On Elastoplastic Damage Modelling in Unsaturated Geomaterials

Solenn Le Pense, Behrouz Gatmiri, Ahmad Pouya

▶ To cite this version:

Solenn Le Pense, Behrouz Gatmiri, Ahmad Pouya. On Elastoplastic Damage Modelling in Unsaturated Geomaterials. 2nd European Conference on Unsaturated Soils (E-Unsat2012), Jun 2012, Naples, Italy. pp.143-149, 10.1007/978-3-642-31343-1_18 . hal-00790238

HAL Id: hal-00790238

https://enpc.hal.science/hal-00790238

Submitted on 21 Feb 2013

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Published in *Unsaturated Soils: Research and Applications*, Proceedings of the 2nd European Conference on Unsaturated Soils, Naples, Italy, 20-22 June 2012. pp 143-149.
Publisher : Springer Berlin Heidelberg
DOI: 10.1007/978-3-642-31343-1_18
Available at http://www.springerlink.com/content/l85w67v2t33n13n1/

On elastoplastic damage modelling in unsaturated geomaterials

Solenn Le Pense, Behrouz Gatmiri and Ahmad Pouya

Abstract. In the context of nuclear waste disposal, the modelling of the behaviour of host rocks and soils still needs improvement. Unsaturated porous geomaterials exhibit particular behaviour when exposed to suction. Their non-linear behaviour may result from two different processes, plasticity which induces irreversible strains and damage which causes a deterioration of their elastic properties. Many elasto-plastic models are now available for unsaturated soils, most of them based on the Barcelona Basic Model (Alonso et al., 1990). They take into consideration a certain number of issues linked with the nature of unsatured soils. Models coupling damage and plasticity have also been proposed for continuous media. Since very few works have attempted to connect these two distinct fields, unsaturated soil mechanics and continuum damage mechanics, this work focuses on the main issues related to the development of a model coupling elasto-plasticity and damage for unsaturated porous media.

Keywords: unsaturated soil, damage, elastoplastic behaviour, porous media

1 Introduction

The purpose of our work is to develop a simple thermodynamically consistent model to describe the hydro-mechanical behaviour of unsaturated geomaterials. The mechanical part will have to take into account the coupling between damage and plasticity phenomena.

In this paper, we will present the main questions raised by this modelling and discuss several options proposed by earlier works.

Solenn Le Pense
Université Paris-Est, Laboratoire Navier (ENPC), France, e-mail: solenn.le-pense@cermes.enpc.fr

Behrouz Gatmiri
University of Tehran, Iran – ANDRA, France, e-mail: Behrouz.Gatmiri@andra.fr

Ahmad Pouya
Université Paris-Est, Laboratoire Navier (IFSTTAR), France, e-mail: ahmad.pouya@enpc.fr

We will limit our study to rate-independent materials and isothermal processes. We will also consider only small strains.

To ensure the thermodynamical consistency of the model we will follow the framework of hyperplasticity developed by Houlsby and Puzrin (2006), in which thermodynamic principles play a major role.

2 Unsaturated geomaterials

It is well known that suction, $s = p_g - p_l$, which is the difference between gas pressure, p_g, and liquid pressure, p_l, has a great impact on unsaturated porous media behaviour.

Indeed, an increase in suction contributes to stiffening the soil against external loading. Moreover, for some soils, a reduction in suction (wetting) for a given confining stress may induce irrecoverable volumetric compression (collapse). Changes in suction may also induce plastic volumetric strains during a drying process.

Suction is closely related to the degree of saturation, S_l, through the water retention curve (WRC), which can exhibit some hysteresis.

Unsaturated soil models are usually extensions of those for saturated soil. The most famous of them is the Cam-clay model, first developed by Roscoe et al (1958) and modified by Roscoe and Burland (1968).

Althought the debate still goes on about the better choice for the stress framework, there is general agreement that two constitutive variables are needed to represent unsaturated soil behaviour.

The Barcelona Basic Model (Alonso et al, 1990), and many other models, use, as constitutive variables, net stress, $\sigma_{\text{net}} = \sigma - p_g \boldsymbol{I}_d$ (with $\boldsymbol{I}_d$ the identity matrix), and suction, s.

Another option is to use, as the first variable, the stress proposed by Bishop (1959), $\sigma^* = \sigma - [p_g - \chi(p_g - p_l)]\boldsymbol{I}_d$, in which $\chi(S_l)$ is a function of the degree of saturation, $\chi(0) = 0$ and $\chi(1) = 1$. The second constitutive variable is usually a function of suction.

