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Can Uncertainty Justify Overlapping Policy Instruments to Mitigate Emissions?

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Abstract

This article constitutes a new contribution to the analysis of overlapping instruments to cover the same emission sources. Using both an analytical and a numerical model, we find that when the risk that the CO₂ price drops to zero and the political unavailability of a CO₂ tax (at least in the European Union) are taken into account, it can be socially beneficial to implement an additional instrument encouraging the reduction of emissions, for instance a renewable energy subsidy. Our analysis has both a practical and a theoretical purpose. It aims at giving economic insight to policymakers in a context of increased uncertainty concerning the future stringency of the European Emission Trading Scheme. It also gives another rationale for the use of several instruments to cover the same emission sources, and shows the importance of accounting for corner solutions in the definition of the optimal policy mix.

Highlights:

- We develop an analytical and a numerical model of the EU energy and carbon markets.
- We add uncertainty on energy demand and focus on instruments for emission reduction.
- We analyze the economic implications of a risk that the CO₂ price drops to zero.
- We find that it can be socially optimal to add an instrument to the EU-ETS.

Keywords: Uncertainty, Policy overlapping, Mitigation policy, Energy policy, EU-ETS, Renewable energy, Corner solutions, Nil CO₂ price, European Union.

JEL codes: Q28, Q41, Q48, Q58

1 Introduction

All countries and regions which have implemented climate policies seem to rely on several policy instruments, some of which cover the same emission sources, rather than a single one¹. For example, in the European Union, CO₂ emissions from the power sector are directly or indirectly covered by the EU Emission Trading System (Ellerman et al., 2010), by energy-efficiency standards and energy-efficiency labels on electric motors and appliances (UE, 2009), by CO₂ or energy taxes (in some Member States), by energy-efficiency obligations² (in some Member States), and by renewable energy power (REP) subsidies, in the form of feed-in tariffs or REP portfolio obligations (in virtually all Member States). This multiplicity of policy instruments is in sharp contrast to the so-called Tinbergen rule (Tinbergen, 1952) which states that achieving a given number of targets requires that policymakers control an equal number of instruments. Unsurprisingly, this multiplicity has generated criticism by some economists who argue that the policy instruments which complement the EU ETS do not reduce CO₂ emissions (which are capped) but reduce the allowance price on the ETS market and generate costly economic distortions (Cf. for instance Böhringer and Keller (2011), Braathen (2007), Fischer and Preonas (2010) or Tol (2010)). Indeed, some abatement options, such as REP sources, are covered by several instruments and benefit from a higher implicit CO₂ price than others, such as coal-to-gas switch. The mix of instruments promoting the same abatement options is therefore suboptimal, at least in a simple economic model, as it disregards the equimarginal principle.

Yet, the multiplicity of policy instruments has been justified by some other economists, on several grounds. First, and most obviously, other policy targets such as air pollution reduction and security of supply are differently impacted by the various CO₂ abatement options. Second, induced technical change may be higher for some options than for others. For instance, the deployment of photovoltaic panels is likely to induce more technical change than coal-to-gas switch (see Fischer and Newell (2008) for a review). Third, the slow diffusion of clean technology justifies implementing more costly but higher potential options, such as photovoltaic panels, before the cheaper but lower potential options, such as coal-to-gas switch (Vogt-Schilb and Hallegatte, 2011). Fourth, some market failures, regulatory failures or behavioral failures may reduce the economic efficiency of market-based instruments and justify additional policy instruments (Gillingham and Sweeney, 2010). For instance, the landlord-tenant dilemma reduces the efficiency of CO₂ pricing and can justify energy-efficiency standards in rented dwellings (de T'Serclaes and Jollands, 2007), while regulatory failures may lead to a too low CO₂ price, or prevent governments to commit to a high enough future CO₂ price (Hoel, 2012).

Our aim is not to discuss these justifications, but to introduce and discuss another rationale: the impact of uncertainty on abatement costs combined to the unavailability of the first-best instrument. It is well known since Weitzman (1974) that under uncertainty, the relative slope of the marginal cost and marginal benefit curve is key to choose between a price instrument (e.g. a CO₂ tax) and a quantity instrument (e.g. a cap-and-trade system, like the EU-ETS). More specifically, in the simplest form of Weitzman's (1974) model, the quantity instrument should be chosen if the marginal benefit curve is steeper than the marginal abatement cost curve while the price instrument should be chosen if the marginal abatement cost curve is steeper. If the marginal benefit curve is completely flat then a tax (set at the expected marginal benefit) is the first-best instrument. In the case of climate change control, most researchers have concluded that on this ground, a tax should be preferred to a cap-and-trade system (e.g. Pizer (1999)). Indeed the marginal benefit curve for CO₂ abatement over a few years period is relatively flat because CO₂ is a stock pollutant (Newell and Pizer, 2003). Actually, this argument is even stronger for policies covering only a small part of total emissions, such as the EU ETS; hence, with an uncertain marginal abatement cost curve, an ETS is less efficient than a tax, i.e. it brings a lower expected welfare.

¹The unconvinced reader is invited to look at the National Communications to the UNFCCC: http://unfccc.int/national_reports/items/1408.php

²Lees (2012) provides a recent survey of these systems in Europe, while Giraudet et al. (2012) discuss the costs and benefits of these systems.

Yet, in the EU, a meaningful CO₂ tax is out of reach because fiscal decisions are made under the unanimity rule, while a cap-and-trade system has been adopted thanks to the qualified majority rule which applies to environmental matters (Convery, 2009). Another main reason why cap-and-trade was chosen was for political economy reason in order to be able to alleviate opposition of e.g. power producers by means of free allocation of emission permits (Boemare and Quirion, 2002).

The fact that the EU ETS is not optimal is illustrated by its history since its introduction in 2005, which shows how volatile the CO₂ price can be: it dropped to virtually zero in 2007 because allowance allocation in phase I was too generous (Ellerman and Buchner, 2008), recovered up to more than 30€/tCO₂ because allocation in phase II was tighter and dropped again sharply in 2009 following the economic crisis, down to 8€/t CO₂ in October 2012. While economists disagree over the marginal benefit of CO₂ abatement, commonly called the "social cost of carbon" (Perrissin Fabert et al., 2012), they would presumably agree that such a price evolution is inefficient: in some periods, the CO₂ price has prompted relatively expensive abatement options (up to 30€/t CO₂) while in other periods, cheaper abatement options have not been implemented. This potentially provides a rationale for correcting the ETS and/or for complementing it. Among the proposed corrections is the introduction of a price cap and a price floor. Since this proposal has been widely debated (e.g. Hourcade and Ghersi (2002)), we will not address it in this paper.

Conversely, to our knowledge only two papers have addressed the role of uncertainty on abatement costs on the effectiveness of multiple instruments. Mandell (2008) show that under some conditions, it is more efficient to regulate a part of emissions by a cap-and-trade program and the rest by an emission tax, than to use a single instrument. Admittedly, under such a mixed regulation, the marginal abatement cost differs across emission sources, which is inefficient, but the emission volume is generally closer to the ex post optimum than under a single instrument: following an increase in the marginal abatement cost, the tax yields too high an emission level while the cap-and-trade system yields a level which is too low, so these inefficiencies partly cancel out.

The other paper is by Hoel (2012, section 9) who studies the opportunity to subsidize REP in case of an uncertain future carbon tax. He studies the case of scientific uncertainty (damages caused by climate change are uncertain) and political uncertainty (the current government knows that there might be a different government in the future, and that this government may have a different valuation of emissions). He shows that scientific uncertainty justifies a subsidy to REP if REP producers are risk-averse. Under political uncertainty, results are more complex. If the current government expects the future government to have a lower valuation of emission reductions than itself, this tends to make the optimal subsidy positive. Hoel (2012) studies the impact of uncertainty, but only when the subsidy is combined to a tax, not when it is combined to an ETS — which is what the present article focuses on.

While we also address the role of uncertainty concerning abatement costs on the effectiveness of multiple instruments, our focus is on whether it makes sense to use several instruments to cover the same emission sources and not to cover different sources, as in Mandell's article (Mandell, 2008). More precisely, we assume that the EU cannot implement a CO₂ tax because of the above-mentioned unanimity rule but can implement an ETS. However some CO₂ abatement options (for illustration, REP) can be incentivised by a price instrument (in this case, a subsidy to REP, e.g. a feed-in tariff). In our model, without uncertainty on the energy demand level (and hence on abatement costs) or if uncertainty is low enough, using the REP subsidy in addition to the ETS is not cost-efficient because there is no reason to give a higher subsidy to REP than to other abatement options. However we find that this uncertainty provides a rationale for using the REP subsidy in addition to the ETS, if it is large enough to entail a risk of a nil CO₂ price³. Even though the first-best policy would be a CO₂ tax, when the latter is unavailable, using both a REP subsidy and an ETS may provide a higher expected welfare than using an ETS alone.

We demonstrate this result using three approaches. Section 2 presents the intuition in a graphical approach. Section 3 develops an analytical model and presents some key analytical

³Since we use an expected welfare maximization model with a subjective probability distribution, we do not distinguish between risk and uncertainty.

results based on the same intuition. Section 4 further completes the model and presents a numerical application on the European power sector. Section 5 concludes.

2 The possibility of a nil CO₂ price: justification and implications for instrument choice

As explained above, the main conclusion of our model is that using a REP subsidy in addition to the ETS improves expected welfare in so far as uncertainty on the demand level is large enough to entail a possibility of a nil CO₂ price, i.e. if there is a possibility that demand for GHG quotas turns out to be so low, compared to its expected value, that the ETS cap becomes non-binding. Such a possibility is not accounted for in Weitzman’s seminal Prices vs. Quantities paper (Weitzman, 1974), or in the related literature. Hence, before presenting our model, we explain in the present section why we consider that this possibility should be accounted for and, using simple graphs, why it qualitatively changes Weitzman’s paper conclusions.

