+	]0;1[	1	1
		Low(-)	_	+	0	0	0

Table 3: Market variables' elasticity with respect to various parameters. ]-1;0[ indicates an elasticity between 0 and -1; ]0;1[ indicates an elasticity between 0 and 1; + or - indicate respectively a positive or negative elasticity; ? indicates an ambiguous sign of the elasticity.

In state +, as one could have expected, more abatements and a higher  $CO_2$  price  $\phi_+$  are triggered by a lower abatement cost, a more elastic electricity demand, more expensive REP, and a higher marginal damage. Moreover, a higher electricity price is triggered by a higher marginal damage, costlier REP, a more elastic electricity demand and, more surprisingly, a lower abatement cost. The explanation is that a lower abatement cost implies a more stringent target (Table 4), which in turn raises the electricity price in state +.

In state +, changes in energy production follow changes in the  $CO_2$  price  $\phi_+$ : lower abatement costs, higher marginal damages and a more elastic electricity demand increase the  $CO_2$  price, which in turn decrease the relative competitiveness of fossil fuel. In state –, the  $CO_2$  price is nil and changes are more sensitive to the REP subsidy: higher abatement costs, higher marginal damages and a more elastic electricity demand increase the optimal REP subsidy, which in turn increase the relative competitiveness of REP.

Comparing Table 4 and Table 3 finally shows that the carbon price and the REP subsidy vary in opposite directions (except when the marginal damage changes). This can be seen in (12). If there is a risk that the carbon price equals zero in the low-demand state of the world, the mitigation efforts induced by the carbon price are no longer sufficient. An additional effort through REP production is necessary, induced by a strictly positive REP subsidy.

## 4. Numerical application: the European electricity sector and allowance market

#### 4.1. Modified model

Having shown some analytical results with a model of a electricity sector alone, we turn to a slightly more complex model to show numerical results calibrated on the European electricity and allowance markets. In this section, we add an explicit allowance supply from non-electricity ETS sectors. We therefore add a composite sector including all the other constrained emitters. The electricity producer can buy emission allowances (e) from the other constrained sectors on the allowance market to comply to the emission constraint. The other ETS sectors are represented by their total abatement cost function, which has the following form:

$$AC_e = \frac{\sigma_e}{2}e^2 - \begin{cases} \iota_e \ e & \text{in state} + \\ 0 & \text{in state} - \end{cases}$$

where  $\sigma_e$  is the slope of the aggregate non-electricity ETS sector marginal abatement cost curve. The intercepts differ in the low demand and the high-demand state of the world. We assume there is a positive correlation between the level of electricity demand and the level of industrial activity. When the electricity demand is low, the industrial activity is also low and the allowance surplus is higher.

Next subsections will detail the data and assumptions made to calibrate the model. Some parameters being subject to a large uncertainty, we use a range of possible values for those parameters and

Description	Dimension	Range
Marginal environmental damage	(€/tCO <sub>2</sub> )	(10.,20.,,30)
Price-elasticity of demand (absolute value)	1	(0.1,0.2,,0.5)
Abatement from the aggregate ETS sector for 15 $€/tCO_2$	(%)	(1,2,,5)
Abatement from the power sector for $15 \in /tCO_2$	(%)	(1,2,,5)
Maximum share of REP in the energy mix	(%)	(10,20,,50)
Standard deviation of demand	(TWh)	(33,49,98)

Table 6: Ranges of parameters used in the numerical simulations for calibration purposes. All possible combinations of parameters were successively simulated.

discuss the distribution of results. For each uncertain parameter, we use a uniform probability distribution and we assume that these parameters are not correlated (except for the electricity demand and the industrial activity levels). Table 5 shows the minimum, median and maximum values of calibrated parameters resulting from the calibration process and used in the simulations.

We performed simulations with all possible combinations of parameters shown in Table 5, without any constraint on the carbon price. We tested the positivity of the carbon price, and if negative in the low-demand state, we conducted other simulations by constraining the carbon price to be equal to zero in the low-demand state. This distinguishes two qualitatively different simulation results. In the first category (subsequently called 2<sup>nd</sup> Best B), the carbon price is strictly positive in the low-demand state and the renewable subsidy is nil. In the second category (subsequently called 2<sup>nd</sup> Best A), the carbon price is nil in the low-demand state and the renewable subsidy is strictly positive. Appendix I details the equations and solution of this model.

## 4.2. Data and assumptions for calibration 4.2.1. Supply functions

The supply curves are tuned so as to match estimated long term marginal production costs functions. According to OECD (2010), the REP production break-even point starts at  $\leq 80/MWh$  and

goes up to  $\leq 160/\text{MWh}$ . This marginal cost is rather a lower bound, as network and intermittency costs tend to raise it. We calibrated the REP supply function slope so as to reach the upper limit of the REP long-term marginal cost at a given percentage of a reference production level. This reference production level is taken equal to the electricity production from REP and fossil fuels in 2008, that is 2,060 TWh (ENERDATA, 2011). For the maximal penetration rate of REP, we took a range of possible percentages, ranging from 10% to 50%. The fossil fuel long term supply curve, set at  $\leq 80/\text{MWh}$  is tuned to an average European CCGT levelized cost of electricity, following OECD (2010).

#### 4.2.2. Demand function

The demand function has been calibrated so as to have a given price-elasticity when the demand equals the average between the 2008 and the 2009 reference production levels (2,060 TWh in 2008 and 1,929 TWh in 2009 (ENERDATA, 2011)). We chose elasticities ranging from -0.1 to -0.5. The demand standard deviation  $\Delta$  between the two states of the world was assumed to be close to the mean absolute deviation from the reference demand in 2008 and 2009. We chose values ranging from +50% to -50% of this value to account for the uncertainty on a possible future shock on demand. We assume each state of demand has a probability of  $\frac{1}{2}$  to occur.

#### 4.2.3. Abatement costs

The slope of the marginal abatement cost curve in the electricity sector has been calculated as follows: given an average  $CO_2$  price of  $\leq 22/tCO_2$  in 2008, we assumed that fuel-switch allowed to abate a range of percentages of the total emissions of the electricity sector in 2008, ranging from 1% to 5%. This is in range with Ellerman and Buchner (2008), reporting an abatement of around 5% at a CO<sub>2</sub> price equal to  $\leq 15/t$ CO<sub>2</sub>. The marginal abatement cost curve of the ETS sector other than electricity was calibrated in the same way, by assuming a certain percentage of abatement in 2008 given the CO<sub>2</sub> price. We assumed abatements ranging from 1% to 5% for both sectors. The intercept of the marginal abatement cost curve for non-electricity sectors in the low-demand state was calculated so as to obtain the difference of allowance over-allocation between 2008 and 2009 when the CO<sub>2</sub> price drops to zero (102 MtCO<sub>2</sub> of allowance surplus in 2008, 241  $MtCO_2$  surplus in 2009; data from Sandbag (2012).