Assuming both the solid phase and the water phase to be incompressible, Houlsby (1997) developed an expression for the rate of work input to an unsaturated granular material. The part relating to change of strain and degree of saturation is given in equation 1. The stress, σ^*, which is work conjugate to the rate of strain, $\dot{\varepsilon}_{ij}$, is similar to Bishop's stress with $\chi(S_l) = S_l$. We will choose this stress in our work since it is in accordance with thermodynamics. We will call it the *average skeleton stress* (eq. 2). The second state variable will be the modified suction, $s^* = \phi s$ (ϕ being the porosity), which is work conjugate to the rate of degree of saturation $\dot{S}_l$.

$$\dot{w} = [\sigma_{ij} - (p_l S_l + (1 - S_l)p_g)\delta_{ij}]\dot{\varepsilon}_{ij} - (p_g - p_l)\phi\dot{S}_l = \sigma^*_{ij}\dot{\varepsilon}_{ij} - s^*\dot{S}_l \tag{1}$$

$$\sigma^* = \sigma - [p_l S_l + (1 - S_l)p_g]\boldsymbol{I}_d = \sigma_{\text{net}} + sS_l\boldsymbol{I}_d \tag{2}$$

In applying the second principle of thermodynamics gives the Clausius-Duhem inequality (considering only elastic strains) (eq. 3). Constitutive equations are derived from Helmholtz free energy, $\mathscr{F}$ (eq. 4).

$$\left(\sigma^* - \frac{\partial \mathscr{F}}{\partial \varepsilon^e}\right) : \dot{\varepsilon}^e - \left(s^* + \frac{\partial \mathscr{F}}{\partial S_l}\right) \dot{S}_l \geqslant 0 \tag{3}$$

$$\sigma^* = \frac{\partial \mathscr{F}}{\partial \varepsilon^e} \;\; ; \;\; s^* = -\frac{\partial \mathscr{F}}{\partial S_l} \tag{4}$$

Concerning the relationship between suction and the degree of saturation, many formulas for the WRC can be found in the literature. Some expressions overlook the hysteresis effects and give a single expression to represent suction as a function of the saturation degree ($s = f(S_l)$) (Brooks and Corey, 1964; Van Genuchten, 1980). More complex models consider hydraulic hysteresis (Wheeler et al, 2003; Sheng et al, 2004).

We assume that hydraulic and mechanical behaviours are decoupled, so that Helmholtz free energy takes the following form : $\mathscr{F} = \mathscr{F}_m(\varepsilon) + \mathscr{F}_l(S_l)$. In the following sections we will focus on the mechanical behaviour.

3 Elasticity

First, we present the elastic part of our model. Having assumed the volumetric and the deviatoric behaviour to be uncoupled, the increment of mechanical work input is given by equation (5).

$$\dot{w}_m = \sigma^* : \dot{\varepsilon} = p^* \dot{\varepsilon}_v + \sigma_d \dot{\varepsilon}_d \tag{5}$$

In which $p^* = \frac{1}{3} tr(\sigma^*)$ is the mean stress, $\sigma_d = \sigma^* - p^* \boldsymbol{I}_d$ the deviatoric stress tensor, $\varepsilon_v = tr(\varepsilon)$ the volumetric strain, and $\varepsilon_d = \varepsilon - \frac{1}{3}\varepsilon_v \boldsymbol{I}_d$ the deviatoric strain tensor.

Experiments have shown that, for soils and rocks, elastic behaviour depends on confinement pressure.

To ensure thermodynamical consistency, elasticity laws will be derived from an energy potential. Several expressions have been proposed for the Helmholtz free energy, such as those by Houlsby (1985), Borja et al (1997) or Houlsby et al (2005). To keep it simple, we will consider a pressure-dependent bulk modulus and a constant shear modulus.

Deriving Helmholtz free energy (eq. 6) gives the constitutive equations (7). A new derivation gives the incremental behaviour law (eq. 8).

$$\mathscr{F}_{m0} = \mathscr{F}_{m0}(\varepsilon_v^e, \varepsilon_d^e) = p_r \kappa \exp\left(\frac{\varepsilon_v^e}{\kappa}\right) + G \varepsilon_d^e : \varepsilon_d^e \tag{6}$$

$$p^* = \frac{\partial \mathscr{F}_{m0}}{\partial \varepsilon_v^e} \;\; ; \;\; \sigma_d = \frac{\partial \mathscr{F}_{m0}}{\partial \varepsilon_d} \tag{7}$$

$$\dot{\sigma}^* = \dot{p}^* \boldsymbol{I}_d + \dot{\sigma}_d = \frac{p^*}{\kappa} \dot{\varepsilon}_v^e \boldsymbol{I}_d + 2G\dot{\varepsilon}_d^e \tag{8}$$

4 Damage

The dissipative behaviour of geomaterials can be due to two different phenomena. The first one is plasticity which causes irreversible strains, the second one is damage, which can be seen as the development of microcracks and results in the deterioration of elastic properties. These behaviours can appear independently or simultaneously depending on the material. (See figures 1-3)