Existing experiences with cap-and-trade systems indicate that an allowance price dropping to zero in an ETS is not unrealistic at all. Indeed, it has happened in some of the most well-known ETS worldwide. In the EU ETS, the CO₂ price dropped to zero at the end of the first period (in 2007). It would have done so in the second period (2008-1012) again without the possibility to bank allowances for the next period (2013-2020) and the likelihood of a political intervention to sustain the price. In the Regional Greenhouse Gas Initiative (RGGI), which covers power plant CO₂ emissions from North-Eastern US states, phase one carbon emissions fell 33% below cap (Point carbon, 2012). Consequently, the price remained at the auction reserve price, below \$2/tCO₂. The cap also turned out to be higher than emissions in the tradable permit program to control air pollution in Santiago, Chile (Coria and Sterner, 2010) and in the UK greenhouse gas ETS (Smith and Swierzbinski, 2007). Even in the US SO₂ ETS, the price is now below \$1/tSO₂ (Schmalensee and Stavins, 2012), vs. more than \$150/tSO₂ ten years before, because new regulations and the decrease in high-sulfur fuels consumption have reduced emissions below the cap.

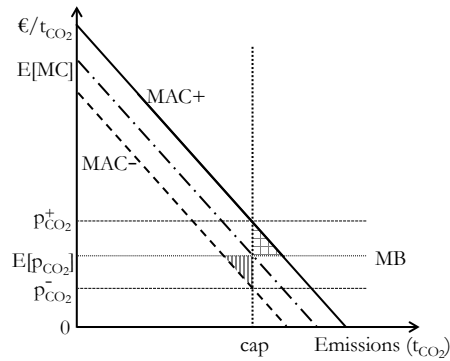
The implications of the possibility of a nil CO₂ price on optimal policy instrument choice are illustrated on Figure 1. For our purpose, it is more convenient to draw the marginal benefit and cost as a function of emissions rather (as in Weitzman’s paper) than as a function of abatement, because we are interested in the uncertainty of unabated emissions. Let’s assume that the Marginal Benefit MB is known with certainty and is perfectly flat. We do not model the uncertainty on the benefit side since it is well known that this uncertainty matters only when correlated with abatement cost (Stavins, 1996, Weitzman, 1974). In our model, as in these two papers, adding (uncorrelated) uncertainty on benefits would not influence the ranking of instruments. Let’s further assume that the marginal abatement cost curve is uncertain and can take with an equal probability two values, $MAC+$ and $MAC-$ ⁴, representing for instance the two extreme cases of a probability distribution. This uncertainty on the MACs captures economic uncertainty, as well as uncertainty on the technological costs (Quirion, 2005). In Figure 1a, uncertainty is lower ($MAC-$ (decreasing dashed line) and $MAC+$ (decreasing solid line) are closer) than in Figure 1b and 1c.

Since the marginal benefit MB is known with certainty and perfectly flat, a price instrument (like a CO₂ tax) is optimal, both ex ante and ex post. On the opposite, a quantity instrument (like an emission cap or the EU-ETS) is generally not optimal ex post because the cap does not follow the (ex post) optimal emission level. Let’s analyze how a risk-neutral policy maker minimizing expected cost (or maximizing expected welfare) would set the cap.

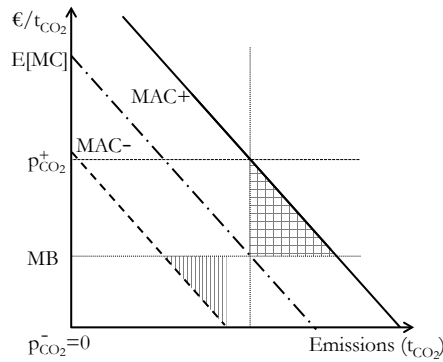
In Figure 1a, with a low uncertainty, the policy maker would set the optimal cap at the intersection between the marginal benefit and the expected marginal abatement cost curve (the dotted-dashed line). This is also the expected emission level under a price instrument. The expected CO₂ price would then equal the marginal benefit⁵, although ex post, the CO₂ price

⁴Noted MC in Weitzman (1974).

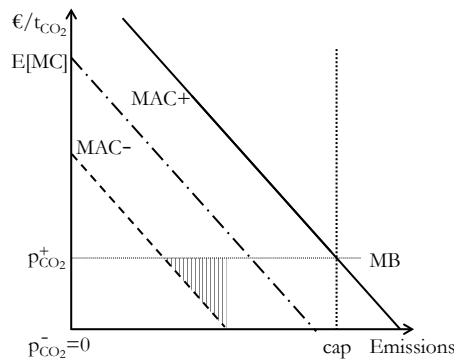
⁵ This equality (in expectation) between the price instrument and the quantity instrument regarding price and quantity is dubbed "certainty equivalence" by Hoel and Karp (2001). They show that while the equivalence prevails with additive uncertainty (a shift of the marginal abatement cost curve as in Weitzman’s original paper), it does



(a) Instrument choice with low uncertainty: the policymaker sets the cap at the intersection of the expected marginal costs and the marginal benefits, minimizing the expected extra cost compared to the *ex-post* optimum (area with vertical lines in the *MAC-* state and area with squares in the *MAC+* state).



(b) Instrument choice with high uncertainty: here setting a cap at the intersection of the expected marginal costs and the marginal benefits does not minimize the total costs. The CO₂ price is nil in the *MAC-* state.



(c) Instrument choice with high uncertainty: setting the cap at the intersection of the *MAC+* marginal costs and the marginal benefits minimizes the costs in the *MAC+* state with no additional costs in the *MAC-* state.

Figure 1: The implications of the possibility of a nil CO₂ price on optimal policy instrument choice.

would be either higher ($p_{\text{CO}_2}^+$) or lower ($p_{\text{CO}_2}^-$) than the expected CO_2 price ($E[p_{\text{CO}_2}]$). The cost of the quantity instrument compared to the price instrument (or to the optimum) is given by the area with squares (in case of a higher than expected cost) or by the area with vertical lines (in case of a lower than expected cost). All this is consistent with Weitzman’s standard model.

Conversely, in Figure 1b which features a large uncertainty, setting the optimal cap at the intersection between the marginal benefit and the expected marginal abatement cost curve (vertical dotted line) does not minimize the expected cost: such a cap would not be binding in the *MAC*-state, but it would entail a significant cost, both in the *MAC*-state (the area with vertical lines) and in the *MAC+* state (the area with squares).

A better solution (Figure 1c) is to set a more lenient cap which equalizes the marginal abatement cost and benefit only in the *MAC+* state: the extra cost compared to the price instrument would then be nil in the *MAC+* state while it would still equal the area with vertical lines in the *MAC*-state. In other words, the policymaker now neglects the *MAC*-state, knowing that in such an eventuality, the cap is non-binding anyway; rather he sets the cap which is optimal in the high-cost state.

Notice in Figure 1c that in the *MAC+* state, the marginal abatement cost equals the marginal benefit; hence the welfare loss from a marginal additional effort would only be of the second order. Conversely, in the low-cost state, the marginal abatement cost is below the marginal benefit; hence the welfare gain from a marginal additional effort would be of the first order. Consequently, an additional policy instrument might improve welfare even if it entails additional abatement in both states of nature, and even if it is imperfect — for example, because it targets only a subset of abatement options, like a REP subsidy.

Having explained the intuition of our main results, we now turn to the presentation of the analytical model.

3 Key analytical results in a stylized power market

In order to examine more in detail the implications on the power sector of a possible nil CO_2 price, we model a stylized European power market in which power demand is uncertain and can have two different levels in the future. This uncertainty can be related to the uncertain marginal abatement cost of the previous section, as a higher/lower power demand will lead to a higher/lower abatement effort for a given emission cap and thus a higher/lower marginal cost.

The next subsection describes the model and presents the equations in a setting with an ETS and a REP subsidy. See A and following for a description of all instrument settings used in the analytical results. Following subsections highlight some key analytical results building on the assumptions presented in previous section.

3.1 Analytical framework and equations

The model represents three types of agents: a regulator, representative power producers and representative consumers. The regulator maximizes an expected welfare function by choosing the optimal level of various instruments depending on the available instrument set: a carbon tax, an emission cap for the power sector or a REP subsidy. The emission cap can be interpreted as a stylized representation of the EU-ETS. The future level of demand is uncertain, with a risk that the carbon price drops to zero in case of low demand. The power market is assumed to be perfectly competitive and we assume a 100% pass-through of the emission allowance.

The model is a two-stage framework. In the first stage, the regulator chooses the level of the various policy instruments, facing an uncertainty about the level of future power demand. In the second stage, the power producers maximize their profit given the carbon tax, the emission cap, the subsidy and the demand function. The model is solved backward. In a first step, we

not under multiplicative uncertainty (a change in the slope of the marginal abatement cost curve). In this paper, we find that even with additive uncertainty on abatement costs, this principle does not prevail if there is a possibility that the price drops to zero.

determine the reaction functions of the producers as a function of policy instruments for various demand states. In a second step, we solve the expected welfare-maximization problem of the regulator over all states to find the optimal levels of the policy instruments.

3.1.1 Step 1: the producer profit maximization problem

We consider two types of power generation: fossil fuels (f) and REP (r). The power producers can also make abatement investments (a) to comply with the emission cap. Those abatements are assumed for simplicity to be independent from the level of fossil-based production. They refer for instance to investments making coal-fueled power plants able to cope with some share of biomass, CCS investments or allowance purchases on the Clean Development Mechanism (CDM) market. p is the electricity wholesale price.

