The transit bottleneck model

Fabien Leurent, Ektoras Chandakas

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Fabien Leurent[a*], Ektoras Chandakas[a]

[a]*Université Paris-Est, Laboratoire Ville Mobilité Transport, Ecole des Ponts ParisTech*
6-8 avenue Blaise Pascal, 77455, Marne la Vallée, Cedex 2, France

Abstract

This paper addresses the issue of passenger waiting and being stored at a station platform, from which point they plan to board route services towards egress stations. Route services are operated using dedicated vehicles of limited capacity. Each route service has a specific set of downstream egress stations and is operated at given frequency using homogeneous vehicles of limited available capacity. The model yields individual waiting time by egress station and the assignment of vehicle capacity to the flows by egress station.

Keywords: mingled queuing

1. Introduction

In large urban areas the transit network is frequently submitted to heavy congestion, especially at the peak hours on working days. Under these circumstances, various links of the network may provide insufficient capacity to the passenger demand. Transposing to traffic flow characteristics, a bottleneck may be created, where these passengers perceive an extended waiting at the station until boarding a vehicle of the service.

The scientific community has provided many approaches to model the total person's capacity constraint through queuing theory; in Gendreau (1984) an unbounded increasing convex delay function is formulated within a theoretical model. Kurauchi et al (2003) approached queuing through a fail-to-board probability, adjusting the waiting time. A bottleneck has been also introduced in dynamic transit assignment models, such as Poon et al (2004), where the queuing time is calculated for a passenger at each time increment through the difference between cumulative arrival and departure curves at a platform node.

The objective of the paper is the formalization of the transit bottleneck model and the mathematical demonstration of the equilibrium. The model is based on the explicit representation of the average number of passengers waiting on a platform for a given egress station. From these passenger stocks is derived the individual probability to board a vehicle of limited capacity that service a given route. The waiting time is calculated after the service competition at the platform from the stock's probability to board a vehicle of the service.

The rest of this paper is in four parts. We begin by formulating the queuing model and developing the necessary mathematical formulas which correspond to a fixed point problem. Then, a mathematical analysis of the fixed point problem is made, by defining the objective function and demonstrating the existence and uniqueness of the solution. Finally, an application instance is used to demonstrate the main characteristics of the transit bottleneck model; the influence of passenger flows on average journey time and destination coordination.

[*] Corresponding author. Tel.: +33-164-152-111; fax: +33-164-152-140
E-mail address: fabien.leurent@enpc.fr

2. The queuing model

2.1. Definitions

Let i be the access station, S_i the set of egress stations j that are serviced by route services $z \in Z_i$. The subset of services that dwell at j coming from i is denoted as $\{z \in (i, j)\}$. During the period H, service z is operated at frequency f'_z by vehicles of residual capacity k'_z at station i (after the egress of the passengers destined to i), yielding an available capacity of $\kappa'_z = f'_z k'_z$ in the period. We assume that a passenger arrives at i under exogenous flow $[q^+_{ij} : j \in J_i]$, yielding entry volumes $H.q^+_{ij}$ by egress station.

Let $f^\ell_{ij} \equiv \sum_{z \in (i,j)} f_{ij}$ be the combined frequency between i and j of the services in Z_i. It is assumed that the platform is shared by the services and that no vehicle can overtake another one, meaning that a time-minimizing passenger is eager to board a relevant vehicle as soon as it has some place available. Notation ℓ designates this set of services where the routes and egress stations make up a connected component in the bipartite graph in $Z^\ell_i \times J^\ell_i$ that links the services to the stations that they serve.

2.2. The unqueued case

If capacity is available to each passenger as soon as he would like from his instant of arrival, then he can board the first relevant vehicle and the average wait time for destination j is:

$$w^\ell_{ij} = \frac{1}{f^\ell_{ij}} \tag{1}$$

Each service in ℓ gets a share f_z / f^ℓ_{ij} of the Hq^+_{ij} volume. The resulting passenger flow, $(Hq^+_{ij} f_z) / f^\ell_{ij}$, yields a number $(Hq^+_{ij}) / f^\ell_{ij}$ of passengers destined to j by vehicle of service z. The assumption that there remains available capacity is that

$$H \sum_{j : z \in (i,j)} \frac{q^+_{ij}}{f^\ell_{ij}} \leq k'_z. \tag{2}$$

This must be checked to ensure that there is no queuing – at least no remaining stock of passengers that keep waiting just after a vehicle relevant to them has left.

