

An exploration of the link between development, economic growth, and natural risk

Stéphane Hallegatte

► To cite this version:

Stéphane Hallegatte. An exploration of the link between development, economic growth, and natural risk. Policy Research Working Paper, 2012, 6216, 10.1596/1813-9450-6216. hal-00802047

HAL Id: hal-00802047 https://enpc.hal.science/hal-00802047v1

Submitted on 8 Oct 2024

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers. L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Policy Research Working Paper 6216

BACKGROUND PAPER TO THE 2014 WORLD DEVELOPMENT REPORT

An Exploration of the Link between Development, Economic Growth, and Natural Risk

Stéphane Hallegatte

The World Bank Sustainable Development Network Office of the Chief Economist October 2012



Policy Research Working Paper 6216

Abstract

This paper investigates the link between development, economic growth, and the economic losses from natural disasters in a general analytical framework, with an application to hurricane flood risks in New Orleans. It concludes that where capital accumulates through increased density of capital at risk in a given area, and the costs of protection therefore increase more slowly than capital at risk, (i) protection improves over time and the probability of disaster occurrence decreases; (ii) capital at risk—and thus economic losses in case of disaster—increases faster than economic growth; (iii) increased risk-taking reinforces economic growth. In this context, average annual losses from disasters grow with income, and they grow faster than income at low levels of development and slower than income at high levels of development. These findings are robust to a broad range of modeling choices and parameter values, and to the inclusion of risk aversion. They show that risktaking is both a driver and a consequence of economic development, and that the world is very likely to experience fewer but more costly disasters in the future. It is therefore critical to increase economic resilience through the development of stronger recovery and reconstruction support instruments.

This paper—prepared as a background paper to the World Bank's *World Development Report 2014: Managing Risks for Development*—is a product of the Office of the Chief Economist, Sustainable Development Network. It is part of a larger effort by the World Bank to provide open access to its research and make a contribution to development policy discussions around the world. Policy Research Working Papers are also posted on the Web at http://econ.worldbank.org. The author may be contacted at shallegatte@worldbank.org.

The Policy Research Working Paper Series disseminates the findings of work in progress to encourage the exchange of ideas about development issues. An objective of the series is to get the findings out quickly, even if the presentations are less than fully polished. The papers carry the names of the authors and should be cited accordingly. The findings, interpretations, and conclusions expressed in this paper are entirely those of the authors. They do not necessarily represent the views of the International Bank for Reconstruction and Development/World Bank and its affiliated organizations, or those of the Executive Directors of the World Bank or the governments they represent.

An exploration of the link between development, economic growth, and natural risk

Stéphane Hallegatte¹

Keyword: Development; Economic growth; Risk; Natural disaster; Economic losses.

JEL: O10; O44; Q01; Q54.

¹The World Bank, Sustainable Development Network, Office of the Chief Economist, Washington D.C., USA (shallegatte@worldbank.org). This paper is a background paper for the 2014 World Development Report "Managing risks for development." The author wants to thank Paolo Avner, Michael Chaitkin, Patrice Dumas, Marianne Fay, Antonin Pottier, and Adrien Vogt-Schilb for useful discussions on the content of this paper. The remaining errors are the author's. The views expressed in this paper are the sole responsibility of the author. They do not necessarily reflect the views of the World Bank, its executive directors, or the countries they represent.

1 Introduction

In spite of increasing investments in disaster risk reduction, economic losses from natural disasters have been growing as fast as economic growth (e.g., for floods in Europe, see Barredo, 2009; at the global scale, with much larger uncertainties, see Miller et al., 2008, Neumayer and Barthel, 2010), or even faster than economic growth (e.g., in the U.S. and for hurricanes, see Nordhaus, 2010; Pielke et al., 2008; at the global scale, Bouwer et al., 2007). Climate change is not responsible of these evolutions (Schmidt et al., 2009; Neumayer and Barthel, 2010; Bouwer, 2011). For instance, the trend in hurricane losses relative to wealth in the US is explained by migrations toward hurricane-prone areas and increasing population wealth (Pielke et al., 2008).

Trends in disaster losses cannot be considered independently from economic growth and the development process. Indeed, many investments that are key for development and growth need to be located in at-risk areas, or have a higher productivity when they are located in at-risk areas.² For instance, international harbors are key to international trade and the growth that comes with it, and create jobs and activities that attract workers in coastal zones in spite of flood risks. In China, for instance, Fleisher and Chen (1997) find that Total Factor Productivity (TFP) is 85 percent higher in coastal regions than in inland regions, and that TFP growth rates are not significantly different in spite of higher investment in inland regions, suggesting a permanent productivity advantage in coastal regions, linked to lower transport costs. Also, cheap waterway transport attracts industrial production close to flood plains, and partly explains why most large cities are located on rivers. Gallup et al. (1998) analyze the impact of geography and transportation costs on productivity and growth, and find that areas with lower transportation costs are more productive; these areas are also often more at risk from floods, because they are on the coast or next to rivers. As an illustration, landlocked countries have higher transportation costs (measured by the shipping costs), and had over the 1965-1990 period a growth rate on average 1 percent lower than coastal countries, which are at risk from coastal floods and storms.

Also, the drivers of economic growth are concentrated in cities, and productivity and productivity growth is larger in cities in part because of agglomeration and concentration externality (Ciccone et al., 1996; Ciccone, 2002; World Bank, 2008; Lall and Deichmann, 2010). Reviewing evidence from eight developing countries, Fields (1975) reports per capita income in urban areas from two to eight times larger than in rural areas. Lu (2002) shows that in China from 1990 to 1999, the urban-rural per capita consumption ratio lies between 1.5 and 5. At the global scale, the World Bank (2008) reports urban-rural

²Productivity is considered here in a broad sense. For instance, the amenities provided by the proximity to water (e.g., near the Floridian beaches) can be considered as increasing the productivity of housing services and leisure activity.

income ratios between 1.5 for developed countries and up to 3 for developing countries, suggesting much higher productivity in cities at all stages of development. And not only are productivity and consumption higher in urban areas, but amenities and infrastructure services are also often superior: among low-income countries with urban population shares of less than 25 percent, access to water and sanitation in towns and cities is around 25 percentage points higher than in rural areas (World Bank, 2008). These differences create strong incentives for rapid rural-urban migration. Confronted with land scarcity and high land costs in large cities, this migration has led to construction in at-risk areas (e.g., Burby et al., 2001; Burby et al., 2006; Lall and Deichmann, 2010). In the most marginal and risky locations, informal settlements and slums are often present, putting a poor and vulnerable population in a situation of extreme risk (e.g., Ranger et al., 2011).

A last interaction between growth and disaster is linked to protection investments, which become both more affordable and profitable as income increases. On average, and with a large variability, there is indeed more protection in high income countries than in low income countries (e.g., on coastal cities, Hanson et al., 2011). This increased protection made possible by higher income influences risks and thus incentives to invest in risky areas.

Trends in disaster losses are therefore linked to rational trade-offs between disaster losses and growth. This paper investigates this relationship between disaster-induced economic losses and development, in an optimal economic growth framework with rational decision-making. Compared with previous investigations of trends in disaster economic losses (e.g., Lewis and Nickerson, 1989; Schumacher and Strobl, 2011), this analysis proposes a more general framework, allowing for multiple production functions and assumptions on protection investments, and provides more general results.

Within this framework, it is found that — even with no change in climate conditions and hazard characteristics, and assuming rational decision-making — risk taking increases with development and natural disasters are likely to become more destructive in the future, even relative to income. The increase in losses arises from an intuitive mechanism: economic growth leads to better protections against natural disasters, which in turn makes it rational to invest more in at-risk areas, worsening the consequences when disasters occur in spite of protections. In most cases, the worsening of disaster consequences dominates the decrease in disaster probability, and average losses increase over time, possibly more rapidly than income.

2 Development and natural risks

It is generally accepted that richer populations invest more to protect themselves from natural hazards. A richer population, however, may also invest more in at-risk areas, increasing exposure to natural hazards. These two trends have opposite impacts on risk, and the resulting trend in risk is thus ambiguous. This trend is investigated in this section with a simple model.

2.1 A general economic growth framework

We assume there are two categories of capital. Capital R represents the capital related to activities that need to be located in areas that are potentially at risk of flooding (e.g., in coastal areas, where storm surge and coastal floods are possible, as well as areas at risk of river floods, high-concentration urban areas at risk of floods in case of heavy precipitations, and hurricane-prone regions). Capital S represents the rest of economy, which can be located in safe locations without loss of productivity. These two capitals are inputs in the production function:

$$Y = e^{\gamma t} F(R, S) \tag{1}$$

where t is time, F is a production function and γ is the exogenous growth in total factor productivity. Classically, we assume that $\partial_R F(R, S) > 0$; $\partial_S F(R, S) > 0$; $\partial_R^2 F(R, S) < 0$ and $\partial_S^2 F(R, S) < 0$ (decreasing returns).