Producers face an aggregate emission cap Ω and benefit from a REP subsidy ρ . ϕ is the carbon price emerging from the allowance market, equal to the shadow value of the emission cap constraint. We assume a 100% pass-through from allowance costs to wholesale price. The producer maximizes its profit Π (Table 1 describes all the variables and parameters).

$$\max_{f,r,a} \Pi(p, f, r, a, \phi) = p \cdot f + (p + \rho) \cdot r - C_f(f) - C_r(r) - AC(a) - PC(f, a, \phi) \quad (1)$$

where $C_f(f)$ and $C_r(r)$ are the production costs from fossil fuel and REP respectively. We assume decreasing returns for REP and constant returns for emitting power plants ($C'_f(f) > 0, C'_r(r) > 0, C''_f(f) = 0$ and $C''_r(r) > 0$). The decreasing returns assumption is due to the fact that the best production sites are used first and that further REP development implies investing in less and less productive sites. On the contrary, emitting technologies such as combined cycles power plants or advanced coal power plants are easily scalable and thus do not generate a scarcity rent (Fischer, 2010, Fischer and Preonas, 2010, Jonghe et al., 2009). $AC(a)$ is the Abatement Cost function of the power producers, independent of fossil or REP production and $PC(f, a, \phi)$ is the allowance Purchasing Cost. The cost functions have a classical linear-quadratic form:

$$\begin{aligned} C_f(f) &= \iota_f \cdot f \\ C_r(r) &= \iota_r \cdot r + \frac{r^2}{2\sigma_r} \\ AC(a) &= \frac{\sigma_a}{2} a^2 \\ PC(f, a, \phi) &= \phi \cdot (\tau \cdot f - a) \end{aligned}$$

With ι_f and ι_r the intercepts (iota like intercept) of the fossil fuel and the REP marginal supply function respectively and σ_r the slope (sigma like slope) of the REP marginal supply function. σ_a is the slope of the marginal abatement cost curve for the power producer and τ is the average unabated emission rate from fossil fuel-based power production. We define a linear downward sloping electricity demand function $d(\cdot)$ (with $d'(\cdot) < 0$) whose intercept depends on the state of the world. We consider two different states s which are equally probable, one with a high demand ($d_+(p)$) and one with a low demand ($d_-(p)$). The demand function is defined as:

$$d(p) = \iota_d \pm \Delta - \sigma_d \cdot p$$

with the intercept being $\iota_d + \Delta$ in the high demand state of the world and $\iota_d - \Delta$ in the low demand state. The equilibrium conditions on the power and the emission market thus depend on the state of the world.

$$f + r = d(p) \quad (2)$$

is the demand constraint. In each state of the world, the power supply has to meet the demand on the power market.

$$\begin{cases} \tau \cdot f_- - a_- < \Omega \\ \phi_- = 0 \end{cases} \quad \text{or} \quad \begin{cases} \tau \cdot f_+ - a_+ = \Omega \\ \phi_+ > 0 \end{cases} \quad (3)$$

	Dimension	Description
f	(MWh)	Electricity from fossil fuels
r	(MWh)	Electricity from REP sources
p	(€/MWh)	Wholesale power price
e	(tCO ₂)	Abatements from the non-power sector
a	(tCO ₂)	Abatements from the power sector
ϕ	(€/tCO ₂)	Carbon price
ρ	(€/MWh)	REP subsidy
Ω	(tCO ₂)	Emission cap
σ_a	(€/tCO ₂ ²)	Slope of the power sector MACC
σ_e	(€/tCO ₂ ²)	Slope of the aggregate CITL sector MACC
σ_d	(MWh ² /€)	Slope of the demand function
σ_r	(MWh ² /€)	Slope of the RE supply function
δ	(€/tCO ₂)	Marginal environmental damage
Δ	(MWh)	Variance of demand
τ	(tCO ₂ /MWh)	Average emission rate of fossil fuels
ι_f	(€/MWh)	Intercept of the fossil fuel supply function
ι_r	(€/MWh)	Intercept of the RE supply function
ι_d	(€/MWh)	Intercept of the demand function
ι_e	(€/tCO ₂)	Intercept of the aggregate CITL sector MACC

Table 1: Variables and parameters used in the models.

expresses the joint constraint on emissions and CO₂ price. In the high-demand state of the world, total emissions cannot be higher than the cap Ω and the CO₂ price is therefore strictly positive. In the low-demand state, we assume that the emission cap constraint is non-binding, hence the CO₂ price is nil.

The first order conditions of the producer maximization problem are the following:

$$p = \iota_f + \tau\phi \quad (4)$$

Fossil fuel producers will equalize marginal production costs with the wholesale market price, net from the price of emissions.

$$\rho + p = \iota_r + \frac{r}{\sigma_r} \quad (5)$$

REP producers will equalize marginal production costs with the wholesale market price, net from the subsidy.

$$\sigma_a a = \phi \quad (6)$$

Fossil fuel producers will equalize the marginal abatement cost with the carbon price.

The values of the market variables (p, f, r, a, ϕ) as a function of policy instruments are found by solving the system of equations (2) to (6). They represent the reaction functions of the power producer.

3.1.2 Step 2: the regulator's expected welfare maximization problem

The regulator, assumed risk-neutral and giving the same weight to consumers and producers for clarity, faces an uncertain future demand and has a limited number of possible policy instruments (i.e. an emission cap and a REP subsidy) to achieve its objective of maximizing the following expected welfare function:

$$\max_{\Omega, \rho} EW(\Omega, \rho) = \sum_{\text{states}} \frac{1}{2} (CS(p) + \Pi(p, f, r, a, \phi) - \text{dam}(f, a) - \rho \cdot r + PC(f, a, \phi)) \quad (7)$$

Where $\frac{1}{2}$ is the probability of the state of the world $+/-$, $CS(p)$ is the consumer surplus and $dam(f, a)$ is the environmental damage function from the GHG emissions. The last two terms of the expected welfare cancel pure transfers between agents included in the profit functions. The consumer surplus CS and the damage function are taken as simple as possible for clarity. In particular, consumer are assumed risk-neutral:

$$CS(p) = \int_0^{d(p)} d^{-1}(q) dq - p \cdot d(p)$$

$$dam(f, a) = \delta \cdot (\tau f - a)$$

With δ the constant environmental damage coefficient (Newell and Pizer, 2003). After having substituted the market variables in the expected welfare function (7) with the reaction functions coming from the producer problem we maximize the expected welfare. The first-order conditions give the optimal levels of the policy instruments across all states (ρ^* and Ω^*).

3.2 Social optimum when the CO₂ price is nil in the low demand state

Proposition 1. *When the CO₂ price is nil in the low-demand state of the world, the optimal renewable subsidy is strictly positive.*

Proof. The optimal levels of the policy instruments across all states are given by solving the first-order conditions of the welfare maximization problem (7) (see C).

$$\Omega^* = \frac{\sigma_a(\Delta + \iota_d + \iota_r \sigma_r - \iota_f(\sigma_d + \sigma_r))\tau + \delta(-1 - \sigma_a(\sigma_d + \sigma_r)\tau^2)}{\sigma_a} \quad (8)$$

$$\rho^* = \frac{\delta \cdot \tau \cdot (1 + \sigma_a(\sigma_d + \sigma_r)\tau^2)}{2 + \sigma_a(2\sigma_d + \sigma_r)\tau^2} \quad (9)$$

knowing that all parameters are positive, and using the reaction functions from the profit maximization problem (1), we can write:

$$0 < \rho^* < \delta \tau \quad (10)$$

Results follow directly. \square

If we consider only one certain state, then we fall back on the first-best optimum characterized by a REP subsidy equal to zero and the emission cap set so as to equalize the carbon price with the marginal damage δ . We see here in (10) that the optimal subsidy is a portion of the marginal environmental damage (see also (12) below).

By substituting the optimal levels of policy instruments in the reaction functions, we obtain the socially optimal level of all market variables for both states of demand (see C). The optimal level of abatement is proportional to the carbon price. There is no abatement (apart from investment in REP) in the low demand state of the world, as we assumed the emission cap to be non-binding (and hence the carbon price to be nil). In both states, the wholesale price level equals the marginal production cost of fossil energy. When the cap is binding and independent abatements help mitigate emissions, equilibrium expressions reflect the substitutions between the various options in order to comply with the cap (direct abatements, fossil fuel reductions and REP).

While in a first-best world the carbon price equals the marginal environmental damage, in this second-best setting, the optimal carbon price (in the high-demand state) is lower because the REP subsidy also reduces emissions. The expected carbon price is equal to:

$$\frac{\phi_+^* + \phi_-^*}{2} = \frac{\delta + \delta \sigma_a \sigma_d \tau^2}{2 + \sigma_a(2\sigma_d + \sigma_r)\tau^2}$$

and can be rewritten as:

$$\frac{\phi_+^* + \phi_-^*}{2} = \frac{\delta}{2 + \frac{\sigma_a \sigma_r \tau^2}{1 + \sigma_a \sigma_d \tau^2}} \quad (11)$$

where it is obvious that the expected carbon price is only a portion of the marginal environmental damage. The term in the denominator expresses the substitutions taking place when the abatement through carbon pricing only is no longer optimal.

Proposition 2. *When the CO₂ price is nil in the low-demand state of the world, the renewable subsidy equivalent in €/tCO₂ is equal to the marginal benefit from abatements minus the expected carbon price.*

Proof. Combining (9) and (11) gives:

$$\frac{\rho^*}{\tau} = \delta - \frac{\phi_+^* + \phi_-^*}{2} \quad (12)$$

The proof follows directly. \square

In (12), $\frac{\rho^*}{\tau}$ is the marginal abatement effort through REP promotion and $\frac{\phi_+^* + \phi_-^*}{2}$ is the marginal abatement effort through carbon pricing. The simple intuition behind this result is that since the expected CO₂ price is below the marginal benefits, the additional instrument, e.g. the REP subsidy, is also used to reduce emissions.

3.3 Expected emissions with various instrument mixes

As mentioned in section 2, in Weitzman's model (Weitzman, 1974) with an additive uncertainty on the marginal abatement cost curve, the expected emissions are the same with a price or a quantity instrument. This is no longer the case in our model.

Proposition 3. *If there is a risk that the CO₂ price equals zero in the low-demand state of the world, expected emissions vary with the instrument mix.*

Proof. We compute expected emissions in three instrument mix settings (see A for a description of all instrument settings used):

- A **first best instrument mix** (M₁), with a unique CO₂ price across all states of the world⁶;
- A **second best instrument mix** (M₂), with an ETS and a REP subsidy;
- A **third best instrument mix** (M₃), with an ETS alone.