2.3. The queued case

If some passengers cannot board the first vehicle relevant to them, then they have to wait for other vehicles to arrive and to supply them with some capacity. Let us assume that there is a fictive, stationary state of passenger traffic on the platform, with a number v_j of passengers waiting for destination j. When a vehicle of service z arrives with available capacity k'_z, there are candidate passengers in number of

$$n_z = \sum_{j \in z} v_j \tag{3}$$

Their individual probability to board, assuming equity among them, is

$$\pi_z = \min\{1, \frac{k'_z}{n_z}\} \tag{4}$$

Then the number of passengers boarding a vehicle of service z to exit at j is $\pi_z v_j$, and the passenger flow during H to j via z is

$$q^{-}_{ijz} = f'_z \pi_z v_j \tag{5}$$

The total flow to j during H is (throughput rate)

$$q^{-}_{ij} = \sum_{j \in z} q^{-}_{ijz} = (\sum_{j \in z} f'_z \pi_z) v_j \tag{6}$$

Let us denote $(f\pi)_j \equiv \sum_{j \in z} f'_z \pi_z$.

Queuing may eventually occur when at least one service has $\pi_z < 1$, meaning saturation of a service route. Let us derive necessary conditions on the π_z and v_j variables that characterize waiting and queuing. A bottleneck model is assumed by egress station j, with arrival rate q^{+}_{ij} on $[0, H]$ and discharge rate q^{-}_{ij} on w_{j0}, in which w_{j0} is the mean wait time between arriving at the queue front end in eligible place and the arrival of a relevant vehicle. A reasonable value could be $w_{j0} = 1/f^{\ell}_{ij}$; let us keep w_{j0} for the moment to have a degree of freedom.

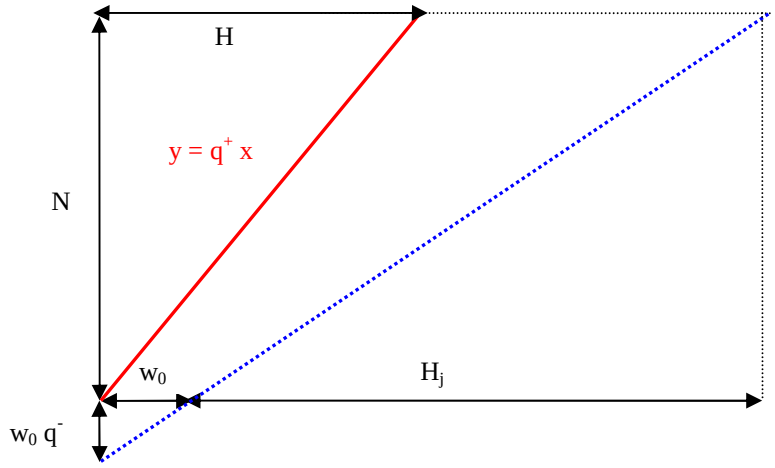


Fig. 1. Cumulative flows at bottleneck

Figure 1 depicts the dynamics of the waiting queue: if $q^{-} < q^{+}$ then there is a 'triangle of waiting' that lies between the cumulated curve of passenger arrival, $y = q^{+}_{ij}.x$, shifted to the right due to the minimum waiting w_{j0}, and the cumulated curve of passenger departure, $y = q^{-}_{ij}(x - w_{j0})$ for $x - w_{j0} \in [0, H_j]$. Flow conservation implies that:

$$N_j = H.q^{+}_{ij} = H_j.q^{-}_{ij}. \tag{7}$$

The average number of passengers waiting for destination j, v_j, is equal to the total time spent at waiting, W^{ℓ}_{ij}, divided by the time interval, $H'_j = w_{j0} + H_j$. Letting $N'_j = N_j + w_{j0} q^{-}_{ij}$ and