	Units	Description	Min	Median	Max
$\sigma_a$	$(\in/\mathrm{MtCO_2}^2)$	Slope of the power sector MACC	0.44	0.81	2.2
$\overline{\sigma_e}$	$(\in/\mathrm{MtCO_2}^2)$	Slope of the rest-of-ETS MACC	0.52	0.95	2.61
$\overline{\sigma_r}$	(GWh <sup>2</sup> /€)	Slope of the demand function	2.58	6.7	12.9
$\overline{\sigma_d}$	(GWh <sup>2</sup> /€)	Slope of the RE supply function	2.49	6.43	12.5
δ	(€/tCO <sub>2</sub> )	Marginal environmental damage	10	15.3	30
$\overline{\Delta}$	(TWh)	Variance of demand	32.8	69.6	98.3
$\tau$	$(tCO_2/MWh)$	Average carbon intensity of fossil fuel-based electricity	0.5	0.5	0.5
λ	-	Probability of the high-demand state	0.5	0.5	0.5
$\iota_f$	(€/MWh)	Intercept of the fossil fuel supply function	80	80	80
$\iota_r$	(€/MWh)	Intercept of the RE supply function	80	80	80
$\iota_d$	(G€/MWh)	Intercept of the demand function	2.19	2.51	2.99
/-	(€/tCO <sub>2</sub> )	Intercept of the rest-of-ETS MACC (state +)	94.6	173	473
$\iota_e$		Intercept of the rest-of-ETS MACC (state –)	0	0	0

Table 5: Values of the calibrated parameters.

We took into account the perimeter of the ETS combustion sector — which includes electricity and heat production — by adding the additional surplus allowances coming from the heat plants (41 MtCO<sub>2</sub> according to Trotignon and Delbosc (2008)).

#### 4.2.4. Additional parameters

We took an average carbon intensity of 0.5 tCO<sub>2</sub>/MWh for fossil production (IEA Statistics, 2011), and a marginal damage between €10 and €30/tCO<sub>2</sub>. The calibration presented in previous paragraphs is very cautious, considering demand and production levels already observed in 2008 and 2009. The increased regulatory risk induced by the introduction of the third ETS phase and possible changes in the future Energy Efficiency Directive are captured through changing the standard deviation of demand and emission surplus from the non electricity ETS sector.

Table 6 synthesizes the range of values used for all parameters subject to a large uncertainty.

## 4.3. Optimal policy instruments and CO<sub>2</sub> price levels

With the parameter ranges shown in Table 5, 50.9% of the simulations display a nil carbon price in the low-demand state and a strictly positive REP subsidy. Figure 2 illustrates Proposition 1. It shows box whisker plots of the optimal emission cap  $\Omega^*$  (Fig. 2a) and the optimal REP subsidy  $\rho^*$  (Fig.

2b) in all simulations with a  $2^{nd}$  Best instrument setting (mix  $M_2^n$ ) and a nil carbon price in the low-demand state. Figure 2c shows a box whisker plot of the expected  $CO_2$  price.

The optimal emission cap ranges from 0.91 to 1.02 GtCO<sub>2</sub>, and the optimal subsidy ranges from  $\leq 2.68$ /MWh to  $\leq 9.93$ /MWh. The optimal expected  $CO_2$  price ranges from €2.97/t $CO_2$  to €13.6/tCO<sub>2</sub>. As a comparison, the actual cap calculated by Trotignon and Delbosc (2008)) amounts to 1.05 GtCO<sub>2</sub>, the actual REP tariff range from €50/MWh to €90/MWh in France and Germany and since summer 2011, the CO<sub>2</sub> price has been in the range ( $\leqslant 3/tCO_2 - \leqslant 13/tCO_2$ ). The relatively low levels of both the expected CO<sub>2</sub> price and the REP subsidy are due to the fact that it is a linear combination of both that equals the marginal damage (see (12)). These values cannot necessarily be directly compared to actual subsidy levels since the latter account for all positive externalities expected from REP support.

#### 4.4. Expected welfare gains from adding a REP subsidu

In order to evaluate the gains from adding a subsidy to the ETS, we compute the expected welfare differences between simulations with different instrument mixes. We compare four settings:

• A first-best instrument mix (M<sub>1</sub>), with a unique CO<sub>2</sub> price across all states of the world;

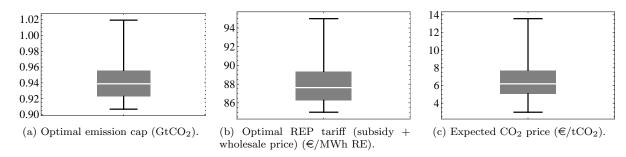


Figure 2: Box whisker plots of the optimal instrument values and expected  $CO_2$  price for all simulations with a  $2^{nd}$  Best instrument setting (mix  $M_2^n$ ) and a nil carbon price in the low-demand state of the world.

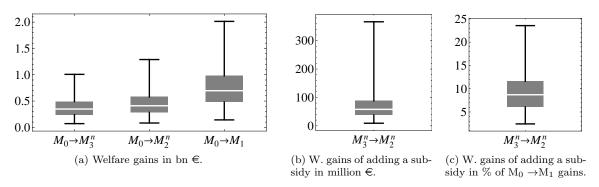


Figure 3: Box whisker plots of expected welfare gains from adding a given instrument mix to a BAU setting with no instrument  $(M_0 \to M_3^n, M_0 \to M_2^n, M_0 \to M_1)$  in bn  $\in$ , and of expected welfare gains from adding a REP subsidy to an ETS  $(M_3^n \to M_2^n)$  in million  $\in$  and in percentage of the expected gains from a carbon tax, in all scenarios where the  $CO_2$  price is nil in the low-demand state of the world.

- A second-best instrument mix (M<sub>2</sub>), with an ETS and a REP subsidy;
- A third-best instrument mix (M<sub>3</sub>), with an ETS alone and a nil CO<sub>2</sub> price in the lowdemand state.
- A business-as-usual setting  $(M_0)$ , with no policy at all.

The gain — or welfare difference — is calculated as the drop in environmental damages minus mitigation costs. Fig. 3 shows box whisker plots of the expected welfare gains from adding a given instument mix compared to the BAU setting ( $M_0$  to  $M_3$ ,  $M_0$  to  $M_2$ ,  $M_0$  to  $M_1$ ) in all scenarios where uncertainty is such that the  $CO_2$  price turns out to be nil in the low-demand state of the world.

Compared to a BAU setting with no instrument (mix  $M_0$ ), The gains from having an ETS and a REP subsidy if there is a risk that the  $CO_2$  price

equals zero in the low-demand state are quite important, ranging from more than  $\in 1.4$  billion to several hundred million  $\in$ . The gains from adding a REP subsidy to an ETS range from ca.  $\in 10$  million to several hundred million  $\in$ . They represent from approximately 3% to 24% of the gains one could expect from a first-best carbon tax.

## 4.5. Expected emissions, productions and prices with various instrument mixes

Following our analysis in section 3 and illustrating Proposition 3, the Fig. 4 presents box whisker plots of expected values of different variables in the simulations with a nil  $CO_2$  price in the low-demand state (superscript n). We computed those values with a 1<sup>st</sup> Best instrument mix (a carbon tax, labeled  $M_1$ ), with a 2<sup>nd</sup> Best setting (ETS + subsidy, labeled  $M_2^n$ ) and in a 3<sup>rd</sup> Best setting (ETS alone, labeled  $M_3^n$ ). Figure 4a presents the expected emissions, Figure 4b the expected  $CO_2$  price, Figure 4c

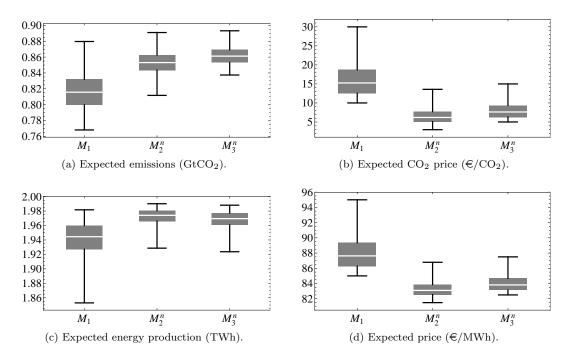


Figure 4: Box whisker plots of the expected values of various variables in simulations  $M_1$  (carbon tax),  $M_2^n$  (ETS + REP subsidy) and  $M_3^n$  (ETS alone) when the  $CO_2$  price is nil in the low-demand state.

the expected energy production and Figure 4d the expected wholesale price.