The uncertainty is assumed to be such as the CO₂ price resulting from an ETS in the low state of demand turns out to be nil (as shown in the model description above). The expected emissions \mathcal{E} are given by:

$$\mathcal{E} = \frac{1}{2} (\tau \cdot f_- - a_-) + \frac{1}{2} (\tau \cdot f_+ - a_+) \quad (13)$$

Let us call \mathcal{E}_X the expected emissions for a given instrument mix M_X. For the previously described mixes, we get:

$$\mathcal{E}_1 = (\iota_d + \iota_r \sigma_r - \iota_f (\sigma_d + \sigma_r)) \tau - \frac{\delta (1 + \sigma_a (\sigma_d + \sigma_r) \tau^2)}{\sigma_a} \quad (14)$$

$$\mathcal{E}_2 = (\iota_d + \iota_r \sigma_r - \iota_f (\sigma_d + \sigma_r)) \tau - \frac{\delta (1 + \sigma_a (\sigma_d + \sigma_r) \tau^2)^2}{\sigma_a (2 + \sigma_a (2\sigma_d + \sigma_r) \tau^2)} \quad (15)$$

$$\mathcal{E}_3 = (\iota_d + \iota_r \sigma_r - \iota_f (\sigma_d + \sigma_r)) \tau - \frac{\delta (1 + \sigma_a (\sigma_d + \sigma_r) \tau^2)}{2\sigma_a} \quad (16)$$

where we see that $\mathcal{E}_1 < \mathcal{E}_2$ and that $\mathcal{E}_2 < \mathcal{E}_3$. \square

⁶Since the marginal benefit is flat, the first-best instrument is always a price instrument, e.g. a carbon tax.

Parameter	Meaning of an increase in the parameter	Sign of partial derivative
σ_a	Higher abatement cost	-
σ_d	More elastic power demand	+
σ_r	Cheaper REP	+
δ	Higher marginal damage	+
Δ	Higher demand variance	-
τ	Higher emission rate of fossil fuels	?

Table 2: Signs of partial derivatives of the boundary function with respect to parameters – indicates a negative partial derivative, + indicates a positive partial derivative and ? indicates an ambiguous sign).

The expected emissions are different in the three settings. They are lower in the second best setting (15) than in the third (16) and even lower with a first best carbon tax (14).

Other variables change when comparing first best, second best and third best. The expected CO₂ price is lowest in the second best setting when it is optimal to implement a REP subsidy along with the emission cap. The drop between first best and second best is mostly due to the nil CO₂ price in the low demand state of the world. When comparing third best and second best, the CO₂ price is lower because another instrument, the REP subsidy, is now also used to reduce emissions.

3.4 Boundary condition for having a nil CO₂ price in the low demand state of the world

As we have seen, the possibility that the CO₂ price becomes nil has important consequences. It is thus useful to know when this possibility occurs. In this aim, we take the value of ϕ_-^* in a model without the additional non-negativity constraint on the carbon price (see D, and A for a description of the instrument settings). When unconstrained, the carbon price in the low demand state has the following form:

$$\phi_-^* = \delta - \frac{\Delta\sigma_a\tau}{1 + \sigma_a(\sigma_d + \sigma_r)\tau^2}$$

Proposition 4. *The carbon price in the low state of demand moves toward zero as mitigation options (abatements and REP) become expensive, uncertainty on the level of the power demand is large, the demand is inelastic and the environmental damage is low.*

Proof. Let us define a boundary function:

$$\Psi(\delta, \Delta, \sigma_a, \tau, \sigma_d, \sigma_r) = \delta - \frac{\Delta\sigma_a\tau}{1 + \sigma_a(\sigma_d + \sigma_r)\tau^2}$$

A lower carbon price in the low state of demand ϕ_-^* is equivalent to having a lower boundary function. The proof then follows from the analysis of the vector of partial derivatives of $\Psi(\cdot)$. Table 2 shows the sign of the partial derivative of $\Psi(\cdot)$ with respect to the parameters. Higher mitigation costs lead to a less stringent emission cap Ω , lowering the carbon price in both states of demand and increasing the risk of a nil CO₂ price. A higher marginal damage and a more elastic power demand (which means higher energy savings for a given change in power price) lead to a more stringent cap. \square

Param.	Meaning of an increase in the parameter	Level of demand (state)	f fossil fuel	r REP	p electricity price	a abate-ments	ϕ CO ₂ price
σ_a	Higher abatement cost	High (+)	+	-]-1;0[i-1]-1;0[
		Low(-)	-	+	0	0	0
σ_d	More elastic power demand	High (+)	?	+]0;1[]0;1[]0;1[
		Low(-)	?	-	0	0	0
σ_r	Cheaper REP	High (+)	?	?]-1;0[]-1;0[]-1;0[
		Low(-)	?	?	0	0	0
δ	Higher marginal damage	High (+)	-	+]0;1[1	1
		Low(-)	-	+	0	0	0

Table 3: Market variables' elasticity with respect to various parameters.]-1;0[indicates an elasticity between 0 and -1;]0;1[indicates an elasticity between 0 and 1; + or - indicate respectively a positive or negative elasticity; ? indicates an ambiguous sign of the elasticity.

Parameter	Meaning of an increase in the parameter	ρ : REP subsidy	Ω : Emission cap
σ_a	Higher abatement cost]0;1[+
σ_d	More elastic power demand]-1;0[-
σ_r	Cheaper REP]0;1[?
δ	Higher marginal damage	1	-

Table 4: Elasticity of instrument variables with respect to various parameters.]-1;0[indicates an elasticity between 0 and -1;]0;1[indicates an elasticity between 0 and 1; + or - indicate respectively a positive or negative elasticity; ? indicates an indeterminate sign of the elasticity.

3.5 Variables' elasticity with respect to parameters

As a preliminary step to the numerical sensitivity analysis presented in Section 4, Table 3 and Table 4 show the sign of the elasticity of all variables with respect to various parameters in the 2nd Best setting (instrument mix M_2 , see A), and indicate whether they are above or below 1.

Proposition 5. *the optimal subsidy ρ^* rises as abatement is more expensive, production from REP sources is cheaper, power demand is less elastic to electricity price and the marginal environmental damage from GHG emissions rises.*

Proof. Table 4 shows the sign of variation of the optimal levels of policy instruments when various parameters change⁷. A positive elasticity indicates a positive variation when a parameter increases, and an absolute elasticity smaller than one indicates that a 1% change in that parameter will cause a less than 1% change in the variable. We see that the elasticity of ρ with respect to σ_a and σ_r is positive but smaller than 1, with respect to σ_d it is negative but smaller than one and the elasticity with respect to δ is 1. The proof follows directly. \square

The explanation of this result is straightforward: more REP should be installed when the environmental damage is higher, when REP are cheaper and when the other ways to reduce emissions, i.e. abatement and energy savings become more expensive. Similarly, a higher abatement cost naturally leads to a less stringent emission cap Ω , while a higher marginal damage and a more elastic power demand (which means higher energy savings for a given change in power price) lead to a more stringent cap. The impact of cheaper REP on the optimal cap is ambiguous: on the one hand, it reduces the overall cost of cutting emissions, leading to a more stringent cap, but on the other hand it pushes to an increased use of the other policy instrument, the subsidy, which minors the importance of the emission cap.

Table 3 shows that in state -, there is no abatement, the carbon price is nil and the power price is solely determined by the supply curve, so the parameters considered in Table 3 have no effect on these variables. However, they have an indirect effect on f_- and r_- since they impact ρ . Hence, the considered parameters increase the amount of REP r_- and they decrease the amount of fossil-fuel electricity f_- when they increase the REP subsidy ρ .

In state +, as one could have expected, more abatements and a higher CO₂ price ϕ_+ are triggered by a lower abatement cost, a more elastic power demand, more expensive REP, and a higher marginal damage. Moreover, a higher power price is triggered by a higher marginal damage, costlier REP, a more elastic power demand and, more surprisingly, a lower abatement cost. The explanation is that a lower abatement cost implies a more stringent target (Table 4), which in turn raises the power price in state +.

In state +, changes in energy production follow changes in the CO₂ price ϕ_+ : lower abatement costs, higher marginal damages and a more elastic power demand increase the CO₂ price, which

⁷Elasticities have been calculated in Mathematica. The Mathematica notebook is available upon request from the contact author

in turn decrease the relative competitiveness of fossil fuel. In state $-$, the CO₂ price is nil and changes are more sensitive to the REP subsidy: higher abatement costs, higher marginal damages and a more elastic power demand increase the optimal REP subsidy, which in turn increase the relative competitiveness of REP.

Comparing Table 4 and Table 3 finally shows that the carbon price and the REP subsidy vary in opposite directions (except when the marginal damage changes). This can be seen in (12). If there is a risk that the carbon price equals zero in the low demand state of the world, the mitigation efforts induced by the carbon price are no longer sufficient. An additional effort through REP production is necessary, induced by a strictly positive REP subsidy.

4 Numerical application : the European power sector and allowance market

4.1 Modified model

Having shown some analytical results with a model of a power sector alone, we turn to a slightly more complex model to show numerical results calibrated on the European power and allowance market. In this section, we add an explicit allowance supply from non-power CITL sectors. We therefore add a composite sector including all the other constrained emitters. The power producer can buy emission allowances (e) from the other constrained sectors on the allowance market to comply to the emission constraint. The other ETS sectors are represented by their total abatement cost function, which has the following form:

$$AC_e = \frac{\sigma_e}{2} e^2 - \begin{cases} \iota_e e & \text{in state } + \\ 0 & \text{in state } - \end{cases}$$

where σ_e is the slope of the aggregate non-electricity ETS sector marginal abatement cost curve. The intercepts differ in the low demand and the high demand state of the world. We assume there is a positive correlation between the level of power demand and the level of industrial activity. When the power demand is low, the industrial activity is also low and the allowance surplus is higher.

Next subsections will detail the data and assumptions made to calibrate the model. Some parameters being subject to a large uncertainty, we use a range of possible values for those parameters and discuss the distribution of results. For each uncertain parameter, we use a uniform probability distribution and we assume that these parameters are not correlated (except for the power demand and the industrial activity levels). Table 5 shows the minimum, median and maximum values of calibrated parameters resulting from the calibration process and used in the simulations.

We performed simulations with all possible combinations of parameters shown in Table 5, without any constraint on the carbon price. We tested the positivity of the carbon price, and if negative in the low demand state, we conducted other simulations by constraining the carbon price to be equal to zero in the low demand state. This distinguishes two qualitatively different simulation results. In the first category (subsequently called 2nd Best B), the carbon price is strictly positive in the low demand state and the renewable subsidy is nil. In the second category (subsequently called 2nd Best A), the carbon price is nil in the low demand state and the renewable subsidy is strictly positive. G details the equations and solution of this model.