$(f\pi)_j \equiv \sum_{j\in z} f'_z \pi_z$, the total wait amounts to the area between the two large triangles minus the area of the small triangle:

$$W_{ij}^{\ell} = \frac{1}{2}\left(\frac{N_j'^2}{q_{ij}^-} - \frac{N_j^2}{q_{ij}^+} - w_{j0}^2 q_{ij}^-\right) \qquad (8)$$

On replacing W_{ij}^{ℓ} with $v_j H'_j = v_j(w_{j0} + H_j)$ and N'_j with $N_j + w_{j0}q_{ij}^-$, we get that

$v_j\left(w_{j0} + \frac{N_j}{q_{ij}^-}\right) = \frac{1}{2}\left(\frac{N_j^2}{q_{ij}^-} + 2w_{j0}N_j + w_{j0}^2 q_{ij}^- - \frac{N_j^2}{q_{ij}^+} - w_{j0}^2 q_{ij}^-\right)$

$v_j \frac{w_{j0}}{N_j} + \frac{1}{(f\pi)_j} - w_{j0} = \frac{1}{2}\left(\frac{N_j}{v_j (f\pi)_j} - \frac{N_j}{q_{ij}^+}\right)$ after dividing by N_j and replacing q_{ij}^- by $v_j (f\pi)_j$

$2v_j \frac{(f\pi)_j w_{j0}}{HN_j} + 2\frac{q_{ij}^+}{N_j}[1 - (f\pi)_j w_{j0}] = \frac{q_{ij}^+}{v_j} - (f\pi)_j$ after multiplying by $2q_{ij}^+ (f\pi)_j / N_j$

At this stage, let us set $w_{j0} = 1/(f\pi)_j$ so as to simplify into

$$\frac{2}{HN_j} v_j = \frac{q_{ij}^+}{v_j} - (f\pi)_j . \qquad (9)$$

This model is taken for all traffic regimes, including the unqueued state in which the left hand side is negligible, yielding $q_{ij}^+ \approx (f\pi)_j v_j = q_{ij}^-$. We expect that $\frac{2}{HN_j} v_j << \frac{q_{ij}^+}{v_j}$, i.e. $2v_j^2 << q_{ij}^+ HN_j = N_j^2$, or

$$v_j << \tfrac{1}{\sqrt{2}} N_j . \qquad (10)$$

Referring to (3) and (7), the π_z depend on the v_j through the n_z. Thus there is a circular dependency between our variables, in other words a Fixed Point Problem (FPP) with respect to vector $\mathbf{v} = [v_j : j \in J_i]$.

In the next section, it is shown that the FPP (3), (7) and (9) has a solution, which is unique. Also included is a computation scheme.

Definition. FPP $\frac{q_{ij}^+}{v_j} - (f\pi)_j \in \{0, \frac{2}{HN_j} v_j\}$ where $[\frac{q_{ij}^+}{v_j} - (f\pi)_j].[\frac{q_{ij}^+}{v_j} - (f\pi)_j - \frac{2}{HN_j} v_j] = 0$.

3. Mathematical analysis of the Fixed Point Problem

The Fixed Point Problem is shown to be equivalent to the first order optimality conditions of a minimization program which is strictly convex. As such program has a solution that is unique, this is also the unique solution to the FPP.

3.1. Admissible set

Denote $\mathbf{v} = [v_j : j \in J_i]$ a vector of passenger stocks by egress station (from entry station i). Define the feasible set as $\mathbf{V} = \{\mathbf{v} \geq 0 : \forall j \in J_i,\ v_j \leq N_j$ and $(f\pi)_j v_j \leq q_{ij}^+\}$.