Consistently with Proposition 3, Figure 4a shows that expected emissions are lower in the  $M_2^n$  setting than in the  $M_3^n$  setting, and the lowest in the  $M_1$  setting. The expected  $CO_2$  price is the lowest in the  $M_2^n$  setting. As a result, the wholesale price is also the smallest in the  $M_2^n$  setting, but expected energy production is the highest.

## 4.6. Shift in the optimal emission cap and $CO_2$ price

In order to discuss the optimization behavior of the social planner, we analyze the optimal instrument levels and carbon price in the second-best setting (labeled  $M_2$ ) for all parameter combinations. For each combination, the uncertainty on the electricity demand is either low enough to get an optimal emission cap that is binding in both states of demand  $(M_2^p)$ , either too high and implies a nil  $CO_2$  price in the low-demand state of the world  $(M_2^n)$ . We then compare the two groups of simulations and show the results as box whisker plots in Fig. 5. Fig. 5a shows the optimal emission cap for all parameter combination, Fig. 5b the REP subsidy, Fig. 5c the  $CO_2$  price in the high-demand

state of the world and Fig. 5d the  $CO_2$  price in the low-demand state of the world.

As already discussed in section 2, Fig. 5a shows a higher emission cap in all  $M_2^n$  scenarios. This is due to the fact that when the  $CO_2$  price turns out to be nil in the low-demand state, no additional mitigation effort is made in this state and the cap is optimized ex-ante on the high demand level. Fig. 5b, 5c and 5d illustrate Proposition 2. If there is a risk that the  $CO_2$  price equals zero as for all  $M_2^n$  scenarios in Fig. 5d, there is a strictly positive subsidy ( $M_2^n$  scenarios in Fig. 5b) and the  $CO_2$  price in the high-demand state of the world drops compared to  $M_2^p$  scenarios(Fig. 5c).

#### 5. Discussion and conclusion

We bring a new contribution to the analysis of the coexistence of several policy instruments to cover the same emission sources. We find that optimizing simultaneously an ETS and e.g. a subsidy to renewable energy power (REP) can improve the welfare compared to a situation with the ETS alone, especially if uncertainty on the level of electricity demand (and hence on the abatement costs) is high enough. In a context of a very low CO<sub>2</sub> price and

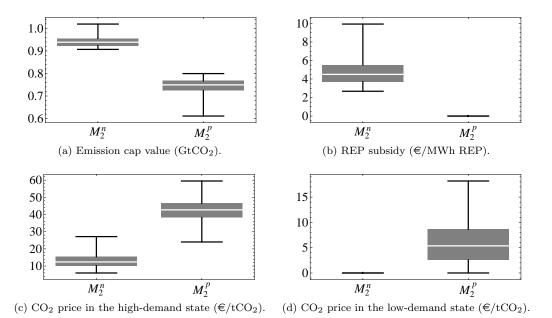


Figure 5: Box whisker plot of various instrument levels and  $CO_2$  price in simulations  $M_2^n$  (ETS + REP subsidy and a nil  $CO_2$  price in the low-demand state) and simulations  $M_2^n$  (ETS + REP subsidy and a strictly positive  $CO_2$  price in the low-demand state).

large anticipated surplus on the EU ETS at least until 2020, these findings justify the addition of other policy instruments aiming at reducing  $\rm CO_2$  emissions covered by the ETS to a possible future revision of the emission cap.

We find that under a reasonable set of parameters, defining simultaneously an emission cap and an overlapping policy instrument, such as a REP subsidy of about €2.7/MWh to €9.9/MWh (corresponding to a tariff ranging from €85/MWh to €95/MWh) can improve welfare by about 2.4% to 23.6% of the total gain of a carbon tax, that is about €9 million/yr to €366 million/yr. This gain is obtained through CO<sub>2</sub> emission reductions alone and does not rely on additional market failures or externalities. The addition of a REP subsidy also increases the total energy production, decreases the electricity price and the CO<sub>2</sub> price and reduces the total expected emissions. Our results are in line with existing literature concerning the decreasing effect of a REP subsidy on the carbon price when it is combined with an emission cap. We however find that under certain circumstances, interactions between a subsidy and an emission cap can reduce emissions and improve welfare, compared to an emission cap alone.

On a more methodological note, our results invite to deepen the reflection on the role of uncertainty. Noticeably, they highlight the possibility of corner solutions (in this case, a zero  $\mathrm{CO}_2$  price), when comparing policy instruments and policy packages. In addition to showing that an optimal policy mix to reduce  $\mathrm{CO}_2$  emissions can contain more than one instrument, we find several key analytical results that qualitatively differ from the literature. For instance, expected emissions are no longer equivalent between policy instruments, even with an additive uncertainty on the marginal abatement cost, and the optimal emission cap no longer depends on all states of nature but only on the high-demand one.

Our results are based on the assumption that the risk of the CO<sub>2</sub> price dropping to zero cannot be excluded. The history of many cap-and-trade systems, including the US acid rain program, RGGI and the EU ETS, fully justifies this assumption, since the allowance price has dropped to virtually zero (or to the floor price) in all these systems. Moreover, uncertainty on the CO<sub>2</sub> price does not only stem from the business cycle, as in our model, but also from uncertainty on future policies, such as the Energy Efficiency Directive whose implementation is currently debated in the EU. Our analysis

brings some economic insight into the debate about the future European policy mix and about whether it is justified or preferable to complement a future revision of the EU-ETS cap with an overlapping instrument.

While developping renewables is a valuable option to mitigate emissions, our results could be obtained with any instrument giving an incentive to reduce emissions in states of the world with low demand levels. Instruments promoting energy efficiency could be equally efficient, provided the actual energy consumption reduction is calculated against the right baseline. One could imagine instruments being more efficient in low-demand states than in high-demand states where mitigation is already incentivized by the positive carbon price, such as efficiency standards based on the mitigation effort. It is hard however to imagine how such instruments would work in practice. Moreover, we explore only one channel of potential interactions, namely uncertainty combined to the unavailability of a carbon tax. Other justifications and effects should be considered when trying to give an accurate picture of the potential efficiency of an instrument addition to the ETS, such as learning or innovation considerations and dynamic or general equlibrium effects for example.

Complementing an ETS with price-like features, such as an auction reserve price or a price floor as argued by Fankhauser et al. (2010) would bring the necessary incentives in the low-demand state. Our results depend however on the second-best framework implied by an inefficient ETS. Optimizing an auction price or a floor price along with the emission ceiling, as in our model, would effectively allow to get back to a first-best framework by imposing a floor at the Pigovian level. If on the contrary one assumes the CO<sub>2</sub> price, the floor price or the auction reserve price to be "too low" (i.e. below the Pigovian level), as does Hoel (2012), our framework becomes relevant again and an additional instrument becomes welfare-improving.

Further aspects could be worth investigating. Modeling banking across trading periods with periodic renegotiation of the cap could mitigate the sub-optimality of the ETS hence the room for complementary policies, but it would seriously complicate the analysis without necessarily providing new insights. Assuming other sources of uncertainty, such as technological or regulatory uncertainty could also have an effect on the outcome, depending on the probability associated with a nil

carbon price. Finally, we focus our analysis on one channel of positive interactions between several mitigation instruments. Completing the picture by incorporating other market failures could bring useful insights on the benefits brought by adding a mitigation instrument to the ETS.