4.2 Data and assumptions for calibration

4.2.1 Supply functions

The supply curves are tuned so as to match estimated long term marginal production costs functions. According to OECD (2010), the REP production break-even point starts at 80 €/MWh and goes up to 160 €/MWh. This marginal cost is rather a lower bound, as network and intermittency costs tend to raise it. We calibrated the REP supply function slope so as to reach the upper limit

	Dimension	Description	Min	Median	Max
σ_a	(€/MtCO ₂ ²)	Slope of the power sector MACC	0.44	0.81	2.2
σ_e	(€/MtCO ₂ ²)	Slope of the rest-of-ETS MACC	0.52	0.95	2.61
σ_r	(GWh ² /€)	Slope of the demand function	2.58	6.7	12.9
σ_d	(GWh ² /€)	Slope of the RE supply function	2.49	6.43	12.5
δ	(€/tCO ₂)	Marginal environmental damage	10	15.3	30
Δ	(TWh)	Variance of demand	32.8	69.6	98.3
τ	(tCO ₂ /MWh)	Average emission rate of fossil fuels	0.5	0.5	0.5
ι_f	(€/MWh)	Intercept of the fossil fuel supply function	80	80	80
ι_r	(€/MWh)	Intercept of the RE supply function	80	80	80
ι_d	(G€/MWh)	Intercept of the demand function	2.19	2.51	2.99
ι_e	(€/tCO ₂)	Intercept of the rest-of-ETS MACC (state +)	94.6	173	473
		Intercept of the rest-of-ETS MACC (state -)	0	0	0

Table 5: Values of the calibrated parameters.

of the REP long-term marginal cost at a given percentage of a reference production level. This reference production level is taken equal to the power production from REP and fossil fuels in 2008, that is 2,060 TWh (ENERDATA, 2011). For the maximal penetration rate of REP, we took a range of possible percentages, ranging from 10% to 50%. The fossil fuel long term supply curve, set at 80 €/MWh is tuned to an average European CCGT levelized cost of electricity OECD (2010).

4.2.2 Demand function

The demand function has been calibrated so as to have a given price-elasticity when the demand equals the average between the 2008 and the 2009 reference production levels (2,060 TWh in 2008 and 1,929 TWh in 2009 (ENERDATA, 2011)). We chose elasticities ranging from 0.1 to 0.5. The demand standard deviation Δ between the two states of the world was assumed to be close to the mean absolute deviation from the reference demand in 2008 and 2009. We chose values ranging from +50% to -50% of this value to account for the uncertainty on a possible future shock on demand.

4.2.3 Abatement costs

The slope of the marginal abatement cost curve in the power sector has been calculated as follows: given an average CO₂ price of 22€/tCO₂, we assumed that fuel-switch allowed to abate a range of percentages of the total emissions of the power sector in 2008, ranging from 1 % to 5 %. Ellerman and Buchner (2008) indicate an abatement of around 5% at a CO₂ price equal to 15€/tCO₂. The marginal abatement cost curve of the ETS sector other than power was calibrated in the same way, by assuming a certain percentage of abatement in 2008 given the CO₂ price. We assumed abatements ranging from 1% to 5% for both sectors. The intercept of the marginal abatement cost curve for non-power sectors in the low demand state was calculated so as to obtain the difference of allowance over-allocation between 2008 and 2009 when the CO₂ price drops to zero (102 MtCO₂ of allowance surplus in 2008, 241 MtCO₂ surplus in 2009; data from Sandbag (2012)). We took into account the perimeter of the CITL combustion sector — which includes power and heat production — by adding the additional surplus allowances coming from the heat plants (41 MtCO₂ according to Trotignon and Delbosc (2008)).

Description	Dimension	Range
Marginal environmental damage	(€/tCO ₂)	(10.,20.,...,30)
Price-elasticity of demand (absolute value)	1	(0.1,0.2,...,0.5)
Abatement from the aggregate ETS sector for 15 €/tCO ₂	(%)	(1,2,...,5)
Abatement from the power sector for 15 €/tCO ₂	(%)	(1,2,...,5)
Maximum share of REP in the energy mix	(%)	(10,20,...,50)
Standard deviation of demand	(TWh)	(33,49,...,98)

Table 6: Ranges of parameters used in the numerical simulations for calibration purposes. All possible combinations of parameters were successively simulated.

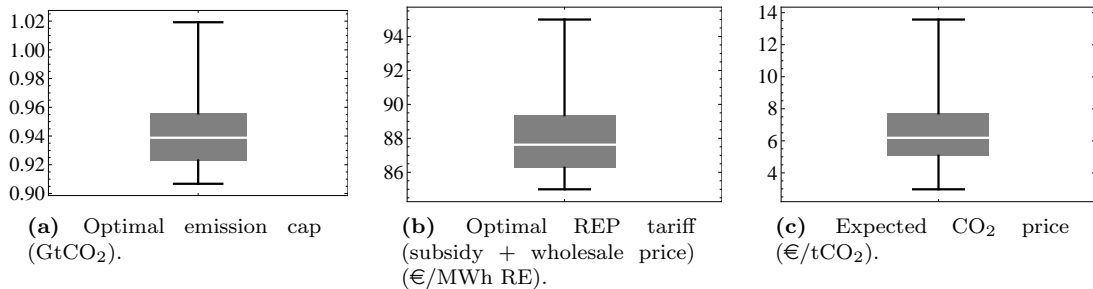


Figure 2: Box whisker plots of the optimal instrument values and expected CO₂ price for all simulations with a 2nd Best instrument setting (mix M₂ⁿ) and a nil carbon price in the low demand state of the world.

4.2.4 Additional parameters

We took an average emission rate of 0.5 tCO₂/MWh for fossil production (IEA Statistics, 2011), and a marginal damage between 10 and 30€/tCO₂. The calibration presented in previous paragraphs is very cautious, considering demand and production levels already observed in 2008 and 2009. The increased regulatory risk induced by the introduction of the third ETS phase and possible changes in the future Energy Efficiency Directive are captured through changing the standard deviation of demand and emission surplus from the non power ETS sector.

Table 6 synthesizes the range of values used for all parameters subject to a large uncertainty.

4.3 Optimal policy instruments and CO₂ price levels

With the parameter ranges shown in Table 5, 50.9% of the simulations display a nil carbon price in the low demand state and a strictly positive REP subsidy. Figure 2 illustrates Proposition 1. It shows box whisker plots of the optimal emission cap Ω^* (Fig. 2a) and the optimal REP subsidy ρ^* (Fig. 2b) in all simulations with a 2nd Best instrument setting (mix M₂ⁿ) and a nil carbon price in the low demand state. Figure 2c shows a box whisker plot of the expected CO₂ price.

The optimal emission cap ranges from 0.91 to 1.02 GtCO₂, and the optimal subsidy ranges from 2.68 to 9.93 €/MWh. The optimal expected CO₂ price ranges from 2.97 to 13.6 €/tCO₂. As a comparison, the actual cap calculated by Trotignon and Delbos (2008) amounts to 1.05 GtCO₂, the actual REP tariff range from 50 to 90 €/MWh in France and Germany and the

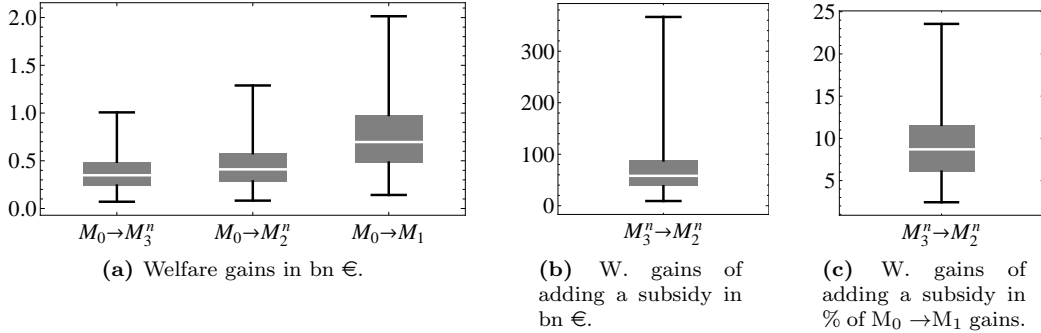


Figure 3: Box whisker plots of expected welfare gains from adding a given instrument mix to a BAU setting with no instrument ($M_0 \rightarrow M_3^n$, $M_0 \rightarrow M_2^n$, $M_0 \rightarrow M_1$) in bn €, and of expected welfare gains from adding a REP subsidy to an ETS ($M_3^n \rightarrow M_2^n$) in bn € and in percentage of the expected gains from a carbon tax, in all scenarios where the CO_2 price is nil in the low demand state of the world.

current CO_2 price is around 6 to 7 €/t CO_2 . The relatively low levels of both the expected CO_2 price and the REP subsidy are due to the fact that it is a linear combination of both that equals the marginal damage (see (12)). These values cannot necessarily be directly compared to actual subsidy levels since the latter account for all positive externalities expected from REP support.

4.4 Expected welfare gains from adding a REP subsidy

In order to evaluate the gains from adding a subsidy to the ETS, we compute the expected welfare differences between simulations with different instrument mixes. We compare four settings:

- A **first best instrument mix** (M_1), with a unique CO_2 price across all states of the world;
- A **second best instrument mix** (M_2), with an ETS and a REP subsidy;
- A **third best instrument mix** (M_3), with an ETS alone and a nil CO_2 price in the low state of demand.
- A **business-as-usual setting** (M_0), with no policy at all.

The gain — or welfare difference — is calculated as the drop in environmental damages minus mitigation costs. Fig. 3 shows box whisker plots of the expected welfare gains from adding a given instrument mix compared to the BAU setting (M_0 to M_3 , M_0 to M_2 , M_0 to M_1) in all scenarios where uncertainty is such that the CO_2 price turns out to be nil in the low demand state of the world.

Compared to a BAU setting with no instrument (mix M_0), The gains from having an ETS and a REP subsidy if there is a risk that the CO_2 price equals zero in the low state of demand are quite important, ranging from more than 1.4 billion € to several hundred millions €. The gains from adding a REP subsidy to an ETS range from a decade to several hundred million €. They represent from approximately 3% to 24% of the gains one could expect from a first best carbon tax.