Lemma 1. *Set* $\mathbf{V}$ *is (a) closed and (b) compact.*

Proof. (a) Let $G_j : \mathbf{v} \mapsto G_j(\mathbf{v}) = (f\pi)_j v_j$. This is a continuous function as $\mathbf{v} \mapsto n_z = \sum_{z \in (i,j)} v_j$ is continuous, as is $n_z \mapsto \pi_z = \min\{1, \frac{k'_z}{n_z}\}$ for $n_z > 0$ and 1 at $n_z = 0$. Thus $\mathbf{v} \mapsto (f\pi)_j$ is continuous as a linear combination of continuous function, and its product by v_j, i.e. G_j, is continuous too. As G_j is continuous and the sets $[0, q_{ij}^+]$ are closed, the reciprocal sets $G_j^{-1}([0, q_{ij}^+])$ are closed. Therefore $\mathbf{V}$ is closed as the intersection of a finite number of closed sets $G_j^{-1}([0, q_{ij}^+])$ or $[0, N_j]$.

(b) As $\mathbf{V} \subset [\mathbf{0}, \mathbf{N}]$ with $\mathbf{N} = [N_j : j \in J_i]$, the multidimensional interval $[\mathbf{0}, \mathbf{N}]$ is bounded hence compact. So is $\mathbf{V}$, a closed subset of $[\mathbf{0}, \mathbf{N}]$.

Lemma 2. *Any solution to the FPP must belong to the admissible set.*

Proof. From (9) and $\mathbf{v} \geq \mathbf{0}$, $q_{ij}^+ - (f\pi^*)_j v_j^* \geq 0$.

Furthermore, $\frac{2}{HN_j} v_j^* \leq \frac{q_{ij}^+}{v_j^*}$ hence $v_j^* \leq \frac{1}{\sqrt{2}} N_j$ meets the second requirement for $\mathbf{v}^*$ to belong to $\mathbf{V}$

3.2. Objective function

Let: $F_j(\mathbf{v}) \equiv (f\pi)_j - \frac{q_{ij}^+}{v_j}$ and $\tilde{F}_j(\mathbf{v}) \equiv \frac{2}{HN_j} v_j - \frac{q_{ij}^+}{v_j} + (f\pi)_j$

These functions have been designed so as to yield cross derivatives that are symmetrical in (v_j, v_r), meaning that there are potential functions f and $\tilde{f}$ from which they are derived:

$\nabla f(\mathbf{v}) = [F_j(\mathbf{v}) : j \in J_i]$ and $\nabla \tilde{f}(\mathbf{v}) = [\tilde{F}_j(\mathbf{v}) : j \in J_i]$.

Lemma 3. Let $\delta_{jr} = 1$ if $j = r$ or $\delta_{jr} = 0$ otherwise. Then:

$$\frac{\partial F_j(\mathbf{v})}{\partial v_r} = \frac{q_{ij}^+}{v_j^2} \delta_{jr} - \sum_{z \in j \cap r} f'_z \frac{\tilde{\pi}_z}{n_z(\mathbf{v})},$$

$$\frac{\partial \tilde{F}_j(\mathbf{v})}{\partial v_r} = \frac{2}{HN_j} \delta_{jr} + \frac{\partial F_j(\mathbf{v})}{\partial v_r},$$

Wherein: $\tilde{\pi}_z(\mathbf{v}) = \pi_z(\mathbf{v}).1_{\{n_z(\mathbf{v}) < k'_z\}}$.

Proof. $\frac{\partial \pi_z}{\partial v_r} = \frac{\partial}{\partial v_r} \min\{1, \frac{k'_z}{n_z}\}$ is equal to 0 if $k'_z \geq n_z(\mathbf{v})$ meaning $\pi_z(\mathbf{v}) = 1$ or else to $\frac{\partial}{\partial v_r} \frac{k'_z}{n_z} = -\frac{k'_z}{n_z^2} \frac{\partial n_z}{\partial v_r} = -\frac{\pi_z}{n_z} 1_{\{z \in r\}}$

On combining, it holds that $\frac{\partial \pi_z}{\partial v_r} = -\frac{\tilde{\pi}_z}{n_z} 1_{\{z \in r\}}$ whatever the case. Consequently,

$$\frac{\partial (f\pi)_j}{\partial v_r} = \sum_{z\in j} f'_z \frac{\partial \pi_z}{\partial v_r} = -\sum_{z\in j} f'_z \frac{\tilde{\pi}_z}{n_z} 1_{\{z\in r\}} = -\sum_{z\in j\cap r} f'_z \frac{\tilde{\pi}_z}{n_z}.$$

Thus both F_j and $\tilde{F}_j$ have derivatives that are symmetrical, i.e. $\frac{\partial F_j}{\partial v_r} = \frac{\partial F_r}{\partial v_j}$ and $\frac{\partial \tilde{F}_j}{\partial v_r} = \frac{\partial \tilde{F}_r}{\partial v_j}$.