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#### Appendix A. Proof of Proposition 3

We compute expected emissions in three instrument mix settings (see Appendix C for a description of all instrument settings used and a reference to the expression of the complete solution):

- A first-best instrument mix, with a unique CO<sub>2</sub> price across all states of the world<sup>9</sup>;
- A second-best instrument mix, with an ETS and a REP subsidy;
- A third-best instrument mix, with an ETS alone.

The uncertainty is assumed to be such as the  $CO_2$  price resulting from an ETS in the low-demand state turns out to be nil (as shown in the model description above). The expected emissions  $\mathcal{E}_e$  are given by:

$$\mathcal{E}_e = \sum_{s \in \text{states}} \mathcal{P}_s \cdot (\tau \cdot f_s - a_s) \tag{A.1}$$

Let us call  $\mathcal{E}_{e,X}$  the expected emissions for a given instrument mix  $X \in [1,2,3]$ , the index referring respectively to the first-best, second-best and third-

<sup>&</sup>lt;sup>9</sup>Since the marginal damage is flat, the first-best instrument is always a price instrument, e.g. a carbon tax.

best mix as described above:

$$\mathcal{E}_{e,1} = \iota_d \tau - \Delta (1 - 2\lambda)\tau + \iota_r \sigma_r \tau$$

$$- \iota_f (\sigma_d + \sigma_r)\tau - \delta (1/(\sigma_a))$$

$$+ (\sigma_d + \sigma_r)(\tau)^2)$$

$$\mathcal{E}_{e,2} = \iota_d \tau - \Delta (1 - 2\lambda)\tau + \iota_r \sigma_r \tau$$

$$- \iota_f (\sigma_d + \sigma_r)\tau$$

$$- \frac{(\delta \sigma_a (\sigma_d + \sigma_r)(\lambda(\sigma_d - \sigma_r) + \sigma_r)(\tau)^4)}{(1 + \sigma_a (\sigma_d + \sigma_r - \lambda \sigma_r)(\tau)^2)}$$

$$- \frac{(\delta (\lambda + \sigma_a (2\lambda \sigma_d + \sigma_r)(\tau)^2))}{(\sigma_a + (\sigma_a)^2 (\sigma_d + \sigma_r - \lambda \sigma_r)(\tau)^2)}$$

$$\mathcal{E}_{e,3} = \iota_d \tau - \Delta (1 - 2\lambda)\tau + \iota_r \sigma_r \tau$$

$$- \iota_f (\sigma_d + \sigma_r)\tau$$

$$- \delta \lambda (1/(\sigma_a) + (\sigma_d + \sigma_r)(\tau)^2)$$
(A.2)

where we see that  $\mathcal{E}_{e,1} < \mathcal{E}_{e,2}$  and that  $\mathcal{E}_{e,2} < \mathcal{E}_{e,3}$ . This result can be linked to the differences i nthe expected carbon price.

$$C = \sum_{s \in \text{states}} \mathcal{P}_s \cdot \phi_s \tag{A.5}$$

The expected carbon price for a given instrument mix  $X \in [1, 2, 3]$  is:

$$C_1 = \delta \tag{A.6}$$

$$C_2 = \delta \cdot \frac{\lambda (1 + \sigma_a \sigma_d(\tau)^2)}{1 + \sigma_a (\sigma_d + \sigma_r - \lambda \sigma_r) \tau^2}$$
(A.7)

$$C_3 = \lambda \delta \tag{A.8}$$

with  $C_2 < C_1$  and that  $C_2 < C_3$ .

#### Appendix B. Proof of Proposition 4

We solve the model by assuming only

$$\begin{cases} \tau \cdot f_+ - a_+ = \Omega \\ \phi_+ > 0 \end{cases}$$

and make no assumption on the level of emissions in the low-demand state. By using the method developed in Appendix E, we compute the difference of the emissions minus the cap:

$$\begin{split} &(\tau \cdot f_{-} - a_{-}) - \Omega = \\ &- (\frac{(2\Delta\sigma_{a}(\sigma_{d} + (-1 + \lambda)\sigma_{r})(\tau)^{3})}{(1 + \sigma_{a}(\sigma_{d} + \sigma_{r} - \lambda\sigma_{r})(\tau)^{2}))} \\ &+ \frac{(\delta\sigma_{a}\sigma_{d}(\sigma_{d} + \sigma_{r})(\tau)^{4})}{(1 + \sigma_{a}(\sigma_{d} + \sigma_{r} - \lambda\sigma_{r})(\tau)^{2})} \\ &+ \frac{(\delta - 2\Delta\sigma_{a}\tau + \delta\sigma_{a}(2\sigma_{d} + \sigma_{r})(\tau)^{2})}{(\sigma_{a} + (\sigma_{a})^{2}(\sigma_{d} + \sigma_{r} - \lambda\sigma_{r})(\tau)^{2})} \end{split}$$

We then compute the partial derivative of this expression with respect to all parameters, and test their positivity.

# Appendix C. Description of the model types and instrument settings used in the analytical and numerical results

Table C.7 links the names used in the text and the instrument settings used in each case. The detailed description of the model framework and the optimal solution calculated using Mathematica are given in the subsequent Appendices. Calculation sheets are available upon request to the authors.

The model used for the analytical results differ slightly from the model used for the numerical results. The numerical model allows for allowance trading by adding an emitting sector from which the electricity producer can buy surplus allowances. The instruments settings and names attached are the same for both models.

Appendix D to Appendix H show the framework and optimal solution for the model used in the analytcal part. Appendix I show the framework and optimal solution for the model used in the numerical part, with the  $M_2^n$  setting. Showing the details of all settings for the model used in the numerical part would be very long and are not shown here. They are available upon request to the authors.

## Appendix D. First Best setting: model with carbon tax

To simulate an economy-wide carbon tax, we add following constraint to the model framework from Section 3:

$$\phi_- = \phi_+$$

The socially optimal level of all market variables for the high-demand state (subscript +) and low demand (subscript -) are:

$$\Omega^{\star} = -\left(\frac{\delta}{\sigma_a}\right) + \Delta\tau + \iota_d\tau - \iota_f\sigma_d\tau - \iota_f\sigma_r\tau + \iota_r\sigma_r\tau - \delta\sigma_d(\tau)^2 - \delta\sigma_r(\tau)^2$$
$$\rho^{\star} = 0$$
$$f_{-}^{\star} = -\Delta + \iota_d + \iota_r\sigma_r - \iota_f(\sigma_d + \sigma_r) - \delta(\sigma_d + \sigma_r)\tau$$

Label	Nature	Instrument setting			$p_{CO_2}^-$	Described in
		Carbon tax	ETS	REP Subsidy		
$M_1$	1 <sup>st</sup> Best	Yes	Useless	Useless	Positive	Appendix D
$M_2^n$	2 <sup>nd</sup> Best	Unavailable	Yes	Yes	Nil	Appendix E
$M_2^p$	$2^{\rm nd}$ Best	Unavailable	Yes	Yes	Positive	Appendix F
$M_3^n$	3 <sup>rd</sup> Best	Unavailable	Yes	Unavailable	Nil	Appendix G
$M_3^p$	3 <sup>rd</sup> Best	Unavailable	Yes	Unavailable	Positive	Appendix H