4.5 Expected emissions, productions and prices with various instrument mixes

Following our analysis in section 3 and illustrating Proposition 3, the Fig. 4 presents box whisker plots of expected values of different variables in the simulations with a nil CO_2 price in the low state of demand (superscript n). We computed those values with a 1st Best instrument mix (a carbon

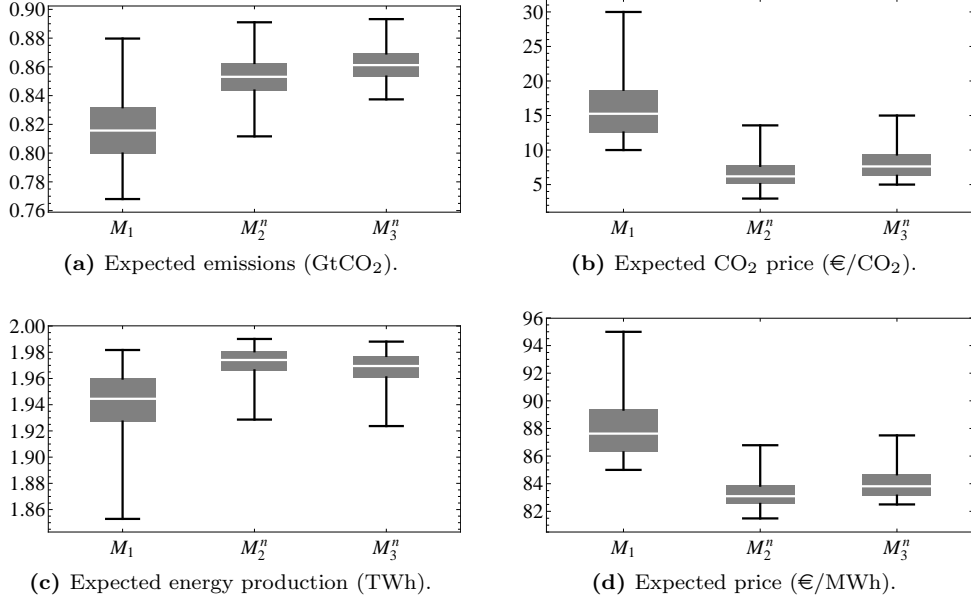


Figure 4: Box whisker plots of the expected values of various variables in simulations M_1 (carbon tax), M_2^n (ETS + REP subsidy) and M_3^n (ETS alone) when the CO₂ price is nil in the low state of demand.

tax, labeled M_1), with a 2nd Best setting (ETS + subsidy, labeled M_2^n) and in a 3rd Best setting (ETS alone, labeled M_3^n). Figure 4a presents the expected emissions, Figure 4b the expected CO₂ price, Figure 4c the expected energy production and Figure 4d the expected wholesale price.

Consistently with Proposition 3, Figure 4a shows that expected emissions are lower in the M_2^n setting than in the M_3^n setting, and the lowest in the M_1 setting. The expected CO₂ price is the lowest in the M_2^n setting. As a result, the wholesale price is also the smallest in the M_2^n setting, but expected energy production is the highest.

4.6 Shift in the optimal emission cap and CO₂ price

In order to discuss the optimization behavior of the regulator, we analyze the optimal instrument levels and carbon price in the second best setting (labeled M_2) for all parameter combinations. For each combination, the uncertainty on the power demand is either low enough to get an optimal emission cap that is binding in both states of demand (M_2^n), either too high and implies a nil CO₂ price in the low demand state of the world (M_2^p). We then compare the two groups of simulations and show the results as box whisker plots in Fig. 5. Fig. 5a shows the optimal emission cap for all parameter combination, Fig. 5b the REP subsidy, Fig. 5c the CO₂ price in the high demand state of the world and Fig. 5d the CO₂ price in the low demand state of the world.

As already discussed in section 2, Fig. 5a shows a higher emission cap in all M_2^n scenarios. This is due to the fact that when the CO₂ price turns out to be nil in the low demand state, no additional mitigation effort is made in this state and the cap is optimized *ex-ante* on the high demand level. Fig. 5b, 5c and 5d illustrate Proposition 2. If there is a risk that the CO₂ price equals zero as for all M_2^n scenarios in Fig. 5d, there is a strictly positive subsidy (M_2^n scenarios in Fig. 5b) and the CO₂ price in the high demand state of the world drops compared to M_2^p scenarios (Fig. 5c).

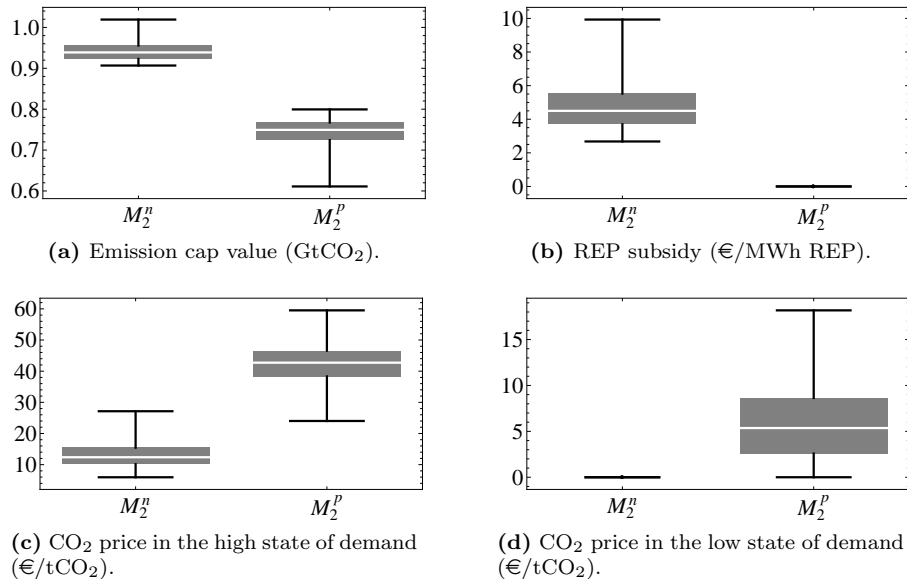


Figure 5: Box whisker plot of various instrument levels and CO₂ price in simulations M_2^n (ETS + REP subsidy and a nil CO₂ price in the low state of demand) and simulations M_2^p (ETS + REP subsidy and a strictly positive CO₂ price in the low state of demand).

5 Conclusion

We bring a new contribution to the analysis of the coexistence of several policy instruments to cover the same emission sources. We find that optimizing simultaneously an ETS and e.g. a subsidy to renewable energy power (REP) can improve the welfare compared to a situation with the ETS alone, especially if uncertainty on the level of power demand (and hence on the abatement costs) is high enough. In a context of a very low CO₂ price and large anticipated surplus on the EU ETS at least until 2020, these findings justify the addition of other policy instruments aiming at reducing CO₂ emissions covered by the ETS to a possible future revision of the emission cap.

We find that under a reasonable set of parameters, defining simultaneously an emission cap and an overlapping policy instrument, such as a REP subsidy of about 2.7 to 9.9 €/MWh (corresponding to a tariff ranging from 85 to 95 €/MWh) can improve welfare by about 2.4% to 23.6% of the total gain of a carbon tax, that is about 9 to 366 million €/yr. This gain is obtained through CO₂ emission reductions alone and does not rely on additional market failures or externalities. The addition of a REP subsidy also increases the total energy production, decreases the power price and the CO₂ price and reduces the total expected emissions. Our results are in line with existing literature concerning the decreasing effect of a REP subsidy on the carbon price when it is combined with an emission cap. We however show that under certain circumstances, interactions between a subsidy and an emission cap can reduce emissions and improve welfare, compared to an emission cap alone.

On a more methodological note, our results invite to deepen the reflection on the role of uncertainty. Noticeably, they highlight of the possibility of corner solutions (in this case, a zero CO₂ price), when comparing policy instruments and policy packages. In addition to showing that an optimal policy mix to reduce CO₂ emissions can contain more than one instrument, we find several key analytical results that qualitatively differ from the literature. For instance, expected emissions are no longer equivalent between policy instruments, even with an additive uncertainty on the marginal abatement cost, and the optimal emission cap no longer depends on all states of nature but only on the high-demand one.

Our results are based on the assumption that the risk of the CO₂ price dropping to zero cannot

be excluded. The history of many cap-and-trade systems, including the US acid rain program, RGGI and the EU ETS, fully justifies this assumption, since the allowance price has dropped to virtually zero (or to the floor price) in all these systems. Moreover, uncertainty on the CO₂ price does not only stem from the business cycle, as in our model, but also from uncertainty on future policies, such as the Energy Efficiency Directive currently debated in the EU. Our analysis brings some economic insight into the debate about the future European policy mix and about whether it is justified or preferable to complement a future revision of the EU-ETS cap with an overlapping instrument.

Further aspects could be worth investigating. Including an allowance floor price for instance should not change qualitatively our results as long as the floor price is below the marginal damage. Modeling banking across trading periods with periodic renegotiation of the cap could mitigate the sub-optimality of the ETS hence the room for complementary policies, but it would seriously complicate the analysis without necessarily providing new insights. Finally, assuming another probability distribution for the future demand level could also have an effect on the outcome, depending on the probability associated with a nil carbon price.

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A Description of the model types and instrument settings used in the analytical and numerical results

Label	Nature	Instrument setting			$p_{\text{CO}_2}^-$	Described in
		Carbon tax	ETS	REP Subsidy		
M_1	1 st Best	Yes	Useless	Useless	Positive	B
M_2^n	2 nd Best	Unavailable	Yes	Yes	Nil	C
M_2^p	2 nd Best	Unavailable	Yes	Yes	Positive	D
M_3^n	3 rd Best	Unavailable	Yes	Unavailable	Nil	E
M_3^p	3 rd Best	Unavailable	Yes	Unavailable	Positive	F

Table 7: Description of the model types and instrument settings

Table 7 links the names used in the text and the instrument settings used in each case. The detailed description of the model framework and the optimal solution calculated using Mathematica are given in the subsequent Appendices. Calculation sheets are available upon request to the authors.