The associated potential functions may be defined in the following way, from a reference point $\mathbf{v}_0$:

$f(\mathbf{v}) = \sum_j \int_{\mathbf{v}_0}^{\mathbf{v}} F_j(v)\,\mathrm{d}v_j$ and $\tilde{f}(\mathbf{v}) = \sum_j \int_{\mathbf{v}_0}^{\mathbf{v}} \tilde{F}_j(v)\,\mathrm{d}v_j$. By construct, $\frac{\partial f}{\partial v_r} = F_r$ and $\frac{\partial \tilde{f}}{\partial v_r} = \tilde{F}_r$.

Theorem 1. (i) The potential function f is convex on $\mathbf{V}$. (ii) The potential function $\tilde{f}$ is strictly convex on $\mathbf{V}$.

Proof. (ii) stems from (i) in a straightforward way, since $\tilde{f} = f + \sum_r \frac{2}{HN_r}(v_r - v_{0r})^2$ in which the first term is convex and the second is strictly convex.

(i) Let us show that the Hessian of f is a positive matrix: $\forall \mathbf{v}' = [v'_r : r \in J_i]$,

$$\sum_{r,j} \frac{\partial^2 f}{\partial v_r\,\partial v_j} v'_r v'_j = \sum_{r,j} \frac{\partial^2 F_j}{\partial v_r} v'_r v'_j$$

$$= \sum_r \frac{q^+_{ir}}{v_r^2} v'^2_r - \sum_{r,j} v'_r v'_j \sum_{z\in j\cap r} f'_z \frac{\tilde{\pi}_z}{n_z(\mathbf{v})}$$

$$= \sum_r \frac{q^+_{ir}}{v_r^2} v'^2_r - \sum_{z\in J_i} f'_z \frac{\tilde{\pi}_z}{n_z} (n'_z)^2 \quad \text{in which } n'_z = \sum_r v'_r = \sum_j v'_j$$

As $\mathbf{v}$ belongs to $\mathbf{V}$, $q^+_{ij} \geq (f\pi)_j v_j$ and $v_j \leq n_z$ for $z \in (i,j)$, hence

$$\sum_r \frac{q^+_{ir}}{v_r^2} v'^2_r \geq \sum_r \frac{\sum_{z\in r} f'_z \pi_z}{v_r} v'^2_r = \sum_{z\in Z_i} f'_z \pi_z \sum_{r\in z} \frac{v'^2_r}{v_r}.$$

On the other hand,

$$\frac{n'^2_z}{n_z} = [\sum_{r\in z} \frac{v'_r}{\sqrt{n_z}}]^2 = [\sum_{r\in z} \frac{\sqrt{v_r}}{\sqrt{n_z}} \frac{v'_r}{\sqrt{v_r}}]^2 \quad \text{to which we apply the Cauchy - Schwarz inequality}$$

$$\leq [\sum_{r\in z} \frac{v_r}{n_z}].[\sum_{r\in z} \frac{v'^2_r}{v_r}] = \sum_{r\in z} \frac{v'^2_r}{v_r} \quad \text{since} \quad \sum_{r\in z} \frac{v_r}{n_z} = 1$$

Thus: $\sum_{r,j} \frac{\partial^2 f}{\partial v_r\,\partial v_j} v'_r v'_j \geq \sum_{z\in J_i} f'_z(\pi_z - \tilde{\pi}_z)(\sum_{r\in z} \frac{v'^2_r}{v_r}) n'^2_z$

$$\geq 0 \quad \text{since } \pi_z \geq \tilde{\pi}_z$$

This ensures that the Hessian matrix of f is positive, hence f is a convex function.