Table C.7: Description of the model types and instrument settings

$$r_{-}^{\star} = \sigma_{r}(\iota_{f} - \iota_{r} + \delta\tau)$$

$$p_{-}^{\star} = \iota_{f} + \delta\tau$$

$$a_{-}^{\star} = \frac{\delta}{\sigma_{a}}$$

$$\phi_{-}^{\star} = \delta$$

$$f_{+}^{\star} = \Delta + \iota_{d} + \iota_{r}\sigma_{r} - \iota_{f}(\sigma_{d} + \sigma_{r})$$

$$- \delta(\sigma_{d} + \sigma_{r})\tau$$

$$r_{+}^{\star} = \sigma_{r}(\iota_{f} - \iota_{r} + \delta\tau)$$

$$p_{+}^{\star} = \iota_{f} + \delta\tau$$

$$a_{+}^{\star} = \frac{\delta}{\sigma_{a}}$$

$$\phi_{+}^{\star} = \delta$$

Solving the profit maximization problem of the producer gives the reaction functions of producers, depending on the level of policy instruments and the state of the world (the first-order conditions are given in (4-6)). Solving the welfare maximization problem of the social planner knowing all the reaction functions gives the following first-order conditions:

$$\left(\frac{\partial EW}{\partial \rho} = 0, \frac{\partial EW}{\partial \Omega} = 0\right) \Rightarrow$$

$$0 = \frac{(\rho^* \sigma_a \sigma_r (\sigma_d + \sigma_r - \lambda \sigma_r)(\tau)^2)}{(1 + \sigma_a (\sigma_d + \sigma_r)(\tau)^2)}$$

$$+ \frac{(\delta(-1 + \lambda)\sigma_a \sigma_r (\sigma_d + \sigma_r)(\tau)^3)}{(1 + \sigma_a (\sigma_d + \sigma_r)(\tau)^2)}$$

$$+ \frac{(\sigma_r (\rho^* + \delta(-1 + \lambda)\tau))}{(1 + \sigma_a (\sigma_d + \sigma_r)(\tau)^2)}$$

$$0 = \frac{(\iota_f \lambda \sigma_a (\sigma_d + \sigma_r)(\tau)^2)}{(1 + \sigma_a (\sigma_d + \sigma_r)(\tau)^2)}$$

$$+ \frac{(\lambda(\delta - (\Delta + \iota_d)\sigma_a \tau))}{(1 + \sigma_a (\sigma_d + \sigma_r)(\tau)^2)}$$

$$+ \frac{(\lambda \sigma_a \tau (-\iota_r \sigma_r + \delta \sigma_d \tau))}{(1 + \sigma_a (\sigma_d + \sigma_r)(\tau)^2)}$$

$$+ \frac{(\lambda \sigma_a (\Omega^* + \delta \sigma_r (\tau)^2))}{(1 + \sigma_a (\sigma_d + \sigma_r)(\tau)^2)}$$

$$+ \frac{(\lambda \sigma_a (\Omega^* + \delta \sigma_r (\tau)^2))}{(1 + \sigma_a (\sigma_d + \sigma_r)(\tau)^2)}$$

$$(E.2)$$

from which we directly derive the optimal level of the policy instruments. By substituting the optimal levels of policy instruments in the reaction functions, we obtain the socially optimal level of all market variables for the high-demand state (subscript +) and low demand (subscript -).

The optimal solution is:

$$\Omega^{\star} = -\left(\frac{\delta}{\sigma_a}\right) + \Delta\tau + \iota_d\tau - \iota_f\sigma_d\tau - \iota_f\sigma_r\tau + \iota_r\sigma_r\tau - \delta\sigma_d(\tau)^2 - \delta\sigma_r(\tau)^2$$

$$\rho^{\star} = -\left(\frac{\left(\delta\sigma_a(-\sigma_r + \lambda(\sigma_d + \sigma_r))(\tau)^3\right)}{\left(1 + \sigma_a(\sigma_d + \lambda\sigma_r)(\tau)^2\right)}\right)$$

$$-\frac{\left(\delta\tau(-(1/2) - \sigma_a\sigma_d(\tau)^2\right)}{\left(1 + \sigma_a(\sigma_d + \lambda\sigma_r)(\tau)^2\right)}$$

$$f_{-}^{\star} = -\Delta + \iota_d - \frac{\left(2(-\iota_r\sigma_r + \iota_f(\sigma_d + \sigma_r))\right)}{\left(2 + \sigma_a(2\sigma_d + \sigma_r)(\tau)^2\right)}$$

$$+\frac{\left((\iota_f - \iota_r)\sigma_a(\sigma_r)^2(\tau)^2\right)}{\left(2 + \sigma_a(2\sigma_d + \sigma_r)(\tau)^2\right)}$$

$$\begin{split} &+ \frac{(2\sigma_{a}\sigma_{d}(\iota_{f}\sigma_{d} - \iota_{r}\sigma_{r})(\tau)^{2})}{(2 + \sigma_{a}(2\sigma_{d} + \sigma_{r})(\tau)^{2})} \\ &+ \frac{(\delta\sigma_{a}\sigma_{r}(\sigma_{d} + \sigma_{r})(\tau)^{2})}{(2 + \sigma_{a}(2\sigma_{d} + \sigma_{r})(\tau)^{2})} \\ &+ \frac{(\sigma_{r}\tau(\delta + 3\iota_{f}\sigma_{a}\sigma_{d}\tau))}{(2 + \sigma_{a}(2\sigma_{d} + \sigma_{r})(\tau)^{2})} \\ &+ \frac{(\sigma_{r}\tau(\delta + 3\iota_{f}\sigma_{a}\sigma_{d}\tau))}{(2 + \sigma_{a}(2\sigma_{d} + \sigma_{r})(\tau)^{2})} \\ &+ \frac{((\iota_{f} - \iota_{r})\sigma_{a}\sigma_{d}\sigma_{r}(\tau)^{2})}{(2 + \sigma_{a}(2\sigma_{d} + \sigma_{r})(\tau)^{2})} \\ &+ \frac{((\iota_{f} - \iota_{r})\sigma_{a}(\sigma_{r})^{2}(\tau)^{2})}{(2 + \sigma_{a}(2\sigma_{d} + \sigma_{r})(\tau)^{2})} \\ &+ \frac{(\delta\sigma_{a}\sigma_{r}(\sigma_{d} + \sigma_{r})(\tau)^{2})}{(2 + \sigma_{a}(2\sigma_{d} + \sigma_{r})(\tau)^{2})} \\ &+ \frac{(\sigma_{r}(2\iota_{f} - 2\iota_{r} + \delta\tau))}{(2 + \sigma_{a}(2\sigma_{d} + \sigma_{r})(\tau)^{2})} \\ &p_{-}^{\star} = \iota_{f} \\ &a_{-}^{\star} = 0 \\ &f_{+}^{\star} = \Delta + \iota_{d} + \iota_{r}\sigma_{r} - \iota_{f}(\sigma_{d} + \sigma_{r}) - \delta(\sigma_{d} + \sigma_{r})\tau \\ &- \frac{(\delta\sigma_{r}\tau)}{(2(1 + \sigma_{a}(\sigma_{d} + \lambda\sigma_{r})(\tau)^{2})} \\ &+ \frac{((\iota_{f} - \iota_{r})\sigma_{a}\sigma_{d}\sigma_{r}(\tau)^{2})}{(2 + \sigma_{a}(2\sigma_{d} + \sigma_{r})(\tau)^{2})} \\ &+ \frac{(\delta\sigma_{a}\sigma_{r}(3\sigma_{d} + \sigma_{r})(\tau)^{2})}{(2 + \sigma_{a}(2\sigma_{d} + \sigma_{r})(\tau)^{2})} \\ &+ \frac{(\delta\sigma_{a}\sigma_{r}(3\sigma_{d} + \sigma_{r})(\tau)^{2})}{(2 + \sigma_{a}(2\sigma_{d} + \sigma_{r})(\tau)^{2})} \\ &p_{+}^{\star} = \frac{(\sigma_{a}(\tau)^{2}(\iota_{f}\lambda\sigma_{r} + \delta\sigma_{d}\tau))}{(1 + \sigma_{a}(\sigma_{d} + \lambda\sigma_{r})(\tau)^{2})} \\ &+ \frac{(\iota_{f} + \delta\tau + \iota_{f}\sigma_{a}\sigma_{d}(\tau)^{2})}{(1 + \sigma_{a}(\sigma_{d} + \lambda\sigma_{r})(\tau)^{2})} \\ &\sigma_{+}^{\star} = \frac{(\delta + \delta\sigma_{a}\sigma_{d}(\tau)^{2})}{(\sigma_{a} + (\sigma_{a})^{2}(\sigma_{d} + \lambda\sigma_{r})(\tau)^{2})} \\ &\phi_{+}^{\star} = \frac{(\delta + \delta\sigma_{a}\sigma_{d}(\tau)^{2})}{(1 + \sigma_{a}(\sigma_{d} + \lambda\sigma_{r})(\tau)^{2})} \end{aligned}$$