The model used for the analytical results differ slightly from the model used for the numerical results. The numerical model allows for allowance trading by adding an emitting sector from which the power producer can buy surplus allowances. The instruments settings and names attached are the same for both models.

Appendices **B** to **F** show the framework and optimal solution for the model used in the analytical part. **G** show the framework and optimal solution for the model used in the numerical part, with the M_2^n setting. Showing the details of all settings for the model used in the numerical part would be very long and are not shown here. They are available upon request to the authors.

B First Best setting: model with carbon tax

To simulate an economy-wide carbon tax, we add following constraint to the model framework from Section 3:

$$\phi_- = \phi_+$$

The socially optimal level of all market variables for the high demand state (subscript +) and low demand (subscript -) are:

$$\begin{aligned}
\Omega^* &= (\sigma_a(\Delta + \iota_d + \iota_r\sigma_r - \iota_f(\sigma_d + \sigma_r))\tau + \delta(-1 - \sigma_a(\sigma_d + \sigma_r)(\tau)^2))/(\sigma_a) \\
\rho^* &= 0 \\
f_-^* &= -\Delta + \iota_d + \iota_r\sigma_r - \iota_f(\sigma_d + \sigma_r) - \delta(\sigma_d + \sigma_r)\tau \\
r_-^* &= \sigma_r(\iota_f - \iota_r + \delta\tau) \\
p_-^* &= \iota_f + \delta\tau \\
a_-^* &= (\delta)/(\sigma_a) \\
\phi_-^* &= \delta \\
f_+^* &= \Delta + \iota_d + \iota_r\sigma_r - \iota_f(\sigma_d + \sigma_r) - \delta(\sigma_d + \sigma_r)\tau \\
r_+^* &= \sigma_r(\iota_f - \iota_r + \delta\tau) \\
p_+^* &= \iota_f + \delta\tau \\
a_+^* &= (\delta)/(\sigma_a) \\
\phi_+^* &= \delta
\end{aligned}$$

C Second Best setting: model with ETS, REP subsidy and a nil CO₂ price in the low state of demand

Solving the profit maximization problem of the producer gives the reaction functions of producers, depending on the level of policy instruments and the state of the world (the first-order conditions are given in (4-6)). Solving the welfare maximization problem of the regulator knowing all the reaction functions gives the following first-order conditions:

$$\frac{\partial EW}{\partial \rho} = 0 \quad \Rightarrow \quad \frac{(\sigma_r(\delta\tau + \delta\sigma_a(\sigma_d + \sigma_r)(\tau)^3 + \rho^*(-2 - \sigma_a(2\sigma_d + \sigma_r)(\tau)^2)))}{(1 + \sigma_a(\sigma_d + \sigma_r)(\tau)^2)} = 0 \quad (17)$$

$$\frac{\partial EW}{\partial \Omega} = 0 \quad \Rightarrow \quad \frac{(\delta + \delta\sigma_a(\sigma_d + \sigma_r)(\tau)^2 + \sigma_a(\Omega^* - (\Delta + \iota_d + \iota_r\sigma_r - \iota_f(\sigma_d + \sigma_r))\tau))}{(1 + \sigma_a(\sigma_d + \sigma_r)(\tau)^2)} = 0 \quad (18)$$

from which we directly derive the optimal level of the policy instruments. By substituting the optimal levels of policy instruments in the reaction functions, we obtain the socially optimal level of all market variables for the high demand state (subscript +) and low demand (subscript -).

The optimal solution is:

$$\begin{aligned}
\Omega^* &= \left(\frac{1}{(\sigma_a)}\right)(\sigma_a(\Delta + \iota_d + \iota_r\sigma_r - \iota_f(\sigma_d + \sigma_r))\tau + \delta(-1 - \sigma_a(\sigma_d + \sigma_r)\tau^2)) \\
\rho^* &= \frac{(\delta\tau(1 + \sigma_a(\sigma_d + \sigma_r)\tau^2))}{(2 + \sigma_a(2\sigma_d + \sigma_r)\tau^2)} \\
f_-^* &= -\Delta + \iota_d - \iota_f(\sigma_d + \sigma_r) + \frac{(\sigma_r(\delta\tau(-1 - \sigma_a(\sigma_d + \sigma_r)\tau^2) + \iota_r(2 + \sigma_a(2\sigma_d + \sigma_r)\tau^2)))}{(2 + \sigma_a(2\sigma_d + \sigma_r)\tau^2)} \\
r_-^* &= \frac{(\sigma_r(2\iota_f - 2\iota_r + \delta\tau + (\iota_f - \iota_r)\sigma_a(2\sigma_d + \sigma_r)\tau^2 + \delta\sigma_a(\sigma_d + \sigma_r)(\tau)^3))}{(2 + \sigma_a(2\sigma_d + \sigma_r)\tau^2)} \\
p_-^* &= \iota_f \\
a_-^* &= 0 \\
\phi_-^* &= 0 \\
f_+^* &= \Delta + \iota_d + \iota_r\sigma_r - \iota_f(\sigma_d + \sigma_r) - \delta(\sigma_d + \sigma_r)\tau - \frac{(\delta\sigma_r\tau)}{(2 + \sigma_a(2\sigma_d + \sigma_r)\tau^2)}
\end{aligned}$$

$$\begin{aligned}
r_+^* &= \frac{(\sigma_r(2\iota_f - 2\iota_r + 3\delta\tau + (\iota_f - \iota_r)\sigma_a(2\sigma_d + \sigma_r)\tau^2 + \delta\sigma_a(3\sigma_d + \sigma_r)(\tau)^3))}{(2 + \sigma_a(2\sigma_d + \sigma_r)\tau^2)} \\
p_+^* &= \frac{(2\delta\tau(1 + \sigma_a\sigma_d\tau^2) + \iota_f(2 + \sigma_a(2\sigma_d + \sigma_r)\tau^2))}{(2 + \sigma_a(2\sigma_d + \sigma_r)\tau^2)} \\
a_+^* &= \frac{(2(\delta + \delta\sigma_a\sigma_d\tau^2))}{(\sigma_a(2 + \sigma_a(2\sigma_d + \sigma_r)\tau^2))} \\
\phi_+^* &= \frac{(2(\delta + \delta\sigma_a\sigma_d\tau^2))}{(2 + \sigma_a(2\sigma_d + \sigma_r)\tau^2)}
\end{aligned}$$

D Second Best setting: model with ETS, REP subsidy and a strictly positive CO₂ price in the low state of demand

We assumed through this paper that the carbon price is nil in the low demand state of the world. This is the case for certain parameter combinations, as discussed in section 3.4. For some other combinations, the carbon price remains positive in both states, and the model is changed as follows. Equation (3) becomes:

$$\begin{cases} \tau \cdot f_- - a_- = \Omega \\ \phi_- > 0 \end{cases} \quad \text{or} \quad \begin{cases} \tau \cdot f_+ - a_+ = \Omega \\ \phi_+ > 0 \end{cases}$$

The optimal solution changes also and becomes:

$$\begin{aligned}
\Omega^* &= \frac{(\sigma_a(\iota_d + \iota_r\sigma_r - \iota_f(\sigma_d + \sigma_r))\tau - \delta(1 + \sigma_a(\sigma_d + \sigma_r)\tau^2))}{(\sigma_a)} \\
\rho^* &= 0 \\
f_-^* &= \iota_d + \iota_r\sigma_r - \iota_f(\sigma_d + \sigma_r) - \delta(\sigma_d + \sigma_r)\tau - \frac{(\Delta)}{(1 + \sigma_a(\sigma_d + \sigma_r)\tau^2)} \\
r_-^* &= \frac{(\sigma_r(\iota_f - \iota_r + \delta\tau - \sigma_a(\Delta - (\iota_f - \iota_r)(\sigma_d + \sigma_r))\tau^2 + \delta\sigma_a(\sigma_d + \sigma_r)(\tau)^3))}{(1 + \sigma_a(\sigma_d + \sigma_r)\tau^2)} \\
p_-^* &= \frac{(\iota_f + \delta\tau + \sigma_a(-\Delta + \iota_f(\sigma_d + \sigma_r))\tau^2 + \delta\sigma_a(\sigma_d + \sigma_r)(\tau)^3)}{(1 + \sigma_a(\sigma_d + \sigma_r)\tau^2)} \\
a_-^* &= \frac{(\delta)}{(\sigma_a)} - \frac{(\Delta\tau)}{(1 + \sigma_a(\sigma_d + \sigma_r)\tau^2)} \\
\phi_-^* &= \delta - \frac{(\Delta\sigma_a\tau)}{(1 + \sigma_a(\sigma_d + \sigma_r)\tau^2)} \\
f_+^* &= \iota_d + \iota_r\sigma_r - \iota_f(\sigma_d + \sigma_r) - \delta(\sigma_d + \sigma_r)\tau + \frac{(\Delta)}{(1 + \sigma_a(\sigma_d + \sigma_r)\tau^2)} \\
r_+^* &= \frac{(\sigma_r(\iota_f - \iota_r + \delta\tau + \sigma_a(\Delta + (\iota_f - \iota_r)(\sigma_d + \sigma_r))\tau^2 + \delta\sigma_a(\sigma_d + \sigma_r)(\tau)^3))}{(1 + \sigma_a(\sigma_d + \sigma_r)\tau^2)} \\
p_+^* &= \frac{(\iota_f + \delta\tau + \sigma_a(\Delta + \iota_f(\sigma_d + \sigma_r))\tau^2 + \delta\sigma_a(\sigma_d + \sigma_r)(\tau)^3)}{(1 + \sigma_a(\sigma_d + \sigma_r)\tau^2)} \\
a_+^* &= \frac{(\delta)}{(\sigma_a)} + \frac{(\Delta\tau)}{(1 + \sigma_a(\sigma_d + \sigma_r)\tau^2)} \\
\phi_+^* &= \delta + \frac{(\Delta\sigma_a\tau)}{(1 + \sigma_a(\sigma_d + \sigma_r)\tau^2)}
\end{aligned}$$

E Third Best setting: model with ETS only and a nil CO₂ price in the low state of demand