The capital R can be affected by *hazards*, like floods and windstorms. If a hazard is strong enough, it causes damages to the capital installed in at-risk areas, and can be labeled as a *disaster*. We assume that in that case, a fraction X of capital R is destroyed. It is assumed that this is the only consequence of disasters.³

These disasters (i.e. hazards that lead to capital destruction) have a probability p_0 of occurring every year, except if protection investments reduce this probability. These protection investments take many forms, depending on which hazard is considered. Flood protections include dikes and seawalls, but also drainage systems to cope with heavy precipitations in urban areas. Windstorm protections consist mainly in building retrofits and stricter building norms, to ensure that old and new buildings can resist stronger winds.

It is assumed that better defenses reduce the probability of disasters, but do not reduce their consequences.⁴ This is consistent with many types of defenses. For instance, seawalls can protect an area up to a design standard of protection but often fail totally if this standard is exceeded; building norms allow houses to resist up to a certain wind speed, but when this wind speed is exceeded, houses are completely damaged and require total

³Disaster fatalities and casualties are not considered in this simple model, assuming that early warning, evacuation and emergency services can avoid them, which is consistent with the observation that disaster deaths decrease with income, at least above a certain income level (Kahn, 2005; Kellenberg and Mobarak, 2008). Human losses could be taken into account if it is assumed that fatalities and casualties can be measured by an equivalent economic loss, which is highly controversial; see a discussion in Viscusi and Aldy (2003). Moreover, additional indirect economic consequences (Hallegatte and Przyluski, 2011; Strobl, 2011) are not taken into account in this first analysis.

⁴This is equivalent to the self-protection of Ehrlich and Becker (1972).

rebuilding. This modeling choice is made without loss of generality, if there is no risk aversion.⁵ In that case, indeed, reducing the probability of a disaster or the consequences in case of disaster is equivalent.

Better defenses are also more expensive, and the annual cost of defenses C increases when the remaining disaster probability p decrease.⁶ The function C(p, R) is assumed twice differentiable, $C(p_0, R) = 0$ (the probability of occurrence is p_0 in the absence of protections), $\partial_R C(p, R) \ge 0$ (it is as or more expensive to protect more capital), $\partial_p C(p, R) \le 0$ (the cost increase when the probability decrease), $\partial_p^2 C(p, R) \ge 0$ and $C(0, R) = +\infty$ (the marginal cost is increasing and it is impossible to reduce the probability to zero) and $\partial_{pR}^2 C(p, R) \le 0$ (the cost of protecting more capital increases when the probability decreases).

Any given year, the economic surplus π is given by:

$$\pi = e^{\gamma t} F(R, S) - C(p, R) - L - r(R + S)$$
(2)

where r is the interest rate, L is the losses from disasters, and is given by a random draw with probability p. If a disaster occurs, losses are equal to XR, i.e. a fraction X of the capital located in the risky area is destroyed. Any given year, the expected loss $\mathbb{E}[L]$ is equal to pXR and the expected output is equal to:

$$\mathbb{E}[\pi] = e^{\gamma t} F(R, S) - C(p, R) - (r + pX)R - rS$$
(3)

Note that in this equation, disaster losses appear as an additional cost of capital at risk, in addition to the interest rate r.

We also define the *risk-free situation* as a situation in which there is no risk, either because $p_0 = 0$ (no hazard), because X = 0 (no vulnerability), or because C(p, R) = 0 (costless protections). In the risk-free situation, there are two capitals R and S, but none of them is at risk.

2.2 Optimal choice of p, R, and S

We assume that a social planner — or an equivalent decentralized decision-making process — decides which amounts of capital R and S are to be located in the risky and safe areas, and the level of protection (p and C(p, R)) that is to be built. Its program is:⁷

⁵The role of risk aversion is investigated in Section 6.

⁶The probability p here includes both the probability that an event exceeds protection capacities, and the defense failure probability, even for weaker events.

⁷This model is more general than the model of Schumacher and Strobl (2011). In the latter, the only decision concerns protection investments that mitigate disaster consequences, and there is no benefit from taking risks and thus no trade-off between safety and income. This model also differs from Hallegatte (2011) in that it is more general on the shape of production and protection cost functions, and it introduces the interest rate to account for the consumption–investment trade-off.

$$\max_{p,R,S} \mathbb{E}[\pi]$$

$$s.t.0 \le p \le p_0$$
(4)

We assume first that there is no risk aversion and we assume that the expected surplus is maximized. From the social planner's perspective, doing so is acceptable if disaster losses remain small compared with aggregated income, consistently with the Arrow-Lind theorem for public investment decisions (Arrow and Lind, 1970). This condition holds if disaster losses can be pooled among a large enough population (e.g., a large country), and with many other uncorrelated risks, i.e. in the presence of comprehensive insurance coverage or post-disaster government support, or if disaster losses can be smoothed over time thanks to savings and borrowing (i.e. self-insurance) or reinsurance. In other terms, the optimal pathways determined by this analysis are valid assuming that the social planner ensures that individual losses remain small thanks to temporal smoothing and redistribution or insurance across individuals. Section 6 investigates the case with risk aversion.

If $p = p_0$, then there is no protection in place — because protections are too expensive — and the situation is highly simplified: disaster risk reduces by a fixed fraction the productivity of the capital at risk. Classically, this reduces the amount of such capital without influencing its growth rate on the balanced growth pathway. In the following, we assume that $p < p_0$.

First order conditions lead to the optimal values of p, R, and S:

$$e^{\gamma t}\partial_R F(R,S) - \partial_R C - (pX+r) = 0$$
⁽⁵⁾

$$e^{\gamma t} \partial_S F(R, S) - r = 0 \tag{6}$$

$$\partial_p C = -XR \tag{7}$$

While the marginal production of capital S is r, the marginal production of capital R is $\partial_R C + pX + r$, i.e. the cost of capital r plus the capital losses due to disasters pX plus the incremental cost of protection $\partial_R C$. The term $\partial_R C + pX + r$ is what we define as the risk-adjusted cost of capital, and it is larger than the risk-free cost of capital, to account for natural risks.⁸

Since pX > 0, $\partial_R C > 0$, and $\partial_R F$ is decreasing, the first equation shows that the presence of risk (X > 0 and C > 0) leads to a reduction in R.

Proposition 1 *The presence of risk and the possibility to protect lead to a reduction in the capital that is located in the risky area, compared with the risk-free situation.*

⁸Equivalently, one can define the risk-adjusted marginal productivity of capital as the marginal productivity of capital reduced by the cost of protection and disaster capital losses: $e^{\gamma t}\partial_R F(R,S) - \partial_R C - pX$.

Taking the derivative of the three equations with respect to time t, one gets:

$$\gamma e^{\gamma t} \partial_R F + [e^{\gamma t} \partial_R^2 F - \partial_R^2 C] \partial_t R + e^{\gamma t} \partial_{RS}^2 F \partial_t S = (X + \partial_{pR}^2 C) \partial_t p \tag{8}$$

$$\gamma e^{\gamma t} \partial_S F + e^{\gamma t} \partial_S^2 F \partial_t S + e^{\gamma t} \partial_{RS}^2 F \partial_t R = 0 \tag{9}$$

$$\partial_t p \partial_p^2 C = -(X + \partial_{pR}^2 C) \partial_t R \tag{10}$$

Equation (10) shows that if $(X + \partial_{pR}^2 C) < 0$, then an increase of at-risk capital (everything else being unchanged) leads to a decrease in protection (i.e. an increase in the probability of occurrence), because the cost of protection then increases more rapidly with R and p than the avoided disaster losses. If $X + \partial_{pR}^2 C = 0$, then the probability of occurrence is independent of R, and thus constant over time even in presence of economic growth.

If we have

$$(X + \partial_{pR}^2 C) > 0 , \qquad (11)$$

then having more capital at risk leads to an increase in protection, and the probability of occurrence p and the amount of capital at risk R evolve in opposite direction. In the following, we assume that this condition is met.

Replacing $\partial_t p$ in Eq. (8) and replacing $e^{\gamma t} \partial_R F$ by $(r + pX + \partial_R C)$ gives:

$$\partial_t R = \frac{\gamma(r(1 - \frac{\partial_{RS}^2 F}{\partial_S^2 F}) + pX + \partial_R C)}{-e^{\gamma t} (\partial_R^2 F - \frac{(\partial_{RS}^2 F)^2}{\partial_S^2 F}) + \partial_R^2 C - \frac{(X + \partial_{pR}^2 C)^2}{\partial_p^2 C}}$$
(12)

Since $\frac{\partial_{RS}^2 F}{\partial_S^2 F} < 1$, the capital at risk increases over time when:

$$-e^{\gamma t} (\partial_R^2 F - \frac{(\partial_{RS}^2 F)^2}{\partial_S^2 F}) > -\partial_R^2 C + \frac{(X + \partial_{pR}^2 C)^2}{\partial_p^2 C}$$
(13)

If this inequality is not satisfied, then development and growth leads to a reduction in capital at risk. It probably corresponds to locations that are too costly to protect against natural disasters and where the optimal solution is "strategic retreat", i.e. the abandonment of human settlements.