3.3. Existence and uniqueness of solution

Theorem 2. (i) Function $\tilde{f}$ has a unique minimum on $\mathbf{V}$. (ii) The FPP has a solution on $\mathbf{V}$, which is unique.

Proof. (i) $\mathbf{V}$ being a closed and compact set, a strictly convex function on it admits a minimum value at some unique point $\mathbf{v}^*$. There, the first order optimality conditions are that

$\nabla\tilde{f}\big|_{\mathbf{v}^*}.(\mathbf{v}-\mathbf{v}^*) \geq 0 \quad \forall \mathbf{v} \in \mathbf{V}^*$, $\mathbf{V}^*$ being a restriction of $\mathbf{V}$ in the vicinity of $\mathbf{v}^*$.

$\forall j \in J_i$, v_j^* cannot be zero since at zero the j-th component of the gradient would be very much negative, indicating that an increase in v_j would enable one to decrease $\tilde{f}$.

Similarly, the condition $q_{ij}^+ = (f\pi^*)_j.v_j^*$ cannot be achieved because in that case $\tilde{F}_j(\mathbf{v}^*) > 0$, meaning that a decrease in v_j would enable one to decrease $\tilde{f}$. Lastly, condition $v_j^* = N_j$ cannot hold because at such a point $\tilde{F}_j = \frac{2}{H} - \frac{1}{H} + (f\pi)_j \geq 0$, meaning that a decrease in v_j would enable one to decrease $\tilde{f}$.

Thus the minimum point of $\tilde{f}$ must be interior to $\mathbf{V}$, which implies that $\nabla\tilde{f} = 0$ at $\mathbf{v}^*$, hence $\mathbf{v}^*$ satisfies the FPP problem. The uniqueness of solution stems from the strict convexity of $\tilde{f}$.

3.4. Solution algorithm

A Newton algorithm is appropriate to solve the minimization program, since the first and second order derivatives are easy to evaluate at each current point. The convergence criterion only involves the norm of the gradient function, $\nabla\tilde{f} = [\tilde{F}_j : j \in J_i]$.

Formula:

$x_{n+1} := x_n + [\nabla^2 \tilde{f}]^{-1}\big|_{x_n}.(\nabla\tilde{f})\big|_{x_n}$, denoting by $[\nabla^2 \tilde{f}] = [\frac{\partial \tilde{F}_j}{\partial v_r} : r, v \in J_i]$ the Hessian matrix.

3.5. Solution revision

If the basic solution to the FPP involves some components v_j^* that are quite small with respect to N_j, then an adapted FPP with $\tilde{F}_j$ replaced by F_j on these components will yield a revised solution with the corresponding egress stations in an unqueued state.

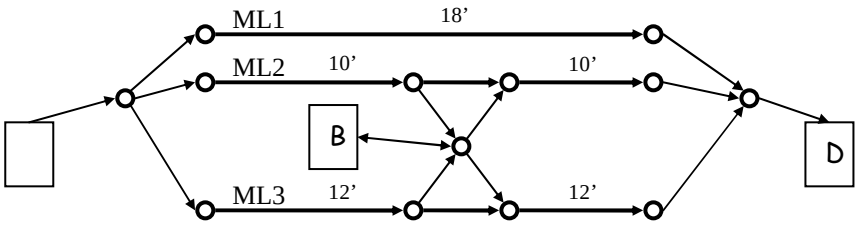
4. An application instance

A classroom instance is used to demonstrate the behaviour of the transit bottleneck model. The example is similar to that developed in Leurent and Askoura (2010) for the comparison of various assignment models under capacity constraints. Three parallel lines connect an origin A with a destination D. The two (ML2, ML3) also serve node B, as shown in figure 2. The following table resumes their operational characteristics.