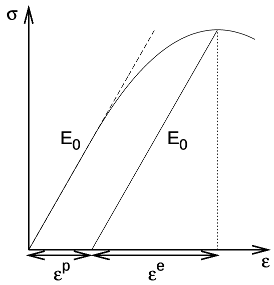

Fig. 1 Plastic behaviour

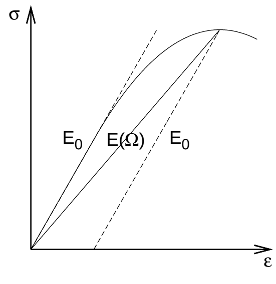

Fig. 2 Brittle behaviour

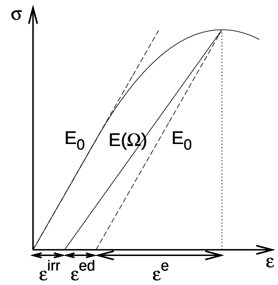

Fig. 3 Coupling of damage and plasticity

Few attempts have been made until now to model damage in unsaturated geomaterials (Shao et al, 2006; Arson and Gatmiri, 2009). For a review on damage modelling in unsaturated geomaterials, see the paper by Arson and Gatmiri (2008).

If some models are based on micromechanical considerations, we choose to use a phenomenological approach and to derive constitutive equations from thermodynamics.

To keep it simple we will consider isotropic damage which will be represented by a scalar damage variable d.

We follow the same method as Ju (1989) and suppose the following form for Helmholtz free energy :

$$\mathscr{F}_m = \mathscr{F}_m(\varepsilon_v^e, \varepsilon_d^e, d) = (1-d)\mathscr{F}_{m0}(\varepsilon_v^e, \varepsilon_d^e) \tag{9}$$

This choice for the change in Helmholtz free energy with damage implies that damage behaviour only depends on elastic strains.

$$\sigma^* = \frac{\partial \mathscr{F}_m}{\partial \varepsilon^e} = (1-d)\frac{\partial \mathscr{F}_{m0}}{\partial \varepsilon^e} \tag{10}$$

$$\tilde{\sigma}^* = \frac{\sigma^*}{1-d} = \frac{\partial \mathscr{F}_{m0}}{\partial \varepsilon^e} \tag{11}$$

We can define a *damaged average skeleton stress* $\tilde{\sigma}^*$ (eq. 11) which is similar to the *effective stress* defined by Kachanov (1958). We can notice that the expression also follows the principle of strain equivalence defined by Lemaitre (1996) which stipulates that :

> Any strain constitutive equation for a damaged material may be derived in the same way as for a virgin material except that the usual stress is replaced by the effective stress.

We define a simple damage criterion (eq. 12), depending on the thermodynamic force conjugated to damage (eq. 13). We assume an associate flow rule (eq. 14).

$$f_d(Y_d, d) = Y_d - C_0 - C_1 d \tag{12}$$

$$Y_d(\varepsilon^e) = -\frac{\partial \mathscr{F}_m}{\partial d} = \mathscr{F}^e_{m0} = p_r \kappa \exp\left(\frac{\varepsilon^e_v}{\kappa}\right) + G\varepsilon^e_d : \varepsilon^e_d \tag{13}$$

$$\dot{d} = \dot{\lambda}_d \frac{\partial f_d}{\partial Y_d} \tag{14}$$

5 Introduction of damage into an elasto-plastic model

We will take as a basis the elasto-plastic model developed by Sheng et al (2004) for unsaturated soils. This model is a variant of the Barcelona Basic Model written in terms of average skeleton stress and suction instead of net stress and suction. It includes hydraulic hysteresis but we will not consider it here.

As proposed by Ju (1989), in order to follow the principle of strain equivalence, damage will be taken into account by introducing damaged stresses ($\tilde{\sigma}^*$) into the equations of the yield surface (eq. 15), the plastic potential (since we are considering a non-associative law) (eq. 16), the flow rule (eq. 17) and the hardening law (eq. 18).

$$f_p(\tilde{\sigma}^*, \tilde{p_0}^*, s) = \tilde{q}^2 - M^2 \tilde{p}^* (\tilde{p_c}^*(\tilde{p_0}^*, s) - \tilde{p}^*) \tag{15}$$

$$g_p(\tilde{\sigma}^*, \tilde{p_0}^*, s) = \zeta \tilde{q}^2 - M^2 \tilde{p}^* (\tilde{p_c}^*(\tilde{p_0}^*, s) - \tilde{p}^*) \tag{16}$$

$$\dot{\varepsilon}^p = \dot{\lambda}_p \frac{\partial g_p}{\partial \tilde{\sigma}^*} \tag{17}$$

$$\dot{\tilde{p}}^*_0 = \frac{\tilde{p_0}^*}{\lambda_0 - \kappa} \dot{\varepsilon}^p_v \tag{18}$$

A representation of the yield surface is given in figures 4 and 5.