To interpret this inequality, we can disregard for now the interactions between R and S (i.e. assuming that $\partial_{RS}^2 F = 0$) and assume that protection costs are independent of R (i.e. $\partial_R^2 C = 0$ and $\partial_{pR}^2 C = 0$). In this case, the capital at risk increases over time when:

$$-e^{\gamma t}\partial_R^2 F > \frac{X^2}{\partial_p^2 C} \tag{14}$$

This inequality is verified if the marginal productivity of the capital at risk decreases more rapidly with R than the marginal cost of protection increases with p. In this same situation, the probability of disaster decreases with economic growth. **Proposition 2** If the concavity of the production function is high enough — or if the convexity of the protection costs is high enough — then economic growth leads to an increase in capital at risk (i.e. an increase in losses when a disaster occurs), and a decrease in the probability of occurrence. If the concavity is lower, then economic growth leads to a decrease in capital at risk (and an increase in disaster probability).

It is interesting to note that — counter-intuitively — the capital at risk increases over time when the concavity is high (i.e. returns of capital at risk are rapidly decreasing), and decreases otherwise.

Note that if development leads to a reduction in capital at risk R and an increase in p, then at one point the economy reaches a situation where there is no protection and $p = p_0$. In such a situation, as already stated, capital at risk R grows at the same rate than in the risk-free situation.

The capital R in the risk-free situation is referred to as R^s , and its evolution is:

$$\partial_t R^s = \frac{\gamma(r(1 - \frac{\partial_{RS}^2 F}{\partial_S^2 F}))}{-e^{\gamma t}(\partial_R^2 F - \frac{(\partial_{RS}^2 F)^2}{\partial_S^2 F})}$$
(15)

Assuming that the capital R increases with economic growth, i.e. that condition (13) is verified, then the comparison of Eqs. (12) and (15) explains how the protection influences the evolution of capital:

- The term pX + ∂_RC in the numerator is the impact of risk and protection marginal costs on marginal productivity; it increases the numerator and accelerates the absolute growth in R.
- The term ∂²_RC in the denominator is the decreasing or increasing return on protection; if the returns on protection are constant (e.g. C(p, R) = RC(p) or C(p, R) = C(p)), then this term does not exist; if the returns are decreasing (i.e. costs are convex and ∂²_RC > 0), then this term increases the denominator and slows down the growth in R; if the returns are increasing, then the growth in R is accelerated.
- The term $-\frac{(X+\partial_{pR}^2C)^2}{\partial_p^2C}$ in the denominator is the impact of the change in protection that is provided if more capital is installed in at risk areas (if the probability of occurrence is fixed, this term does not appear). Since $\partial_p^2 C > 0$, this term is negative and reduces the denominator and thus accelerate the growth in R.

2.3 Trend in average annual losses

The average economic losses due to disasters are equal to $\mathbb{E}[L] = pXR$.

$$\partial_t \mathbb{E}[L] = X R \partial_t p + p X \partial_t R \tag{16}$$

It can be rewritten:

$$\partial_t \mathbb{E}[L] = \partial_t p \left[\partial_p C - \frac{p \partial_p^2 C}{1 + \frac{\partial_{pR}^2 C}{X}} \right]$$
(17)

Since $\partial_p C < 0$ and $\partial_p^2 C > 0$, and under condition (11), then $\mathbb{E}[L]$ and p evolve in opposite directions.

Proposition 3 If $X + \partial_{pR}^2 C > 0$, then average annual disaster losses increase when the probability of occurrence decreases over time.

This result highlights the need to consider the combination of exposure (R) and probability (p) to investigate risks. In particular, a reduction in the probability of occurrence does not mean that average losses decrease; on the opposite, this general analysis suggests that under mild conditions a decrease in the probability of occurrence leads to an increase in average losses, because of the increase in capital at risk.

3 Special cases for protection costs

We already made assumptions on the shape of C(p, R), but it is useful to explore two extreme cases for the dependence of C(p, R) to R.

In a first case, we can consider a coast or a river, where additional capital investments are done at a fixed density and are thus using additional land, which in turn requires additional protection. In such a case, the protected areas increases proportionally with the invested capital in the risky zone, and $C(p, R) = R\tilde{C}(p)$. This is the case in Florida, or in the south of France, where population density is lower and flood exposure increases mainly through the construction of individuals houses, at low density. This situation can be labeled "horizontal" or "area-increasing" accumulation.

In a second case case, we consider a given risky areas, which is protected against coastal floods and where investment takes place. In such a case, the risky and protected area does not increase with investments, and the cost of protection is independent of the amount of protected capital: $C(p, R) = \tilde{C}(p)$. This is notably the case where additional investments take place through higher concentration and density, on a given area. Examples of such places are the Netherlands, New Orleans, or Manhattan in New York City. This situation can be labeled "vertical" or "density-increasing" accumulation.

Let us explore the consequence of these two patterns on flood risks.

3.1 Horizontal accumulation

In that case, we can assume that the protection cost function has the form $C(p, R) = R\tilde{C}(p)$. The marginal cost and benefit of protection are equal (Eq. (7)), which means $R\partial_p\tilde{C}(p) = -XR$. Therefore, p is independent of R and constant over time.

Deriving the previous equation with respect to R gives $X + \partial_{pR}^2 C = 0$, and we can rewrite Eq. (5) as:

$$e^{\gamma t}\partial_R F(R,S) = \overbrace{\tilde{C}(p) + pX + r}^{r'}$$
(18)

Since $\tilde{C}(p)$ is constant and positive, the risk-adjusted marginal productivity r' is also a constant, larger than r. In this case, R evolves like a capital without risk, but with a larger interest rate r' instead of r. Since marginal productivity needs to be larger, the amount of capital is lower in presence of decreasing returns ($R < R^s$), i.e. risk leads to a reduction in capital R. With classical production functions (CES or Cobb-Douglas), the capital at risk R increases at the same rate as economic growth. Since p is constant, $\partial_t \mathbb{E}[L] = pX \partial_t R$, and average annual losses grow at the same rate as capital at risk and as risk-free economic growth.

Proposition 4 In horizontal-accumulation locations — i.e. where flood exposure increases because the developed area at risk is expanded and where protection costs increase therefore proportionally with protected capital — rational decision-making leads to annual flood losses growing at the same rate as economic growth, with a constant flood probability, regardless of how protection costs vary with the residual probability of occurrence.

3.2 Vertical accumulation

In that case, we can assume that $C(p, R) = \xi p^{-\nu} + C_0$. Assuming $p < p_0$, we can use Eq. (7) to find:

$$p = \left(\frac{RX}{\nu\xi}\right)^{-\frac{1}{1+\nu}} \tag{19}$$

We have $\partial_{pR}^2 C = 0$, and thus $X + \partial_{pR}^2 C > 0$ if there is risk, so that condition (11) is always verified. As a consequence, we know that p and R evolve in opposite directions: if capital increases (resp. decreases), the protection is improved (resp. weakened) and disaster probability decreases (resp. decreases). We are in the situation where p decreases and R increases when condition (13) is verified, and it can be rewritten:

$$-e^{\gamma t}(\partial_R^2 F - \frac{(\partial_{RS}^2 F)^2}{\partial_S^2 F}) > \frac{X^2 p^{\nu+2}}{\nu(\nu+1)\xi}$$
(20)

Also, we know from the general analysis that $\partial_t \mathbb{E}[L]$ is positive and average disaster losses are increasing over time.

Proposition 5 In locations where flood exposure rises as a result of increased density in a given protected area (e.g., New Orleans), rational decision-making results in a continuous increase in annual flood losses, when flood probability decreases over time.

To know whether capital at risk and flood losses grow more or less rapidly than economic growth, assumptions are needed on the shape of the production function. This is what is investigated in the next section.

4 Special cases for the production function

In this section, we keep the "vertical-accumulation" assumption, i.e. $C(p, R) = \xi p^{-\nu} + C_0$. All qualitative results are however valid if $C(p, R) = \xi R^c p^{-\nu} + C_0$, where $0 \le c < 1$.