# Appendix F. Second Best setting: model with ETS, REP subsidy and a strictly positive CO<sub>2</sub> price in the low-demand state

We assumed through this paper that the carbon price is nil in the low-demand state of the world. This is the case for certain parameter combinations,

as discussed in section 3.4. For some other combinations, the carbon price remains positive in both states, and the model is changed as follows. Equation (3) becomes:

$$\begin{cases} \tau \cdot f_- - a_- = \Omega \\ \phi_- > 0 \end{cases} \quad \text{or} \quad \begin{cases} \tau \cdot f_+ - a_+ = \Omega \\ \phi_+ > 0 \end{cases}$$

The optimal solution changes also and becomes:

$$\begin{split} \Omega^{\star} &= -(\frac{\delta}{\sigma_a}) + \iota_d \tau - \iota_f \sigma_d \tau \\ &- \iota_f \sigma_r \tau + \iota_r \sigma_r \tau - \delta \sigma_d(\tau)^2 - \delta \sigma_r(\tau)^2 \\ \rho^{\star} &= 0 \\ f^{\star}_- &= \iota_d + \iota_r \sigma_r - \iota_f (\sigma_d + \sigma_r) - \delta (\sigma_d + \sigma_r) \tau \\ &- \frac{(\Delta)}{(1 + \sigma_a (\sigma_d + \sigma_r)(\tau)^2)} \\ r^{\star}_- &= -(\frac{(\Delta \sigma_a \sigma_r(\tau)^2)}{(1 + \sigma_a (\sigma_d + \sigma_r)(\tau)^2)} \\ &+ \frac{((\iota_f - \iota_r) \sigma_a \sigma_d \sigma_r(\tau)^2)}{(1 + \sigma_a (\sigma_d + \sigma_r)(\tau)^2)} \\ &+ \frac{(\delta \sigma_a \sigma_r (\sigma_d + \sigma_r)(\tau)^2)}{(1 + \sigma_a (\sigma_d + \sigma_r)(\tau)^2)} \\ &+ \frac{(\delta \sigma_a \sigma_r (\sigma_d + \sigma_r)(\tau)^2)}{(1 + \sigma_a (\sigma_d + \sigma_r)(\tau)^2)} \\ p^{\star}_- &= \frac{(\sigma_a (-\Delta + \iota_f (\sigma_d + \sigma_r))(\tau)^2)}{(1 + \sigma_a (\sigma_d + \sigma_r)(\tau)^2)} \\ &+ \frac{(\delta \sigma_a (\sigma_d + \sigma_r)(\tau)^3)}{(1 + \sigma_a (\sigma_d + \sigma_r)(\tau)^2)} \\ &+ \frac{(\delta \sigma_a (\sigma_d + \sigma_r)(\tau)^2)}{(1 + \sigma_a (\sigma_d + \sigma_r)(\tau)^2)} \\ \phi^{\star}_- &= \delta - \frac{(\Delta \tau)}{(1 + \sigma_a (\sigma_d + \sigma_r)(\tau)^2)} \\ f^{\star}_+ &= \iota_d + \iota_r \sigma_r - \iota_f (\sigma_d + \sigma_r)(\tau)^2) \\ f^{\star}_+ &= \iota_d + \iota_r \sigma_r - \iota_f (\sigma_d + \sigma_r)(\tau)^2) \\ r^{\star}_+ &= -(\frac{(\iota_r \sigma_a \sigma_d \sigma_r(\tau)^2)}{(1 + \sigma_a (\sigma_d + \sigma_r)(\tau)^2)}) \\ &+ \frac{(\sigma_a (\Delta + \iota_f \sigma_d) \sigma_r(\tau)^2)}{(1 + \sigma_a (\sigma_d + \sigma_r)(\tau)^2)} \\ &+ \frac{(\sigma_a (\Delta + \iota_f \sigma_d) \sigma_r(\tau)^2)}{(1 + \sigma_a (\sigma_d + \sigma_r)(\tau)^2)} \end{split}$$

$$\begin{split} &+\frac{\left((\iota_f-\iota_r)\sigma_a(\sigma_r)^2(\tau)^2\right)}{(1+\sigma_a(\sigma_d+\sigma_r)(\tau)^2)} \\ &+\frac{\left(\delta\sigma_a\sigma_r(\sigma_d+\sigma_r)(\tau)^3\right)}{(1+\sigma_a(\sigma_d+\sigma_r)(\tau)^2)} \\ &+\frac{\left(\sigma_r(\iota_f-\iota_r+\delta\tau)\right)}{(1+\sigma_a(\sigma_d+\sigma_r)(\tau)^2)} \\ &+\frac{\left(\sigma_a(\Delta+\iota_f\sigma_d)(\tau)^2\right)}{(1+\sigma_a(\sigma_d+\sigma_r)(\tau)^2)} \\ &+\frac{\left(\iota_f+\delta\tau\right)}{(1+\sigma_a(\sigma_d+\sigma_r)(\tau)^2)} \\ &+\frac{\left(\sigma_a(\tau)^2(\iota_f\sigma_r+\delta(\sigma_d+\sigma_r)\tau)\right)}{(1+\sigma_a(\sigma_d+\sigma_r)(\tau)^2)} \\ &a_+^\star = \frac{\delta}{\sigma_a} +\frac{\left(\Delta\tau\right)}{(1+\sigma_a(\sigma_d+\sigma_r)(\tau)^2)} \\ &\phi_+^\star = \delta +\frac{\left(\Delta\sigma_a\tau\right)}{(1+\sigma_a(\sigma_d+\sigma_r)(\tau)^2)} \end{split}$$