To simulate a third best setting with no REP subsidy, we add following constraint to the model framework from Section 3:

$$\rho = 0$$

The socially optimal level of all market variables for the high demand state (subscript +) and low demand (subscript -) are:

$$\begin{aligned}\Omega^* &= (\sigma_a(\Delta + \iota_d + \iota_r\sigma_r - \iota_f(\sigma_d + \sigma_r))\tau + \delta(-1 - \sigma_a(\sigma_d + \sigma_r)(\tau)^2))/(\sigma_a) \\ \rho^* &= 0 \\ f_-^* &= -\Delta + \iota_d + \iota_r\sigma_r - \iota_f(\sigma_d + \sigma_r) \\ r_-^* &= (\iota_f - \iota_r)\sigma_r \\ p_-^* &= \iota_f \\ a_-^* &= 0 \\ \phi_-^* &= 0 \\ f_+^* &= \Delta + \iota_d + \iota_r\sigma_r - \iota_f(\sigma_d + \sigma_r) - \delta(\sigma_d + \sigma_r)\tau \\ r_+^* &= \sigma_r(\iota_f - \iota_r + \delta\tau) \\ p_+^* &= \iota_f + \delta\tau \\ a_+^* &= (\delta)/(\sigma_a) \\ \phi_+^* &= \delta\end{aligned}$$

F Third Best setting: model with ETS only and a positive CO₂ price in the low state of demand

To simulate a third best setting with no REP subsidy, we add following constraint to the model framework from D:

$$\rho = 0$$

The socially optimal level of all market variables for the high demand state (subscript +) and low demand (subscript -) are:

$$\begin{aligned}f_-^* &= -\delta\tau(\sigma_d + \sigma_r) - \frac{\Delta}{\sigma_a\tau^2(\sigma_d + \sigma_r) + 1} + \iota_d - \iota_f(\sigma_d + \sigma_r) + \iota_r\sigma_r \\ r_-^* &= \frac{\sigma_r(\delta\sigma_a\tau^3(\sigma_d + \sigma_r) + \delta\tau - \sigma_a\tau^2(\Delta - (\iota_f - \iota_r)(\sigma_d + \sigma_r)) + \iota_f - \iota_r)}{\sigma_a\tau^2(\sigma_d + \sigma_r) + 1} \\ p_-^* &= \frac{\delta\sigma_a\tau^3(\sigma_d + \sigma_r) + \delta\tau + \sigma_a\tau^2(\iota_f(\sigma_d + \sigma_r) - \Delta) + \iota_f}{\sigma_a\tau^2(\sigma_d + \sigma_r) + 1} \\ a_-^* &= \frac{\delta}{\sigma_a} - \frac{\Delta\tau}{\sigma_a\tau^2(\sigma_d + \sigma_r) + 1} \\ \phi_-^* &= \delta - \frac{\Delta\sigma_a\tau}{\sigma_a\tau^2(\sigma_d + \sigma_r) + 1} \\ f_+^* &= -\delta\tau(\sigma_d + \sigma_r) + \frac{\Delta}{\sigma_a\tau^2(\sigma_d + \sigma_r) + 1} + \iota_d - \iota_f(\sigma_d + \sigma_r) + \iota_r\sigma_r \\ r_+^* &= \frac{\sigma_r(\delta\sigma_a\tau^3(\sigma_d + \sigma_r) + \delta\tau + \sigma_a\tau^2(\Delta + (\iota_f - \iota_r)(\sigma_d + \sigma_r)) + \iota_f - \iota_r)}{\sigma_a\tau^2(\sigma_d + \sigma_r) + 1}\end{aligned}$$

$$\begin{aligned}
p_+^* &= \frac{\delta\sigma_a\tau^3(\sigma_d + \sigma_r) + \delta\tau + \sigma_a\tau^2(\Delta + \iota_f(\sigma_d + \sigma_r)) + \iota_f}{\sigma_a\tau^2(\sigma_d + \sigma_r) + 1} \\
a_+^* &= \frac{\delta}{\sigma_a} + \frac{\Delta\tau}{\sigma_a\tau^2(\sigma_d + \sigma_r) + 1} \\
\phi_+^* &= \delta + \frac{\Delta\sigma_a\tau}{\sigma_a\tau^2(\sigma_d + \sigma_r) + 1}
\end{aligned}$$

G Model with allowances from non-power ETS sectors and nil CO₂ price in the low state of demand

Section 4 extends the model and allows for allowance trading by adding an emitting sector from which the power producer can buy surplus allowances. This surplus is labeled e and its supply is modeled by a linear mac curve. The profit maximization problem becomes:

$$\max_{f,r,a,e} \Pi(p, f, r, a, e, \phi) = p \cdot f + (p + \rho) \cdot r - C_f(f) - C_r(r) - AC(a) - AC_e(e) - PC_e(f, a, e, \phi)$$

with

$$AC_e(e) = \frac{\sigma_e}{2}e^2 - \begin{cases} \iota_e \cdot e & \text{in the low demand state} \\ 0 & \text{in the high demand state} \end{cases}$$

The allowance purchasing cost is modified as follows:

$$PC_e(f, a, e, \phi) = \phi \cdot (\tau \cdot f - a + e)$$

and (3) becomes:

$$\begin{cases} \tau \cdot f_-^* - a_-^* < \Omega - e_-^* \\ \phi_-^* = 0 \end{cases} \quad \text{or} \quad \begin{cases} \tau \cdot f_+^* - a_+^* = \Omega - e_+^* \\ \phi_+^* > 0 \end{cases}$$

The welfare maximization problem becomes:

$$\max_{\Omega, \rho} EW(\Omega, \rho) = \sum_{\text{states}} \frac{1}{2} (CS(p) + \Pi(p, f, r, a, e, \phi) - \text{dam}_e(f, a, e) - \rho \cdot r + PC_e(f, a, e, \phi))$$

where $\text{dam}_e(\cdot)$ is the modified environmental damage function:

$$\text{dam}_e(f, a, e) = \delta \cdot (\tau f - a - e)$$

The optimal solution of this problem is the following:

$$\begin{aligned}
\Omega^* &= -\frac{(\delta(\sigma_a + \sigma_e))}{(\sigma_a\sigma_e)} + (\Delta + \iota_d + \iota_r\sigma_r - \iota_f(\sigma_d + \sigma_r))\tau - \delta(\sigma_d + \sigma_r)\tau^2 \\
\rho^* &= \frac{(\delta\tau(\sigma_a + \sigma_e + \sigma_a\sigma_e(\sigma_d + \sigma_r)\tau^2))}{(2(\sigma_a + \sigma_e) + \sigma_a\sigma_e(2\sigma_d + \sigma_r)\tau^2)} \\
f_-^* &= -\Delta + \iota_d - \iota_f\sigma_d - \iota_f\sigma_r + \iota_r\sigma_r - \frac{(\delta\sigma_r(\sigma_d + \sigma_r)\tau)}{(2\sigma_d + \sigma_r)} + \frac{(\delta(\sigma_a + \sigma_e)(\sigma_r)^2\tau)}{((2\sigma_d + \sigma_r)(2(\sigma_a + \sigma_e) + \sigma_a\sigma_e(2\sigma_d + \sigma_r)\tau^2))} \\
r_-^* &= \frac{(\sigma_r(2(\iota_f - \iota_r)(\sigma_a + \sigma_e) + \delta(\sigma_a + \sigma_e)\tau + (\iota_f - \iota_r)\sigma_a\sigma_e(2\sigma_d + \sigma_r)\tau^2 + \delta\sigma_a\sigma_e(\sigma_d + \sigma_r)(\tau)^3))}{(2(\sigma_a + \sigma_e) + \sigma_a\sigma_e(2\sigma_d + \sigma_r)\tau^2)} \\
p_-^* &= \iota_f \\
a_-^* &= 0 \\
e_-^* &= \frac{(\iota_e)}{(\sigma_e)}
\end{aligned}$$

$$\phi_-^* = 0$$

$$f_+^* = \Delta + \iota_d + \iota_r \sigma_r - \iota_f (\sigma_d + \sigma_r) - \delta (\sigma_d + \sigma_r) \tau - \frac{(\delta (\sigma_a + \sigma_e) \sigma_r \tau)}{(2(\sigma_a + \sigma_e) + \sigma_a \sigma_e (2\sigma_d + \sigma_r) \tau^2)}$$

$$r_+^* = \frac{(\sigma_r (2(\iota_f - \iota_r) (\sigma_a + \sigma_e) + 3\delta (\sigma_a + \sigma_e) \tau + (\iota_f - \iota_r) \sigma_a \sigma_e (2\sigma_d + \sigma_r) \tau^2 + \delta \sigma_a \sigma_e (3\sigma_d + \sigma_r) (\tau)^3))}{(2(\sigma_a + \sigma_e) + \sigma_a \sigma_e (2\sigma_d + \sigma_r) \tau^2)}$$

$$p_+^* = \frac{(2\iota_f (\sigma_a + \sigma_e) + \iota_f \sigma_a \sigma_e (2\sigma_d + \sigma_r) \tau^2 + 2\delta \tau (\sigma_a + \sigma_e + \sigma_a \sigma_d \sigma_e \tau^2))}{(2(\sigma_a + \sigma_e) + \sigma_a \sigma_e (2\sigma_d + \sigma_r) \tau^2)}$$

$$a_+^* = \frac{(2\delta (\sigma_a + \sigma_e + \sigma_a \sigma_d \sigma_e \tau^2))}{(\sigma_a (2(\sigma_a + \sigma_e) + \sigma_a \sigma_e (2\sigma_d + \sigma_r) \tau^2))}$$

$$e_+^* = \frac{(2\delta (\sigma_a + \sigma_e + \sigma_a \sigma_d \sigma_e \tau^2))}{(\sigma_e (2(\sigma_a + \sigma_e) + \sigma_a \sigma_e (2\sigma_d + \sigma_r) \tau^2))}$$

$$\phi_+^* = \frac{(2\delta (\sigma_a + \sigma_e + \sigma_a \sigma_d \sigma_e \tau^2))}{(2(\sigma_a + \sigma_e) + \sigma_a \sigma_e (2\sigma_d + \sigma_r) \tau^2)}$$