4.1 Capital at risk and safe capital are separable inputs in the production function

If F(R, S) = f(R) + g(S), then $\partial_{RS}^2 F = 0$, and all equations can be simplified. The evolution of the capital at risk is:

$$\partial_t R = \frac{\gamma(r + pX + \partial_R C)}{-e^{\gamma t} \partial_R^2 f(R) + \partial_R^2 C - \frac{(X + \partial_{pR}^2 C)^2}{\partial_p^2 C}}$$
(21)

In absence of risk and protection, the evolution would be:

$$\partial_t R^s = \frac{\gamma r}{-e^{\gamma t} \partial_R^2 f(R^s)} \tag{22}$$

We can also calculate $\partial_t S$ as:

$$\partial_t S = \frac{\gamma r}{-e^{\gamma t} \partial_S^2 g(S)} \tag{23}$$

If we now assume that $f(R) = \lambda R^{\mu}$ and $g(S) = \alpha \lambda S^{\mu}$, we can solve Eq. (6):

$$S = (\alpha \lambda \mu)^{\frac{1}{1-\mu}} e^{\frac{\gamma t}{1-\mu}} r^{\frac{-1}{1-\mu}}$$
(24)

and we have:

$$\frac{\partial_t S}{S} = \frac{\partial_t R^S}{R^S} = \frac{\gamma}{1-\mu} \tag{25}$$

So, with this shape of production function, a productivity growth at rate γ leads to a risk-free economic growth at rate $\frac{\gamma}{1-\mu}$ (i.e. R^s and S grow at rate $\frac{\gamma}{1-\mu}$).

With
$$p = \left(\frac{RX}{\nu\xi}\right)^{-\frac{1}{1+\nu}}$$
, we can rewrite Eq. (5):
 $e^{\gamma t}\lambda\mu R^{\mu-1} = r + pX = r + X\left(\frac{X}{\nu\xi}\right)^{-\frac{1}{1+\nu}} R^{-\frac{1}{1+\nu}}$

This equation cannot be solved analytically, but its behavior in special cases can be analyzed.

4.1.1 Low development level

If productivity is low, capital is also limited (R is small) and thus the probability of occurrence of a disaster is large (p is large).⁹ In such an extreme situation, the capital interest rate r is small in the at-risk area compared with the flood-related capital losses pX, and Eq.(26) can be simplified by removing r, leading to the solution:

$$R(t) = R_0 e^{\frac{\gamma}{1+\nu} - \mu} t$$
(27)

(26)

If $\mu > \frac{\nu}{1+\nu}$, R decreases and p increases over time. It is likely that this situation corresponds to locations that are abandoned over time, since they are too expensive to protect against natural disasters.

In the situation in which protection improves over time, $\mu < \frac{\nu}{1+\nu}$, the growth rate of capital at risk is $\frac{\gamma}{\frac{\nu}{1+\nu}-\mu}$, which is always larger than $\frac{\gamma}{1-\mu}$. In this case, therefore, the capital at risk grows more rapidly than the safe capital and risk-free income. The relative vulnerability of the economy can be measured by the amount of disaster losses when a disaster occurs divided by income or by the "fraction at risk", i.e. the share of capital at risk *R* in total capital *R* + *S*. This vulnerability is increasing over time, as shown in Fig.1 with the fraction at risk. Interestingly, the growth in capital at risk is more rapid when ν is smaller, i.e. when the convexity of protection costs is lower and protection costs increase slowly with the protection level.

Average losses pXR have a growth rate equal to:

$$\gamma_L = \frac{\gamma}{1 - \mu \frac{1 + \nu}{\nu}} \tag{28}$$

⁹We assume here that p_0 is large, i.e. that the area-at-risk has a large flood probability in absence of protection, and that $p < p_0$. If productivity is so low that $p = p_0$, then pX can be replaced by p_0X , which is independent of R. Then, Eq. (26) can be simplified by replacing its right-hand-side by $r + p_0X = r'$. In this situation, the capital at risk is lower than in absence of risk ($R < R^s$), but it grows at the same rate. When development increases productivity, there is a time when protection is such that $p < p_0$, and the following calculation holds if p_0 is large enough.



Figure 1: Evolution of the "fraction at risk", i.e. the share of capital at risk R in total capital R+S, as a function of time. The fraction at risk increases with development, until it stabilizes at high development level. Calculations using numerical values from New Orleans (see Section 5) and $\alpha = 2$.

In the situation in which protection improves over time, $\mu < \frac{\nu}{1+\nu}$, average losses increase over time, and they increase more rapidly than $\frac{\gamma}{1-\mu}$ and thus more rapidly than risk-free economic growth, i.e. the growth rate of R^s and S (in other terms, the income elasticity of average annual losses is larger than one).

So, in this case, annual disaster losses grow more rapidly than risk-free economic growth at low levels of development, when disaster losses dominate the interest rate in the assessment of the cost of capital, and when the returns on capital at risk are decreasing rapidly, more rapidly than a limit value defined by the shape of the production costs.

In that case, the economic surplus (generated by the at-risk capital) is $\mathbb{E}[\pi] = e^{\gamma t} F(R, S) - C(p, R) - (r + pX)R$ and is growing at the same rate as losses, i.e. $\frac{\gamma}{1-\mu\frac{1+\nu}{\nu}}$. It means that the process of increasing risk taking leads to a growth in economic surplus that is more rapid than risk-free economic growth. So, increasing risk-taking is also a driver of economic growth (even though the presence of risk leads to a lower output, see proposition 1).

Note that this case is equivalent to the case explored in Hallegatte (2011) where the total amount of capital (R + S) is fixed at an exogenous level K, and is independent of the risk level (which is equivalent to r = 0 provided that $R \leq K$). It is a situation in which there is no consumption–investment trade-off, and in which the capital at risk can keep increasing more rapidly than growth, until all the capital is located in the risky area. What follows shows that accounting for the consumption–investment trade-off changes significantly the results at high level of development.

4.1.2 High development level

At a high development level, capital productivity is large, then the amount of capital at risk R is large.¹⁰ As a consequence, the probability of occurrence (p) is small. In such an extreme situation, the capital interest rate r is large compared with flood-related capital losses pX, and Eq.(26) can be simplified by removing pX, leading to the risk-free solution:

$$R(t) = R_0 e^{\frac{\gamma}{1-\mu}t} \tag{29}$$

At high level of development, when disaster probability is very low, the capital at risk grows at the same rate than economic growth. This is why the fraction at risk stabilizes at high income, as shown in Fig.1.

¹⁰Using a *reductio ad absurdum* argument, it is easy to show from Eq. (26) that R tends toward infinity when productivity grows. Assume that R is bounded when $t \to +\infty$. In this case, the left-hand-side of Eq. (26) tends toward infinity when t increases, so the right-hand-side has to do the same. In that case, rbecomes negligible over time, and the solution of Eq. (26) tends toward Eq. (27), which is not bounded when $t \to +\infty$. This is in contradiction with our initial hypothesis.

When productivity is high, the average losses pXR has a growth rate equal to:

$$\gamma_L \underset{t \to +\infty}{\longrightarrow} \frac{\gamma}{1 - \mu} \frac{\nu}{(1 + \nu)} \tag{30}$$

Average annual losses are thus growing less rapidly than economic growth at high level of development, but they never decrease in absolute terms. At high development level, the growth rate in annual disaster losses is equal to economic growth multiplied by a "protection factor" ($\nu/(1+\nu)$), which depends only on the shape of the protection costs and is lower than one.

The protection factor is also the income elasticity of average disaster losses (in a given region, for a given hazard). It should not be confused with the income elasticity of disaster damages (when a disaster occurs), which is larger than one.

In that case, the different terms of the economic surplus from at-risk capital ($\mathbb{E}[\pi] = e^{\gamma t}F(R,S) - C(p,R) - (r + pX)R$) are growing at different rates. The production $e^{\gamma t}F(R,S)$ and capital cost rR are growing at a rate $\frac{\gamma}{1-\mu}$. The protection costs and average losses pXR are growing at a rate $\frac{\gamma}{1-\mu} \frac{\nu}{1+\nu}$, i.e. more slowly than production. It means that when productivity tends to the infinity, the economic surplus is growing at the rate $\frac{\gamma}{1-\mu}$, i.e. at the rate of risk-free economic growth.

4.1.3 Development and disaster trends

This analysis leads to three conclusions, concerning the trends in capital at risk, average annual losses, and economic surplus.

Proposition 6 If (i) capital at risk and safe capital can be separated in the production function, (ii) protection costs are independent of the amount of capital to protect (vertical accumulation on a given area), and (iii) capital returns are decreasing more rapidly than a threshold that depends on the convexity of protection cost (i.e. if $\mu < \nu/(1 + \nu)$), then economic losses in case of disaster (R) grow more rapidly than risk-free economic growth, but their rate of growth converges toward the rate of risk-free economic growth as development proceeds. The relative vulnerability of the economy (the "fraction at risk") is thus increasing over time.