## Appendix G. Third Best setting: model with ETS only and a nil CO<sub>2</sub> price in the low-demand state

To simulate a third-best setting with no REP subsidy, we add following constraint to the model framework from Section 3:

$$\rho = 0$$

The socially optimal level of all market variables for the high-demand state (subscript +) and low demand (subscript -) are:

$$\Omega^{\star} = -\left(\frac{\delta}{\sigma_{a}}\right) + \Delta\tau + \iota_{d}\tau - \iota_{f}\sigma_{d}\tau - \iota_{f}\sigma_{r}\tau + \iota_{r}\sigma_{r}\tau - \delta\sigma_{d}(\tau)^{2} - \delta\sigma_{r}(\tau)^{2}$$

$$\rho^{\star} = 0$$

$$f_{-}^{\star} = -\Delta + \iota_{d} + \iota_{r}\sigma_{r} - \iota_{f}(\sigma_{d} + \sigma_{r})$$

$$r_{-}^{\star} = (\iota_{f} - \iota_{r})\sigma_{r}$$

$$p_{-}^{\star} = \iota_{f}$$

$$a_{-}^{\star} = 0$$

$$\phi_{-}^{\star} = 0$$

$$f_{+}^{\star} = \Delta + \iota_{d} + \iota_{r}\sigma_{r} - \iota_{f}(\sigma_{d} + \sigma_{r}) - \delta(\sigma_{d} + \sigma_{r})\tau$$

$$r_{+}^{\star} = \sigma_{r}(\iota_{f} - \iota_{r} + \delta\tau)$$

$$p_{+}^{\star} = \iota_{f} + \delta\tau$$

$$a_{+}^{\star} = \frac{\delta}{\sigma_{a}}$$

$$\phi_{+}^{\star} = \delta$$

# Appendix H. Third Best setting: model with ETS only and a positive CO<sub>2</sub> price in the low-demand state

To simulate a third-best setting with no REP subsidy, we add following constraint to the model framework from Appendix F:

$$\rho = 0$$

The socially optimal level of all market variables for the high-demand state (subscript +) and low demand (subscript -) are:

$$\begin{split} \Omega^{\star} &= -(\frac{\delta}{\sigma_a}) + \iota_d \tau - \iota_f \sigma_d \tau - \iota_f \sigma_r \tau \\ &+ \iota_r \sigma_r \tau - \delta \sigma_d(\tau)^2 - \delta \sigma_r(\tau)^2 \\ \rho^{\star} &= 0 \\ f^{\star}_- &= \iota_d + \iota_r \sigma_r - \iota_f (\sigma_d + \sigma_r) - \delta (\sigma_d + \sigma_r) \tau \\ &- \frac{(\Delta)}{(1 + \sigma_a (\sigma_d + \sigma_r)(\tau)^2)} \\ r^{\star}_- &= \frac{(\sigma_a (-\Delta + (\iota_f - \iota_r) \sigma_d) \sigma_r(\tau)^2)}{(1 + \sigma_a (\sigma_d + \sigma_r)(\tau)^2)} \\ &+ \frac{((\iota_f - \iota_r) \sigma_a (\sigma_r)^2(\tau)^2)}{(1 + \sigma_a (\sigma_d + \sigma_r)(\tau)^2)} \\ &+ \frac{(\delta \sigma_a \sigma_r (\sigma_d + \sigma_r)(\tau)^3)}{(1 + \sigma_a (\sigma_d + \sigma_r)(\tau)^2)} \\ &+ \frac{(\sigma_r (\iota_f - \iota_r + \delta \tau))}{(1 + \sigma_a (\sigma_d + \sigma_r)(\tau)^2)} \\ p^{\star}_- &= \frac{(\sigma_a (-\Delta + \iota_f (\sigma_d + \sigma_r))(\tau)^2)}{(1 + \sigma_a (\sigma_d + \sigma_r)(\tau)^2)} \\ &+ \frac{(\delta \sigma_a (\sigma_d + \sigma_r)(\tau)^3)}{(1 + \sigma_a (\sigma_d + \sigma_r)(\tau)^2)} \\ d^{\star}_- &= \frac{\delta}{\sigma_a} - \frac{(\Delta \tau)}{(1 + \sigma_a (\sigma_d + \sigma_r)(\tau)^2)} \\ \phi^{\star}_- &= \delta - \frac{(\Delta \sigma)}{(1 + \sigma_a (\sigma_d + \sigma_r)(\tau)^2)} \\ f^{\star}_+ &= \iota_d + \iota_r \sigma_r - \iota_f (\sigma_d + \sigma_r) - \delta (\sigma_d + \sigma_r) \tau \\ &+ \frac{(\Delta)}{(1 + \sigma_a (\sigma_d + \sigma_r)(\tau)^2)} \\ r^{\star}_+ &= \frac{(\Delta \sigma_a \sigma_r (\tau)^2)}{(1 + \sigma_a (\sigma_d + \sigma_r)(\tau)^2)} \\ &+ \frac{((\iota_f - \iota_r) \sigma_a \sigma_r (\sigma_d + \sigma_r)(\tau)^2)}{(1 + \sigma_a (\sigma_d + \sigma_r)(\tau)^2)} \\ &+ \frac{((\iota_f - \iota_r) \sigma_a \sigma_r (\sigma_d + \sigma_r)(\tau)^2)}{(1 + \sigma_a (\sigma_d + \sigma_r)(\tau)^2)} \end{split}$$

$$\begin{split} &+\frac{\left(\delta\sigma_{a}\sigma_{r}(\sigma_{d}+\sigma_{r})(\tau)^{3}\right)}{\left(1+\sigma_{a}(\sigma_{d}+\sigma_{r})(\tau)^{2}\right)}\\ &+\frac{\left(\sigma_{r}(\iota_{f}-\iota_{r}+\delta\tau)\right)}{\left(1+\sigma_{a}(\sigma_{d}+\sigma_{r})(\tau)^{2}\right)}\\ &p_{+}^{\star}=\frac{\left(\Delta\sigma_{a}(\tau)^{2}\right)}{\left(1+\sigma_{a}(\sigma_{d}+\sigma_{r})(\tau)^{2}\right)}\\ &+\frac{\left(\iota_{f}\sigma_{a}(\sigma_{d}+\sigma_{r})(\tau)^{2}\right)}{\left(1+\sigma_{a}(\sigma_{d}+\sigma_{r})(\tau)^{2}\right)}\\ &+\frac{\left(\delta\sigma_{a}(\sigma_{d}+\sigma_{r})(\tau)^{2}\right)}{\left(1+\sigma_{a}(\sigma_{d}+\sigma_{r})(\tau)^{2}\right)}\\ &+\frac{\left(\iota_{f}+\delta\tau\right)}{\left(1+\sigma_{a}(\sigma_{d}+\sigma_{r})(\tau)^{2}\right)}\\ a_{+}^{\star}&=\frac{\delta}{\sigma_{a}}+\frac{\left(\Delta\tau\right)}{\left(1+\sigma_{a}(\sigma_{d}+\sigma_{r})(\tau)^{2}\right)}\\ \phi_{+}^{\star}&=\delta+\frac{\left(\Delta\sigma_{a}\tau\right)}{\left(1+\sigma_{a}(\sigma_{d}+\sigma_{r})(\tau)^{2}\right)} \end{split}$$

# Appendix I. Model with allowances from non-electricity ETS sectors and nil $CO_2$ price in the low-demand state

Section 4 extends the model and allows for allowance trading by adding an emitting sector from which the electricity producer can buy surplus allowances. This surplus is labeled e and its supply is modeled by a linear mac curve. The profit maximization problem becomes:

$$\max_{f,r,a,e} \Pi(p, f, r, a, e, \phi) = p \cdot f + (p + \rho) \cdot r$$
$$-C_f(f) - C_r(r) - AC(a)$$
$$-AC_e(e) - PC_e(f, a, e, \phi)$$

with

$$AC_e(e) = \frac{\sigma_e}{2}e^2 - \begin{cases} \iota_e \cdot e & \text{low demand} \\ 0 & \text{high demand} \end{cases}$$