Table 1. Operational characteristics of the service routes

Route	Frequency (veh/h)	Vehicle Capacity	Service Capacity
ML1	5	200	1000
ML2	10	250	2500
ML3	5	300	1500

Fig. 2. The service network of the application instance



4.1. Passenger traffic flow states

The model presented calculated the expected travel time of an access – egress station couple in the basis of the passenger stock (for the waiting time) and the weighted average of the flow assigned to the transit services under strict capacity constraints. Therefore, when dealing with at least two parallel routes, we can observe three distinct flow states which are illustrated in the figures 3(a) and 3(b), where the dashed line corresponds to the in-vehicle travel time and the continuous line to the average journey time (in-vehicle travel time + waiting time).

(I) The uncongested state (0A), where a weak passenger flow is not facing any capacity constraints

(II) The semi-congested state (AB), when at least one of the services is saturated, while others have some available capacity to accommodate the additional flow. Even though, the access – egress flow q_{ij}^{+} can be transferred within the reference period H, there is an impact in both the expected waiting time and the in-vehicle travel time.

(III) The congested state (B+), where all the services are saturated and the access – egress flow q_{ij}^{+} cannot be served within the reference period, H. The insufficient capacity results in a significant increase of the expected waiting time for the access – egress couple.

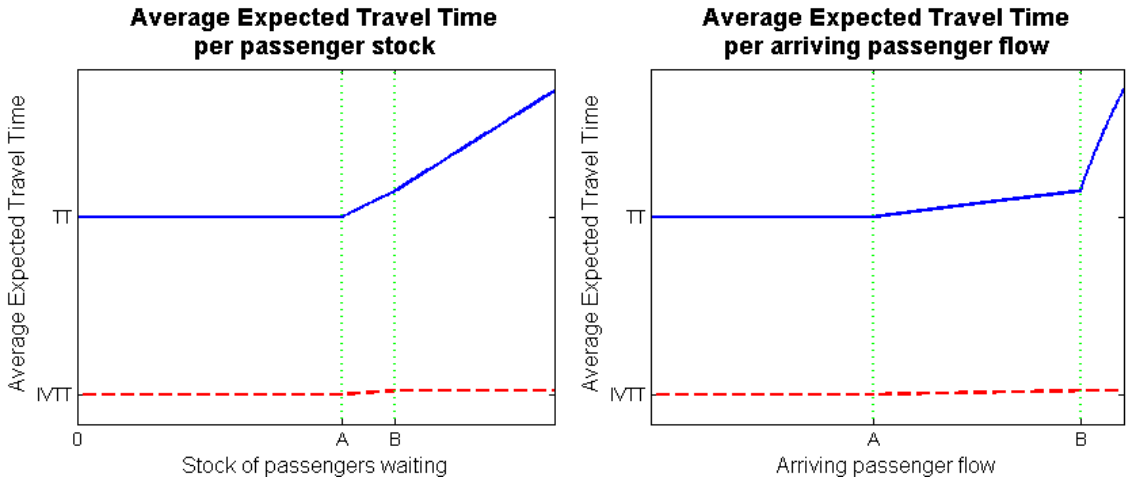


Fig. 3. Average Expected Travel Time and In-vehicle travel time per (a) stock of passengers waiting and (b) exogenous passenger flow

The passenger stock for each destination is calculated by a Newton – Raphson algorithm with a slight underestimation. That is attributed the objective function developed due to some mathematical

approximation of the queue. Nonetheless, the difference observed is insignificant for the calculation of the expected waiting time and the flow share of the transit services.

4.2. Destination coordination

When multiple access – egress flows share the same transit service, we observe a competition among their respective partial passenger stocks at boarding. At unsaturated conditions, it corresponds to the relative proportion of the frequencies. When at least one of the routes is saturated, the passenger flows are reassigned while there is a coordination of the partial passenger stocks per egress station. The behavior of the bottleneck model at congested regime is demonstrated using the three route network described previously. While the exogenous passenger flow of the A->D access – egress station couple is constant at a low level ($q^+_{AD} = 2000$), we increase the A->B passenger flow, q^+_{AB}, until all routes are saturated.