Proposition 7 In terms of annual average disaster losses, there is a disaster bell-shaped curve in relative terms, i.e. a situation in which disaster losses are growing more rapidly than risk-free economic growth at low stages of development and then keep growing in absolute terms but more slowly than risk-free economic growth at higher productivity. At high productivity, the growth rate of annual losses is the risk-free economic growth rate reduced by a "protection factor" that depends only on the convexity of protection costs.

Proposition 8 The presence of risk reduces the economic surplus. But because of increasing risk-taking, the growth rate of economic surplus is larger than risk-free economic growth at low development level, and it tends to risk-free economic growth at high development level.

As already stated, this result can be generalized to cases where $C(p, R) = \xi R^c p^{-\nu} + C_0$ and $0 \le c < 1$. If c = 1, then protection is constant and average losses and capital at risk grow at the same rate than economic growth.

Importantly, we do not need capital to be "more productive" in at-risk areas to have capital at risk increasing more rapidly than risk-free economic growth and the capital located in safe areas. We only need a separable production function with two categories of capital (R and S), which are imperfectly substituable and have decreasing returns, and R related to activities located in risky areas. In that case, there is an incentive to invest in at-risk areas to benefit from high marginal returns at low capital levels. Results are then independent of the relative productivity α of the two capitals. The productivity ratio (α) determines the ratio between the two capitals (S/R) at high development level.

4.2 Capital at risk and safe capital are substituable inputs in a Cobb-Douglas function

If R and S are imperfectly substituable, we can assume that $F(R, S) = \lambda R^{\mu_1} S^{\mu_2}$. This section demonstrates that this situation is similar to the situation where R and S are separable in the production function.

In this case, the marginal productivity of S gives us:

$$S = \left(\frac{e^{\gamma t}\lambda\mu_2}{r}\right)^{\frac{1}{1-\mu_2}} R^{\frac{\mu_1}{1-\mu_2}}$$
(31)

With $p = \left(\frac{RX}{\nu\xi}\right)^{-\frac{1}{1+\nu}}$, we have:

$$pX = X \left(\frac{X}{\nu\xi}\right)^{-\frac{1}{1+\nu}} R^{-\frac{1}{1+\nu}}$$
(32)

and the marginal productivity of R gives us:

$$\lambda \mu_1 \left(\frac{\lambda \mu_2}{r}\right)^{\frac{\mu_2}{1-\mu_2}} e^{\frac{\gamma t}{1-\mu_2}} R^{\frac{\mu_1+\mu_2-1}{1-\mu_2}} = r + X \left(\frac{X}{\nu\xi}\right)^{-\frac{1}{1+\nu}} R^{-\frac{1}{1+\nu}}$$
(33)

Here, we can use the same approach as before.

4.2.1 Low development level

At low level of development, and using the same assumptions on p_0 , pX is larger than r, and the equation can be approximated by assuming that $r \ll pX$, which gives:

$$R(t) = R_0 e^{\frac{\gamma}{1 - (\mu_1 + \mu_2) - \frac{1 - \mu_2}{1 + \nu}}t}$$
(34)

So *R* is increasing if $1 - (\mu_1 + \mu_2) > \frac{1-\mu_2}{1+\nu}$, i.e. if $\nu > \frac{\mu_1 + 2\mu_2}{1-\mu_1 - \mu_2}$. Using classical values for decreasing return (i.e. $\mu_1 + \mu_2 \approx 0.3$), and assuming that the capital at risk and the safe capital have the same exponent, it leads to $\nu > 0.64$, which is the case if protection costs are convex.

Since economic growth in absence of risk would be $\frac{\gamma}{1-(\mu_1+\mu_2)}$, the capital at risk increases more rapidly than risk-free economic growth. Average losses $\mathbb{E}[L] = pXR$ are growing at a rate:

$$\gamma_L = \frac{\gamma}{1 - (\mu_1 + \mu_2) - \frac{1 - \mu_2}{1 + \nu}} \frac{\nu}{\nu + 1}$$
(35)

Average losses increase more rapidly than risk-free economic growth if:

$$\frac{\gamma}{1 - (\mu_1 + \mu_2) - \frac{1 - \mu_2}{1 + \nu}} \frac{\nu}{\nu + 1} > \frac{\gamma}{1 - (\mu_1 + \mu_2)}$$
(36)

If R is increasing, then the denominator is positive, and this inequality is always verified. So, in this setting, at low level of development and under mild conditions insuring that the probability of occurrence decreases with time, average disaster losses increase more rapidly than risk-free economic growth.

4.2.2 High development level

At high level of development, pX is very small compared with r, and the equation can be solved by assuming at pX = 0:

$$R(t) = R_0 e^{\frac{\gamma}{1 - (\mu_1 + \mu_2)}t}$$
(37)

Which is also the rate of risk-free economic growth. Average losses $\mathbb{E}[L] = pXR$ are then growing at a rate:

$$\gamma_L = \frac{\gamma}{1 - (\mu_1 + \mu_2)} \frac{\nu}{\nu + 1}$$
(38)

In this case, the growth rate of disaster losses is lower than the rate of risk-free economic growth. Indeed, the growth rate in annual disaster losses is equal to economic growth multiplied by the same "protection factor" $\nu/(1+\nu)$ as in the case of a separable production function.

Proposition 9 The case where R and S are substituable inputs in a Cobb-Douglas function is equivalent to the case where the production function is separable.

4.3 Capital at risk and safe capital are perfect substitutes, but with different productivity

In that case, R and S are the same kind of capital, but the capital located in at risk areas is more productive than the same capital located in safe areas. It means that $F(R, S) = f(R + \alpha S)$, with $\alpha < 1$ to ensure that there is an interest to invest in at-risk areas. Then we have:

$$e^{\gamma t}\partial_R F(R,S) = e^{\gamma t} f' = r + \partial_R C + pX$$
(39)

$$e^{\gamma t} \partial_S F(R, S) = e^{\gamma t} \alpha f' = r \tag{40}$$

which means:

$$\frac{1}{\alpha} = \frac{r + \partial_R C + pX}{r} \tag{41}$$

Unless R = 0 or S = 0. If both are positive and $C(p, R) = \xi p^{-\nu} + C_0$, then we have $p = \left(\frac{RX}{\nu\xi}\right)^{-\frac{1}{1+\nu}}$ and \overline{R} defined by:

$$X\left(\frac{X\overline{R}}{\xi\nu}\right)^{-\frac{1}{1+\nu}} = r\left(\frac{1}{\alpha} - 1\right) \tag{42}$$

$$\overline{R} = \left[\frac{1}{r\left(\frac{1}{\alpha} - 1\right)}\right]^{(1+\nu)} X^{\nu}(\xi\nu)$$
(43)

In this case, R and p are constant, if R and S are non-zero.

If S = 0, then R grows with time, until it reaches \overline{R} . Then, S grows. In such a case, development starts by accumulating capital in at-risk areas, where productivity is higher. But more capital at risk means increased protection. And since protection costs increase non-linearly, the protection cost required to do so increases nonlinearly with time, until the capital marginal productivity adjusted by the protection costs is the same in at-risk areas and in safe areas. Then, development proceeds by accumulating only in safe areas, and the capital at risk and average disaster losses are then constant over time. This formalization of the production function does not reproduce the observed dynamics of capital, showing that it misses important characteristics of the link between safe capital and capital at risk (including the fact that some sectors benefit from being located in at-risk areas while others do not, and the agglomeration and congestion effects). However, this result assuming perfect substituability supports the idea that what matters is the substitution between capital at risk and safe capital and their returns, more than the difference in productivity.

5 Numerical application to New Orleans

We apply these formulations to the case of New Orleans, using the following illustrative assumptions:

- The capitals are separable, and we consider only the capital R in the flood-prone area of New Orleans. the capital S is the capital located in safe areas in the rest of the region or the country. The interest rate is r = 5 %.
- The area is fixed, and the protection costs depend only on the probability of occurrence p, not on the amount of capital to protect: C(p, R) = ξp^{-ν} + C₀. In that case, condition (11) is always verified, and p and R evolve in opposite directions.
- In absence of protection, the city would be flooded every year ($p_0 = 1$).
- The cost of protecting New Orleans against category-3 storms is about \$3 billion in investment, and we assume a 10% per year maintenance cost; the probability of such a storm is one out of 50 years. The annualized protection cost is C(1/50) =\$450 million per year, taking into account the cost of capital and maintenance costs.
- Protecting New Orleans against category-5 hurricane floods would cost about \$30 billion, with also a 10% per year maintenance cost, and the probability of such a storm is one out of 200 years, so that C(1/200) = \$4.5 billion per year.¹¹

Even though each of these assumptions can be discussed, they provide an order of magnitude for the cost of protecting the city. Using these assumptions, we have:

$$C(p) = \xi \left(p^{-\nu} - p_0^{-\nu} \right) , \qquad (44)$$

with $p_0 = 1$, $\nu = 1.66$, and $\xi = 6.8 \cdot 10^{-4}$.