The allowance purchasing cost is modified as follows:

$$PC_e(f, a, e, \phi) = \phi \cdot (\tau \cdot f - a + e)$$

and (3) becomes:

$$\begin{cases} \tau \cdot f_{-}^{\star} - a_{-}^{\star} < \Omega - e_{-}^{\star} \\ \phi_{-}^{\star} = 0 \end{cases}$$
 or 
$$\begin{cases} \tau \cdot f_{+}^{\star} - a_{+}^{\star} = \Omega - e_{+}^{\star} \\ \phi_{+}^{\star} > 0 \end{cases}$$

The welfare maximization problem becomes:

$$\max_{\Omega,\rho} EW(\Omega,\rho) = \sum_{\text{states}} \frac{1}{2} [CS(p) + \Pi(p, f, r, a, e, \phi) - dam_e(f, a, e) - \rho \cdot r + PC_e(f, a, e, \phi)]$$

where  $dam_e(\cdot)$  is the modified environmental damage function:

$$dam_e(f, a, e) = \delta \cdot (\tau f - a - e)$$

The optimal solution of this problem is the following:

$$\begin{split} \Omega^{\star} &= -(\frac{(\delta(\sigma_{a} + \sigma_{e}))}{(\sigma_{a}\sigma_{e})}) \\ &+ (\Delta + \iota_{d} + \iota_{r}\sigma_{r} - \iota_{f}(\sigma_{d} + \sigma_{r}))\tau \\ &- \delta(\sigma_{d} + \sigma_{r})(\tau)^{2} \\ \\ \rho^{\star} &= \frac{(\delta\tau(\sigma_{a} + \sigma_{e} + \sigma_{a}\sigma_{e}(\sigma_{d} + \sigma_{r})(\tau)^{2}))}{(2(\sigma_{a} + \sigma_{e}) + \sigma_{a}\sigma_{e}(2\sigma_{d} + \sigma_{r})(\tau)^{2})} \\ f^{\star}_{-} &= -\Delta + \iota_{d} - \iota_{f}\sigma_{d} - \iota_{f}\sigma_{r} \\ &+ \iota_{r}\sigma_{r} - \frac{(\delta\sigma_{r}(\sigma_{d} + \sigma_{r})\tau)}{(2\sigma_{d} + \sigma_{r})} \\ + \frac{(\delta(\sigma_{a} + \sigma_{e})(\sigma_{r})^{2}\tau)}{(2(\sigma_{a} + \sigma_{e}) + \sigma_{a}\sigma_{e}(2\sigma_{d} + \sigma_{r})(\tau)^{2})} \\ r^{\star}_{-} &= \frac{((\iota_{f} - \iota_{r})\sigma_{a}\sigma_{e}\sigma_{r}(2\sigma_{d} + \sigma_{r})(\tau)^{2})}{(2(\sigma_{a} + \sigma_{e}) + \sigma_{a}\sigma_{e}(2\sigma_{d} + \sigma_{r})(\tau)^{2})} \\ + \frac{(\delta\sigma_{a}\sigma_{e}\sigma_{r}(\sigma_{d} + \sigma_{r})(\tau)^{3})}{(2(\sigma_{a} + \sigma_{e}) + \sigma_{a}\sigma_{e}(2\sigma_{d} + \sigma_{r})(\tau)^{2})} \\ + \frac{((\sigma_{a} + \sigma_{e})\sigma_{r}(2\iota_{f} - 2\iota_{r} + \delta\tau))}{(2\sigma_{e} + \sigma_{a}(2 + 2\sigma_{d}\sigma_{e}(\tau)^{2} + \sigma_{e}\sigma_{r}(\tau)^{2}))} \\ p^{\star}_{-} &= \iota_{f} \\ a^{\star}_{-} &= 0 \\ e^{\star}_{-} &= \frac{(\iota_{e})}{(\sigma_{e})} \\ \phi^{\star}_{-} &= 0 \\ f^{\star}_{+} &= \Delta + \iota_{d} + \iota_{r}\sigma_{r} - \iota_{f}(\sigma_{d} + \sigma_{r}) - \delta(\sigma_{d} + \sigma_{r})\tau \\ - \frac{(\delta(\sigma_{a} + \sigma_{e})\sigma_{r}\tau)}{(2(\sigma_{a} + \sigma_{e}) + \sigma_{a}\sigma_{e}(2\sigma_{d} + \sigma_{r})(\tau)^{2})} + \sigma_{r}(\iota_{f} - \iota_{r}) \\ + \sigma_{r} \frac{(3\delta(\sigma_{a} + \sigma_{e})\tau)}{(2(\sigma_{a} + \sigma_{e}) + \sigma_{a}\sigma_{e}(2\sigma_{d} + \sigma_{r})(\tau)^{2})} \\ p^{\star}_{+} &= \iota_{f} + \frac{(3\delta(\sigma_{a} + \sigma_{e})\tau)}{(2(\sigma_{a} + \sigma_{e}) + \sigma_{a}\sigma_{e}(2\sigma_{d} + \sigma_{r})(\tau)^{2})} \\ p^{\star}_{+} &= \iota_{f} + \frac{(2\delta(\sigma_{a} + \sigma_{e})\tau)}{(2(\sigma_{a} + \sigma_{e}) + \sigma_{a}\sigma_{e}(2\sigma_{d} + \sigma_{r})(\tau)^{2})} \\ p^{\star}_{+} &= \iota_{f} + \frac{(2\delta(\sigma_{a} + \sigma_{e})\tau)}{(2(\sigma_{a} + \sigma_{e}) + \sigma_{a}\sigma_{e}(2\sigma_{d} + \sigma_{r})(\tau)^{2})} \\ p^{\star}_{+} &= \iota_{f} + \frac{(2\delta(\sigma_{a} + \sigma_{e})\tau)}{(2(\sigma_{a} + \sigma_{e}) + \sigma_{a}\sigma_{e}(2\sigma_{d} + \sigma_{r})(\tau)^{2})} \\ p^{\star}_{+} &= \iota_{f} + \frac{(2\delta(\sigma_{a} + \sigma_{e})\tau)}{(2(\sigma_{a} + \sigma_{e}) + \sigma_{a}\sigma_{e}(2\sigma_{d} + \sigma_{r})(\tau)^{2})} \\ \end{pmatrix}$$

$$+\frac{(2\delta\sigma_{a}\sigma_{d}\sigma_{e}(\tau)^{3})}{(2(\sigma_{a}+\sigma_{e})+\sigma_{a}\sigma_{e}(2\sigma_{d}+\sigma_{r})(\tau)^{2})}$$

$$a^{\star}_{+}=\frac{(2\delta(\sigma_{a}+\sigma_{e})+\sigma_{a}\sigma_{d}\sigma_{e}(\tau)^{2}))}{(\sigma_{a}(2(\sigma_{a}+\sigma_{e})+\sigma_{a}\sigma_{e}(2\sigma_{d}+\sigma_{r})(\tau)^{2}))}$$

$$e^{\star}_{+}=\frac{(2\delta(\sigma_{a}+\sigma_{e})+\sigma_{a}\sigma_{e}(2\sigma_{d}+\sigma_{r})(\tau)^{2}))}{(\sigma_{e}(2(\sigma_{a}+\sigma_{e})+\sigma_{a}\sigma_{e}(2\sigma_{d}+\sigma_{r})(\tau)^{2}))}$$

$$\phi^{\star}_{+}=\frac{(2\delta(\sigma_{a}+\sigma_{e})+\sigma_{a}\sigma_{e}(2\sigma_{d}+\sigma_{r})(\tau)^{2})}{(2(\sigma_{a}+\sigma_{e})+\sigma_{a}\sigma_{e}(2\sigma_{d}+\sigma_{r})(\tau)^{2})}$$