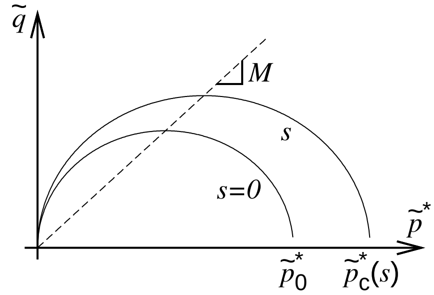

Fig. 4 Yield surface in the ($\tilde{p}^*$, $\tilde{q}$) plane

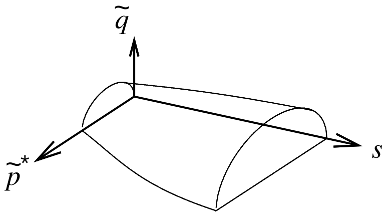

Fig. 5 Form of the yield surface in the ($\tilde{p}^*$, s, $\tilde{q}$) plane

6 Conclusion

We have proposed a way to model the coupled elasto-plastic behaviour of damaged geomaterials. To consider the effect of suction, we chose to use, as the stress framework, the average skeleton stress (eq. 2) with modified suction. In terms of elasticity, constitutive laws are derived from Helmholtz free energy to ensure thermodynamical consistency. The principle of effective stress and the principle of strain equivalence are used to develop the damage part of the model. Plasticity is supposed to take place in the undamaged part of the material, so that the damaged average skeleton stress is used in the plasticity equations.

So, we have presented here the bases of a model that can represent the coupling of damage and plasticity when modelling the behaviour of unsaturated geomaterials. The next step of our work will be to develop the corresponding numerical model and to implement it into a finite element code in order to validate it.

References

Alonso E, Gens A, Josa A (1990) A Constitutive model for partially saturated soils. Géotechnique 40(3): 405–430

Arson C, Gatmiri B (2008) On damage modelling in unsaturated clay rocks. Physics and Chemistry of the Earth 33: S407–S415

Arson C, Gatmiri B (2009) A mixed damage model for unsaturated porous media. Comptes Rendus Mécanique 337(2): 68–74

Bishop A (1959) The effective stress principle. Teknisk Ukeblad 39: 859–863

Borja R, Tamagnini C, Amorosi A (1997) Coupling plasticity and energy-conserving elasticity models for clays. Journal of geotechnical and geoenvironmental engineering 123(10): 948–957

Brooks R, Corey A (1964) Hydraulic properties of porous media. Hydrology Papers / Colorado State University 3

Houlsby G (1985) The use of a variable shear modulus in elastic-plastic models for clays. Computers and Geotechnics 1(1): 3–13

Houlsby G (1997) The work input to an unsaturated granular material. Géotechnique 47(1): 193–196

Houlsby G, Puzrin A (2006) Principles of hyperplasticity: an approach to plasticity theory based on thermodynamic principles. Springer Verlag

Houlsby G, Amorosi A, Rojas E (2005) Elastic moduli of soils dependent on pressure: a hyperelastic formulation. Géotechnique 55(5): 383–392

Ju J (1989) On energy-based coupled elastoplastic damage theories: constitutive modeling and computational aspects. International Journal of Solids and Structures 25(7): 803–833

Kachanov L (1958) Time of the rupture process under creep conditions. Isv Akad Nauk SSR Otd Tekh Nauk 8: 26–31

Lemaitre J (1996) A course on damage mechanics. Springer Verlag

Roscoe K, Burland J (1968) On the generalized stress–strain behaviour of wet clay. Engineering plasticity 3: 539–609

Roscoe K, Schofield A, Wroth C (1958) On the yielding of soils. Géotechnique 8(1): 22–53

Shao J, Jia Y, Kondo D, Chiarelli A (2006) A coupled elastoplastic damage model for semi-brittle materials and extension to unsaturated conditions. Mechanics of materials 38(3): 218–232

Sheng D, Sloan S, Gens A (2004) A constitutive model for unsaturated soils: thermomechanical and computational aspects. Computational Mechanics 33(6): 453–465

Van Genuchten M (1980) A closed-form equation for predicting the hydraulic conductivity of unsaturated soils. Soil Science Society of America Journal 44(5): 892–898

Wheeler S, Sharma R, Buisson M (2003) Coupling of hydraulic hysteresis and stress-strain behaviour in unsaturated soils. Géotechnique 53(1): 41–54