For New Orleans, we assume that 50% of capital at-risk is lost in case of floods. This large fraction translates the facts that part of the city is under normal sea level and stayed flooded for weeks after Katrina hit the city, amplifying damages to houses and buildings, and that floods occur through dike failure, leading to high water velocity and large damages (RMS, 2005).

¹¹State officials estimated the cost of Category 5 protection between \$2.5 and \$32 billion (Carter, 2005; Revkin and Drew, 2005; Schwartz, 2005). More recent and detailed estimates by Louisiana Coastal Protection and Restoration (LACPR, led by the U.S. Army Corps of Engineers) reach even larger values.

The production function is $Y = e^{\gamma t} F(K) = \lambda e^{\gamma t} R^{\mu}$, with $\mu = 0.3$ and $\gamma = 0.015$ (total factor productivity grows by 1.5% per year). The risk-free growth rate, i.e. the growth in the capital S located in safe areas in the rest of the country, is $\frac{\gamma}{1-\mu} \approx 2.1\%$ per year.

The variable Y is the local GDP in the flood-prone areas of New Orleans. Using a New Orleans exposed population of 500,000 people, and the GDP per capita of New Orleans (\$24,000 per year in 2009), we have a city GDP of \$12 billion. To estimate λ , we solve numerically Eq. (26) to find R as a function of λ , and we chose λ so that the income in absence of disaster is $Y = \lambda R^{\mu} = 12 billion (in economic data, protection expenditures are included in income). The value is $\lambda = 3.53$. We find that the optimal capital at risk in New Orleans is R = \$59 billion, i.e. about 5 times the local income. Losses in case of flood would be about \$30 billion, which is consistent with data for the flood due to Katrina (removing losses due to wind) (RMS, 2005). The optimal probability of flood is found at 2.2% per year, i.e. a return period of 45 years (which is also close to the return period of a category-3 hurricane, i.e. the current protection level in the city).

Then, we can solve numerically Eq. (26) for a series of t, to investigate the dependence of risk to income. We can calculate the trend in R(t) and in average annual loss. We find that R is growing at a rate 2.4% against 2.1% for the growth without risk. Average annual losses due to floods are growing at a rate 1.5%, slower than economic growth. Thanks to increased risk taking, the economic surplus π is growing at a rate 2.2%, i.e. more rapidly than risk-free economic growth.

In the case of New Orleans, therefore, a rational decision-maker would make average disaster losses increase less rapidly than economic growth (1.5 vs. 2.1%), but would increase capital at risk more rapidly than risk-free growth (2.4 vs 2.1%). The consequence is that the average cost of disasters is decreasing relative to regional or national wealth, but the consequences in case of disasters are increasing, even relatively to wealth. It means that the New Orleans region evolves toward fewer disasters with consequences that are growing relative to income, leading to an increased need for recovery and reconstruction support.

One can investigate how this result depends on the local income in New Orleans (and thus on total factor productivity), assuming that everything else remains unchanged (including protection costs C(p) and the fraction of capital at risk lost in case of flood X). Results are reproduced in Fig. 2 and 3. Figure 2 shows the annual growth rate of capital at risk R, of average annual losses (pXR), and of the economic surplus, as a function of local income Y. The vertical line shows the current income in New Orleans, and the horizontal line the rate of risk-free economic growth. At very low productivity, the growth in capital at risk R would be 4.6% per year, i.e. more than twice the rate of risk-free economic growth. In this situation, the annual probability of flood would exceed 80% (see Fig. 3). This growth then converges toward the rate of risk-free economic growth



Figure 2: Annual growth rate of capital at risk R, average annual disaster losses, and economic surplus growth rate, as a function of local income. The horizontal line is the rate of risk-free economic growth. Growths in capital at risk and in economic surplus are more rapid than economic growth at early development stage, and these growth rates converge toward economic growth rate over time. Growth in average annual losses is faster than risk-free economic growth at low income and slower at high income. The vertical dashed line shows the current income in New Orleans.



Figure 3: Optimal disaster probability of occurrence in New Orleans, as a function of local income.

(the horizontal dashed line) as development proceeds. The economic surplus growth rate is also larger than the risk-free economic growth rate at low development level, and it converges toward it as development proceeds.

Figure 1 shows how the fraction at risk — i.e. the fraction of capital located in at risk area — increases with income, assuming $\alpha = 2$ (i.e., that in absence of risk, this fraction would be 38 percent, according to Eq. (24)).

For average annual disaster losses, the growth rate is 2.8% per year at low development level, i.e. 40% faster than economic growth. At an income of 40 million USD per year in the city, the growth rate of annual losses is equal to the rate of economic growth (2.1%), and this growth rate keeps decreasing until 1.3% per year, which is economic growth (2.1%) corrected by the "protection factor", equal to 0.62 in the case of New Orleans.

In this case, therefore, the income elasticity of average disaster losses would be equal to 0.62 (i.e. a 1% growth in the US would lead to a 0.62% growth in average annual losses) and the income elastricity of disaster losses (when a disaster occurs) would be equal to 1.1 (i.e. a 1% growth in the US would lead a 1.1%-increase in the size of disasters when they occur).

6 Taking into account risk aversion

The present analysis does not include risk aversion, following Arrow and Lind (1970). It thus assumes that the social planner who determines the appropriate level of risk ensures that (i) aggregate losses remain limited compared with national income; (ii) there is risk sharing across individuals in the country to avoid large individual losses¹²; and (iii) there is temporal smoothing of disaster losses, through savings and borrowing (self-insurance) or reinsurance. In absence of these elements, risk aversion needs to be taken into account. This is the case, for instance, in small countries where the entire economy can be affected (as in Grenada after hurricane Ivan in 2004 where losses reached 200% of GDP) and where the risk-free level of capital R (R^s) would be large compared with the rest of the economy.

To take into account risk aversion, we need to introduce an utility function, which we assume to depend on the economic surplus $u(\pi)$ and to have decreasing returns $u'(\pi) > 0$ and $u''(\pi) < 0$. The utility cost of disasters can be approximated by the insurance premium δ that the region would be ready to pay to avoid all losses, which is defined by:

$$u(\pi_0 - \delta) = pu(\pi_0 - L) + (1 - p)u(\pi_0)$$
(45)

where π_0 is the surplus in absence of disaster and is equal to $e^{\gamma t}F(R,S) - C(p,R) - r(R+S)$. This equation defines a function $\delta(p, R, S, t)$, which replaces pXR in Eq. (3) when risk aversion is accounted for.

$$\mathbb{E}[u] = \overbrace{e^{\gamma t} F(R,S) - C(p,R) - r(R+S) - \delta(p,R,S,t)}^{\widetilde{\pi}}$$
(46)

And the maximization program becomes:

$$\max_{p,R,S} \pi$$

$$s.t.0 \le p \le p_0$$
(47)

We use a constant relative risk aversion (CRRA) utility function under the form $u = \frac{\pi^{1-\rho}}{1-\rho}$, then we have:

$$\delta = \pi_0 - \left(p(\pi_0 - RX)^{1-\rho} + (1-p)\pi_0^{1-\rho} \right)^{\left(\frac{1}{1-\rho}\right)}$$
(48)

Since $\delta > pRX$ in presence of positive risk aversion, the taking into account of risk aversion makes perceived risk larger and creates a non-linearity between R and risk. To go further, the optimization program can be solved numerically with the parameters and

¹²Since fatalities and casualties cannot be shared, it means that forecasts and early warning systems reduce human losses, as is observed in most developed countries where economic losses have increased while human losses have decreased.



Figure 4: Evolution of the "fraction at risk", i.e. the share of capital at risk R in total capital R+S, as a function of time, with and without risk aversion. Risk aversion reduces the fraction at risk at all development levels. Calculations using numerical values from New Orleans (see Section 5) and $\alpha = 2$.

functional forms from Section 5 on New Orleans, and using the same methodology to calibrate λ . Since risk aversion introduces total income in the equations of R and p, it creates a link between R and S even when the two capitals are separable in the production function. It means that the value of S (i.e. α) also needs to be calibrated. In practice, the value of S depend on how disaster risks in New Orleans are shared with risk-free capital (or risk that is subject to a risk independent of hurricane risk). As an illustration, equations are solved assuming that $\alpha = 2$.

Results for the fraction at risk are presented in Fig. 4, for a risk aversion $\rho = 2$. It shows that risk aversion leads to locating less capital in at risk areas, at all development levels.