We focus on the share of each transit service for the A->D flow, illustrated in the following figure. At unsaturated conditions the routes ML1, ML2, ML3 are assigned the 25%, 50%, 25% of the A->D flow, according to the route frequencies. Nevertheless, once a transit route is at capacity, an increase in the waiting time and a change in the flow proportions is observed. At the example developed, ML2 is the first to be saturated for $q^+_{AB} = 2250$ *passengers* to destination B, and its flow proportion for the q^+_{AD} flow is reduced ($p^{ML2}_{AD} = 0{,}45$ for $q^+_{AB} = 2582$). The saturation of ML3 creates a competition between the B and D partial stocks at A for the transit routes ML2 and ML3. Therefore, ML1 accommodates the flow unable to board to the other routes and its flow proportion increases from $p^{ML1}_{AD} = 0{,}27$ (for $q^+_{AB} = 2582$) to $p^{ML1}_{AD} = 0{,}51$ (for $q^+_{AB} = 3300$). When all services are at capacity, the boarding competition reduces the probability of the stock to board ML2 and ML3 and therefore the proportion of q^+_{AD} passing from ML1 increases to $p^{ML1}_{AD} = 0{,}57$ for $q^+_{AB} = 5000$.

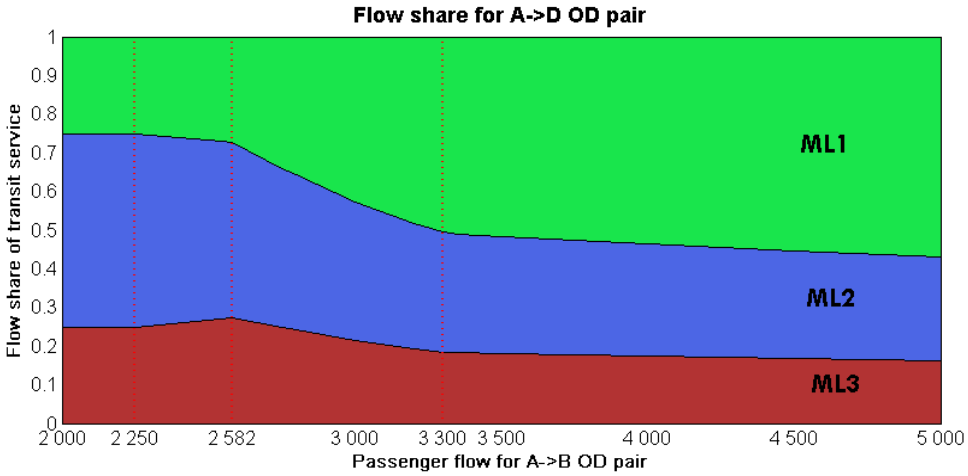


Fig. 4. The cumulative passenger flow share of each transit service

5. Conclusion

The paper develops a model for taking account of the effect of a vehicle's total capacity to a passenger's route choice and travel time. The effects are considered explicitly through the formation of a passenger stock

per egress station at a platform. Even though there is not an exact physical explanation, it portrays an average mingled passenger queue at the arrival of a vehicle. Congestion occurs when the stock cannot be absorbed during the reference period, so that a passenger is unable to access the first vehicle that arrives after his joining the queue.

The transit bottleneck model is used within a flow assignment model in a transit network such that described in Leurent et al (2011). As part of an elaborate cost-flow function, it is used to evaluate the effect if the exogenous flow on the average waiting time and the route share – by extension the average in-vehicle travel time – by imposing strict capacity constraints. The bottleneck does not guarantee the conservation of flow even though the difference is insignificant. Nevertheless, the flow assignment is made at a network level, with a relaxation of the strict capacity constraints, where the conservation of flow at each node is guaranteed.

We proved that the transit bottleneck model can be treated as a fixed point problem, of which we have shown the existence and uniqueness of a solution. Moreover, we proposed an outline of a solution algorithm, using the Newton Raphson algorithm. Finally, an application instance is provided in order to analyse the behaviour of the model. We determined three traffic flow states – uncongested, semi-congested and congested state – with different characteristics and we illustrated how the destination coordination may alter the route share of all connected routes in saturated conditions.

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