Figure 5 shows that risk aversion has an ambiguous impact on the probability of occurrence: at low development level, the capital at risk is so much smaller with risk aversion that it is optimal to increase the probability of occurrence; at higher development level, risk aversion leads to better protection and to a decrease in the probability of occurrence. Finally, Fig. 6 shows that capital at risk still grow more rapidly than income at all development levels, and the growth rate converges toward the risk-free growth rate. At high development level and in this simulation, the capital at risk is lower but increases more



Figure 5: Evolution of the annual probability of occurrence, without risk aversion and with risk aversion ($\rho = 2$).

rapidly with risk aversion than without risk aversion. At high development level, average annual losses grow at a lower rate than risk-free economic growth, like in the case without risk aversion.

Numerical simulations suggest therefore that the qualitative results in the case without risk aversion remain valid with risk aversion. An exploration of various values of risk aversion (ρ) and of various risk sharing level (modeled through α here) confirms that results are robust to the presence of risk aversion.

7 Conclusion and discussion

This paper proposes an economic analytical framework to analyze the trade-off between disaster losses and investment returns in areas at risk from natural hazards, assuming an optimal growth pathway and rational (and perfectly informed) decision-making. Under conditions that ensure that protection improves over time, the presence of risk and the possibility to protect against disasters lead to a lower amount of capital in risky area (compared with the risk-free situation), but it also increases the growth rate of capital at risk where protection costs increase less rapidly than the amount of protected capital (i.e. where investments are at least partly done by increasing capital density and concentration).



Figure 6: Evolution of capital at risk R and average annual disaster losses as a function of time, without risk aversion and with risk aversion ($\rho = 2$).

By improving protection, economic development drives the economy toward more risky behaviors (i.e. a growing share of capital is installed in at-risk areas), and — reciprocally — this increasing risk-taking is found to accelerate economic growth. Along an optimal growth pathway, increasing risk taking is thus both a driver and a consequence of economic development.

By focusing on average losses that mix probability and loss (e.g., Toya and Skidmore, 2007; Rashky, 2008), econometric studies have left out of their analysis the potential increase in damages when a disaster occur. Current trends in disaster losses appear however consistent with the prediction of fewer but larger disasters (e.g., Etkin, 1999; Nordhaus, 2010; Bouwer et al., 2007; Pielke et al., 2008; Bouwer, 2011; Schumacher and Strobl, 2011). These results are also in line with UN-ISDR (2009), which observes that poor countries suffer from frequent and low-cost events, while rich countries suffer from rare but high-cost events. This trend is illustrated by the Japanese case. Thanks to protection investments, the country can cope without any damage with frequent earthquakes and tsunamis that would cause disasters in any other place of the world. But this resilience allows for higher investments in at-risk areas, and exceptional quakes like the recent Tohoku Pacific earthquake can then lead to immense losses. There is thus a potential negative side-effect from higher protection against disaster, in the form of an increasing vulnerability to exceptional events. In absence of fatalities and human losses (e.g., thanks to early warning and evacuation), this increase in vulnerability is however found optimal, even in presence of risk aversion.

The paper suggests that natural disasters will become less frequent but more costly with development and economic growth, and this result has some policy-relevant consequences. In particular, it means that development requires more resilience, i.e. an improved ability to deal with and recover from rare events, which exceed the protection capacity. The Tohoku Pacific earthquake could thus be an illustration of the type of events the world will have to deal with in the future. Such a trend toward larger disasters translates into a strong and increasing need for crisis management and post-disaster support, through (1) forecasts and early warning to mitigate human losses (e.g., Subbiah et al., 2008; Hallegatte 2012a); (2) rainy-day funds and insurance and reinsurance schemes to support reconstruction (e.g., Ghesquiere and Mahul, 2010; de Forges et al., 2011; Kunreuther and Michel-Kerjan, 2012); and (3) new international instruments for post-disaster support and solidarity (e.g., Linnerooth-Bayer et al. 2009).

The analysis proposed in this paper is still incomplete, and additional developments are needed. When considering large-scale disasters, for instance, the taking into account of indirect losses would also become relevant. These indirect losses are due to limits to reconstruction capacity (see, e.g., Hallegatte et al., 2007; Hallegatte 2012b) and to ripple-effects through supply chains (e.g., Henriet et al., 2012) and to other secondary effects (e.g., Lall and Deichmann, 2010). Because indirect losses are an externality (one indi-

vidual's decisions on risk-taking have an impact on the damages he or she suffers from, but also on the secondary damages on others), a social planner may want to influence risk-related decisions. And since indirect impacts appear to be growing nonlinearly with direct impacts (Hallegatte et al., 2007; Loaya et al., 2009), and because increased productivity may have been obtained at the expense of resilience (Henriet et al., 2012; Hallegatte 2012b), taking indirect losses into account could influence the long-term relationship between development and natural risks.¹³

But more importantly, the assumption of rational and perfectly informed decisionmaking is clearly an oversimplification of reality. The analysis of disaster protection and risk in such a framework suggests that — in a normative context — it is optimal from an economic perspective to increase risk taking with development (provided that fatalities can be avoided) and disasters should become rarer and more costly as income grows. But interpreting real-world disaster loss data series would require a positive approach, where realistic decision-making is considered. To do so, the analysis needs to include market and behavior imperfections that influence individuals' and firms' risk-related decisions, such as:

- Information and transaction costs: Since the information on natural hazards and risk is not always easily available, households and businesses may decide not to spend the time, money and effort to collect them, and disregard this information in their decision-making process (Magat et al., 1987; Camerer and Kunreuther, 1989; and Hogarth and Kunreuther, 1995).
- Moral hazard: Since insurance and post-disaster support are often available in developed countries, households and firms in risky areas do not pay the full cost of the risk, and this effect may lead economic actors to take more risk than what is socially optimal (e.g., Burby et al., 1991; Laffont, 1995).
- Irrational behaviors and biased risk perceptions: Individuals do not always react rationally when confronted to small probability risks, and they defer choosing between ambiguous choices (Tversky and Shafir 1992; Trope and Lieberman, 2003). Moreover, they have trouble to take into account events that have never occurred before (the "bias of imaginability", see Tversky and Kahneman, 1974). Finally, private and public investment decisions do not always adequately take long and very long-term consequences into account (for public decisions, see Michel-Kerjan, 2008; for private decisions, see Kunreuther et al. 1978, and Thaler, 1999).

¹³In particular, indirect losses cannot be modeled through an increase in X, the fraction of at-risk capital that is destroyed in case of disaster, because indirect losses increase nonlinearly with direct losses and thus depend nonlinearly on the fraction at risk R/S. The effect of indirect losses appears therefore similar to the one of risk aversion, and is likely to lead to the same qualitative results than in this analysis.

These factors are likely to increase risk taking in most instances, and would result in disaster economic losses that grow more rapidly than what is suggested by the analysis proposed in this paper. So, even though it might be optimal to see fewer and larger disasters in the future, the current increase in disaster losses is very likely to exceed what is optimal. The difference between the individuals' and firms' risk taking and the optimal risk level estimated in the current analysis provides a justification for public policies that regulate risk taking (e.g., zoning policies, information campaign, technological risk regulations).

8 References

Arrow, K., and R. Lind, 1970. Uncertainty and the Evaluation of Public Investment Decisions. American Economic Review 60, no.3: 364–78.

Barredo, J.I., 2009. Normalised flood losses in Europe: 1970–2006. Nat. Hazards Earth Syst., 9, 97–104.

Bouwer, L. M., R. P. Crompton, E. Faust, P. Höppe, and R. A. Pielke Jr., 2007. Confronting disaster losses. Science, 318, 753.

Bouwer, L. M., 2011. Have disaster losses increased due to anthropogenic climate change?. Bull. Amer. Meteor. Soc., 92, 39–46.

Burby, R. J., Beverly A. Cigler, Steven P. French, Edward J. Kaiser, Jack Kartez, Dale Roenigk, Dana Weist, and Dale Whittington, 1991. Sharing Environmental Risks: How to Control Governments' Losses in Natural Disasters. Boulder, CO: Westview.

Burby R. J, Nelson A. C., Parker D., and Handmer J., 2001. Urban Containment Policy and Exposure to Natural Hazards: Is There a Connection? Journal of Environmental Planning and Management 44(4), p. 475–490.

Burby R.J., Birch E. L., and Wachter S. M., 2006. The problems of containment and the promise of planning. In Rebuilding urban places after disaster: lessons from Hurricane Katrina. University of Pennsylvania Press.

Camerer, C. and H. Kunreuther, 1989. Decision Processes for Low Probability Events: Policy Implications. Journal of Policy Analysis and Management, 8, 565-592.

Carter, N.T., 2005. New Orleans Levees and Floodwalls: Hurricane Damage Protection, CRS Report for Congress, Order Code RS22238, September 6, 2005.

Ciccone, A., 2002. Agglomeration effects in Europe, European Economic Review 46 (2002) 213–227

Ciccone, A., and R. E. Hall, 1996. Productivity and the density of economic activity, The American Economic Review, 86(1), 54–70

Ehrlich, I., Becker, G.S., 1972. Market insurance, self-insurance, and self-protection. The Journal of Political Economy 80, 623-648. Etkin D., 1999. Risk transference and related trends: driving forces towards more mega-disasters. Environmental Hazards 1, 69–75

Fields, G., 1975. Rural-Urban Migration, Urban Unemployment and Underemployment, and Job-Search Activity in LDCs, Journal of Development Economics 2(2), 165-87.

Fleisher, B. M., and J. Chen, 1997. The coast—non-coast income gap, productivity and regional economic policy in China. J. Comp. Econ. 25, 2:220–236.

Gallup, J.L., J. D. Sachs, and A. D. Mellinger, 1998. Geography and Economic Development, National Bureau of Economic Research Working Paper No. 6849.

Ghesquiere, Francis, and Olivier Mahul, 2010. Financial Protection of the State against Natural Disasters: A Primer. Policy Research Working Paper Series 5429, World Bank, Washington, DC.

Hallegatte, S., 2011. How economic growth and rational decisions can make disaster losses grow faster than wealth, Policy Research Working Paper 5617, The World Bank.

Hallegatte, S., 2012a. A Cost Effective Solution to Reduce Disaster Losses in Developing Countries: Hydro-Meteorological Services, Early Warning, and Evacuation, Policy Research Working Paper 6058, The World Bank.

Hallegatte, S., 2012b. Modeling the roles of inventories in the assessment of natural disaster economic costs: application on hurricane Katrina in Louisiana, Policy Research Working Paper 6047, The World Bank.

Hallegatte, S., J.-C. Hourcade, P. Dumas, 2007, Why economic dynamics matter in assessing climate change damages : illustration on extreme events, Ecological Economics 62(2), 330-340.

Hallegatte S., Przyluski, V., 2011. The Economics of Natural Disasters: Concepts and Methods, Policy Research Working Paper 5507, The World Bank, Washington D.C.

Hanson, S., R. Nicholls, R.,N. Ranger, N., S. Hallegatte, S., J. Corfee-Morlot, J., C. Herweijer, C. and J. Chateau, J., 2011. A global ranking of port cities with high exposure to climate extremes., Climatic Change, 104: 89-111.

Henriet, F., S. Hallegatte, L. Tabourier, 2011. Firm-Network Characteristics and Economic Robustness to Natural Disasters, Journal of Economic Dynamics and Control, 36, 150-167.

Hogarth, R., and H. Kunreuther, 1995. Decision Making Under Ignorance: Arguing with Yourself. Journal of Risk and Uncertainty, 10, 15-36.

Kahn, M., 2005. The death toll from natural disasters: the role of income, geography, and institutions. Review of Economics and Statistics 87 (2), 271–284.

Kellenberg, D. K., and A. M. Mobarak, 2008: Does rising income increase or decrease damage risk from natural disasters? J. Urban Econ., 63, 722–802.

Kunreuther, H., R. Ginsberg, L. Miller, P. Sagi, P. Slovic, B. Borkan, and N. Katz, 1978. Disaster Insurance Protection: Public Policy Lessons. John Wiley and Sons, New York.

Kunreuther, H., and E. Michel-Kerjan. 2012. Impact of Behavioral Issues on Green Growth Policies and Weather-Related Disaster Reduction in Developing Countries. Paper presented at the Green Growth Knowledge Platform inaugural conference, Mexico City, January 12–13.

Laffont, J.J., 1995. Regulation, moral hazard and insurance of environmental risks, Journal of Public Economics 58(3), 319–336

Lall S. V, and Deichmann U., 2010. Density and disasters: economics of urban hazard risk. Policy Research Working Paper 5161, The World Bank, Washington, D.C.

Linnerooth-Bayer, J., Warner, K., Bals, C., Höppe, P., Burton, I., Loster, T., Haas, A., 2009. Insurance, developing countries and climate change. The Geneva Papers on Risk and Insurance-Issues and Practice 34, 381-400.

Loayza, N., Olaberria, E., Rigolini, J. and Christiansen, L., 2009. Natural Disasters and Growth - Going Beyond the Averages. World Bank Policy Research Working Paper 4980. Washington, DC: World Bank.

Lewis, T., and D. Nickerson, 1989. Self-insurance against natural disasters, Journal of Environmental Economics and Management, 16, 209–223.

Lu D., 2002. Rural-urban income disparity: impact of growth, allocative efficiency, and local growth welfare. China Econ Rev., 13(4), 419–429.

Magat, W., K. W. Viscusi, and J. Huber, 1987. Risk-dollar trade-offs, risk perceptions, and consumer behaviour. In: Learning About Risk [Viscusi, W. and W. Magat (eds.)]. Harvard University Press, Cambridge, MA, 83-97.

Michel-Kerjan, E., 2008. Disasters and public policy: Can market lessons help address government failures. Proceedings of the 99th National Tax Association conference, Boston, MA.

Miller S, Muir-Wood R, Boissonnade A, 2008. An exploration of trends in normalized weather-related catastrophe losses. In: Diaz HF, Murnane RJ (eds) Climate extremes and society. Cambridge University Press, Cambridge, 225–347

Neumayer, E., F. Barthel, 2010. Normalizing economic loss from natural disasters: A global analysis, Climatic Change, submitted.

Nordhaus, William D., 2010. The Economics of Hurricanes and implications of global warming, Climate Change Economics, 1, 1–20.

Pielke, Jr., R. A, C. Landsea, M. Mayfield, J. Laver, and R. Pasch, R, 2005. Hurricanes and global warming. Bulletin of American Meteorological Society, 86(11), 1571–1575

Pielke, R. A., Jr., Gratz, J., Landsea, C. W., Collins, D., Saunders, M. A., and Musulin, R., 2008. Normalized hurricane damages in the United States: 1900–2005, Natural Hazards Review, 9(1), 29–42.

Ranger N., S. Hallegatte, S. Bhattacharya, M. Bachu, S. Priya, K. Dhore, F. Rafique, P. Mathur, N. Naville, F. Henriet, C. Herweijer, S. Pohit, J. Corfee-Morlot, 2011. A Preliminary Assessment of the Potential Impact of Climate Change on Flood Risk in Mumbai, Climatic Change, 104(1), 139–167.

Rashky, P., 2008. Institutions and the losses from natural disasters. Natural Hazards and Earth System Sciences 8, pp. 627-634

Revkin, A.C. and C. Drew, "Engineers' warnings and pleas for money went unheeded," The New York Times, Sept. 2, 2005.

RMS, 2005. Hurricane Katrina: Profile of a Super Cat, Lessons and Implications for Catastrophe Risk Management, available on www.rms.com

Schmidt, S., C. Kemfert, and P. Höppe, 2009: Tropical cyclone losses in the USA and the impact of climate change: A trend analysis based on data from a new approach to adjusting storm losses. Environ. Impact Assess. Rev., 29, 359–369.

Schumacher, I., Strobl, E., 2011. Economic development and losses due to natural disasters: The role of hazard exposure. Ecological Economics.

Schwartz, J., 2005. Full flood safety in New Orleans could take billions and decades, The New-York Times, November 29, 2005

Strobl, E., 2011. The economic growth impact of hurricanes: evidence from U.S. coastal counties, Review of Economics and Statistics, 93(2), 575–589.

Subbiah, A., Bildan, L., Narasimhan, R. (2008). Background Paper on Assessment of the Economics of Early Warning Systems for Disaster Risk Reduction. World Bank Group for Disaster Reduction and Recovery. Washington DC, USA.

Thaler, R., 1999. Mental accounting matters. Journal of Behavioral Decision Making, 12, 183-206

Toya H., and Skidmore, M., 2007. Economic development and the impact of natural disasters. Economic Letters, 94, 20–25.

Trope, Y. and N. Liberman, 2003. Temporal construal. Psychological Review. 110 (3), 403-421.

Tversky A., and D. Kahneman, 1974. Judgment under Uncertainty: Heuristics and Biases, Science, New Series, 185(4157), 1124–1131.

Tversky, A. and E. Shafir, 1992. Choice under conflict: the dynamics of deferred decision. Psychological Science, 3(6), 358-361.

UN-ISDR, 2009: Risk and poverty in a changing climate: Invest today for a safer tomorrow. United Nations International Strategy for Natural Disaster Reduction Global Assessment Rep. on Disaster Risk Reduction, 207 pp.

Viscusi, W. K., and J. E. Aldy, 2003. The value of a statistical life: a critical review of market estimates throughout the world. Journal of Risk and Uncertainty 5, 5–76.

World Bank, 2008. Reshaping Economic Geography. The World Development Report 2009. The World Bank. Washington, D.C., USA.